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Spring 2016

# An Investigation of the Duality Between Art and Math

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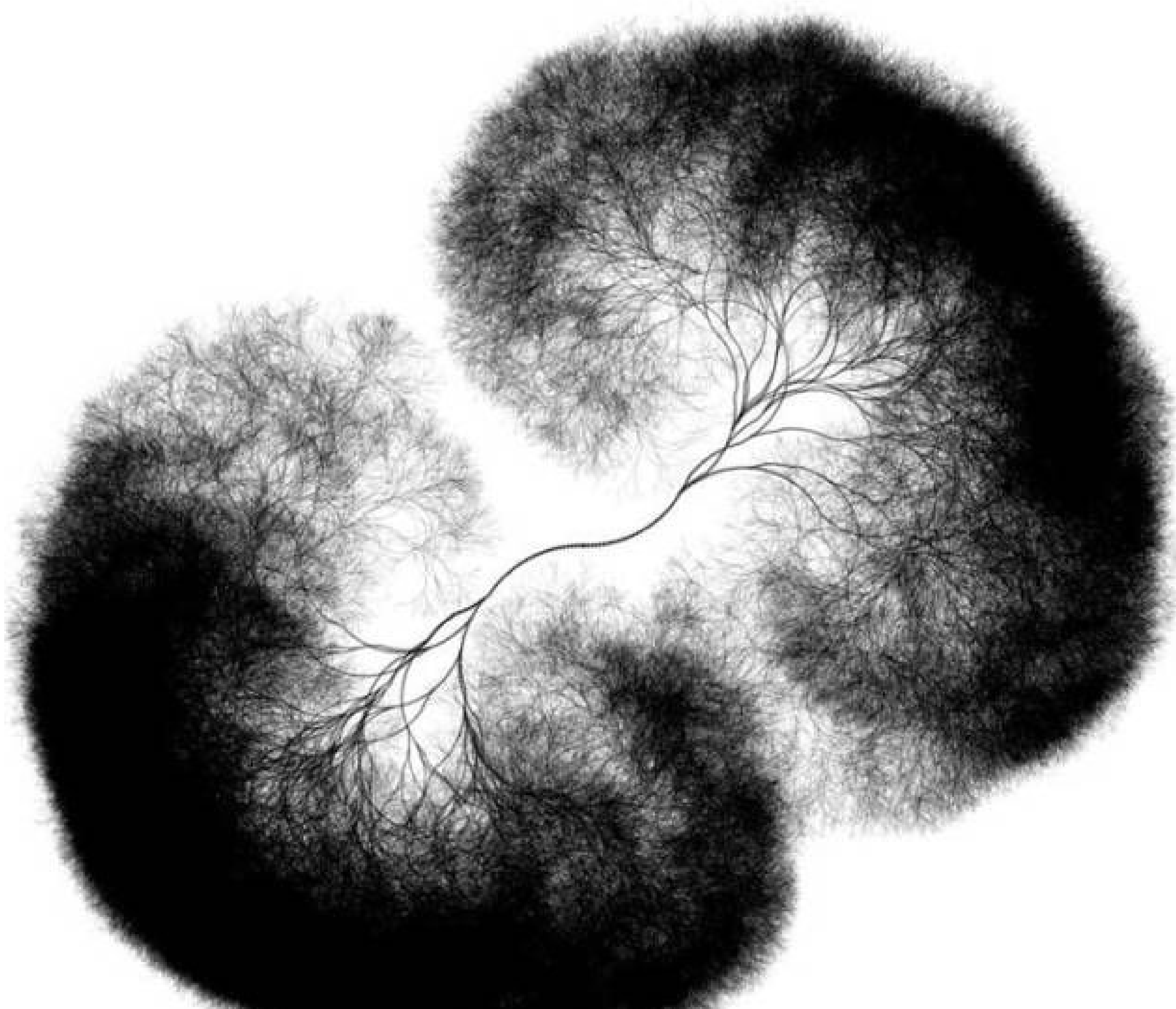
# An Investigation of the Duality Between Art and Math

By Hope Hickman



[Fig 1] Maddox, Anthony. *Neuron Fractal*. Digital image. *Art and Science Journal*. N.p., n.d. Web. <http://www.artandsciencejournal.com/post/19130472373/anthony-mattox>.

**An Introduction**

As a child, the world is full of mystery and allure. The workings of the world have yet to unveil themselves through science, and imagination seems a stronger force than the scientific method. Discovery happens daily through the investigation of trial and play. There is no means to an end when a child sifts through the leaves in his or her backyard, enamored by the multitude of slugs and bugs and gooey gunk taking up rent in the mass of decay. The innocent mind wanders aimlessly and what it picks up while on that voyage is often profound.

In the years of one's youth, there are moments when doors to the adult world hurl themselves open, possibly during the unveiling of fractions, the discovery of the water cycle, or learning of hibernation.

Fourth grade was the year of rocks: sedimentary, igneous, metamorphic, tectonic plates, Pangea. It was very overwhelming as a child to find out that mountains are the result of an ever changing crust, one that is twisting and turning under the feet of all people and all cities.

The idea of nothingness arises in a child's mind very early on. "What was when I was not born yet?". Spatial identification is complicated, "If I am four feet tall, how tall is a two hundred foot building and how big is the earth?". The questions come at such a fast pace and answers are flung haphazardly into the forefront of young minds. Without much order, the layers of life unfold.

This beautiful process of learning is a component of human life which propels children into the adult world, the world of business, science, technology, politics and art. Without the luster of learning, human development may stand still.

The ability to ask questions, to dig in the metaphorical dirt of man and come to conclusions without the intention of solving some specific problem is what makes man into human. When one is a child, one does not differentiate between biology and chemistry. One looks at the world with large glossy eyes in awe of the subject matter that is existence. One creates mud castles and doodles and lines up macaroni shells. Love, happiness, and anger coincide with plant biology, mathematics, and engineering. The two mesh together in an educational experience. Learning to build a fire does not necessarily include a strong knowledge of chemistry, physics, or math. However, a good fire depends on the fuel, proper air flow and geometry. So much is learned through multidisciplinary actions.

The human experience is reliant on our passions and our minds. Human development, science, technology, mathematics, engineering, what are these things without the human experience? Although disciplines are a necessary way to categorize information, where is the line drawn between two things as perpendicular as art and mathematics? Can art, humanities and social sciences expand and leak into those pure disciplines?

I bring up a child's mind for comparison because children easily blur these lines. There is lesser duality in an innovative mind between art and mathematics. When boxes exist that allocate science, exploration and questioning to a particular discipline, we are limiting our possibilities for exploration in all realms of humanity.

Mathematics can improve art. Art can improve mathematics. Math and art can combine to synthesize great frontiers. They are different, and are perceived as divergent but the duality between the two is not as simple as black and white.

**The Presence Math Has Had in Art : Trigonometry to Topology**

For thousands of years math has been shaking hands with art, providing both inspiration and structure. Within some works of art, it is obvious. Intricate geometries, repeating patterns, and intentional proportions reveal themselves at first glance. For others, the relationship is initially unrecognizable, whilst the math connection is fiercely sound. In some cases mathematics is a necessity for construction. In others mathematical concepts simply provide inspiration. Either way, be it clear or hidden, conceptual or direct, in nearly every branch of fine art, one can find examples of this interconnection.

Look back to the Medieval era (500 CE- 1500 CE) when architects built skyscrapers of stone and glass with simple tools such as compasses, right angled square bodies, and brute force. Here, mathematics was critical to aesthetics. During the construction of these structures laborers would not refer to proofs or theoretical work. None of the physical work included axioms, theorems or deductions. However, the main mathematical application of medieval cathedral construction was still that of practical geometry.

Even though masonry was definitely a craft, and despite the empirical nature of a stonemason's work, a huge component of the job was based directly in geometry. In fact, stonemasonry was known at this time as the art of geometry. Look at the following images, that of the Milan Cathedral in Italy and the Burgos Cathedral. Without the eloquence of mathematics, these cathedrals would cease to exist as works of art. They would instead be merely thick, brute, utilitarian walls with roofs.

[Fig 2 ] Milan Cathedral. Digital image. *Destination 360*. N.p., n.d. Web. <http://www.destination360.com/europe/italy/milan-cathedral>.

[Fig 3 ] Burgos Cathedral. Digital image. *History Hub*. N.p., n.d. Web. <http://www.thehistoryhub.com/wp-content/uploads/2014/08/Burgos-Cathedral-Pictures.jpg>.

The Burgos Cathedral of gothic style may be of particular interest to mathematicians. Built in Burgos Spain in 1221, the tallest tower of the Burgos Cathedral is an octagonal tower known as the Cimborio (spanish for dome). In 2011, architect, Dirk Huylebrouk, in collaboration with mathematicians Antonio Redondo Buitrago and Encarnacion Reyes Iglesias, analyzed the orthogonal geometry of the Cimborio vault. Their findings clearly show examples of the cordovan proportion and tesseracts within the Cimborio vault's roof.

[Fig 4 ] Burgos cimborio. Digital image. *Flickr Hive*. N.p., n.d. Web.
<Burgos Cathedral. Digital image. History Hub. N.p., n.d. Web. .>.

For those who are unfamiliar, the Cordovan proportion, $C=(2-2)1/2$, is the ratio between the radius R and side length L of a regular octagon. The Cordovan proportion was introduced by R. de la Hoz in 1973, seven hundred and fifty two years after the vault was made. This Cordovan proportion lends itself to an entire family of Cordovan polygons.

The tesseract, also a component to the Cimborio, is a 4-D analog to the square. It is known as the 4-D hypercube. Hidden orthographic projections of the tesseract are embedded with rosettes into the structure of the Cimborio roof. There is even a fractal pattern to the combinations of rosettes and tesseracts that Huylebrouk brings to the attention of the reader.[1]

Aside from the interesting math, the beauty of Huylebrouk's analysis lies in the fact that the Cimborio in the Burgos Cathedral is only one example of interesting geometry hidden in Cathedral art. Notre Dame, Chartres, St. Paul's, Cologne and many more have similar shapes and ratios within their walls, within their windows and within their structure. Many, of which, have yet to be discovered.

Within cathedrals, it is fairly evident that some sort of mathematical game is being played. I say this because of the symmetry, the structure, and the functionality of the buildings. However, the mathematics that are intertwined with art are often times much more subtle. For example, the use of the golden ratio within two dimensional composition.

Composition is often times one of the most challenging and most important components of a piece of art. Many painters are taught to follow either the rule of thirds, or the golden ratio to achieve a pleasing composition within their work.

Neither is very difficult to understand. Both the law of thirds and the golden ratio determine the placement of the piece's focal points. When an artist wants to follow the law of thirds, he or she must take the space that the piece will be occupying and divide it by thirds both horizontally and vertically. Then he or she places the focal points along the dividing lines.

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[1] Huylebrouck, Dirk, Antonia Redondo Buitrago, and Encarnación Reyes Iglesias. "Octagonal Geometry of the Cimborio in Burgos Cathedral." *Octagonal Geometry of the Cimborio in Burgos Cathedral*. Paperity, Feb. 2011. Web. 25 Feb. 2016. <http://paperity.org/p/8949024/octagonal-geometry-of-the-cimborio-in-burgos-cathedral>.

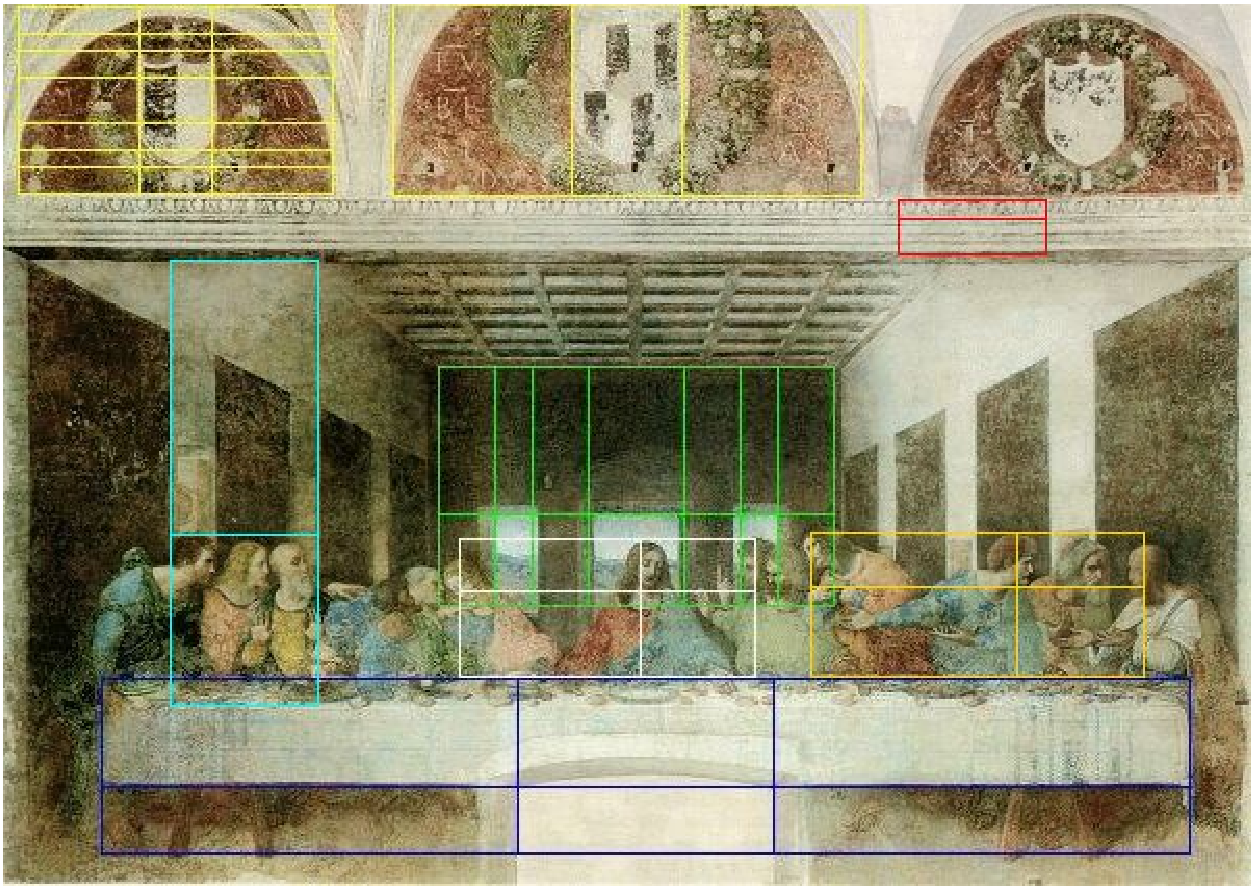

[Fig 5 ] Slazinik, Courtney. Rule of thirds. Digital image. *Click It Up A Notch*. N.p., n.d. Web. <http://clickitupanotch.com/2010/09/rule-of-thirds/>.

Notice that each dividing line, by itself, divides the piece into two parts. The area of these two can then be related in a 2:1 ratio. On one side there are two thirds and on the other side, there is one third.

Following the golden ratio is similar except the golden ratio is not as simple as a 2:1 ratio. Instead of dividing the space by thirds. The artist uses a ratio of approximately 1.618034 :1 to determine the most compositionally pleasing dividing lines.

In the Renaissance period (1300 CE-1700 CE) the golden ratio was known as the “divine proportion”. Leonardo, Botticelli, Michelangelo, Raphael, and many more used the golden ratio in their pieces. However, the golden ratio dates back even further than Leonardo Da Vinci.

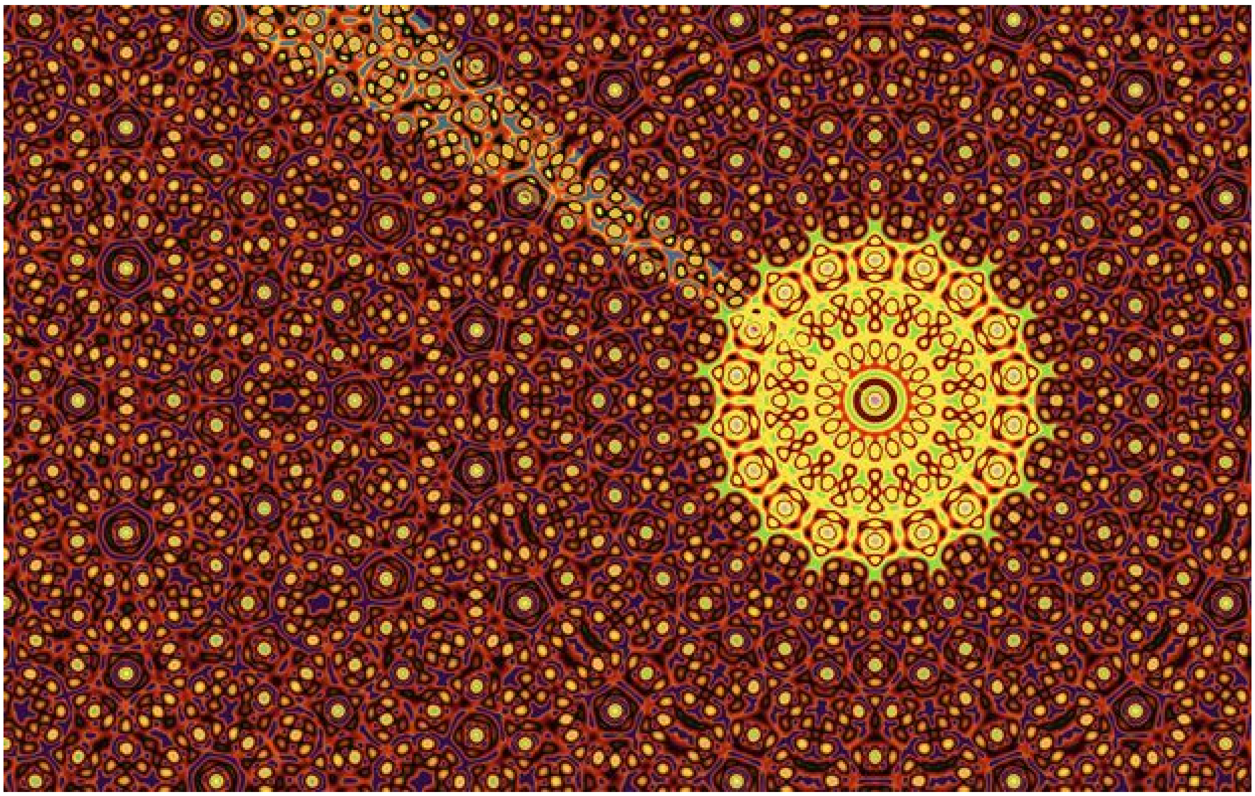
Around 500 BCE, Phidias, a greek sculptor and mathematician, is known to have used the golden ratio in the design of sculptures for the Parthenon, a famous former temple in Greece.



[Fig 6]Meisner, Gary. *The Last Supper*. Digital image. *Goldennumber.net*. N.p., n.d. Web. <http://www.goldennumber.net/art-composition-design/>.

It was not until 1200 CE that Leonardo Fibonacci discovered the mathematical sequence that converges to the golden ratio, and not until the 1500's did it pick up the name divine proportion. The common mathematical name phi was given in the 1900's. Since then The Golden Ratio has continued to appear in both science and math.

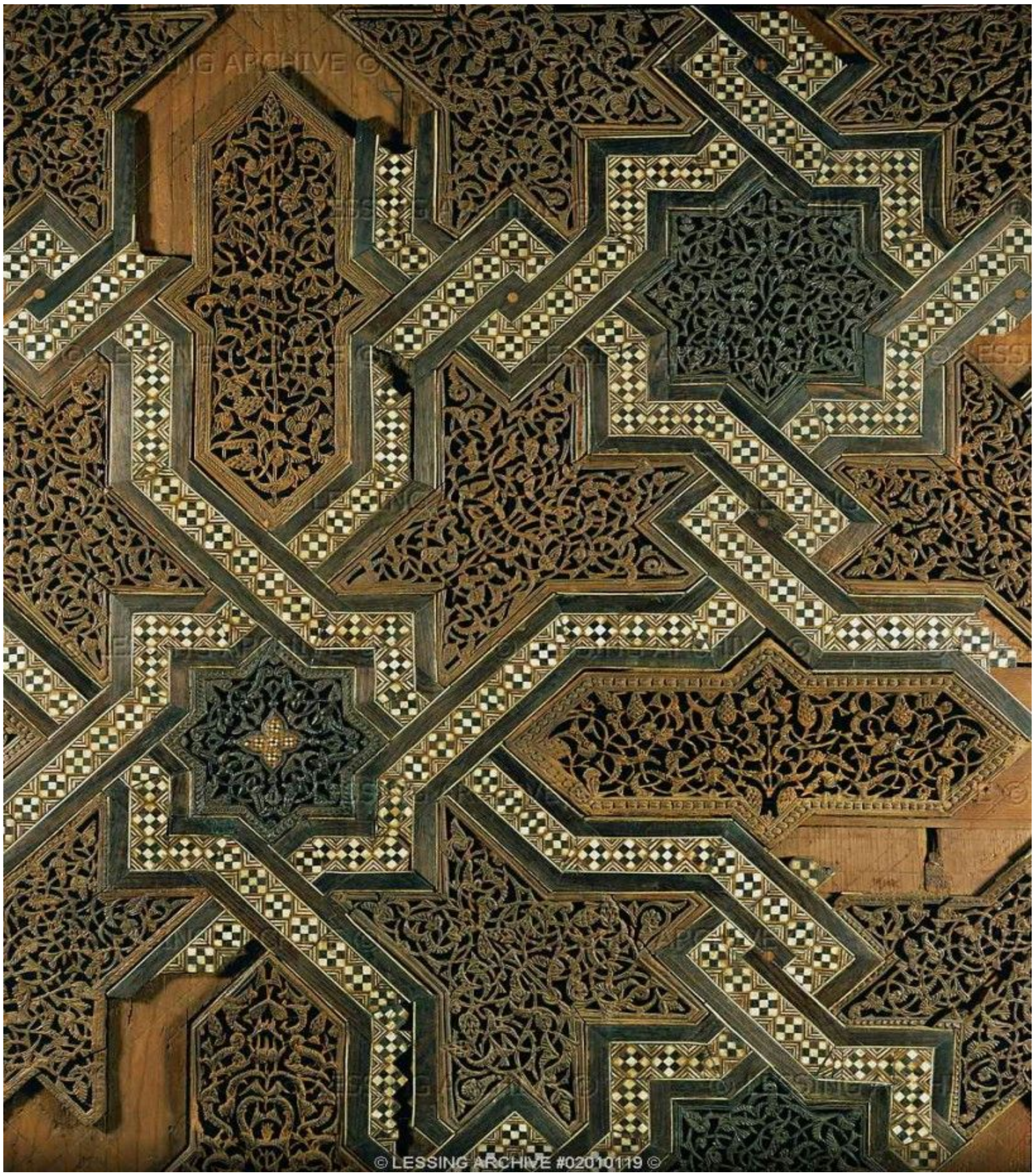
It contributed to Roger Penrose's discovery in the 1970's of Penrose tilings, which is a non-periodic tiling generated by an aperiodic set of prototiles. Penrose tiles were the first to allow fivefold rotational symmetry.  Phi also held a large role in the discovery of quasicrystals, structures that are ordered but non periodic and found in nature. Quasicrystals were discovered in the 1980's, and unlocked a whole new world. The previously accepted tenet of solid state physics was blown away upon discovering quasicrystal atomic structures in matter.



[Fig 7 ] Quasicrystals in art. Digital image. *Crystallography*. N.p., n.d. Web. <http://www.iycr2014.org/events/lectures/chai-and-why-science-cafe-quasicrystals-from-impossibility-to-nobel-prize>.

Interestingly enough, quasicrystals seem to have been found in even older art than some cathedrals. Islamic art, which is considered to have been produced from the 7th century onwards, almost always exhibits geometric patterns. Figurative imagery is not often included due to religious beliefs. The Hadith and many Islamic authorities discourage depictions of both humans and animals. Because of this, Islamic geometric patterns have evolved into quite spectacular and intricate formulations.

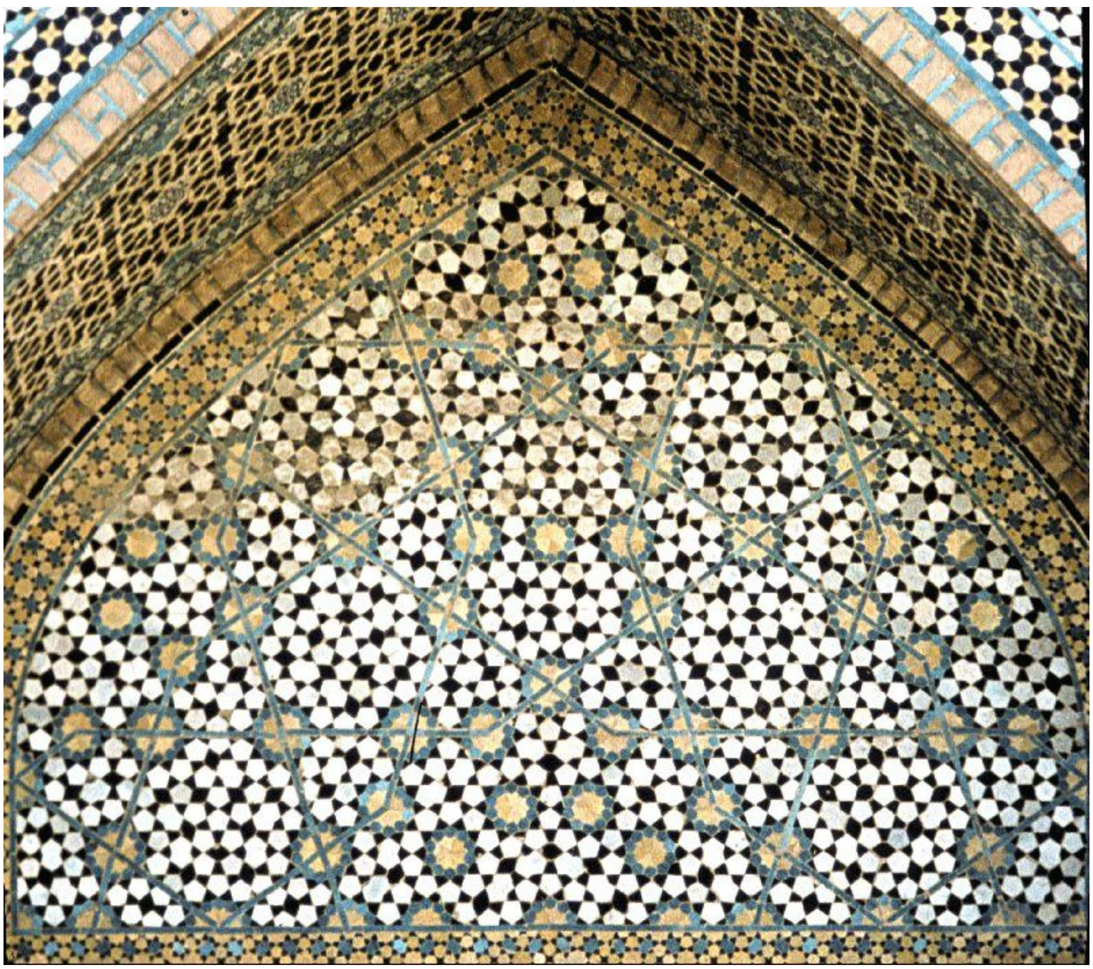
These patterns are often created using combinations of repeating circles and squares as a foundation. However, they often branch off so severely, that the above statement may be too terse. These simple geometries interlace and overlap and often create bundles of intricate patterns and shapes with complex rotational symmetries.



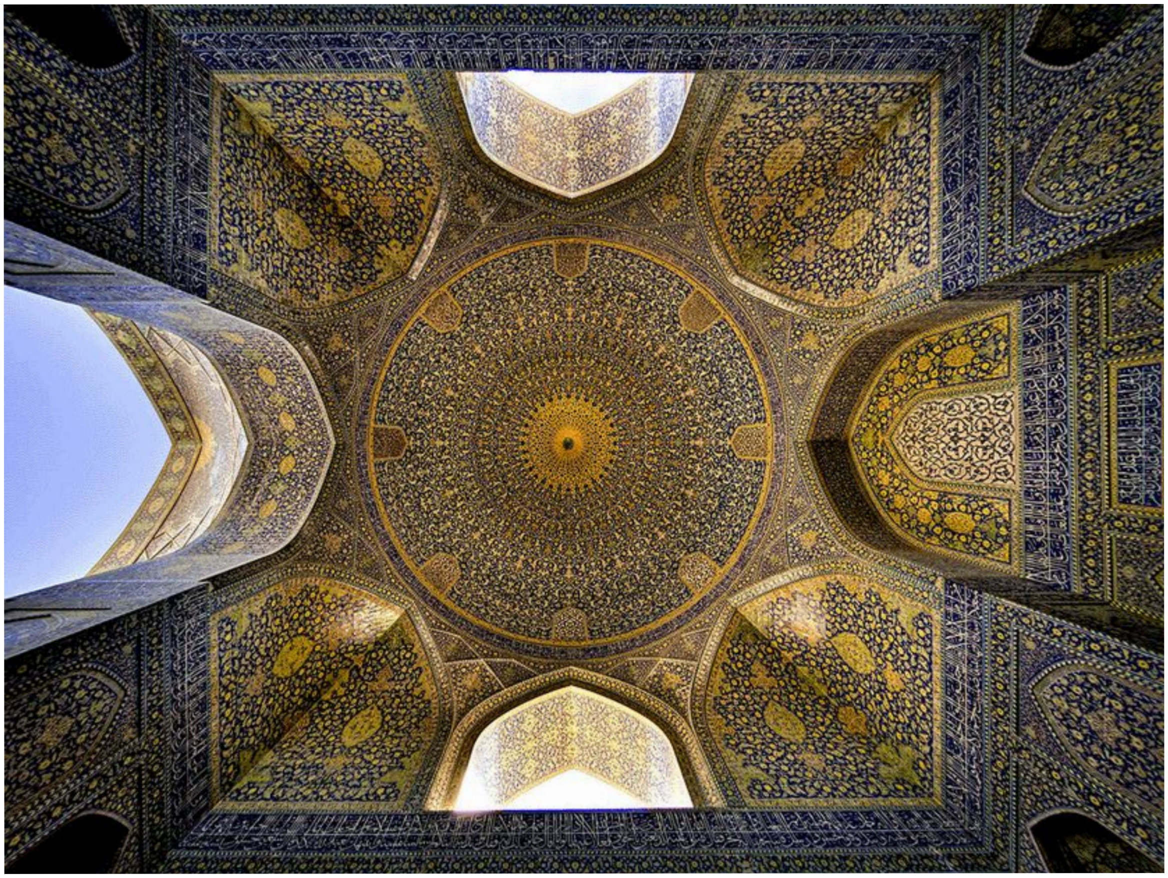

[Fig 8 ] Islamic Woodworking. Digital image. *The Masterpiece Minibar*. N.p., n.d. Web. <http://islamic-arts.org/2012/the-masterpiece-minbar/>.

They occur in a variety of forms in Islamic art including carpets, tilework, Persian girih, vaulting, stone screens, ceramics, leather, stained glass, woodwork and metalwork. With each craft and each generation, Islamic geometric patterning evolved.

By the 1600's the techniques used in Islamic art were sophisticated enough to make somewhat quasi-periodic patterns similar to the penrose tilings. The prime example being that of a girih design on the Darb-i-Imam shrine in Isfahan, Iran. The shrine was made in 1453.



[Fig 9 ] Dar-I Iman Shrine. Digital image. *John Baez's Stuff*. N.p., n.d. Web. <http://math.ucr.edu/home/baez/week247.html>.



[Fig 10 ] Shah Mosque. Digital image. *The Wonders of Islamic Architecture*. N.p., n.d. Web. <http://tribune.com.pk/story/871868/the-wonders-of-islamic-architecture/>.

One artist particularly interested in quasicrystals as a source of inspiration is Tony Robbin. Born in Washington in 1943, Robbin works with computer visualizations, painting and sculpture. He is particularly fascinated with the idea of the fourth dimension and has developed his own method of sculpture that mimics quasicrystal structures. He currently holds a patent for his method.

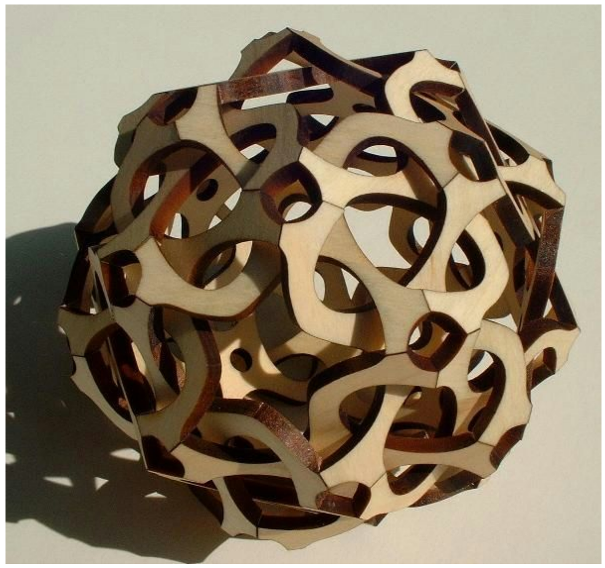
Robbin's "quasi-crystal" sculptures are based on the idea of quasicrystal geometry; they display non-repeating geometric patterns though they are made up of identical, repeating elements. Below is a picture of his work at COAST (*Center for Art, Science, and Technology)* at the Danish Technical University.

[Fig 11 ] Tony Robbin's Sculpture at COAST. Digital image. *Art*. N.p., n.d. Web. <http://www.domica.ch/art.html>.

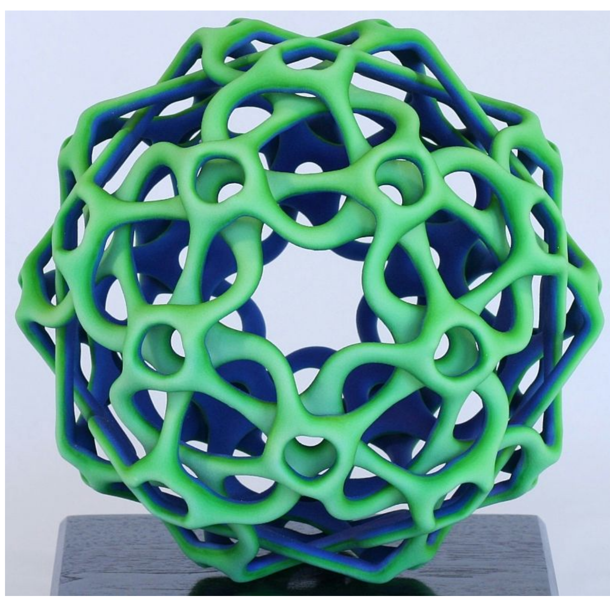
Another rather famous artist, known for his mathematically inspired sculptural work is George W. Hart. Hart was born in 1955. He is an American geometer, sculptor, mathematician and computer scientist. He received a B.S. in Mathematics from MIT, an M.A. in linguistics from Indiana University , and a Ph.D. in Electrical Engineering and Computer Science from MIT.

Hart's art is known for its mathematical depth. His pieces often center around novel polyhedron structures and the algorithms which produce them. Hart has discovered multiple algorithms which produce new classes of polyhedra. He then uses sculpture to present them to the world. Hart often implements them with the aid of a 3-D printer.

Below are two of Hart's sculptures. The first is made out of wood and can geometrically be understood as a subset of the first stellation of the rhombic triacontahedron. It is composed of thirty parts which lie on the face planes of a rhombic triacontahedron. The second was created using a 3-D printer.



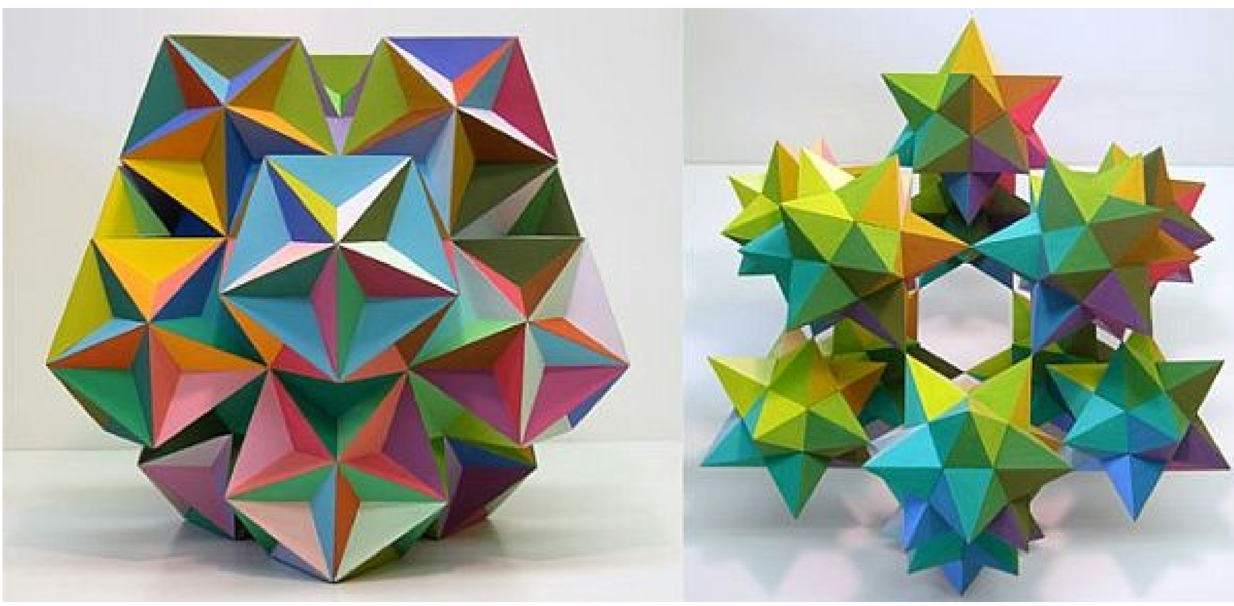
[Fig 12 a] Hart, George W. Rhombic Triacontahedron. Digital image. *Knot Structured*. N.p., n.d. Web. <http://georgehart.com/sculpture/knot-structured.html>.



[Fig 12 b] Hart, George W. Rhombic Triacontahedron. Digital image. *Knot Structured*. N.p., n.d. Web. <http://georgehart.com/sculpture/knot-structured.html>.

Similar to Hart's work is some of Morton C. Bradley's. Bradley was born in 1912 and died in 2004. Most of Bradley's geometric sculptures involved polyhedra based constructions. He had an affinity for color and would paint his pieces in as organized a manner as the mathematics which governed his creations. Bradley graduated from Harvard in 1933 with a degree in the arts.

Below are two of Bradley's sculptures. The geometric forms are each based on twelve copies of a Kepler-Poinsot polyhedron, with twelve great dodecahedra on the left and twelve small stellated dodecahedra on the right.They are made of painted, 2-ply Strathmore paper.



[Fig 14] Bradley, Morton. Kepler-Poinsot Polyhedron. Digital image. *MoMath*. N.p., n.d. Web. <http://momath.org/home/math-monday-02-01-10/>.

The list of sculptural works relating to mathematics goes on and on. The following are topologically inspired sculptures, paintings, fiber arts, computer generated images, buildings and more.

[Fig 15 ] Hild, Eva. Untitled. Digital image. *Azurebumble: Sculpture*. N.p., n.d. Web. <https://azurebumble.wordpress.com/category/sculpture/page/2/>.

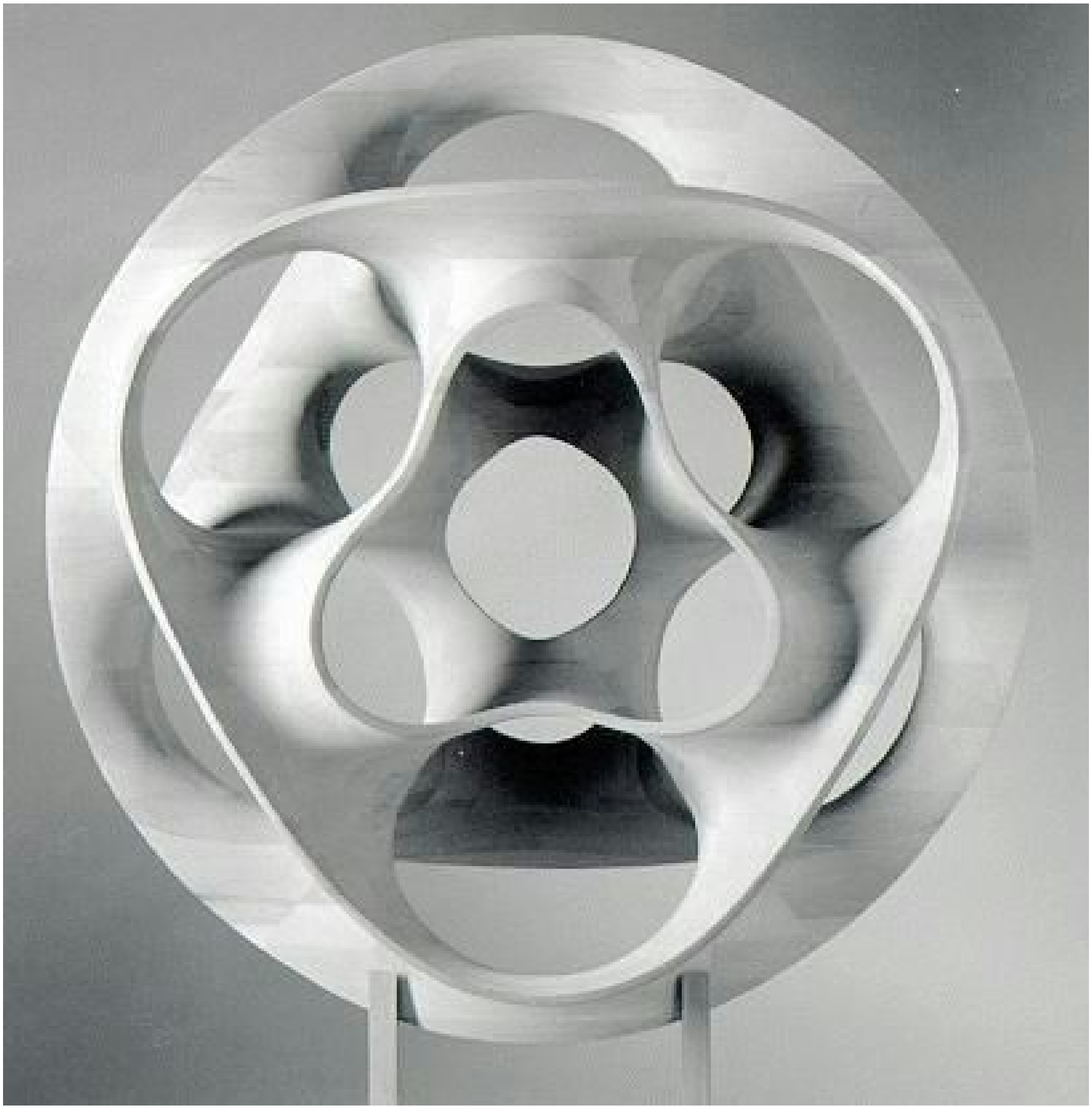
Above is one of Eva Hild's many topologically inspired pieces. Her ceramic pieces are made of stoneware and are handbuilt using thin slabs of clay. The pieces investigate and challenge the perception of inner and outer spaces.

[Fig 16 ] Untitled. Digital image. *John Mason | Ceramics*. N.p., n.d. Web. <https://taylordantuma.wordpress.com/2014/09/04/john-masons/>.

Above is a ceramic structure built by the American artist John Mason. John Mason is well known for investigating simple mathematical concepts. He often includes rotation, symmetry and repeating units. His art is viewed as algorithmic.

[Fig 17 ] Séquin, Carlos H. *TorusKnot5.3*. Digital image. *Art Geometry and Abstract Sculpture*. N.p., n.d. Web. <http://www.eecs.berkeley.edu/~sequin/ART/Science_Sculpture/TorusKnot5_3.JPG>.

Above is a bronze sculpture by Carlo H. Séquin. Séquin is a graduate professor in computer science at University of California Berkley. He is constantly producing topologically interesting sculptures. Many are 3-D printed before cast in bronze. Above is a torus knot.



[Fig 18 ] Collins, Brent. 6-Storey Scherk Minimal Surface. Digital image. *Collins Sculptures*. N.p., n.d. Web. <http://www.cs.berkeley.edu/~sequin/SCULPTS/collins.html>.

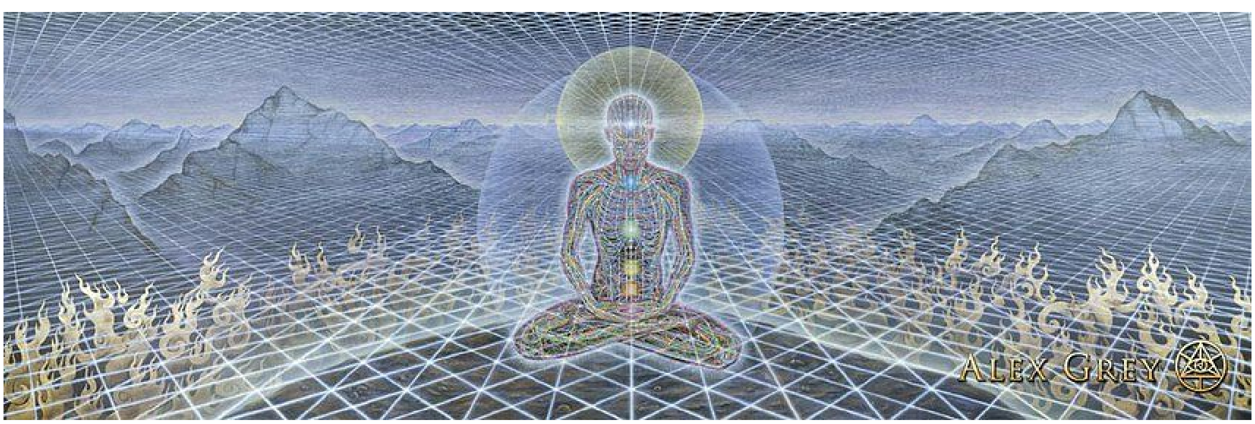
This can be understood as a 6-storey Scherk Minimal Surface, wound into a toroidal loop. It is a wooden sculpture by the artist Brent Collins. Collins has taken a liking to saddle surfaces and has been highly inspired by mathematics. The inspiration for his pieces is read quite clearly throughout his work.

[Fig 19 ] 6-Storey Scherk Minimal Surface. Digital image. *Bradford Hansen-Smith*. N.p., n.d. Web. <http://www.bridgesmathart.org/art-exhibits/bridges2005/BradfordHansenSmith.html>.

This is the work of Bradford Hansen-Smith, a sculptor who has invested much time and effort investigating circles and simple geometries. He has been able to create wonderfully intricate sculptures from folding and intertwining paper circles.

[Fig 20 ] Johnson, Crockett. *Point Collineation on the Triangle*. Digital image. *Mathematical Paintings of Crockett Johnson*. N.p., n.d. Web. <http://www.bridgesmathart.org/art-exhibits/bridges2005/BradfordHansenSmith.html>.

This is one of Crockett Johnson's many mathematical paintings. The painting is titled "Point Collineation in the Triangle" and is inspired by the famous mathematician Euler. The piece refers to projective geometry.




[Fig 21] Grey, Alex. *Theologue*. Digital image. N.p., n.d. Web. <http://www.newmanayurveda.com/>.

The above painting is one of Alex Grey's many works. Grey is fascinated by sacred geometry, an idea that ascribes spiritual meaning to certain geometries.

[Fig 22] Friedman, Tom. *Theologue*. Digital image. *SAATCHI Gallery*. N.p., n.d. Web. <http://www.saatchigallery.com/aipe/tom_friedman.htm>.

This is the work of Tom Friedman who is more whimsical in his sculpture. He is merely inspired by mathematics. Although his work doesn't permeate into high level mathematics some of his work is fairly algorithmic.

[Fig 23 ] Caltrava, Santiago. *City of Arts and Sciences*. Digital image. *Best Interior Designers*. N.p., n.d. Web. <http://www.bestinteriordesigners.eu/top-interior-designers-santiago-calatrava/>.

Santiago Calatrava is an architect, engineer, and sculptor. He is well known across the globe for pushing architectural boundaries. His work is both topologically inspired, geometrically inspired and organically inspired. The connection between his work and mathematics is evident through both the outstanding structural engineering and artistic flow of his buildings.

[Fig 24 ] Mixed Media Chairs. Digital image. *Unbranded Designs*. N.p., n.d. Web. <http://www.unbrandeddesigns.com/blog/category/furniture-in-the-wild/>.

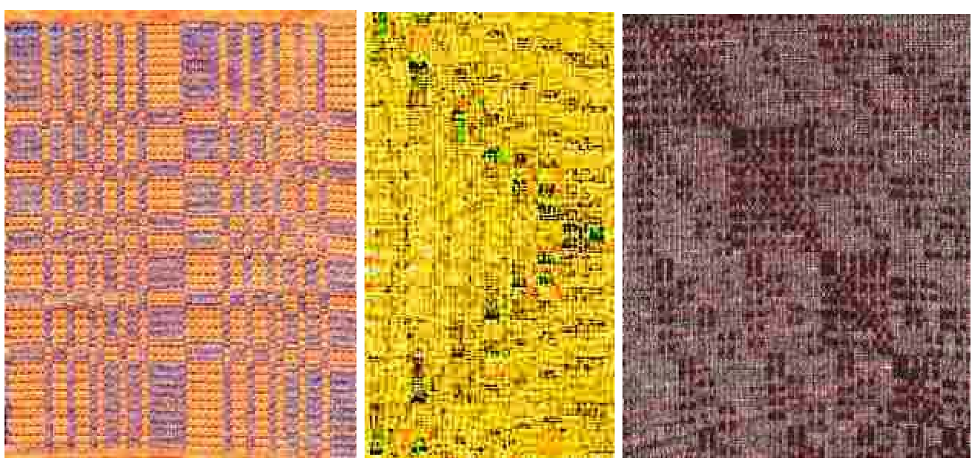
Here is the work of Joris Laarman. Laarman experiments with a broad pool of 3-D printers and materials. The patterns are synthesized using computer software programs. Much of his work is a combination of digital fabrication and craftsmanship.

[Fig 25 ] *Computational Oragami*. Digital image. *Inhabitat*. N.p., n.d. Web. <http://inhabitat.com/computational-origami-is-paper-art-that-folds-itself/origami-erik-demaine2/>.

The sculptures here are made of folded paper, created by sculptor/ mathematician Erik Demaine. Demaine is a professor of computer science at MIT. As a child prodigy, Demaine began college at the age of 12. His PhD dissertation documented much in the field of computational origami, the mathematical study of paper folding. The above images are a byproduct of his studies.

[Fig 26 ] Ernest, John. Maquette For Relief Mural at International Union of Architects. Digital image. *Wikipedia*. N.p., n.d. Web. <https://en.wikipedia.org/wiki/John_Ernest#/media/File:John_Ernest_maquette_for_mural_1961.JPG>.

This is one of John Ernest's works. Ernest is an abstract painter transfixed with mathematics. His particular interests lay in graph theory. Alongside artist Anthony Hill, Ernest made contributions studying crossing numbers of complete graphs.



[Fig 27] Dietz, Ada. Polynomial Expansions in Textiles. Digital image. *Fiber Arts*. N.p., n.d. Web. <http://fiberarts.org/design/articles/algebra.html>.

In 1946 Ada Dietz, an American weaver, developed a threading pattern based on a cubic binomial expansion. Dietz is most known for her detailed written study, *Algebraic Expressions in Handwoven Textiles*, written in 1949. It includes multiple weaving schemes all based on the expansion of multivariate polynomials.  Above are a few examples. On the far right is an example of $(a+b)^3$. In the middle is an example of $(a+b+c+d+e+f+g+h)^2$. On the left is an example of $(a+b+c+d+e+f)^2$.

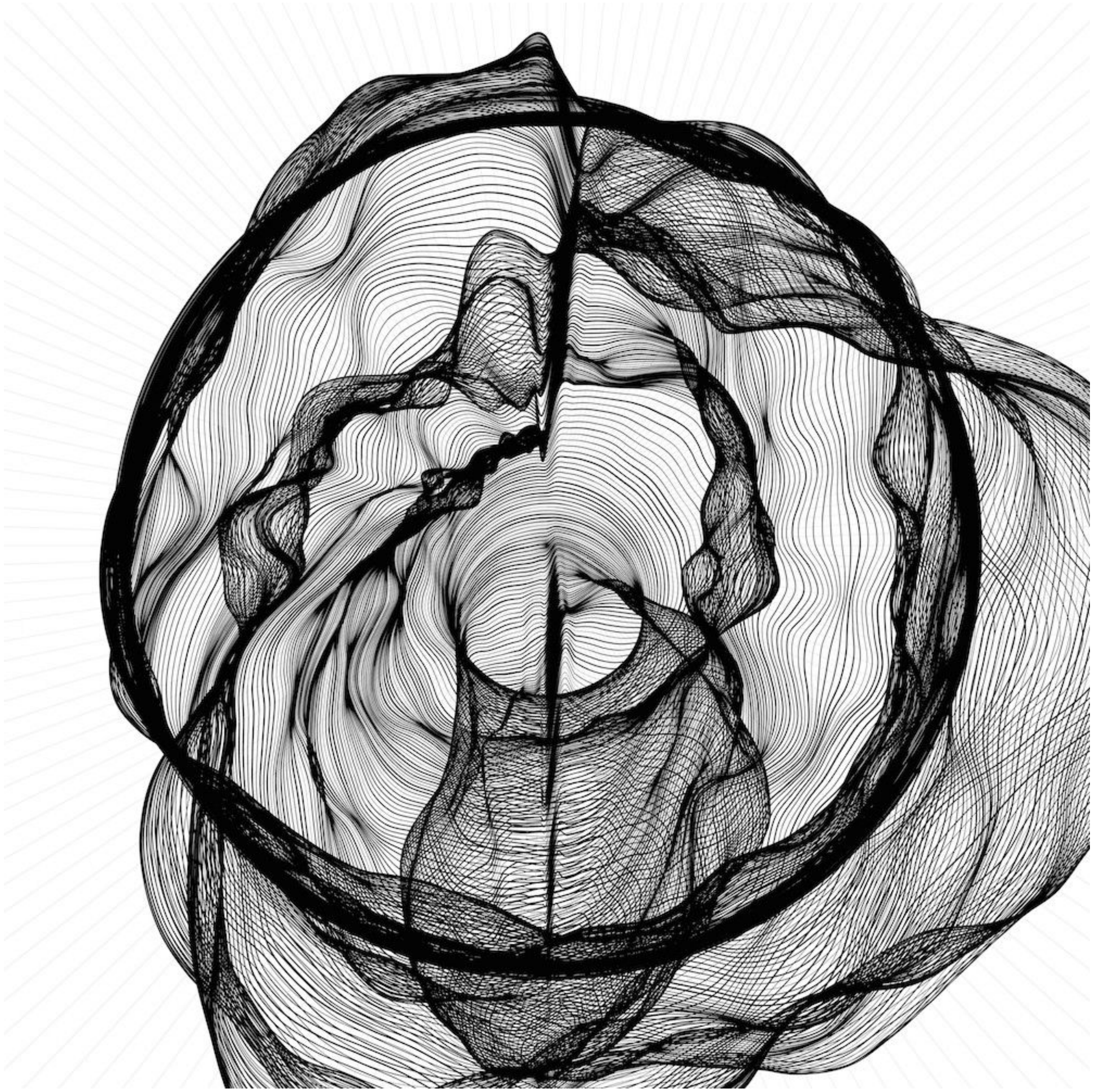
[Fig 28 ] Meyer, Gabriele. Crochet Hyperbolic Surface. Digital image. *Mathematical Art Galleries*. N.p., n.d. Web. <http://gallery.bridgesmathart.org/exhibitions/2014-joint-mathematics-meetings/gabriele_meyer>.

This is the work of Gabriele Mayer inspired by the work of Dr. Diana Tamina. It is a crocheted hyperbolic plane. Essentially the piece is one continuous spiral that interacts with itself organically. It is not surprising to find that crocheting, along with all fiber arts, is intrinsically mathematical. The structure, pattern and number system associated with stitching all provide strong evidence to support the fact.

Dr. Daina Taimina is a professor of mathematics at Cornell. In 1997 she discovered that through traditional crochet, one could model hyperbolic space. Before Taimina, many believed that materially addressing this aberrant geometry was not possible. In fact, before the century, many mathematicians had tried to prove this type of geometry impossible.

This style, which is now called hyperbolic crocheting, has been revealing quite interesting results both scientifically and artistically since it developed. Below is work done by the Institute of Figuring. It is part of the Crochet Coral Reef project, a project that intersects mathematics, art, and biology to respond to the oceanic crisis worldwide.

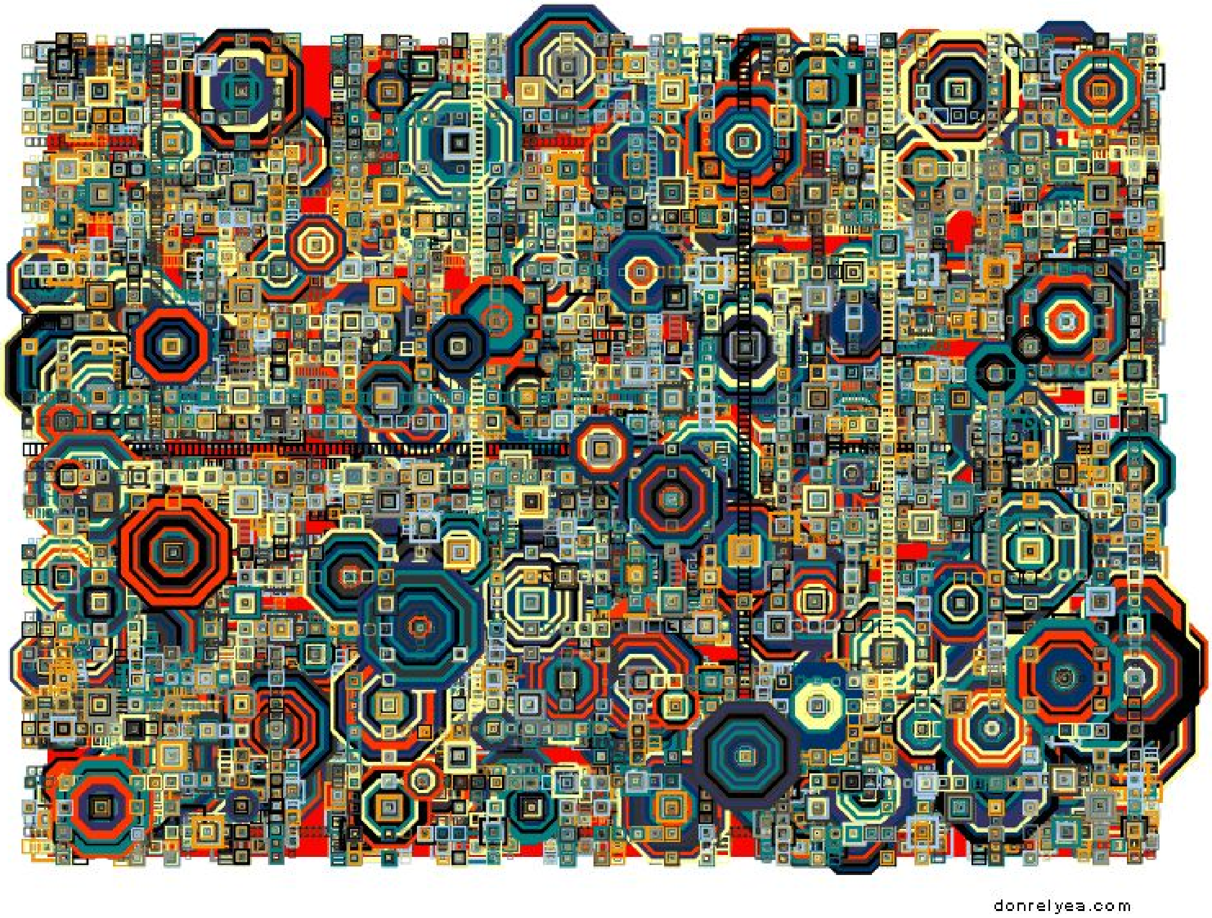
[Fig 29] Hyperbolic Crochet Reefs. Digital image. *Mathematical Art Galleries*. N.p., n.d. Web. <http://hellohart.com/tag/anneke-wiese/>.



[Fig 30]*Death Pop*. Digital image. *Zen Bullets*. N.p., n.d. Web. <http://zenbullets.com/prints/LP18_1_s14.php>.

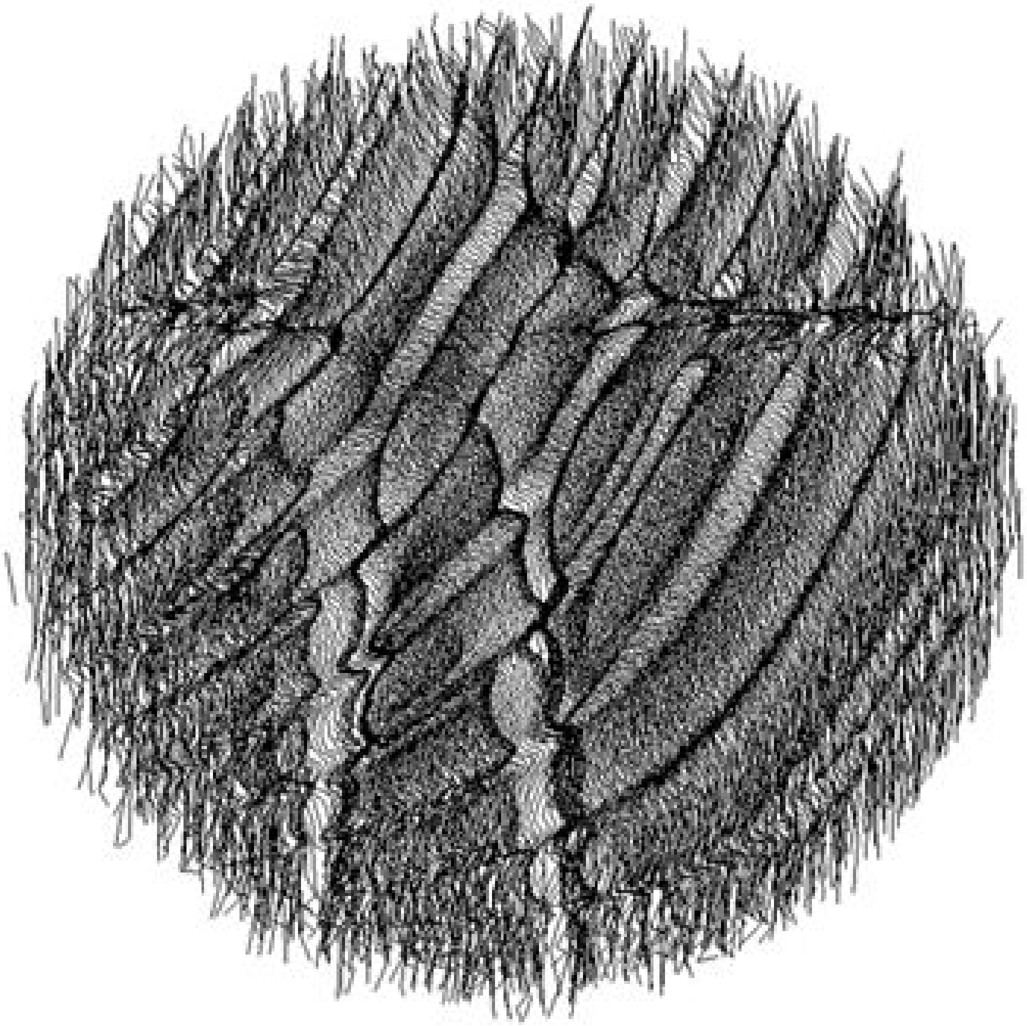
This image is a prime example of a popular new computer generated form of artwork, often referred to as algorithmic art. Algorithmic art is also known as “computer generated art”.

This was created by Matt Pearson. It is a part of his Death Pop series which explores the sound and form of vinyl.



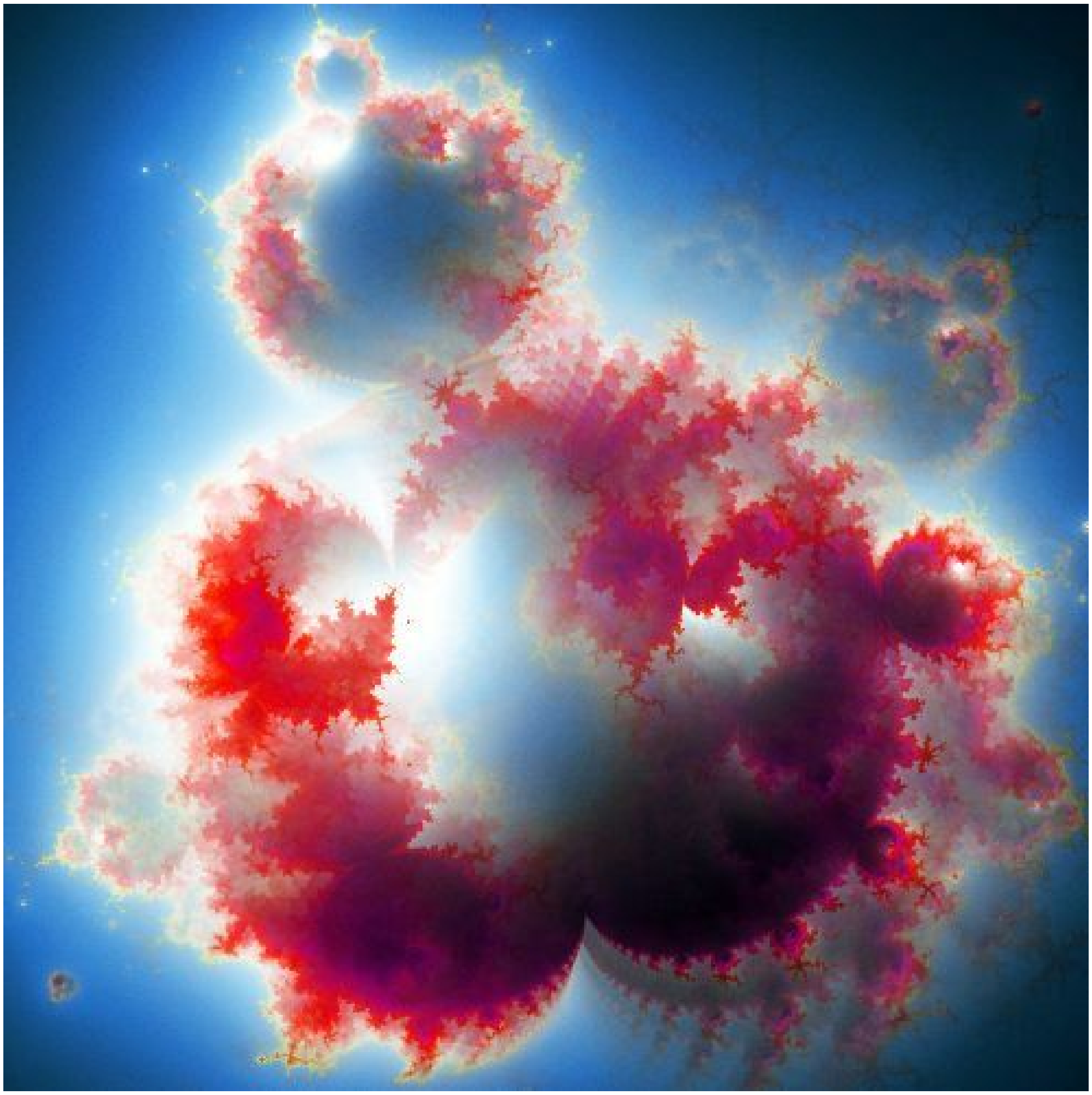

[Fig 31 ] Hillbert Space Filling Curve Abstract Geometric Art. Digital image. *Don Rylea*. N.p., n.d. Web. <http://www.donrelyea.com/hilbert_algorithmic_art_menu.htm>.

This piece by Don Relyea is also algorithmic art. The algorithm here is generated from a manipulation of the Hilbert space filling curve.



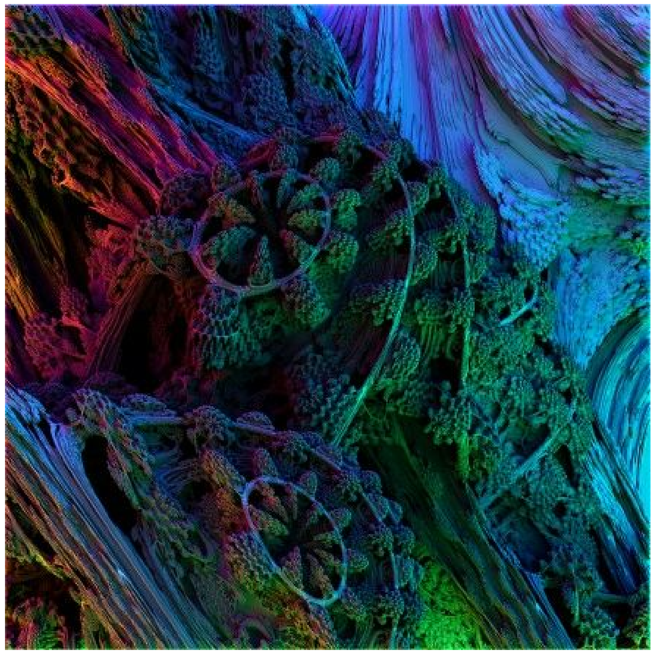
[Fig 32 ] Ceric, Vlatko. Algorithmic Art. Digital image. *Croatia World Network*. N.p., n.d. Web. <http://www.croatia.org/crown/articles/9293/1/Profdr-Vlatko-Eeriae-and-his-algorithmic-art-in-New-York-on-Nov-1st-2007.html>.

This is a piece that I am particularly drawn to. It was made by Dr. Vlatko Ceric, in the beginning of his artistic career. Ceric worked in computer modeling and simulation. He began producing images in the 1970's. He was a bit of a pioneer for algorithmic art.

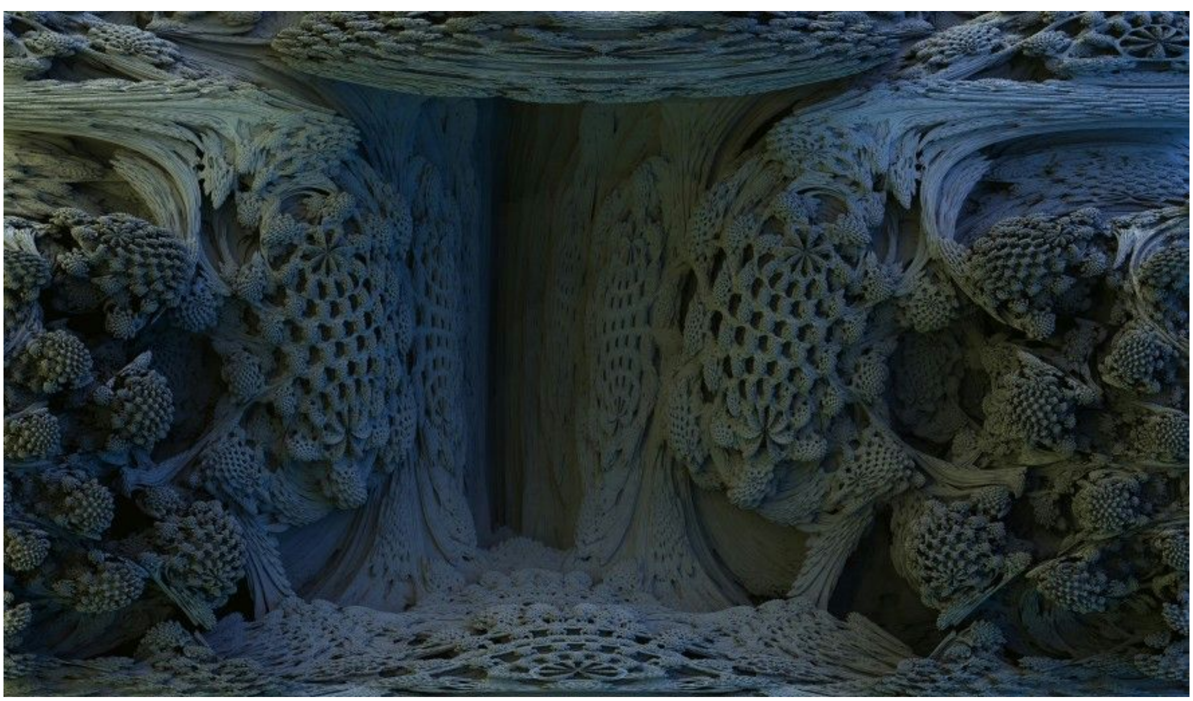


[Fig 33] Mandelbulb. Digital image. *Above Top Secret*. N.p., n.d. Web. <http://www.abovetopsecret.com/forum/thread542140/pg1>.

This image is one of many images from newly discovered algorithms by Daniel White and Paul Nylander. In 2009, the two managed to create the Mandelbulb, a 3-D version of the Mandelbrot set. Words can not describe the art that results from this purely mathematical process. Below are renderings of the Mandelbulb surface. What I find particularly interesting is that neither Daniel White nor Paul Nylander were seeking out art in their venture. Both the Mandelbrot and the Mandelbulb are, however, works of art.



[Fig 34 a] Surface of Mandlebulbs. Digital image. *Skytopia*. N.p., n.d. Web. <http://www.skytopia.com/project/fractal/mandelbulb.html>.



[Fig 34 b] Surface of Mandlebulbs. Digital image. *Skytopia*. N.p., n.d. Web. <http://www.skytopia.com/project/fractal/mandelbulb.html>.

**The Beauty of Mathematics**

Beauty is defined as a combination of qualities such as shape, color, or form, that pleases the aesthetic senses. Based on the shape, color and form of the previously listed artworks, it is safe to say that mathematics is capable of generating beauty. Here I will argue that mathematical results can themselves be beautiful.

The process of investigating mathematics is a highly creative act. It is not merely number crunching. It is a colorful discipline that spans many, very large areas, all of which interact. When one sits down to delve into higher level math the mind lights on fire with neural activity. When studying algebra and geometry the mind grasps around metaphors and real life examples to grapple with the density of fundamental theorems. When studying statistics and numerics, the brain allocates buckets and bins and filing cabinets to hold compounding information. To imagine probabilities and limits it creates branches and fractals.The mind twists around concepts looking at every angle until a three dimensional object of understanding starts to form.

During mathematical research, there is rarely a formula for success and, arguably, never a step by step guide. In order to find appropriate steps, creativity is necessary. Often times, the process takes years of research. Sometimes it opens up more questions than answers, pushing the investigation back to the drawing board, where a new approach has to be synthesized.

Many mathematicians consider their work an art form, and since it is undoubtedly a craft that requires great discipline and creativity, this is hard to argue. The British philosopher and mathematician Bertrand Russell expressed this idea through these words:

> "Mathematics, rightly viewed, possesses not only truth, but supreme beauty — a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as poetry."[2]

Russell highlights a very interesting component to mathematical beauty, its stern perfection. In painting and music, creativity and intuition are seen aesthetically in the finalized product, that which is presented to the audience. The creativity and intuition that results in mathematical beauty is relatively behind the scenes. It is within the pursuit of mathematics that the mathematician must rely on intuition and creativity. For the finalized product, it is reasonable to say that the results will be conclusive. The results will be certain, proven, perfect. The

[2] "Quote By Bertrand Russell (Earl Russell)." *Quotery*. n.d. Thu. 24 Mar. 2016. <http://www.quotery.com/quotes/mathematics-rightly-viewed-possesses-not-only-truth-but-supreme-beauty/>.

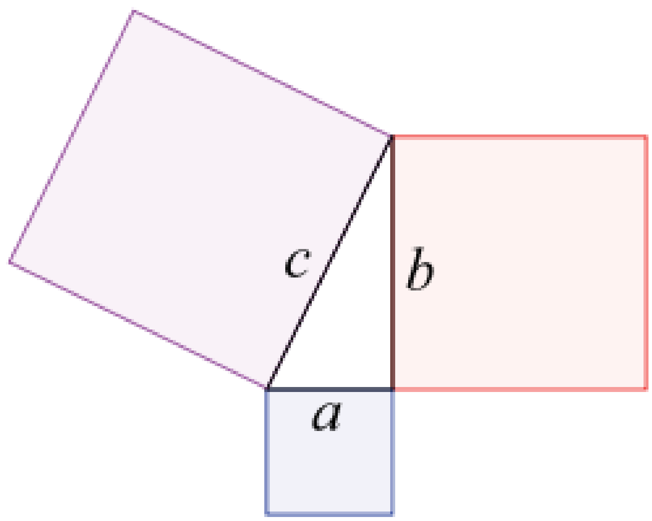
discrepancy between where and when creativity and intuition is employed should not play a part, however, in determining the beauty of the final product.

A prime example of this is with photography. Photography is identified as a fine art. A photographer is an artist. However, a photographer is merely uncovering something that already exists. Yes, many photographers heavily alter their images, and many stage their photographs, but not all. Those who do not are not seen as lesser artists. The images that exist in this world are not the photographer's images. The photographer did not create man nor matter nor cameras but through their intuition and creativity they are able to capture the beauty of their surroundings. The final product is a product of art yet often times is only a discovery of something in existence.

The aesthetic quality of a photograph can be assessed. The lighting, the composition, the emotion and material all play key roles in determining the "beauty" of a photograph. In mathematics, the beauty of results lies in the succinctness, unexpectedness and application to multiple problems across multiple fields of study. The original insight that developed the results, the number of assumptions that have been made, and the connection results have towards the real world all play a part in their elegancy and beauty.

Mathematical proofs can exhibit beauty through their form, their length and their organization. Often times there are different ways to express the same theorem or postulate, some methods being derived in surprising ways. For example, the Pythagorean theorem, which relates the three sides of a right triangle, has over three hundred different published proofs to its name.

It states that the square of the hypotenuse is equal to the sum of the squares of the other two sides. In relationship to the image below, the pythagorean theorem states that $a^2 + b^2 = c^2$. Below are two well known proofs of the Pythagorean theorem.



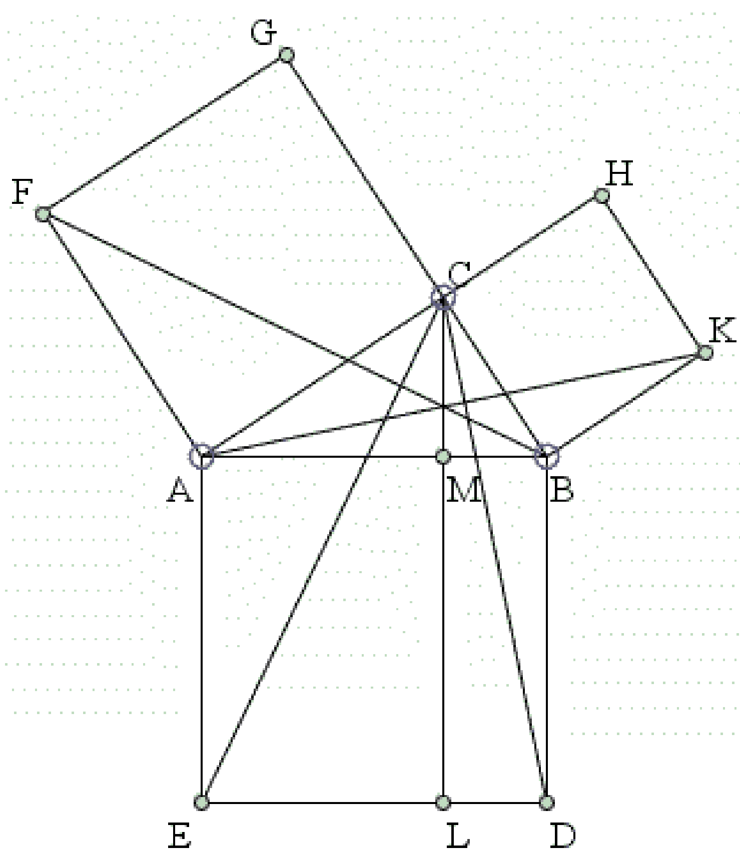

[Fig 35 a] *Bride's Chair*. Digital image. *Cut The Knot*. N.p., n.d. Web. <http://www.cut-the-knot.org/pythagoras/>.

**Proof by Euclid**

This is one of the Pythagorean theorems most famous proofs. It was discovered by Euclid (323–283 BCE), who is often referred to as the “father of geometry”. The configuration, used below to describe his logic, is known as the Bride’s Chair.

This proof implements the Side Angle Side postulate which states that if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then these two triangles are congruent.

The reader also needs to be familiar with the term “altitude” of a triangle. In geometry, an altitude of a triangle is a line segment through a vertex and perpendicular to (i.e. forming a right angle with) a line containing the base (the opposite side of the triangle). The reader also should know that the length of the base of a triangle multiplied by its altitude is equal to twice that of the area of the triangle.




[Fig 35 b] *Bride's Chair*. Digital image. *Cut The Knot*. N.p., n.d. Web. <http://www.cut-the-knot.org/pythagoras/>.

The proof goes as follows.

Notice that

AE=AB
AF=AC
$\angle BAF = \angle BAC + \angle CAF = \angle CAB + \angle BAE = \angle CAE$

By the Side Angle Side postulate, it follows that the triangle ABF is congruent to the triangle AEC: $\Delta ABF = \Delta AEC$.

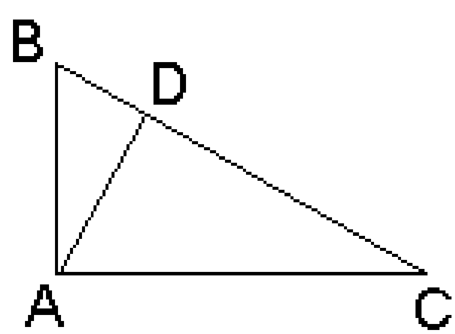
$\Delta ABF$ has base AF and the altitude from B is equal to AC. Its area therefore equals half that of the square on the side AC.

Also, $\Delta AEC$ has base AE and the altitude from C is equal to AM, where M is the point of intersection of AB with the line CL parallel to AE. Thus the area of $\Delta AEC$ equals half that of the rectangle AELM. So, the area $AC^2$ of the square on side AC equals the area of the rectangle AELM.

Similarly, the area $BC^2$ of the square on side BC equals that of rectangle BMLD. Finally, the two rectangles AELM and BMLD make up the square on the hypotenuse AB.

**Proof using altitude alone**

We start with the original right triangle, now denoted ABC, and need only one additional construct - the altitude AD.




The triangles ABC, DBA, and DAC are similar which leads to two ratios:
AB/BC = BD/AB and AC/BC = DC/AC.

Written another way these become
$AB \cdot AB = BD \cdot BC$ and $AC \cdot AC = DC \cdot BC$

Summing up we get

$$AB \cdot AB + AC \cdot AC = BD \cdot BC + DC \cdot BC$$
$$AB \cdot AB + AC \cdot AC = (BD + DC) \cdot BC$$
$$AB^2 + AC^2 = BC^2$$

Though both proofs lead to the same conclusion, they tell much different stories. In Euclid's proof, the beauty lies in the spatial association and extended meaning of the Pythagorean theorem. It illustrates a deep geometric relationship between the three sides of a triangle. The proof is creative and literally reaches outside of the box. The second proof which relies on the altitude of a right triangle is also beautiful. It is highly algebraic compared to the former. The second proof is more digestible but it is less graphic. The beauty here lies in concision.

Many theorems have been proved repeatedly. The existence of infinitely many prime numbers, the evaluation of zeta(2), Euler's polyhedra formula, the fundamental theorem of algebra, and quadratic reciprocity, to name a few. All the different approaches to these proofs illustrate the vast creativity that exists in mathematics, as well as the pursuit to better understand an idea. Within each proof, one can find a moment of beauty, clarity, concision, and often times surprise. For the reader's enjoyment, below is a very small handful of beautiful proofs. The first being a well known proof by contradiction, proving the irrationality of the square root of two.

**Irrationality of $\sqrt{2}$**

Before beginning the proof, the reader should be aware of a preliminary statement that will be used. Coming directly from the theorem which states that the parity of an integer equals the parity of its square, it is known that if an integer( $x$ ) is even, its square root( $\sqrt{x}$ ), is also even, so long as $\sqrt{x}$ is an integer.

The proof goes as follows:

Assume that $\sqrt{2}$ is, to the contrary, rational.

Then, $\sqrt{2} = \frac{p}{q}$ **,**where $p$ and $q$ are integers whose greatest common denominator is 1.

Squaring both sides of the equation $\sqrt{2} = \frac{p}{q}$ yields $2 = \frac{p^2}{q^2}$

Which means that $2$ divides $p^2$ and from the preliminary statement, we know that $p$ is an even integer. So, by definition of even integer, we can write $p = 2k$ for some integer $k$.

Notice also that $2q^2 = p^2 = (2k)^2 = 4k^2$. So, $q^2 = 2k^2$.

Which means that, similarly to $p$, $2$ divides $q^2$ and from the preliminary statement, we know that $q$ is an even integer. So, by definition of even integer, we can write: $q = 2t$ for some integer $t$.

To restate, if $\sqrt{2}$ is rational and equal to $\frac{p}{q}$, both $q = 2t$, and $p = 2k$ for some integers $t, k$.

Notice that this is a contradiction to the original statement $\sqrt{2}$ is rational and equal to $\frac{p}{q}$. If both $q = 2t$, and $p = 2k$ than the greatest common denominator of $p$ and $q$ is $2$, not 1.

Therefor, $\sqrt{2}$ must be irrational.

Proofs by contradiction are often times fun and tricky. They wrap around a false idea, under the logical pretense that the idea is true. Then, once the path takes the reader to a falsehood, the foundation of that false idea crumbles and the reader discovers that it was never true to begin with. Proofs by contradiction are quite dramatic in this regard.

The above proof has a very beautiful structure to it. The first time one reads through the statements, it is often tricky to follow. The proof seems circular in logic. In a way, it is.

However, the circle that is being drawn will never be complete. You can never get back to the original statement, so long as you follow that specific logical path. In other words, the imaginary circle is broken by reality, which is quite a beautiful idea in and of itself.

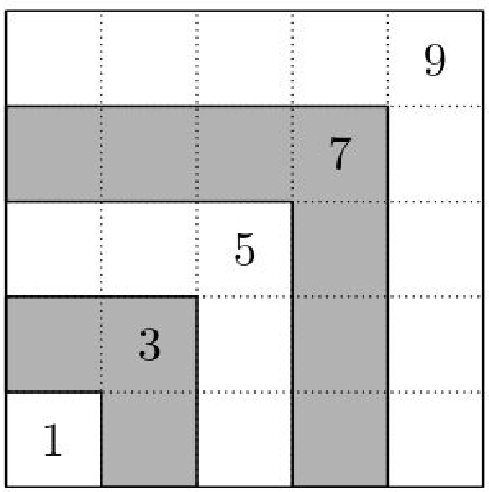
**Adding odd numbers, the proof that**

$$\sum_{k=1}^{n}(2k-1)=n^2$$

The above equation is equivalent to the summation of $n$ odd terms starting at 1 and increasing by 2 with each term.

$$1+3+5+7+9+11+13+.....+(2n-1) \;=\; n^2$$

The summation can be proved easily through induction, but the picture proof uncovers a completely new way of looking at the equation and unveils why the summation is true in a very simple and beautiful way.




[Fig 36 ] Picture Proof. Digital image. N.p., n.d. Web. <http://web.mit.edu/18.098/book/extract2009-01-21.pdf>.

Each term in the sum $\sum_{k=1}^{n}(2k-1)$ adds one odd number to the existing summation. If we represent theses odd numbers as the area of an L shaped piece, each piece extends the square above by one unit on each side. Adding an $n$ -number of terms, means adding an $n$-number of L shaped pieces, making an $n$ by $n$ square whose area is $n^2$.

**The Binomial Theorem and Pascal's Triangle**

The binomial theorem describes the expansion of powers of any binomial.

Below are a few examples.

$(a+b)^2 = a^2 + 2ab + b^2$
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

The formulaic expression that dictates the binomial theorem is as follows:

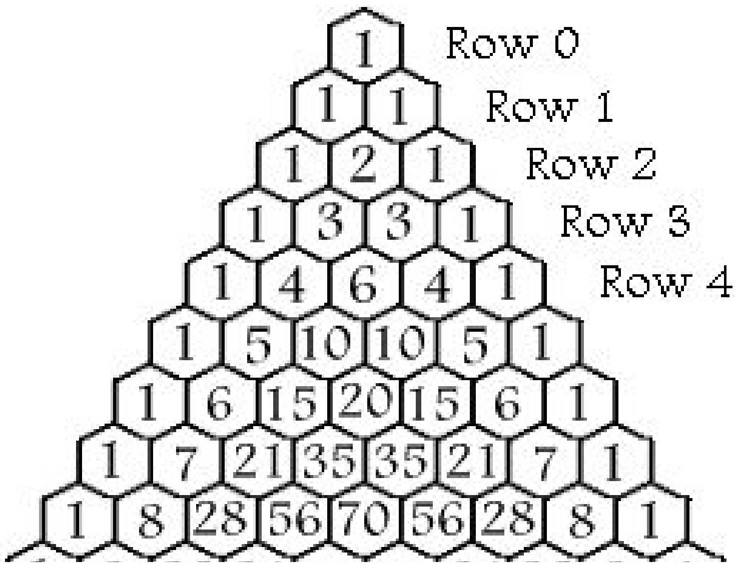
$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Note that the binomial coefficient, often referred to as "n choose k", is:

$$\binom{n}{k} = {}_nC_k = \frac{n!}{(n-k)!\,k!}$$

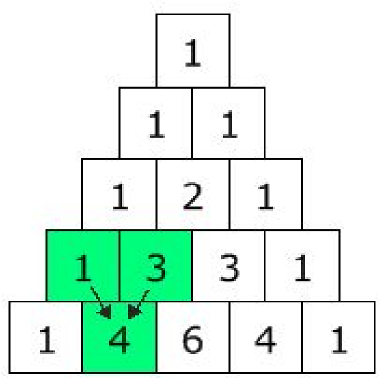
These coefficients for varying n and k can be arranged quite nicely to form Pascal's Triangle, as seen below. The row number represents the n value of the polynomial (whatever power the binomial is being raised to) and the elements within the row correspond directly to the coefficients of the polynomial.

In relationship to the formulaic expression above, from left to right, the elements correspond to increasing k values. The first element being $k = 0$, where k increments by one, each element.

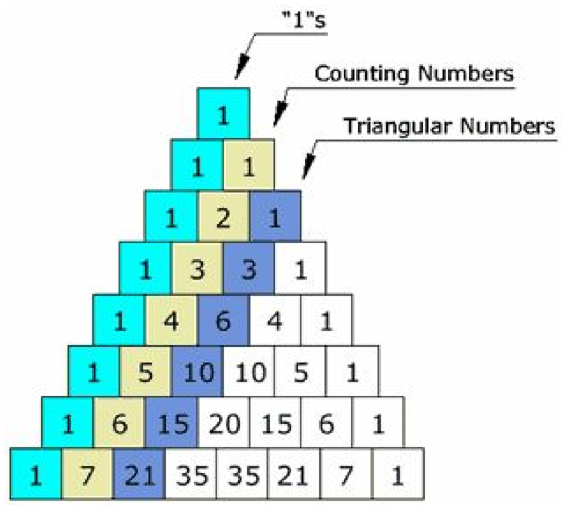



[Fig 37 ] Pascal's Triangle. Digital image. *All You Ever Wanted To Know About Pascal's Triangle*. N.p., n.d. Web. <http://ptri1.tripod.com/>

Pascal's triangle is quite beautiful as a mathematical construct not merely due to its application to polynomials. Within Pascal's triangle there are interesting patterns and sequences. Notice that each element, aside from the top tier element (1), can be synthesized by summing the two elements uppermost left and right of it, such as 1+3 = 4, highlighted in green below.



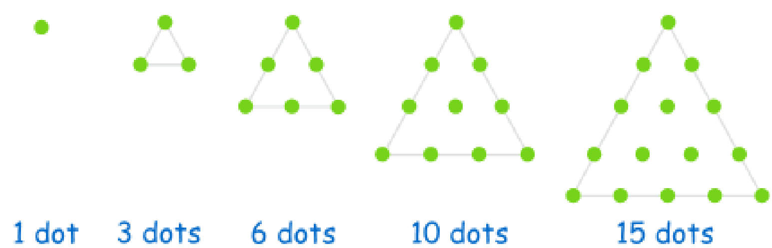

[Fig 38 a] Pascal's Triangle Info. Digital image. *Math Is Fun*. N.p., n.d. Web. <http://www.mathsisfun.com/pascals-triangle.html>.




[Fig 38b] Pascal's Triangle Info. Digital image. *Math Is Fun*. N.p., n.d. Web. <http://www.mathsisfun.com/pascals-triangle.html>.

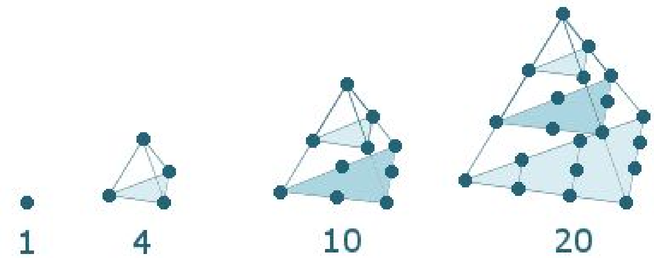
Notice also the diagonal sequences within the triangle. The first diagonal is obviously composed of all 1's. The second is composed of the counting numbers and the third the triangular numbers.

Triangular numbers are a sequence of numbers relating to the units used to create two dimensional pyramids. They can be depicted easily as dots.



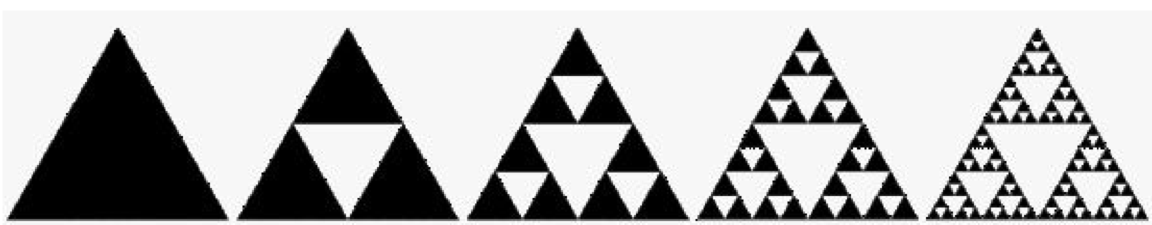

[Fig 38 c] Pascal's Triangle Info. Digital image. *Math Is Fun*. N.p., n.d. Web. <http://www.mathsisfun.com/pascals-triangle.html>.

The fourth diagonal, similarly can be depicted as a sequence of tetrahedral numbers. Tetrahedral numbers are a bit more difficult to understand. If you were to sum the triangle numbers together as you increased their size you would generate the tetrahedral numbers. Imagine stacking the dot triangle images on top of each other, creating three dimensional pyramids. The number of dots needed to create that configuration as it increases in size generates the tetrahedral number sequence. Below is a visual aid.

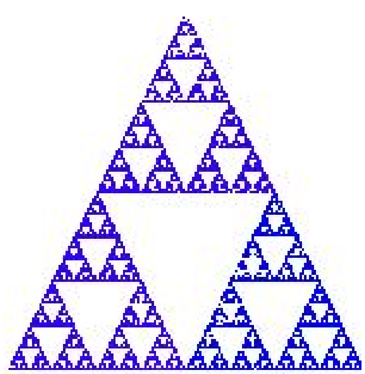



[Fig 38 d] Pascal's Triangle Info. Digital image. *Math Is Fun*. N.p., n.d . Web. <http://www.mathsisfun.com/pascals-triangle.html>.

If you were to color the odd units on Pascal's triangle black and the even white, you would end up with the Sierpinski Triangle, created using an ever repeating pattern of triangles. Refer to the images below, the pattern continues forever to generate the Sierpinski Triangle, in blue.



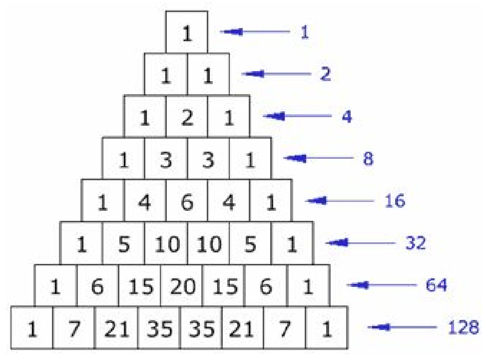
[Fig 39 a] Sierpinksi Triangle. Digital image. *Math Is Fun*. N.p., n. d. Web. <http://www.mathsisfun.com/pascals-triangle.html>.



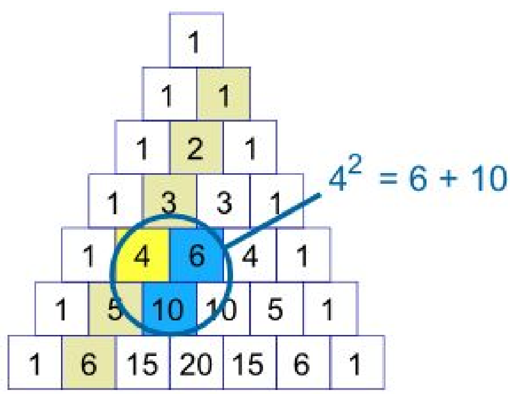
[Fig 39 b] Sierpinksi Triangle. Digital image. *Math Is Fun*. N.p., n.d. Web. <http://www.mathsisfun.com/pascals-triangle.html>.

Notice also, if you were to sum each row of Pascal's Triangle, the resulting set is composed of all powers of 2:

$$\{2^0, 2^1, 2^2, 2^3, 2^4, ....\} = \{1, 2, 4, 8, 16, ...\}$$




[Fig 38 e] Pascal's Triangle Info. Digital image. *Math Is Fun*. N.p., n.d . Web. <http://www.mathsisfun.com/pascals-triangle.html>.




[Fig 38 f] Pascal's Triangle Info. Digital image. *Math Is Fun*. N.p., n.d. Web. <http://www.mathsisfun.com/pascals-triangle.html>.

And the square of each number in the second diagonal is equal to the sum of that to the right and that to the bottom-right.

This is only the beginning. Pascal's triangle has a clear line of symmetry, can generate the Fibonacci Sequence, can model the probability of flipping a coin, and much more. For being such a simply constructed device, the complexity and interesting structure hidden within are phenomenally beautiful. Again, there is a delicate balance of creativity and complexity paired with simplicity and concision creating a wonderfully entertaining and artistic product.

**Geometric series**

A geometric series is a series with a constant ratio between successive terms. Put in layman's terms, an added list of numbers, continually multiplied by the same number. An example would be the following.

$$\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{8}+\tfrac{1}{16}+\ldots\ldots=\sum_{n=1}^{\infty}(\tfrac{1}{2})^n$$

You can think of this sum as the halving of a carrot. You begin with one carrot and cut it in half. You push one half off to your left. You take the other half of the carrot and repeat the exact same process. As you continue to cut, the pile to your left will grow but will never be greater than one carrot. And with imaginatively precise cutlery, you will be able to continue to cut the carrot forever, always adding smaller and smaller halves of the carrot pieces to the pile at your left.

Through this analogy it is easy to conceptualize that the geometric series shown is equivalent to one unit. Notated mathematically:

$$\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{8}+\tfrac{1}{16}+\ldots\ldots=\sum_{n=1}^{\infty}(\tfrac{1}{2})^n=1$$

Notice, all geometric series can be written in a summation notation as follows:

$$\sum_{n=0}^{\infty} ar^n$$ where $a$ and $r$ are simply constants.

As it turns out, any geometric series, whose ratio between successive terms $r$ is somewhere between -1 and 1, has a convergent sum equivalent to:

$$\frac{a}{1-r}$$

For further investigation, let us take a look at the expansion of the finite geometric series, $S$.

$$S = \sum_{n=0}^{k} ar^n = a + ar + ar^2 + ar^3 + .....ar^k$$

Notice what happens to the finite geometric series $S$ when it is multiplied by the constant $r$.

$$rS = ar + ar^2 + ar^3 + ar^4 + .... + ar^{(k+1)}$$

If $rS$ is then subtracted from $S$, an exact value for $S$ is deduced.

$$S - rS = a - ar^{(k+1)}$$

$$S(1-r) = a(1-r^{(k+1)})$$

$$S = \frac{a(1-r^{(k+1)})}{(1-r)}$$

However, this only provides an exact value for *finite* geometric series. For infinite geometric series, the $k$ value is approaching infinity. Therefore, we take the limit of $S$ as $k$ approaches infinity.

$$\sum_{n=0}^{k} ar^n = \frac{a(1-r^{(k+1)})}{(1-r)}$$

$$\lim_{k\to\infty} \sum_{n=0}^{k} ar^n = \lim_{k\to\infty} \frac{a(1-r^{(k+1)})}{(1-r)}$$

$$\sum_{n=0}^{\infty} ar^n = \lim_{k\to\infty} \frac{a(1-r^{(k+1)})}{(1-r)}$$

Notice that if $r$ was equal to or greater than 1 then this series would continue to grow. However, for the range of converging $r$ values, this is not the case.

$$\text{for } -1 < r < 1$$

$$\lim_{k\to\infty} r^{(k+1)} = 0$$

So,

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

The explanation above hinges on the fact that we know the geometric series converges and S can be thought of as a constant value. The process used here is not necessarily typical. There are many different convergent series whose values require significantly more work to find. The explanation is concise and very direct. Something incredibly abstract as an infinite sum is unearthed in an organized, easy to follow way. For that reason, this explanation is very aesthetically pleasing.

**Euler's identity**

Arguably the most beautiful equation in existence, Euler's identity combines five of the most interesting constants into a single statement.

$$e^{i\pi} + 1 = 0$$

What makes Euler's identity so phenomenal is that it's components are mathematically dense and somewhat abstract numbers that, when arranged in the specific combination, create an equivalence statement. Based only on the constants themselves, this relationship is not at all intuitive.

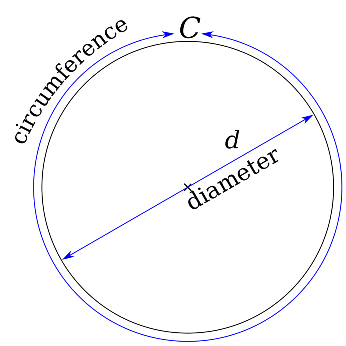
$e$ can be explained as the base rate of growth shared by all continually growing processes. It shows up whenever systems grow exponentially and continuously, such as population, radioactive decay, and interest calculations. Mathematically speaking, $e$ is equivalent to the limit below:

$$e = \lim_{n\to\infty}(1 + \tfrac{1}{n})^n$$

$i$ is equivalent to the square root of negative one. Put into symbols,

$$i = \sqrt{-1}\,.$$

$\pi$ can be explained as the number equal to the circumference of a circle divided by it's diameter. $\pi = \frac{C}{d}$




[Fig 40 ] *Pi*. Digital image. *Wikipedia*. N.p., n.d. Web. <https://en.wikipedia.org/wiki/Pi>.

1 is the multiplicative identity within the real and complex numbers, meaning any number multiplied by 1 is itself. And, 0 is the additive identity in the real and complex numbers, meaning any number plus 0 is itself.

$1, 0, i, e, \pi$ are some of the most important mathematical numbers in existence. They are seemingly unrelated, and yet, when you combine the five constants, $e^{i\pi} + 1 = 0$. You acquire a statement in perfect harmony.

Within the mathematical world, there exists stern perfection, concision, deep networks of interrelations and profound structure. Mathematics not only prescribes to our aesthetic senses by engaging our eyes and ears in interesting analysis. It also reaches deep into our thought processes and synthesizes beautiful structures of understanding. Due to the nature of mathematical investigation and what it produces, it is evident that mathematical results can, themselves, be beautiful.

**Conclusion**

Mathematics and art have multiple overlapping qualities. Those of which include creation, intuition, discipline, interest, innovation and beauty. Yet most see the two as diametrically opposed. From the surface, this is easy to understand. In grade school students' math classes contain set rules with difficult concepts. As they age, the information compounds and they are expected to recall previous material. There are homeworks and exams and if a student fails, they are considered unintelligent. Art classes, on the other hand, have fewer rules and less boundaries with simpler concepts. The student does not have to memorize past material. Instead, he or she creates art using their intuition. As long as the student participates, he/she is rewarded with good grades and even if their work is subpar, the student is not labelled incompetent.

In the adult world, artists are viewed romantically, often times as enigmatic, with a greater understanding of human nature and emotion. By contrast, mathematicians are depicted as brainiacs, incredibly intelligent and logical. Mathematics is often paired with unemotional logic while art is paired with emotions and romanticism. Mathematics is paired with rules and set answers whereas art is paired with boundless creativity. Given that these quick overview terms are divergent, it seems appropriate to label art and mathematics as opposites.

However, it is not as simple as one might think. The subjects have a relationship, in fact they have many. As shown earlier in the text, mathematics generates art through patterns, forms, and concepts and can produce art through algorithmic computer processes.

Likewise, art can illuminate mathematics, such as the images of the Mandelbulb space(pg 32), the Hilbert space filling curve(pg 30) and the Sierpinski triangle(pg 46). When either an artist or a mathematician pushes the limits of a mere diagram, deeper understanding and or new frontiers for investigation may be uncovered. An artistic representation of some mathematical process, be it sculptural or two dimensional, can provide a visual that brings to life complicated ideas that feel weighed down by lengthy proofs and complex notation. The works of Crockett Johnson (pg.22 ) and Dr. Diana Tamina (pg. 27) are prime examples.

In both the construction and finalized product of an artwork, mathematical questions may arise. In many instances the artist may not be able to answer these questions, nor realize the significance of their work. A good example of this would be the quasicrystal structures found in Islamic tiling (pg12). In such instances, an interdisciplinary relationship is highly beneficial to the progress of the piece and scientific investigation. In some cases, the work may need the attention of trained mathematicians, engineers, or software designers to provide interesting practical problems to solve.

Another similarity between the two disciplines, art and math, is their omnipresence. Both the common phrase ,"art is everywhere" and the common phrase, "math is everything" hold a powerful truth about the world around us. It is true that both art and beauty exist in the natural world. The human mind translates much of what our aesthetic senses pick up as "art"; early mornings in August, the way water beads on the roof of your car, birds aligning on telephone poles. Art is seen from the natural world around us and drawn from the natural world.

Likewise, mathematics is seen in the real world through symmetries, fractals, spacial relations and such. Mathematics is drawn from the real world and mathematics is applied to the real world. Notice that this is slightly different from other fields of study such as chemistry, biology, physics and history. Those of which study the world around us. Biology does not create new biology, nor does physics create new physics. In these fields, scientists analyze their surroundings and attempt to make sense of it. In mathematics and art, the world is a medium for play. New mathematical concepts are created and works of art are created for reasons that extend beyond analysis and explanation. Mathematics and art can both be used to reflect on and describe the world around us, but they also posses a life of their own. They may create worlds unfamiliar to us.

For that reason, mathematics and art are both highly explorative. The mathematician and the artist both create and the true difference between art and mathematics is not that one is logical while the other beautiful. In fact a recent study (2014) from University College London[3] showed that the same areas in our brain that recognizes beautiful art, lights up with neural activity when an individual is shown equations like Euler's Formula and the Pythagorean theorem.

Mathematics is equally as creative and beautiful as art, and many pieces of art are as complicated, logical and as developed as a complex mathematical proof. The real difference between art and math lies subtly in the foundation of both disciplines. Art is based more strongly on intuition. Although an artist may heavily base his or her work on some specific topic, the resulting product travels through the artist's hands and the editing process is driven by intuition. The artist inputs his or her style into the work. They may deviate far from a specific path and his or her pieces may develop by branching along the individual's interests and intrinsic drive.

Mathematics, does often require intuition and stylization in notative capacities. However, the intuitive component is eclipsed by the mathematical investigation. Here, the process of creation relies heavily on logical construction and the mathematician does not branch off based

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[3] Zeki S, Romaya JP, Benincasa DMT and Atiyah MF (2014) The experience of mathematical beauty and its neural correlates. *Front. Hum. Neurosci*. **8**:68. doi: 10.3389/fnhum.2014.00068

on his or her own intuition. Rather, they are then led by the math down an unknown road. Each step requires questioning and creative insight.

Despite this singular fundamental difference, the two fields are still not opposites. More importantly, this singular opposite helps create a relationship that is capable of fostering academic progress. The links that exist between art and math accompanied with the difference in production yields an intense potential to expose findings to all fields of the academic world.

Through art, intuition can lead to new creation. Through math, analysis can lead through explanation, new creation and understanding. Together, the intuition of the artist and the analysis of the mathematician can be combined. Topics of interest such as spacial relations, symmetry, patterns, measurements, geometry, topology, combinatorics, converging and complex series all have very visual counterparts. Thus, given the visual aspect of art, an artistic investigation is capable of intuitively delving into these subjects to help explain, create and expand these mathematical concepts, pulling from all facets of some synthesized world to create a well-rounded understanding of the subspace, opening doors to the new and unknown.

Only a very slim subset of the population is capable of reading through complex mathematics. For that reason, much of pure math has very little exposure to the academic community outside of the mathematicians therein. Likewise, art is a bit of a black sheep. It is far separated from the hard sciences of the academic world despite being present as a field of study at most public universities. All-in-all, there are walls put up around both disciplines that shield other fields from easily accessing the pivotal research, techniques and findings within the two highly creative topics. But as explained above, if the two were to combine, these walls have the potential to really break down.

Imagine a university that allows collaborative research between artists and mathematicians. Imagine math graduate students working in collaboration with graduate students in studio art to present their thesis work in a visually stimulating way, possibly an entire gallery of scientific and mathematical findings put to sculpture, video, drawing, prints or painting. The painstaking and tedious role of reading through extensive mathematical findings dissolves into an entertaining trip through a world that so few people truly understand. Such a scenario would allow stepping stones between the world of mathematics and those analyzing the human world. Researchers from all fields could find relations with the artworks presented.

The study of art and the phenomenal skill of translation and visual communication that educated artists possess is so fundamental to spreading knowledge. The fact that this process and artists across the world are not being heavily utilised to convey information in an academic setting is baffling.

It is critical that academics continue to integrate disciplines that are not diametrically opposed, specifically math and art. No matter how illusive the compatibility between these subjects may seem at first, the real relationship is binding and beneficial. Both mathematics and art are beautiful and both are excellent propellants in research and discovery.

All-in-all, as humankind develops, it is critical to begin looking at the world in an innovative fashion that includes not only pure science but human intuition and craft. As the child learns through multi-disciplinary investigation, so should the scientist, the engineer, the artist and the mathematician. Though specialization within all fields is critically important to discovery and thought provoking analysis, a holistic approach to learning and investigation will inevitably unveil new frontiers. Humankind has so much still to learn and so much yet to discover that it would be foolish to begin putting boxes around disciplines. Particularly, art and mathematics need no line dividing the two.

To sever the two disciplines from one another would be limiting the possibilities of discovery. To  support and emphasize this investigation would mean potentially unearthing a new basin of knowledge. Few would disagree that mathematics holds the answers to the world around us. But even fewer realize that it takes art and human intuition to let mathematics relinquish that grasp.

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