# Catastrophe Modeling with Financial Applications 

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Catastrophe Modeling with Financial Applications
Honors Research Project


#### Abstract

Catastrophe modeling is used to prepare for losses caused by natural catastrophes such as earthquakes, hurricanes, or tornadoes and man-made catastrophes such as terrorism. Modeled data can be used to create a comprehensive distribution of possible disasters. The distribution gives probabilities of potential catastrophes of different severities occurring over a certain time frame. Calculating potential losses and probability of those losses occurring allows insurance companies to plan and reserve enough money to protect themselves from catastrophic events. Using a catastrophe case study posted online from the Casualty Actuarial Society and R software, this paper shows the use of statistical techniques to create an Exceedance Probability plot for possible losses from a set of hurricanes with varying loss severity (CAS 18). The creation of the probability plot will then be used on a set of data called "SP500_2000to2015_SM" to show how the use of catastrophe modeling can apply to financial data.


## Initial Catastrophe Model

| Event <br> $\left(\mathrm{E}_{\mathrm{i}}\right)$ | Description | Annual <br> probability of <br> occurrence <br> $\left(\mathrm{p}_{\mathrm{i}}\right)$ | Loss <br> $\left(\mathrm{L}_{\mathrm{i}}\right)$ |
| :---: | :---: | :---: | :---: |
| 1 | Category 5 Hurricane | 0.002 | $\$ 10,000,000$ |
| 2 | Category 4 Hurricane | 0.005 | $\$ 5,000,000$ |
| 3 | Category 3 Hurricane | 0.010 | $\$ 3,000,000$ |
| 4 | Category 2 Hurricane | 0.020 | $\$ 2,000,000$ |
| 5 | Category 1 Hurricane | 0.030 | $\$ 1,000,000$ |

Table 1 Hurricane Table
Table 1 above from the CAS case study shows a set of 5 different categories of hurricanes that can occur independently throughout a given year (Olson 7). In this example, each hurricane has a probability of occurrence over a one-year period that is listed in the column labeled "Annual probability of occurrence $\left(\mathrm{p}_{\mathrm{i}}\right)$ ". The column labeled "Loss $\left(\mathrm{L}_{\mathrm{i}}\right)$ " states the losses that are incurred when each individual hurricane occurs. A probability table for possible losses can be made from the above data to see the probabilities of losses if multiple hurricanes were to occur over a year.

To create a probability table for possible losses from the above set of hurricanes, an assumption must first be made. The table will cover all the possible losses that can occur in a one-year timespan assuming that each type of hurricane can only occur once throughout the year. In one year, there are a total of thirty-two different combinations of the above hurricanes that could occur with possible losses that range from $\$ 0$ (no hurricane occurs) to $\$ 21$ million (each hurricane occurs). The probability of each combination must be calculated to create the table. Every combination consists of each category of hurricane either occurring or not occurring. For each combination, multiplying the probabilities of each outcome for the individual hurricane categories (did the hurricane occur or not) will give an overall probability for that combination
happening during the year. For example, the probability of a $\$ 15$ million loss due to category 5 and category 4 hurricanes would be calculated as $\mathrm{P}($ Category $5 \cap$ Category $4 \cap$ Not Category 3 $\cap$ Not Category $2 \cap$ Not Category 1). Since each of these events occur independently of each other, it can be written as $\mathrm{P}($ Category 5) $* \mathrm{P}($ Category 4$) * \mathrm{P}($ Not Category 3$) * \mathrm{P}($ Not Category 2)* $\mathrm{P}($ Not Category 1$)$. Using the respective probabilities from Table 1., the probability of occurrence for a $\$ 15$ million loss from the specific combination of a category 5 and category 4 hurricane, $(.002)^{*}(.005)^{*}(.990)^{*}(.980)^{*}(.970)$, equals $9.410940 \mathrm{e}-06$. This calculation can be done for each of the thirty-two possible combinations of hurricanes to finish the table, but this takes a lot of time and can only be applied to the probabilities and losses given in Table 1. Using the software R , the table can be created at a much faster rate and is not limited to the specific numbers from the table.

| Hurricane Loss Probability Table (\#1 in Appendix: Code) | > All 10 Sort |  |  |  |  |  | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | [1, ] | - | 0 | 0 | 0 | - All.size | 9.345158e-01 |
|  | ${ }_{[2,1]}$ | $\bigcirc$ | $\bigcirc$ | - | $\bigcirc$ | $1 \quad 1$ | 2.890257e-02 |
|  | [3, | - | - |  | 2 | ${ }_{3}$ | $1.907175 \mathrm{e}-02$ $9.43953 \mathrm{e}-03$ |
| Using the software R, the table to the right, Table 2, | ${ }_{[5,1}^{[4,}$ | $\bigcirc$ | - | ${ }^{3}$ | 2 | 1 3 | 9.439553e-03 |
|  | [6, ] | 0 | 0 | 3 | - | 14 | $2.919449 \mathrm{e}-04$ |
| was made to show all possible combinations of the 5 differen | [7, ] | $\bigcirc$ | 5 | ${ }_{3}$ | 0 | - 5 | 4.696059e-03 <br> 1.926439 e |
| categories (code uses loss severity) and their respective | [9, ] | 0 | 5 | 。 | - | 16 | $1.452389 \mathrm{e}-04$ |
|  | [10, ] 111, | O | ${ }_{5}^{\circ}$ | ${ }^{3}$ | 2 | $1{ }_{0}$ | 5.958060e-06 |
| probabilities that can be made using the information from | [12, ${ }^{\text {[12, }}$ | - | 5 | 3 | - | $0^{8}$ | 4.743494e-05 |
|  | [14, ] | $\bigcirc$ | 5 | 3 | - | 19 | 2.964060e-06 |
|  | [15,] | 10 | 0 | - | $\bigcirc$ | 010 | $1.872777 \mathrm{e}-03$ |
| multiply the probabilities of the individual hurricane events in | ${ }_{[16,1}$ | 10 | 5 | ${ }^{3}$ | ? | 10 | 5.689609e-07 |
|  | [18 | 0 | 5 | 3 | 2 | 111 | $2.9940000-08$ |
| each combination to calculate the probability of the | [19, ] | 10 | $\bigcirc$ | $\bigcirc$ | 2 | $\bigcirc 12$ | 3.821994e-05 |
| combination occurring (All Prob). Each row not only contain | ${ }_{[21,1}$ ] | 10 | - | - | 2 | 3 | 1.891694e-05 |
| combination occurring (All.Prob). Each row not only contains | [22,] | 10 | 0 | 3 | - | 114 | 5.850600e-07 |
| the probability of the combination occurring, but also the total | [23,] | 10 | 5 | ${ }_{3}$ | ${ }_{2}$ | 15 | 9.410940e-06 $3.860600-07$ |
|  | [25,] | 10 | 5 | - | 0 | 16 | 2.910600e-07 |
| severity of the loss (All.Size) and the individual hurricane | ${ }_{[27,1}^{[26,]}$ | 10 | 5 | ${ }^{3}$ | 2 | 17 | 1.920600e-07 |
| losses that sum up to the total severity. It can be from both the | ${ }_{[29,]}^{[29,]}$ | 10 | 5 | 3 | ${ }_{2}$ | 18 | 9.506000e-08 <br> 5.940000 e |
|  | [30,] | 10 | 5 | 3 | 0 | 19 | $2.9400000-09$ |
| Table 2 and the graph of All.Prob below (Graph 1) seen that | $\begin{aligned} & {[31,]} \\ & {[32,]} \end{aligned}$ | $10$ | 5 | 3 | 2 | 20 | $\begin{aligned} & 1.9400 \\ & 6.0000 \\ & \hline \end{aligned}$ | the higher the loss, the less likely the combination occurs. This

[^0] Table
is due to the fact that the higher category hurricanes have lower probabilities of occurrence.

## Notes About R Model

The loss probability table is great for checking combination probabilities for each of the thirty-two different combinations of hurricanes. Since the code uses vectors that contain the probabilities of occurrence and loss amounts for each individual hurricane category, the numbers can be modified for any other example that includes five category hurricanes. The severities and probabilities will be altered in the table depending on the numbers in the vectors used to create the table. The total number of categories for hurricanes can also be changed if a model requires more or less than five categories.

The loss probability table does contain multiple rows that have the same total loss severity. For example, hurricanes with losses [10,5] sum up to the same total loss severity as hurricanes with losses [10,3,2]. The probability that a specific loss severity occurs in one year for a severity that can only happen from one combination of hurricanes, such as $\$ 4$ million (\$3 and \$1 million losses), is the probability listed in the table for that given combination. If the loss severity can result from multiple combinations like the $\$ 15$ million shown before, the probabilities can be added together since they are mutually exclusive (they cannot happen at the same time) to calculate the probability that that specific loss severity occurs during the year. After you find the probability for each specific loss as shown in the table to the right (Rows are losses and Columns are probabilities of those losses), you can calculate the exceedance probability curve.

|  | $[, 1]$ | $[, 2]$ |
| ---: | ---: | ---: |
| $[1]$, | 0 | $9.345158 \mathrm{e}-01$ |
| $[2]$, | 1 | $2.890255 \mathrm{e}-02$ |
| $[3]$, | 2 | $1.907175 \mathrm{e}-02$ |
| $[4]$, | 3 | $1.002940 \mathrm{e}-02$ |
| $[5]$, | 4 | $2.919449 \mathrm{e}-04$ |
| $[6]$, | 5 | $4.888703 \mathrm{e}-03$ |
| $[7]$, | 6 | $1.511970 \mathrm{e}-04$ |
| $[8]$, | 7 | $9.583794 \mathrm{e}-05$ |
| $[9]$, | 8 | $5.039900 \mathrm{e}-05$ |
| $[10]$, | 9 | $1.467060 \mathrm{e}-06$ |
| $[11]$, | 10 | $1.873745 \mathrm{e}-03$ |
| $[12]$, | 11 | $5.795088 \mathrm{e}-05$ |
| $[13]$, | 12 | $3.821994 \mathrm{e}-05$ |
| $[14]$, | 13 | $2.009900 \mathrm{e}-05$ |
| $[15]$, | 14 | $5.850600 \mathrm{e}-07$ |
| $[16]$, | 15 | $9.797000 \mathrm{e}-06$ |
| $[17]$, | 16 | $3.030000 \mathrm{e}-07$ |
| $[18]$, | 17 | $1.920600 \mathrm{e}-07$ |
| $[19]$, | 18 | $1.010000 \mathrm{e}-07$ |
| $[20]$, | 19 | $2.940000 \mathrm{e}-09$ |
| $[21]$, | 20 | $1.940000 \mathrm{e}-09$ |
| $[22]$, | 21 | $6.000000 \mathrm{e}-11$ |

Table 3 Probability Table for each size of Loss

## Exceedance Probability for Hurricanes

Once the probability for each size of loss is known, exceedance probabilities can be calculated. The exceedance probability of a certain size of loss is the probability that a loss of that size or more occurs (CAS 18). Since each event is mutually exclusive, for each event the exceedance probability is calculated by adding up the probability of the event happening and every even with a loss greater than its own (Olson 5). This would take an extensive amount of time by hand, but in R it can be done much faster. The table of exceedance probabilities can be seen to the right and the plot of the probabilities (Exceedance Probability Curve) can be seen below. Each point on the curve shows the probability that in the time span of one year a loss of that severity of loss or greater will occur.

|  | $[, 1]$ | $[, 2]$ |
| ---: | ---: | ---: |
| $[1]$, | 0 | $1.000000 \mathrm{e}+00$ |
| $[2]$, | 1 | $6.548425 \mathrm{e}-02$ |
| $[3]$, | 2 | $3.658170 \mathrm{e}-02$ |
| $[4]$, | 3 | $1.750995 \mathrm{e}-02$ |
| $[5]$, | 4 | $7.480547 \mathrm{e}-03$ |
| $[6]$, | 5 | $7.188602 \mathrm{e}-03$ |
| $[7]$, | 6 | $2.299899 \mathrm{e}-03$ |
| $[8]$, | 7 | $2.148702 \mathrm{e}-03$ |
| $[9]$, | 8 | $2.052864 \mathrm{e}-03$ |
| $[10]$, | 9 | $2.002465 \mathrm{e}-03$ |
| $[11]$, | 10 | $2.000998 \mathrm{e}-03$ |
| $[12]$, | 11 | $1.272529 \mathrm{e}-04$ |
| $[13]$, | 12 | $6.930200 \mathrm{e}-05$ |
| $[14]$, | 13 | $3.108206 \mathrm{e}-05$ |
| $[15]$, | 14 | $1.098306 \mathrm{e}-05$ |
| $[16]$, | 15 | $1.039800 \mathrm{e}-05$ |
| $[17]$, | 16 | $6.010000 \mathrm{e}-07$ |
| $[18]$, | 17 | $2.980000 \mathrm{e}-07$ |
| $[19]$, | 18 | $1.059400 \mathrm{e}-07$ |
| $[20]$, | 19 | $4.940000 \mathrm{e}-09$ |
| $[21]$, | 20 | $2.000000 \mathrm{e}-09$ |
| $[22]$, | 21 | $6.000000 \mathrm{e}-11$ |

Table 4 Hurricane Exceedance
Probabilities


Graph 1 Hurricane Exceedance Probability Curve

## Financial Data - S\&P 500 Stock Market Prices 2000-2015 (\#2 in Appendix: Code)

The loss probability table and exceedance probability curve used for the hurricane model has applications other than catastrophe modeling. The stock market is continually changing and losses can occur when the price of stock decreases. The loss probability model can function in the realm of stocks similar to how it did with catastrophes. Instead of using categories of hurricanes as independent events in the model, companies that sell stock will be used. For this paper, the stock data of Microsoft, Disney, Amazon, Bank of America, and McDonalds from the 2000-2015 S\&P 500 will be used (SP500).

Since stock prices vary for most companies, it is important to look at how prices change percentage wise. Percent change of price can be viewed by taking the log-difference of the stock price data. After the stock data for each company is modified to log-difference, the loss probability table can be used to calculate probabilities of total percent decrease for combinations of company stock percent decreases.

As an example, with the companies listed above, a probability of occurrence for a percent decrease is set at .01 . To find the percent decrease for each company that corresponds with a probability of occurrence of .01 , a normal distribution with mean equal to the mean of the logdifferenced data and standard deviation equal to the standard deviation of the log-differenced data is fitted over the data from $-10 \%$ to $10 \%$ as an estimate for each company. Then, the 1 -st percentile is calculated and used as the percent decrease in the loss probability model. The logdifferenced data with normal overlays can be seen below for each company.



Bank of
America

## Financial Data Loss Probability Table - Using Normal Estimates (\#3 in Appendix: Code)

|  | MSFT | DIS | AMZN | BAC | MCD | All.Size | All. Prob |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [1 | -0.04659933 | -0.04615067 | 137897 | -0.07203936 | -0.03495377 | -0.28112210 | 0 |
| [ | -0.04659933 | -0.04615067 | -0.08137897 | -0.07203936 | 00 | -0.24616833 | 9 |
| [3, ] | -0.04659933 | 00 | 97 | -0.07203936 | -0.03495377 | -0.23497143 | 9 |
|  | 0 | -0.04615067 | 7 | -0.07203936 | -0.03495377 | 7 | 9 |
|  | 3 | -0.04615067 | -0.08137897 | 0 | -0.03495377 | 4 | 9 |
|  | -0.04659933 | 0 | 97 | -0.07203936 | 00 | 66 | 07 |
|  | -0.04659933 | -0.04615067 | 00 | 6 | -0.03495377 | -0.19974312 | 9 |
|  | 0.00000000 | -0.04615067 | -0.08137897 | -0.07203936 | 0.00000000 | -0.19956901 | 7 |
|  | 0.00000000 | 0.00000000 | -0.08137897 | -0.07203936 | -0.03495377 | -0.18837210 | $9.80100 \mathrm{e}-07$ |
| [10, | -0.04659933 | -0.04615067 | -0.08137897 | 0.00000000 | 0.00000000 | -0.17412897 | 9.80100e-07 |
|  | -0.04659933 | -0.04615067 | 0.00000000 | -0.07203936 | 0.00000000 | -0.16478936 | $9.80100 \mathrm{e}-07$ |
| [12,] | -0.04659933 | 0.00000000 | -0.08137897 | 0000000 | -0.03495377 | -0.16293207 | -07 |
| [13 | 0.00000000 | -0.04615067 | -0.08137897 | 0 | -0.03495377 | 1 | $9.80100 \mathrm{e}-07$ |
| 1 | -0.04659933 | 0 | 0 | -0.07203936 | -0.03495377 | 5 | $9.80100 \mathrm{e}-07$ |
| [15, ] | 0.0000000 | 0.00000000 | -0.08137897 | -0.07203936 | 0.00000000 | -0.15341833 | 9.70299e-05 |
| [16, ] | 0.00000000 | -0. | 0.00000000 | -0.07203936 | -0.03495377 | -0.15314380 | $9.80100 \mathrm{e}-07$ |
| [17, ] | -0.04659933 | 0.00000000 | -0 | 0000 | 0.00000000 | -0.12797830 | 9.70299e-05 |
| [18, ] | -0.04659933 | -0.04615067 | 00 | 0 | -0.03495377 | -0.12770376 | $9.80100 \mathrm{e}-07$ |
| [19, ] | 0.00000000 | -0.04615067 | -0.08137897 | 000000 | 00 | -0.12752964 | $9.70299 \mathrm{e}-05$ |
|  | -0.04659933 | 0.00000000 | 00 | -0.07203 | 00 | -0.11863869 | $9.70299 \mathrm{e}-05$ |
| [21, ] | 0.00000000 | -0.04615067 | 0.00000000 | -0.07203936 | 00 | 3 | 9.70299e-05 |
| [22,] | 0.00000000 | 0 | -0.08137897 | 0.00000000 | -0.03495377 | 4 | 9.70299e-05 |
|  | 0.00000000 | 0.00000000 | 00000 | -0.07203936 | -0.03495377 | -0.10699313 | 9.70299e-05 |
|  | -0.04659933 | -0.04615067 | 000000 | 00000000 | 0.00000000 | -0.09275000 | 0299e-0 |
| [2 | -0.04659933 | 0.00000000 | 0.00000000 | 0000000 | -0.03495377 | -0.08155309 | 9.70299e-0 |
| [26, ] | 0.00000000 | 0.00000000 | -0.08137897 | 0.00000000 | 0.00000000 | -0.08137897 | 9.60596e-0 |
| [27, ] | 0.00000000 | -0.04615067 | 0.00000000 | 0.00000000 | -0.03495377 | -0.08110444 | $70299 \mathrm{e}-0$ |
| [28, | 0.00000000 | 0.00000000 | 0.00000000 | -0.07203936 | 0.00000000 | -0.07203936 | $9.60596 \mathrm{e}-03$ |
| [29, | -0.04659933 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | -0.04659933 | $9.60596 \mathrm{e}-03$ |
| [30, | 0.00000000 | -0.04615067 | 0.00000000 | 0.00000000 | 0.00000000 | -0.04615067 | $9.60596 \mathrm{e}-03$ |
| [31, | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | -0.03495377 | -0.03495377 | 9.60596e-03 |
| [32,] | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | $9.50990 \mathrm{e}-01$ |

Table 5 Financial Loss Combination Probability Table - Normal Estimates
Using . 01 as the probability of occurrence for each individual event and the 1-st percentile values for the percent decrease severities, the above table shows the probabilities for each combination of possible percent decreases for the five companies. For this data, since the loss is a negative percent, as the total percent value gets smaller, the probability decreases. This is the opposite of what happened with the hurricane data because the smaller the value for the financial data, the larger the percent decrease. This is shown in the graph below (Graph 2) which plots the values for All.Prob for the financial data.

## Exceedance Probability for Financial Data - Normal Estimates

Exceedance probabilities can also be calculated for the percent losses of the financial data. Since there are no duplicate percent loss values, the probabilities can be calculated straight from the loss combination probability table. The exceedance probabilities in the table below and the exceedance curve represent the same idea for the financial data as it did for the hurricane
data. Each point on the exceedance curve is the probability that over a year time span, the percent loss will be equal to or greater than that value.

$$
\begin{array}{rrr} 
& {[, 1]} & {[, 2]} \\
{[1,]} & 0.00000000 & 1.0000000000 \\
{[2,]} & 0.03495377 & 0.0490099501 \\
{[3,]} & 0.04615067 & 0.0394039900 \\
{[4,]} & 0.04659933 & 0.0297980299 \\
{[5,]} & 0.07203936 & 0.0201920698 \\
{[6,]} & 0.08110444 & 0.0105861097 \\
{[7,]} & 0.08137897 & 0.0104890798 \\
{[8,]} & 0.08155309 & 0.0008831197 \\
{[9,]} & 0.09275000 & 0.0007860898 \\
{[10,]} & 0.10699313 & 0.0006890599 \\
{[11,]} & 0.11633274 & 0.0005920300 \\
{[12,]} & 0.11819003 & 0.0004950001 \\
{[13,]} & 0.11863869 & 0.0003979702 \\
{[14,]} & 0.12752964 & 0.0003009403 \\
{[15,]} & 0.12770376 & 0.0002039104 \\
{[16,]} & 0.12797830 & 0.0002029303 \\
{[17,]} & 0.15314380 & 0.0001059004 \\
{[18,]} & 0.15341833 & 0.0001049203 \\
{[19,]} & 0.15359245 & 0.0000078904 \\
{[20,]} & 0.16248341 & 0.0000069103 \\
{[21,]} & 0.16293207 & 0.0000059302 \\
{[22,]} & 0.16478936 & 0.0000049501 \\
{[23,]} & 0.17412897 & 0.0000039700 \\
{[24,]} & 0.18837210 & 0.0000029899 \\
{[25,]} & 0.19956901 & 0.0000020098 \\
{[26,]} & 0.19974312 & 0.0000010297 \\
{[27,]} & 0.20001766 & 0.0000010198 \\
{[28,]} & 0.20908274 & 0.0000000397 \\
{[29,]} & 0.23452277 & 0.0000000298 \\
{[30,]} & 0.23497143 & 0.0000000199 \\
{[31,]} & 0.24616833 & 0.0000000100 \\
{[32,]} & 0.28112210 & 0.0000000001
\end{array}
$$

Table 6 Financial Exceedance Probabilities Normal Estimates


Graph 2 Financial Exceedance Curve - Normal Estimates

## Weighted Financial Data Loss Probability Table (\#4 in Appendix: Code)

It is important to diversify investment portfolios so that if a company's stock value that you own decreases, the portfolio is not affected as much as it would be if the entire portfolio consisted of the one company's stock. This can be shown in the loss probability table by multiplying the company's individual percent decreases by weights. The weights represent the percentage of each company's stock in a portfolio. The stock percent decreases only affect the percentage of that company's weight in the portfolio. To show this, two different sets of weight values are tested for the same five companies with the same values as before. The two sets of weights and their respective loss probability tables are below.

Weight 1: $($ Microsoft $=.4$, Disney $=.1$, Amazon $=.3$, Bank of America $=.15$, McDonalds $=.05)$

|  | MSFT | DIS | AMZN | BAC | MCD | All.Size |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 63 | -0.004615067 | 41369 | 8059 | 7688 | . 060 | 0 |
|  | 1863 | -0.004615067 | -0.0244136 | -0.0108059 | 0 | -0.058474394 | 9 |
|  | -0.0186 | 0.000000000 | -0.02441369 | -0.0108059 |  |  |  |
|  | -0.01863973 | 0.000000000 | -0 |  |  |  |  |
| [5, | 18 | -0. | -0 | 0.0000000 | -0. |  |  |
|  | -0.01863973 | -0. | -0 |  |  |  |  |
| [7, | -0 | 0.000000000 | -0 | . 0.0000000 | -0 |  |  |
|  | -0.01 | 0.000000000 | -0 | 0.0000000 |  | -0.043053423 | 9.70299e-05 |
| [9 | 0.00000000 | -0 | -0. | -0 | -0 |  |  |
|  | 0.00000000 | -0.0 | -0 | -0.0108059 |  |  |  |
|  |  |  | -0 |  |  |  |  |
|  | -0.01 | -0.00 | 0.00000000 | -0.0108059 | -0.001747688 | -0.03 |  |
|  | 0.00000000 | 0.000000000 | -0.020 | -0. | 0.000000000 | -0. |  |
|  | -0.018 | -0.00 | 0.00000000 | -0.0108059 | 0.000000000 | -0.034060702 |  |
|  | -0.0 | . 0 |  | -0. |  |  |  |
|  | 0.00000000 | -0.00 | -0 |  |  |  |  |
|  | -0.0 | 0.00 | 0.0 | -0.010 |  | -0. |  |
|  | 0.00000000 | -0. | -0 | 0.0000000 |  | -0.029028759 |  |
|  | 0.00000000 | 0.000000000 | -0. | . 0000000 | -0.001747688 | -0.026161380 |  |
|  | -0.018 | -0.00 | 0. | 0.0000000 | -0 | -0. | $9.80100 \mathrm{e}-07$ |
|  | 0.00000000 | 0.000000000 | -0. |  |  | -0.024413692 |  |
|  | -0. | -0.00 | 0. | 0.0000000 |  | -0.023254798 | $9.70299 \mathrm{e}-05$ |
|  | -0. |  |  |  | -0.001747688 | -0.020387419 |  |
|  | -0.01 | 0.0 |  |  |  |  |  |
|  | 0.00000000 | -0.0 | 0.00000000 | -0.0 | -0. | -0.017168659 |  |
|  | 0. | -0.00 |  | -0. |  | -0. |  |
|  | 0. | 0. | . 0 | -0.0108059 | -0.001747688 | -0.010 | .70299e-05 |
| [28,] | 0.000000 | . 0000 | 0.00000000 | -0.0 |  | 0. |  |
|  | 0. | -0. | 0.00000000 | 0.0000000 | -0.001 | 0. | . $70299 \mathrm{e}-05$ |
| [3 | 0.00000000 | -0.004615 | 0. | 0.0000000 | 0 | -0. | .60596e-03 |
|  | 0.0 | 0.0 | . 000 | . 0 | -0.001747688 | -0.001747688 |  |
| [32,] | 0.000000 | 0.000000 | 0.00000 | 0.0000 | 0.000000 | 0.00000 | $9.50990 \mathrm{e}-01$ |

Table 7 Financial Data Loss Combination Probability Table - Weight 1 Normal Estimates

\section*{Weight 2: $($ Microsoft $=.1$, Disney $=.3$, Amazon $=.2$, Bank of America $=.15$, McDonalds $=.25)$ <br> |  | MSFT | DIS | AMzN | BAC | MCD | All.Size | b |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 59933 | 8452 | 7579 | -0.0108059 | 08738441 | -0.054325274 | 0 |
|  | 0.000000000 | -0.0138452 | -0.01627579 | -0.0108059 | -0.008738441 | -0.049665341 | 9 |
| [3 | -0.004659933 | -0.0138452 | -0.01627579 | -0.0108059 | 000000000 | -0.045586833 | 9 |
|  | -0.004659933 | -0.0138452 | -0.01627579 | 0.0000000 | -0.008738441 | -0.043519370 | 09 |
| [5, ] | 0.000000000 | -0.0138452 | -0.01627579 | -0.0108059 | . 00000000 | -0.040926900 | 7 |
|  | -0.004659933 | 0000000 | -0.01627579 | -0.0108059 | -0.008738441 | -0.040480073 | 09 |
| [7, ] | 0.000000000 | -0.0138452 | -0.01627579 | 0.0000000 | -0.008738441 | -0.038859437 | 7 |
| [8, ] | -0.004659933 | -0.0138452 | 0.00000000 | -0.0108059 | -0.0087384 | -0.0380 | 000e-09 |
| [9 | 0.000000000 | . 0000000 | -0.01627579 | -0.0108059 | -0.008738441 | . 03582014 | 07 |
| [10, ] | -0.004659933 | -0.0138452 | -0.01627579 | 0.0000000 | 0.0 | -0. | 7 |
| , ] | 0.000000000 | -0.0138452 | 0.00000000 | -0.0108059 | -0.008738441 | -0.03338954 | -07 |
| [2,] | -0.004659933 | 0.0000000 | -0.01627579 | -0.0108059 | 0.000000000 | -0.031741632 | 7 |
| , ] | 0.000000000 | -0.0138452 | -0.01627579 | 0.0000000 | 0.000000000 | -0.030120996 | 05 |
| [4, ] | -0.004659933 | . 0000000 | -0.01627579 | 0.0000000 | -0.008738441 | -0.029 | -07 |
| ,] | -0.004659933 | -0.0138452 | 0.00000000 | -0.0108059 | 0.000000000 | -0.029311038 | 07 |
| [16, ] | -0.004659933 | -0.0138452 | 0.00000000 | 0.0000000 | -0.008738441 | -0.02724357 | -07 |
| , ] | 0.000000000 | . 0000000 | -0.01627579 | -0.0108059 | 00 | -0.027081699 | 5 |
| [18, ] | 0.000000000 | 0.0000000 | -0.01627579 | 0.0000000 | -0.008738441 | -0.025014236 | $9.70299 \mathrm{e}-05$ |
| , | 0.000000000 | -0.0138452 | 0.00000000 | -0.0108059 | 000 | -0.024651105 | 9.70299e-05 |
| [20, ] | -0.004659933 | 0.0000000 | 0.00000000 | -0.0108059 | -0.008738441 | -0.024204278 | 析 |
| [21,] | . 000000000 | -0.0138452 | 0.00000000 | 00 | -0.008 | -0. | $9.70299 \mathrm{e}-05$ |
| [22, ] | -0.004659933 | 0.0000000 | -0.01627579 | 0.0000000 | 0.000000000 | -0.020935728 | $9.70299 \mathrm{e}-05$ |
| [23,] | 0.000000000 | 0.0000000 | 0.00000000 | -0.0108059 | -0.008738441 | -0 |  |
| [24, ] | -0.004659933 | -0.0138452 | 0.00000000 | 0.0000000 | 0.000000000 | -0.018505134 | 9.7 |
| [25,] | 0.000000000 | 0.0000000 | -0.01627579 | 0000000 | 0000000 | 0.016275795 | 03 |
| [26,] | -0.004659933 | 0.0000000 | 0.00000000 | -0.0108059 | 0.000000000 | -0.015465837 | 9.7 |
| [27, ] | 0.000000000 | -0.0138452 | 0.00000000 | . 0000000 | 0.000000000 | -0.013845201 | 6e-03 |
| $[28$, | -0.004659933 | 0.0000000 | 0.00000000 | 0.0000000 | -0.008738441 | -0.013398374 | 9.70299e-05 |
| [29,] | 0.000000000 | 0.0000000 | 0.00000000 | -0.0108059 | 0.000000000 | -0.010805904 | . $60596 \mathrm{e}-03$ |
| [30,] | 0.000000000 | 0.0000000 | 0.00000000 | 0.0000000 | -0.008738441 | -0.008738441 | -03 |
| [31, ] | -0.004659933 | 0.0000000 | 0.00000000 | 0.0000000 | 0.000000000 | -0.004659933 | -03 |
|  | 0.000000000 | 0.0000000 | 0.00000000 | 0. | 0.000000000 |  |  |

Table 8 Financial Data Loss Combination Probability Table - Weight 2 Normal Estimates

For the weighted loss probability tables, it can be seen that the combination percentage decreases were larger values than they were unweighted. This means the total portfolio percentage decrease was less than that of the unweighted. Although the total percentage decreases changed, the probabilities did not. This means the weight of the portfolio has no effect on the probability that the percent decreases occur.

## Exceedance Probability for Weighted Financial Data

The exceedance probabilities and curves can be calculated for both sets of weights using the probabilities from the loss combination tables since there are no duplicate percent losses. The exceedance probabilities and curves for both sets of weights can be seen below. Each point on the exceedance curve is the probability that over a year time span, the percent loss will be equal to or greater than that value.

|  |  | $[, 1]$ |
| ---: | ---: | ---: |
| $[1]$, | 0.060222082 | 0.0000000001 |
| $[2]$, | 0.058474394 | 0.0000000100 |
| $[3]$, | 0.055607015 | 0.0000000199 |
| $[4]$, | 0.053859327 | 0.0000010000 |
| $[5]$, | 0.049416178 | 0.0000010099 |
| $[6]$, | 0.047668490 | 0.0000019900 |
| $[7]$, | 0.044801111 | 0.0000029701 |
| $[8]$, | 0.043053423 | 0.0001000000 |
| $[9]$, | 0.041582352 | 0.0001000099 |
| $[10]$, | 0.039834663 | 0.0001009900 |
| $[11]$, | 0.036967285 | 0.0001019701 |
| $[12]$, | 0.035808390 | 0.0001019800 |
| $[13]$, | 0.035219596 | 0.0001990099 |
| $[14]$, | 0.034060702 | 0.0001999900 |
| $[15]$, | 0.031193323 | 0.0002009701 |
| $[16]$, | 0.030776447 | 0.0002019502 |
| $[17]$, | 0.029445635 | 0.0002989801 |
| $[18]$, | 0.029028759 | 0.0003960100 |
| $[19]$, | 0.026161380 | 0.0004930399 |
| $[20]$, | 0.025002486 | 0.0004940200 |
| $[21]$, | 0.024413692 | 0.0100999801 |
| $[22]$, | 0.023254798 | 0.0101970100 |
| $[23]$, | 0.020387419 | 0.0102940399 |
| $[24]$, | 0.018639731 | 0.0199000000 |
| $[25]$, | 0.017168659 | 0.0199009801 |
| $[26]$, | 0.015420971 | 0.0199980100 |
| $[27]$, | 0.012553592 | 0.0200950399 |
| $[28]$, | 0.010805904 | 0.0297010000 |
| $[29]$, | 0.006362755 | 0.0297980299 |
| $[30]$, | 0.004615067 | 0.0394039900 |
| $[31]$, | 0.001747688 | 0.0490099501 |
| $[32]$, | 0.000000000 | 1.0000000000 |
| $[10$ |  |  |

Table 9 Weight 1 Exceedance Probabilities Normal Estimates

|  | $[, 1]$ | $[, 2]$ |
| ---: | ---: | ---: |
| $[1]$, | 0.054325274 | 0.0000000001 |
| $[2]$, | 0.049665341 | 0.0000000100 |
| $[3]$, | 0.045586833 | 0.0000000199 |
| $[4]$, | 0.043519370 | 0.0000000298 |
| $[5]$, | 0.040926900 | 0.0000010099 |
| $[6]$, | 0.040480073 | 0.0000010198 |
| $[7]$, | 0.038859437 | 0.0000019999 |
| $[8]$, | 0.038049479 | 0.0000020098 |
| $[9]$, | 0.035820140 | 0.0000029899 |
| $[10]$, | 0.034780929 | 0.0000039700 |
| $[11]$, | 0.033389546 | 0.0000049501 |
| $[12]$, | 0.031741632 | 0.0000059302 |
| $[13]$, | 0.030120996 | 0.0001029601 |
| $[14]$, | 0.029674169 | 0.0001039402 |
| $[15]$, | 0.029311038 | 0.0001049203 |
| $[16]$, | 0.027243575 | 0.0001059004 |
| $[17]$, | 0.027081699 | 0.0002029303 |
| $[18]$, | 0.025014236 | 0.0002999602 |
| $[19]$, | 0.024651105 | 0.0003969901 |
| $[20]$, | 0.024204278 | 0.0003979702 |
| $[21]$, | 0.022583642 | 0.0004950001 |
| $[22]$, | 0.020935728 | 0.0005920300 |
| $[23]$, | 0.019544345 | 0.0006890599 |
| $[24]$, | 0.018505134 | 0.0007860898 |
| $[25]$, | 0.016275795 | 0.0103920499 |
| $[26]$, | 0.015465837 | 0.0104890798 |
| $[27]$, | 0.013845201 | 0.0200950399 |
| $[28]$, | 0.013398374 | 0.0201920698 |
| $[29]$, | 0.010805904 | 0.0297980299 |
| $[30]$, | 0.008738441 | 0.0394039900 |
| $[31]$, | 0.004659933 | 0.0490099501 |
| $[32]$, | 0.000000000 | 1.0000000000 |
| $[10$ |  |  |

Table 10 Weight 2 Exceedance Probabilities Normal Estimates


Graph 3 Weight 1 Exceedance Curve Normal Estimates


Graph 4 Weight 2 Exceedance Curve Normal Estimates

## Financial Data - More Accurate Distribution (\#5 in Appendix: Code)

To get an estimate of the percent decrease values used in the loss probability tables for the financial data when the probability of loss was set to .01 , a normal distribution was used. Based on the overlays of the normal distributions for each of the companies, it appears as though it is too heavy tailed of a distribution to use to get an estimate. The standardized student's t distribution is similar to the normal distribution, but with lower degrees of freedom it tends to have lighter tails. After testing the standardized student's t distribution with the same mean, standard deviation, and degrees of freedom 3, it appears to be a much better distribution to use to estimate the 1 -st percentiles for the five companies (Mimoto Lecture 25). Below are the standardized student's t overlays for the company percentage plots.



Financial Data Loss Probability Table - Using standardized student's t estimates (\#6 in

## Appendix: Code)

|  | MSFT | DIS | AMZN | BAC | MCD |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 6 | -0.05175132 | -0.09123836 | 6 | 4 | -0.31513130 | 00000e-10 |
|  | -0.05221926 | -0.05175132 | -0.09123836 | 6 | 0.0000000 |  | 0000e-09 |
| [3, | -0. |  |  |  |  |  |  |
| [4, ] | 0.00000000 | -0.05175132 |  |  |  |  |  |
|  | -0 | 32 |  |  | 4 | -0 | $9.90000 \mathrm{e}-09$ |
|  | -0.05221926 |  |  | -0 |  | -0 | $9.80100 \mathrm{e}-07$ |
|  | -0.05221926 | -0.05175132 |  | 6 | -0 | -0 | $9.90000 \mathrm{e}-09$ |
|  | 0.00000000 | -0.05175132 |  | -0.08071596 |  |  |  |
|  |  |  |  |  |  |  |  |
| [ | - |  |  |  |  |  |  |
| 11,] | -0 | -0.05175132 |  | -0 |  |  | -07 |
|  | -0 | 0.00000000 | -0.09 |  | -0.0392064 |  | $9.80100 \mathrm{e}-07$ |
|  | 0.00000000 | -0.05175132 | -0 |  | -0.0392064 | -0 | $9.80100 \mathrm{e}-07$ |
|  | -0.05221926 | 00 |  | -0 | -0 | -0 | $9.80100 \mathrm{e}-07$ |
|  | 0.00000000 | 0.00000000 | -0.09123836 | -0.08071596 |  |  | -05 |
|  | 0.00000000 | -0.05175132 |  | 6 | -0 |  | . $80100 \mathrm{e}-07$ |
|  | -0 |  | 0.09 |  |  |  | -05 |
|  | -0. | -0.05175132 | 0.00000000 |  | -0.0392064 | -0.1 | .80100e-07 |
|  | 0.00000000 | -0.05175132 | -0.09123836 |  |  | -0. |  |
|  | -0. |  |  | -0 |  |  |  |
|  |  |  |  |  |  |  |  |
|  | 0.00000000 |  | -0.09123836 |  |  |  |  |
|  | 0. |  |  | -0.08071596 | -0.0392064 |  | . $70299 \mathrm{e}-05$ |
|  | -0.0 | . 0 |  |  |  | -0.10397057 |  |
|  | -0.05 |  | 0. |  | -0. | -0 |  |
|  | 0.0000 | 0. | -0.0 |  |  | -0. |  |
|  | 0 | -0. |  | 0.00000000 | -0. | -0.09095771 |  |
|  | 0. | 00 | 00 | -0.08071596 | 0 | -0.080715 | $9.60596 \mathrm{e}-03$ |
|  | -0.0522 | 0.00000000 | 0000000 | 0.00000000 | 0.0000000 | -0.052219 | -0 |
|  | 0.000000 | -0.05175132 | 0.00000000 | 0.00000000 | 0.0000000 | -0.051751 | .60596e-03 |
|  | 0.0000000 | 0.00000000 | 0.0 | 0.00000000 | -0.0392064 | -0. |  |
| 32,] | 0.0 | . | . | . | 0.0000000 |  |  |

Table 11 Financial Data Loss Combination Probability Table - standardized student's testimates

Using the standardized student's $t$ distribution decreased the combination percent decreases so that the total portfolio would decrease by a larger amount. This was to be expected because the standardized student's $t$ distribution has lighter tails, so the value of the company's individual percent decreases became smaller as the 1 -st percentile moves more to the left on the log-differenced financial plot. The standardized student's $t$ distribution overall appears to be better than the Normal distribution to estimate the percent decreases for the financial data when creating a loss probability table.

## Exceedance Probabilities for Financial Data - Standardized student's t

Since the standardized student's t looks to be a better fit for the financial data to estimate the percent losses at the 1 -st percentile, new exceedance probability curves can be made to show a better picture of the exceedance probabilities for the unweighted and weighted sets of data. Again, since there are no duplicates of percent losses in the loss combination probability tables, the exceedance probabilities can be calculated using the probabilities from that table. The new exceedance curve and the probabilities are shown below.

$$
\begin{array}{rrr} 
& {[, 1]} & {[, 2]} \\
{[1,]} & 0.00000000 & 1.0000000000 \\
{[2,]} & 0.03920640 & 0.0490099501 \\
{[3,]} & 0.05175132 & 0.0394039900 \\
{[4,]} & 0.05221926 & 0.0297980299 \\
{[5,]} & 0.08071596 & 0.0201920698 \\
{[6,]} & 0.09095771 & 0.0105861097 \\
{[7,]} & 0.09123836 & 0.0104890798 \\
{[8,]} & 0.09142565 & 0.0008831197 \\
{[9,]} & 0.10397057 & 0.0007860898 \\
{[10,]} & 0.11992236 & 0.0006890599 \\
{[11,]} & 0.13044476 & 0.0005920300 \\
{[12,]} & 0.13246728 & 0.0004950001 \\
{[13,]} & 0.13293522 & 0.0003979702 \\
{[14,]} & 0.14298968 & 0.0003009403 \\
{[15,]} & 0.14317697 & 0.0002039104 \\
{[16,]} & 0.14345762 & 0.0002029303 \\
{[17,]} & 0.17167368 & 0.0001059004 \\
{[18,]} & 0.17195433 & 0.0001049203 \\
{[19,]} & 0.17214162 & 0.0000078904 \\
{[20,]} & 0.18219607 & 0.0000069103 \\
{[21,]} & 0.18266402 & 0.0000059302 \\
{[22,]} & 0.18468654 & 0.0000049501 \\
{[23,]} & 0.19520893 & 0.0000039700 \\
{[24,]} & 0.21116072 & 0.0000029899 \\
{[25,]} & 0.22370564 & 0.0000020098 \\
{[26,]} & 0.22389293 & 0.0000010297 \\
{[27,]} & 0.22417358 & 0.0000010198 \\
{[28,]} & 0.23441533 & 0.0000000397 \\
{[29,]} & 0.26291204 & 0.0000000298 \\
{[30,]} & 0.26337998 & 0.0000000199 \\
{[31,]} & 0.27592490 & 0.0000000100 \\
{[32,]} & 0.31513130 & 0.0000000001
\end{array}
$$

# Weight 1 Financial Data Loss Probability Table - Using standardized student's t estimates (\#7 in Appendix: Code) 

|  | MSFT | DIS | AM2N | BAC | MCD | All.Size | A11. Prob |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 08877 | 32 | -0.02737151 | 39 | 6032 | 7 | $1.00000 \mathrm{e}-10$ |
|  | -0.0208877 |  |  |  |  |  |  |
|  | -0.0208877 | 0.00000000 |  |  |  |  |  |
|  | -0.020 | 0.00000000 |  | -0. | 0. | -0.0603 |  |
| [5, | . 220 | -0.005175132 | -0.0273715 | 000 | -0.00196032 | -0.055394663 |  |
| [6, ] | -0.0208877 | -0.005175132 | -0.0273715 |  | 0.00000000 |  |  |
| [7, | -0.0208877 | 0.000000000 |  |  | -0.0010 |  |  |
|  | . 02 | 0.0 |  |  |  |  |  |
|  | 0.0000000 | -0.00517513 |  | -0.01210739 | -0 |  |  |
| 10 | 0.0000000 | -0.005175132 | -0.02737151 | -0.01210739 |  |  |  |
|  |  |  |  |  |  |  |  |
|  | -0.0208877 |  |  |  |  |  |  |
| [13 | 0.0000000 | 0.00 | -0 | -0.01 | 0.00000000 | -0.039478903 |  |
|  | -0. | -0. | 0.00000000 | -0.01210739 | 00000 | -0.038170229 |  |
|  | -0 | 0.00000000 |  | -0 | -0. |  |  |
|  | 0.0000000 | -0.00 |  |  |  |  |  |
|  | -0. |  |  | -0.010 |  |  |  |
|  |  | -0 | -0. |  |  |  |  |
|  | 0.0000000 |  | -0 |  | -0.00196032 |  |  |
|  | -0.0 | -0 |  |  |  |  |  |
|  | . 0 |  | -0.0 |  |  |  |  |
|  | -0. | -0. | 0.00000000 | 0.00000000 | 0.000 | -0.02606283 |  |
|  | -0. |  |  | . 00000000 | -0.00196032 | -0.02284802 |  |
|  | -0.020 |  |  | 0.00 |  | -0. |  |
|  | . | -0 |  | 0.0 |  |  |  |
|  | 0.0 | -0. |  | 0.0 |  |  |  |
| [27, | 0. | 0.000000000 | 0 | -0.0 | -0.0 | -0. |  |
|  | 0.0000000 | 0.000000000 | 0.0000000 | -0.01210739 |  | -0.01 |  |
|  | 0.0000000 | -0.005 | 0.0000 | 0.00000000 | -0.0019603 | -0.007 |  |
|  | 0.0 | -0.0 | 0.0 | . |  |  |  |
|  | 0.0 | 0.000000000 | 0. | . | -0.00 | -0. |  |
| [32,] | . 000 | 0.000000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.000000000 | 50 |

Table 113 Financial Data Weight 1 Loss Combination Probability Table - standardized student's $t$

## Weight 2 Financial Data Loss Probability Table - Using standardized student's t estimates

## (\#7 in Appendix: Code)

|  |  | DIS | AMZN | BAC |  | ize |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 21926 | -0.01552539 | -0.01824767 | -0.01210739 | -0.009801599 | . 060903987 | 0 |
| [2 | 0.000000000 | -0.01552539 | -0.01824767 | -0.01210739 | -0.009801599 | -0.055682061 | $9.90000 \mathrm{e}-09$ |
| [3, ] | -0.005221926 | -0.01552539 | -0.01824767 | -0.01210739 | 0.000000000 | -0.051102387 | 90000e-09 |
| [4, | -0.00522 | -0.015525 | -0.01824767 | 000000 | -0.009801599 | 92 | $9.90000 \mathrm{e}-09$ |
| [5, ] | 0.000000000 | -0.01552539 | -0.01824767 | -0.01210739 | 0.000000000 | -0.045880462 | 7 |
| [6, ] | -0.005221926 | 0.00000000 | -0.01824767 | -0.01210739 | -0.009801599 | -0.045378592 | 09 |
| [7,] | 0.000000000 | -0.01552539 | -0.01824767 | 0.00000000 | -0.009801599 | -0.043574666 | 0100e-07 |
| [8, ] | -0.005221926 | -0.01552539 | 000 | -0.01210739 | -0.009801599 | -0.042656314 | $9.90000 \mathrm{e}-09$ |
| [9, ] | 0.000000000 | . 00000000 | -0.01824767 | -0.01210739 | -0.009801599 | -0.040156666 | $9.80100 \mathrm{e}-07$ |
| [10, ] | -0.005221926 | -0.01552539 | -0.01824767 | 0.00000000 | 0.000000000 | 93 | 07 |
| [11,] | 0.000000000 | -0.01552539 | 0.00000000 | -0.01210739 | -0.009801599 | -0.037434389 | 7 |
| [12,] | -0.005221926 | 0.00000000 | -0.01824767 | -0.01210739 | 0.000000000 | -0.035576993 | .80100e-07 |
| [13, ] | 0.000000000 | -0.01552539 | -0.01824767 | 0.00000000 | 0.000000000 | -0.033773067 | 5 |
| [14,] | -0.005221926 | . 00000 | -0.01824767 | 0000000 | -0.009801599 | . 033271197 | 7 |
| [15, ] | -0.005221926 | -0.01552539 | 0.00000000 | -0.01210739 | 0.000000 | -0.032854715 | 07 |
| [16, ] | -0.005221926 | -0.01552539 | 0.0000000 | 0.00000000 | -0.009801599 | -0.030548920 | 0100e-07 |
| [17, ] | 0.000000000 | 0.00000000 | -0.01824767 | -0.01210739 | 0.000000000 | -0.030355067 | 5 |
| [1 | 0.000000000 | 0.00000000 | -0.01824767 | 0.00000000 | -0.00980159 | -0.02 | $9.70299 \mathrm{e}-05$ |
| [19,] | 0.000000000 | -0.01552539 | . 00000000 | -0.01210739 | 0.000000000 | -0.027632789 | 05 |
| [20, ] | -0.005221926 | 0.00000000 | 0.00000000 | -0.01210739 | -0.009801599 | -0.027130920 | $9.80100 \mathrm{e}-07$ |
| [21, ] | 00 | -0.01552539 | 00 | 0.00000000 | -0.009801599 | -0.025326994 | 5 |
| [22,] | -0.005221926 | 0.0 | -0.01824767 | 0.00000000 | 0. | -0.023469598 | 5 |
| [23, ] | 0.000000000 | 0.00000000 | 0.00000000 | -0.01210739 | -0.009801599 | -0.021908994 | $9.70299 \mathrm{e}-05$ |
| [24, ] | -0.005221926 | -0.01552539 | 0.00000000 | 0.00000000 | 0.000000000 | -0.020747321 | $9.70299 \mathrm{e}-05$ |
| [25, ] | 0.000000000 | 0.00000000 | -0.01824767 | 0.00000000 | 0.000000000 | -0.018247672 | $9.60596 \mathrm{e}-03$ |
| [26,] | -0.005221926 | 0.00000000 | 0.00000000 | -0.01210739 | 0.000000000 | -0.017329320 | 9.70299e-05 |
| [27, ] | 0.000000000 | -0.01552539 | 0.00000000 | 0.00000000 | 0.000000000 | -0.015525395 | 3 |
| [28, ] | -0.005221926 | 0.00000000 | 0.00000000 | 0.00000000 | -0.009801599 | -0.015023525 | . 7 |
| [29,] | 0.000000000 | 0.00000000 | 0.0000000 | -0.01210739 | 0.000000000 | -0.012107395 | 9.60596e-03 |
| [30,] | 0.000000000 | 0.00000000 | 0.0000000 | 0.00000000 | -0.009801599 | -0.009801599 | 9.60596e-03 |
| [31, ] | -0.005221926 | 0.00000000 | 0.00000000 | 0.00000000 | 0.000000000 | -0.005221926 | 9.60596e-03 |
| , | 0.00000000 | 0000 | 0. | . | ,00000000 |  | $9.50990 \mathrm{e}-01$ |

Table 124 Financial Data Weight 2 Loss Combination Probability Table - standardized student's t

```
            [,1] [,2]
        [,1] [,2]
    [1,] 0.067502057 0.0000000001 [1,] 0.060903987 0.0000000001
    [2,] 0.065541738 0.0000000100 [2,] 0.055682061 0.0000000100
    [3,] 0.062326926 0.0000000199 [3,] 0.051102387 0.0000000199
    [4,] 0.060366606 0.0000010000 [4,] 0.048796592 0.0000000298
    [5,] 0.055394663 0.0000010099 [5,] 0.045880462 0.0000010099
    [6,] 0.053434343 0.0000019900 [6,] 0.045378592 0.0000010198
    [7,] 0.050219531 0.0000029701 [7,] 0.043574666 0.0000019999
    [8,] 0.048259211 0.0001000000 [8,] 0.042656314 0.0000020098
    [9,] 0.046614354 0.0001000099 [9,] 0.040156666 0.0000029899
[10,] 0.044654035 0.0001009900 [10,] 0.038994993 0.0000039700
[11,] 0.041439223 0.0001019701 [11,] 0.037434389 0.0000049501
[12,] 0.040130549 0.0001019800 [12,] 0.035576993 0.0000059302
[13,] 0.039478903 0.0001990099 [13,] 0.033773067 0.0001029601
[14,] 0.038170229 0.0001999900 [14,] 0.033271197 0.0001039402
[15,] 0.034955417 0.0002009701 [15,] 0.032854715 0.0001049203
[16,] 0.034506960 0.0002019502 [16,] 0.030548920 0.0001059004
[17,] 0.032995098 0.0002989801 [17,] 0.030355067 0.0002029303
[18,] 0.032546640 0.0003960100 [18,] 0.028049271 0.0002999602
[19,] 0.029331828 0.0004930399 [19,] 0.027632789 0.0003969901
[20,] 0.028023154 0.0004940200 [20,] 0.027130920 0.0003979702
[21,] 0.027371508 0.0100999801 [21,] 0.025326994 0.0004950001
[22,] 0.026062835 0.0101970100 [22,] 0.023469598 0.0005920300
[23,] 0.022848023 0.0102940399 [23,] 0.021908994 0.0006890599
[24,] 0.020887703 0.0199000000 [24,] 0.020747321 0.0007860898
[25,] 0.019242846 0.0199009801 [25,] 0.018247672 0.0103920499
[26,] 0.017282526 0.0199980100 [26,] 0.017329320 0.0104890798
[27,] 0.014067714 0.0200950399 [27,] 0.015525395 0.0200950399
[28,] 0.012107395 0.0297010000 [28,] 0.015023525 0.0201920698
[29,] 0.007135451 0.0297980299 [29,] 0.012107395 0.0297980299
[30,] 0.005175132 0.0394039900 [30,] 0.009801599 0.0394039900
[31,] 0.001960320 0.0490099501 [31,] 0.005221926 0.0490099501
[32,] 0.000000000 1.0000000000 [32,] 0.000000000 1.0000000000
```

Table 135 Weight 1 Exceedance Probabilities Standardized Student's $t$ Estimates

Table 16 Weight 2 Exceedance Probabilities Standardized Student's t Estimates


Graph 6 Weight 1 Exceedance Curve Standardized Student's t Estimates


Graph 7 Weight 2 Exceedance Curve Standardized Student's $t$ Estimates

## Conclusion

Over the course of working on this project with the case model/financial data, I learned how to analyze the data to produce, read, and interpret both loss combination probability models and exceedance probability curves. These types of models are used by actuaries and statisticians that work with catastrophic and financial data so that they can predict the amount of money they need to have reserved in case of catastrophes or stock market changes (CAS 12). Using the exceedance probability curves, they can find the probability that a certain loss or greater will occur over a specific period. At the beginning of the project I attempted to generate the models by hand using pencil and paper, but after finding out the amount of work that is put into making the models, it was decided that using a program software to generate them would be a lot more efficient. Programming these models using the software R has taught me a lot about the language, which is becoming more popular in the world of statistics because it is free to use. Learning about the tools used by actuaries in the professional field to complete this project has only assured me more that I am moving into a field that is right for me.

## References

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Olson, Erin, and Jason Kundrot. Catastrophe Model Facilitators Guide. CAS, 2014.
"SP500 For 15 Yrs." Redirect to Secure Connection, gozips.uakron.edu/~nmimoto/pages/datasets/SP500.txt.

```
# Cat Modeling Project
# - Jeremy
#
###########################################
```

\#-------------------------------------------
\# 1. Hurricane Model
$\mathrm{n}<-5$
$1<-\operatorname{rep}(\operatorname{list}(0: 1), \mathrm{n})$
All.Comb <- expand.grid(l) \#Create a grid of all possible combinations
names(All.Comb) <- c("10M", "5M", "3M", "2M", "1M") \#Rename columns to corresponding
loss values

Prob <- $\mathrm{t}\left(\mathrm{apply}\right.$ (All.Comb, 1 , function(x) $\mathrm{x} * \mathrm{c}(.002, .005, .01, .02, .03)+(1-\mathrm{x})^{*} \mathrm{c}(.998, .995$, $.99, .98, .97)$ )) \#Change each element to its corresponding probability(loss/no loss)

Size <- $\mathrm{t}(\mathrm{apply}($ All.Comb, 1 , function(x) $\mathrm{x} * \mathrm{c}(10,5,3,2,1))$ ) \#Change each element to its loss value

All.Prob <- apply(Prob, 1, prod) \#Multiply row probabilities to get overall chance event occurs All.Size <- apply(Size, 1, sum) \#Sum each row for total sum of losses

All.Unsorted <- cbind(Size, All.Size, All.Prob) \#Combine individual losses, total loss, and overall probability
All.Sorted <- All.Unsorted[order(All.Size),] \#Sort from smallest total loss to largest total loss All.Sorted
\#Calculate Probability of each loss occuring
ProbLoss <- matrix ( $0,22,2$ ) \# Table with Probabilities of each loss for(i in 0:21)
\{

ProbLoss $[i+1,1]<-\mathrm{i}$
\}
for(i in 0:21)
\{
for(j in 1:32)
\{
if(All.Sorted[j,6] == i)
\{
$\operatorname{ProbLoss}[i+1,2]=(\operatorname{ProbLoss}[i+1,2]+$ All.Sorted $[j, 7])$
\}
\}
\}

```
ProbLoss
#Calculate Exceedance Probabilities
Exceedance <- matrix (0,22,2)
for(i in 0:21)
{
    Exceedance[i+1,1] <- i
}
for(i in 0:21)
{
    for(j in 0:21)
    {
        if(ProbLoss[j+1,1] >= i)
        {
            Exceedance[i+1,2] = Exceedance[i+1,2] + ProbLoss[j+1,2]
    }
    }
}
#Plot Exceedance Probability Curve
Exceedance
plot(y = Exceedance[,2], x = Exceedance[,1], ylim = c(0,.05), type = 'o', xlab = "Loss (Millions)"
, ylab = "Exceedance Probability") #plot All.Prob
#---------------------------------------
# 2. After loading SP500 Dataset
ls()
dim(D.ad)
# install.packages("quantmod")
library(quantmod)
#----- MICROSOFT
plot(D.ad[, "MSFT"])
plot(log(D.ad[, "MSFT"]))
plot(diff(log(D.ad[, "MSFT"])))
hist(diff(log(D.ad[, "MSFT"])))
hist(diff(log(D.ad[, "MSFT"])), 200, xlim=c(-.1, .1))
```

```
Y <- diff(log(D.ad[, "MSFT"]))[-1] #Y is log-difference of AAPL
head(Y)
mean(Y)
sd(Y)
x <- seq(-.1,.1, .01)
plot(x, dnorm(x, mean(Y), sd(Y)), type="l", col="red")
hist(diff(log(D.ad[, "MSFT"])), 200, xlim=c(-.1, .1), freq=FALSE)
lines(x, dnorm(x, mean(Y), sd(Y)), type="l", col="red") # overlay pdf of normal
VaR.msft <- qnorm(.01, mean(Y), sd(Y))
#----- DISNEY
plot(D.ad[, "DIS"])
plot(log(D.ad[, "DIS"]))
plot(diff(log(D.ad[, "DIS"])))
hist(diff(log(D.ad[, "DIS"])))
hist(diff(log(D.ad[, "DIS"])), 200, xlim=c(-.1, .1))
Y <- diff(log(D.ad[, "DIS"]))[-1] #Y is log-difference of AAPL
head(Y)
mean(Y)
sd(Y)
x <- seq(-.1,.1, .01)
plot(x, dnorm(x, mean(Y), sd(Y)), type="l", col="red")
hist(diff(log(D.ad[, "DIS"])), 200, xlim=c(-.1, .1), freq=FALSE)
lines(x, dnorm(x, mean(Y), sd(Y)), type="l", col="red") # overlay pdf of normal
VaR.dis <- qnorm(.01, mean(Y), sd(Y))
#----- AMAZON
plot(D.ad[, "AMZN"])
plot(log(D.ad[, "AMZN"]))
plot(diff(log(D.ad[, "AMZN"])))
```

hist(diff(log(D.ad[, "AMZN"])))
$\operatorname{hist}(\operatorname{diff}(\log (\mathrm{D} . \mathrm{ad}[$, "AMZN"])), 200, xlim=c(-.1, .1))

Y <- diff( $\log ($ D.ad[, "AMZN"]) $)[-1]$ \#Y is log-difference of AAPL head(Y)
mean( Y )
$\operatorname{sd}(\mathrm{Y})$
$\mathrm{x}<-\operatorname{seq}(-.1, .1, .01)$
$\operatorname{plot}(x, \operatorname{dnorm}(x, \operatorname{mean}(Y), \operatorname{sd}(Y))$, type="l", col="red")
$\operatorname{hist}(\operatorname{diff}(\log (\mathrm{D} . \operatorname{ad}[$, "AMZN"])), 200, xlim=c(-.1, .1), freq=FALSE)
lines(x, dnorm(x, mean(Y), sd(Y)), type="l", col="red") \# overlay pdf of normal
VaR.amzn <- qnorm(.01, mean(Y), sd(Y))

```
#---- BANK OF AMERICA
plot(D.ad[, "BAC"])
plot(log(D.ad[, "BAC"]))
plot(diff(log(D.ad[, "BAC"])))
hist(diff(log(D.ad[, "BAC"])))
hist(diff(log(D.ad[, "BAC"])), 200, xlim=c(-.1, .1))
```

$\mathrm{Y}<-\operatorname{diff}(\log (\mathrm{D} \cdot \mathrm{ad}[$, "BAC"])$)[-1]$ \#Y is log-difference of AAPL
head(Y)
mean( Y )
$\operatorname{sd}(\mathrm{Y})$
$\mathrm{x}<-\operatorname{seq}(-.1, .1, .01)$
$\operatorname{plot}(\mathrm{x}, \operatorname{dnorm}(\mathrm{x}, \operatorname{mean}(\mathrm{Y}), \operatorname{sd}(\mathrm{Y}))$, type="l", col="red")
$\operatorname{hist}(\operatorname{diff}(\log (\mathrm{D} . \operatorname{ad}[$, "BAC"] $)), 200, \operatorname{xlim}=c(-.1, .1)$, freq=FALSE $)$
lines( $x, \operatorname{dnorm}(x, \operatorname{mean}(Y), \operatorname{sd}(Y))$, type="l", col="red") \# overlay pdf of normal
VaR.bac <- qnorm(.01, mean(Y), sd(Y))

```
#------ MCDONALDS
plot(D.ad[, "MCD"])
plot(log(D.ad[, "MCD"]))
plot(diff(log(D.ad[, "MCD"])))
hist(diff(log(D.ad[, "MCD"])))
hist(diff(log(D.ad[, "MCD"])), 200, xlim=c(-.1, .1))
```

$\mathrm{Y}<-\operatorname{diff}(\log (\mathrm{D} \cdot \mathrm{ad}[$, "MCD"]))[-1] \#Y is log-difference of AAPL
head(Y)
mean(Y)
sd(Y)
$\mathrm{x}<-\operatorname{seq}(-.1, .1, .01)$
$\operatorname{plot}(\mathrm{x}, \operatorname{dnorm}(\mathrm{x}, \operatorname{mean}(\mathrm{Y}), \operatorname{sd}(\mathrm{Y}))$, type="l", col="red")
hist(diff(log(D.ad[, "MCD"])), 200, xlim=c(-.1, .1), freq=FALSE)
lines( $x, \operatorname{dnorm}(x, \operatorname{mean}(Y), \operatorname{sd}(Y))$, type="l", col="red") \# overlay pdf of normal
VaR.mcd <- qnorm $(.01$, mean $(\mathrm{Y}), \operatorname{sd}(\mathrm{Y}))$
\#-----------------------------------------
\# 3. Combine VaR like Hurricanes (Using Normal Distribution)
$\mathrm{n}<-5$
$1<-\operatorname{rep}(\operatorname{list}(0: 1), \mathrm{n})$
All.Comb <- expand.grid(1) \#Create a grid of all possible combinations
names(All.Comb) <- c("MSFT", "DIS", "AMZN", "BAC", "MCD") \#Rename columns to
corresponding company names

Prob <- t (apply(All.Comb, 1 , function(x) $\mathrm{x}^{*} \mathrm{c}(.01, .01, .01, .01, .01)+(1-\mathrm{x}) * \mathrm{c}(.99, .99, .99, .99$, .99))) \#Change each element to its corresponding probability(loss/no loss)

Size <- t (apply(All.Comb, 1, function(x) $\mathrm{x} * \mathrm{c}($ VaR.msft, VaR.dis, VaR.amzn, VaR.bac, VaR.mcd))) \#Change each element to its loss value

All.Prob <- apply(Prob, 1, prod) \#Multiply row probabilities to get overall chance event occurs All.Size <- apply(Size, 1, sum) \#Sum each row for total sum of losses

All.Unsorted <- cbind(Size, All.Size, All.Prob) \#Combine individual losses, total loss, and overall probability
All.Sorted <- All.Unsorted[order(All.Size),] \#Sort from smallest total loss to largest total loss All.Sorted
\#Calculate Probability of each percent loss occuring All.SortedDec <- All.Unsorted[order(-All.Size),]

ProbLoss <- matrix (0, 32, 2) \# Table with Probabilities of each loss for(i in 1:32)
\{
ProbLoss[i,1] <- -All.SortedDec[i,6]
ProbLoss[i,2] <- All.SortedDec[i,7]
\}
ProbLoss
\#Calculate Exceedance Probabilities
Exceedance <- matrix $(0,32,2)$
for(i in 0:31)
\{
Exceedance $[i+1,1]<-\operatorname{ProbLoss}[i+1,1]$
for( j in $0: 31$ )
\{
if $(\operatorname{ProbLoss}[j+1,1]>=\operatorname{ProbLoss}[i+1,1])$
\{
Exceedance $[\mathrm{i}+1,2]=$ Exceedance $[\mathrm{i}+1,2]+\operatorname{ProbLoss}[\mathrm{j}+1,2]$
\}
\}
\}
\#Plot Exceedance Probability Curve
Exceedance
plot $(\mathrm{y}=$ Exceedance[,2], $\mathrm{x}=$ Exceedance[,1], ylim $=\mathrm{c}(0, .05)$, type = ' o ', xlab $=$ "Percent Loss" , ylab = "Exceedance Probability") \#plot All.Prob
\#---------------------------------
\# 4. Combine VaRs with weights of each holdings. (Normal Distribution)
weight1 <- c(.4,.1,.3,.15,.05) \#Set 1 percentage of total portfolio
VarWeight 1 <- weight 1 *c(VaR.msft, VaR.dis, VaR.amzn, VaR.bac, VaR.mcd) \#Apply Set1 weights to percent decreases
weight2 <- c(.1,.3,.2,.15,.25) \#Set 2 percentage of total portfolio
VarWeight2 <- weight2*c(VaR.msft, VaR.dis, VaR.amzn, VaR.bac, VaR.mcd)\#Apply Set2 weights to percent decreases
n <- 5
$1<-\operatorname{rep}(\operatorname{list}(0: 1), n)$
All.Comb <- expand.grid(1) \#Create a grid of all possible combinations
\#\#Weighted Combination1\#\#
names(All.Comb) <- c("MSFT", "DIS", "AMZN", "BAC", "MCD") \#Rename columns to corresponding company names

Prob <- $\mathrm{t}(\operatorname{apply}($ All.Comb, 1, function( x$) \mathrm{x} * \mathrm{c}(.01, .01, .01, .01, .01)+(1-\mathrm{x}) * \mathrm{c}(.99, .99, .99, .99$, .99))) \#Change each element to its corresponding probability(loss/no loss)

Size $<-\mathrm{t}($ apply(All.Comb, 1, function( x$) \mathrm{x} *$ VarWeight1)) \#Change each element to its loss value

All.Prob <- apply(Prob, 1, prod) \#Multiply row probabilities to get overall chance event occurs All.Size <- apply(Size, 1, sum) \#Sum each row for total sum of losses

All.Unsorted <- cbind(Size, All.Size, All.Prob) \#Combine individual losses, total loss, and overall probability
All.Sorted1 <- All.Unsorted[order(All.Size),] \#Sort from smallest total loss to largest total loss All.Sorted1
\#\#Weighted Combination2\#\#
names(All.Comb) <- c("MSFT", "DIS", "AMZN", "BAC", "MCD") \#Rename columns to corresponding company names

Prob <- $\mathrm{t}($ apply(All.Comb, 1 , function(x) $\mathrm{x} * \mathrm{c}(.01, .01, .01, .01, .01)+(1-\mathrm{x}) * \mathrm{c}(.99, .99, .99, .99$, .99))) \#Change each element to its corresponding probability(loss/no loss)

Size <- t (apply(All.Comb, 1, function(x) $\mathrm{x} *$ VarWeight2)) \#Change each element to its loss value

All.Prob <- apply(Prob, 1, prod) \#Multiply row probabilities to get overall chance event occurs All.Size <- apply(Size, 1, sum) \#Sum each row for total sum of losses

All.Unsorted <- cbind(Size, All.Size, All.Prob) \#Combine individual losses, total loss, and overall probability
All.Sorted2 <- All.Unsorted[order(All.Size),] \#Sort from smallest total loss to largest total loss All.Sorted2
\#plot All.Prob for Weight1 vs Weight2

```
plot(x = All.Sorted1[,7], y = -All.Sorted1[,6], type = 'o', ylab = 'Percent Loss', xlab =
'Probability', main = 'Weight1 vs Weight2 using Normal estimates', sub = 'Black = Weight1 &
Red = Weight2')
lines(x = All.Sorted2[,7], y = -All.Sorted2[,6], col = "red", type = 'o')
###Exceedance Probability for Weight1
#Calculate Probability of each percent loss occuring
ProbLoss <- matrix (0, 32, 2) # Table with Probabilities of each loss
for(i in 1:32)
{
    ProbLoss[i,1] <- -All.Sorted1[i,6]
    ProbLoss[i,2] <- All.Sorted1[i,7]
}
```


## ProbLoss

```
#Calculate Exceedance Probabilities
```

\#Calculate Exceedance Probabilities
Exceedance1 <- matrix (0,32,2)
Exceedance1 <- matrix (0,32,2)
for(i in 0:31)
for(i in 0:31)
{
{
Exceedance1[i+1,1] <- ProbLoss[i+1,1]
Exceedance1[i+1,1] <- ProbLoss[i+1,1]
for(j in 0:31)
for(j in 0:31)
{
{
if(ProbLoss[j+1,1] >= ProbLoss[i+1,1])
if(ProbLoss[j+1,1] >= ProbLoss[i+1,1])
{
{
Exceedance1[i+1,2] = Exceedance1[i+1,2] + ProbLoss[j+1,2]
Exceedance1[i+1,2] = Exceedance1[i+1,2] + ProbLoss[j+1,2]
}
}
}
}
}
}
\#Plot Exceedance Probability Curve
\#Plot Exceedance Probability Curve
Exceedance1
Exceedance1
plot(y = Exceedance1[,2], x = Exceedance1[,1], ylim = c(0,.05), type = 'o', xlab = "Percent Loss"
plot(y = Exceedance1[,2], x = Exceedance1[,1], ylim = c(0,.05), type = 'o', xlab = "Percent Loss"
, ylab = "Exceedance Probability") \#plot All.Prob
, ylab = "Exceedance Probability") \#plot All.Prob
\#\#\#Exceedance Probability for Weight2
\#\#\#Exceedance Probability for Weight2
\#Calculate Probability of each percent loss occuring
\#Calculate Probability of each percent loss occuring
ProbLoss <- matrix(0, 32, 2) \# Table with Probabilities of each loss
ProbLoss <- matrix(0, 32, 2) \# Table with Probabilities of each loss
for(i in 1:32)
for(i in 1:32)
{
{
ProbLoss[i,1] <- -All.Sorted2[i,6]
ProbLoss[i,1] <- -All.Sorted2[i,6]
ProbLoss[i,2] <- All.Sorted2[i,7]
ProbLoss[i,2] <- All.Sorted2[i,7]
}

```
}
```


## ProbLoss

\#Calculate Exceedance Probabilities
Exceedance2 <- matrix ( $0,32,2$ )
for(i in 0:31)
\{
Exceedance2[i+1,1] <- ProbLoss $[i+1,1]$
for $(\mathrm{j}$ in $0: 31)$
\{
if(ProbLoss[j+1,1] >= ProbLoss $[i+1,1])$
\{
Exceedance2[i+1,2] = Exceedance2[i+1,2] + ProbLoss[j+1,2] \}
\}
\}
\#Plot Exceedance Probability Curve Exceedance2
$\operatorname{plot}(\mathrm{y}=$ Exceedance2[,2], $\mathrm{x}=$ Exceedance2[,1], ylim $=\mathrm{c}(0, .05)$, type $=$ ' o ', xlab $=$ "Percent Loss" , ylab = "Exceedance Probability") \#plot All.Prob
\#--------------------------------
\# 5. Use other distribution than Normal (Normal vs standardized student t)
\#Install Garch package
library(fGarch)
\#----- MICROSOFT
plot(D.ad[, "MSFT"])
plot(log(D.ad[, "MSFT"]))
plot(diff(log(D.ad[, "MSFT"])))
hist(diff(log(D.ad[, "MSFT"])))
hist(diff(log(D.ad[, "MSFT"])), 200, xlim=c(-.1, .1))

Y <- diff( $\log ($ D.ad[, "MSFT"] $)$ )[-1] \#Y is log-difference of AAPL
head(Y)
$\mathrm{x}<-\operatorname{seq}(-.1, .1, .01)$
$\operatorname{plot}(x, \operatorname{dstd}(x$, mean $=\operatorname{mean}(Y), \operatorname{sd}=\operatorname{sd}(Y), n u=3)$, type="l", col="red")
hist(diff(log(D.ad[, "MSFT"])), 200, xlim=c(-.1, .1), freq=FALSE)
lines $(x, \operatorname{dstd}(x$, mean $=$ mean $(Y), \operatorname{sd}=\operatorname{sd}(Y), n u=3)$, type="l", col="red") \# overlay pdf of standardized student's t

VaR2.msft <- qstd(.01, mean(Y), sd(Y))

```
#----- DISNEY
plot(D.ad[,"DIS"])
plot(log(D.ad[,"DIS"]))
plot(diff(log(D.ad[, "DIS"])))
hist(diff(log(D.ad[, "DIS"])))
hist(diff(log(D.ad[, "DIS"])), 200, xlim=c(-.1, .1))
```

Y <- diff(log(D.ad[, "DIS"]))[-1] \#Y is log-difference of AAPL
head(Y)
mean(Y)
$\operatorname{sd}(\mathrm{Y})$
$\mathrm{x}<-\operatorname{seq}(-.1, .1, .01)$
$\operatorname{plot}(\mathrm{x}, \operatorname{dstd}(\mathrm{x}$, mean $=$ mean $(\mathrm{Y}), \operatorname{sd}=\operatorname{sd}(\mathrm{Y}), \mathrm{nu}=3)$, type="l", col="red")
hist(diff(log(D.ad[, "DIS"])), 200, xlim=c(-.1, .1), freq=FALSE)
lines $(x, \operatorname{dstd}(x$, mean $=\operatorname{mean}(Y), \operatorname{sd}=\operatorname{sd}(Y), n u=3)$, type="l", col="red") \# overlay pdf of
standardized student's t
$\operatorname{VaR} 2 . \operatorname{dis}<-\mathrm{qstd}(.01, \operatorname{mean}(\mathrm{Y}), \operatorname{sd}(\mathrm{Y}))$
\#----- AMAZON
plot(D.ad[, "AMZN"])
plot(log(D.ad[, "AMZN"]))
plot(diff(log(D.ad[, "AMZN"])))
$\operatorname{hist}(\operatorname{diff}(\log (D \cdot a d[, ~ " A M Z N "])))$
hist(diff(log(D.ad[, "AMZN"])), 200, xlim=c(-.1, .1))

```
Y <- diff(log(D.ad[, "AMZN"]))[-1] #Y is log-difference of AAPL
head(Y)
mean(Y)
sd(Y)
x <- seq(-.1,.1, .01)
plot(x, dstd(x, mean = mean(Y), sd = sd(Y), nu = 3), type="l", col="red")
hist(diff(log(D.ad[, "AMZN"])), 200, xlim=c(-.1, .1), freq=FALSE)
lines(x, dstd(x, mean = mean(Y), sd = sd(Y), nu = 3), type="l", col="red") # overlay pdf of
standardized student's t
VaR2.amzn <- qstd(.01, mean(Y), sd(Y))
#----- BANK OF AMERICA
plot(D.ad[, "BAC"])
plot(log(D.ad[, "BAC"]))
plot(diff(log(D.ad[, "BAC"])))
hist(diff(log(D.ad[, "BAC"])))
hist(diff(log(D.ad[, "BAC"])), 200, xlim=c(-.1, .1))
Y <- diff(log(D.ad[, "BAC"]))[-1] #Y is log-difference of AAPL
head(Y)
mean(Y)
sd(Y)
x <- seq(-.1,.1,.01)
plot(x, dstd(x, mean = mean(Y), sd = sd(Y), nu = 3), type="l", col="red")
hist(diff(log(D.ad[, "BAC"])), 200, xlim=c(-.1, .1), freq=FALSE)
lines(x, dstd(x, mean = mean(Y), sd = sd(Y), nu = 3), type="l", col="red") # overlay pdf of
standardized student's t
VaR2.bac <- qstd(.01, mean(Y), sd(Y))
#------ MCDONALDS
plot(D.ad[, "MCD"])
```

```
plot(log(D.ad[, "MCD"]))
plot(diff(log(D.ad[, "MCD"])))
hist(diff(log(D.ad[, "MCD"])))
hist(diff(log(D.ad[, "MCD"])), 200, xlim=c(-.1, .1))
```

$\mathrm{Y}<-\operatorname{diff}(\log (\mathrm{D} \cdot \mathrm{ad}[$, "MCD"]))[-1] \#Y is log-difference of AAPL
head(Y)
mean(Y)
$\operatorname{sd}(\mathrm{Y})$
$\mathrm{x}<-\operatorname{seq}(-.1, .1, .01)$
$\operatorname{plot}(\mathrm{x}, \operatorname{dstd}(\mathrm{x}$, mean $=$ mean( Y$)$, sd = sd(Y), nu = 3), type="l", col="red")
$\operatorname{hist}(\operatorname{diff}(\log (D \cdot a d[, ~ " M C D "])), 200, x \lim =c(-.1, .1)$, freq=FALSE)
lines $(x, \operatorname{dstd}(x$, mean $=\operatorname{mean}(Y), s d=\operatorname{sd}(Y), n u=3)$, type="l", col="red") \# overlay pdf of
standardized student's t
VaR2.mcd <- qstd(.01, mean(Y), sd(Y))
\#----------------------------------------
\# 6. Combine VaR like Hurricanes (Using standardized student's t Distribution)
$\mathrm{n}<-5$
$1<-\operatorname{rep}(\operatorname{list}(0: 1), n)$
All.Comb <- expand.grid(l) \#Create a grid of all possible combinations
names(All.Comb) <- c("MSFT", "DIS", "AMZN", "BAC", "MCD") \#Rename columns to
corresponding company names

Prob <- t (apply(All.Comb, 1 , function(x) $\mathrm{x}^{*} \mathrm{c}(.01, .01, .01, .01, .01)+(1-\mathrm{x}) * \mathrm{c}(.99, .99, .99, .99$, .99))) \#Change each element to its corresponding probability(loss/no loss)

Size $<-\mathrm{t}$ (apply(All.Comb, 1, function(x) $\mathrm{x} * \mathrm{c}($ VaR2.msft, VaR2.dis, VaR2.amzn, VaR2.bac, VaR2.mcd))) \#Change each element to its loss value

All.Prob <- apply(Prob, 1, prod) \#Multiply row probabilities to get overall chance event occurs All.Size <- apply(Size, 1, sum) \#Sum each row for total sum of losses

All.Unsorted <- cbind(Size, All.Size, All.Prob) \#Combine individual losses, total loss, and overall probability
All.Sorted <- All.Unsorted[order(All.Size),] \#Sort from smallest total loss to largest total loss

All.Sorted
\#Calculate Probability of each percent loss occuring All.SortedDec <- All.Unsorted[order(-All.Size),]

ProbLoss <- matrix (0, 32, 2) \# Table with Probabilities of each loss for(i in 1:32)
\{
ProbLoss[i,1] <- -All.SortedDec[i,6]
ProbLoss[i,2] <- All.SortedDec[i,7]
\}
ProbLoss
\#Calculate Exceedance Probabilities
Exceedance <- matrix $(0,32,2)$
for(i in 0:31)
\{
Exceedance $[\mathrm{i}+1,1]<-\operatorname{ProbLoss}[\mathrm{i}+1,1]$
for( j in $0: 31$ )
\{
if $(\operatorname{ProbLoss}[j+1,1]>=\operatorname{ProbLoss}[i+1,1])$
\{
Exceedance $[\mathrm{i}+1,2]=$ Exceedance $[\mathrm{i}+1,2]+\operatorname{ProbLoss}[\mathrm{j}+1,2]$
\}
\}
\}
\#Plot Exceedance Probability Curve
Exceedance
plot ( $\mathrm{y}=$ Exceedance[,2], $\mathrm{x}=$ Exceedance[,1], ylim $=c(0, .05)$, type $=$ ' o ', $\mathrm{xlab}=$ "Percent Loss" , ylab = "Exceedance Probability") \#plot All.Prob
\#---------------------------------
\# 7. Combine VaRs with weights of each holdings (standardized student's $t$ Distribution)
weight1 <- c(.4,.1,.3,.15,.05)
VarWeight1 <- weight $1 * c($ VaR2.msft, VaR2.dis, VaR2.amzn, VaR2.bac, VaR2.mcd)
weight2 <- c(.1, 3,.2,.15,.25)
VarWeight2 <- weight2*c(VaR2.msft, VaR2.dis, VaR2.amzn, VaR2.bac, VaR2.mcd)
$\mathrm{n}<-5$
$1<-\operatorname{rep}(\operatorname{list}(0: 1), n)$

```
##Weighted Combination1##
names(All.Comb) <- c("MSFT", "DIS", "AMZN", "BAC", "MCD") #Rename columns to
corresponding company names
```

Prob <- $\mathrm{t}(\operatorname{apply}($ All.Comb, 1, function $(\mathrm{x}) \mathrm{x} * \mathrm{c}(.01, .01, .01, .01, .01)+(1-\mathrm{x}) * \mathrm{c}(.99, .99, .99, .99$, .99))) \#Change each element to its corresponding probability(loss/no loss)

Size <- $\mathrm{t}(\operatorname{apply}($ All.Comb, 1 , function(x) $\mathrm{x} *$ VarWeight1)) \#Change each element to its loss value

All.Prob <- apply(Prob, 1, prod) \#Multiply row probabilities to get overall chance event occurs All.Size <- apply(Size, 1, sum) \#Sum each row for total sum of losses

All.Unsorted <- cbind(Size, All.Size, All.Prob) \#Combine individual losses, total loss, and overall probability
All.Sorted1 <- All.Unsorted[order(All.Size),] \#Sort from smallest total loss to largest total loss All.Sorted1
\#\#Weighted Combination2\#\#
names(All.Comb) <- c("MSFT", "DIS", "AMZN", "BAC", "MCD") \#Rename columns to corresponding company names

Prob <- $\mathrm{t}\left(\right.$ apply(All.Comb, 1 , function(x) $\mathrm{x}^{*} \mathrm{c}(.01, .01, .01, .01, .01)+(1-\mathrm{x}) * \mathrm{c}(.99, .99, .99, .99$, .99))) \#Change each element to its corresponding probability(loss/no loss)

Size <- $\mathrm{t}(\operatorname{apply}($ All.Comb, 1, function(x) $\mathrm{x} *$ VarWeight2)) \#Change each element to its loss value

All.Prob <- apply(Prob, 1, prod) \#Multiply row probabilities to get overall chance event occurs All.Size <- apply(Size, 1, sum) \#Sum each row for total sum of losses

All.Unsorted <- cbind(Size, All.Size, All.Prob) \#Combine individual losses, total loss, and overall probability
All.Sorted2 <- All.Unsorted[order(All.Size),] \#Sort from smallest total loss to largest total loss All.Sorted2

```
###Exceedance Probability for Weight1
#Calculate Probability of each percent loss occuring
ProbLoss <- matrix(0, 32, 2) # Table with Probabilities of each loss
for(i in 1:32)
{
    ProbLoss[i,1] <- -All.Sorted1[i,6]
    ProbLoss[i,2] <- All.Sorted 1[i,7]
```

```
}
ProbLoss
#Calculate Exceedance Probabilities
Exceedance1 <- matrix (0,32,2)
for(i in 0:31)
{
    Exceedance1[i+1,1] <- ProbLoss[i+1,1]
    for(j in 0:31)
    {
        if(ProbLoss[j+1,1] >= ProbLoss[i+1,1])
        {
        Exceedance1[i+1,2] = Exceedance1[i+1,2] + ProbLoss[j+1,2]
    }
    }
}
#Plot Exceedance Probability Curve
Exceedance1
plot(y = Exceedance1[,2], x = Exceedance1[,1], ylim = c(0,.05), type = 'o', xlab = "Percent Loss"
, ylab = "Exceedance Probability") #plot All.Prob
###Exceedance Probability for Weight2
#Calculate Probability of each percent loss occuring
ProbLoss <- matrix(0, 32, 2) # Table with Probabilities of each loss
for(i in 1:32)
{
    ProbLoss[i,1] <- -All.Sorted2[i,6]
    ProbLoss[i,2] <- All.Sorted2[i,7]
}
```


## ProbLoss

```
\#Calculate Exceedance Probabilities
Exceedance2 <- matrix \((0,32,2)\)
for(i in 0:31)
\{
```

```
Exceedance2[i+1,1] <- ProbLoss[i+1,1]
```

Exceedance2[i+1,1] <- ProbLoss[i+1,1]
for(j in 0:31)
for(j in 0:31)
{
{
if(ProbLoss[j+1,1] >= ProbLoss[i+1,1])
if(ProbLoss[j+1,1] >= ProbLoss[i+1,1])
{
{
Exceedance2[i+1,2] = Exceedance2[i+1,2] + ProbLoss[j+1,2]
Exceedance2[i+1,2] = Exceedance2[i+1,2] + ProbLoss[j+1,2]
}
}
}

```
}
```

\#Plot Exceedance Probability Curve
Exceedance2
$\operatorname{plot}\left(\mathrm{y}=\right.$ Exceedance2[,2], $\mathrm{x}=$ Exceedance2[,1], ylim $=\mathrm{c}(0, .05)$, type $=$ ' $\mathrm{o}^{\prime}$, xlab $=$ "Percent Loss" , ylab = "Exceedance Probability") \#plot All.Prob


[^0]:    Table 2 Hurricane Loss Combination Probability

