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## ESSAY

# SOME LESSONS ABOUT THE LAW FROM SELF-REFERENTIAL PROBLEMS IN MATHEMATICS

*John M. Rogers\** and *Robert E. Molzon\*\**

If systems of law can be analogized to axiomatic systems of mathematical logic in a tight enough fashion, then some conclusions of twentieth-century mathematicians regarding number theory systems may usefully be applied to legal systems as well. In particular, legal theorists must become comfortable with the incompleteness of legal systems, no matter how carefully constructed, in the same way that mathematicians and philosophers have become comfortable with the incompleteness of axiomatic systems of number theory. The analogy sheds light on some currently perceived problems with our constitutional system.

We first describe briefly mathematician Kurt Gödel's brilliant Incompleteness Theorem of 1931, and explore some of its general implications. We then attempt to draw a parallel between axiomatic systems of number theory (or of logic in general) and systems of law, and defend the analogy against anticipated objections. Finally, we reach two types of conclusions. First, failure to distinguish between language and metalanguage in mathematical self-referential problems leads to fallacies that are highly analogous to certain legal fallacies. Second, and perhaps more significantly, Gödel's theorem strongly suggests that it is impossible to create a legal system that is "complete" in the sense that there is a derivable rule for every fact situation. It follows that criticisms of constitutional systems for failure to determine every answer are unfair: they demand more than any legal system can give. At best they are antilegal in the sense that rejection of such constitutional systems on such a ground would require the rejection of all constitutional systems.

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## I. SELF-REFERENCE AND GÖDEL'S THEOREM

Consider the following sentence.

*This sentence is false.*

If we think about the preceding sentence, we have difficulty ascertaining whether it is true or false — a quixotic, perhaps wasteful, endeavor. But thinking about the very difficulty of ascertaining whether the sentence is true or false may provide some interesting insights.

The paradox stems from the self-referential nature of the sentence. The sentence says something about itself. Of course a sentence can say something about itself and be easily determined either true or false. For instance:

*This sentence has five words.*

But intuition suggests that if an expression *can* refer to itself, a seeming paradox can be constructed. For instance:

*I am lying.*

*Every assertion in this article is misleading.*

*This statement cannot be proved true.*<sup>1</sup>

It is possible that every consistent *system* of statements that is expressive enough to include self-referential statements includes some statements that turn on themselves in this way — statements that cannot be proved true or false within the system.

Number theory has been used to examine whether consistent systems always include undecidable propositions. Number theory is the branch of mathematics dealing with relationships between integers. In a particular "system," propositions must be expressed in a certain way, and a particular set of axioms (assumed truths) and rules is used to generate a set of properly expressed statements that are "true." From a limited number of axioms, the number theorist — like the mathematician generally — develops (proves) other statements (theorems).

In 1931 German number theorist Kurt Gödel did a revolutionary thing. He proved that if a number theory system's set of axioms is complex enough to include simple arithmetic, then there are true statements within the system that cannot be reached using the axioms and rules of the system. In other words, he proved that such systems have formally undecidable propositions.<sup>2</sup> He did this by demonstrat-

1. This last example is perhaps true, but somewhat difficult to prove!

2. Kurt Gödel, *Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I*, 38 MONATSHEFTE FÜR MATHEMATIK UND PHYSIK 173-98 (1931), translated in KURT GÖDEL, ON FORMALLY UNDECIDABLE PROPOSITIONS OF PRINCIPIA MATHEMATICA AND RELATED SYSTEMS (B. Meltzer trans., 1962).

ing that self-referential statements of number theory could be expressed within such number theory systems, and that indeed a self-referential statement could always be expressed that was unprovable in the same way that

*This sentence cannot be proved.*  
cannot be proved.

### *Gödel and the Paradox of Self-Reference*

In this section we elaborate on some of the problems and ideas connected with self-reference, and then briefly explain how Gödel successfully analyzed one particular type of self-reference. By discussing self-reference in the abstract setting of logic before considering legal self-reference problems, we hope to present the underlying problem free of legal aspects that might tend to conceal the more fundamental difficulties.

The "liar's paradox" (as in "I am lying") is actually quite old. It appears in Paul's Letter to Titus.<sup>3</sup> Epimenides the Cretan is reported to have said, "All Cretans are liars."<sup>4</sup> This statement presents a paradox because Epimenides himself was a Cretan and the statement was taken to mean that every statement by a Cretan is false.<sup>5</sup>

The Epimenides paradox is an example of a semantic self-referential statement in which an assertion in some way refers to itself. A discussion of semantic self-reference must first distinguish between formal language and metalanguage — languages that are both usually present in mathematical logic. Statements made in the formal language are expressed in the symbols of the language following the formation rules of the language. The metalanguage, on the other hand, is the language used to describe the rules, symbols, and statements of the formal language. The formal language should not be considered in some sense better than the metalanguage. The two languages may use the same symbols or not. For example, the rules of syntax of English are themselves written in English.

An example might help explain the distinction. Suppose we take the game of chess as our formal language; it exists in the form of the board, the pieces, and the rules for moving the pieces. The

3. Titus 1:12.

4. DOUGLAS HOFSTADTER, *METAMAGICAL THEMAS: QUESTING FOR THE ESSENCE OF MIND AND PATTERN* 7 (1985).

5. Even so taken, the statement could be nonparadoxical if we simply conclude that it is false, on the assumption that some other Cretan statement is true. J.L. MACKIE, *TRUTH, PROBABILITY, AND PARADOX* 276 (1973). The necessity of the assumption demonstrates logical problems with this resolution. *Id.* at 276-85. The problem is presented more purely by the simple statement, "This statement is a lie."

metalanguage will be English and we may use English to describe the functions of the pieces and the rules. Now suppose we hypothesize a certain position from which White can mate Black in five moves. One could “prove” this possibility by moving the pieces and thus demonstrating that mate is indeed always possible in five moves. Since Black has only a finite number of possible moves after any move of White, one could actually carry out this proof, although it would obviously be very cumbersome. Alternatively, one could describe in words the moves White must make to force mate in five moves. Each of these proofs is equally valid and, in this case, the metalanguage proof is in some respects better than the formal language proof.

Now we describe the paradox, Richard’s Paradox, that eventually led Gödel to one of his main results.<sup>6</sup> Suppose we take simple arithmetic, with the operations of addition and multiplication, as our formal language, and we begin to make a list of arithmetic statements, such as what it means for a number to be even or prime. Next we label our list so that each definition, statement, and property is assigned a number. Our labeling system thus serves as a metalanguage because it is a system that refers to the formal language.

We now make a definition. A number shall be called “Richardian” if it does not satisfy the property it describes. For example, suppose the definition of an even number is labeled 5. The 5 is Richardian because it is not even. On the other hand, suppose the definition of prime number<sup>7</sup> is labeled 13. Then 13 would not be Richardian because it does satisfy the definition of prime. The definition of a Richardian number must appear somewhere on our list and it must be labeled with some number, which we shall refer to as  $r$ . Is this number  $r$  Richardian or not? Some thought should convince the reader that the number  $r$  is Richardian if and only if it is not Richardian. This is Richard’s Paradox.

The logical difficulty with Richard’s Paradox is that the formal language and the metalanguage have been confused. The formal language is the language of arithmetic and the metalanguage is the labeling system. The property “Richardian” is not an arithmetic property; it is not a property of numbers with the operations of addition and multiplication. Rather, it is a property that depends on the metalanguage. In this case a solution to the paradox is found in the realization that the two languages have been mixed.

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6. This version of Richard’s Paradox is drawn substantially from ERNEST NAGEL & JAMES R. NEWMAN, *GÖDEL’S PROOF* 60-63 (1958). Indeed, Nagel and Newman’s admirable book supports much of the following description of Gödel’s proof.

7. A number is prime if it is divisible only by itself and 1.

Now we consider Gödel's approach to self-reference. As Richard's Paradox illustrates, mixing a formal language with its metalanguage poses fundamental difficulties. One of Gödel's accomplishments was demonstrating that a formal language, arithmetic, could serve as its own metalanguage. He did this by describing a correspondence between statements in arithmetic and statements in the metalanguage. In fact, Gödel described this relationship so precisely that one can think of it as a machine that, when given a metamathematical statement as input, churns out a mathematical statement. The machine can also run in reverse, so that if one plugs in a mathematical statement, out pops the corresponding metamathematical statement.

Once this machine was constructed, Gödel formulated a special, and somewhat complicated, arithmetic relationship, which we shall refer to as  $G$ . If the arithmetic relationship is plugged into the machine (running in reverse), the machine produces the following metamathematical statement:

*The formula  $G$  cannot be derived in the language of arithmetic.*

It should now be apparent why formula  $G$  is so special: the corresponding metamathematical statement referred to  $G$  itself! Note that we have said nothing about the truth or falsity of  $G$ . It is simply a well-formed statement to be understood as we understand a sentence that is grammatically correct without judging the truth of the meaning.

The formula  $G$  has another significant property, however. If one could derive  $G$  by a sequence of valid arithmetic statements, then the negation of  $G$  could be derived. But one cannot derive both a statement and its negation if arithmetic is consistent. Hence  $G$  cannot be derived. Therefore the metamathematical statement "The formula  $G$  cannot be derived in the language of arithmetic" must be a true statement. The inescapable conclusion:  $G$  is true but it cannot be derived.

In other words, the concepts of truth and derivation are not at all equivalent. This may not come as a surprise to a lawyer, but it certainly was a shock to mathematicians.<sup>8</sup>

Gödel's theorem can be applied to any consistent system that is expressive enough to serve as its own metalanguage.<sup>9</sup> His theorem has

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8. To understand the significance of what Gödel proved, one must keep in mind that it is not at all surprising that arithmetic contains a statement about numbers which cannot be derived from axioms. The surprise comes from the proof that arithmetic contains a *true* statement that cannot be derived.

9. Many texts state and prove Gödel's result on incompleteness in the language of an arithmetic containing the operations of addition and multiplication. This is an example of a "first order" language in which quantification is allowed only on individual variables. In dealing with a legal system, we are concerned with incompleteness in higher order languages. In fact, Gödel's

enormous significance for the way in which we, as reasoning humans, look at meaning and truth. The significance lies not only in Gödel's conclusion, but also in the way that the conclusion was reached. In a remarkable book, Professor Douglas Hofstadter has made Gödel's reasoning, as well as the fundamental significance of it, intuitively accessible to those of us who do not think about number theory every day.<sup>10</sup> Hofstadter showed that the genius reflected in Gödel's proof is in astonishing ways mirrored in the music of Johann Sebastian Bach and in the drawings of M.C. Escher. Hofstadter also explored a broad range of areas of intellectual endeavor where ideas reflected in Gödel's proof provide insight, from grammar and translation of foreign languages to molecular biology and artificial intelligence. One is left with the realization that problems associated with self-reference tell us something significant about the meaning of "truth" and the extent to which truth exists (or can be proved).

## II. A LEGAL ANALOGY TO GÖDEL

Perhaps we can derive comparable insights about the law. To draw conclusions about legal systems from Gödel's discovery, we need to analogize legal systems to axiomatic systems. This essay attempts to do so.<sup>11</sup> First, we show how law can be looked at as a system. We then analogize "truth" in logic to "the law" in legal analysis. Finally, we assume that consistency, properly understood, is as essential to law as it is to axiomatic systems.

### A. *The Law as a System*

At first it may seem implausible to think about "the law" as something that can be examined for precise characteristics in the way that a system of number theory or a microbiological process can. We all know that laws change from time to time, they come from various sources, they conflict, and their application depends on the whims of all sorts of people of varying degrees of intelligence, honesty, and interest.

At a high level of abstraction, however, it is easy to describe the

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incompleteness result holds in any (higher order) language which is expressive enough to describe arithmetic with addition and multiplication. This would include almost any language which is complex enough to describe a legal system. See, e.g., C. Smorynski, *The Incompleteness Theorems*, in *HANDBOOK OF MATHEMATICAL LOGIC* 821, 825-29 (Jon Barwise ed., 1977).

10. See DOUGLAS R. HOFSTADTER, *GÖDEL, ESCHER, BACH: AN ETERNAL GOLDEN BRAID* (1979).

11. The analogy has been suggested elsewhere, notably in DANIEL KORNSTEIN, *THE MUSIC OF THE LAWS* 124-28 (1982), and John M. Farago, *Intractable Cases: The Role of Uncertainty in the Concept of Law*, 55 N.Y.U. L. REV. 195, 224-29 (1980).

law as a system. A legal rule merely is a statement that if certain actions or circumstances are found to exist or to have existed, a certain action (or inaction) will be required (or permitted, or not required, or not permitted) by whatever force supports the requirement. For instance, one such statement might be: if a citizen earns \$50,000 in 1990, he must pay the government \$10,000 in taxes. Another might be: if anyone drives a vehicle on Main Street, she may not drive faster than thirty-five miles per hour. Or: if anyone intentionally rapes a minor, he will be hanged. Or: when someone has a duty of care and violates that duty, thereby proximately causing injury to another, she must pay the victim an amount equal to his injury. Thus we have a well-defined form for a statement of law.

### B. "The Law" Analogized to "Truth"

We must be careful in analogizing between legal systems and axiomatic systems of logic.<sup>12</sup> For instance, if we evaluate a statement in logic for "truth," then what is the value for which legal statements are evaluated? Asked differently, what does it mean to say that a rule of law is "true"? In Euclidean geometry it was possible to test certain theorems for truth by checking them against objective reality. Within the limits of measurement, for instance, all right triangles conform to the Pythagorean Theorem. One of the axioms or fundamental assumptions of Euclidean geometry is that if there is a point outside of a line, only one line parallel to the first line can be drawn through the point. In the nineteenth century, mathematicians developed a "non-Euclidean" geometry — including a whole collection of theorems and corollaries — in which multiple parallel lines were assumed to be able to pass through the point. Perhaps one reason that non-Euclidean geometry developed so recently was that there was no obvious way of checking its theorems against objective reality. This makes the theorems no less "true" (assuming all of the relevant axioms), but the meaning of "true" may be a little different for some. Truth may just mean "consistent with all other axioms and proven and provable theorems."

One perhaps overly positivistic analogue to truth when speaking about law is what a court with jurisdiction would decide if presented with the case. Just because no court has resolved or will resolve a

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12. We do not argue that law is a calculus in the way that Roy Stone has. See Roy L. Stone-de Montpensier, *The Compleat Wrangler*, 50 MINN. L. REV. 1001 (1966); Roy L. Stone, *Affinities and Antinomies in Jurisprudence*, 1964 CAMBRIDGE L.J. 266, 281-85. We are only identifying some commonalities between consistent systems of laws and other consistent systems of propositions.



dispute between two parties obviously does not mean that there is no law. Even before there is a dispute, there can be law. But the most accurate definition of law is often in ultimate practical terms what the lawyer thinks the legal system will allow or require. Just like geometric theorems that only occasionally may be verified by mapping onto spatial reality, legal truth may only occasionally be pinned down by a court decision. But the whole body of legal results consistent with those decisions may be said to constitute what is law.<sup>13</sup> The lawyer who advises on the law is in effect saying what a court would hold if it were to decide the issue.<sup>14</sup> To a large extent this is what we mean when we say, "The law is . . ."

### C. Consistency

If the law can be compared to a body of correspondences between fact situations and results, only some of which are represented by court decisions, in what sense can it be said that the correspondences are consistent with each other? Stated otherwise, do the rules of law produce consistent results? In logic it is fundamental that a statement and its negation cannot both be true. A hypothesis is disproved, for instance, by demonstrating that the hypothesis necessarily implies both  $X$  and not- $X$ . A system of statements could be invented in which both  $X$  and not- $X$  are true, but it could soon be proved that *all* statements in the system are true, and the system would not be of much use.<sup>15</sup>

Similarly, a system of law in which a statement in the form of a rule is both a rule of law and *not* a rule of law would not be very

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13. More precisely but more abstractly, we should talk not of "courts" but of legitimate decisionmaking authority. A legal system can exist without courts, but there has to be someone to apply the law sometime. Even in a legal system where there is no superior enforcement authority, such as the public international legal system, cases are "decided" in a sense. The international legal system can be described as a system of reciprocal entitlements, and that system can be examined for "cases" in which disputes are resolved. See Anthony D'Amato, *Is International Law Really "Law"?*, 79 Nw. U. L. REV. 1293 (1985).

14. Compare Ronald Dworkin's somewhat more subjective definition: "A proposition of law may be asserted as true if it is more consistent with the theory of law that best justifies *settled* law than the contrary proposition of law." RONALD DWORIN, *TAKING RIGHTS SERIOUSLY* 283 (1977) (emphasis added). But see H.L.A. Hart's criticism of the definition of law as simply a prediction of what courts will hold. H.L.A. HART, *THE CONCEPT OF LAW* 138-44 (1961).

15. See, e.g., NAGEL & NEWMAN, *supra* note 6, at 50-51; RAYMOND SMULLYAN, *FOREVER UNDECIDED: A PUZZLE GUIDE TO GÖDEL* 57-58 (1987).

useful.<sup>16</sup> It contradicts the very way we think about law.<sup>17</sup> How can some action be required in certain circumstances, if the same action is also not required in identical circumstances? When explaining *stare decisis* to first-year law students, one of us sometimes says that "consistency may be the hobgoblin of little minds, but it is the glory of the law." Perhaps it is more accurate to say that consistency is essential to thinking about law.

Once while teaching tort law, one of us tried to get a student to distinguish two cases decided in opposite ways. After some initial attempts that did not hold up, the student agreed for the sake of argument that the facts of the two cases were not distinguishable in any relevant way. He nonetheless continued to maintain that both cases were decided correctly. "Why should we let logic get in the way of fair results?" he asked. This is a discussion-stopper. If identical cases can be decided correctly in inconsistent ways, there is not much left to the inquiry into what the law is. That such a question is a discussion-stopper shows the extent to which consistency is built into the very nature of the way we think about law. Of course, the student may have meant only that a superficial or seeming logic should not interfere with reliance upon distinctions that can be felt but that are difficult to articulate. If so, the discussion is still hard to continue, as inarticulable distinctions are hard to talk about, but at least the underlying assumption of consistency is preserved.

Moreover, the many ways in which lawyers are accustomed to talking about "inconsistency" in the law do not really conflict with the widespread assumption that legal systems are consistent. For instance there are many *misapplications* of the law. The law may still be assumed consistent even though different judges make inconsistent determinations on indistinguishable facts. Most lawyers will argue that one of the decisions was simply wrong. By analogy, the system of arithmetic is consistent even though some people add wrong. In fact the whole system of court appeals is designed to achieve greater levels of consistency under the rubric of "correcting errors." Of course not all errors are corrected, and inconsistencies remain in fact. This observation, however, does not undermine the assumption that at least one of two inconsistent decisions is wrong.<sup>18</sup> Again analogizing to arith-

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16. Stone has argued that "the law is consistent and the law must be consistent, just as, and in the same way as, and for the same reasons as, mathematics and logic must be consistent." Stone-de Montpensier, *supra* note 12, at 1015. We do not argue that law, mathematics and logic "must" be consistent. But if one *assumes* consistency in logic or mathematics then one inevitably encounters true statements which cannot be derived. One cannot exist without the other.

17. See, e.g., DWORKIN, *supra* note 14, at 27, 73-74.

18. See *id.* at 279-90.

metic, even though some addition errors are never caught, wrong sums are still wrong.

But often, one might argue, there are inconsistent results in cases each decided "rightly," for instance in different jurisdictions, or at different points in history. Contributory negligence may be an absolute bar to recovery for negligence in Virginia but not in Ohio. State sales tax may have been deductible from income for federal income tax purposes several years ago, but not today. Such "inconsistencies" trouble us little, however. The very fact of the difference in jurisdiction or the difference in time reconciles the results. In the case of the different jurisdictions, the different results are but *consistent* applications of the higher rule (metarule) that a state through its governmental institutions should be able to determine its own tort law. In the case of the different points in time, the different results are but consistent applications of the higher law (metarule) that Congress can modify income tax liability from year to year.

Of course, seeming inconsistencies in the law may make it difficult to determine whether one of two rules is simply wrong, or whether some relevant distinctions (like differences in jurisdiction) render the rules consistent. When the Supreme Court interprets the Constitution in a way apparently different from how it has done so before, for example, those who defend the new decision argue either that the previous decisions were wrong or that the new decision is consistent with the previous decisions, perhaps because of changed circumstances.<sup>19</sup> Each possibility is in any event consistent with the assumption that rules of law in the legal system must be consistent.<sup>20</sup>

Finally, at times the law may not require one action or decision only, but instead permits a particular range of choices. The actor or decisionmaker in this situation is said to have discretion.<sup>21</sup> When discretion is exercised in different ways on identical facts, a host of issues

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19. Even when the Supreme Court has given solely prospective effect to a new constitutional decision, articulable distinctions can render consistent the different treatment of activity before and after the new decision. George P. Fletcher, *Paradoxes in Legal Thought*, 85 COLUM. L. REV. 1263, 1272-80 (1985).

20. For a contrary argument, see PETER SUBER, *THE PARADOX OF SELF-AMENDMENT* 295-96 (1990). Suber accepts some contradiction in the law in order to validate constitutional self-amendment, in the face of arguments that such self-amendment is inherently contradictory. See Alf Ross, *On Self-Reference and a Puzzle in Constitutional Law*, 78 MIND 1 (1969). Suber is then forced to rationalize as to when the law requires consistency and when it does not. SUBER, *supra*, at 296-303. This is the weakest part of Suber's otherwise rigorous analysis. While a detailed critique of Suber is beyond the scope of this article, we suggest that a sounder response to Ross could be made by relying upon a precise and limited definition of consistency to avoid the contradiction Ross asserted.

21. See generally John M. Rogers, *A Fresh Look at Agency "Discretion,"* 57 TUL. L. REV. 776, 776-80 (1983).

involving "consistency" arises.<sup>22</sup> But unless discretion is legally narrowed by previous decisions within the range of choices, there is still *legal* consistency even when indistinguishable facts yield different results. This results from the very fact that the law permits a range of choices.<sup>23</sup>

### III. PAYING ATTENTION TO LEVELS

#### A. *Language and Metalanguage in the Law*

The law can thus be considered as a system of consistent statements. Can law also serve as its own metalanguage? A positive answer seems to result inescapably from the need for rules to determine whether particular well-formed statements are rules of law. Just as arithmetic contains statements about numbers that are not only undervivable but also false, many properly formed statements of law are inaccurate (we might say false) statements — statements that are merely in the proper form. For instance: if anyone fails to pay assessed taxes, he will be hanged. Which statements out of the set of all properly formed statements are rules of law? To determine this, we need something analogous to the axioms and rules of number theory used to determine which theorems are "true."

What we have are rules for creating rules of law. These are more frequently called procedures or processes, but because we can put them in the form of statements of law, they can be called rules as well.<sup>24</sup> For instance, a rule for creating a rule might be: if a majority of both legislative houses votes for a tax law, and the chief executive does not veto the bill, the tax will be the law. Another might be: if the Supreme Court finds the tax to be excessive, the tax will not apply. Thus we have law serving as its own metalanguage.

We can keep going. How do we determine which rules for creating rules are true, and which are false? Again, we have rules for making this determination. For instance: if the state constitution says that the chief executive has the power to veto legislation, then no bill shall be-

22. See John E. Coons, *Consistency*, 75 CAL. L. REV. 59 (1987).

23. The logical analogy would be "If *X*, then *Y* or *Z*." Both "*X* and *Y*" and "*X* and *Z*" are consistent with each other — as long as "*X* and *Y*" does not imply "If *X*, then not *Z*." Stated in legal terms, if the law truly permits *Y* or *Z* result whenever there is fact *X*, then *Z* result on fact *X* is legally consistent with *Y* result on fact *X*.

Other definitions of consistency may lead to other types of "inconsistent law." See SUBER, *supra* note 20, at 170-73.

24. H.L.A. Hart calls them "secondary rules" and categorizes them as rules of recognition, rules of change, and rules of adjudication. See HART, *supra* note 14, at 77-96. Professor Franck refers to rules about rules as "secondary rules of process." See Thomas M. Franck, *Legitimacy in the International System*, 82 AM. J. INTL. L. 705, 752 (1988). Professor Ross refers to "rules of competence." See Ross, *supra* note 20, at 1-2.

come law without his or her signature. Then how do we determine who will interpret the constitution, or indeed which constitution to interpret? If we keep stepping back, we will find rules determined by rules determined by rules determined by rules and so on.<sup>25</sup> Where will it stop? Who provides the ultimate axioms?<sup>26</sup> Before looking for an answer, it may be helpful to examine a realistic example to show that viewing law as a hierarchical system of rules and metarules is not only descriptively accurate, but it also helps us to think clearly about at least some difficult legal issues.

### B. *United States v. Nixon*

A case that involved a potential conflict of fundamental constitutional power illustrates the hierarchical nature of legal rule determination, while helping to suggest the source of the ultimate axiom(s). *United States v. Nixon*<sup>27</sup> presented the issue of whether the President had to comply with a court-ordered subpoena *duces tecum* directing him to produce certain tape recordings and documents relating to conversations with aides and advisers. The constitutional law teacher might ask a student as Question 1, "Are the President's communications privileged in the context presented by the case?" The trial court's answer, affirmed by the Supreme Court, is no.

Question 2 is "Who decides Question 1?" In other words, does the Constitution give the President an absolute privilege, determined by himself, or can the *court* determine that the President should not have the privilege in some cases? The President argued that his privilege was absolute under the Constitution; the Supreme Court disagreed.

Question 3 is "Who decides Question 2?" In other words, who has the power to make a final determination of the meaning of the Constitution? The answer is the Supreme Court, at least where the case is justiciable. The question was apparently settled long ago in the famous case of *Marbury v. Madison*.<sup>28</sup> President Nixon could not really be accused of having challenged this accepted principle; his lawyers did not do so. Indeed, that they litigated the case in the Supreme Court amounted to an acceptance of the Supreme Court's power to

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25. For an engaging discussion of these levels, see Stuart Banner, *Please Don't Read This Title*, 50 OHIO ST. L.J. 243, 245-46, 249-56 (1989); see also Farago, *supra* note 11, at 205-06.

26. According to Alf Ross, "the series of authorities cannot be infinite," and therefore there must be a "highest authority whose competence is not derived from any other authority." Ross, *supra* note 20, at 2.

27. 418 U.S. 683 (1974).

28. 5 U.S. (1 Cranch) 137 (1803).

decide. Such an accusation would have mixed levels by confusing Question 2 with Question 3.

The really interesting question is Question 4: "Who decides Question 3?" In other words, what is to keep other branches from themselves asserting the final power to interpret the Constitution? We can only answer this question by asking what would happen if such an assertion were made in a particular case. The people with true power would support one side or the other. The people who would ultimately support or repress a revolution, and win, are the operative American polity. Stated another way, the polity is the American people exercising power individually and collectively through their various private and public institutions.<sup>29</sup>

The real genius of Chief Justice John Marshall in *Marbury* was that he decided his Question 2 in *favor* of the political interests that then controlled the other two branches of government. The Democrat-Republican Jefferson had just been elected to succeed the Federalist Adams as President, and a Federalist appointee to a judgeship (*Marbury*) sought to obtain his commission from the new Democrat-Republican Secretary of State (Madison). Chief Justice Marshall (a Federalist and Madison's predecessor as Secretary of State) held that the Supreme Court lacked jurisdiction in that case because the statute granting jurisdiction was unconstitutional. Because the Court ruled in favor of the politically dominant Jeffersonians, it was impossible for the President or Congress to refuse to comply. Chief Justice Marshall thereby resolved Question 3 in favor of the Supreme Court. This resolution, effectively unchallenged, was accepted by the people and their institutions. Because the polity accepts this, no President would dare violate a Supreme Court's clear order. Impeachment, perhaps rebellion, stands in his way. But if the polity *were* to accept a President's (or Congress') authority to interpret the Constitution differently from the Supreme Court, then we would simply have a different constitutional system.

In theory we could keep stepping to higher levels, and never stop. Who tells the polity what to do?<sup>30</sup> Fortunately it is unnecessary to

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29. To refer to the polity simply as "the people" would be a little misleading, since a bare majority of the population might not have the wherewithal to change a ruling power with which they disagreed. Those with the determination, ingenuity, funds, numbers, and other resources sufficient to change or retain ruling power ultimately determine whether a constitutional system will remain or change.

For example, a dictatorship can have a constitutional system of law, and until it is overthrown, we may say that the polity of the country has acquiesced in the dictatorship. But we would be reluctant to say that a majority of the people necessarily supports the dictatorial constitutional system.

30. The same question could perhaps be stated, "What makes the polity's acceptance of law

continue to provide the answer, and it would be difficult in any event to obtain agreement. Common ground may be possible, however, as to the ultimate nature of law. At least in some sense it reflects human reasoning. The idea that law exists "out there," not promulgated by humans but discovered by them, sounds passé, a sort of pre-Holmesian myth that modern legal thinking has outgrown. We think instead of laws that govern human behavior as coming ultimately from people — rulers, legislatures, courts. Even law based upon custom presumably reflects antecedent human decisions as to what behavior is required. To the extent that law does in fact reflect the reasoning of man, we should be able to use knowledge about the way in which people think to help ascertain the nature of law and to examine the limits of law.

### C. *Legal Fallacies Analogous to the Confusion of Language and Metalanguage*

Some legal confusion is highly analogous to the mixture of levels found in Richard's Paradox. Consider for example, the problem of revising the U.S. Constitution. Under Article V, two thirds of the states may call for a constitutional convention. Cries of alarm have risen when that number recently was approached,<sup>31</sup> based primarily on fears that such a convention would produce a document significantly different from our present Constitution.<sup>32</sup> Reassurance should come, however, from the Article V requirement that the product of such a convention must have subsequent ratification by the legislatures of, or conventions in, three fourths of the states; this would be an extraordinary burden for any drastic change to shoulder.<sup>33</sup>

A typical response to this reassurance, however, is to point out that the present Constitution superseded the Articles of Confederation without going through the amendment procedures required by the Articles.<sup>34</sup> What is to keep the new convention from providing its own procedures for putting the new constitution into effect, without regard to the limits of the present Constitution, just as the present Constitution did without regard to the limits of the Articles of Confederation?

Stated another way, the question is whether it is the law that the

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determinative?" Peter Suber's treatment of H.L.A. Hart's *The Concept of Law* deals with the possible paradox of saying acceptance makes acceptance determinative. SUBER, *supra* note 20, at 65-72.

31. See RUSSELL L. CAPLAN, CONSTITUTIONAL BRINKSMANSHIP 73-89 (1988).

32. *Id.* at vii-viii.

33. See PAUL J. WEBER & BARBARA A. PERRY, UNFOUNDED FEARS 8, 117, 119 (1989).

34. See CAPLAN, *supra* note 31, at 141-42. Actually, this point is debatable, although most writers take the position that the present Constitution cannot be viewed as an amendment of the Articles of Confederation. See SUBER, *supra* note 20, at 327-30.

Constitution cannot be changed without subsequent approval by the legislatures of three fourths of the States.<sup>35</sup> The answer requires us to step outside of the system. To what extent does the polity, the ultimate determiner of the Constitution, give the Constitution the power to limit its own change?<sup>36</sup> In this case, one might strongly argue that the polity of this nation, after 200 years, demands compliance with the present Constitution far more than the polity of the United States of the 1780s demanded compliance with the Articles of Confederation.<sup>37</sup> In effect, a bloodless revolution occurred during the period of the adoption of the present Constitution, as the polity accepted the new one and rejected the Articles.

Conceivably there will be such a bloodless revolution — contrary to the present Constitution — in connection with a new constitutional convention.<sup>38</sup> But it is also conceivable that any group of people could gather and propose a new U.S. Constitution,<sup>39</sup> and that it would “catch on.” The American polity’s acceptance of the present Constitution as the law of the land prevents such a result,<sup>40</sup> and there is no reason to expect the present Constitution not to bind in this respect any less than in the other respects in which we have accepted it over the past 200 years.

Careful attention to levels may also help us to defend against attacks on the legitimacy of the constitutional system we have. One such attack involves the alleged indeterminacy of judicial determinations: constitutional interpretation cannot be “determined,” and in particular it cannot be determined by the original intent of the Framers. The indeterminacy attack thus requires a rejection of interpretivism — the idea that judges can and should decide constitutional cases in accordance with the Framers’ intent. The asserted indeterminacy

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35. Professor Suber has examined this very question at extraordinary length, thereby bringing to light remarkable insights about the nature of constitutional systems. SUBER, *supra* note 20; see also Norbert Hoerster, *On Alf Ross's Alleged Puzzle in Constitutional Law*, 81 MIND 422 (1972); Joseph Raz, *Professor A. Ross and Some Legal Puzzles*, 81 MIND 415 (1972); Ross, *supra* note 20.

36. See SUBER, *supra* note 20, at 49-72; Ross, *supra* note 20, at 4, 21-24.

37. See CAPLAN, *supra* note 31, at 142-46; WEBER & PERRY, *supra* note 33, at 14-29.

38. In Kentucky, for instance, the state legislature provided for a referendum on a constitution prepared by a Constitution Revision Assembly. The procedure did not follow the procedure for amendment provided in the current state constitution. The highest court of the state nonetheless upheld the procedure, reasoning that the state constitutional procedures for amendment were not exclusive, but also recognizing that “[w]hen the people vote on the proposed Constitution it will be an expression of the inalienable right of the ultimate sovereign to reform the government.” *Gatewood v. Matthews*, 403 S.W.2d 716, 721-22 (Ky. 1966).

39. Some are described by SUBER, *supra* note 20, at 330-31.

40. We assume that the very process of instituting a constitutional convention would not materially alter the polity’s support of the current Constitution because the convention would be created pursuant to that very Constitution.



can be used as a weapon to undermine the consensus of the American polity about the limited role of courts in our constitutional system.<sup>41</sup> Put very simply, the idea is that courts are not really constrained by the "law," so why not accept the judiciary as a political branch just like the legislature or the executive? Careful attention to levels shows the fallacy of at least some of these arguments.

Mark Tushnet, for instance, criticizes interpretivism on the ground that the theory is conceptually unable to provide the constraints on potential judicial tyranny that its advocates claim it offers.<sup>42</sup> Under the "liberal" view, the advantage of interpretivism is that unelected judges will be unable to construe the Constitution to say whatever they want, and thereby possibly abuse power, because they will be constrained by the meaning of the Constitution attributable to the Framers.<sup>43</sup> Tushnet argues that in order to figure out what the Framers really intended, we must make assumptions about how a present day situation would have been treated in the past. Because it is not readily apparent which portions of the Framers' world view to "modernize," a degree of indeterminacy affects the interpretive endeavor:

The first step is an argument that interpretivism must rest on an account of historical knowledge more subtle than the naive presumption that past attitudes and intentions are directly accessible to present understanding. The second step identifies the most plausible such account, the view — sometimes called hermeneutics — that historical understanding requires an imaginative transposition of former world views into the categories of our own . . . . The project of imaginative transposition can be carried through in a number of different ways, with a number of different results, none of which is more "correct" than the others.<sup>44</sup>

This indeterminacy is said to undermine the "liberal" goal of confining the power of the unelected branch. According to Tushnet, if there is no determinacy there is no substantial constraint: "The existence of such an indeterminacy means that interpretivism, unless it falls back on nonliberal assumptions, cannot constrain judges sufficiently to carry out the liberal project of avoiding judicial tyranny."<sup>45</sup> Stated

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41. See, e.g., Mark V. Tushnet, *Following the Rules Laid Down: A Critique of Interpretivism and Neutral Principles*, 96 HARV. L. REV. 781 (1983). Other examples are collected in Ken Kress, *Legal Indeterminacy*, 77 CAL. L. REV. 283, 302 n.67 (1989); see also Paul Brest, *Interpretation and Interest*, 34 STAN. L. REV. 765 (1982); Arthur S. Miller, *Toward a Definition of "The" Constitution*, 8 U. DAYTON L. REV. 633, 650-53 (1983). See generally Kress, *supra*; John B. McArthur, *Abandoning the Constitution: The New Wave in Constitutional Theory*, 59 TUL. L. REV. 280, 298-99 (1984).

42. Tushnet, *supra* note 41, at 793.

43. E.g., ROBERT BORK, *THE TEMPTING OF AMERICA* (1990).

44. Tushnet, *supra* note 41, at 793; see also *id.* at 801 ("[T]he need to identify functional equivalents over time necessarily imports significant indeterminacy . . .").

45. *Id.* at 793.

more strongly, “[t]he interpretivists’ premise of determinate intentions is essential to their project of developing constraints on judges.”<sup>46</sup>

Attention to the distinction between rules and metarules exposes the fallacy in this line of argument. Choice at one level does not constrain choice at a conceptually distinct level. The problem with Tushnet’s argument is its assumption that no significant constraint can exist without determinacy. It is not true, generally, that judges are without constraint if the law permits more than one conclusion. As we have seen, a law can permit a range of results and still provide constraints. This happens frequently when agencies exercise discretion, or when judges apply different sentences for similar conduct. No one would argue that a denial of all discretion is required to put any significant restraints on judges.

Why should it be any different at the level of constitutional interpretation? If interpretivism *is* indeterminate — that is, if the interpretive endeavor may lead to one of several possible results — it does not follow that *any* result is possible. If an interpretive technique is sufficiently indeterminate to permit ten or twelve interpretations of a particular clause, that still leaves millions of interpretations that are *not* permitted. Surely it is consistent with the project of liberal constitutional theory to preclude those millions.<sup>47</sup>

Now suppose we ask what the rule is for determining whether a constitutional rule of interpretation (interpretivism) is a rule of law. As the answer, we have posited the following rule about a rule about rules: what the polity is willing to accept determines how the Supreme Court interprets the Constitution. In other words, the interpretivist would argue that the polity does (or should) impose on the Court the requirement to interpret the Constitution according to the intent of the Framers. That requirement may permit the Court to exercise discretion (by choosing one of the results permitted by the hermeneutic endeavor), but the polity will not permit the Court to come to conclusions outside the range of those choices.<sup>48</sup> Thus the “indeterminacy”

46. *Id.* at 800.

47. None of this demonstrates that interpretivism *is* the law, but only that it could logically and consistently be the law even though indeterminate.

48. Tushnet’s critique of neutral principles founders on the same rock. Neutral principles theory requires that judges decide cases on the basis of general principles applied consistently in all similar cases. Because there is frequently more than one way to justify a case, a consistency requirement will not determine absolutely how a subsequent case will be resolved. Thus, the requirement of consistency is not always determinate: “[T]here are always a number of justificatory principles available to make sense of case 1 and a number of techniques to select the ‘true’ basis of case 1.” Tushnet, *supra* note 41, at 811.

Assuming this is so, it simply does not follow that a neutral principles requirement provides no constraint on judges. The analogy to agency or judicial discretion again is helpful. Thinking of the rule for determining whether a constitutional rule of interpretation (in this case neutral

criticism of interpretivism (and of neutral principles) simply fails adequately to distinguish indeterminacy from absence of constraint.

In both examples — the constitutional convention and the interpretivism critique — the failure to recognize levels is highly analogous to Richard's failure to distinguish between language and metalanguage.<sup>49</sup> The need for ratification by three fourths of the states as a requirement at the constitutional interpretation level is not undermined by the fact that such ratification is not binding upon the polity that accepts and thereby determines what the constitution shall be — a conceptually higher level. And the possibility that a constitutional system that permits a range of choices (interpretivism) can actually *be* the choice of the polity (at a conceptually higher level) does not undermine the validity of the choice.

#### IV. LESSONS FROM THE INHERENT INCOMPLETENESS OF LEGAL SYSTEMS

We have a somewhat deeper point to make. Gödel demonstrated that within certain systems, a language *could* serve as its own metalanguage. Gödel showed how to construct an arithmetical formula *G* that *represented* the metamathematical statement: "The formula *G* is not demonstrable."<sup>50</sup> Gödel proved *G* metamathematically (i.e., outside the system), thereby proving that true statement *G* could not be reached within the system. Because this is so, such systems are not complete. If a system is sufficiently complex to permit self-description, and the system is consistent,<sup>51</sup> then *G* can always be constructed, and the system is incomplete.

We have already seen that statements about the law can be in the form of laws. The parallel to Gödel's proof will be complete if we can determine that legal rules are expressive enough to refer to themselves. The true insight of Gödel's proof was his ability to demonstrate that propositions of number theory could be made to refer to themselves.

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principles) is a rule of law, again we can posit: what the polity is willing to accept determines how the Supreme Court interprets the Constitution. In other words, the neutral principles advocate would argue that the polity imposes on the Court the requirement to interpret the Constitution according to neutral principles. That requirement may permit the Court to exercise discretion (by choosing one of the results permitted by a neutral principle), but the polity will not permit the Court to come to conclusions outside the range of those choices — conclusions that no neutral principle can justify. So it is not true that "[n]eutral application requires that we be able to identify *the* principle that justified the result in case 1 in order to be sure that it is neutrally applied in case 2." *Id.* (Once again, this does not demonstrate that neutral principles is the law, but only that it could logically and consistently be the law even though indeterminate.).

49. See *supra* notes 6-7 and accompanying text.

50. NAGEL & NEWMAN, *supra* note 6, at 85.

51. See *supra* note 9 and accompanying text.

It is in fact far easier to conclude that legal rules can be self-referential.<sup>52</sup> We have already seen that statements about the law can be in the form of law. The following examples not only demonstrate the possibility of legal self-reference, but also suggest the Gödelian uncertainty that can occur when there is such self-reference.

### A. *Self-proving Rules*

First of all, in some situations a proviso or an exception to a rule is stated in self-referential terms. For instance, a legal rule like "All residents except university students must pay a six percent sales tax" sometimes appears in the form, "All residents must pay a six percent sales tax, but this provision does not apply to university students." Such self-reference is generally effective,<sup>53</sup> and raises only esoteric theoretical problems.<sup>54</sup>

For example, Section 6 of the Twentieth Amendment to the U.S. Constitution states: "This article shall be inoperative unless it shall have been ratified as an amendment to the Constitution by the legislatures of three-fourths of the several States within seven years from the date of its submission."<sup>55</sup> For many reasons, this sentence may not be operative — that is, it may not state a rule of law. It may be contained in an amendment that has merely been suggested by a newspaper editor. Or if the sentence is in an amendment proposed by a member of Congress, it may not have obtained a two-thirds majority in both Houses of Congress, or may otherwise have failed to meet the Article V requirements for amending the Constitution. In these circumstances the sentence is accurate regardless of whether three fourths of

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52. Professor Farago easily concludes that law is complex enough to include arithmetic. See Farago, *supra* note 11, at 226-27. He also explains why legal systems are not reducible to a finite recursive process, and that they are thus not able in that way to "escape the consequences of a legal Gödel." *Id.* at 228.

Farago, however, finds something like a way out of the Gödelian legal uncertainty: intractable cases can be decided by the exercise of judicial discretion. *Id.* at 236-39. The trouble with this escape is that it amounts to another metarule, one that is just as vulnerable as any other to destructive self-reference if there is no meta-metarule. For example, can a judge decide whether a judge cannot exercise discretion to decide an "intractable" case? See Banner, *supra* note 25, at 246-47.

53. This is because every possible hypothesis about whether such a self-referential rule is a rule of law or not leads to the conclusion that the rule *is* a rule of law. In logic, an analogous statement would be, "This statement is either true or false." If the statement is assumed true, then of course it *is* true. But even if the statement were assumed for the moment to be false, by its terms we would be led to the conclusion that it is nonetheless true. So the statement must be true.

54. See SUBER, *supra* note 20, at 190-93 (considering the paradoxical nature of "self-repealing" laws).

55. U.S. CONST. amend. XX, § 6.

the States have ratified the amendment within seven years. The amendment is simply inoperative.

If, on the other hand, the external requirements for operability (the Article V requirements) are met, two possibilities exist: three-fourths ratification occurred within seven years, or it took longer than seven years. If the former, then the amendment is operative and Section 6 accurately states the law (since the conditions for inoperativeness are not met). If the latter, then *by virtue of Section 6 itself* the amendment is not operative. Again, Section 6 accurately states the law (since the amendment is inoperative when the conditions for inoperativeness are met). The self-referential statement makes itself true.<sup>56</sup>

### B. *Ineffective Bootstrapping*

A different type of self-reference occurs when rules seek to affirm themselves. The analogous statement from the realm of logic would be, "This statement is true." How do we know whether the statement is true? Assume for the moment that the statement is true, then by its terms it is true. But if we assume for the moment that the statement is false, then it is stating something that by assumption is not true, and therefore the statement is false. The statement thus tells us nothing more about whether it is true or false than we already knew. One might well ask why such statements are ever made. Yet analogous attempts at self-affirmation appear with some frequency in the law.

In *Quern v. Jordan*,<sup>57</sup> the Supreme Court decided that state sovereign immunity reflected in the Eleventh Amendment does not preclude a federal court from ordering state officials to send certain notices. In doing so the Court stated that 42 U.S.C. section 1983 was not intended to override the sovereign immunity of the states.<sup>58</sup> Justice Brennan, concurring, criticized the majority's conclusion regard-

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56. Of course this might not be the result if Article V of the Constitution were different. The effect of the self-referential statement may depend on the content of the higher level rules. For instance, a different Article V might have an additional sentence that states: "All amendments to this Constitution shall have no procedural requirements for passage other than those listed in this Article, and any additional such requirements contained in any particular amendment shall be a nullity and severable." If this were the case, § 6 of Amendment XX might not be self-proving; indeed, it might well not be a rule of law at all, because the amendment would come into effect even if it took longer than seven years to get three-fourths ratification. The results of a particular self-referential rule can thus be changed by stepping out of the system and changing the higher level rules.

It appears, however, that regardless of how Article V were rewritten, a self-proving statement could be constructed. For instance, in the example of the modified Article V, a self-proving rule could be constructed if § 7 were added to the amendment saying, "This amendment shall not be operative if any part of this amendment, including this section, is found inoperative." While § 6 might not be operative, § 7 would be.

57. 440 U.S. 332 (1979).

58. 440 U.S. at 338-41.

ing section 1983 as “gratuitous” and “patent[ ] dicta.”<sup>59</sup> The majority responded in a footnote that it could not see how its statement in the text “can be characterized as ‘dicta.’”<sup>60</sup> The footnote thus in effect said, “and this statement is holding and not dictum.” In our judicial system where holdings are binding on lower courts in ways that dicta are not, is a statement by the Supreme Court any more holding by virtue of a statement that “this opinion is holding”?

If we oversimplify for the moment, and assume that a court makes law when it holds something, and does not make law when it engages in *obiter dictum*, then the Court in *Quern* in effect was trying to say, “This opinion is the law.” But any court, engaging in the most wild irrelevancies, could append “and this opinion is the law” and we would easily deny that such irrelevancies would bind. In order to evaluate the binding nature of the opinion, including the self-referential statement itself, we have to look at the same factors that determine whether the opinion is binding without regard to the self-referential statement. The footnote logically adds nothing.

Of course we could step out of the system and change the rules so that the footnote would be effective. For instance, if binding holding were redefined to include any matter that a court designates to be holding, then the *Quern* footnote would be of great, perhaps determinative, significance.<sup>61</sup>

In actual fact, the rules may be somewhere in between. That is, dictum is perhaps not entirely legally irrelevant. To the extent that Supreme Court dictum generally has *some* legal weight, the footnote will to that limited extent increase the binding force of what otherwise might have been dictum in the text. The counterargument would be that the statement in the text already had at least the weight accorded to dictum, and additional dictum could add no more. But if the footnote dictum somehow makes the statement in the text more like holding, there *is* arguably an effect from the footnote. If so, the Court arguably could have further bolstered its opinion by adding a footnote to its footnote, saying that the original footnote was holding! Then footnote could be placed after footnote after footnote, and so on.

A British version of the problem in *Quern* occurred in *London Street Tramways Co. v. London County Council*.<sup>62</sup> The House of Lords found that a decision of the House of Lords upon a question of

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59. 440 U.S. at 350 (Brennan, J., concurring).

60. 440 U.S. at 341-42 n.12.

61. Under the new rules, though, a new self-referential statement that would be ineffective could be created.

62. 1898 App. Cas. 375.

law is conclusive and binds the House in subsequent cases, and that an erroneous decision can be set right only by an Act of Parliament.<sup>63</sup> There was little discussion of the logic of this action. Could the House of Lords, in a subsequent decision, hold that the House need not be bound by erroneous decisions, thus reversing one ruling and paving the way for the reversal of another? There would be no contradiction in such a ruling, which simply would hold that the earlier holding that the House is bound by an erroneous decision itself was erroneous.<sup>64</sup>

The potential absurdities involved in thinking about dictum regarding holding can perhaps be brushed away by observing that the bottom line is arrived at simply by asking when the court has the power to extend its own power by redefining when its decisions are binding. As we have seen in the discussion of *Marbury*, the answer must lie in the source of the court's power. When the Supreme Court in effect stated that "we have the power to interpret the Constitution," it was ultimately the polity's acquiescence that rendered the rule a rule of law. In other words, we have to step out of the system to find the answer.<sup>65</sup>

Thus if we are careful in our attention to levels, we can be comfortable with this type of legal self-reference as well. But some examples

63. 1898 App. Cas. at 381.

64. The *London Street Tramways* case was analyzed in Stone-de Montpensier, *supra* note 12. Stone noted that critics of *London Street Tramways* argued that the decision heaved itself up by its own bootstraps and that the arguments were circular. See Stone-de Montpensier, *supra* note 12, at 1015. Stone defended the *Tramways* decision by arguing that an opposite decision (i.e., "the decisions of this House are not binding upon the House") would involve a paradox like the Epimenides (liar's) paradox: "It seems to me also that Gödel's results would also apply, for the rule [that House of Lords decisions are not binding], even though concerned with consistency, is not provable within the legal system. If it were provable, it would be refutable; since the rule is not internally provable, it is irrefutable." *Id.*

Stone stuck to his position that the *London Street Tramways* decision could not have been otherwise without being paradoxical, despite a subsequent House of Lords Practice Statement that may in effect have amounted to an overruling of *Tramways*. Roy Stone-de Montpensier, *Logic and Law: The Precedence of Precedents*, 51 MINN. L. REV. 655, 661 (1967). Stone argued that the Practice Statement was limited and self-contradictory, and that the holding of *London Street Tramways* was necessary for law to be consistent. *Id.* at 674. Despite multiple references to Gödel, Stone argued that "the legal system may be complete and decidable and . . . consistent." *Id.* at 670.

Stone's argument leads to the conclusion that *stare decisis* is the only attitude that a court in a legal system can adopt. J.C. Hicks, *The Liar Paradox in Legal Reasoning*, 29 CAMBRIDGE L.J. 275, 288 (1971). One wonders whether Stone would find the American legal system, in which the Supreme Court overrules itself upon occasion, to be a true legal system.

We argue directly to the contrary, that legal systems are inherently incomplete, and that identifying Gödelian undecidability in a system does not invalidate either the rule or the system that can produce it. Such undecidabilities can always be identified in sufficiently expressive consistent systems.

65. Another example of a self-bolstering rule is a *California Evidence Code* provision that limits the applicability to that very code of a particular rule of interpretation (that statutes in derogation of the common law should be interpreted narrowly). Banner, *supra* note 25, at 244-45.

of legal self-reference lead rather directly to the legal counterpart of undecidability.

### C. *Tangled Loops of Law*

We have looked at self-referential rules that deny their own applicability on certain conditions and thus are self-proving. We have also looked at self-referential rules that seek to affirm themselves, but can only be judged by stepping out of the system. Still others rules are self-referential in ways that tie themselves into undecidable loops analogous to Gödel's *G*.

Comparable loops exist in the law. Such loops can be disguised in a pair of mutually self-referential statements. Consider the following two sentences:

*The following statement is false.*

*The previous statement is true.*

The familiar conflict of law conundrum called *renvoi* provides an analogy to this type of hidden, tangled self-reference. If state *A*'s conflict of law rules in a particular situation require application of the law of state *B*, including *B*'s conflict of law rules, and state *B*'s conflict of law rules in a particular situation require application of the law of state *A*, including *A*'s conflict of law rules, then it is impossible to determine whose law to apply.<sup>66</sup> It does not follow that no law should be applied. One of the two rules has to be changed, however, to resolve the issue.

Nevertheless, that an undecidable proposition *can* be created does not undermine the whole idea of a conflict of law system, or even a conflict of law system which permits reference to the conflict rules of another jurisdiction.<sup>67</sup> Indeed, Gödel's theorem at least suggests (and by analogy proves) that all systems of law permit the construction of undecidable propositions. Therefore, if a rule's undecidability undermines any legal system in which such a rule is possible, then it is impossible to have a legal system. So if our human nature, our ability to reason, requires a consistent legal system, we *must* be willing to admit the possibility of undecidable rules.

66. See RESTATEMENT (SECOND) OF CONFLICT OF LAWS § 8 cmt. j, illus. 2 (1971).

67. Hicks suggests that "the whole logical impasse would be avoided" with advantage "by . . . regarding all choice-of-law rules as referring to the domestic rules [i.e., the law minus choice-of-law] of the country concerned." Hicks, *supra* note 64, at 289. The solution is an example of a "Theory of Types" in which all, or certain kinds of, self-referring statements are simply banished from the system. *Id.* at 279-80. Such an answer is intuitively unsatisfying. See HOFSTADTER, *supra* note 10, at 21-23. It is not at all uncommon, for instance, for lawyers to think of and apply a conflict-of-law rule that refers to the whole law of a foreign jurisdiction. See RESTATEMENT (SECOND) OF CONFLICT OF LAWS § 8(2) (1971).



The law can refer to itself and result in undecidability within the system not only in reciprocal situations like that of the renvoi problem. For instance, consider a statute that states, "This statute cannot be reviewed by a court." This statement certainly could be codified within the law, so it is well formed. But could it be proved from statutory law? It seems not, since reality tells us that courts sometimes review statutes that purport not to be reviewable. On the other hand, the statement may well be true. It just cannot be proved.

An interesting example of such undecidability is found in the Vienna Convention on the Law of Treaties.<sup>68</sup> The treaty deals in article 53 with the question of whether there are some rules of public international law from which sovereign states may *not* derogate by treaty:

A treaty is void if, at the time of its conclusion, it conflicts with a peremptory norm of general international law. For the purposes of the present Convention, a peremptory norm of general international law is a norm accepted and recognized by the international community of States as a whole as a norm from which no derogation is permitted and which can be modified only by a subsequent norm of general international law having the same character.<sup>69</sup>

Does this article state a rule of law for parties to the Vienna Convention? First of all, one might argue that the article states a rule of law only if the article itself is a peremptory norm. Otherwise, a subsequent treaty could merely be inconsistent with article 53 itself, thereby supersede article 53, and in doing so render article 53 of no binding effect. But article 53 might also be interpreted to say that new peremptory norms cannot arise by treaty.<sup>70</sup> Because the Vienna Convention is a treaty, article 53 might plausibly be read as: "This peremptory norm cannot be a peremptory norm." Other interpretations of article 53 are also possible, albeit with similar problems,<sup>71</sup> but for our purposes it really doesn't matter, because at least the Vienna Con-

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68. Vienna Convention on the Law of Treaties, *opened for signature* May 23, 1969, 1155 U.N.T.S. 331 (entered into force Jan. 27, 1980).

69. *Id.* at 344.

70. Otherwise the article seems to contradict itself. How can it be that a peremptory norm cannot be derogated from by a treaty, but can be modified by a treaty? See *Summary Records of the 890th Meeting*, [1966] 1 Y.B. Intl. L. Commn. 310, U.N. Doc. A/CN.4/SER.A/1966 (remarks of Mr. Rosenne regarding the commentary to what is now article 53).

71. The International Law Commission's official commentary to what is now article 53 states: "As a modification of a [peremptory norm] would today most probably be effected through a general multilateral treaty, the Commission thought it desirable to indicate that such a treaty would fall outside the scope of the article." International Law Commn., *Official Commentary to the Draft Articles on the Law of Treaties, Article 50 [now Article 53]*, 61 AM. J. INTL. L. 285, 411 (1967). Special Rapporteur Sir Humphrey Waldock favored retention of this sentence in the commentary, however, "[d]espite the apparent logical inconsistency," on the ground that a general multilateral treaty could have the effect of amending a peremptory norm "if it gained sufficient acceptance by States." *Summary Records of 890th Meeting*, *supra* note 70, at 310 (remarks of Mr. Waldock).

vention *could* have been written clearly to that effect.<sup>72</sup>

Again, that such an undecidable proposition can exist does not undermine the Vienna Convention, or the idea that there is or ought to be an international legal system that includes a law of treaties, or indeed the idea that peremptory norms exist. Again by analogy to Gödel's theorem, under *any* system it is possible to construct such undecidable rules. This is in the very nature of a system of consistently generated rules of law.

#### D. *A Critique of Critiques*

Our proposition is thus that systems or bodies of law are inherently incomplete in the sense that they contain rules of law that cannot be derived within the system. We have tried to provide support for the necessary premises for such a conclusion: that the law can be analogized to the systems that Gödel examined, that the very word "law" implies a requirement of consistency, and that legal rules are sufficiently expressive to serve as their own metalanguage. It follows that constitutional systems are by nature incomplete in the sense that some law is not derivable through the rules that make up the constitutional system. Our point is that this is a characteristic of constitutional systems, and indeed of legal systems in general, and that it therefore serves as no basis for criticism of any particular constitutional system.<sup>73</sup>

A number of seemingly clever critiques of legal theories ignore this reality. One sees the fallacy again and again in varying forms. The three examples that we expose here involve critiques of legal positivism, of interpretivism and neutral principles, and of law and economics theory. There are others, but these three show the versatility of the trick. In each case the target theory is attacked, in effect, for not being able to prove itself.<sup>74</sup>

Our first example is Ronald Dworkin's argument against the positivism of H.L.A. Hart. Dworkin argues that Hart's positivism leads to unanswerable questions, whereas Dworkin vigorously asserts that even

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72. For instance, by adding the words, "and not by multilateral treaty."

73. Professor Suber reaches a somewhat different conclusion in his examination of constitutional provisions for constitutional amendment: "the logical case that self-amendment is paradoxical is strong and resists all obvious solutions; but . . . law can absorb and tolerate even a real . . . logical paradox because law is what is accepted as law, not a logical system afflicted with entangling content." SUBER, *supra* note 20, at 45.

74. For a somewhat analogous critique of Stanley Fish's deconstruction, with only passing reference to Gödel, see Pierre Schlag, *Fish v. Zapp: The Case of the Relatively Autonomous Self*, 76 GEO. L.J. 37 (1987).

hard questions of law have answers.<sup>75</sup> As the analogy to Gödel suggests, however, to say that all questions of law have answers does not lead to the conclusion that such answers are always derivable.

Dworkin attributes to Hart the thesis that “in every nation which has a developed legal system, some *social* rule or set of social rules exists within the community of its judges and legal officials, which rules settle the limits of the judge’s duty to recognize any other rule or principle as law.”<sup>76</sup> At pains to refute this, Dworkin noted that Hart accepts that social rules of recognition may be “uncertain” in some cases, so that a social rule “does not settle every issue that might arise about what counts as law.”<sup>77</sup> Dworkin refers to Hart’s inquiry about whether the English Parliament can bind itself into the future. That is, if the “rule of recognition” of England is that whatever Parliament enacts is law, what if Parliament enacts a law that limits its own ability to repeal the law (i.e., an “entrenching” law)? The rule of recognition cannot answer how a court would treat such a law. This uncertainty, per Dworkin, undermines Hart’s thesis:

It simply does not fit the concept of a social rule, as Hart uses that concept, to say that a social rule may be uncertain in the sense that Hart now has in mind. If judges are in fact divided about what they must do if a subsequent Parliament tries to repeal an entrenched rule, then it is not uncertain whether any social rule governs that decision; on the contrary, it is certain that none does.<sup>78</sup>

Dworkin thereby purports to have stated a “counterexample” to the thesis that “some social rule always exists that stipulates necessary and sufficient conditions for what judges must recognize as law.”<sup>79</sup> We must therefore look elsewhere to find the answers — answers that do exist — to constitutional questions.

In this case the “uncertain” example that Dworkin uses is transparently one of the self-referential problems that can be suggested for *any* rule for determining what the law is.<sup>80</sup> Can Parliament pass a law stating, “This law cannot be changed”?<sup>81</sup> That Parliament cannot answer this question should now come as no surprise.<sup>82</sup> We might construct a similar example to test whether “Dworkin’s rights thesis” is

75. For the latter point, see RONALD DWORKIN, *A MATTER OF PRINCIPLE* 119-45 (1985).

76. DWORKIN, *supra* note 14, at 59-60.

77. *Id.* at 61 (referring to HART, *supra* note 14, at 144).

78. *Id.* at 60.

79. *Id.* at 62.

80. Dworkin supposes that the entrenchment clause example is neither extraordinary nor rare, but does not support the supposition. *Id.* at 62.

81. The problem so stated is remarkably similar to the question of whether the House of Lords can bind itself into the future. See *supra* text accompanying notes 62-64.

82. This of course does not mean that there is no answer. We must step out of the system to

the rule for determining what the law is.<sup>83</sup>

Our second example of failure to recognize this involves the contention that "original intent" should determine constitutional interpretation. Just as it is impossible for Parliament to determine that the rule of Parliament is the rule of law, it is also impossible for original intent to determine whether original intent is the law. Yet this characteristic of any secondary rule has been used to attack original intent as a basis for determining the meaning of constitutions and statutes.

Such an attack is found in Mark Tushnet's alternative bases for rejecting original intent and neutral principles. He argues first that neither original intent nor neutral principles could sufficiently limit the power of judges to meet the "liberal" goal of individual liberty.<sup>84</sup> He argues in the alternative, assuming that there *is* some constraint on courts' decisionmaking scope, that such constraint must come from a common "underlying societal perspective,"<sup>85</sup> "shared conceptions,"<sup>86</sup> "shared societal understandings,"<sup>87</sup> a "community of understanding."<sup>88</sup> These are unexceptionable descriptions of how the polity ultimately governs the interpretation of the constitution. Tushnet's remarkable idea is that if there *is* in fact a community of understanding, then there is no need to protect individual, out-of-mainstream thought:

[W]e must develop a shared system of meanings to make either interpretivism or neutral principles coherent. But in developing such a system, we will destroy the need for constitutional theory, predicated as that need is on liberal individualism; the problem identified by Hobbes, Locke, and liberal thought in general disappears in a society in which such a shared understanding exists.<sup>89</sup>

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get the answer, though. The source ought to be whoever *gave* Parliament the power. Who did? We would say the British polity, but others might posit the Queen, or God.

83. For us to attempt to do so here would potentially subject us to the criticism that the example we chose could not really be a product of that system. A Dworkin disciple, for instance, could explain how our example did not reflect Dworkin's rights thesis. If we are right about the analogy to Gödel's theorem, however, we need not construct such an example. We can say with confidence that if his system is consistent, such examples exist.

Professor Farago has demonstrated this in his elaborate and telling criticism of Dworkin's argument that all cases have right answers. See John M. Farago, *Judicial Cybernetics: The Effects of Self-Reference in Dworkin's Rights Thesis*, 14 VAL. U. L. REV. 371 (1980). Farago nonetheless prefers Dworkin's rights thesis to Hart's positivism, essentially because Dworkin's theory leaves fewer cases unanswerable. *Id.* at 420-23. This does nothing to validate Dworkin's incompleteness criticism of Hart, however.

84. See Tushnet, *supra* note 41, at 786-821; *supra* notes 42-48 and accompanying text.

85. Tushnet, *supra* note 41, at 825.

86. *Id.*

87. *Id.* at 826.

88. *Id.*

89. *Id.* at 826-27.

In other words, for interpretivism or neutral principles to work to protect individualism, we must have a society in which there is no individualism to protect.

It may be sufficient to answer this argument by saying that Tushnet is simply mixing levels again. There can be a shared societal understanding at the level of the polity, perhaps derived ultimately from the way we think, that requires either interpretivism or neutral principles, while protecting at the next level down — constitutional interpretation — different views present in a mixed or individualistic society. Indeed, if every governmental theory to protect minority views could be challenged on the ground that the theory itself required the absence of minority views (consensus), governmental protection of most individual rights would be impossible.<sup>90</sup>

The argument is in reality a version of self-destructive legal self-reference. Tushnet has in effect ascertained within interpretivism and

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90. The interpretivist or the neutral principles advocate might try to avoid the problem by refusing to defend this particular undecidable self-referential rule. She might argue in the alternative that the polity had created a rule more akin to: "This rule permits disagreement with any statement other than this one." Certainly we could have a broad — almost universal — societal consensus on certain overriding "constitutional" principles — "first order" views like *tolerance* of different political, social, religious, and moral views that we could then call "second level" views — while contemplating a wide lack of consensus on the second level views.

One might muster considerable support for this alternative response. The remarkable nature of our republic is that our people do, by and large, have a broad consensus on such core fundamental constitutional principles, a shared societal understanding that transcends the varied bases of widely divergent potential factions. To argue that governmental tolerance of opposing ideas requires broad agreement to be effective as a policy certainly does not undermine the very notion of tolerance. We just have agreement at one level, and disagreement at another level.

Similarly, if there is a broad consensus that courts should decide cases in a legally consistent fashion, or that courts should follow the intent of the Framers (at some level of abstraction or other), it is perfectly consistent to support that consensus for the purpose of protecting differences on a host of other less fundamental issues. Arguing that such limits on courts are inherently contradictory because there must be either consensus or not-consensus ignores the possibility that there is consensus on the first order level on protecting not-consensus at the second order level. The irony (to be charitable) is that such an inherent contradiction argument helps to destroy the presumably valuable first order consensus.

That the argument tends to destroy consensus about fundamental societal assumptions regarding our Constitution is illustrated by Tushnet's deprecation of answers of judicial nominees to questions by Senators. According to him, nominees "ought to know better" than to answer "yes" to the questions: "Do you intend to apply the law rather than make it?" and "Will you apply the words of the Constitution in the way that the framers intended?" Tushnet, *supra* note 41, at 781. But asking such questions is merely one way that the polity imposes its fundamental rules upon judges. To suggest that judges should properly answer "no" is in effect to urge the polity to modify (one might say give up on) such constraints upon the judiciary.

These are the intellectual underpinnings of tyranny. If one wanted subtly to wean the American polity away from (relatively) popular control of government, to inure the polity to control of the government by an elite, the first thing to do would be to attack general attitudes of the people that constrain the role of the least representative branch of government. If the people are willing to accept the courts as legislatures, then we can have a constitution in which courts are legislatures, just as we can have a constitution in which an archduke rules if the polity accepts it. In contrast, if one wanted to preserve politically responsible government, it would be better to urge senators to seek reaffirmation of those constraints the polity may justly impose on judges.

neutral principles the possibility of saying, "This rule permits disagreement with any statement, including this one." The polity cannot impose this rule on the basis of a necessary consensus because then disagreement could not exist with respect to this very rule.

Again, the ability to identify such a statement says nothing about the relative merits of a constitutional system in which interpretivism or neutral principles is the law. By analogy to Gödel, such an undecidable statement *always* appears in any practical legal system. Otherwise a consistent system of constitutional law would be impossible.

Our third example of the fallacious use of a self-destruction argument is Arthur Leff's otherwise trenchant criticism of law and economics theory<sup>91</sup> as represented in Richard Posner's *Economic Analysis of Law*.<sup>92</sup> Leff identifies Posner's "most key" passage as a definition of efficiency to mean "exploiting economic resources in such a way that human satisfaction as measured by aggregate consumer willingness to pay for goods and services is maximized."<sup>93</sup> Leff characterizes this as implying that "you can tell what people want, and how much they want it, by seeing what they buy, and how much they pay for it."<sup>94</sup> Leff then outlines a paradox:

But if one thoroughly accepts the idea that the results people actually achieve for themselves are the ones that, among the available alternatives, they wanted to achieve, then one trembles on the edge of a worrisome paradox. For it is then also possible to state matters this way: whatever people achieve for themselves, they perceive it as the best they can do. If what they want can be determined only by seeing what they "buy," then what they "buy" must (by definition) be what they wanted (again, given the available choices).<sup>95</sup>

The "paradox" is that our society has created a political process, with resulting legislation and administration, that interferes with the market, and therefore gets in the way of achieving the very goal, aggregate human happiness, that a more market-oriented system would cause.<sup>96</sup> How can society have "bought" a system that does not result in optimum happiness (what society wants), when people's wants are defined by what they buy? Leff has thus in effect discerned the following state-

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91. Arthur A. Leff, *Economic Analysis of Law: Some Realism about Nominalism*, 60 VA. L. REV. 451 (1974).

92. RICHARD A. POSNER, *ECONOMIC ANALYSIS OF LAW* (1972).

93. Leff, *supra* note 91, at 462 (quoting POSNER, *supra* note 92, at 4).

94. *Id.*

95. *Id.* at 463.

96. *Id.*

ment in Posner's scheme: "This rule maximizes what society wants, even if society doesn't want this rule."

It should now be clear that this is but another example of a "tangled loop of law," identifiable in any legal scheme. That Leff has found one in law and economics theory does little to undermine that theory. Law and economics theory, to put it another way, cannot be used to demonstrate law and economics theory, but theories generally cannot prove themselves, and it is unfair to demand that they do so.

To be fair to Leff, it should be noted that he purports to argue not that law and economics theory is therefore internally inconsistent, but rather that one must step out of the system ("[o]ne needs an extracultural standing place") to evaluate the system's validity.<sup>97</sup> Our point, though, is that this is no more true, nor damning, for law and economics theory than for any other.

None of this is meant to argue affirmatively in favor of Hart's positivism, or interpretivism, or law and economics theory. We have to be careful of clever criticisms, however, that at bottom are not very telling. In each of these three cases, a legal theory was criticized for failing to prove itself as a rule within its own system. A key self-referential statement was identified as "contradictory." It was not really contradiction, however, but undecidability within the advocated legal theory. Finding such an undecidability cannot invalidate a consistent system, however, since every consistent system has one.

Undecidabilities can of course be very disturbing. It is troubling, for instance, that a system of tolerance cannot answer the question whether intolerance will be tolerated. But it is simply unfair to criticize a regime of tolerance for its failure to provide an answer to the question. The analogy to Gödel shows that such a "weakness" is a universal characteristic of legal schemes in which the law can refer to itself in a meaningful way.

### CONCLUSION

We can view the law as a hierarchical system of rules that generates consistent rules of law — analogous to a formal system like logic or number theory that generates a set of consistent and true statements. Each is related in some sense to the way we think. To the extent that each permits self-reference, and the system of law certainly seems to, lessons from the analogy to number theory can be drawn.

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97. *Id.* at 464. Leff proceeded to do so. He challenged Posner's analysis of the efficiency of criminal punishment, for instance, by questioning Posner's assumption that the justification for the punishment of criminal violations is deterrence. *Id.* at 464-66.

Careful attention to the hierarchical nature of the legal system can resolve certain legal fallacies by distinguishing between language and metalanguage. Analogy to Gödel's theorem teaches a more fundamental lesson: any sufficiently expressive formal system must contain undecidable propositions. Identifying such undecidable propositions in a constitutional or other legal system cannot undermine that system, since any legal system must contain rules that cannot be determined to be the law or not to be the law. Those who attack constitutional systems on grounds that fail to recognize this characteristic of all legal systems are unfairly criticizing — and indeed undermining — such systems.