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# A Management Model for Specification of Groundwater Withdrawal Permits

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The Massachusetts Water Management Act was enacted in 1986 to preserve the State's water resources. The intent of the Act was to allow for sustained economic growth while protecting the natural environment by minimizing the occurrence of low stream flows. As a result of the act, a permit must be obtained for new water withdrawals (including increases on existing withdrawals) of more than 0.1 million gallons per day (0.00438 $m^3/s$). The permits specify the degree to which applicants may withdraw water, and reserve the right to curtail use during low flow seasons. A linear programming model is presented that is capable of assisting regulatory agencies in specifying details of permits for groundwater use. The model links groundwater withdrawals with surface streamflow, considering consumptive use and interbasin transfers. The optimization minimizes the depletion of streamflow below a standard while honoring the statistical distribution of allowed withdrawals permitted each applicant. The results specify the amount and timing of allowed withdrawals throughout the year.

## INTRODUCTION

The interaction between ground and surface waters has long been recognized, and the depletion of surface water flow by groundwater pumping can have a significant impact on low stream flows. Flow depletion can be either direct depletion of flow from the stream or the reduction of groundwater flow to the stream. To reduce the adverse impact of groundwater pumping on low stream flows several states regulate pumping rates at critical times.

In 1986, Massachusetts passed the Water Management Act (WMA) in an effort to preserve its water resources. One objective of the Act is to provide a regulatory means of managing the development of both the surface and groundwater resources of the State, so that continued and sustainable economic growth is allowed, while still protecting the natural environment by setting minimum stream flow standards. This paper describes the Massachusetts Act, and presents a model capable of assisting regulatory officials in formulating policy and in implementing controls.

## BACKGROUND

### *Previous Research*

Early research in the area of well and stream interaction include *Theis* [1941], *Glover and Balmer* [1954], and *Hantush* [1965]. Their results were for an infinite homogeneous isotropic aquifer, with fully penetrating stream and well. *Jenkins* [1968*a*] introduced a stream depletion factor, which is a lumped aquifer parameter capable of describing the complete aquifer-well relationship, and applied the principle of superposition to obtain results for nonsteady state pumping rates. *Jenkins'* [1968*a*] concept was used recently by *Wallace et al.* [1990] and in Massachusetts [*Massachusetts Department of Environmental Management (Mass. DEM)*, 1987*b*]. *Hantush* [1967], *Jenkins* [1968*b*], and *Moulder and Jenkins* [1969] made further refinements to eliminate the usual assumption of a straight river of infinite length.

Models that determine the volume of water depleted from a stream during a time period from the volume pumped during previous time periods were presented by *Maddock* [1974], *Morel-Seytoux and Daly* [1975], and *Morel-Seytoux* [1975*a*, *b*]. All of these models rely on the linearity of the aquifer systems, assuming, for unconfined aquifers, that the drawdowns are small compared to the thickness of the aquifers.

Conjunctive ground/surface water models have been available specifically for the management of water resources, and some have addressed streamflow depletion. In a review of distributed-parameter groundwater management models, *Gorelick* [1983] cited models that specifically included the dynamic interaction between wells, aquifers, and streams. Most of the approaches are similar to the works of either *Jenkins* [1968*a*] or *Morel-Seytoux and Daly* [1975]. These include *Taylor* [1970] and *Taylor and Luckey* [1974], both of which used the stream depletion method of *Jenkins* [1968*a*]. The approach presented by *Morel-Seytoux and Daly* [1975] uses linear influence coefficients generated from a finite difference model of the stream aquifer system.

*Morel-Seytoux* [1975*a*], *Illangasekare and Morel-Seytoux* [1982, 1986], and *Young et al.* [1986] have addressed the area of conjunctive-use management subject to the institutional constraints of western water law. *Young and Bredehoeft* [1972] used linear programming in conjunction with a simulation model to allocate water so that stream depletion would be limited. *Peralta et al.* [1988*a*, 1990] used conjunctive use management models to plan the optimal spatial distribution of crops for an interconnected river-aquifer system. In their models stream-stage and groundwater levels were dynamically affected by streamflow and pumping during the optimization period. *Peralta et al.*



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[1988*b*] refined the linear influence coefficient approach of modeling an aquifer-surface water system and applied that model to evaluate the potential impact of recharge basins on the optimal extraction of groundwater from the Grand Prairie Aquifer. *Hantush and Marino* [1989] modeled an idealized three-well system, attempting to maximize withdrawals while maintaining streamflow.

Only a few studies have been published concerning conjunctive groundwater management in the Eastern United States. These include well pumping simulation models designed to determine the availability of water supply to communities during drought [*Mass. DEM*, 1987*a*, *b*].

### *Massachusetts Legislation*

The WMA recognizes that groundwater and surface water resources are interconnected, and therefore must be managed together. Simply stated, the objectives of the Water Management Act are to manage the water resources of the State so that continued and sustainable economic growth is allowed, and the natural environment is protected. Environmental protection is measured by the maintenance of minimum streamflows, while sustainable economic growth is interpreted to mean allowance of increased use of both ground and surface water.

In response to this act, the Massachusetts Department of Environmental Protection (DEP), established a permit system for all new (or increased) water withdrawals exceeding 0.1 million gallons per day (mgd) (0.00438 $m^3/s$). The permit system is intended to help ensure an appropriate balance among competing water withdrawals and to protect the water resource itself. Permits for new withdrawals may be denied if the new withdrawal, combined with all existing withdrawals, causes streamflow to drop below a preestablished minimum. The minimum streamflow standard is set by the Department of Environmental Management (DEM) to protect established withdrawals and the natural environment (e.g., fisheries).

To address the issue of economic growth, the DEP reserves the right to require curtailments in well withdrawals during times of low flow, yet allow higher withdrawals at other times. While no specific details were given as to how and when these curtailments might occur, a frequency distribution is included in each permit showing the anticipated curtailments. Figure 1 shows a generic permit diagram, the form of which is described by shape parameters P1, P2, and P3, which are specified for each applicant. The DEP computes the values of P1, P2, and P3 using an algorithm that estimates the streamflow duration curve (based on the drainage area, basin characteristics, and all upstream withdrawals) for the river basin at the point of the withdrawal. This type of permit only shows what might happen statistically over time and does not indicate when during the year, or for how long the user might be required to curtail withdrawals.

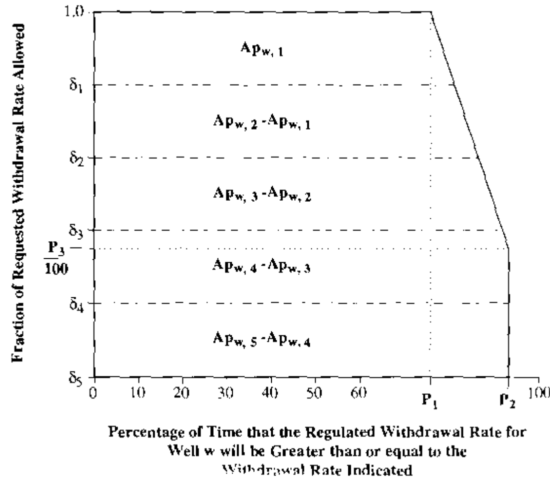


Fig. 1. Typical DEP permit for well *w* showing shape parameters (P1, P2, P3) and discretization levels.

## GROUNDWATER/SURFACE WATER INTERACTION MODEL

This section describes the development of a descriptive groundwater/surface water interaction model that is used in the next section as part of a prescriptive management model. The model used in this analysis was developed first by *Theis* [1941] and then again by *Hantush* [1965] in a slightly different analytical form. *Jenkins* [1968*a*] later summarized both of their works and discussed the application of these models to intermittent pumping scenarios.

While the linear influence coefficient approach to modeling aquifer-surface water interaction has been widely used in water management studies [*Morel-Seytoux and Daly*, 1975; *Morel-Seytoux*, 1975; *Illangasekare and Morel-Seytoux*, 1982, 1986; *Peralta et al.*, 1988*a*, *b*], it was not used in this approach because detailed aquifer modeling for each permit applicant was beyond the scope of this study. The lumped parameter model summarized by *Jenkins* [1968*a*] represented the most practical approach, even though it does not allow for modeling the impact of river stage evolution on aquifer discharge to the river. This aspect is thought to be of minimal concern for this study.

The model is based on the assumption that groundwater pumping rates are constant for a specified period of time. In addition, the aquifer is unconfined, isotropic, homogeneous, and semi-infinite in areal extent; the river is straight and infinite in length; the drawdown due to the well is small compared to the thickness of the aquifer; water is released instantaneously from storage; and the well and river are fully penetrating.

### *Steady-Pumping Model*

For an unconfined aquifer the rate of water depletion from the river $Qr$ is defined by

$$Qr = Qw \ \text{erfc} \ [(\text{SDF}/(4t))^{0.5}] \tag{1}$$

where $Qw$ is the rate that water is pumped out of the well, $t$ is the time since pumping began, SDF is a single parameter that completely describes the aquifer, and erfc is the complementary error function which is defined by

$$\text{erfc} \ (y) = \frac{2}{\sqrt{\pi}} \int_y^\infty \exp \ (-y'^2) \ dy' \tag{2}$$

The parameter SDF, or stream depletion factor, is defined as

$$\text{SDF} = d^2 n/(Kb) \tag{3}$$

where $d$ is the distance between the river and the aquifer, $n$ is the aquifer's effective porosity (or specific yield), $b$ is the aquifer's thickness, and $K$ is the aquifer's hydraulic conductivity.

*Jenkins* [1968*a*] also reformulated *Hantush*'s [1965] formulation for the volume of water depleted from the river, $Vr$:

$$Vr = 4Qwt\{i^2 \operatorname{erfc}[(\mathrm{SDF}/(4t))^{0.5}]\} \tag{4}$$

where $i^2$ erfc is the second repeated integral of the complementary error function:

$$i^2 \operatorname{erfc}(y) = \int_y^\infty \int_{y'}^\infty \operatorname{erfc}(y'')\, dy''\, dy' \tag{5}$$

In (4), SDF represents the time of steady pumping required for 28% of the volume pumped to be depleted or diverted from the stream.

Thus for steady pumping rates, the depletion from the river on both a flow rate and volume basis can be described by (1) and (4). For notational convenience the dimensionless function $F(t)$, which is dependent on the aquifer parameters as well as time, is defined as

$$F(t) = 4\{i^2 \operatorname{erfc}[(\mathrm{SDF}/(4t))^{0.5}]\} \tag{6}$$

This function is used below to relate the period and magnitude of pumping to the cumulative volume of stream depletion.

### *Nonsteady Pumping Model*

The above relationships are useful for steady pumping rates. The effect of varying pumping rates can be determined by applying the principle of superposition to the steady pumping model. This approach is justified because drawdowns in the aquifer are assumed to be small compared to the thickness of the aquifer, and the resulting groundwater differential equation is linear. Assuming that the pumping rate will be constant during a specified time period, $\Delta t$, a discrete time pumping rate, $Qw_i$, can represent the constant pumping rate during period $i$.

Assuming that pumping begins at period one, at the end of the first period the cumulative volume of stream depletion is due only to the rate of withdrawal during that period:

$$Vr_1 = Qw_1 \Delta t F(\Delta t) \tag{7}$$

The cumulative volume of stream depletion up to the end of the second period is determined by the following: first, adding the effects of the withdrawal rate from the first period as if it were allowed to continue through the second period; second, adding the effects of a fictitious source that started injecting water into the well at the end of the first period so as to cancel out the effect of continuing the first withdrawal rate beyond the end of the first period; and finally, adding the effects of the second period's withdrawal rate. In general, the volume of flow depleted from the river from a single well at the end of period $i$, $Vr_i$, can be expressed as

$$Vr_i = \Delta t \sum_{k=1}^{i} \{Qw_k[(i-k+1)F((i-k+1)\Delta t) - (i-k)F((i-k)\Delta t)]\} \tag{8}$$

provided $F(t) = 0$ for all time $t \leq 0$ and all time steps equal $\Delta t$.

The average rate of stream depletion, $\bar{Q}r_i$, over a time period $i$ can be determined from

$$\bar{Q}r_i = (Vr_i - Vr_{i-1})/\Delta t \tag{9}$$

Combining (8) and (9) yields

$$\bar{Q}r_i = \sum_{k=1}^{i} \{Qw_k[(i-k+1)F((i-k+1)\Delta t) - 2(i-k) \cdot F((i-k)\Delta t) + (i-k-1)F((i-k-1)\Delta t)]\} \tag{10}$$

which can be rewritten as

$$\bar{Q}r_i = \sum_{k=1}^{i} \{Qw_k C_{i-k}\} \tag{11}$$

where the stream depletion coefficients $C_{i-k}$ are defined as

$$C_{i-k} = (i-k+1)F((i-k+1)\Delta t) - 2(i-k)F((i-k)\Delta t) + (i-k-1)F((i-k-1)\Delta t) \tag{12}$$

For this definition to be valid, $F(t)$ must be redefined to

$$F(t) = 4\{i^2 \operatorname{erfc}[(\mathrm{SDF}/(4t))^{0.5}]\} \qquad t > 0$$
$$F(t) = 0 \qquad t \leq 0 \tag{13}$$

This linear model, relating the well pumping rates to stream depletion, can be rewritten as

$$\bar{Q}r_i = \sum_{j=0}^{i-1} \{Qw_{i-j} C_j\} \tag{14}$$

where

$$C_j = (j+1)F((j+1)\Delta t) - 2(j)F(j\Delta t) + (j-1)F((j-1)\Delta t) \tag{15}$$

The $C_j$ coefficients describe the fractions of the withdrawal $j$ periods ago that will be depleted from the stream during the present period (note that $j = i - k$). The definition of $C_j$ as presented in (15) is more compact and suitable for interpretation and modification than in its previous forms, particularly when return flows are included (see next section).

### *Consumptive Use and Return Flows*

The above stream depletion model must be modified to account for consumptive use and return flows to the stream. If $\beta_c$ is the fractional consumptive use of an applicant, then $(1 - \beta_c)$ is the fraction of water available to be returned to the stream through surface water discharge from a wastewater treatment plant or through groundwater flow from septic system(s).

The variable $\beta_w$ represents the fraction of a community's water use that is served by a wastewater treatment plant discharging to the same river basin. It is assumed that the discharge is close to where the stream is depleted by well pumping and the water returned to the stream through the wastewater treatment plant does not reduce streamflow. The fraction of a community's withdrawal that is return flow is therefore $\beta_w(1-\beta_c)$. If the community's water system's storage is small enough to give the water system a short retention time compared to length of the pumping period $\Delta t$, then $\beta_w(1-\beta_c)Qw_i$ is the amount of water returned to the stream during period $i$. This amount is no longer depleted from the stream during period $i$, and it may be subtracted from $\bar{Q}r_i$. The resulting depletion model considers only return flow via wastewater treatment facilities:

$$\bar{Q}r_i = \sum_{j=0}^{i-1} \{Qw_{i-j}C_j\} - \beta_w(1-\beta_c)Qw_i \qquad (16)$$

It can be rearranged to yield

$$\bar{Q}r_i = \sum_{j=1}^{i-1} \{Qw_{i-j}C_j\} + Qw_i(C_0 - \beta_w(1-\beta_c)) \qquad (17)$$

If $\beta_s$ represents the fraction of a community located within the river basin that uses individual septic systems, then $\beta_s(1-\beta_c)$ is the fraction of a community's withdrawal that is returned as septic flow. Septic flow usually spends a significant amount of time flowing through the ground before discharging to a surface water. Therefore the impact on streamflow due to septic systems is assumed to be the average of the previous year's septic return flow. Thus the rate of stream depletion will be decreased by the average of the community's septic return flows. The resulting depletion model considers only return flow via septic systems:

$$\bar{Q}r_i = \sum_{j=0}^{i-1} \{Qw_{i-j}C_j\} - \beta_s(1-\beta_c)/Np \sum_{j=0}^{Np-1} Qw_{i-j} \qquad (18)$$

where $Np$ is the number of periods in a year. Equation (18) can be rearranged to yield

$$\bar{Q}r_i = \sum_{j=0}^{Np-1} \{Qw_{i-j}[C_j - \beta_s(1-\beta_c)/Np]\} + \sum_{j=Np}^{i-1} \{Qw_{i-j}C_j\} \qquad (19)$$

Equations (18) and (19) model the return flow over the previous year of pumping and are therefore valid after 1 year of pumping, when $i \geq N_p$. When the model is cast in the descriptive form (see next section, The Descriptive Model, (20), (21), and (22)), the limitation $i \geq N_p$ may be dropped without introducing error.

### *Descriptive Model*

The final model incorporates the effects of both types of return flows and consumptive use. Equation (14) represents the descriptive model, with the exception that the stream depletion coefficients $C_j$ are now defined by

$$C_0 = F(\Delta t) - \beta_w(1-\beta_c) - \beta_s(1-\beta_c)/Np \qquad (20)$$

$$C_j = (j+1)F((j+1)\Delta t) - 2(j)F(j\Delta t) + (j-1)F((j-1)\Delta t) - \beta_s(1-\beta_c)/Np \qquad (21)$$

$$j = 1 \cdots Np - 1$$

$$C_j = (j+1)F((j+1)\Delta t) - 2(j)F(j\Delta t) + (j-1)F((j-1)\Delta t) \qquad (22)$$

$$j = Np \cdots i - 1$$

The limiting case for these coefficients occurs when there is no return flow and when the distance between the well and stream is zero. In this case the value of SDF goes to zero and $C_j$ is defined by

$$C_j = 1 \quad j = 0$$
$$C_j = 0 \quad j \neq 0 \qquad (23)$$

This model can be extended to a multiple well system by summing the effects of all the individual wells during period $i$. Further detail can be found in the work by F. A. Mueller (unpublished manuscript, 1990).

## MANAGEMENT MODEL

The intent of the management model is to optimally implement the permit conditions (in the form of the DEP withdrawal permit diagrams) so that all permit applicants will know when, and to what extent, they must curtail well withdrawals. The goal of the model is to minimize the streamflow depletion subject to the permitted withdrawals and the other physical constraints on the surface/groundwater system, represented by the model developed in the previous section. The model is divided into 13 four-week decision periods for each year. For each decision period, each permit applicant would be told its allowable withdrawal rate.

The main decision variables in the problem are $\alpha_{w,i}$, which describe the fraction of the requested withdrawal rate the applicant is allowed to take from well $w$ during decision period $i$. If the requested withdrawal rate from well $w$ is $Q_w$, then the regulated (or permitted) volume of water that could be withdrawn from well $w$ during decision period $i$ would be $\alpha_{w,i}Q_w$ (equivalent to the symbol $Qw_i$ used in the previous section).

Other decision variables are used to keep track of (1) the stream depletion due to well withdrawals over time, (2) the amount that the depleted streamflow is above and below the streamflow standard, and (3) how the main decision variables conform to the shape of the DEP permits. The allowed withdrawals during each decision period of a year are constrained to be the same from year to year.

### *Minimize Surface Water Depletion*

The degree of protection to the environment is measured by the changes in both the duration and the amount that the streamflow is below the minimum streamflow standard at the nearest downstream gaging station. This change is due to the effects of new withdrawals and is represented by the increase in the area below the minimum streamflow standard and above the flow duration curve. Figure 2 shows the

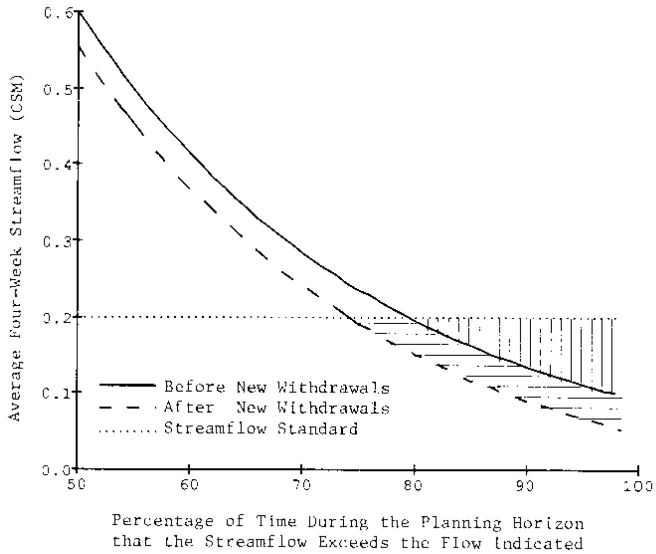


Fig. 2. Effects of new well withdrawals on the streamflow duration curve.

"before" and "after" flow duration curves, where the horizontally cross-hatched area is a measure of the degradation to the environment resulting from increased withdrawals. Thus a measure of the streamflow-protection objective of the WMA is to minimize the increase in this area, which is equivalent to minimizing the entire cross-hatched area.

The objective function is therefore to minimize the area under the standard and above the depleted (new) streamflow duration curve:

$$\text{minimize} \sum_{i=1}^{N} QSn_i \qquad (24)$$

where $QSn_i$ is the amount of flow by which the streamflow is below the standard during period $i$ and $N$ is the number of decision periods in the planning horizon. All decision periods are of equal size. In developing this measure, the following assumptions were made: (1) the depleted streamflow, as computed at the nearest downstream gaging station, is representative of the upstream watershed; (2) the streamflow is a stationary random process so that future streamflow patterns may be predicted from historical streamflow patterns; (3) the planning horizon, or time period over which the analysis is conducted, is large enough to accurately represent the streamflow in a statistical sense; and (4) the response time for changes in surface water hydraulics is assumed to be much shorter than the 4-week decision period.

*Physical Constraints*

The first set of constraints computes the relationship between $\bar{Q}r_i$, the average amount of stream depletion during decision period $i$, and the water withdrawn from all of the $M$ wells in the river basin during the previous decision periods. This relationship is given in (14) for a single well, and for $M$ wells is defined by

$$\bar{Q}r_i = \sum_{w=1}^{M} \sum_{j=0}^{i-1} \{(Q_w \alpha_{w,i-j}) C_{w,j}\} \qquad i = 1 \cdots N \qquad (25)$$

where $C_{w,j}$ is defined by (20), (21), and (22) for each well $w$ having its own SDF.

The second set of constraints measures the positive or negative difference between the depleted streamflow and the streamflow standard during each decision period $i$:

$$Qf_i - \bar{Q}r_i - \text{STD} = QSp_i - QSn_i \qquad i = 1 \cdots N \qquad (26)$$

where $Qf_i$ is the average streamflow during decision period $i$ and STD is the minimum streamflow standard. $QSp_i$ and $QSn_i$ measure the amount that the resultant streamflow is above or below the standard, respectively, and only one can be positive. Rearranging yields

$$QSp_i - QSn_i + \bar{Q}r_i = Qf_i - \text{STD} \qquad i = 1 \cdots N \qquad (27)$$

*Permit Constraints*

A set of constraints is included to force the main decision variable values to approximate the shape of the permits. This approximation is achieved by first, dividing the permit for each well $w$ into several discretization levels; second, determining the area above the discretization level and below the permit curve; and last, forcing the values of the decision variables to conform to the areas specified. The discretization levels and areas are illustrated in Figure 1 for an example with five discretization levels. The resulting permit shape will look more like a series of steps, rather than a straight line slope. As is shown in Figure 1, $Ap_{w,l}$ is defined as the area above discretization level $\delta_{w,l}$ and below the permit curve for well $w$. The $Ap_{w,l}$ areas are determined from the values of the main decision variables, $\alpha_{w,i}$, by a series of constraints. The first type of constraint computes the positive or negative distance between the values of the decision variables and each discretization level:

$$\alpha_{w,i} - \delta_{w,l} = Sp_{w,l,i} - Sn_{w,l,i} \qquad (28)$$

$$w = 1 \cdots M, \; l = 1 \cdots L - 1, \; i = 1 \cdots N$$

where $Sp_{w,l,i}$ and $Sn_{w,l,i}$ are dummy variables representing, respectively, the positive and negative differences between the decision variable $\alpha_{w,i}$ and the discretization level $\delta_{w,l}$, and $L$ is the number of discretization levels used. There is no constraint for the $L$th (or last) discretization level in (28) because $\delta_{w,L}$ will always be zero, in which case the difference will always be positive and equal to $\alpha_{w,i}$.

For each well, the area between each discretization level and the value of the decision variable is determined by adding all of the positive differences between the decision variable values and the discretization levels and relating them to the areas in the DEP permits. The resulting constraints are

$$\sum_{i=1}^{N} \alpha_{w,i} \geq (Ap_{w,L})N/100 \qquad w = 1 \cdots M \qquad (29)$$

$$\sum_{i=1}^{N} Sp_{w,l,i} \le (Ap_{w,l})N/100 \quad (30)$$

$$w = 1 \cdots M, \; l = 1 \cdots L - 1$$

where $Ap_{w,l}$ (expressed as a percentage) is the area between the DEP permit and the discretized level $(\delta)_{w,l}$ for well $w$. Equation (29) places a lower bound on the time average of the allowed fractional withdrawal rate for each permit applicant. This constraint insures that, for each well, the average allowed withdrawal over the planning horizon will equal or exceed that which is allowed by the permit (i.e., the area under the permit). Equation (30) places an upper bound on the time average of the $Sp_{w,l,i}$ variables for each well. Acting together these constraints force the values of the decision variables $\alpha_{w,i}$ to fit the shape of the DEP permit for each well. In both equations, the factor $N/100$ converts a nondimensional fraction into a percent, consistent with Figure 1. Strict equality constraints were not used in (29) and (30) to avoid posing a problem with an infeasible solution.

In addition to the above, bounds are placed on many of the decision variables. The bound on the allowable fraction withdrawal is 100%:

$$\alpha_{w,i} \le 1, \qquad i = 1 \cdots N \quad w = 1 \cdots M \quad (31)$$

The upper bound to the stream depletion rate during a decision period is the average flow rate in the stream during the period:

$$\bar{Q}r_i \le Qf_i, \qquad i = 1 \cdots N \quad (32)$$

Finally, nonnegativity is imposed on all decision variables:

$$\alpha_{w,i}, \; QSn_i, \; QSp_i, \; \bar{Q}r_i, \; Sp_{w,l,i}Sn_{w,l,i} \ge 0 \quad (33)$$

*Summary*

The entire formulation is to minimize (24); subject to (25), and (27) through (33). The linear formulation stated above has on the order of $LMN$ constraints and $2LMN$ variables, which is quite large, even for small-scale problems. Steps were taken to reduce the formulation's size, the first of which requires the decision variables for each applicant to be the same from year to year by forcing the $i$ indices of $\alpha_{w,i}$ to vary only from 1 to 13. Whenever an index greater than 13 is called for, it is replaced by the index corresponding to the same decision period of the year. The second step eliminates all constraints from (25) and (27) that represent decision periods during which the streamflow standard is not threatened by the withdrawals. The application of these steps significantly reduces the size of the linear formulation. The degree of size reduction depends on the number of decision periods in the planning horizon during which the minimum streamflow standard may be violated.

## APPLICATION TO CHARLES RIVER BASIN

*Background*

Data were gathered on the basin characteristics and permit applicants for the Charles River Basin including streamflow records, permit applications, permits issued, and the characteristics of the aquifers. Details pertaining to the data can

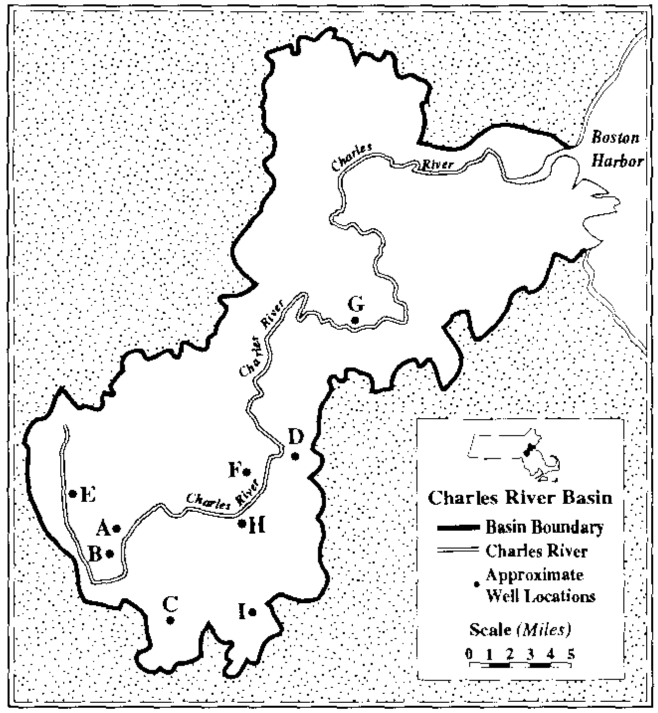


Fig. 3. Charles River Basin showing approximate locations of wells.

be found in the work by F. A. Mueller (unpublished manuscript, 1990). There were a total of nine applicants for water withdrawals in the Charles River Basin. Approximate locations of the nine wells are shown in Figure 3. Pertinent information included type (ground or surface withdrawal), location, and intended use (agriculture, commercial, industrial, municipal, residential, unaccounted for, or other) (N. Fennessey, unpublished data, 1990). The requested withdrawal rate and the estimated consumptive use for each applicant are shown in the second and third columns from the left of Table 1. The DEP permit specifications (P1, P2, and P3) are shown in the three rightmost columns of Table 1. From these specifications, the appropriate discretization levels, $\delta_{w,l}$, and the corresponding areas $Ap_{w,l}$ were computed. In this application, five discretization levels ($L = 5$) were used; $\delta_{w,5}$ was always set to zero. The remaining discretization levels were set at 0.2, 0.4, 0.6, and 0.8, unless P3 was greater than 20% in which cases $\delta_{w,4}$ was set at P3/100 and the remaining discretization levels were set to equally subdivide the interval between $\delta_{w,4}$ and 1.0.

The minimum streamflow standard (STD) set by the DEM for the Charles River Basin is 0.21 cfs/mi$^2$ (0.0023 m$^3$ s$^{-1}$ km$^{-2}$). The drainage area contributing to the Dover gage is 184 mi$^2$ (476.6 km$^2$), which results in a minimum streamflow standard of 38.6 cfs or 25 mgd (1.095 m$^3$/s). Twenty years (1965–1985) of streamflow data were used from the U.S. Geological Survey gage at Dover.

The aquifer characteristics, distance between the point of withdrawal and the stream, hydraulic conductivity, thickness, and specific yield, were combined according to (3) to determine the SDF for each withdrawal point. The distances were estimated from maps and also checked, when possible, against data from other sources [*Walker et al.*, 1975, 1977; *Mass. DEM*, 1988]. The aquifer parameter values of hydraulic conductivity, thickness, and specific yield were deter-

TABLE 1. Permit Application Data Summary

| Applicant | Requested Withdrawal Rate $Q_w$, mgd | Consumptive Use $100\beta_c$, % | Return Flows: In-Basin Septic $100\beta_s$, % | Return Flows: In-Basin WWTP $100\beta_w$, % | Return Flows: Out of Basin, % | Stream Depletion Factor SDF, days | DEP Permit Specifications: P1, % | P2, % | P3, % |
|---|---|---|---|---|---|---|---|---|---|
| A | 0.47 | 7 | 50 | 0 | 50 | 3.7 | 50 | 77 | 0 |
| B | 0.66 | 100 | 0 | 0 | 100 | 1.8 | 50 | 55 | 0 |
| C | 0.81 | 10 | 52 | 48 | 0 | 12.5 | 59 | 100 | 39 |
| D | 0.20 | 10 | 0 | 100 | 0 | 0.05 | 62 | 100 | 9 |
| E | 1.22 | 11 | 7 | 85 | 8 | 0.02 | 64 | 100 | 37 |
| F | 0.11 | 10 | 64 | 36 | 0 | 2.2 | 66 | 100 | 10 |
| G | 1.50 | 11 | 10 | 0 | 90 | 0.025 | 66 | 100 | 57 |
| H | 0.21 | 8 | 100 | 0 | 0 | 0.4 | 54 | 100 | 8 |
| I | 0.33 | 8 | 40 | 0 | 60 | 0.95 | 50 | 86 | 0 |

DEP, Massachusetts Department of Environmental Protection; WWTP, wastewater treatment plant; mgd, million gallons per day (equals 0.0438 $m^3/s$).

mined from pump test data found in new source approval reports, if they were available, or were estimated from other sources. For most wells, the aquifer's saturated thickness was available from the permit application, or it was estimated along with the remaining parameters from U.S. Geological Survey studies of the basin [*Walker et al.*, 1975, 1977]. The aquifer's transmissivity (square meters/day) was estimated directly from the aquifer thickness and the aquifer yields reported by *Walker et al.* [1975, 1977]. The estimated aquifer thicknesses, hydraulic conductivities and transmissivities ranged from 8 to 25 m, 7 to 40 m/day, and 200 to 1500 $m^2$/day, respectively. Storativity values were assumed to be 0.2 m/m. The resulting values for SDF for the applicants are given in the seventh column from the left of Table 1. Where one applicant listed more than one withdrawal point, a single SDF value was determined by either averaging values or, if summertime use was dominant for one well, using that value.

The amount and types of return flows for each permit applicant were determined from estimated consumptive use and from a river basin inventory and analysis [*Mass. DEM*, 1988]. The parameters used to compute the return flows in the model are shown in the fourth, fifth, and sixth columns from the left of Table 1.

### *Results*

Allowed withdrawals for the nine applicants resulting from the optimization are shown in Table 2, and are illustrated in Figure 4. Overall, the DEP allows 81% of all requested withdrawals on an average annual basis. For these results, the value of the objective function indicates that the fractional increase in streamflow depletion below the standard is 0.24, or 24%.

The shape of the applicant's permits were specified (by the values of P1, P2, and P3) and were approximated in the formulation by a series of steps. Examples of these approximate permit shapes are illustrated in Figure 5 for applicants A, B, and C. The width of the steps in Figure 5 is 7.7% which represents 4 weeks of one year.

The streamflow duration curve that is predicted under these permit conditions and scheduled curtailments is illustrated in Figure 6. As can be seen from the figure, the after streamflow duration curve approaches the before curve as the flow drops below the standard. This situation indicates that the impact of withdrawals is reduced whenever the streamflow is below the standard.

The application of this management model using data for the Charles River Basin had the following number of parameters: nine permit applicants ($M$), five permit discretization levels ($L$), and 20 years of streamflow data. The size reduction techniques described earlier, allow the 20 years of streamflow data to be incorporated into the model without using 260 decision periods ($N$). This was achieved by eliminating from the formulation all constraints and variables associated with computing streamflow depletion during decision periods when the streamflow would not be depleted

TABLE 2. Percentage of Requested Withdrawals Allowed by DEP Permits

| Permit Applicant | Decision Period: 5 | 6 | 7 | 8 | 9 | 10 | 11 | Overall |
|---|---|---|---|---|---|---|---|---|
| A | 97 | 71 | 44 | 0 | 0 | 0 | 14 | 63 |
| B | 81 | 1 | 0 | 0 | 0 | 0 | 0 | 52 |
| C | 100 | 92 | 69 | 47 | 56 | 73 | 100 | 87 |
| D | 100 | 100 | 92 | 55 | 16 | 38 | 74 | 83 |
| E | 100 | 100 | 96 | 78 | 41 | 54 | 84 | 89 |
| F | 100 | 100 | 100 | 61 | 18 | 41 | 81 | 85 |
| G | 100 | 100 | 99 | 89 | 59 | 69 | 89 | 93 |
| H | 100 | 93 | 80 | 45 | 15 | 32 | 60 | 79 |
| I | 100 | 78 | 57 | 14 | 0 | 0 | 36 | 68 |

Allowed withdrawals for all applicants were 100% for periods 1, 2, 3, 4, 12, and 13.

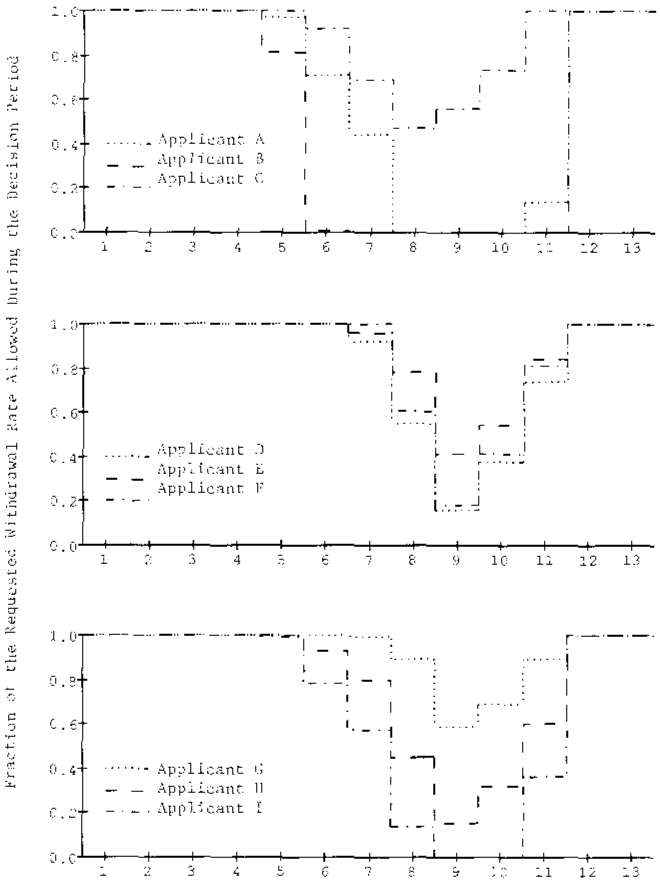


Fig. 4. Distribution of allowed withdrawals over the year.

below the standard. The reduced formulation had 546 constraints and 1101 variables. The problem was solved in 956 iterations using LINDO, requiring 3.2 min of CPU time on a VAX 11/780.

An alternative analysis was performed using 26 two-week decision periods per year. However, the computation time was approximately 5 times longer. In addition, two other advantages of the 4-week periods are apparent: the regulatory burden is lessened, and the resulting allowed withdrawals are not as drastically different from one period to the next.

*Discussion*

The ability of the model and DEP Permits to minimize the depletion of the streamflow below the standard is excellent. The area between the before and after streamflow duration curves that is below the minimum streamflow standard is only 24% of what it would have been if the applicants were allowed to withdraw all of the water they requested all of the time. This number represents the best that can be done with the permits issued by the DEP.

The data shown in Table 2 indicate that the DEP allows all of the permit applicants at least 50% of their requested withdrawal rate on an average annual basis. The applicants receiving the least and most of their requested withdrawals are B and G, respectively. Applicant B is allocated only 52% because there is no return flow; all of its withdrawal is evaporated. Applicant G is allocated 93% of its withdrawal rate for two reasons. The first is that the DEP permit is based on the minimum streamflow standard which in turn is based on drainage area above the withdrawal point. Applicant G's withdrawal has the largest drainage area of all applicants, and therefore the impact of the withdrawal on the streamflow will not be as significant. The second reason is that the DEP does not account for the fact that 90% of applicant G's wastewater is discharged out of basin.

The distributions of the allowed withdrawals over the year shown in Table 2 and Figure 4 indicate that withdrawals are being curtailed from four to seven decision periods of the year. These curtailments begin as early as the fifth decision period (starting on April 23) and end as late as decision period 11 (ending on November 4). The majority of the curtailments occur during decision periods 8, 9, and 10, which represent the time period from July 16 to October 7, the low flow season for most rivers in Massachusetts. The curtailments are spread out over several decision periods because of the shape required by the DEP permits.

According to the results applicant B is required to begin curtailments during decision period 4 which begins on April 23. This result may seem unusual because there is normally plenty of streamflow in April. This result makes sense, however, because some of the water pumped from the ground during this time period will not deplete water from the stream until a much later time.

The degree of curtailment selected by the management model is not limited by the selection of the discretization levels. Neither the number of steps nor the level of each step

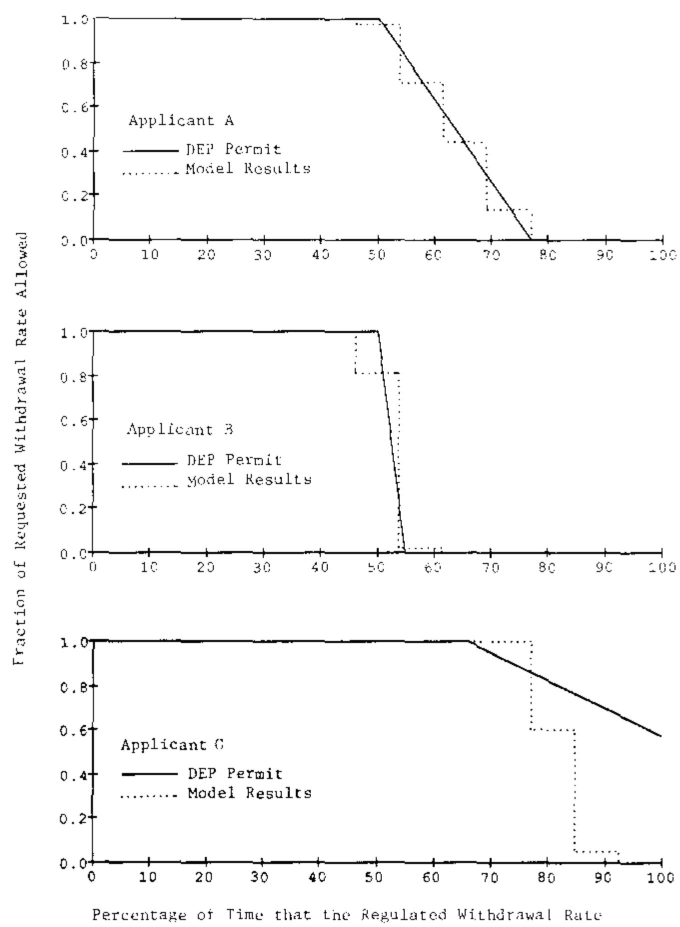


Fig. 5. Permit shapes specified by DEP and resulting from the model for applicants A, B, and G.

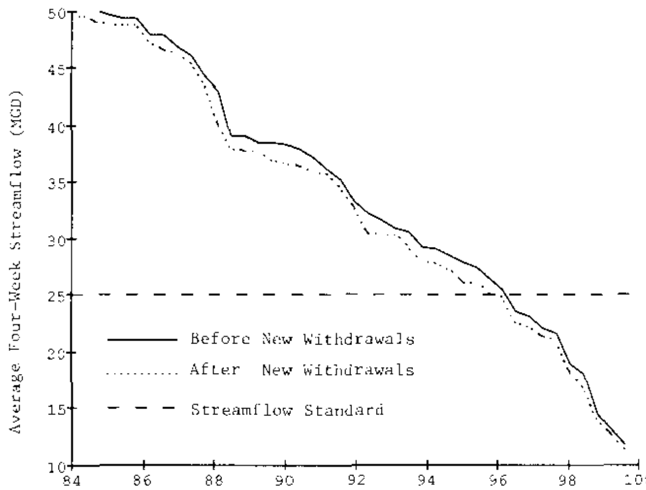


Fig. 6. Effect of the allowed withdrawals on the streamflow duration curve.

(as shown in the permit shapes in Figure 5 or as listed in Table 2) is necessarily the same as the discretization levels. This is evident in the results for applicants A, B, D, F, H, and I, where each of these applicants had discretization levels of 0.8, 0.6, 0.4, 0.2, and 0.0. With the exception of 0% allocations, there are only two instances where the allowed percentage of requested withdrawal coincided with a discretization level (i.e., 60 and 80% for applicant H).

Even though the SDF values are small compared to the 28 day time step, the effect of pumping is distributed over more than one decision period. For example, for a SDF of 1.8 days, approximately 75% of the withdrawal during a decision period will be depleted from the stream during that period. During the following decision period the effect is approximately 14%. In addition, these values do not account for delayed return flow from septic systems.

### *Potential Use*

The model could be used to assist in the actual development of permits by testing various conditions before their issuance. In this mode the relative impact on the various applicants could be compared. In addition, the effect on the depletion of low streamflows could be examined. Alternatively, it could be useful in providing guidance to the DEP in applying the permit conditions during low flow times.

The management model provides considerably more guidance than the DEP permits, in that it shows the best times to curtail use, and the amount that must be curtailed so that streamflow depletion is minimized. This information is useful in providing advance warning to users, particularly since the response times for streamflow depletion (caused by pumping) are sometimes lengthy.

Limitations in the use of the model are also important. The assumptions of an unconfined, isotropic, homogeneous and semi-infinite aquifer along an infinite straight stream render the model a simplification of any real-world application. In addition, the model assumes that the impact of all the withdrawals and return flows is seen at the gaging station, as opposed to being distributed along the length of the stream. This assumption could have an effect on the relative curtailments allotted to the various applicants. The linearity assumption for the aquifer may not be valid at all locations and during all time periods. However, the intent of the study was to address the formulation of policy by the DEP. The results show that the use of systems analysis techniques using straightforward models can assist both policy makers and decision makers in concentrating on the important issues.

Despite its limitations, the model has considerable merit in illustrating the relative differences among applicants for well water withdrawal permits. In addition, the results of the model show how the timing of curtailed withdrawals can help reduce the depletion of streamflow.

## SUMMARY AND CONCLUSIONS

The management model presented in this paper was developed to give guidance to the Massachusetts DEP in applying the Water Management Act. Enforcement of the act is accomplished through the issuance of permits which specify how much water an applicant may withdraw during times of normal, or above average, streamflow, and the percentage of time that they may have to curtail their permitted withdrawals during times of low flow. The DEP permits specify, in a statistical sense, how the allowed withdrawals should be implemented, but do not designate when curtailments should be made. The management model specifies, for each decision period, the percentage of requested withdrawal that would be allowed.

Results of the application of the model to the permitting process for the Charles River show that the model has potential for use in providing guidance to the DEP in both establishment of future permits and in the implementation of those that already exist.

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