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A New Algorithm for the Numerical Computation of Gravitational Waves

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* A New Algorithm for the Numerical Computation of Gravitational Waves

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*Introduction

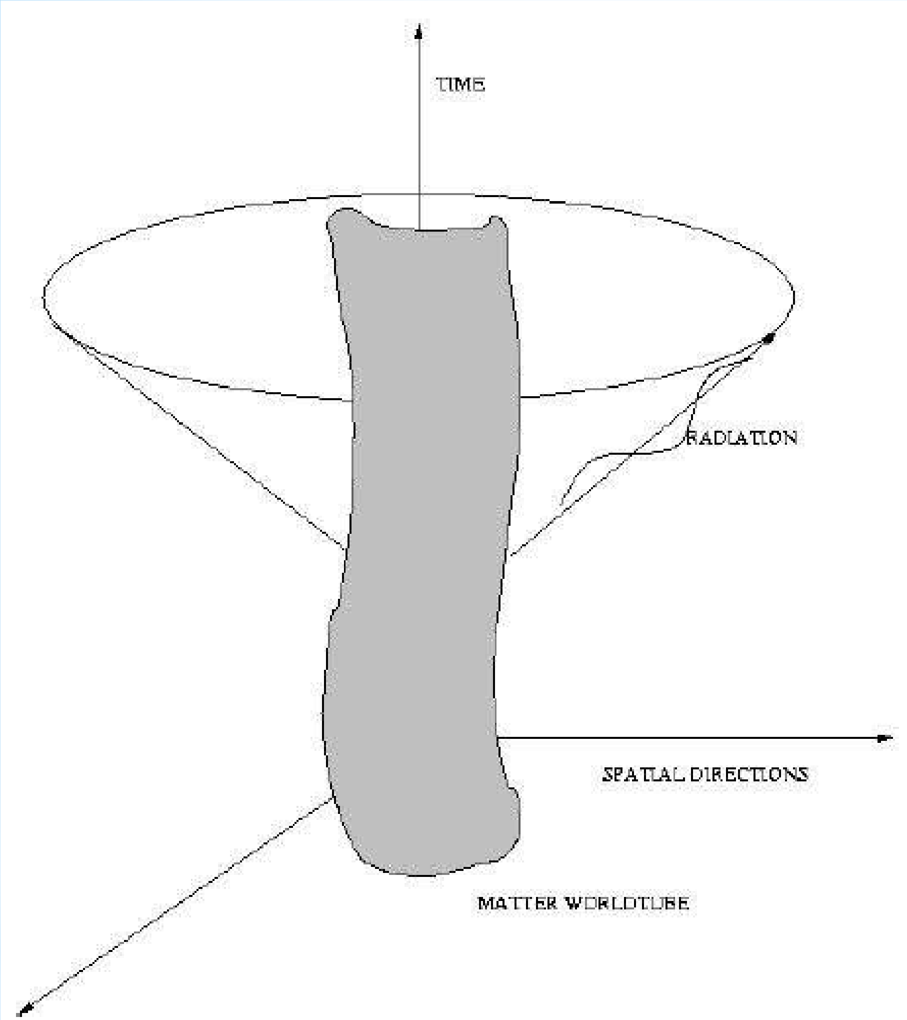
Image of black hole merger. Data from the Chandra X-Ray Observatory and the Hubble Space telescope

* Gravitational Wave Astronomy can "see" with gravitational waves:
  * Colliding black holes and galaxies,
  * The birth of a black hole in a supernova
  * The growth pains of our universe
  * The structure of spacetime

*Challenges

Image of a tumbleweed.
Endangering LIGO

* Gravitational wave detectors are looking for perturbations ~0.001x the width of a proton
* Accurate gravitational waveforms are essential!
* The calculation of gravitational waves is tough!
* General relativity defines properly gravitational radiation only at future null infinity, but mathematically it is estimated at a finite radius.

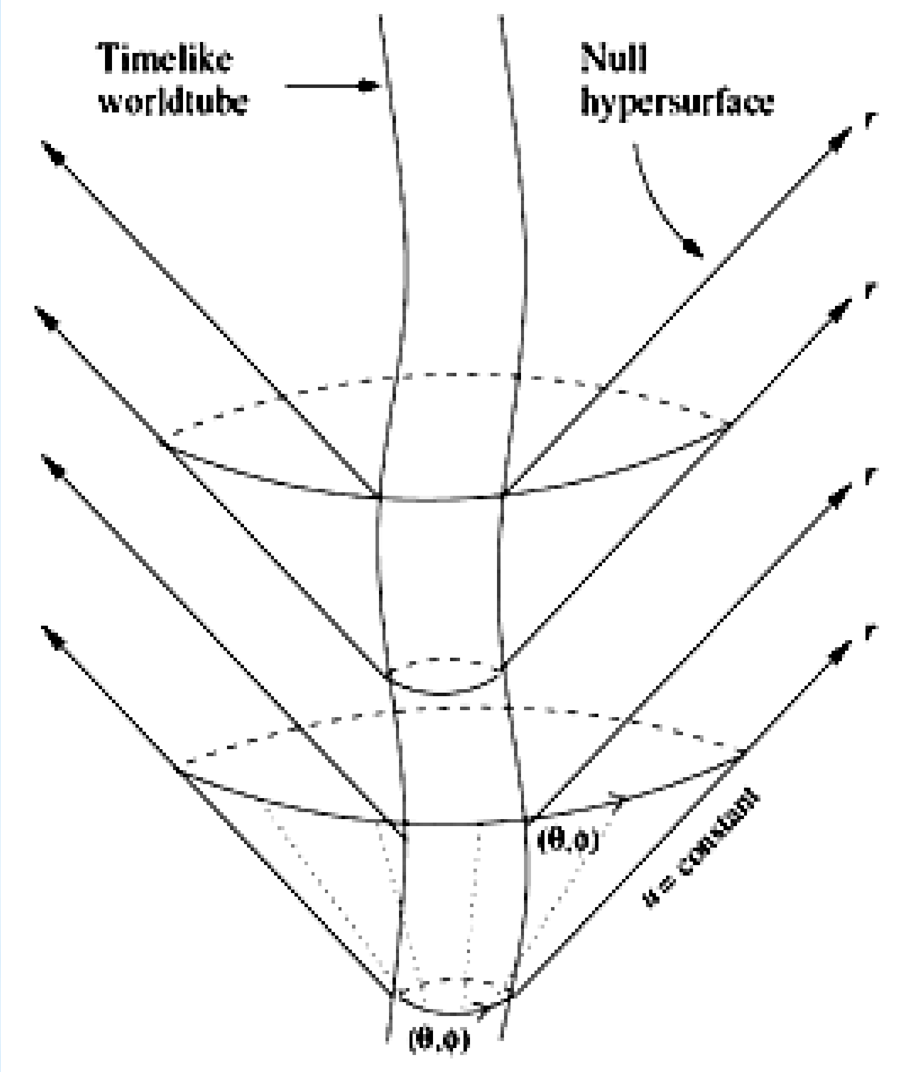


* Our Cauchy-characteristic method satisfies the detection criteria required by Advanced LIGO.
* Covers all the space & time
  1. Close to the merger - Cauchy region
  2. Towards infinity - characteristic
* Compactified radial coordinate extends the grid to null infinity

$$r \to x = r/(r+R_E)$$

* No artificial outer boundary
* Accurate computation of the signal

*Highlights



* Boundary data
  1. Free initial characteristic data
  2. Time dependent Cauchy data
* Evolution propagates the field:
  1. Time integration
  2. Radial Marching
* Waveform computation
  * Conformal expansion at infinity
  * Computation of curvature

*Description

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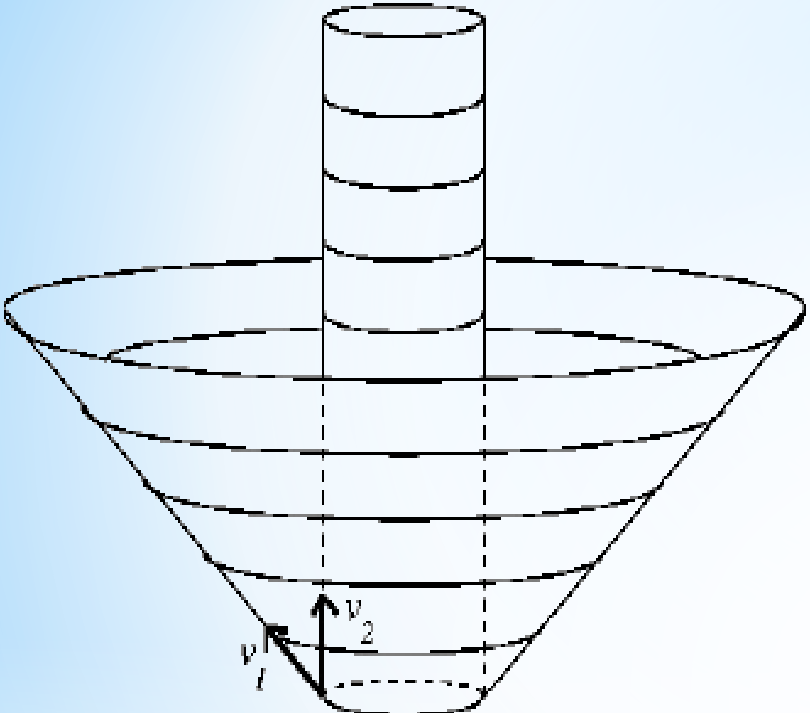

* The spacetimes one can “build” are the spacetimes with Cauchy surfaces
* Find the unique solution which depends continuously upon the initial data.
* Is a null-timelike boundary well-posed?
* Split the problem in two:
  1. Whole-space problem -periodic
  2. Half-space problem -time boundary
* Analyze each individual problem

*Problems

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* Modified Wave Equation

$$g^{ab}\nabla_a\nabla_b\Phi = S(\Phi, \partial_c\Phi, x^c)$$

$$(t = \tilde{t} - \tilde{x},\ x = \tilde{t} + \tilde{x})$$

$$\Phi(t,x) \rightarrow = \Phi(t,x) + g(t)$$

$$\Phi = e^{ax}\Psi,\ a > 0$$

$$\Phi = e^{st+i(\omega_1 x + \omega_2 y)}$$

* The wave equation in characteristic coordinates *(t, x)* allows solution freedom independent of initial data
* Method – introduce the a-term
* Analyze stability against perturbations
* Prove stability against lower-order terms analyzing the Fourier modes
* Conclude well-posedness for a>0

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$$2(\Psi_x + a\Psi)_t = \left((1-x)^2\Psi_x\right)_x + \Psi_{yy} + b\left(\left((1-x)^2\Psi_x\right)_y + \left((1-x)^2\Psi_y\right)_x\right) + F(t,x,y)$$

$$\Psi(0,x,y) = f(x,y), \quad -\infty < x, y < \infty\,, \quad t \geq 0,$$

* For a ≤ 0, the problem is ill-posed. Numerical instability is evident and the runs quickly crash.
* For a = 1 and |b| < a, there are no growing modes.
* For |b| > a, there are exponentially growing modes but the runs are numerically stable and convergent.
* In all simulations for a > 0, the wave remains smooth and there is no sign of numerical instability.

*Whole-space Simulations

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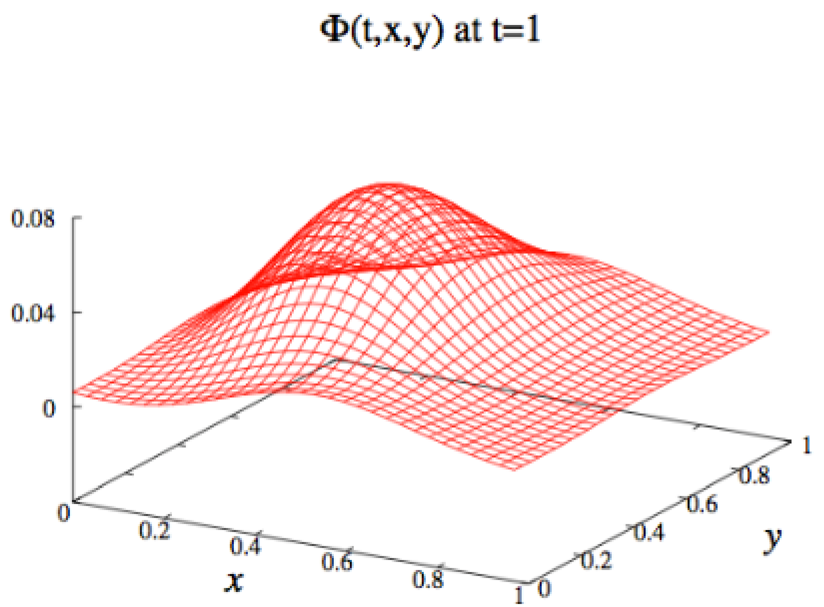
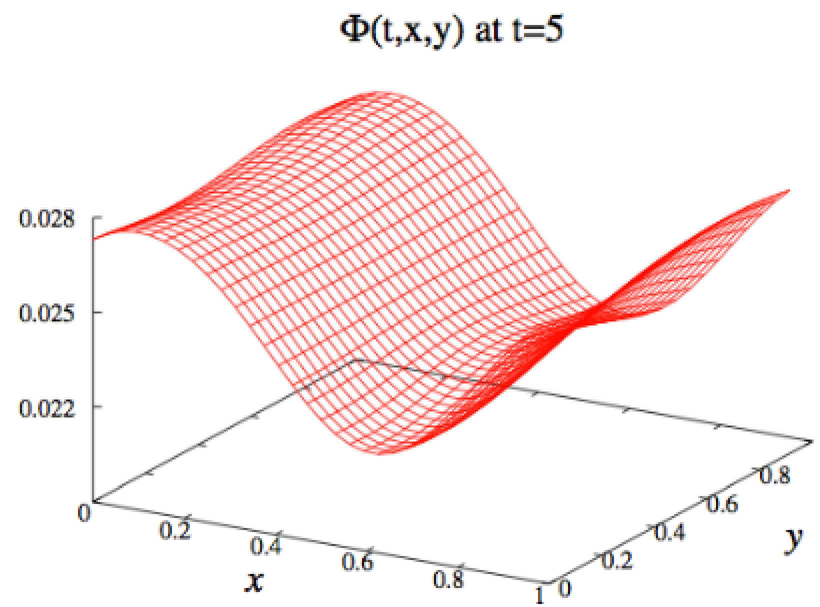
Early time behavior for b = .1 and a = 1 with initial data and no source.

* at t = 1 (left plot) the wave has undergone little change
* at t = 5 (right plot) it begins to homogenize in the y-direction.





*Whole-space Simulations

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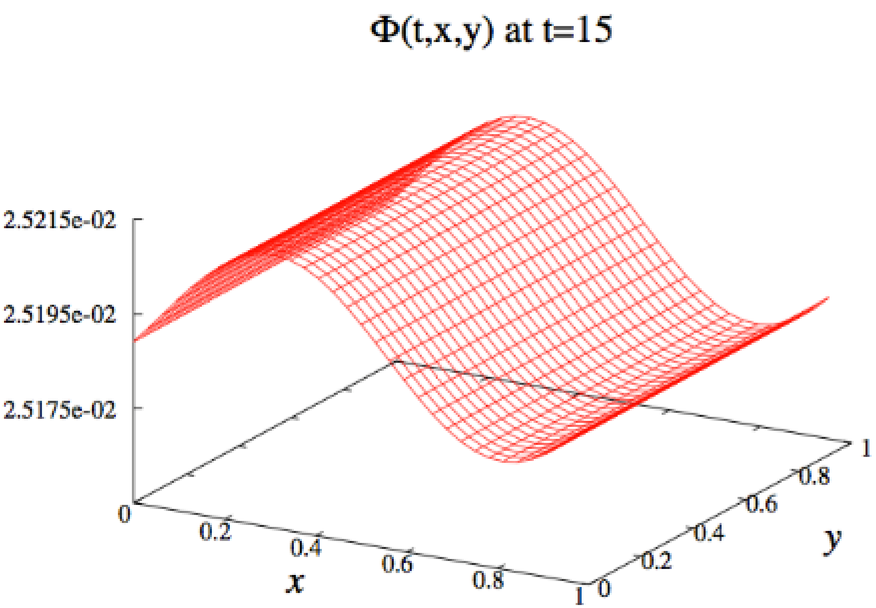
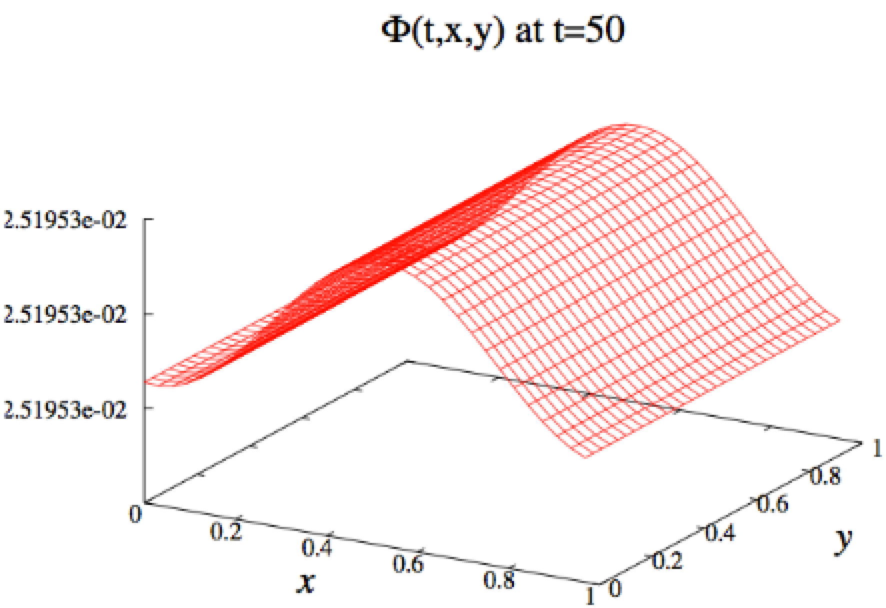
*Whole-space simulations

Late time behavior for b = .1 and a = 1 with initial data and no source.

* at t = 15 (left plot) the wave has become uniform in y direction
* at t = 50 (right plot) the traveling wave decayed towards a constant solution





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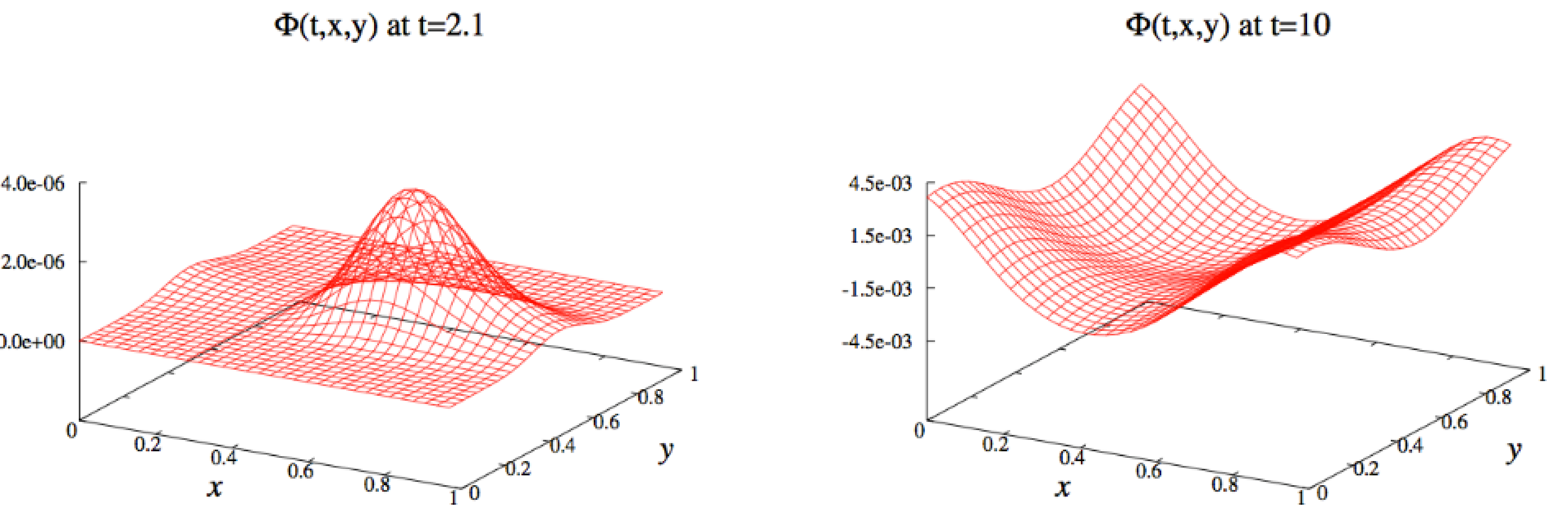
Early time behavior for b = .1 and a = 1 with source and no initial data.

* at t = 2.1 (left plot) the signal "instantaneously" propagates along the x (null) line
* at t = 10 (right plot) the wave begins to homogenize in the y-direction.



Shining a ray in the forward direction immediately gets illuminated from behind

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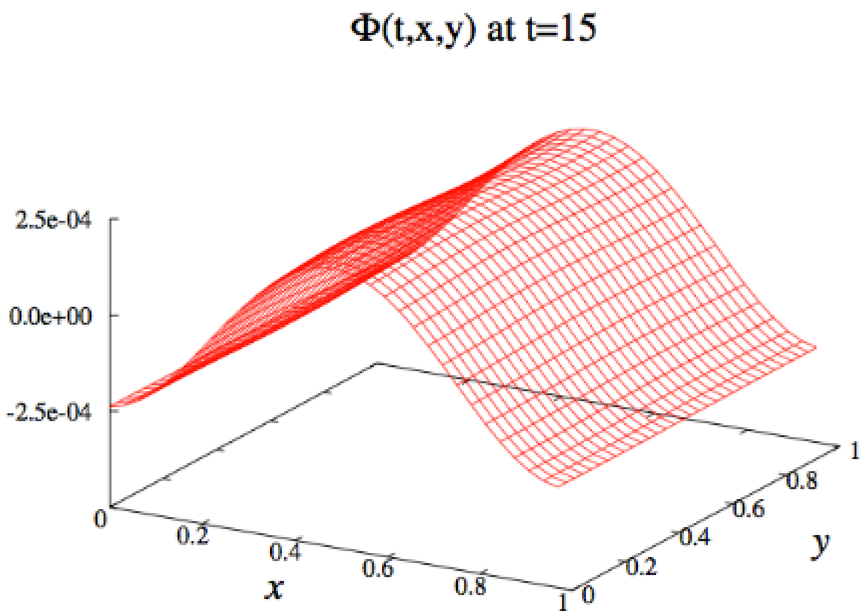
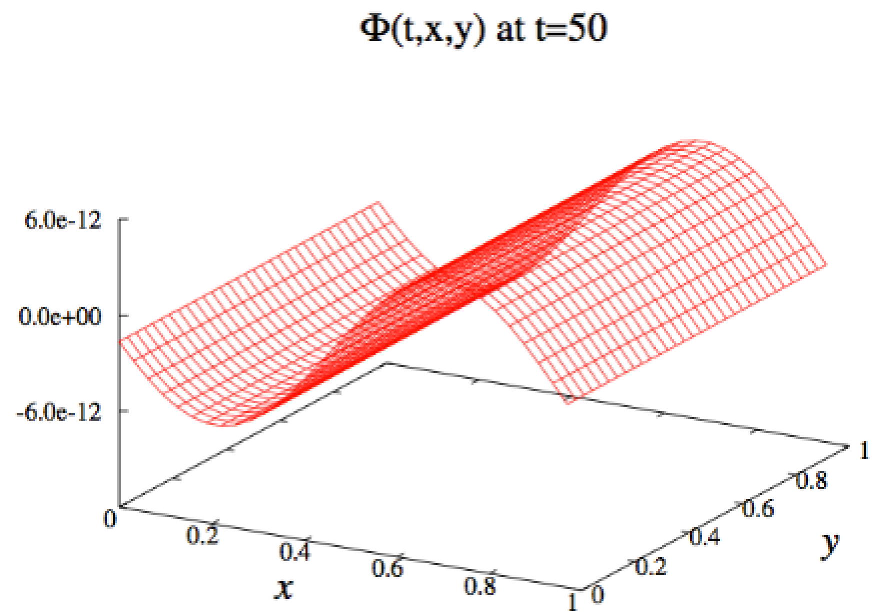
Late time behavior for b = .1 and a = 1 with initial data and no source.

* at t = 15 (left plot) the wave has become uniform in y direction
* at t = 50 (right plot) the traveling wave decayed continuoulsy





*Whole-space simulations

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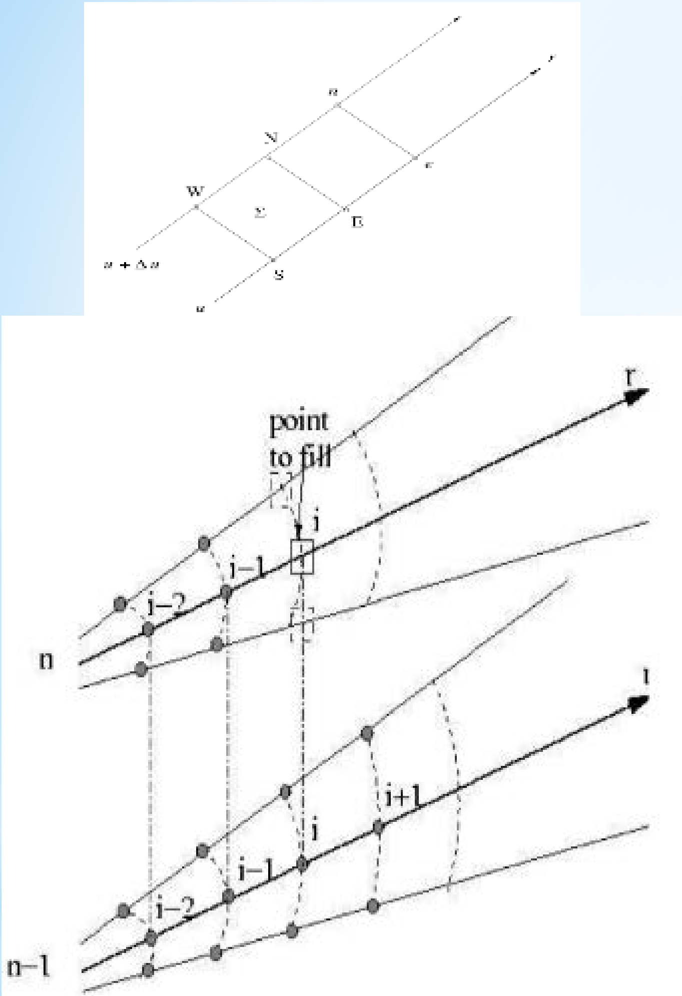
Half-space Algorithm



* Modified marching based on the parallelogram rule
* The only stable algorithm, 2-levels
  1. Time update to the present level
  2. Step-up to infinity in r direction
* Instabilities can arise from lower order terms
* Cured by adding angular dissipation

$$\epsilon h^3 \partial_y^4 (\lambda \partial_t - \partial_x)\Phi, \quad \epsilon = const > 0.$$

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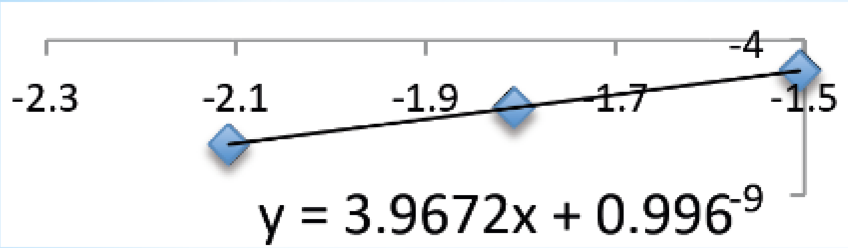
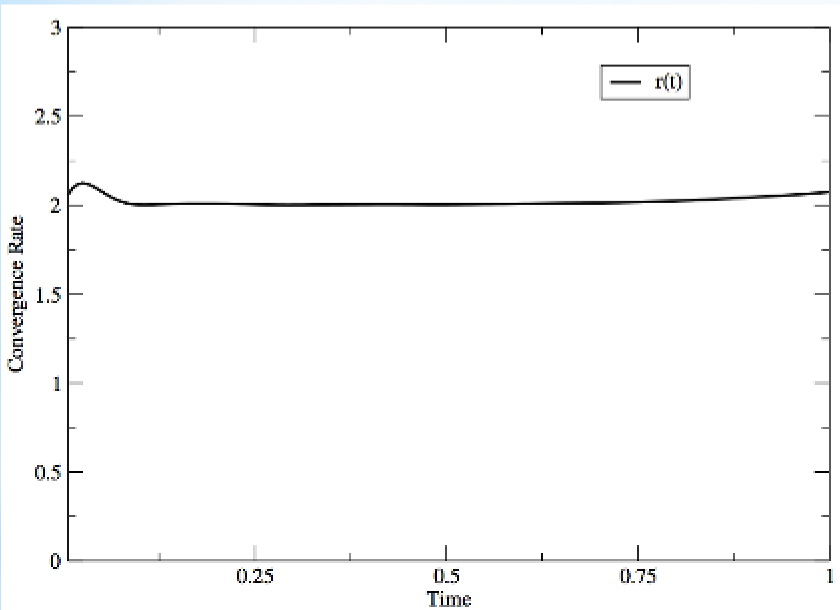
*Convergence

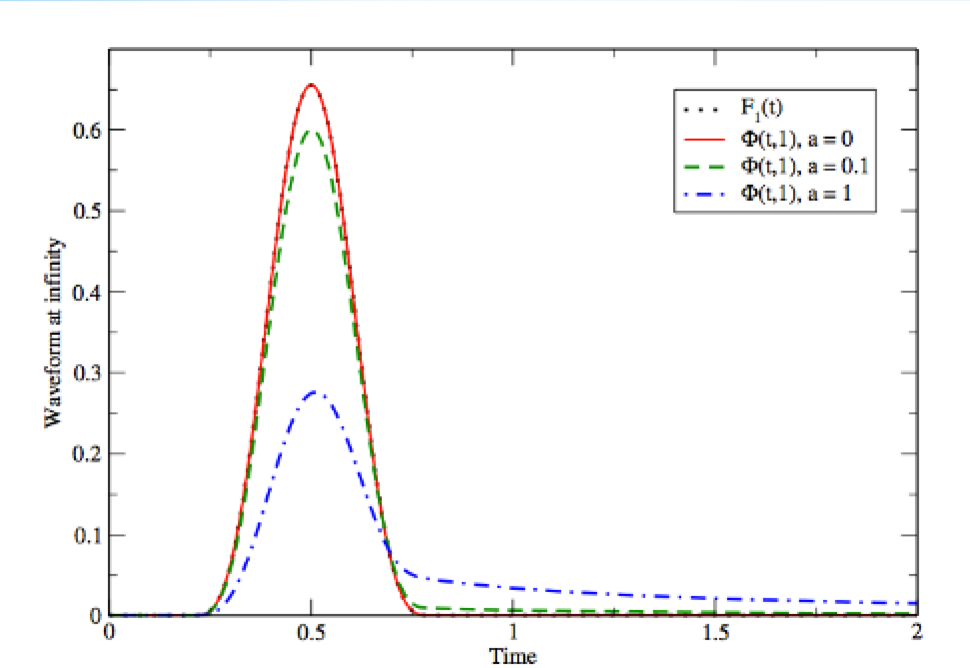


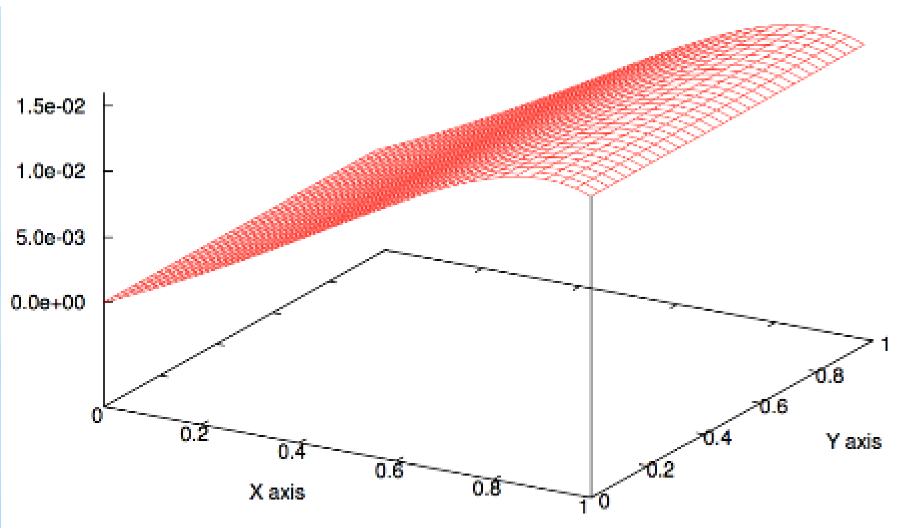


* The algorithm is accurate
* Relative error 10^{-3}% for N= 64x64

$$\Phi = e^{st}\phi(x)\cos(\omega y), \quad s = -\frac{\omega^2}{a}$$

* Local convergence 4^{th} order
* Global convergence 2^{nd} order

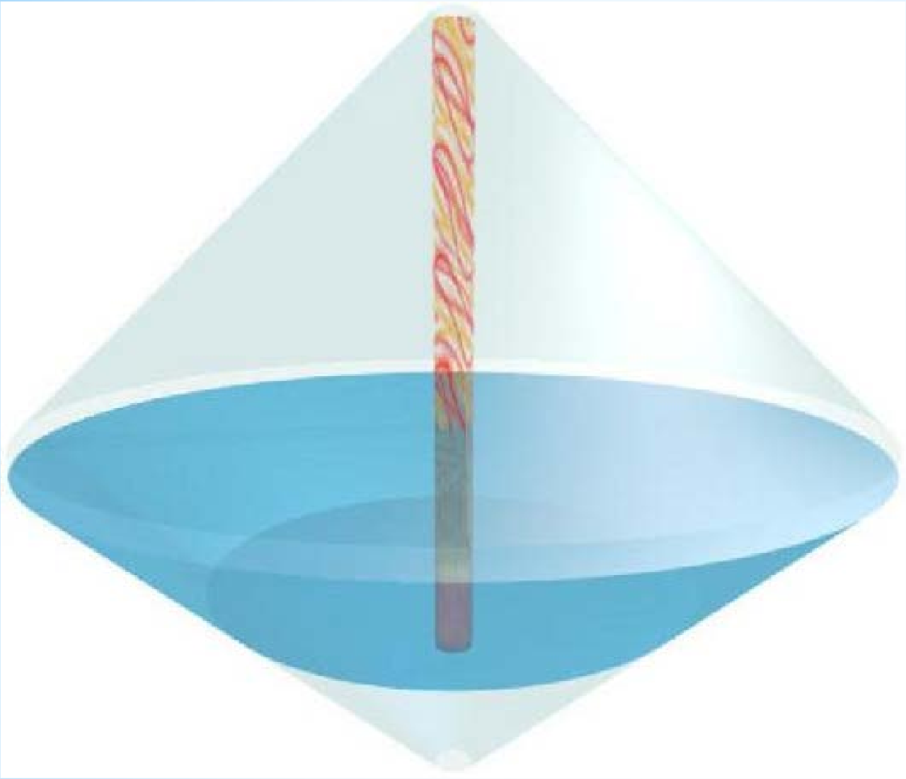




* The a-term produces
  1. A damping of the outgoing waveform due to the a-term
  2. A tail to the waveform which decays in time
  3. A distinctively almost horizontal slope near the outer boundary (infinity)

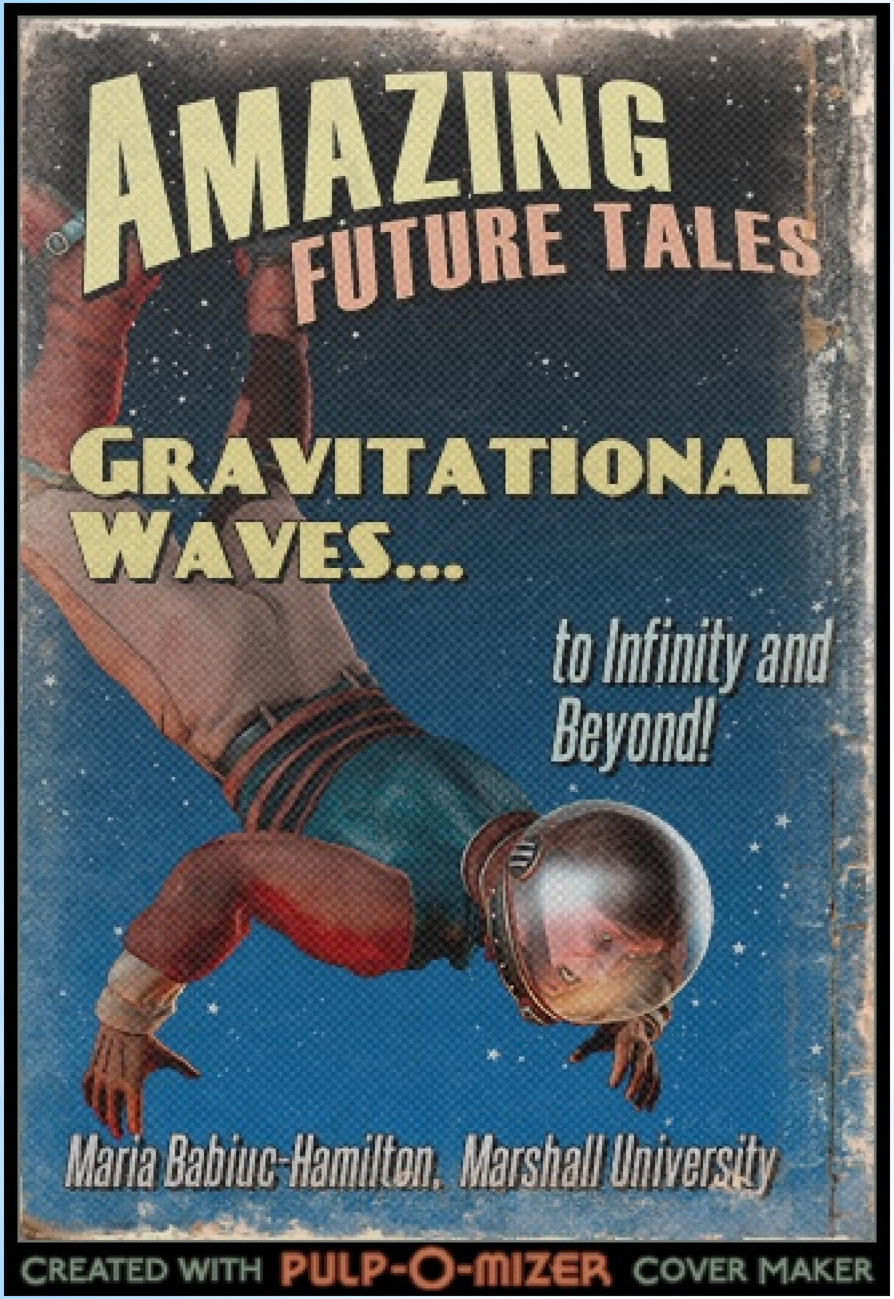
*The a-term

*Marching to Infinity



Conformal Infinity

* The well-posedness of the null-timelike problem for the Einstein equations has not yet been established.
* This proof of well-posedness of the corresponding problem for the quasilinear wave equation is a first step toward treating the gravitational case



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*Thank you