Presumptions and Modal Logic: A Hohfeldian Approach

John P. Finan

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THE DIFFICULTY of distinguishing between an inference and a presumption, a difficulty that bedevils tort and evidence teachers, (see Appendix I) among others, may be dispelled by a study of the deontic nature of permissible inferences and presumptions.[1] Using scholastic terminology, an inference is a function of the intellect, not the will. Therefore, deontic notions of permission and duty seem foreign to inference. However, deontic notions are legitimate, because the law, in assigning a fact finding function to judge and jury, uses deontic notions in assigning fact finding competence.[2] Thus, the statement that an inference is not permissible means that insufficient evidence has been introduced to permit the jury to find the fact in question. It does not matter whether the jury, by applying its collective intelligence, would draw the inference. Their incompetence to draw the inference is not a function of rationality,[3] but of a rule of law that deprives them of competence.

A presumption is the opposite of a permissible inference; the jury is directed to find the presumed fact regardless of its assessment of the probative force of the evidence. That is because the law directs that a fact be found, that an inference be drawn, absent rebuttal.[4] If the presumption is

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[1] Deontic notions are treated by Hohfeld in his analysis of rights and privilege, HOHFELD, FUNDAMENTAL LEGAL CONCEPTIONS, ch. 1 (W. COOK ed. 1919). *See also* R. DIAS, JURISPRUDENCE, ch. 2 (4th ed. 1976).His treatment of jural relations also includes the notion of power, which is not deontic and is perhaps alethic. See Appendix II for a catalogue of Hohfeld's relations and their formulation in terms of modal logic. A square of opposition for deontic relationships is also included in Appendix II for the convenience of the reader in drawing immediate inferences.

[2] Furthermore logical operators used in examining truth functional compound statements may be used in examining compound statements whose components are not truth functional. *See* A. ROSS, DIRECTIVES AND NORMS at 139-43 (1968). On p. 143 Ross states:

> It is to be sure, correct that the logical connectives and inferences which operate in directive speech cannot, if we accept that directives are without truth-value, be interpreted as truth-functions and truth relations. Since they do actually function in directive speech the natural question is whether they might be *interpreted in some other way*. If it could be shown that the connectives, as they function in directive discourse, are definable by means of value tables analogous to those we know from ordinary logic, the only difference being that the two indefinables are interpreted as referring not to truth and falsity, but to another pair of values, then it would be unreasonable not to characterize the relations defined in this way as logical. Deontic logic would then be on a level with the traditional propositional calculus—a new interpretation of the same formal system, of equal status with the old. (Emphasis in original).

[3] In saying that rationality is irrelevant, we of course refer to the jury's assessment of rationality. The reason for the command not to draw the inference is often, though not always, based on the rationality of a higher court or legislature, which has determined that, if a jury should find a fact, it would not be acting rationally. In a case without a jury, a certain legislative body or superior court has decided that the trial judge may not find a given fact.

[4] At least in civil cases. *See* McCORMICK, EVIDENCE, § 342 (2d ed. 1972).

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conclusive, then the rebuttal is irrelevant because the command to draw the inference is unconditional. Thus there are three classifications:

1) impermissible inferences;
2) mandatory inferences; and
3) jury questions (inference is neither impermissible nor mandatory), or judge questions (in cases without a jury, if appellate court and legislature permit but do not mandate the inference).

The term "permissible inference" is used to refer to the third classification.

Usually the direction is conditional: *If you find the basic fact, you are directed to find the ultimate (operative) fact.* Sometimes the presumed fact is an intermediate evidentiary fact, in which case the presumption is a so-called tactical presumption. However, for economy of style, we will refer to the presumed fact as an ultimate fact, except in those cases where our analysis would be affected by the characterization of the presumed fact.

The term ultimate fact is not easy to define. The following definitions are found in Black's Law Dictionary:[5]

> Facts necessary and essential for decision by court. Those facts found in that vaguely defined field lying between evidential facts on the one side and the primary issue or conclusion of law on the other, being but the logical results of the proofs, or in other words, mere conclusions of fact.

Consider the following: Ultimate facts are facts which are of consequence to the determination of an action, not because they support a further inference, but because under the substantive law and the pleadings they are of consequence in and of themselves. An ultimate fact is a fact which is a necessary but not necessarily a sufficient condition of recovery or conviction.

An evidentiary fact is a necessary condition of liability only if basing an inference on it is the *only possible* way to prove a fact necessary to the outcome of a case. It is well nigh inconceivable that an evidentiary fact will ever be logically necessary, although it may be practically necessary, given the difficulty of obtaining other proof. Even if necessary, however, an evidentiary fact would not fit the above definition of an ultimate fact because, by definition, its function is to provide the basis for a further inference.

An element of a crime is a necessary condition of conviction and hence an ultimate fact. If there is only one element, it is also a sufficient condition. However, if there is more than one element, as there usually is, then it is not a sufficient condition.

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[5] BLACK'S LAW DICTIONARY 1691-92 (4th ed., rev. 1968).

A permissible inference is symbolized as follows:[6]

> P H (x,[7] p, y)
> It is permissible that x (the trier of fact) find the ulti-
> mate fact.[8]

Every inference requires a basic fact. So we have:
> B F   )[9]   P H (x, p, y)
> If the basic fact is established, then, it is permissible
> that x (the trier of fact) find the ultimate fact.

A mandatory inference is symbolized as follows:
> O H (x, p, y)
> It is obligatory that x (the trier of fact) find the ulti-
> mate fact. Adding the basic fact, we have:
> B F   )   O H (x, p, y)
> If the basic fact is established, then it is obligatory that
> x (the trier of fact) find the ultimate fact.

The above is a conclusive presumption. A rebuttable presumption is symbolized as follows:

> (B F and[10] $\overline{R}$)[11]   )   O H (x, p, y)
> If basic fact is established and the ultimate fact is not
> rebutted, then it is obligatory that x (the trier of fact)
> find the ultimate fact.

Using equivalent terminology, a permissible inference equals either

> P H (x, p, y)
>   or
> $\overline{O}$ $\overline{H}$ (x, p, y)

The latter means, it is not obligatory that x not find the ultimate fact. This formulation does not warn us of difficulty. However, difficulty becomes

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[6] The reader is directed to Appendix II and to the following articles: Mullock, *Holmes on Contractual Duty*, 33 PITT. L. REV. 471 (1972), and Anderson, *The Logic of Hoh-feldian Propositions*, 33 PITT. L. REV. 29 (1972), which will provide the necessary theoretical background for an understanding of the remainder of this article. Note that this article follows Anderson's article in usinig H, instead of M which Professor Mullock uses, for the 3-place predicate.

Note also that this article uses D more broadly than does Professor Mullock, as explained in Appendix II which should be consulted at this time.

[7] x = trier of fact. (jury or judge as the case may be).

[8] A more accurate, but certainly awkward, translation reads:
It is permissible (P) that the trier of fact (x) bring about (H) the social condition (p) with respect to the person who will suffer a legal detriment (y) if the inference is drawn. For convenience we will refer to y as the defendant, with the understanding that this is not always the case (*e.g.*, if defendant is asserting an affirmative defense).

[9]) = implies or more precisely, . . .) . . . = If . . . then . . .

[10] "And" is usually symbolized but since its symbolization can not be reproduced, the full word is used.

[11] — = not. $\overline{R}$ = not R = the presumption is not rebutted.

apparent when we consider the negative formulation of a permissible inference, the so called impermissible inference.

The negative is formulated as follows:

$\overline{P}$ H (x, p, y) which is equivalent to O $\overline{H}$ (x, p, y).[12]

The former, in non-symbolic language reads: It is not permissible that x find the ultimate fact. The latter, in non-symbolic language reads: It is obligatory that x not find the ultimate fact.

Stated more conventionally, the former may be translated to read:

    I. The jury is not permitted to find the ultimate fact,
       and the latter may be translated to read:

    II. The jury is obliged (has a duty) not to find the
       ultimate fact.

Translation No. II above probably is the best rendition to demonstrate the inaccuracy of the formulation. It is inaccurate because the jury *lacks the power* to find the ultimate fact; and is senseless to speak of a duty *not* to create a legal relationship if one lacks the power to create it. *It is the judge who has the duty, the duty not to empower the jury* to find the ultimate fact. Thus, formulated accurately, a *permissible* inference is a mandatory one; the *judge* has a *duty,* formulated as follows:

    I   O $\overline{H}$ (j, Dp,[13] y)
          or
    II $\overline{P}$ H (j,[14] Dp, y)

    I = it is obligatory that the judge not empower the
       jury to find the ultimate fact.

    II = it is not permissible that the judge empower the
       jury to find the ultimate fact.

The judge's duty is twofold: he has a duty not to empower the jury and he has a duty to find that the ultimate fact is false. His second duty may be formulated as follows:

    O H (j, $\overline{p}$, y)
       or
    $\overline{P}$ $\overline{H}$ (j, $\overline{p}$, y)

What of presumptions? With respect to "impermissible inferences," it is clear that the jury has no power to find the ultimate fact. Thus, it is

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[12] *See* Appendix II, p.19.

[13] Note that this a mixed modality involving a duty not to create a power. If power is symbolized Z H (x, Dp, y) and if Dp is shorthand for Z H (x, Dp, y), then O $\overline{H}$ (j, Dp, y) is shorthand for O $\overline{H}$ (j, Z H (x, Dp, y),y). *See* Appendix II.

[14] j = judge.

senseless to speak of a duty not to find the fact. With presumptions, however, the matter is more complex.

If a jury is required to find the ultimate fact, given a finding that the basic fact exists, then, unless the ultimate fact is rebutted by a preponderance of the evidence (or other requisite quantum), we have a so called "tough presumption." If we are dealing with a tough presumption and the judge finds, as a matter of law, that the basic fact has been established and that the evidence rebutting the ultimate fact is insufficient to support a jury verdict, then the case does *not* go to the jury, and the jury has *no power* to find that the ultimate fact is not true. Hence it would be senseless to speak of their duty not to find the ultimate fact. On the other hand, if there is enough evidence to support a jury verdict that the basic fact is true but not enough evidence to obtain a peremptory ruling that the basic fact is true, and if there is enough evidence rebutting the ultimate fact to support a jury finding that the ultimate fact is false, but not enough to obtain a peremptory ruling, then the jury has the power either to find or not to find the ultimate fact under the following instructions:

> If you find that the proponent has proved the basic fact by a preponderance of the evidence (or other requisite quantum) and you find that the opponent has failed to rebut the ultimate fact by a preponderance of the evidence (or other requisite quantum), then you are *directed* to find that the ultimate fact is true.

If the jury should find that the basic fact is true and that the rebutting evidence is insufficient to carry the opponent's burden of rebutting the truth of the ultimate fact then it has the *duty* to find that the ultimate fact is true, but it also has the *power to breach that duty,* at least under a general verdict. Thus, if a jury *should find that the ultimate fact does not exist,* it would have exercised *a power to affect legal relations and at the same time have breached a duty.*

The notion of a power coexisting with a duty not to exercise the power is not an unfamiliar one in the law.[15] For example, when a creditor in a suretyship situation releases a principal debtor with a "reservation of rights," the release is treated as a covenant not to sue. In this situation, the creditor has both the *duty* not to sue, and the *power* to sue.[16] Thus, presumptions must be broken down into two categories. The *first* is where the jury lacks power, as when the judge finds an ultimate fact as a matter of law without submitting an issue to the jury. The *second* is where the jury, though having the power not to, has a duty to find the ultimate fact. Here the jury *may*[17] breach its

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[15] *See* Appendix II.

[16] *See also* RESTATEMENT (SECOND) OF CONTRACTS § 170, Illustration 1 (Tent. Draft Nos. 1-7 rev. & ed. 1973), where an obligee, prior to the obligors receiving notification of an assignment, has the power to discharge the debtor-obligor by receiving payment. Surely the obligee, who has assigned his claim, has a duty not to discharge the obligor.

[17] Has the power to breach its duty.

duty. In the first situation, the formulation of the legal relationships would be $\overline{Z}$ H (x, Dp, y).[18] In the second situation, where the jury has the power, but the duty not to exercise the power, the appropriate formulation would require two modal propositions as follows:

> 1) Z P (x, Dp, y)
>
> x has the power to create a certain modal (in this case deontic) relation affecting y.
>
> 2) $\overline{P}$ H (x, Dp, y)
>
> x is not permitted to create a certain modal (in this case deontic) relation affecting y.

A glance at the last two formulations suggests why we have substituted Z for P as the modal operator respecting power. It also suggests that the word *permitted* can be used in two senses. First, one may be *permitted* to bring about a deontic relation in the sense that if one does a certain *act in law,* the legal system attaches a legal consequence, that is, it creates the deontic relationship. Second, one may be *permitted* to bring about a deontic relation in the sense that if one brings it about by performing the necessary *act in law,* he breaches no duty. Using *permitted* in the first sense, one is *permitted* if he is able to affect legal relationships in a given legal context.[19] The *second sense* of the word *permitted* (the more conventional one, we submit) is that, to say that one is *permitted to bring* about a deontic relation means that he *breaches no duty in doing the act in law* to which the legal system attaches legal consequences.

There is no quarrel with using P in the former sense to express the notion of power, but we are doubtful that the use of the letter P in that sense signifies a deontic relation. Would M, the alethic operator for possibility be more appropriate?[20] The difficulty we have with M is that we are talking about legal possibility, not logical possibility. It is not my intention in this presentation to even attempt to solve this difficulty. I would ask the reader to reflect whether: (1) P is correct as used by Professor Mullock, 2) M is correct, or 3) there is some other substitution for Z which more accurately reflects the nature of the modal operator for legal power.

Query: Until we can identify with confidence a particular modal operator, would it not be better to use the modal variable Z? Query: does the system R mentioned in the Anderson article help?[21] If we say that "p relevantly implies q," means that "p implies q in a relevant legal system," do we

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[18] *See* Appendix II for the meaning of modal operator Z. Mullock *supra* note 6, would use P.

[19] The notion of a given legal context is critical. Is that what the system R means by a relevant condition? May we use M to designate ability, as L is used to designate necessity within a relevant legal system; *i.e.,* if a holder does not take for value, it necessarily follows under the U.C.C. that he is not a holder in due course?

[20] *Id.*

not use "implies" in a stronger sense than material implication? May we not say that a result necessarily follows, or necessarily does not follow, in a given legal system?

If "relevant necessity" may be used to express a statement of law expressed hypothetically; if juristic fact x then legal consequence y necessarily follows; then may not "relevant possibility" be used to express power? Relevant possibility means legal ability or legal possibility. Thus, Z H (x, Dp, y) means x has the legal ability, or it is legally possible that x bring about a given deontic relation affecting y. Does Z $=$ M (or perhaps Mr^{22} to distinguish logical possibility, M, from legal possibility)?

APPENDIX I
The Semantic Muddle (Unraveled?)

The following is a list of names given to various inferences and presumptions. The prolixity of names and the penchant of writers to use the names interchangeably is partially responsible for the confusion in this area. The use of the term "prima facie case" to designate II, III or IV is of course a serious obstacle to understanding; as is a similar use of res ipsa loquitur.

I   Natural Inference
     "Roll your own" inference

II Permissible Inference
     Standardized Inference
     Permissive Presumption
     Presumption of Fact
     Prima Facie Case

III Mandatory Inference — rebuttable by enough evidence to support a jury verdict
     Weak Presumption
     "Burst the Bubble" Presumption
     Tender Presumption
     "Smoke em out" Presumption
     Prima Facie Case

IV Mandatory Inference — rebuttable by enough evidence to carry burden of persuasion
     Strong Presumption
     Tough Presumption
     Prima Facie Case

V Conclusive Presumption
     Mandatory Inference — Nonrebuttable

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[21] Anderson, *supra* note 6, at 34.
[22] M, subscript r.

APPENDIX II

Hohfeldian relationships are basically two-term relationships. A table of the relations follows:

	PRIMARY RELATIONSHIPS		SECONDARY RELATIONSHIPS	
JURAL CORRELATIVES	Right	Privilege	Power	Immunity
	Duty	No-right	Liability	Disability
JURAL OPPOSITES	Right	Privilege	Power	Immunity
	No-right	Duty	Disability	Liability

The above jural correlatives each involve two terms; x (an agent) and y (a patient). The same holds true for privilege, sometimes called a liberty.[23] If x has a privilege, y has no right that x will not do the privileged act. Rights and privileges and their correlatives and opposites make up the primary jural relations. The secondary relations also involve two terms. An agent and a patient. Thus, if x has a power, y has a liability to have his legal relationships affected by the exercise of the power. If x has an immunity, y has a disability to affect the legal relations between himself and x in the domain where the immunity exists.

Professor Anderson, in using the insights culled from Hohfeld, suggests that there are more than two terms in Hohfeldian relationships, that there are at least three terms;[24] for every relationship involves an agent, a patient and a state of affairs. The agent and patient are symbolized by x and y, the state of affairs by p; x, y and p are enclosed within parenthesis. In Anderson's formulation, preceding the parenthesis we have a three-place predicate, H, which joins the three relationships together. Thus H (x, p, y) gives us our atomic formula over which deontic operators may range, the deontic operators being O for obligation and P for permission. Thus, x is obliged to pay y would be symbolized, O H (x, p, y). x is permitted to pay y would be symbolized by P H (x, p, y).

The Anderson article confines itself to the primary relationships whose primitive term is duty ("right," "no right" and "privilege" are definable in terms of duty). The secondary relationship's primitive term is power, (immunity, disability and liability may be defined in terms of power). There is no modal notation in the Anderson article[26] for the notion of power. However, in the article by Professor Mullock,[27] a symbol is introduced, D, which symbolizes a deontic relationship. Thus, if x instead of paying p, brings about the deontic relationship that he (x) ought to pay p (*i.e.,* exercises a power),

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[23] *See* DIAS *supra* note 1, at 35.

we symbolize that by saying H (x, Dp, y), which is a shorthand for H (x, O H (x, p, y), y). In the Mullock article, [28] D is used as a deontic variable ranging over O (obligation) and P (permission). In this article we use the term D more broadly, as explained below.

If one exercises a power, he frequently brings about a *deontic* relationship, but he also may bring about a power relationship. Thus, if y has a power of acceptance and he exercises that power by performing a jural act, which under applicable law constitutes acceptance, he creates a *duty* (a deontic relationship) owing to him from the offeror; and if a bilateral contract is involved, a corresponding duty owing from him to the offeror. Prior to the offer, however, x had the power to make an offer and thus to create a legal relationship between himself and y. However, that relationship was not deontic. It involved neither permission nor obligation; rather it involved a power. x, by exercising his power to make an offer, *creates in y another power* and in himself a corresponding liability. It is not clear what is the correct modal operator symbolizing power. We suggest that Z[29] be used. Professor Mullock uses P.[30] However, it is submitted that if P is used in the sense of permission, this is an inaccurate symbolization of power because we frequently have in law, as well as in morals, situations where an individual (x) has a power but a duty not to exercise that power.[31] M (symbolizing possibility) is, we submit, a better substitution instance for Z than is P,[32] but it is entirely possible that the appropriate substitution for Z is an as yet undeveloped modal operator signifying power.

A power may be a power to create a deontic relationship, or the power to create a secondary relationship; a relationship defined in terms of power. Thus we use Dp to symbolize not only deontic relationships but also power relationships (be they alethic or some other modality). Thus, if x has the *power* to create a *power* (i.e., the power to make an offer which will empower the offeree to create a deontic relationship by acceptance), his power would be symbolized Z H (x, Dp, y). The offeree's *power* to create a *deontic* relationship would also be symbolized Z H (x, Dp, y). The suggested symbolism does not distinguish between the power to create a power (alethic?) relationship and a power to create a deontic relationship. For purposes of this article on presumptions such a distinction does not seem to be necessary.

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[25] Anderson, *supra* note 6.

[26] *Id.*

[27] Mullock, *supra* note 6, at 475.

[28] *Id.*

[29] Z is a modal variable ranging over P and O, M and L, and also X an undefined modal operator signifying power. L is the Lukasiewicz modal operator for monadic necessity, M is the Lukasiewicz modal operator for monadic possibility. *See* D. SNYDER, MODAL LOGIC (1971) where the notations for LUKASIEWICZ's modal operators are set forth.

[30] Mullock, *supra* note 6, at 471, 477.

[31] *See* DIAS, *supra* note 1, at 56.

[32] *See* p. 24-25 *infra.*

However, if greater refinement is thought necessary, Ap instead of Dp might be used for power relationships.

We have treated a *power to create a power* (or some other secondary relationship defined in terms of power) as itself a *power*. Not all agree. It is stated by Stone[33] that "A's 'power' " to mail a letter to B offering to buy Whiteacre from B for 1,000 pounds" is not a power in the Hohfeldian sense, since it does not "change B's legal relations."[34] It is merely a "physical ability" and a "legal privilege (or liberty)."[35] In order to take this (Stone's) view, one would have to exclude *power* from the definition of a *legal relationship*. But surely, we submit, one who has a power to make a will containing a power of appointment has the power to create a legal relationship between the donee of the power and those who are liable to the donee's exercise of it. Referring to Stone, quoted above,[36] Dias states that Stone "seems to assume that liability [to the exercise of a power] necessarily contemplates a change for the worse. That is not what Hohfeld meant by it."[37]

Resolution of this dispute is necessary in dealing with the notion of the duty of a judge not to exercise his power to empower the jury to create a deontic relationship;[38] for if the judge lacks the power to empower the jury,[39] it makes little sense to speak of his duty not to do the impossible.

In siding with the position that a *power to create a power* is itself a *power*, we have created a difficulty in symbolizing power which would not be present if Stone's definition were accepted. If one has a power to create a power and the *created power* is itself a *power* to create a deontic relationship, we must have a notational device to represent two power relationships and one deontic relationship. This creates difficulties which are not present when Dp symbolizes only a deontic relationship. By enlarging the definition of Dp to include "power" relationships, we have created the following difficulty.

When Dp is a symbol for a "power" relationship, Dp by definition[40] equals Z H (x (y, Dp, x), y). Thus Dp is found in both the definiens and the definiendum. Obviously the power to create another power is a power to create some other power than the one being exercised. Dp in the definiendum represents a different power than does Dp in the definiens. It might be desirable to introduce a notation to make this distinction, but we believe that context makes the meaning clear and that the introduction of an additional

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[33] J. STONE, LEGAL SYSTEMS AND LAWYERS' REASONING 147 (1964).

[34] *Id.*   [35] *Id.*   [36] *Id.*

[37] DIAS, *supra* note 1, at 48 n.1.

[38] *See* p. 5 *infra.*

[39] For a discussion of a *power* and *a duty not to exercise it, see* DIAS, *supra* note 1, at 56-58. For a full discussion, read DIAS at 47-58.

[40] It should be remembered that Dp is a variable which represents either a "different power" or a deontic relationship.

notation would be an unnecessary refinement, at least for the purpose of this article which is limited to exploring the nature of presumptions.

Hohfeld's primary relationships formulated in terms of deontic logic follow. Notice that there are two formulations for each relationship and that each of these formulations is equivalent to its companion formulation. Notice also that negatives are added in accordance with modern corrections.[41]

Right	**Privilege**
O H (x, p, y)	P H (x, p, y)
$\overline{P}\,\overline{H}$ (x, p, y)	$\overline{O}\,\overline{H}$ (x, p, y)
Duty to do	**No Right**
O H (x, p, y)	(y has no right that x not do)
$\overline{P}\,\overline{H}$ (x, p, y)	P H (x, p, y)
	$\overline{O}\,\overline{H}$ (x, p, y)
No Right	**Duty (not to do)**
(y has no right that x do)	$\overline{P}$ H (x, p, y)
$\overline{O}$ H (x, p, y)	O $\overline{H}$ (x, p, y)
P $\overline{H}$ (x, p, y)	

The above table is reproduced below using non-symbolic language.

(Demand) Right	**Privilege**
y has a demand right that x do p — x is not permitted not to do p for y	x is permitted to do p, "against" y — x has no duty not to do p "against" y
Duty	**No Right**
x has a duty to do p for y — x is not permitted not to do p for y	x is permitted to do p against y — x is not obliged not to do P "against" y
No Right	**Duty**
y has no demand right that x do p — x is permitted not to do p for y	x is not permitted to do p against y — x is obliged not to do p against y

So *duty* is different when it is a *correlative of demand right* than when it is the *opposite of privilege*. Duty means respectively:

$$\text{O H}\,(x, p, y) — \text{O}\,\overline{\text{H}}\,(x, p, y)$$

$$\overline{\text{P}}\,\overline{\text{H}}\,(x, p, y) — \overline{\text{P}}\,\text{H}\,(x, p, y)$$

No right is different when it is the *opposite of a demand right* than when it is the *correlative of privilege*. *No right* means respectively:

$$\overline{\text{O}}\,\text{H}\,(x, p, y) — \overline{\text{O}}\,\overline{\text{H}}\,(x, p, y)$$

$$\text{P}\,\overline{\text{H}}\,(x, p, y) — \text{P H}\,(x, p, y)$$

One advantage of formulating basic jural relations in symbolic terms is that a square of opposition can be created. The obvious advantage of a square is that it allows the drawing of immediate inferences. The square for the primary relations follows:

SQUARE OF OPPOSITION

A	E
$\text{O H}\,(x, p, y)$	$\text{O}\,\overline{\text{H}}\,(x, p, y)$
$\overline{\text{P}}\,\overline{\text{H}}\,(x, p, y)$	$\overline{\text{P}}\,\text{H}\,(x, p, y)$
I	**O**
$\text{P H}\,(x, p, y)$	$\overline{\text{O}}\,\text{H}\,(x, p, y)$
$\overline{\text{O}}\,\overline{\text{H}}\,(x, p, y)$	$\text{P}\,\overline{\text{H}}\,(x, p, y)$

A	E
Duty (to do)	Duty (not to do)
I	**O**
Privilege (to do)	Privilege (not to do)

The above meets the criteria for a square because:

A and E can both be false but not both true.

I and O can both be true but not both false.

A and O cannot both be true or both be false, they are contradictories.

E and I cannot both be true or both be false, they too are contradictories.

An example of an immediate inference using the above is:
If it is false that x has a duty to do p (*i.e.*, if A is false); then x has a privilege not to do p (*i.e.*, if O

is true). In symbolic form this inference reads:

$$\overline{O}\, H\,(x, p, y) \qquad) \qquad P\, \overline{H}\,(x, p, y)$$

Although the development of deontic logic greatly aids in analysis, it should be noted that according to Dias, Jeremy Bentham (1748 - 1832), "evolved a 'deontic logic' with which to demonstrate the relationship between command, prohibition and permission."[42]

<div align="center">Decided aspects—obligative</div>

Command (C) Positive directive (Do X)		Prohibition (P) Negative directive (Do not do X)
Non-prohibition (NP) Negative non-directive (Permission to do X)		Non-command (NC) Positive non-directive (Permission not to do X)

<div align="center">Undecided aspects—unobligative[43]</div>

SECONDARY RELATIONSHIPS IN SYMBOLIC FORM

$P\, H\,(x, Dp, y)$
x has power over y

$\overline{P}\, H\,(x, Dp, y)$
y has an immunity against x's power

$P\, H\,(x, Dp, y)$
y has a liability to x's power

$\overline{P}\, H\,(x, Dp, y)$
x has a disability to affect y's right

$\overline{P}\, H\,(x, Dp, y)$
x has a disability to affect y's rights

$P\, H\,(x, Dp, y)$
y is liable to x's power