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A previous analysis of a linear current source of finite length embedded in a conducting medium of infinite extent is extended to linear current sources of (1) infinite length and (2) semi-infinite length. Electric and magnetic field expressions are derived, and the results are numerically evaluated for frequencies in the ULF/ELF bands. For convenience, some of the results are presented in a dimensionless form. A comparison is made between the electromagnetic fields produced by linear current sources of finite and infinite length, and it is shown that there is a relative enhancement in the electric field near the source of finite length. It is also found that an optimum frequency exists for the electric field produced by a linear current source of infinite length at which the field amplitude is a maximum at a fixed observation point. Some practical applications of our results are suggested.

INTRODUCTION

Shortly after the end of the First World War a pair of articles appeared in the scientific literature describing experiments and theoretical work on the electromagnetic fields produced in, on, and above the sea by submerged cables carrying alternating current [*Drysdale*, 1924; *Butterworth*, 1924]. As described in the article by *Drysdale* [1924], and in greater detail in a later paper by *Wright* [1953], this work was undertaken as a part of a British Navy project called "Leader Gear," which was concerned with the use of the cable-generated electromagnetic fields for navigation. No further articles appeared on the fields produced by submerged cables until after the end of the Second World War, when Von Aulock prepared two lengthy reports on the subject [*Von Aulock*, 1948, 1953]. With the exception of a useful summary of Von Aulock's results by *Kraichman* [1976] and several particularly pertinent articles by *Wait* [1952, 1959, 1960, 1969], little research has since been carried out on the electromagnetic fields produced by submerged cables. This is unfortunate, because it is our belief that the fields could have important applications in undersea communication. The primary objective of this paper is to take the work just described through a further stage of development and thus provide an improved theoretical basis for studies of the feasibility of the use of the fields from submerged cables for undersea communication.

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To achieve this objective, we start with our previous work [*Inan et al.*, 1982] on a linear current source of finite length submerged in an infinite, isotropic, homogeneous, time-invariant, and conducting medium and consider the case when the observation distance from the source is much smaller than the length of the source and is far from the ends of the source (i.e., linear current source of infinite length), and the case when the observation distance from the source is much smaller than the length of the source and is near one of the ends of the source (i.e., linear current source of semi-infinite length). We obtain expressions for the electric and the magnetic fields for both cases and evaluate them numerically at ultralow and extremely low frequencies (ULF/ELF; frequencies less than 3 kHz).

Our results are valid under the following assumptions: (1) All the fields vary with time as exp $(i\omega t)$, (2) the source is infinitely thin in cross section, which is a valid assumption as long as the shortest distance from the observation point to the surface of the assumed cylindrical current source is much greater than the radius of the source [*Wait*, 1952], (3) all parts of the source wire carry equal current at a given instant of time, which is a good assumption for an insulated source wire at sufficiently low frequencies [*Wait*, 1952], and (4) the displacement current term is neglected in comparison to the conduction current term, which is justified for frequencies less than about 100 MHz in the case of seawater [*Inan et al.*, 1982].

DERIVATION OF THE FIELD COMPONENTS

In the preceding work by *Wait* [1952] and *Inan et al.* [1982] a linear current source of finite length submerged in a conducting medium of infinite extent is considered. In the cylindrical coordinate system (ρ, ϕ, z) the source wire extends from l_1 to l_2 along the z axis. The source current is the real part of $Ie^{i\omega t}$ taken to be in the z direction. The electric field components at the observation point P(ρ, z) are given by the following expressions:

$$E_\rho = -I\left[\frac{\partial Q(r)}{\partial \rho}\right]_{l=l_1}^{l_2} \tag{1}$$

$$E_z = -I\int_{l_1}^{l_2} P(r)\,dl - I\left[\frac{\partial Q(r)}{\partial z}\right]_{l=l_1}^{l_2} \tag{2}$$

with the third component $E_\phi = 0$. The functions $P(r)$ and $Q(r)$ in the above expressions are defined as [*Sunde*, 1968]

$$P(r) = \frac{i\omega\mu}{4\pi}\frac{e^{-\gamma r}}{r} \qquad Q(r) = \frac{e^{-\gamma r}}{4\pi\sigma r}$$

where

$$\gamma = (1+i)\beta \qquad \beta = (\omega\mu\sigma/2)^{1/2} = 1/\delta$$

$$r = [\rho^2 + (z-l)^2]^{1/2}$$

σ and μ being the conductivity and the permeability and γ and δ being the propagation constant and the skin depth of the surrounding conducting medium.

Suppose now that we let l_1 go to $-\infty$ and l_2 go to $+\infty$; i.e., consider a linear current source of infinite length. For this case, $E_\rho = 0$ and the only nonzero electric field component becomes

$$E_z = -I\int_{-\infty}^{+\infty} P(r)\,dl \tag{3}$$

Thus the electric field of a linear current source of infinite length aligned in the z direction has only a z component. This expression can be written as

$$E_z = -\frac{i\omega\mu I}{2\pi}\int_0^{+\infty}\frac{e^{-(1+i)u}}{u}\,dx \tag{4}$$

where

$$u = (a^2 + x^2)^{1/2} \qquad a = \beta\rho \qquad x = \beta(z-l)$$

As shown in the appendix, this integral can be replaced by the modified Bessel function of the second kind of order zero, $K_0(\gamma\rho)$, yielding

$$E_z = -\frac{\gamma^2 I}{2\pi\sigma}K_0(\gamma\rho) \tag{5}$$

which is the same expression as that derived by *Von Aulock* [1948] and by *Wait* [1959]. To derive the magnetic field, we substitute the electric field expression into the appropriate Maxwell's equation, and we obtain the following expression for the single magnetic field component B_ϕ:

$$B_\phi = -\frac{i}{\omega}\frac{\partial E_z}{\partial \rho}$$

which yields

$$B_\phi = \frac{\gamma\mu I}{2\pi}K_1(\gamma\rho) \tag{6}$$

where $K_1(\gamma\rho)$ is the modified Bessel function of the second kind of order 1. This latter expression for the magnetic field was also derived by *Von Aulock* [1948] and by *Wait* [1959].

At distances much larger than the skin depth of the surrounding conducting medium, the modified Bessel functions in (5) and (6) can be approximated by

$$K_\nu(\gamma\rho) \simeq \left(\frac{\pi}{2\gamma\rho}\right)^{1/2} e^{-\gamma\rho} \tag{7}$$

and both E_z and B_ϕ can be calculated directly using this expression.

We now concentrate our attention on the electric and the magnetic fields produced near the ends of a linear current source of infinite length. We let l_1 go to $-\infty$ while keeping l_2 finite, which gives a linear current source of semi-infinite length. Starting with (1) and (2), replacing l_1 by $-\infty$, and generally following a similar approach to the one used by *Wait* [1952] and by *Inan et al.* [1982] and using the results of the appendix, we obtain the following three expressions for the nonzero field components produced in the surrounding conducting medium:

$$E_\rho = \frac{\rho I}{4\pi\sigma}\frac{e^{-\gamma r_2}}{r_2^3}(\gamma r_2 + 1) \tag{8}$$

$$E_z = \frac{i\omega\mu I}{4\pi}\left[\left(\sinh^{-1}\frac{x_2}{a} - E_c(a, x_2) - \mathrm{Ker}_0\,(2^{1/2}a)\right)\right.$$

$$\left. - i[E_s(a, x_2) + \mathrm{Kei}_0\,(2^{1/2}a)]\right]$$

$$+\frac{I}{4\pi\sigma}\frac{e^{-\gamma r_2}}{r_2^3}(\gamma r_2+1)(z-l_2) \tag{9}$$

$$B_\phi = -\frac{\rho\mu I}{4\pi\delta^2}\{[M_c(a, x_2)+M_s(a, x_2)+N_c(a, x_2)] + i[M_c(a, x_2)-M_s(a, x_2)-N_s(a, x_2)]\} - \frac{\mu I}{2(2)^{1/2}\pi\delta}[\mathrm{Ker}_0'(2^{1/2}a)+i\,\mathrm{Kei}_0'(2^{1/2}a)] \tag{10}$$

where

$$r_2=[\rho^2+(z-l_2)^2]^{1/2} \qquad x_2=\beta(z-l_2)$$

Note that Ker_0, Kei_0, Ker_0', and Kei_0' are the Kelvin functions of zeroth order and their derivatives with respect to their argument, and E_c, E_s, M_c, M_s, N_c, and N_s are defined in the previous works [*Computation Laboratory of Harvard University*, 1949; *Wait*, 1952; *Inan et al.*, 1982] as follows:

$$E_c(a, x_2)=\int_0^{x_2}\frac{1-e^{-u}\cos u}{u}\,dx \tag{11a}$$

$$E_s(a, x_2)=\int_0^{x_2}\frac{e^{-u}\sin u}{u}\,dx \tag{11b}$$

$$M_c(a, x_2)=\int_0^{x_2}\frac{e^{-u}\cos u}{u^2}\,dx \tag{11c}$$

$$M_s(a, x_2)=\int_0^{x_2}\frac{e^{-u}\sin u}{u^2}\,dx \tag{11d}$$

$$N_c(a, x_2)=\int_0^{x_2}\frac{e^{-u}\cos u}{u^3}\,dx \tag{11e}$$

$$N_s(a, x_2)=\int_0^{x_2}\frac{e^{-u}\sin u}{u^3}\,dx \tag{11f}$$

When the observation point is on the axis of the semi-infinite wire (i.e., $\rho=0$ and $z>l_2$), the two components E_ρ and B_ϕ become zero, and the only nonzero component E_z (which is along the axis of the source wire) simplifies to the form

$$E_z=\frac{i\omega\mu I}{4\pi}\left\{[\gamma_e+\ln(2^{1/2})+\ln x_2-E_c(0, x_2)] - i\left[E_s(0, x_2)-\frac{\pi}{4}\right]\right\}+\frac{I}{4\pi\sigma\delta^2}\frac{e^{-(1+i)x_2}}{x_2^2}[(1+i)x_2+1] \tag{12}$$

In this last equation, γ_e is Euler's constant, which, to four significant figures, is given by $\gamma_e=0.5772$. This constant is usually denoted by the symbol γ, but we have added the subscript e to distinguish it from the γ used here for the propagation constant.

EFFECTS OF INSULATION

In the preceding derivation of the field expressions produced by linear current sources we assume that the current is uniform throughout the length of the conducting wire comprising the actual source and that it flows into the conducting medium only from the ends of the wire. For this condition of uniform current to apply, it is of course necessary to have insulation all along the wire except at the ends of it. Without insulation the current in the wire would flow into the surrounding conducting medium from all parts of the wire's surface, and uniform current flow along the length of the wire could not be achieved.

Extending the results by *Sunde* [1968], *Wait* [1952] states that for low frequencies the propagation constant of the current flowing in the source wire is determined largely by the electrical characteristics of the insulation and gives the following approximate expression for the current $I(l)$ at a point in the wire located a distance l away from the generator terminals:

$$I(l)\simeq I_0 e^{-\Gamma l} \tag{13}$$

where I_0 is the current assumed to be entering the wire at the generator terminals. The propagation constant Γ in this expression is given approximately by

$$\Gamma\simeq i(\varepsilon_i\mu_i\omega^2)^{1/2} \tag{14}$$

where ε_i and μ_i are the permittivity and permeability of the insulation material. It follows from these expressions that the current along an insulated wire of length L will be essentially uniform provided $|\Gamma L|\ll 1$. Suppose we take polyethylene to be a typical insulating material. For this material we can write $\varepsilon_i=\varepsilon_r\varepsilon_0$, with $\varepsilon_r=2.25$ (the relative dielectric constant of polyethylene), and $\mu_i=\mu_0$. Substituting the known values of ε_0 and μ_0, we obtain

$$|\Gamma|\simeq(3.14\times10^{-8}f)\ \mathrm{m}^{-1}$$

f being the frequency of the source current. For $f=1$ Hz, $|\Gamma|\simeq3.14\times10^{-5}\ \mathrm{km}^{-1}$ and so the length of the wire can be as long as several hundreds of kilometers and yet the condition $|\Gamma L|\ll1$ is still satisfied. Thus for low frequencies and typical insulating materials, it is reasonable to assume that an alternating current is uniform throughout the length of the wire for wire lengths varying from several tens to several hundreds of kilometers. It also follows that the displacement currents flowing through the insulation are negligible and the current mostly flows into the medium from the ends of the wire.

It is also shown analytically by *Wait* [1952] and stated by *Kraichman* [1976] that the electric and magnetic field expressions for linear current sources of both finite and infinite length (with the wire assumed to be of negligible thickness) are unaffected by the properties of the insulating material on the wire as long as the ratio of the radius of the insulation to the wavelength of the surrounding conducting medium is much less than 1. This condition can be written in the form

$$b(f\mu\sigma)^{1/2} \ll 1 \qquad (15)$$

where b is the radius of insulation. For seawater ($\sigma = 4$ S m^{-1}, $\mu = \mu_0 = 4\pi \times 10^{-7}$ H m^{-1}) this condition becomes

$$2.24 \times 10^{-3} b f^{1/2} \ll 1$$

which will of course be satisfied for low frequencies and reasonable thickness of insulation.

COMPUTATION OF THE FIELDS

Plots of the variations with distance of the amplitudes of E_z and B_ϕ produced in seawater by a linear current source of infinite length are given by *Von Aulock* [1948] and *Kraichman* [1976] for a few selected frequencies and for limited ranges of amplitude and distance. These data play such an important role in the study of the electric and magnetic fields produced in seawater by submerged linear current sources that we have extended the Von Aulock/Kraichman curves over a number of decades in frequency, amplitude, and distance.

As is shown in the appendix, (5) and (6) can be written in the following forms:

$$E_z = -\frac{i\omega\mu I}{2\pi}\left[\mathrm{Ker}_0\,(2^{1/2}a) + i\,\mathrm{Kei}_0\,(2^{1/2}a)\right] \qquad (16)$$

$$B_\phi = -\frac{\mu I}{2^{1/2}\pi\delta}\left[\mathrm{Ker}_0'\,(2^{1/2}a) + i\,\mathrm{Kei}_0'\,(2^{1/2}a)\right] \qquad (17)$$

where the real and imaginary parts are separated. It follows that E_z and B_ϕ can be evaluated either by using tables of the Kelvin functions of zeroth order and their derivatives [*Lowell*, 1959; *Young and Kirk*, 1964] or by using numerical integration techniques for arguments that are not tabulated in these tables (see the appendix).

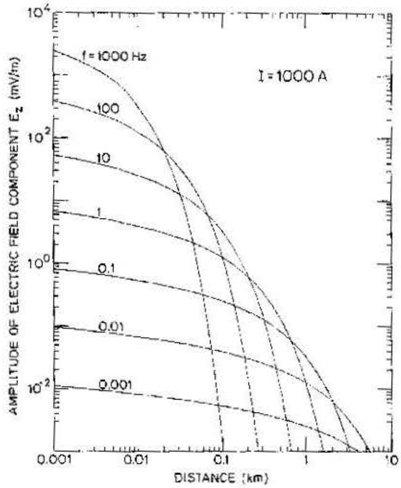

Fig. 1. Variation with perpendicular distance of the amplitude of the electric field produced in seawater by a linear current source of infinite length carrying an alternating current of amplitude 1000 A. The electric field amplitudes for an arbitrary current I can be derived from the values given above by multiplying by $I/1000$.

The curves we obtained for the variation of the amplitude of the electric field component E_z with frequency and perpendicular distance from the linear current source of infinite length are shown in Figure 1, and the corresponding curves for the amplitude of the magnetic field component B_ϕ are shown in Figure 2. It is clearly seen in both figures that at a constant frequency the rate of attenuation per unit distance increases with increasing distance. It is also observed that this rate becomes larger with increasing frequency (i.e., 55 dB per wavelength [*Inan et al.*, 1982]).

There is an interesting feature of the electric field variation that particularly distinguishes it from the magnetic field variation: at a fixed observation distance the electric field first increases, reaches a maximum value, and then decreases with increasing frequency, whereas the magnetic field decreases monotonically with increasing frequency. This feature can be seen more clearly when the variation of the amplitude of the electric field is plotted against frequency, as shown in Figure 3, and it implies that for a given

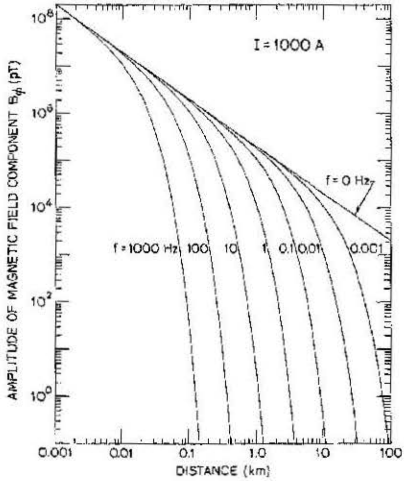

Fig. 2. Variation with perpendicular distance of the amplitude of the magnetic field produced in seawater by a linear current source of infinite length carrying an alternating current of amplitude 1000 A. The magnetic field amplitudes for an arbitrary current I can be derived from the values given above by multiplying by $I/1000$.

perpendicular distance between the source and the field point there is an optimum frequency at which the amplitude of the electric field is a maximum at the field point.

Shown in Figures 4 and 5 are comparisons of the electric and the magnetic fields produced by a linear current source of finite length (length 100 m) [*Inan et al.*, 1982] with those produced by a linear current source of infinite length. The variation with distance (along the perpendicular axis through the center of the finite source) of the ratio of the amplitudes of the electric fields produced by the finite (100 m) and infinite sources is plotted in Figure 4. Because of the choice of the central axis to show the distance variation, the E_z component for the finite source is also the total electric field, and we already know that the E_z component for the infinite source is everywhere the same as the total field. Thus the ratio of the E_z components shown is also the ratio of the total electric fields. Figure 5 shows the variation of the ratio of the amplitudes of the total magnetic field (i.e., its single component B_ϕ) for the same two sources along the same axis.

Figure 6 shows contours of the maximum values reached by the two components of the electric field, E_ρ (solid curves) and E_z (dashed curves), around the end A of a linear current source of semi-infinite length during one cycle of the current in the source (the amplitude and the frequency of the current are assumed to be 1000 A and 100 Hz). As can be seen, there are some regions in the figure where E_ρ is larger than E_z, but for large distances, E_z becomes by far the largest component. Note also that as the point of observation begins to move toward the other end of the source, i.e., the end at infinity, E_z contours become parallel to the source, and the electric field values tend toward those produced by a linear current source of infinite length.

PARAMETRIC APPROACH

It is possible to present the above results in a dimensionless form by normalizing all the distances by the skin depth of the surrounding conducting medium [*Fraser-Smith and Bubenik*, 1979]. For a linear current source of infinite length, amplitudes of the field components given by (16) and (17) can be written in the following parametric form:

$$|E_z^p| = \frac{\pi\sigma\delta^2}{I}|E_z| = [\mathrm{Ker}_0^2\,(2^{1/2}a) + \mathrm{Kei}_0^2\,(2^{1/2}a)]^{1/2} \qquad (18)$$

$$|B_\phi^p| = \frac{2^{1/2}\pi\delta}{\mu I}|B_\phi| = [\mathrm{Ker}_0'^2\,(2^{1/2}a) + \mathrm{Kei}_0'^2\,(2^{1/2}a)]^{1/2} \qquad (19)$$

The right-hand sides of these two parametric equations have been evaluated, and the data are plotted in Figure 7. The curve with crossed circles gives the variation of the amplitude of the electric field in parametric form, as given by (18), and the other curve (open circles) gives the variation of the amplitude of the magnetic field in parametric form, as given by (19). The variable along the horizontal axis in this figure, $2^{1/2}a$, is the argument of the Kelvin functions; it is essentially the perpendicular distance from the source in terms of skin depths. The two curves in Figure 7 are useful for rapid computation of the amplitudes of either the electric or the magnetic fields produced in any arbitrary conducting medium of infinite extent by a linear current source of infinite length over a very wide range of source currents and frequencies. For example, if the observation point is 100 m away from the source and the source frequency is 1 Hz, we have $2^{1/2}a \simeq 0.56$, and using Figure 7 we obtain approximately 1.0 and 1.6 for the parametric electric and magnetic field quantities. Knowing that $\delta \simeq 251.6$ m for $f = 1$ Hz, $\sigma = 4$ S m^{-1}, and $\mu = 4\pi \times 10^{-7}$ H m^{-1}, and using

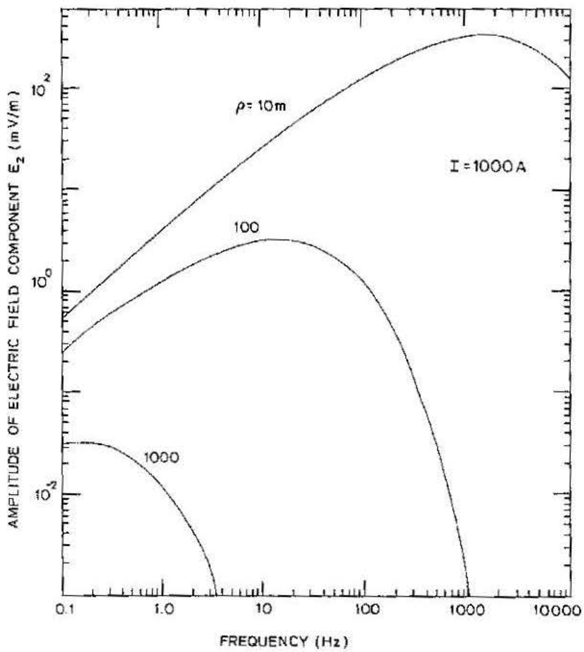

Fig. 3. Variation with frequency of the amplitude of the electric field produced in seawater by a linear current source of infinite length carrying an alternating current of amplitude 1000 A at a fixed perpendicular distance from the source. The electric field amplitudes for an arbitrary current I can be derived from the values given above by multiplying by $I/1000$.

$I = 1000$ A, we obtain $|E_z| \simeq 1.26$ mV m^{-1} and $|B_\phi| \simeq 1.80 \times 10^6$ pT. These particular values can be checked against the values of $|E_z|$ and $|B_\phi|$ plotted in Figures 1 and 2.

We have also plotted in Figure 8 the amplitude of the electric field in the alternate parametric form given as

$$|E_z^p|_\rho = \frac{2\pi\sigma\rho^2}{I}|E_z|$$
$$= (2^{1/2}a)^2[\mathrm{Ker}_0^2\,(2^{1/2}a) + \mathrm{Kei}_0^2\,(2^{1/2}a)]^{1/2} \qquad (20)$$

This equation is basically the same as (18), except that in (18) the term multiplying $|E_z|$ on the left-hand side is not a function of the perpendicular distance ρ and the corresponding term on the left-hand side of the above equation is not a function of frequency f. In other words, the single curve seen in Figure 8 is a parametric form of the curves seen in Figure 3; it contains all their information on the variation of the amplitude of the electric field versus frequency for a fixed observation point, as well as many additional data of the amplitude of the electric field at other observation distances. The maximum value of the electric field expression occurs for $2^{1/2}a \simeq 2.17$. This is a useful result to remember, because one can easily calculate the optimum frequency for a fixed observation point by just using this simple relation. Next the data plotted in Figure 8 show that the maximum value of the parametric electric field expression $|E_z^p|$ is approximately 0.83, from which the actual value of the electric field amplitude at the optimum frequency can easily be obtained.

DISCUSSION AND CONCLUSIONS

The first objective in deriving the field expressions for linear current sources of finite [*Inan et al.*, 198

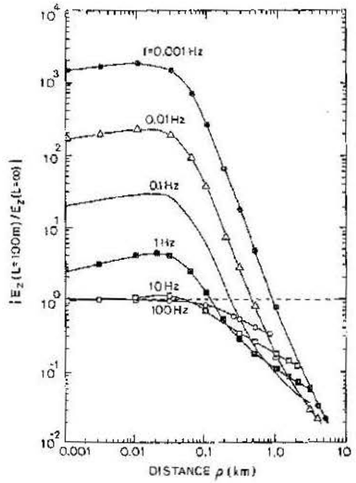

Fig. 4. Variation with perpendicular distance of the ratio of the amplitude of the z component of the electric field produced by a linear current source of finite length (100 m) to the amplitude of the z component of the electric field produced by a linear current source of infinite length, both carrying alternating currents of the same amplitude and frequency. The particular perpendicular axis along which the variation is shown passes through the center of the finite source. The surrounding conducting medium (seawater, $\sigma = 4$ S m^{-1}) is infinite in extent. Each curve refers to a different frequency in the range 0.001–100 Hz as shown in the figure.

and semi-infinite length was to investigate the possibility of producing large electromagnetic fields near their open ends (i.e., point electrodes) for undersea communication at ULF/ELF frequencies. A second objective was to derive enough theory and numerical data to provide the basis for the preliminary design of undersea cable assemblies that would have the capability of producing measurable ULF/ELF signals throughout limited regions of the ocean without the cables producing the signals necessarily being located in those regions. To some extent these objectives have been achieved; comparatively large electric and, to a lesser extent, magnetic fields can be produced at ULF/ELF frequencies in the vicinity of the point electrodes. However, it is found that the same frequency versus range constraint applies as is encountered in other methods of electromagnetic signal propagation in seawater. Communication to large ranges can only be achieved by using low frequencies but because the rate of data transfer is directly pro-

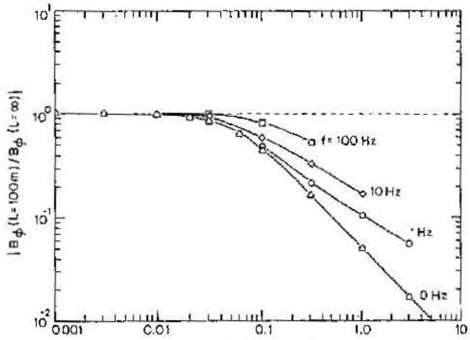

Fig. 5. Variation with perpendicular distance of the ratio of the amplitude of the magnetic field produced by a linear current source of finite length (100 m) to the amplitude of the magnetic field produced by a linear current source of infinite length, both carrying alternating currents of the same amplitude and frequency. The particular perpendicular axis along which the variation is shown passes through the center of the finite source. The surrounding conducting medium (seawater, $\sigma = 4$ S m^{-1}) is infinite in extent. Each curve refers to a different frequency in the range 0–100 Hz as shown in the figure.

portional to the signal frequency, the amount of information transmitted over the large ranges may be too small to be useful.

Comparing the ULF/ELF fields produced in sea-

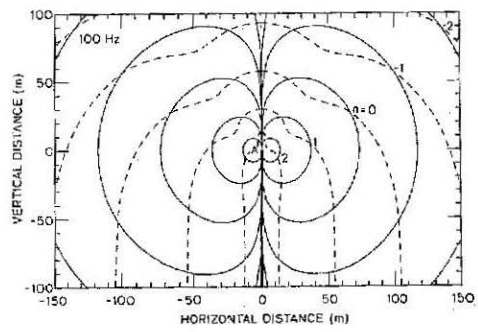

Fig. 6. Variation of the amplitudes of the ρ and z components of the electric field, E_ρ and E_z, produced in seawater by a linear current source of semi-infinite length (extending from $-\infty$ to A) and carrying an alternating current of amplitude 1000 A and frequency 100 Hz. The solid contours in this figure correspond to E_ρ, the dashed contours correspond to E_z, and the values of n shown for each contour give the numerical amplitude of the corresponding electric field in units of 10^n mV m^{-1}. Note that the field patterns are cylindrically symmetric about the axis defined by the source.

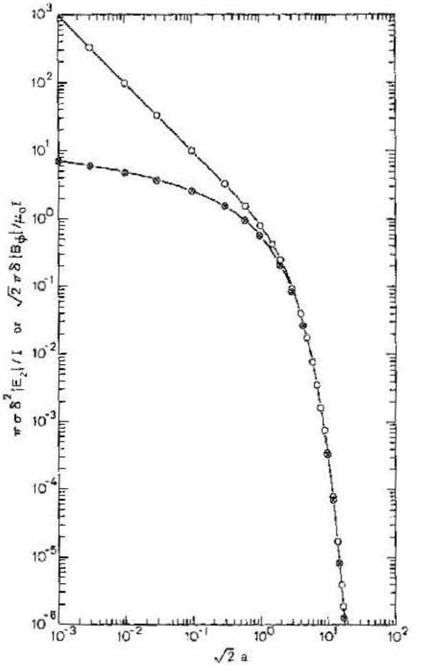

Fig. 7. Variation with $2^{1/2}a$ (where $a = \rho_\delta = \rho/\delta$) of the parametric expressions for the amplitude of the electric field (crossed circles) and the magnetic field (open circles) produced in a conducting medium of infinite extent by a linear current source of infinite length carrying an alternating current of amplitude I (equations (18) and (19)).

water by cables of finite and semi-infinite length with the fields produced by a cable of infinite length, our data show that at short distances from the point electrodes, i.e., distances less than about 1 km, the first two sources can produce greatly enhanced electric fields. Our data indicate that the increase is confined largely to frequencies in the ULF range and, within this range, it becomes larger as the frequency is decreased. At 0.001 Hz, for example, an increase of nearly 2000 times is observed at a perpendicular distance of 10 m from the center of the source of length 100 m (see Figure 4), and this increase is nearly 10 times larger than the increase at 0.01 Hz. It is observed that this increase begins to decline rapidly with distance once some limiting distance has been exceeded. It is obvious from the results that point electrodes have at best no advantage over infinite sources at distances much above 1 km and, at worst, they may produce much smaller fields than the infinite sources for some geometries and frequencies. We conclude therefore that point electrodes can be used most advantageously for undersea communication only over short distances.

The possible situations where such communication could be used are not necessarily trivial. For example, the ULF/ELF electric fields around the gap between two point electrodes [*Inan et al.*, 1982], or near a single point electrode, could provide a low-data rate, nonacoustic means of communication with an undersea or seafloor receiver. At the short distances under consideration (distances less than 1 km) there is likely to be no difficulty providing communication with divers [*Inan et al.*, 1982; *MacLeod*, 1977]. As a final note on the possible communication applications of point electrodes, we observe that long range is not always a desirable feature of a communication system, and under some circumstances the limited ranges for signal propagation that we have discussed here could be a decided advantage.

It is also important to note that in the case of a linear current source of infinite length, for a fixed observation point, there exists an optimum frequency at which the amplitude of the electric field becomes maximum. For example, for an observation distance

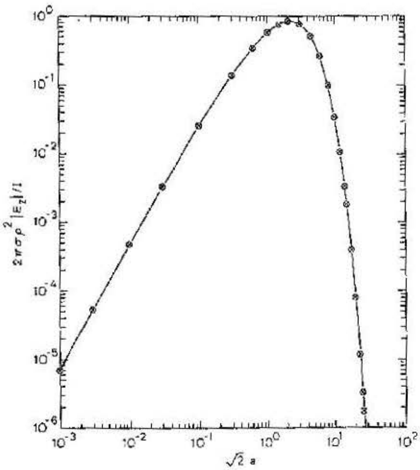

Fig. 8. Variation with $2^{1/2}a$ (where $a = \rho_\delta = \rho/\delta$) of the parametric expression, given by (20), for the amplitude of the electric field produced in a conducting medium of infinite extent by a linear current source of infinite length carrying an alternating current of amplitude I. Note that the maximum parametric amplitude (0.83) occurs for $2^{1/2}a \simeq 2.17$.

of 1 km, $f_{opt} \simeq 0.15$ Hz, and for 100 m, $f_{opt} \simeq 15$ Hz. The bandwidth is very limited at these frequencies, but there may well be special applications in which maximum electric field is more important than bandwidth.

Turning now to the electromagnetic fields produced at large distances from the sources (distances of the order of 10 km, or greater), our data show, as we have commented above, that cables of infinite length appear to be the best choice if long-distance undersea communication using electromagnetic fields from cable sources is the goal. It can be seen from Figures 1 and 2 that the maximum range that can be achieved with an alternating current of 1000 A with present electric and magnetic field sensors is of the order of 0.1–4 km for frequencies in the range 1000–1 Hz and of the order of 4–100 km for frequencies in the range 1–0.001 Hz. We assume that the cables are in a sea of infinite extent in quoting these results; in practice they will have to be located on the seafloor and both the sea surface and the seafloor will influence the maximum ranges that can be achieved. The effects of the sea surface can be neglected unless the sea is electrically shallow [*Fraser-Smith and Bubenik*, 1979], but the seafloor, the effective conductivity of which is generally lower than that of the seawater, will increase the ranges over which the fields can be measured.

The maximum range that can be achieved at any frequency can be made larger either by increasing the amplitude of the current or by increasing the sensitivity of the sensors used to measure the electromagnetic fields. However, (1) our assumed current (1000 A) is already substantial and a significant increase would entail large increases in power, and (2) the near-vertical nature of the slopes of most of the curves in Figures 1 and 2 as they approach the lower horizontal axes implies that even major increases in sensor sensitivity would have only a small effect on maximum range (even when no allowance is made for the presence of a background noise). The range can always be increased by reducing the frequency, but the lowest frequencies considered in Figures 1 and 2 are already extraordinarily low by present undersea communication standards, which severely restricts the rate of data transfer, and there is the further disadvantage that background noise increases rapidly with decreasing frequency throughout the ULF range. We conclude that maximum ranges of 1–10 km are likely to be the limit for a single cable of infinite length in an infinite extent of seawater and that undersea communication by means of the electromagnetic fields from ULF/ELF currents in undersea cables is necessarily restricted to special applications of limited range using either a single cable of infinite length (transverse range 1–10 km) or an array of such cables (transverse ranges of a few tens to a few hundreds of kilometers, depending on the number of cables) [*Inan et al.*, 1982].

APPENDIX

In equation (4) we have an integral of the form

$$I_1 = \int_0^\infty \frac{e^{-(1+i)u}}{u}\, dx \tag{A1}$$

where $u = (a^2 + x^2)^{1/2}$, which can also be written as

$$I_1 = \int_1^\infty \frac{e^{-(1+i)at}}{(t^2 - 1)^{1/2}}\, dt \tag{A2}$$

From *Lebedev* [1972, p. 119] we have the following integral expression for the modified Bessel function of the second kind of order ν:

$$K_\nu(z) = \frac{\pi^{1/2}}{\Gamma(\nu + 1/2)} \left(\frac{z}{2}\right)^\nu \int_1^\infty e^{-zt}(t^2 - 1)^{\nu - 1/2}\, dt \tag{A3}$$

provided Re $z > 0$ and Re $\nu > -\frac{1}{2}$. Here Γ is the gamma function defined as

$$\Gamma(n) = \int_0^\infty y^{n-1} e^{-y}\, dy$$

with Re $n > 0$. Choosing $\nu = 0$ and knowing that $\Gamma(\frac{1}{2}) = \pi^{1/2}$ yields

$$K_0(z) = \int_1^\infty \frac{e^{-zt}}{(t^2 - 1)^{1/2}}\, dt \tag{A4}$$

Substituting $z = a(1 + i)$ in (A4) and comparing it with (A2), we obtain

$$K_0[a(1 + i)] = I_1 \tag{A5}$$

In other words, the integral I_1 is a zeroth-order modified Bessel function of the second kind of complex argument. Since $\gamma = (1 + i)\beta$ and $a = \beta\rho$, we have

$$I_1 = K_0(\gamma\rho) = \int_0^\infty \frac{e^{-(1+i)u}}{u}\, dx \tag{A6}$$

We can also write the following expression [*Dwight*, 1961]:

$$i^{-\nu} K_\nu(\alpha i^{1/2}) = \mathrm{Ker}_\nu\, \alpha + i\, \mathrm{Kei}_\nu\, \alpha \tag{A7}$$

where $\mathrm{Ker}_\nu\, \alpha$ and $\mathrm{Kei}_\nu\, \alpha$ are Kelvin functions of the second kind of order ν [*Young and Kirk*, 1964]. From complex variables we know that $i^{1/2}$

$(1+i)/2^{1/2}$ and by letting $\nu = 0$, it follows that

$$K_0(\gamma\rho) = \mathrm{Ker}_0\,(2^{1/2}a) + i\,\mathrm{Kei}_0\,(2^{1/2}a) \tag{A8}$$

We have now expressed the integral I_1 in terms of known Kelvin functions. If we decompose I_1 into its real and imaginary parts, we obtain

$$\begin{aligned} \mathrm{Ker}_0\,(2^{1/2}a) &= \int_0^\infty \frac{e^{-u}\cos u}{u}\,dx \\ \mathrm{Kei}_0\,(2^{1/2}a) &= -\int_0^\infty \frac{e^{-u}\sin u}{u}\,dx \end{aligned} \tag{A9}$$

If we now let the upper limits of the first two integrals given by (11) go to infinity and compare the results with the integrals given by (A9), we can write

$$\begin{aligned} \mathrm{Ker}_0\,(2^{1/2}a) &= \int_0^\infty \frac{1}{u}\,dx - E_c(a,\infty) \\ \mathrm{Kei}_0\,(2^{1/2}a) &= -E_s(a,\infty) \end{aligned} \tag{A10}$$

Similarly, if we let the upper limits of the last four integrals given by (11) go to infinity and compare the results with the derivatives of the Kelvin functions, we obtain

$$\begin{aligned} \mathrm{Ker}_0'\,(2^{1/2}a) &= \frac{a}{2^{1/2}}\,[M_c(a,\infty) + M_s(a,\infty) + N_c(a,\infty)] \\ \mathrm{Kei}_0'\,(2^{1/2}a) &= \frac{-a}{2^{1/2}}\,[M_c(a,\infty) - M_s(a,\infty) - N_s(a,\infty)] \end{aligned} \tag{A11}$$

From (A7) it also follows that

$$K_1(\gamma\rho) = i\,\mathrm{Ker}_1\,(2^{1/2}a) - \mathrm{Kei}_1\,(2^{1/2}a) \tag{A12}$$

where $\mathrm{Ker}_1\,(2^{1/2}a)$ and $\mathrm{Kei}_1\,(2^{1/2}a)$ can be expressed in terms of the derivatives of Kelvin functions of zeroth order using recurrence formulas as [*Dwight*, 1961]

$$\begin{aligned} \mathrm{Ker}_1\,(2^{1/2}a) &= \frac{1}{2^{1/2}}\,[\mathrm{Ker}_0'\,(2^{1/2}a) - \mathrm{Kei}_0'\,(2^{1/2}a)] \\ \mathrm{Kei}_1\,(2^{1/2}a) &= \frac{1}{2^{1/2}}\,[\mathrm{Ker}_0'\,(2^{1/2}a) + \mathrm{Kei}_0'\,(2^{1/2}a)] \end{aligned} \tag{A13}$$

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