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Testing a model for the well-posedness of the Cauchy-characteristic problem in Bondi coordinates

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Testing a model for the well-posedness of the Cauchy-characteristic problem

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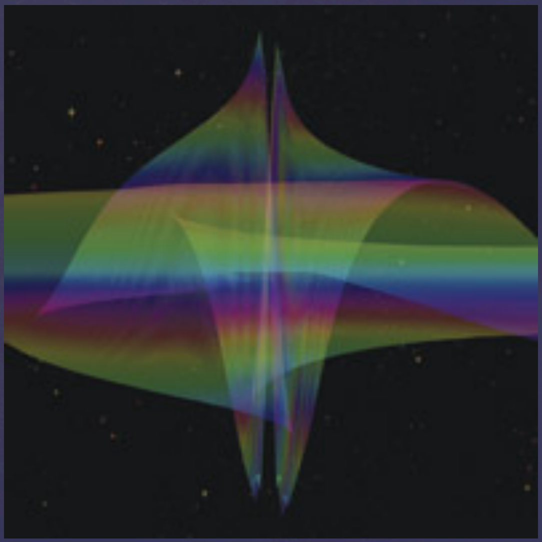
15th Annual East Coast Gravity Meeting and Josh Goldberg Fest
Syracuse University, Syracuse, NY, April 20-22, 2012



- What can we see with gravitational waves:
  - Colliding black holes and galaxies,
  - The birth of a black hole in a supernova
  - The growth pains of our universe
- Gravitational waves are unambiguous measured only at future null infinity

Background

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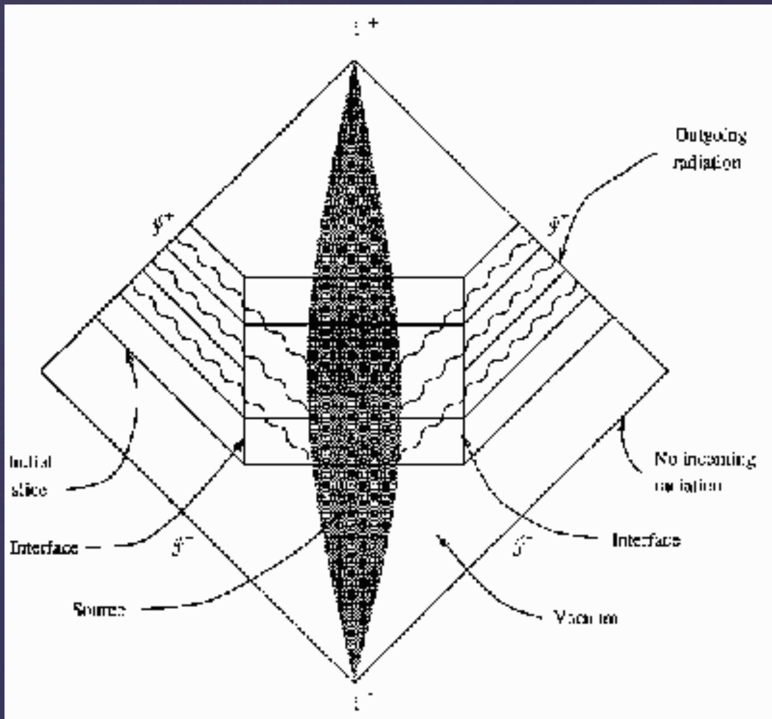
Formulation



- Cauchy-characteristic method covers all spacetime by combining 2 regions
  1. A timelike (Cauchy) close to BBH
  2. A null (characteristic) far field.
- Cauchy-characteristic initial-value
- Outward radial evolution
- Compactified radial coordinate
- Accurate gravitational radiation.

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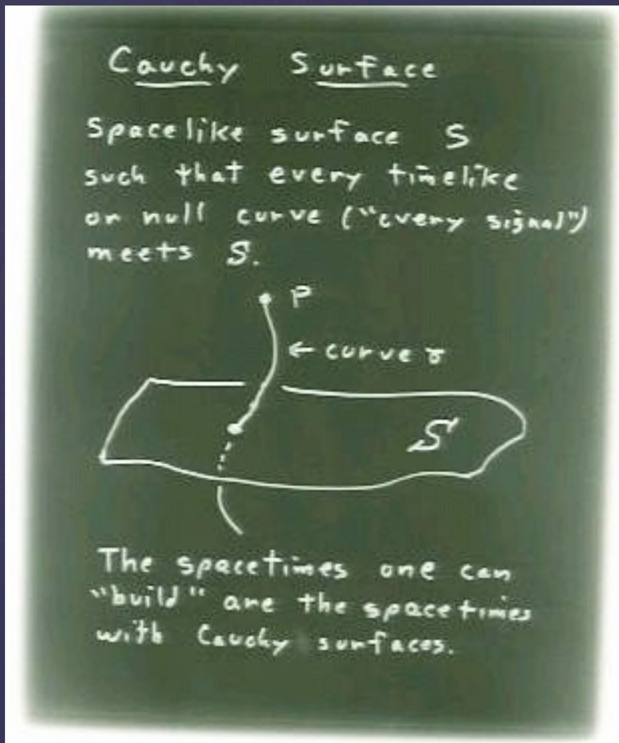
Question



- Is the null-timelike problem well-posed?
- "As a general rule, it is considerably more difficult in the null case to write down formulae which say what one wants to say."
  R. Geroch, Asymptotic Structure of Spacetime
- H-O Kreiss and J. Winicour, "The well-posedness of the null-timelike problem for quasilinear wave", CQG 28, 2011

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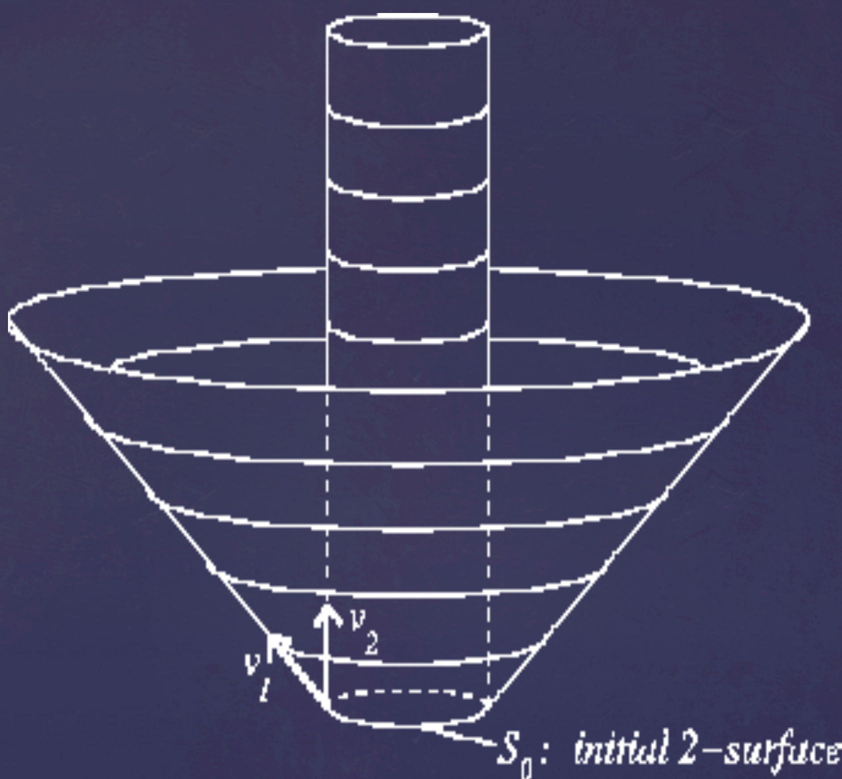

- Timelike & null initial boundary
- Split the problem into:
  - Cauchy problem
  - Half-plane (strip) problem
- Show that each individual problem is well posed.
- Analyze stability against lower order perturbations.

Approach

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- **Real thing:** solve Einstein Equation of general relativity in Bondi-Sachs metric coordinates and calculate the gravitational waves.
- **Model:** solve the quasilinear wave equation in null-timelike compactified coordinates, on an asymptotically flat background with source, gived data on the timelike and initial boundary.

$$g^{ab}\nabla_a\nabla_b\Phi = S(\Phi, \partial_c\Phi, x^c)$$

Description

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Method

- Change of variables: $\Phi = e^{ax}\Psi,\ a > 0$
- 1+1 wave equation in characteristic coordinates:

$$t = \tilde{t} - \tilde{x}, x = \tilde{t} + \tilde{x}$$

$$\partial_t \partial_x \Phi = S -> \partial_t(\partial_x + a)\Psi = F, F = e^{-ax}S, \Psi(0,x) = e^{-ax}f(x)$$

- Energy estimates weighted norm is well-posed:

$$E = \frac{1}{2}\int dx e^{-2ax}\left((\partial_x\Phi)^2 + a^2\Phi^2\right)$$

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- Discretization of the wave equation:

$$\partial_t \left(D_{0x} U + aU \right) = LOT + S(t,x,y)$$

$$U_0 = F(x,y),\ (x,y) \in [0,2\pi), U(x,y) = U(x+2\pi, y+2\pi)$$

- Discrete Fourier Transform:

$$U(t,x,y) = \frac{1}{N} \sum_0^{N-1} \sum_0^{N-1} \hat{U}(t,\omega_1,\omega_2) e^{i(\omega_1 x + \omega_2 y)},\ \hat{U}(t,\omega_1,\omega_2) = \frac{1}{N} \sum_0^{N-1} \sum_0^{N-1} U(t,x,y) e^{-i(\omega_1 x + \omega_2 y)}$$

- Equation to evolve:

$$\partial_t \hat{U} = p \cdot \hat{U} + \hat{S} / d$$

Algorithm

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Stability

- When *Re(p)<0,* the solution decays exponentially fast
- When *Re(p)>0,* the solution grows exponentially fast
- When *Im(p)≠0,* the solution has oscillatory growing modes
- Even for *Re(p)<0,* time integration stability

$$\mathrm{Re}(p) = -\frac{a\omega_{21}^2 + 2b\omega_{10}\omega_{20} + \ldots}{a^2 + \omega_{10}^2}, \quad \mathrm{Im}(p) = -\frac{-\omega_{10}\omega_{21}^2 + 2ab\omega_{20} + \ldots}{a^2 + \omega_{10}^2}$$

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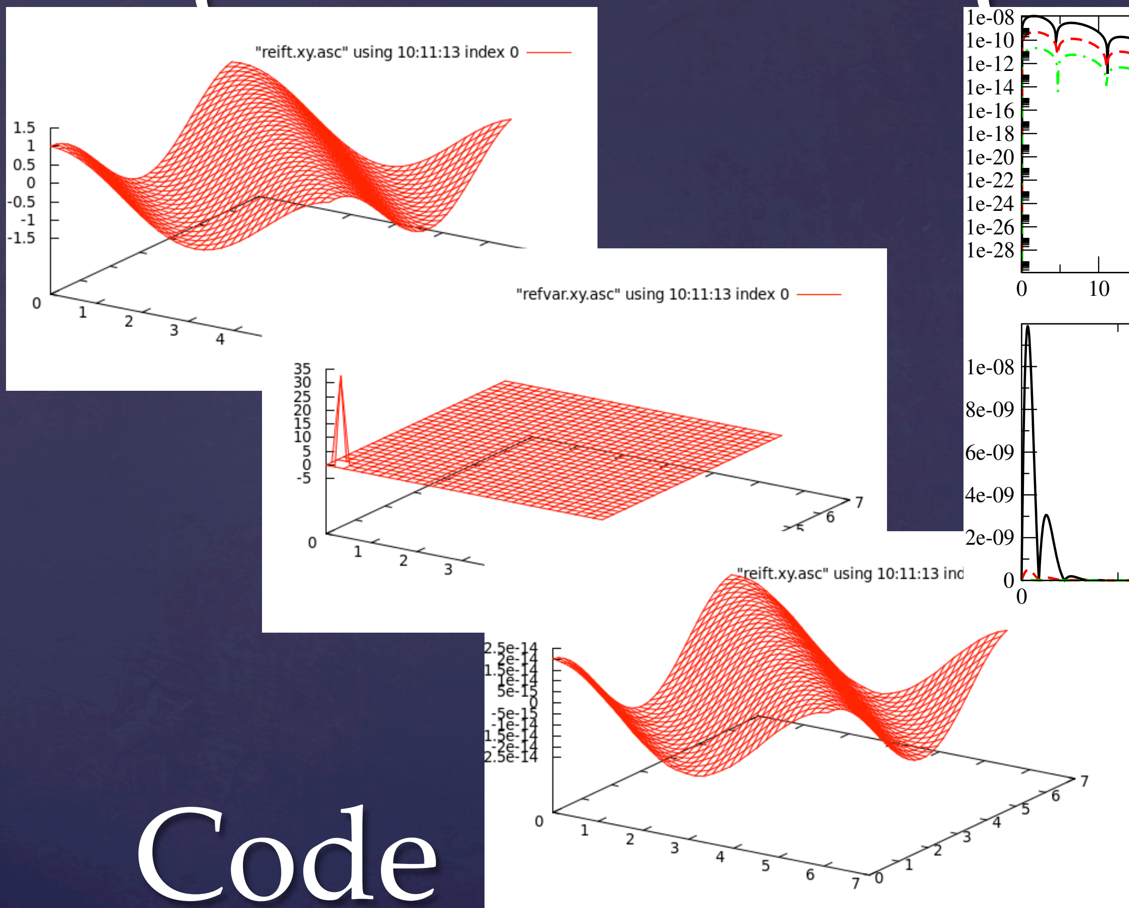
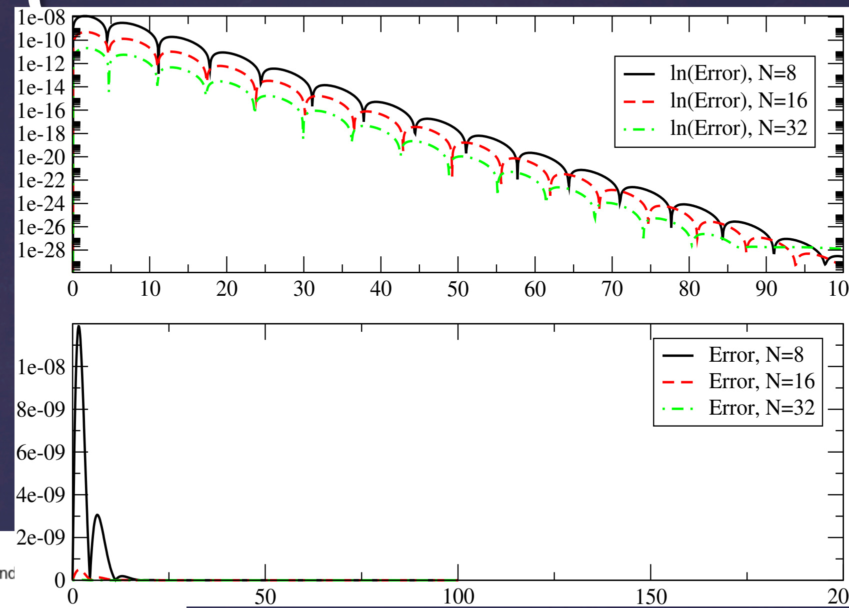
- Evolved Variable



- Error



Code

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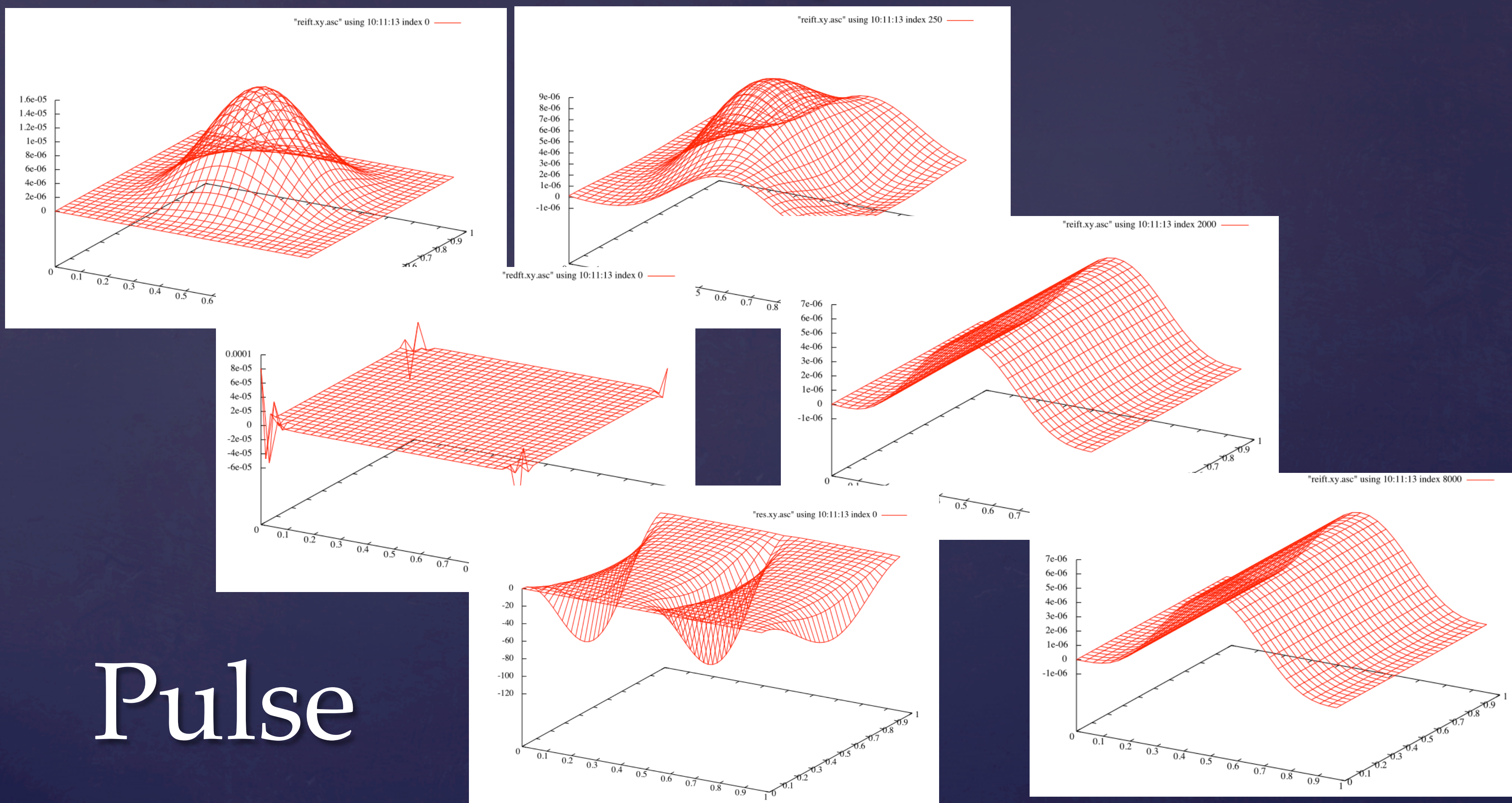
Start Variable

Evolved Variable



Pulse

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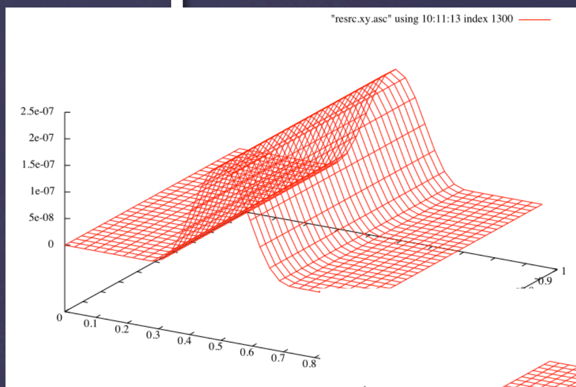
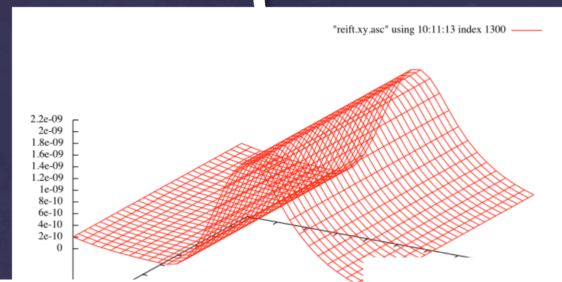
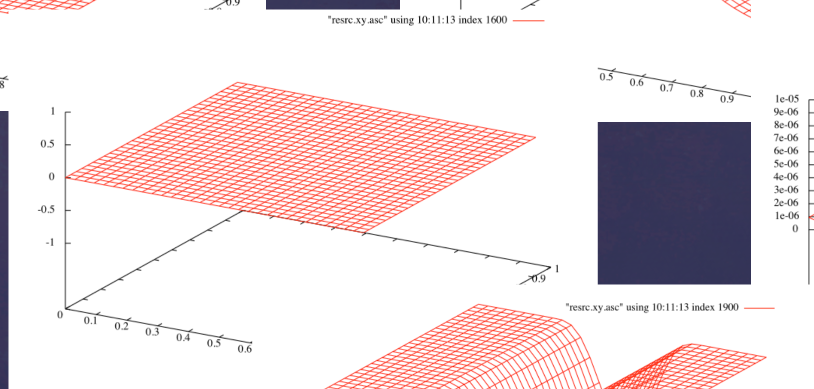
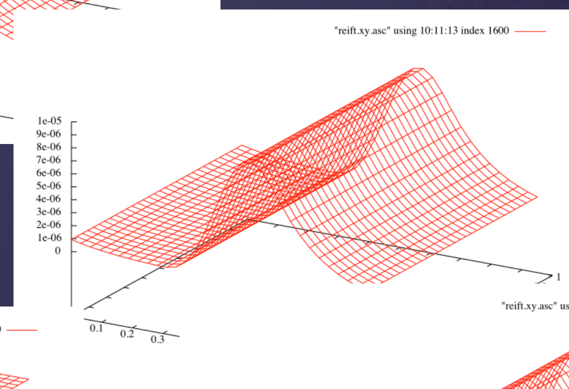
Source

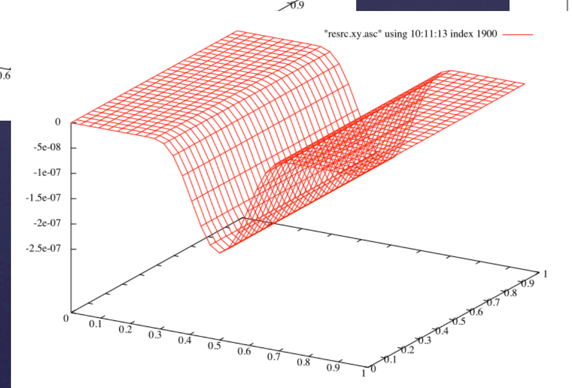
Solution

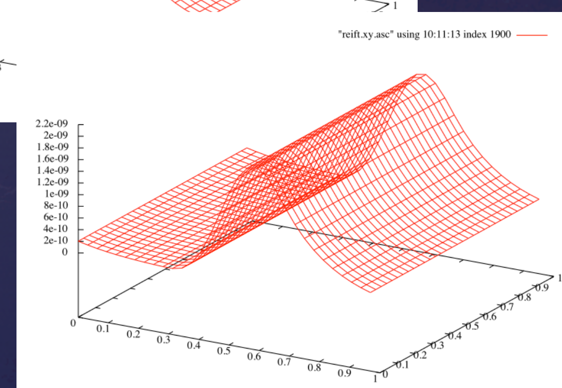












Source

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Checklist

- **Goal:** prove well-posedness of quasilinear equation in the null-characteristic domain
- ✓ Prove well-posedness of Cauchy problem
- ✓ Study the effect of lower order perturbations
- ➢ Apply boundary on the initial characteristics
- ➢ Restrict the problem to timelike-null domain
- ➢ Extend the analysis to the quasilinear wave

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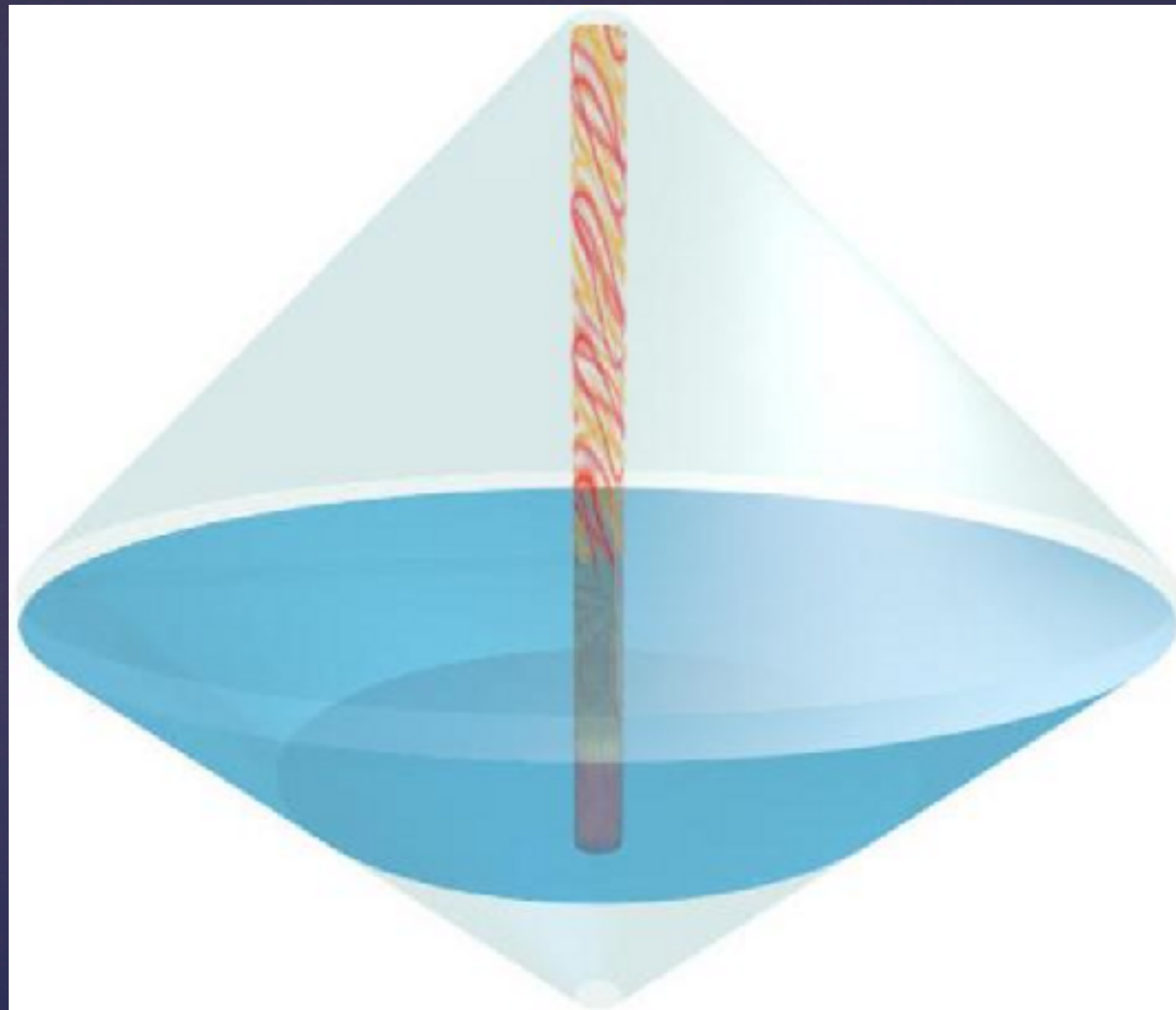

"Conformal Infinity", Jorg Frauendiener, 2004

Thank You

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