20th GR and 10th Amaldi Conference on Gravitational Waves, 7-13 July 2013, Warsaw, Poland

WELL-POSEDNESS OF CHARACTERISTIC EVOLUTION IN BONDI COORDINATES

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Abstract

❖Gravitational waves carry information about their source, and their detection will uncover facets of our universe, otherwise invisible.

❖Recently, we made publicly available a waveform computation tool, the ***PITT code***, as part of the Einstein Toolkit open software for relativistic astrophysics. The code implements the "characteristic method," which computes the gravitational waves infinitely far from their source in terms of compactified light cones.

❖We proved that our code produces waveforms that satisfy the demands of next generation detectors. However, the main problem is that the well-posedness of the Einstein equations in characteristic formulation is not proven.

Here we present our progress towards developing and testing a new computational evolution algorithm based on the well-posedness of the characteristic evolution.

- We analyze the well-posedness of the problem for quasi-linear scalar waves propagating on an asymptotically flat curved space background with source, in null Bondi-Sachs coordinates.
- We design a new numerical boundary and evolution algorithm, and proved that is stable both numerically and analytically.
- We built and run numerical tests to confirm the well-posedness and stability properties of the new algorithm.

The knowledge gained from the model problems considered here should be of benefit to a better understanding of the gravitational case. A new characteristic code based upon well-posedness would be of great value.

$$g^{ab}\nabla_a\nabla_b\Psi = S(\Phi,\partial_c\Psi,x^c), r -> x = r/(R+r)$$

$$g_{ab}dx^a dx^b = -(e^{2\beta}W - r^{-2}h_{AB}W^AW^B)dt^2$$
$$-2e^{2\beta}dtdr - 2h_{AB}W^B dtdx^A + r^{-2}h_{AB}dx^A dx^B$$

The simplified model wave equation in characteristic coordinates: ($t \Longrightarrow t-x$, $x \Longrightarrow t+x$) is:

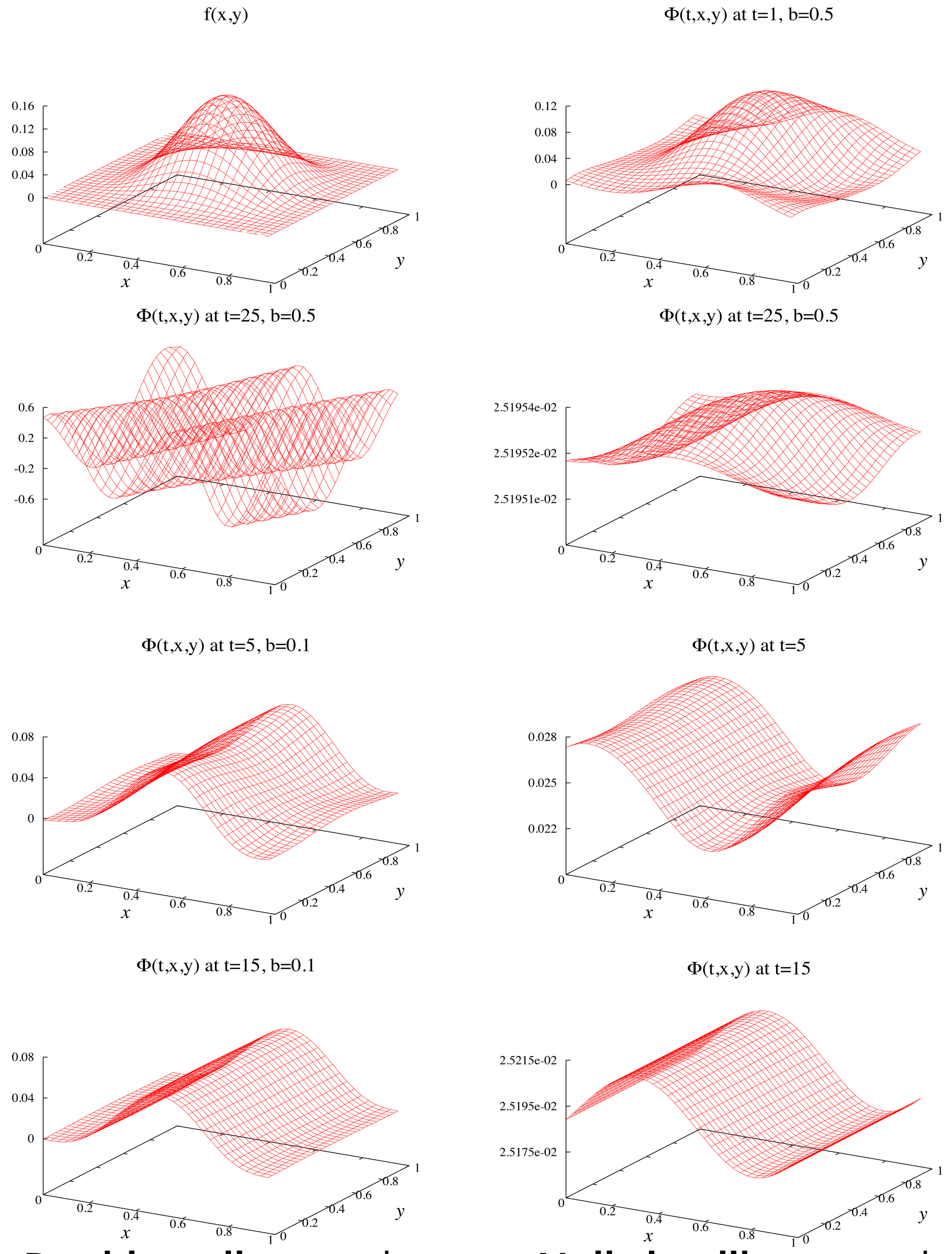
$$\Psi = e^{ax}\Phi, a>0 \Rightarrow \partial_t(\partial_x\Phi + a\Phi) =$$
$$\partial_x((1-x)^2\partial_x\Phi) + \partial_y^2\Phi - 2b\partial_y\Phi + S$$

We developed computational evolution algorithms and proved the numerical stability by the Fourier-Laplace and the energy method.

Whole Space Simulations

We design two numeric Fourier algorithms:

- the **double-null case**, with both t and x directions as characteristics
$$(\partial_x\Phi + a\Phi) = (\partial_x^2 + \partial_y^2)\Phi - 2b\partial_y\Phi$$
- the **null-timelike case**, with the t direction timelile, and the *x*-direction characteristic
$$\partial_t(\partial_x\Phi + a\Phi) = \partial_y^2\Phi - 2b\partial_y\Phi$$

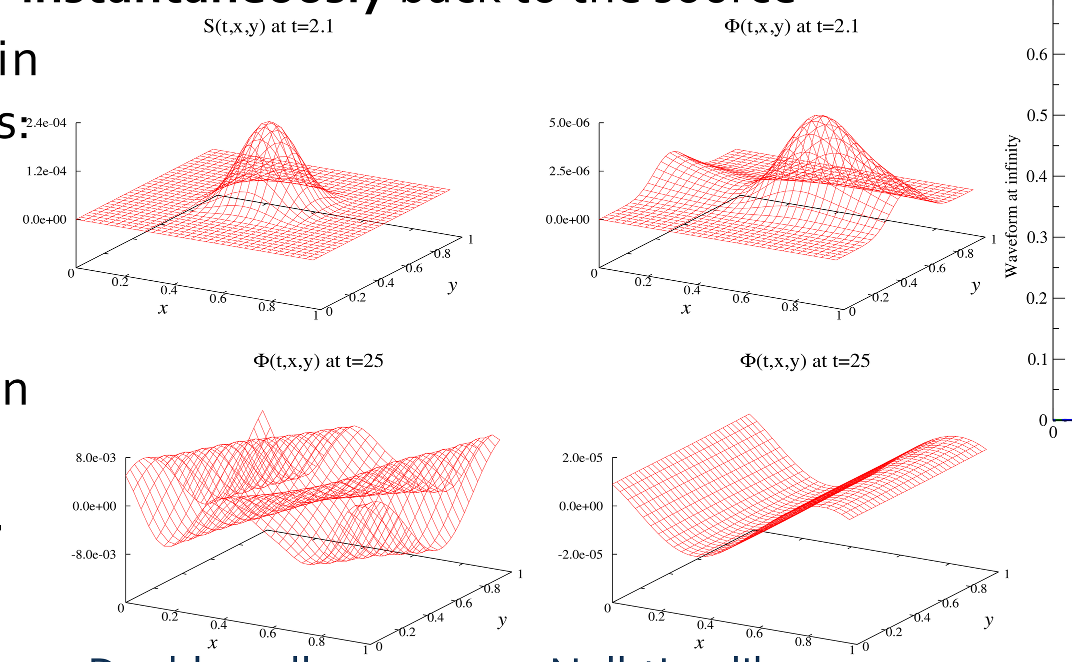


Double-null case: the wave becomes uniform in *y* and the shape is frozen.

Null-timelike case: the wave becomes uniform in *y* and travels in x direction.

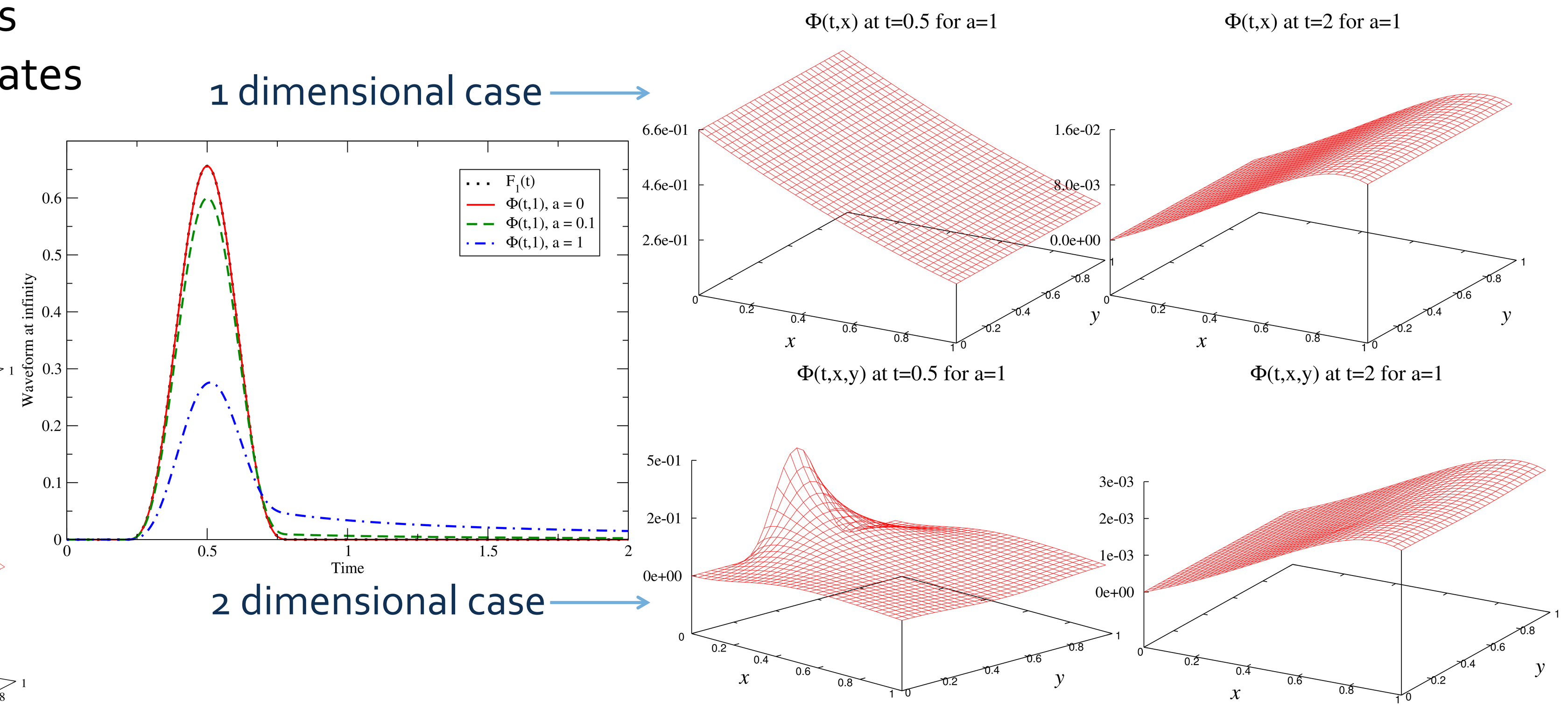
Periodic boundary conditions allow characteristics to form closed timelike curves

The simulations show that signal propagates **instantaneously** back to the source



Double-null case   Null-timelike case

The effect of the *a*-term: damping at the peak, and tail during decay



Strip Problem Simulations

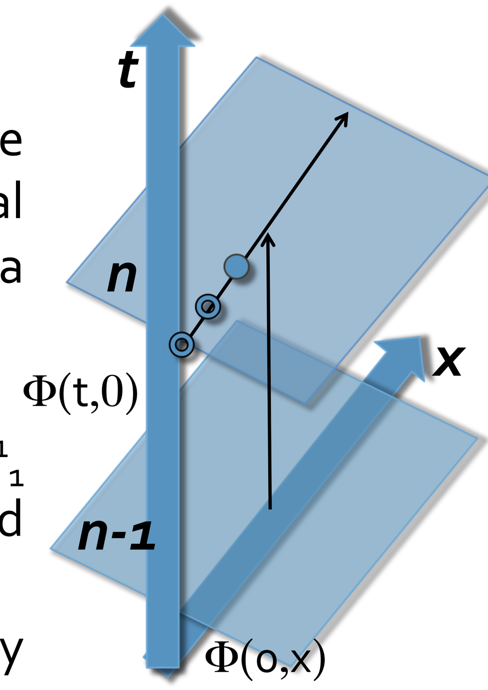
❖The lack of periodicity in the characteristic *x*-direction makes the Fourier approach unusable.

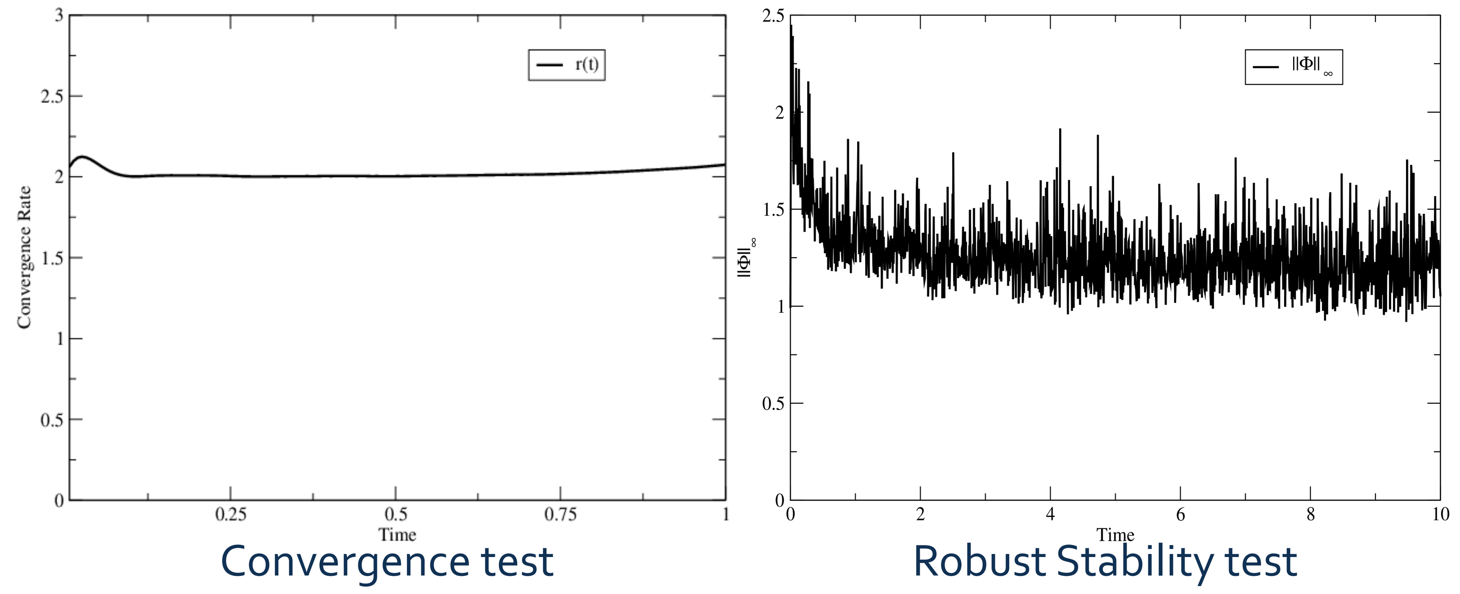
❖The method of lines for time is not applicable.

❖We construct a marching algorithm along the outgoing characteristics, obtained from the integral identity satisfied by a scalar wave at the corners of a characteristic parallelogram

❖The matching algorithm proceeds as follows:

- With Φ^n_j given on time level t_n, Φ^{n+1}_0 and Φ^{n+1}_1 given on time level t_{n+1}, we determine Φ^{n+1}_2 and stepping forward Φ^{n+1}_3, Φ^{n+1}_4, Φ^{n+1}_5 ...
- No boundary condition is needed at infinity points x=1, they are on an ingoing characteristic
- The initial data Φ^0_j is supplied by the characteristic initial data Φ(0,x)
- The boundary data Φ^n_0 is supplied by the Dirichlet boundary data Φ(t,0).
- A start-up algorithm is used to obtain Φ^n_1





Convergence test   Robust Stability test

Conclusions

❖The main result is that numerical stability is controlled by the condition *a*>0, an important feature which had been overlooked in treatments of the characteristic initial value problem for the wave equation. We tested the finite difference code for the whole space and for the initial-boundary value problem in a strip.

❖The pure Cauchy problem was implemented with periodic boundary conditions so that characteristics formed closed timelike curves. This gave rise to a signal that propagates well-posed instantaneously back to the source.

❖The null evolution code for the strip problem, with timelike inner boundary and characteristic outer boundary, implements the marching algorithm to integrate on the characteristics.

Future Plans

❖Next step is to extend this treatment to the simulation of gravitational waves in the full nonlinear context of general relativity.

Acknowledgements

NSF grants PHY-0854623 and PHY-1201276 to the University of Pittsburgh
NSF grant PHY-0969709 to Marshall University.