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Comment on "Long-Time Dynamics via Direct Summation of Infinite Continued Fractions"

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In their recent work Cai, Sen, and Mahanti [1] proposed a method to evaluate the long-time behavior of dynamical correlation functions based in the continued fraction formalism [2]. The authors argued that the long-time behavior of the correlation function $a(t)$ can be inferred by directly evaluating the inverse Laplace transform of the continued fraction $a(z)$ with the knowledge of its first few coefficients Δ_n's and the use of some extrapolation scheme (in the form $\Delta_n = n^{\phi}$) for the remaining coefficients.

As a test ground for their method the authors used the subsurface spin correlation function of the $S=\frac{1}{2}$ semi-infinite XY chain at infinite temperature, which has a known analytic solution [3]. By using the first five Δ's and the ansatz $\Delta_n = (\Delta_5 - \Delta_4)(n-4) + \Delta_4$ for $n > 5$, they claimed that their result is in "very good" agreement with the exact one, as shown in Fig. 4 of their paper [1].

To check the validity of their method we exactly evaluated the first 100 coefficients of the continued fraction from the exact result (Eq. 2.12 of Ref. [3]). The results are shown in Fig. 1 together with the ansatz used in [1]. To our surprise the dependence of Δ_n with n turned out to be quite different than the one proposed. In fact for large n it has an exponential dependence with n and not the linear form used by Cai, Sen, and Mahanti. The correlation function obtained with their method does not agree at all with the exact result except for short times ($Jt < 4$). A closer look at the correlation function with a finer scale reveals remarkable discrepancies (see Fig. 2), with the relative error becoming rather large at longer times. The ansatz in [1] deviates so much from the exact Δ's at higher orders, hence the ensuing $a(t)$ is necessarily very different than the exact one, especially at long times.

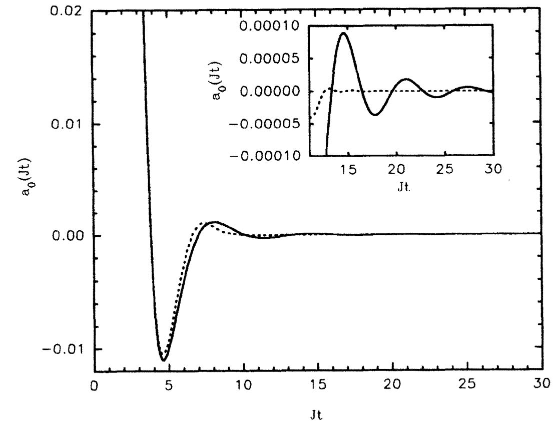

FIG. 2. The subsurface spin correlation function for the semi-infinite XY chain at infinite temperature. The solid lines are from the exact result [3]. The dashed lines are obtained by using the ansatz of [1]. The inset uses a finer scale to show the discrepancies between the two results at longer times.

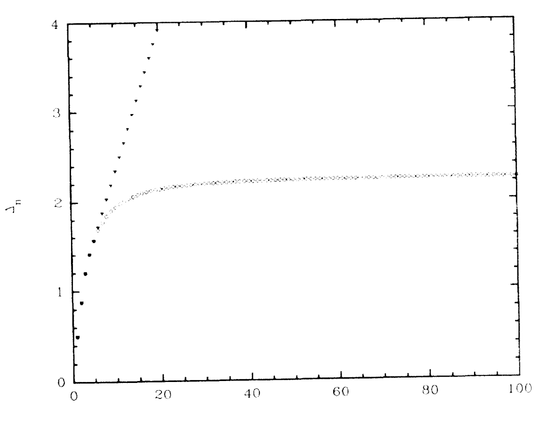

FIG. 1. The coefficients of the continued fraction (Δ_n) versus n of the subsurface spin correlation function for the semi-infinite XY chain at infinite temperature. The circles are the exact results obtained from [3] while the triangles are from the ansatz used by Cai, Sen, and Mahanti.

A preliminary analysis of some nontrivial cases (Δ_n not linear in n) shows that in order to make a reliable extrapolation for the Δ's one needs first to obtain a large enough number of exact Δ's so that they are already in the asymptotic regime. A detailed discussion on this point will be given elsewhere [4].

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