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A Hyperbolic Solver for Black Hole Initial Data in Numerical Relativity

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Initial Data in Numerical Relativity

The constraints are formulated as elliptic equations

- The Conformal Method
- The Conformal Thin Sandwich
- The Gluing Technique

The constraints are formulated as parabolic equations

- The quasi-spherical method

The constraints are formulated as strongly hyperbolic equations

- The strongly hyperbolic method

A Different Approach to Initial Data for Black Holes

The Strongly Hyperbolic Method

- The 3D surfaces are foliated into 2D surfaces with unit normal $\hat{n}_i$
- The momentum constraints form a system of first-order PDEs:

$$\mathcal{L}_{\hat{n}}\mathbf{K} - \hat{D}^l\mathbf{k}_l + F_{\mathbf{K}} = 0$$
$$\mathcal{L}_{\hat{n}}\mathbf{k}_i + \mathbf{K}^{-1}(\boldsymbol{\kappa}\hat{D}_i\mathbf{K} - 2\mathbf{k}^l\hat{D}_i\mathbf{k}_l) + F_{\mathbf{k_i}} = 0$$

This system stays strongly hyperbolic if: $\mathbf{K}\boldsymbol{\kappa} < \mathbf{0}$

- The Hamiltonian constraint reduces to an algebraic equation:

$$\boldsymbol{\kappa} = (2\mathbf{K})^{-1}\left(2\mathbf{k}^l\mathbf{k}_l - \frac{1}{2}\mathbf{K}^2 - \boldsymbol{\kappa}_0\right)$$

The extrinsic curvature is regained from the constrained variables $\mathbf{K}, \mathbf{k}_j, \boldsymbol{\kappa}$

$$K_{ij} = \boldsymbol{\kappa}\hat{n}_i\hat{n}_j + 2\hat{n}_{(i}\mathbf{k}_{j)} + \mathbf{K_{ij}}.$$

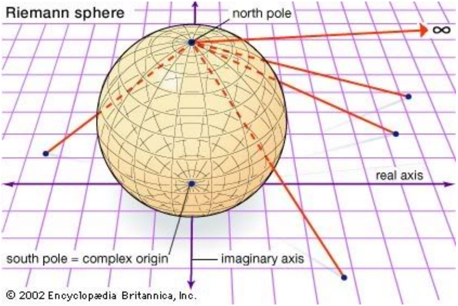
Required Initial Data

Calculated from the 4D metric in Kerr-Schild form

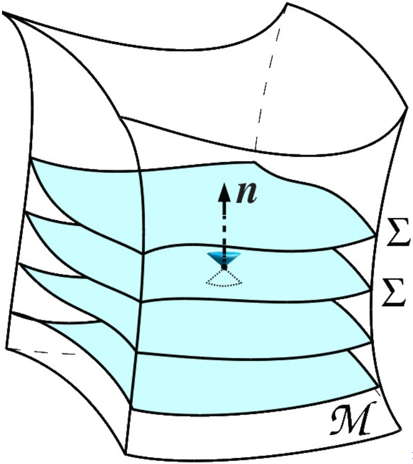
1. Initial data given on the whole 3D surface
   - The 2D metric, lapse and shift and their radial derivatives
   - The trace-free part of the 3D extrinsic curvature
2. Extra initial data given only on the initial 2D foliation
   - The 3D extrinsic curvature
   - The 3D intrinsic curvature scalar
3. The choice of 2D foliation
   - Sphere
   - Oblate Spheorid

2D Sphere + 1D Radius

Stereographic Projection = conformal (angle-preserving) map of the sphere onto the topologically equivalent complex plane



The hyperbolic equations are evolved radially inward by integrating from r_{max} to r_{min}



The 2D+1 Computational Grid

- The unit sphere metric is:

$$q_{ab} = q_{(a}\,\bar{q}_{b)} = \Omega^2\,\delta_{ab}$$

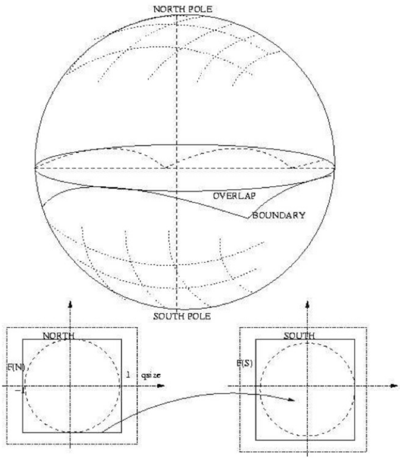
- The complex dyad is

$$q_a = \Omega(d\zeta)_a,\ \zeta = e^{-i\phi}\cot\tfrac{1}{2}\theta$$

- The angular derivatives become Newton-Penrose operators ð, $\bar{ð}$

All the variables are projected onto the compex dyad (arXiv:160105386)

The PittNull Stereographic Module
Finite difference algorithm for the ð, $\bar{ð}$ operators



The Numerical Algorithm

Startup

- Read in the 4D metric and in cartesian coordinates
- Decide on the best foliation by a radial direction ρ
- Calculate all the necessary quantities
- Interpolate the quantities onto the sphere (oblate spheroid)
- Transform from cartesian to stereographic coordinates

Solver

- Calculate the spin-weighted fields
- Calculate the principal part and the forcing terms
- Proceed with the radial integration

Recovery

- Calculate the extrinsic curvature from the evolved variables

The Radial Integration

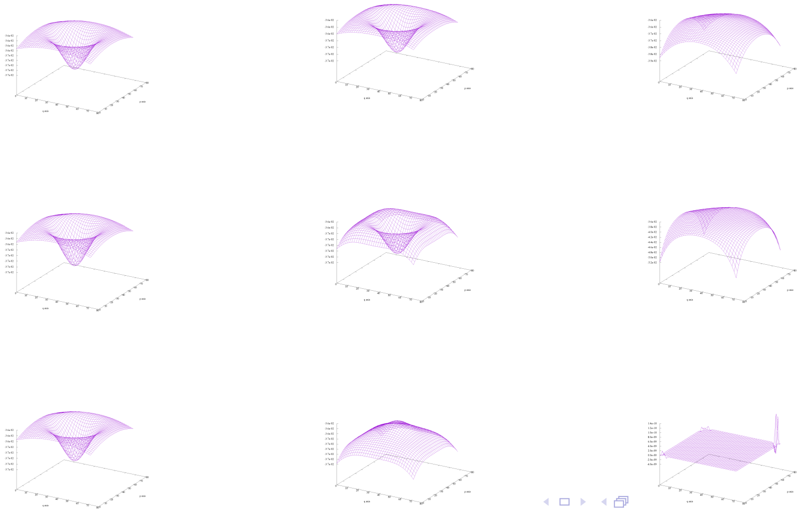
Homemade Runge-Kutta 2^{nd} and 4^{th} order integrator

- Flexible, can be stopped at singularities and started again
- The algebraic equation for κ is updated at intermediary steps
- Mask and N/S interpolation of the *RHS* before each integration step
- Dissipation of the evolved variables after each integration step
  $\partial_r \rightarrow \partial_r + \epsilon\eth\bar{\eth}$ or $\partial_r \rightarrow \partial_r - \epsilon^4\eth^2\bar{\eth}^2$
- Incorporates internally the radial dependence of the integration step by changing the integration variable

$$dr \leq r\Omega d\zeta \rightarrow ds = \frac{dr}{r} \propto d\zeta \; and \; \partial_r \rightarrow e^{-s}\partial_s \; , s = \ln r$$

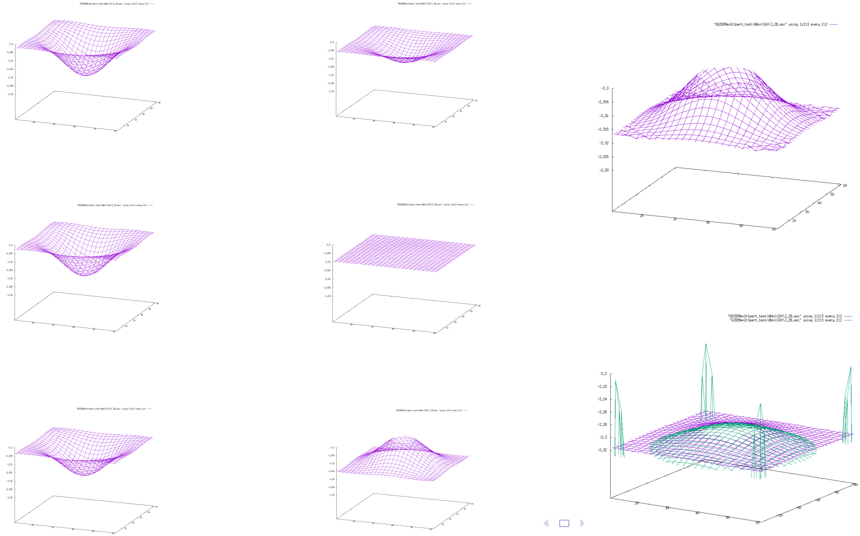
Variable **K** for perturbed Schwarzchild

$r_{max} = 10M$, $dr = 0.01$, $d\zeta = 0.03$

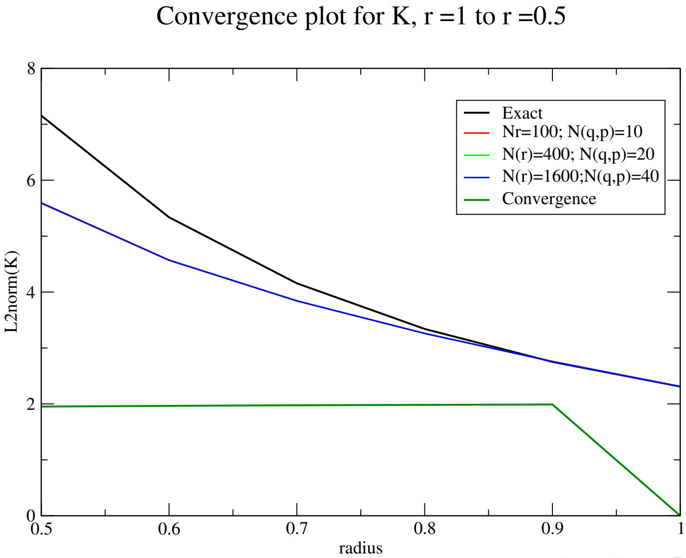


Variable **K** for perturbed Schwarzchild

$r_{max} = 1M$, $dr = 0.01$, $d\zeta = 0.03$



Exact Schwarzchild, $r_{max} = 1M$, $dr \leq 1/4 d\zeta^2$



Future Work

Perfect the integration grid such that $dr \to 0$ as $r \to 0$

Implement and test the 4^{th} order integrator

Fine tune the dissipation and the interpolation

Implement new test cases and test the code

Implement and test the recovery routines

Release the code for comparison with other ID approaches