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Further investigation of the interference minimums in the low-frequency electromagnetic fields produced by a submerged vertical magnetic dipole

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The quasi-static electromagnetic fields generated along the sea surface by a submerged vertical magnetic dipole are evaluated numerically using exact expressions and the results are plotted in a parametric form for source depths varying from 2 to 14 seawater skin depths δ. The curves show that there is a minimum in the amplitude of the vertical component of the magnetic field for horizontal distances from the source in the range 9–14δ and for dipole depths ranging from 2 to 8δ, with the deepest minimum occurring at a horizontal distance of $\rho_{\min} \simeq 11.07\delta$ when the dipole is at a critical depth of $d_c \simeq 4.22\delta$. There also exists a similar minimum point in the variation along the surface of the amplitude of the total electric field for horizontal distances from the source in the range 10–20δ and dipole depths ranging from 4 to 23δ, with the deepest minimum occurring at a horizontal distance of $\rho_{\min} \simeq 12.95\delta$ when the dipole is at a depth of $d_c \simeq 9.38\delta$. Both minimums are due to the strong destructive interference between the direct and the lateral wave components of the fields. No such minimum point exists for the variation of the amplitude of the horizontal component of the magnetic field.

1. INTRODUCTION

In a theoretical study of the quasi-static ULF/ELF magnetic fields generated at the surface of an infinitely deep sea by submerged harmonic magnetic dipoles, *Fraser-Smith and Bubenik* [1976] discovered some unexpected and surprisingly sharp minimums in their computed field amplitudes for both vertical magnetic dipole (VMD) and horizontal magnetic dipole (HMD) sources. The characteristics of the minimums depended strongly on the dipole type, on the magnetic field component, and on the frequency and depth of the dipole sources, but when they appeared they were always located at the point of transition to the power law decline with increasing distance that is typical for the field components at large distances. Figure 1, reproduced from *Fraser-Smith and Bubenik* [1976], shows some examples of these minimums as they appeared in the original data plots. *Bannister* [1984*a*, *b*] later derived new approximate expressions for the field components produced by the VMD and confirmed the existence of the Fraser-Smith and Bubenik minimums analytically. He also stated that the physical phenomenon behind the deep minimums was destructive interference between the direct and lateral wave components of the field, in agreement with the earlier attribution of *Bubenik and Fraser-Smith* [1978] and *Fraser-Smith and Bubenik* [1979]. (The "lateral" and "up-and-over" wave designations used by *Bannister* [1984*a*, *b*] and by *Fraser-Smith and Bubenik* [1979], respectively, are essentially the same; see *Staiman and Tamir* [1966] for a general discussion of the properties of the lateral wave). Because the minimums have potential application in experiments to verify the extensive theory that has now been developed for the fields produced by submerged dipole sources, and in the interpretation of the fields produced by such sources in practice, we here analyze the fields produced at the surface of a sea of infinite depth by a submerged VMD in greater detail and obtain more information about the minimums observed in both its magnetic and electric field components.

Theoretical expressions for the quasi-static subsurface-to-surface propagation of the electromagnetic fields produced by submerged VMD have

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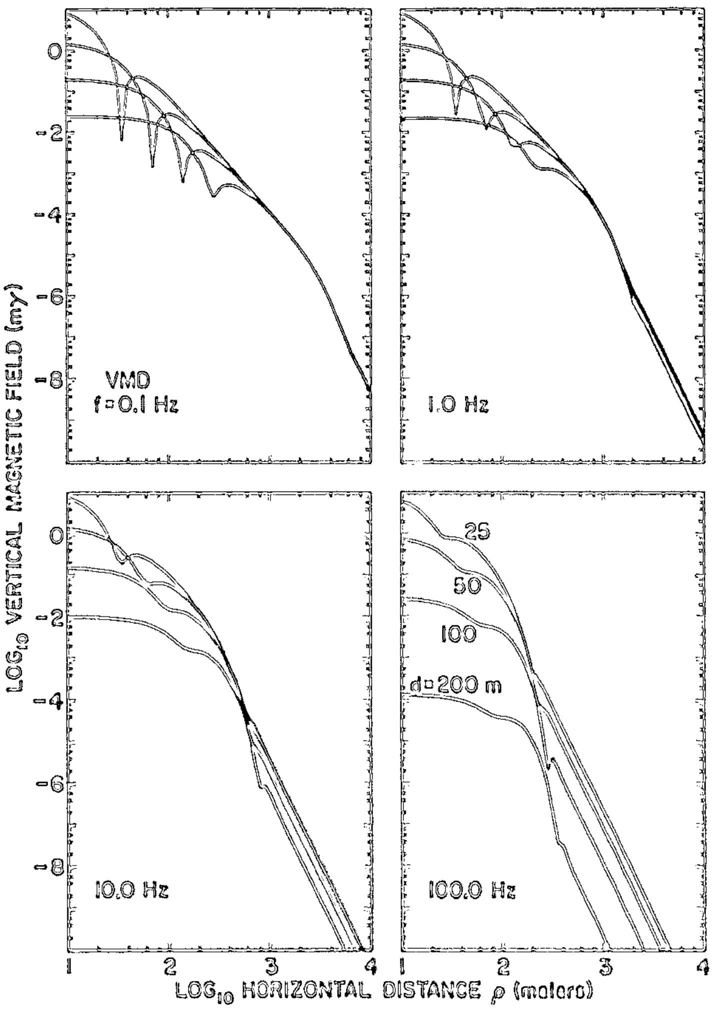

Fig. 1. Original magnetic field data computed by *Fraser-Smith and Bubenik* [1976] for the fields produced on the sea surface by a VMD submerged at depth *d*. The minimums that are the subject of this paper are best seen in the curves for a frequency of 100 Hz; they appear as small "jiggles" in the curves for horizontal distances in the range 200–600 m.

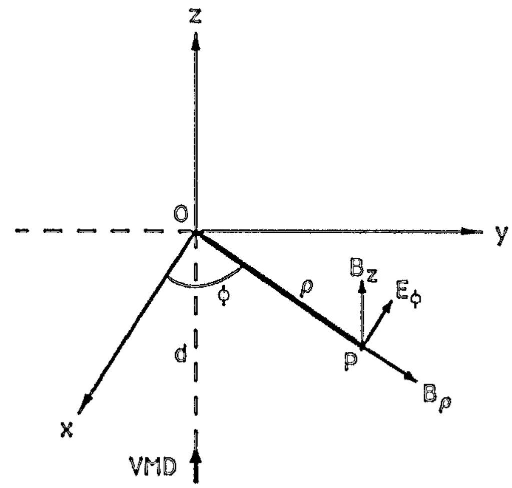

Fig. 2. The geometry employed in computing the electric and magnetic field components B_ρ, E_ϕ, and B_z at point P on the sea surface ($z = 0$) due to a VMD dipole source located at a depth d below the surface.

been tabulated by a number of authors [e.g., *Wait and Campbell*, 1953; *Baños*, 1966; *Sinha and Bhattacharya*, 1966; *Kraichman*, 1976; *Fraser-Smith and Bubenik*, 1976; *Bannister*, 1984*a*, *b*]. Our calculations were made by using both the Sommerfeld integral expressions of *Fraser-Smith and Bubenik* [1976] and the analytical expressions derived by *Sinha and Bhattacharya* [1966]. We also used (1) the approximate analytical expressions of *Bannister* [1984*a*], and (2) some numerical values for the horizontal component of the electric field computed independently by D. M. Bubenik (personal communication, 1989) to confirm the results of our computations.

The numerical data are presented in a parametric form by normalizing all the distances to the skin depth of the conducting medium (seawater in this case), which means that the results can be used for all frequencies in the range for which the quasi-static approximation is valid. Since the approximation requires only that the source-receiver distances be much smaller than a free space wavelength, and the largest such distances considered in this work are of the order of 24 seawater skin depths, the approximation is not particularly restrictive and our results will certainly be valid for all frequencies in the VLF range (3–30 kHz) and below. The upper medium (air) is assumed to be nonconducting ($\sigma = 0$ S/m) and both media are assumed to be nonmagnetic ($\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m). The displacement currents in both media have been neglected, which is a good assumption for the quasi-static case. Because of our use of a parametric approach, the results of our work are not only applicable to the case of the air-sea interface but they can also be used for the air-Earth interface, or any other interface between a semi-infinite nondissipative and a semi-infinite dissipative media separated by a plane boundary.

2. FIELD COMPONENTS

The geometry employed in our computations is shown in Figure 2. The seawater is assumed to be a homogeneous conducting medium of conductivity σ occupying the region $z < 0$ of a cylindrical coordinate system (ρ, ϕ, z). The source is a VMD located at a depth of d below the sea surface at the point (0, 0, $-d$); it is assumed to have a dipole moment m that varies harmonically with time t at the angular frequency ω. Such a dipole could consist of a small insulated wire loop of area dA carrying a current I exp $(i\omega t)$, in which case $m = IdA$ (we follow convention in suppressing the time factor exp $(i\omega t)$ in the field expressions). The resulting three non-zero components of the rotationally symmetric electromagnetic fields, B_ρ, E_ϕ, and B_z, are evaluated on the sea surface at $(\rho, \phi, 0)$.

The exact Sommerfeld integral expressions for the three field components have the following form [*Baños*, 1966]:

$$B_\rho = \frac{\mu_0 m}{2\pi} \int_0^\infty \frac{\lambda e^{-ud}}{\lambda + u} J_1(\lambda\rho)\lambda^2 \, d\lambda \qquad (1)$$

$$E_\phi = \frac{-i\omega\mu_0 m}{2\pi} \int_0^\infty \frac{e^{-ud}}{\lambda + u} J_1(\lambda\rho)\lambda^2 \, d\lambda \qquad (2)$$

$$B_z = \frac{\mu_0 m}{2\pi} \int_0^\infty \frac{e^{-ud}}{\lambda + u} J_0(\lambda\rho)\lambda^3 \, d\lambda \qquad (3)$$

where $u^2 = \lambda^2 + \gamma^2$, $\gamma^2 = i\omega\mu_0\sigma$, and $J_0(\lambda\rho)$ and $J_1(\lambda\rho)$ are the Bessel functions of the first kind of orders zero and one, respectively. Analytical expressions derived by *Sinha and Bhattacharya* [1966] for the three field components can be written in the form

$$\begin{aligned}
B_\rho = \frac{\mu_0 m}{4\pi}\{ &-2e^{-p}R^{-3}L\{[ap + 3(a+\tau)] \\
&+ p^{-2}[L^2p^3a + p^2a(10L^2 - 3) + 9pa(5L^2 - 2) \\
&+ 15(a+\tau)(7L^2 - 3)]\} \\
&+ R^{-3}\{L^2(s - 40\tau) - [15\tau^5 - 27\tau^3 + \tau(12 + a^2)]\}I_0K_0 \\
&- R^{-3}\{L^2(s + 2ap) - [15\tau^5 + 3\tau^3 - \tau(2 - a^2)]\}I_1K_1 \\
&+ R^{-3}\{L^2t + L[2\tau(\alpha - 6\beta) + w - 6\beta\tau p^{-2}(20 - 35\tau^2)] \\
&+ [2\beta\tau(4\tau^2 - 3) - w]\}I_0K_1 \\
&- R^{-3}\{L^2t - L[2\tau(\beta - 6\alpha) + w - 6\alpha\tau p^{-2}(20 - 35\tau^2)] \\
&- [w + 2\alpha\tau(3 - 4\tau^2)]\}I_1K_0\}
\end{aligned} \qquad (4)$$

$$\begin{aligned}
E_\phi = \frac{-i\omega\mu_0 m}{4\pi}\{ &2e^{-p}p^{-2}R^{-2}[L^2p^2a + pa(6L^2 - 1) \\
&+ 3(a+\tau)(5L^2 - 1)] \\
&+ R^{-2}[L\tau(v - 5)]I_0K_0 - R^{-2}[L\tau(v + 5)]I_1K_1 \\
&+ R^{-2}[L(w - 2\beta\tau) + 6\beta p^{-2}\tau(5\tau^2 - 4)]I_0K_1 \\
&- R^{-2}[L(w - 2\alpha\tau) - 6\alpha p^{-2}\tau(5\tau^2 - 4)]I_1K_0\}
\end{aligned} \qquad (5)$$

$$\begin{aligned}
B_z = \frac{\mu_0 m}{4\pi}\{ &2e^{-p}p^{-2}R^{-3}\{15\tau^2(1+p)(1 - 7L^2) \\
&+ 3a^2(2 - 15L^2) + (5L^2 - 1)[6 - p(a^2 - 6)] \\
&- 5pa^2L^2 - p^2[2 + L^2(a^2 - 12)] + 2L^2p^3\} \\
&+ LR^{-3}[d_1 + 2a^2 - 24]I_0K_0 - LR^{-3}[d_1 - 70\tau^2]I_1K_1 \\
&+ LR^{-3}[e_1 + \beta(20\tau^2 - 6) + 6\beta L^{-1}p^{-2}f]I_0K_1 \\
&- LR^{-3}[e_1 + \alpha(20\tau^2 - 6) - 6\alpha L^{-1}p^{-2}f]I_1K_0\}
\end{aligned} \qquad (6)$$

where

$$p = \gamma R \qquad a = \gamma\rho \qquad L = d/R$$

$$\tau = \rho/R \qquad \gamma = (1+i)/\delta$$

$$\delta = (2/\omega\mu_0\sigma)^{1/2} \qquad R = (\rho^2 + d^2)^{1/2}$$

$$d_1 = 4a^2 - 10a^2\tau^2 + 120\tau^2 - 105\tau^4$$

$$e_1 = 22a\tau - a^3\tau - 45a\tau^3$$

$$f = 40\tau^2 - 8 - 35\tau^4$$

$$s = 10a^2\tau - 8\tau + 90\tau^3 - 3ap$$

$$t = a^3 - 3a + 45a\tau^2$$

$$v = 15\tau^2 + a^2 - 7$$

$$w = a(6\tau^2 - 1)$$

$$I_n = I_n(\alpha) = I_n\left[\frac{\gamma}{2}(R - d)\right]$$

$$K_n = K_n(\beta) = K_n\left[\frac{\gamma}{2}(R + d)\right]$$

In the above expressions, $I_n(\alpha)$ and $K_n(\beta)$ are modified Bessel functions of the first and second kinds of order n and with complex arguments α and β. Following *Dwight* [1961], these Bessel functions can be expressed in terms of the Kelvin functions

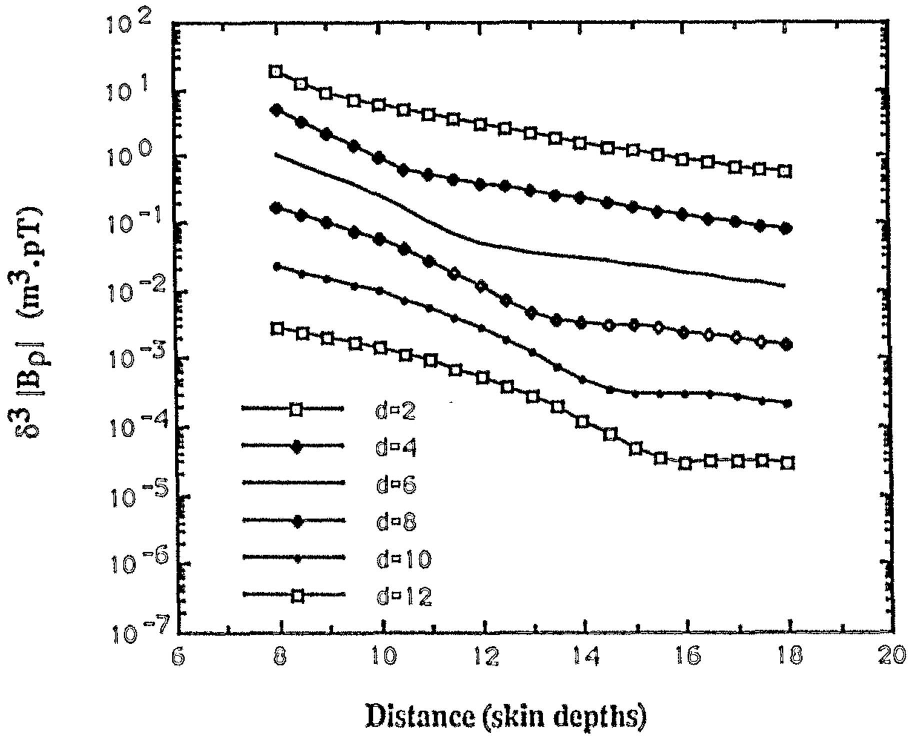

Fig. 3. Variation with distance along the sea surface of the amplitude of the horizontal magnetic field component B_ρ produced by a VMD submerged at various depths d in the range 2–12 seawater skin depths.

$ber_n(x)$, $bei_n(x)$, $ker_n(x)$, $kei_n(x)$ and their derivatives (denoted by a prime) as follows:

$$I_0(\alpha) = ber_0(\alpha_m) + ibei_0(\alpha_m)$$

$$I_1(\alpha) = bei_1(\alpha_m) - iber_1(\alpha_m)$$

$$K_0(\beta) = ker_0(\beta_m) + ikei_0(\beta_m)$$

$$K_1(\beta) = -kei_1(\beta_m) + iker_1(\beta_m)$$

where

$$ber_1(\alpha_m) = [ber_0'(\alpha_m) - bei_0'(\alpha_m)]/\sqrt{2}$$

$$bei_1(\alpha_m) = [ber_0'(\alpha_m) + bei_0'(\alpha_m)]/\sqrt{2}$$

$$ker_1(\beta_m) = [ker_0'(\beta_m) - kei_0'(\beta_m)]/\sqrt{2}$$

$$kei_1(\beta_m) = [ker_0'(\beta_m) + kei_0'(\beta_m)]/\sqrt{2}$$

and where $\alpha_m = |\alpha|$ and $\beta_m = |\beta|$. The values of these Kelvin functions and their derivatives, both of order zero, are tabulated by *Lowell* [1959] for a wide range of arguments. However, because it was not feasible for us to automate our computations while using these printed tables, we wrote a computer program to evaluate the necessary Kelvin functions and their derivatives for any arguments α_m and β_m. After checking the results given by this program against the data in the Lowell tables, we incorporated it into our field computation programs.

We computed the electromagnetic field components in two different ways: (1) by numerical integration of the Sommerfeld integrals in (1)–(3) as modified by *Fraser-Smith and Bubenik* [1976] and using the techniques described by *Bubenik* [1977]; (2) by evaluating the analytical expressions (4)–(6) derived by *Sinha and Bhattacharya* [1966]. Both sets of equations are exact, so our results should be more accurate than those published previously by *Bannister* [1984*a*] and *King and Brown* [1984], where use was made of approximate expressions. This is true especially for the range of observer distances where the direct and the lateral wave components of the fields interfere with one another.

The explicit equations derived by *Sinha and Bhattacharya* [1966] are lengthy, but they are useful because the direct and lateral wave components of the fields can be explicitly identified in them. More specifically, the terms that are multiplied by the exponential term e^{-p} correspond to the direct wave component, whereas the terms that are multiplied by the product of the modified Bessel functions I_n

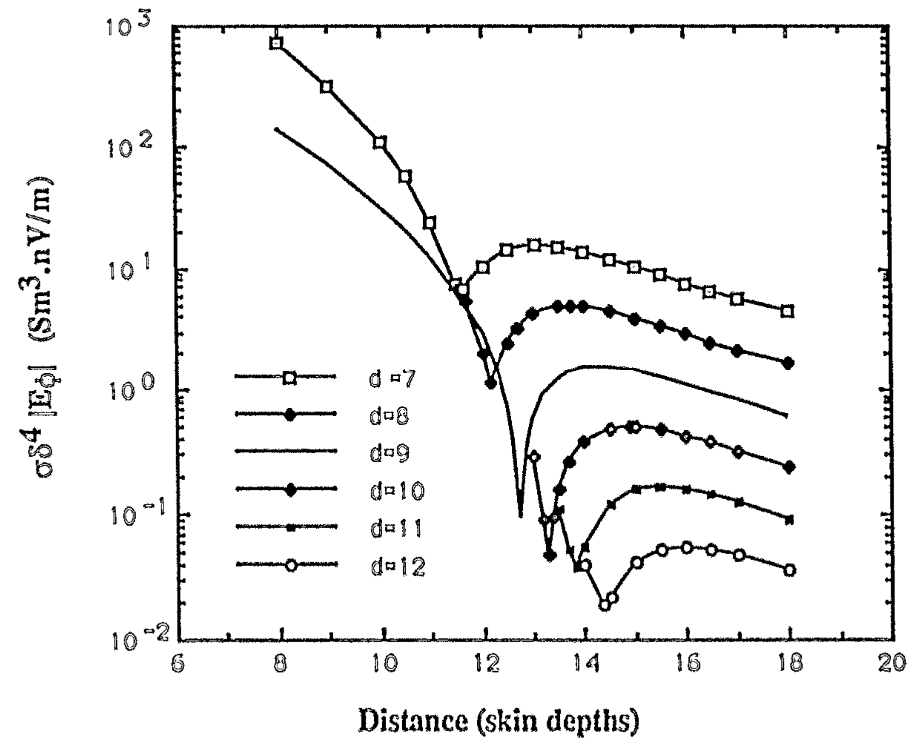

Fig. 4. Variation with distance along the sea surface of the amplitude of the horizontal electric field component E_ϕ produced by a VMD submerged at various depths d in the range 7–12 seawater skin depths.

and K_n correspond to the lateral wave component. Thus the two wave components can easily be extracted and computed separately without approximation. This is not the case for the Sommerfeld integral expressions given by (1)–(3), where the two wave components cannot be separated from one another unless approximations are made [e.g., *Bannister*, 1984*a*, *b*; *King*, 1985].

We take the magnetic dipole moment to be unity ($m = 1$ A m^2), with the implication that the computed field values should be multiplied by the actual dipole moment if they are to be used for a VMD of arbitrary dipole moment. The units for the electric and magnetic field components are chosen to be nanovolt per meter and picotesla (1 pT = 1 milligamma).

3. NUMERICAL RESULTS AND DISCUSSION

The principal results of our computations are presented in Figures 3, 4, and 5, which show the variation with normalized horizontal distance ρ/δ of the amplitudes of the three nonzero field components in their parametric form, $\delta^3|B_\rho|$, $\sigma\delta^4|E_\phi|$, and $\delta^3|B_z|$. The overall distance range is restricted to $6 \leq \rho/\delta \leq 20$, since that is the range in which the minimums of interest are located. For each field component a set of curves are plotted for a range of integer values of the normalized depth d/δ; the range varies for each component and depends once again on the occurrence of the minimums.

Examining each of the figures in turn, we find little evidence of a minimum in the curves showing the distance variation of the horizontal magnetic field component B_ρ (Figure 3). However, there is a point of inflection in the curves that moves progressively out to larger distances from the source as the dipole depth is increased and which does in fact become a very weak minimum for $d = 12\delta$.

The curves in Figures 4 and 5 show a well-defined minimum in the horizontal electric field component E_ϕ (Figure 4), and, as expected from the earlier studies by *Fraser-Smith and Bubenik* [1976] and *Bannister* [1984*a*, *b*], in the vertical magnetic field component B_z (Figure 5). The horizontal location of the minimum point, ρ_{min}, depends on the depth of the VMD and it is quite different for the two field components. For E_ϕ, ρ_{min} ranges approximately from 10 to 20 skin depths as the dipole depth varies from 4 to 23 skin depths (only part of this variation is covered by the data shown in the figure), whereas ρ_{min} for the B_z component ranges approximately

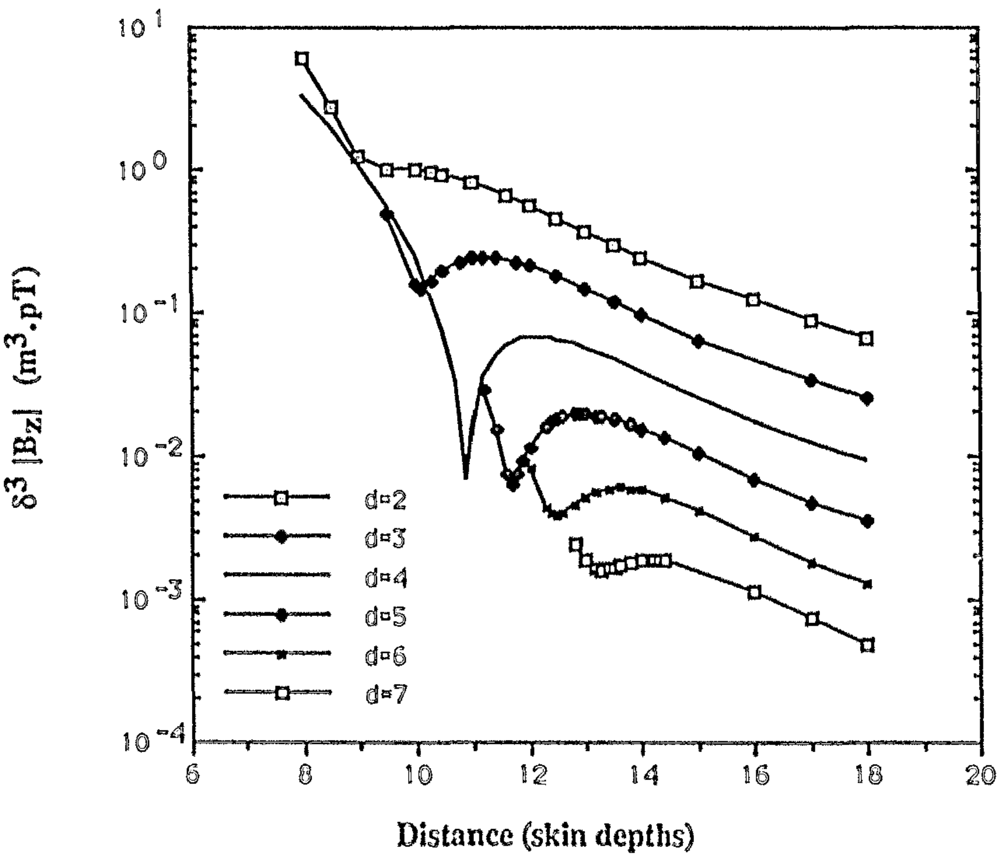

Fig. 5. Variation with distance along the sea surface of the amplitude of the vertical magnetic field component B_z produced by a VMD submerged at various depths d in the range 2–7 seawater skin depths.

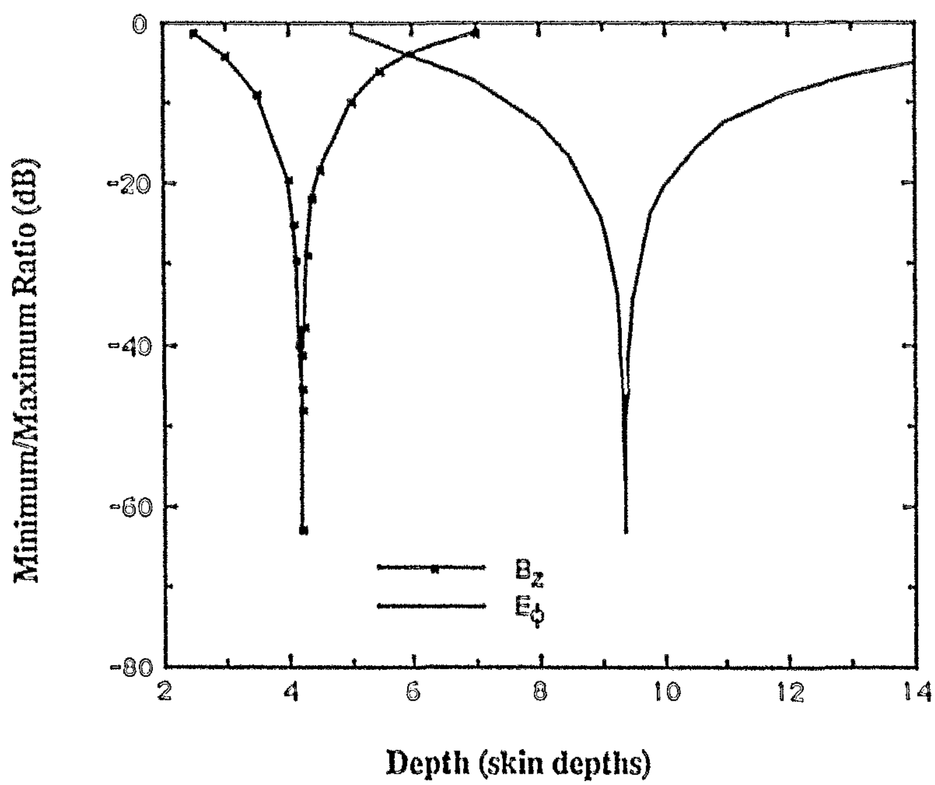

Fig. 6. Variation with normalized depth of the minimum/maximum amplitude ratios for the magnetic and electric field components B_z and E_ϕ.

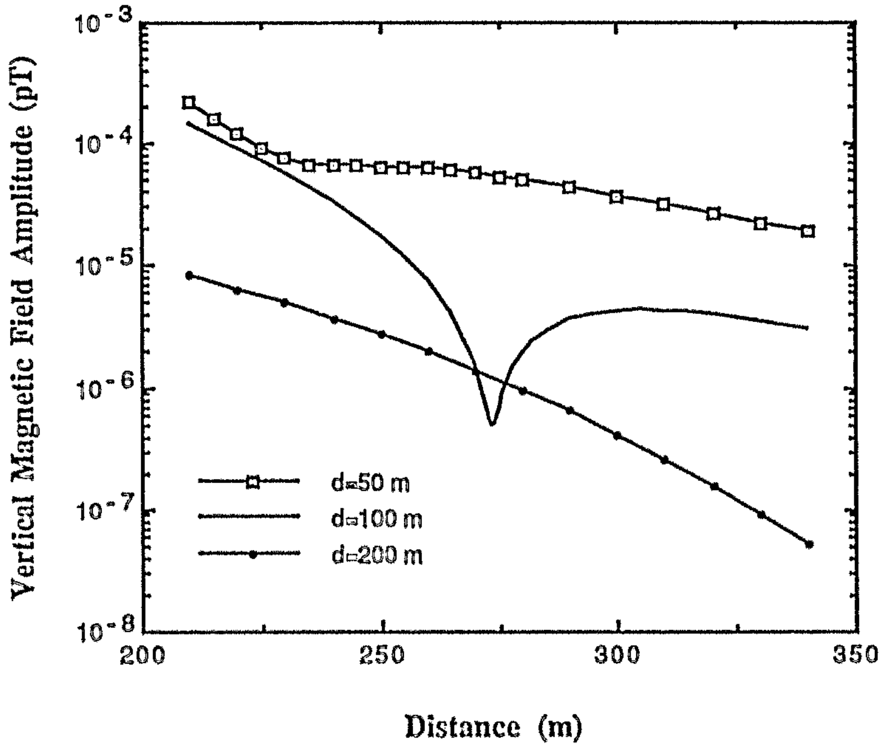

Fig. 7. Variation of the amplitude of the vertical magnetic field component B_z with horizontal distance from the VMD source. The field amplitudes for three different depths d are illustrated, and the data are presented in nonparametric form.

from 9 to 14 skin depths as the dipole depth varies from 2 to 8 skin depths. The curves for both E_ϕ and B_z have no further interference minimums for dipole depths outside the above ranges.

In addition to their variation of location with dipole depth, the minimums also vary in strength. We will measure this strength by taking the ratio (measured in decibels) of the minimum amplitude to the succeeding maximum amplitude as distance is increased. In Figure 6 we plot the variations of these amplitude ratios as functions of the normalized dipole depth for both E_ϕ and B_z. The curves show that there exists a sharply defined critical depth d_c for each of the two field components for which the amplitude ratio is a minimum. For E_ϕ the critical depth is 9.38δ, and the horizontal distance corresponding to the critical depth is 12.95δ; the amplitude ratio for this minimum is close to -63 dB. Similarly, for the vertical component of the magnetic field B_z the critical depth of the dipole is 4.22δ, and the horizontal distance corresponding to the critical depth is 11.07δ; the amplitude ratio for this minimum is also close to -63 dB. As seen in Figure 6, the ratio of minimum to maximum amplitude can change drastically with only a very small change in the depth of the dipole near the critical depth d_c.

Our primary goal in presenting the dipole field data in parametric form is to compress the quantity of data required to illustrate all the significant variations in the field components while maintaining maximum generality. The parametric data can be easily converted into actual values by simple hand computations. To illustrate, we have converted some of the parametric data in Figure 5 (which cover the amplitude of the vertical component of the magnetic field) into actual field amplitudes measured in picoteslas for a source frequency of 100 Hz and for dipole depths of 50, 100, and 200 m. The results are shown in Figure 7. At 100 Hz the skin depth δ is approximately 25m (assuming $\sigma = 4.0$ S/m), and the normalized amplitudes $\delta^3|B_z|$ for the dipole depths of 2, 4, and 8 skin depths must be divided by $\delta^3 \simeq (25)^3$ to obtain the actual amplitudes of B_z in picoteslas. The horizontal range of the curves in Figure 7 varies from 210 to 340 m, which corresponds approximately to the normalized range varying from 8.4 to 13.6 in Figure 5. The one minimum that can be seen in Figure 7 occurs for a dipole depth of 100 m and the minimum point is at a horizontal distance of 273.7 m from the source. For this one specific curve the ratio of the minimum amplitude to the following maximum amplitude is close to 0.116, or −18.7 dB. Note that this is the same minimum that was studied in some detail by *Fraser-Smith and Bubenik* [1976] and by *Bannister* [1984*a*, *b*]. Its location differs slightly in Bannister's analysis due to the approximate analytical expressions that were used in his computations.

The numerical results presented here can be easily extended to larger horizontal distances by using the available asymptotic expressions. These asymptotic expressions are valid under the two conditions $|\gamma\rho| \gg 1$ and $\rho \gg d$ and can be obtained by taking the first two terms of the asymptotic expressions of the modified Bessel functions of large arguments and substituting them in place of the products of the modified Bessel functions appearing in the exact expressions given by (4)-(6), yielding [*Sinha and Bhattacharya*, 1966]

$$B_\rho \simeq \frac{\mu_0 m}{2\pi} \frac{3e^{-\gamma d}}{\gamma \rho^4}$$

$$E_\phi \simeq \frac{-i\omega\mu_0 m}{2\pi} \frac{3e^{-\gamma d}}{\gamma^2 \rho^4} \tag{7}$$

$$B_z \simeq \frac{-\mu_0 m}{2\pi} \frac{9e^{-\gamma d}}{\gamma^2 \rho^5}$$

Note that the terms corresponding to the direct wave component in (4)-(6) are those that have the exponential multiplier e^{-p}, and they can be neglected in the asymptotic range because of the large exponential attenuation. The terms that correspond to the lateral wave component of the field take the forms given in (7) in the asymptotic range, and there they comprise the total field. These latter expressions are identical to the ones tabulated by *Bannister* [1984*a*, Table 1] for $\gamma_0 = 0$.

4. CONCLUSIONS

Our computations verify that there is a sharp minimum in the amplitude of the vertical magnetic field produced on the sea surface by a VMD submerged in seawater. The location and depth of the minimum vary with the depth of the VMD, but in general it is located at horizontal distances in the range of 9–14 seawater skin depths for dipole depths in the range of 2–8 skin depths. We fail to detect a significant minimum in the variation of the amplitude of the horizontal magnetic field component, but there is another sharp minimum in the variation of the amplitude of the horizontal electric field component (which also represents the total electric field in this case). The characteristics of this latter minimum also vary with the depth of the VMD, but in general it is located at horizontal distances in the range of 10–20 skin depths for dipole depths in the range of 4–23 skin depths. Clearly the minimums in the electric and magnetic field components differ both in the horizontal distances at which they occur and in their variations with dipole depth.

We have assumed a sea of infinite depth, and we have only considered field measurements on the sea surface in this work. However, we know that interference minimums are also observed above sea surface in the fields produced by submerged dipoles [*Fraser-Smith and Bubenik*, 1979] and similarly in the fields produced beneath the sea by submerged dipoles [*Bubenik and Fraser-Smith*, 1978]. It is possible that these other minimums, which are not confined to VMD sources, could be as sharp or sharper than those we have discussed here. Furthermore, it is not clear what effect a seafloor might have on the minimums, although it is known that a seafloor can sometimes create major changes in the fields produced by dipole sources [*Fraser-Smith et al.*, 1987]. (It does not appear that minimums are produced in the electromagnetic fields propagating through the seafloor from dipole sources located on the seafloor [*Fraser-Smith et al.*, 1988]). In a shallow sea it is conceivable that the sea surface minimums we have described here for a submerged VMD will be partly or wholly filled in due to a wave component propagating through the seafloor and then up to the receiver. Under such conditions, measurements on the field amplitudes in the minimums may provide information about the effective electrical conductivity of the seafloor.

For some time it has been evident that more experiments to measure the electromagnetic fields produced in, on, and above the sea by harmonic dipole sources are desirable, since there is now a large body of theory that has little experimental backing [*Fraser-Smith et al.*, 1987]. Sharp minimums such as those described in this paper could be particularly useful in such studies, and in other experimental work, because they provide an unambiguous and comparatively precise point of reference that can greatly enhance the accuracy of the measurements.

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