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Price Signaling in a Two-Market Duopoly

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PRICE SIGNALING IN A TWO-MARKET DUOPOLY

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PRICE SIGNALING IN A TWO-MARKET DUOPOLY

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Thesis

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ABSTRACT

Within any industry, firms typically produce related products over multiple subsequent periods in an attempt to build consumer loyalty and achieve continued sales. Apple releases new iPhones and car companies produce new models every year, relying on consumers believing each new product is of high quality. Firms rely on the spillover effects from previous markets, where firms are able to more easily demonstrate their product's quality to the consumers before purchase. The goal is to find a range of prices which allows the high quality firm to distinguish its type to consumers via the price p^H and if spillover effects in subsequent markets can occur. We look at a duopoly of two firms, of high and low qualities, where each firm produces a product in an initial market and a second, related product in a subsequent market. Using each firm's expected profits, based on Bayesian probabilities, we analyze a firm's mimicking strategy to find the range of p^H that allows for a separating equilibrium and spillover effects. In a second market where firms are the same qualities as in the first market, the high quality firm experiences spillover effects and can signal its quality with a lower price than in the first market. When firms change qualities in the second market, no spillover effect occurs and the newly high quality firm must increase p^H from the previous market in order to separate.

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CHAPTER I

INTRODUCTION

Within any industry, it is very rare for a firm to produce a single product in a single period (also called the market) and then leave the industry entirely. Most firms will produce related products over multiple subsequent markets in an attempt to build consumer loyalty and achieve continued sales. Apple has shown this with the release of yearly iPhone models. Car companies like Ford and Toyota release new models each year to drive sales. In most markets, consumers are initially unaware of the product's quality until after they have purchased and used the product. Because of this, firms must find a way to signal their product's quality to consumers and ensure that they are able to sell higher quality goods at higher prices. If a firm, such as Apple, wishes to get some sort of control within an industry over many periods/markets they will want to convince consumers they produce strictly high quality products.

The high quality firms will need to separate themselves from the lower quality firms and allow consumers to identify their products as high quality before purchasing them. The prices charged by each type of firm will constitute a revealing, or *separating*, equilibrium where the high quality firm is able to reveal its true quality to consumers. By having consumers perceive their products as high quality, firms should be able to take advantage of their reputation and achieve separation in subsequent markets. If the high quality firms are unable to reveal their quality, the low quality firms can take advantage of the product quality ambiguity and their lower price to sell to more consumers.

When product quality is unobservable before purchase a firm will necessarily charge a price above the marginal cost, the cost of producing an additional unit [1, 2, 3]. For Bertrand competition where firms select prices and consumers determine the quantity to purchase, firms have positive expected profits when either the rivals' marginal cost is unknown [2], or the number of firms is unknown [3].

If a high quality firm can reveal its type across multiple stages and multiple products, the firm can take advantage of spillover effects (see *Appendix B*). Think of this as being well liked by a teacher because they enjoyed teaching your older sibling. If spillover effects are present, the firm can take advantage of any previous market results and adjust its pricing strategy accordingly. Its the idea of a firm using its reputation because of previous results that motivates why this study is important.

Bagwell and Riordan [4] looked at how charging an unnecessarily high initial price affects a high quality firm's ability to reveal its true quality to consumers across subsequent markets. A demand function was determined, based on a firm's type (quality), price chosen, and the beliefs of consumers. The demand function created a horizontal parabola as a function of the ratio of informed to uninformed consumers. Any price inside of the parabola would lead to the low quality firm attempting to mimic high quality - charging a price expected to be from a high quality firm. By charging a price which is too high to be profitable, the high quality firm is able to reveal its type in an initial market and take advantage of the spillover effects - they can charge lower prices in subsequent markets and still be able to separate from any low quality firms [4, 5].

If a high quality firm can still signal their quality while being able to use a lower price than before, they have an incentive to charge a higher separating price in the initial market to allow the signaling price to fall over subsequent markets. Therefore looking at using price as a signal over multiple markets has real applications to many industries, especially in technology and automotive markets.

Another paper written by Bester and Demuth [6] looked at price competition involving a duopoly between an incumbent and entrant firm are the only two firms. They assume the incumbent is fully informed, while consumers are uninformed, of the entrant firm's quality. Their model utilized reaction functions - functions that determine the best reaction price to the prices chosen by rivals - and found that the entrant firm may not have to distort its price to signal its true quality.

Baye and Harbaugh [7] discussed a monopolistic firm which produces two products of differing qualities. Instead of looking at a market with multiple firms they focused on a scenario involving one firm that produces two products, one of "good" and one of "best" quality, in a single market. The interest of the paper lies in whether a single firm selling products of different qualities gives the same profits as it would if each product was sold by a different firm. When product types cannot be observed, Baye and Harbaugh found that there is no qualitative difference between a single producer with multiple products and a multi-firm market where each produces one product. Therefore, their findings ideas can influence the creation and evaluation of our model.

A comparative price signaling equilibrium was also considered, which was defined as an equilibrium where consumers infer a product's quality through the observation of both prices. This allows the consumers to base their decisions on the pair of prices, rather than just the price of the single good. Baye and Harbaugh found that if consumers wish to purchase less of the higher priced product, there is no need to raise the price of the high quality product above the full-information price. The price for the high quality product must be set to guarantee equilibrium demand for the high quality good does not exceed that for the low quality good. The same condition is true for a firm that faces interdependent demands.

Janssen and Dubovik [8] studied an oligopolistic market (only a small number of firms) in which heterogeneous information about prices and product quality exists. They assumed three groups on consumers exist where they consumers know both the product prices and quality, know only prices, or know neither price nor quality. Firms play a mixed strategy, meaning they randomize their price selections, over a curve of price-quality combinations. The equilibrium is characterized by better price and quality combinations being associated with lower prices.

Daughety and Reinganum [9] looked at price competition among firms producing products which differ in costs and consumer satisfaction (valuation). By using profit functions which chooses the best response to its rivals' strategies, as well as providing a price which makes mimicking less profitable. They found that the incomplete information (the quality of a firm's product is private information known only by the firm itself) causes both qualities of firms to charge higher prices than in the full information game.

In an oligopoly where firms are either of high or low quality, Janssen and Roy [10] show that a certain price charged by a high quality firm will allow them to signal their quality and provide a symmetric fully-revealing (separating) equilibrium. The range of reasonable prices for each type of firm is known as its *price support*, which is usually determined by factors of production. For example, the price support for the high quality firm is bounded by consumer valuations and the firms' production costs. The research found that because of out-of-equilibrium beliefs, neither type of firm could gain through charging prices outside of the price supports. Therefore, mimicking was not a profitable option for either type of firm. In this case, it was noted that the price support changed based on the percentage of consumers who choose to buy a good. The magnitude of this change is determined by how different the price is from the consumer valuation of a high quality product. They were able to find that, under certain conditions, a symmetric fully-revealing equilibrium exists where low quality firms play a mixed strategy of prices within the low quality price support while high quality firms all charge a common deterministic price.

Janssen and Roy also briefly discussed pooling equilibria, which would occur when the price supports for the low and high quality firms are not disjoint.[1] This

[1] After refining these equilibria using the D1 Criterion, Janssen and Roy found that none of these equilibria survive.

means that there is at least one price, or a range of prices, which lies within the price support for both a low and high quality firm. A price in this interval could feasibly be chosen by both quality of firms without being part of a mimicking strategy. We necessarily need the price supports to be disjoint in a separating equilibrium, where firms are prevented from fooling consumers by mimicking another firm type's pricing strategy. This would not provide any type of signal to the consumer, which gives a pooling equilibria where neither type is revealed via prices to the consumer before purchase. Therefore if the price supports shared an interval that were viable for both types of firms to charge, a pooling equilibrium could occur.

In addition to using price as a signal, many prior studies have chosen to include advertising levels as a signal as well. Advertising allows firms to introduce their products to the consumers before they see a price tag in the store. Any level of advertising leads to additional direct costs to the firms, so its use as a signal is certainly plausible. By spending money on advertising in hopes that it will draw a larger consumer base and increase sales, firms can potentially use their advertisement expenditure to signal that their product is high quality.

A study done by Paul Milgrom and John Roberts [11] was based on repeat sales where both price and advertising are variable and may potentially be used as signals. Milgrom and Roberts used sequential equilibria, where a sequence of signals is being sent to slowly allow for the type to be observed by the consumer. The analysis used profit as a function of price, beliefs, and advertisement. It was found that a non-zero level of advertising was not viable for the low quality firm, and that a positive high quality advertising level could allow for a separating equilibrium. Some of the separating equilibria will be refined using the Intuitive Criterion as defined by Cho and Kreps [12]. In addition to separating equilibria, pooling equilibria can exist but will not survive the Intuitive Criterion.

As an addition to the previous analysis done in [11], Linnemer [13] extended the research to durable goods which do not generate immediate repeat purchases. Some examples of durable goods include houses and cars. Instead of looking at multiple firms, Linnemer focused on a monopoly which produces either a high quality or low quality good. It was shown that a high price paired with dissipative advertising (advertising that is dispersed slowly after initial release) will provide an efficient signal. However, when the proportion of informed consumers is low, quality can be signaled only through setting a high price.

Also following the work done in [11], Fluet and Garella [14] looked at both monopoly and duopoly markets where advertising and prices were used as signals. They wanted to determine the issue, if any, of using price or advertising to signal quality. Additionally, Fluet and Garella wanted to find whether advertising has a positive or negative effect on a firm's profit and signaling ability. After constructing expected profit functions following ideas used in [11], they found that the level of advertising depends on the difference in magnitude of the firm qualities (i.e. how much better is the high quality product than the low quality one?). When there is little difference between the two qualities advertising is necessary to invoke a separating equilibrium, while a large difference leads to smaller advertising levels. Signaling through high prices is costly when rivals are able to take away the market share (number of consumers who purchase the high quality good) of the high priced firm.

Hao Zhao [15] wanted to find the optimal pricing and advertising strategies of a firm when introducing a new product to uninformed consumers. After determining a demand function with inputs of advertising and price, Zhao considered two cases: when consumers know the product's quality before buying, and where consumers are uncertain. When the quality is known by consumers, the high quality firm will advertise more than the low quality firm if it is more efficient in producing high quality goods. In the case where consumers are uncertain, the high quality firm will is willing to advertise instead of being perceived as low quality. Therefore, in a separating equilibrium the high quality firm will spend less on advertising and charge a higher price. If the firm has a truly high quality product, it should allow price to be the signal and only conduct modest advertising.

In a paper written by Makoto Abe [16], price and advertising levels were used as signals in a single market study between a national brand and a private-label clone.[2] Abe studied both separating and pooling equilibria where national brands were able to set themselves apart from the clone company through setting unique levels of both price and advertising. Abe used the firms' utility functions to describe the utility (profit) a firm receives based on the sales it has for a given price and advertisement level.

[2]A national brand can be thought of as the "name brand" product, where the private-label clone is similar to a "knock-off".

In discussing the conditions for separating equilibria, both envy and non-envy situations were considered where the low quality firm wishes to deviate and pretend to be high quality or not deviate respectively. In the envy case, where the low quality firm would be willing to deviate, the high quality firm can separate with any positive level of advertising a_0 that would reduce the low quality firm's profits. It was found that unique levels of price and advertising by both low and high quality firms can provide a unique separating equilibrium in both the envy and non-envy situations.

Additionally, pooling and hybrid equilibria (see *Appendix B*) were discussed. In an approach similar to the separating case, these equilibria were found to exist. Upon further refinement by the Intuitive Criterion (see *Appendix B*) as defined by Cho and Kreps, none of these equilibria survive and therefore are not discussed at length. In conclusion, Abe found that only a single unique separating equilibrium of advertising levels and prices survived the intuitive Criterion.

Finally, Pires and Catalão-Lopes [17] focused on advertising expenditure as the signaling device, in a multi-product economy in which products are sequentially released in the market. Using gross profit functions (expected profits minus the level of advertising expenditure) as the payoffs of the firms, Pires and Catalão-Lopes compared the maximum difference in profits from mimicking and discussed the equilibrium possibilities for the first market. In the first market it was found that the type of equilibrium, either pooling or separating, was dependent on the distribution of types within the market (i.e. the probability that the firm was of high or low quality).

In the second stage, Pires and Catalão-Lopes used a correlation coefficient, a number between zero and one, to relate the qualities of the goods between the first and second markets. The closer the coefficient was to one, the more likely the second product was to have the same quality as the first. The addition of the correlation was crucial to creating a realistic setup to study any information spillovers for any subsequent markets.

The second market contained two possible scenarios: where the quality of the first product was observed before purchasing the second product, and where the quality of the first product was not observed in time for the second product to be released. Both of these cases can occur in a real market place, especially with durable goods (houses, cars, etc.) where the quality may not become apparent before the next model is released. In the situation where quality was observed, the second market acted just as the first market - there was no consequence, or spillover effect, of advertising expenditure used in the first market. However when the quality of the first good is not observed, the equilibria in the second market varied depending on the level of the correlation coefficient and initial distribution of firm types.

The following research will be a mixture of the papers discussed above. A duopoly (where two firms sell to the entire market) will be used, where each firm is either of high or low quality. We assume that each firm produces a product of the same quality as the firm itself - a high quality firm produces a high quality product, and vice versa. Therefore, a firm is only able to produce a product which matches its quality - a low quality firm cannot produce a high quality good, for example.

Because consumers are typically unable to observe the total amount of advertising or observe different levels of advertising, we choose to only use price as a signal and neglect the effects of advertising. The price of a product is always observed and is ideally the same across consumers, making price a more accurate and controllable signal for the firms to use. The previous literature mentioned above has dealt with either price or a mixture of advertisement and price as a signal in only one market. Only one paper [17] discussed signaling via advertising and prices in two related markets.

The goal of our research is to find conditions based on firm costs and consumer valuations that will guarantee the existence of a separating equilibrium in two sequential duopolistic markets. We will use the same structure and assumptions as the single market model used in [10]. This model will then be adapted to a second market, utilizing the updated probabilities found via the correlations coefficient from [17]. This will be done for two related markets in which the firms are actively trying to sell a product.

The basic assumptions and single stage model creation (Chapter 3) come from the [10] and follow the same guidelines. As part of the equilibrium analysis, we must consider each quality of firm's incentives to mimic, and out-of-equilibrium beliefs. By discussing the out-of-equilibrium beliefs of the consumers, we are able to identify whether a firm is able to effectively fool consumers through mimicking a firm of the opposite quality. This culminates in the consumers identifying a mimicking firm, and adjusting their purchase strategy if possible.

The mimicry analysis be done using the same expected profit comparisons that Janssen and Roy used. The expected profits used will be based on those from [10], with slight adjustments, as discussed in Chapter 3. Using the probability that the other firms are of high quality, a firm can determine its expected profit at both a high and low quality price. A comparison of these two expected profits can then be used to determine whether mimicking (charging a price within the price support of the opposite quality firm) is beneficial. If deviating to such a price results in a higher expected profit, the firm will choose to mimic. If deviating results in a lower expected profit, the firm will use a price indicative of its true quality.

When looking at the second stage, we will extend the single stage model used in [10] to include a second market (period) via the correlation coefficient used in [17]. We will assume that the first market ends in a separating equilibrium, allowing consumers and firms to observe the quality of all products from the first market.

At the beginning of the second market, we derive the new expected profits of each quality of firm by adjusting the probability that a firm is high quality. This is done through the correlation coefficient. A firm is believed to be high quality in the second market with higher probability if they were revealed to be high quality in the previous market. Likewise, being revealed as low quality reduces consumer confidence that the firm is high quality in the second market - they are believed to be high quality with less probability than in the first market.

The rest of the paper will be structured as follows. The model and basic assumptions will be discussed in Chapter 3. Chapter 4 will examine the three possibilities of the single market case, where the separating equilibria will be presented. A model for the second market will be discussed in Chapter 5. Two cases will be considered, where the firms do not change qualities and the case where they both change types. These will be covered in Sections 5.1 and 5.2 respectively. Chapter 6 will provide numerical examples to demonstrate the theoretical work and its application to real scenarios. Chapter 7 will review our findings and suggest possibilities for future research.

CHAPTER II

BAYESIAN GAME DESCRIPTION

A Bayesian Game relies on *imperfect information* and the use of Bayesian probabilities to find an equilibrium. The imperfect information typically stems from the imbalance of information about a player. In our case, the players are the firms which are active in the market. The imperfect information comes from the firms knowing only their own qualities and costs; they don't know this information about the other firm.

This gives us a signaling game in which a high quality firm's profit relies on its ability to signal its quality to consumers while deterring low quality firms from mimicking them. The stronger the signal, in our case a price, the more likely it is that the firm can distinguish itself above the other firm and separate from firms of different qualities. The goal of our signaling game is to find constraints which will provide a separating equilibrium that allows consumers to observe a product's quality before purchase and prevent firms from pretending to be of a different quality.

Many times the source of the imperfect information is assumed as part of nature. Nature is seen as a separate player which assigns a type (quality) to each player. Nature plays no other role than to 'choose' the qualities of our firms, which is based on the probability a firm is of high quality. If there are multiple stages Nature can assign types for both firms and for all steps at the beginning, or assign types to the firms at the beginning of each stage. In our game, the firms are assigned a quality (low or high) as their type from this probability. They both produce a product of the quality they are as assigned by the probability (Nature).

Each player (firm) needs to make decisions based on educated guesses. The uncertainty within the game because of imperfect information causes the firms to use expected profits (based off of the probability other firms are of high quality) to make decisions (mimicking strategy and price choices). These decisions are the key to finding the Bayesian Perfect Equilibrium of the game - a set of one high quality and one low quality price that would be played, resulting in consumers purchasing products and ending the current market (period).

The firms simultaneously select the price which they wish to charge in the first market, based on their true quality and chosen mimicking strategy. As the firms choose their strategies and release products with chosen prices into the market, consumers decide from which firm to purchase. The consumers, upon observing the prices, may adjust their prior beliefs, or *priori*, about the qualities of the firms. This updating is done via Bayes' Rule (see *Appendix B*).

The consumers then decide which price to pay based on their beliefs. If the high quality firm can provide a strong enough pricing signal, consumers will become informed of the product types even before purchase. This gives what is called a revealing or separating equilibrium, because the firms qualities are revealed to the consumers. If the signal is not strong enough, we will have a pooling equlibrium where product qualities are not revealed.

A high quality firm being able to charge a price to induce a separating equilibrium will allow consumers to purchase a high quality good which is certainly preferred over a low quality good. If they are able to purchase a high quality product at a price which is below their valuation of the good, the consumer will feel as they found a bargain. Therefore, they will benefit from the price being below their maximum valuation.

If not, consumers are unable to discern the product types before purchase which leads to a pooling equilibrium (all qualities are pooled together). In a pooling equilibrium, consumers are unable to tell which type of product they are paying for. Additionally, consumers may receive a product of the opposite quality they intended to purchase if a firm was able to mimic another. Therefore, consumers will choose to pay the lower price of the two in order to minimize their displeasure if the product turns out to be of low quality.

If the game has only one stage, all players earn a payoff after this step and Bayesian Nash Equilibria (BNE) can be refined through the use of any chosen criterion. We use the Intuitive Criterion as outlined in [12]. The refining of BNEs is done to eliminate any BNE which are not sequentially rational (see *Appendix B*). If there are multiple stages, play continues with the first player making another move (firms choose a second market price) which is then followed by the process described above. At the end of each market, payoffs are calculated and equilibria are found for each firm. We only consider two markets, so the game ends after one repetition.

CHAPTER III

MODEL

This paper will focus on a Bertrand duopoly where price is the only means of signaling a firm's type to the consumers. For simplicity, we assume that there are only two firms, each is active in a two-period markets, and each supply one product per period. We also assume that the second product is released after the first with enough time for consumers to observe the quality of the first good.[1]

We will assume the firm types are independently drawn from a common distribution, which is accurately reflected in consumer beliefs, which produces a high quality type with probability α, and a low quality type with probability $1-\alpha$ in both markets. For simplicity, we will refer to these types as H-type and L-type. We will also use a subscript $t = 1, 2$ to denote the market. The consumer's beliefs for the first market are $\mu_1^H = \alpha$ and $\mu_1^L = 1 - \alpha$ for a good being of high and low quality, respectively. A firm's type is private information, meaning that only the firm knows its type. Therefore, a single firm's type is not known by either the competing firm or the consumers. Each player in the game has the following type space: $\{H_1^x, H_2^x\}, \{H_1^x, L_2^x\}, \{L_1^x, L_2^x\}, \{L_1^x, H_2^x\}$ where the superscript $x = A, B$

[1] The introduction of the second good may come before consumers can observe the quality of the first good. This is reasonable for durable goods, which take more time to reveal their quality. This scenario will not be discussed.

denotes the firm. The type space is simply the four possible quality combinations that a firm can have over the two periods. One of these four type profiles is assigned to each firm by probability, which decides their qualities in each market as it begins.

For the basic model[2], we make some simplifying assumptions about the signaling game with price being the only way firms can signal their product's quality to consumers. First, we assume there are no other frictions in the market, so only price is considered by the consumers when buying a good. All consumers in the market buy a good, either from of H or L quality, with probability one. We will assume that firms of the same type have identical marginal costs (see *Appendix B*) for both markets such that $c^H = c_1^H = c_2^H$ and $c^L = c_1^L = c_2^L$. It is also assumed that consumers have a constant valuation across all markets for each type of good. Therefore $V^H = V_1^H = V_2^H$ and $V^L = V_1^L = V_2^L$. The unit costs and consumer valuations are related in the following ways:

$$\begin{aligned} c^H &> c^L \geq 0 \\ V^H &> V^L \\ V^L &> c^L \\ V^H &> c^H \end{aligned}$$

These assumptions on costs and valuations are in line with those used by Janssen and Roy. The prices that can be charged by each firm are determined using the inequalities above. It is assumed that for any market $t = 1, 2$ and for any firm of type

[2]For reference, a variable list of all variables included in the paper is included in *Appendix A*.

$\tau \in \{L, H\}$, the price is set:

$$p_t^\tau \in [0, V^H].$$

This should be intuitive, given that a firm charging above the highest consumer valuation will not be able to sell any product to consumers.

High quality firms will charge a common deterministic price $p^H \in [c^H, V^H]$. A proof of this claim is contained in *Appendix C*. The low quality firm's profit, as found by Janssen and Roy [10], is

$$\pi^{L*} = \alpha(\bar{p}^L - c^L) = \alpha(p^H - (V^H - V^L) - c^L) \tag{3.1}$$

A low quality firm will set prices within a price support $p^L \in [\underline{p}^L, \bar{p}^L]$ where, in accordance with the assumptions of Janssen and Roy [10],

$$\bar{p}^L = p^H - (V^H - V^L) \tag{3.2}$$

and

$$\underline{p}^L = \pi^{L*} + c^L \implies \underline{p}^L = \alpha(p^H - (V^H - V^L)) + c^L(1 - \alpha). \tag{3.3}$$

Janssen and Roy noted that if $V^H - p^H = V^L - \bar{p}^L$, consumers are indifferent between buying a low quality product at its highest price $\bar{p}^L$ and a high quality product at its price p^H to obtain (3.2). The low quality firm sets $\bar{p}^L$ in (3.2) so that the difference in p^H and $\bar{p}^L$ is equal to the difference in consumer valuations for the different types of products. Therefore, consumers will be willing to pay a price $\bar{p}^L$ that is exactly this difference below p^H because they are indifferent about which type of product they receive. The lower end of the low quality price support, $\underline{p}^L$, as found in (3.3) is the lowest possible price that would lead to non-negative profits.

The sequencing of the game is as follows: The game begins when the quality of each firm, and therefore the product, is determined by the given probability for both markets. Next, after learning of their quality, each firm will choose their first market price p_1^x to be charged. After consumers observe p_1^x from each firm, they decide how much to buy of the first product and from which firm. The final profits for each firm are then calculated, and the first period ends. To begin the second period, the consumer beliefs about the quality of each firm, and therefore the product quality, in the second market is adjusted via the correlation coefficient ρ. These adjustments are done based on each firm's quality as revealed in the first period. Meanwhile, firms also update their beliefs for the second market. This update is done the same for firms and consumers alike, so consumer beliefs of a firm being high quality match that of the firms. Now, after learning of their quality, each firm chooses a price p_2^x to be charged in the second market. Finally, consumers observe p_2^x from each firm, form their expectations, and decide how much to buy and from which firm. Profits are then computed for each firm and the second period ends.

The payoff for each firm is its expected per-unit profit. Throughout the paper we make assumptions modeled after those of Janssen and Roy [10], but with slight modifications. We assume that the expected profit for a high quality firm is

$$E(\pi^{H_t*}) = \frac{\mu_t^H}{2}(p^H - c^H)$$

while the expected profit for any low quality firm is assumed to be

$$E(\pi^{L_t*}) = \mu_t^H(p^L - c^L)$$

where μ_t^τ represents the firm's priori beliefs about the distribution of firm types at the beginning of the t^{th} market. The derivation of these expected profits is explained in more detail in the next section. Consumer payoffs do not directly play a role because this game focuses on the firm's ability to signal its type to the consumer. However, they do factor into the decision made on which price to pay.

Because we are considering two markets, we must take into account changes in consumer beliefs after the first good is purchased. It should be noted that the probabilities used above will be altered slightly in the second market when the quality of the first good is observed. Once the previous product's quality is able to be observed, the prior beliefs are then changed to posterior beliefs via the correlation coefficient. The correlation coefficient ρ relates the qualities of the products from each market.

The following changes in beliefs are taken from Pires and Catalão-Lopes. In the second market $\mu_1^H = \alpha$ is replaced by $\mu_2^H = \alpha + \rho(1 - \alpha)$ if the first product was observed to be high quality. One can think of these beliefs as consumers believing they underestimated the portion of high quality firms after seeing a high quality good in the first market. If the first product was observed as low quality, $\mu_1^H = \alpha$ is replaced by $\mu_2^H = \alpha - \rho\alpha$. This stems from the idea that consumers feel they overestimated the probability that a firm was high quality in the first market. These changes are in line with the expectation that seeing a high quality good in the previous market should lead to an increase in a consumer's belief that firms are high quality in the second market, while observing a low quality good will have the opposite effect. Hence, firms who were shown to be high quality in the first market benefit from the separating equilibrium while low quality firms suffer. Firms also update their beliefs in this way, which changes the expected profits in the second market. These adjustments lend credence to the idea that spillover effects can occur in periods that follow a previous market that ends in a separating equilibrium.

CHAPTER IV

EQUILIBRIA IN A SINGLE MARKET

Before the first stage of the Bayesian game, we must discuss consumer preferences as well as create the framework for expected payoffs for the firms. It is very important to remember that a firm's type is not known, so only the price of the product is observed by the consumer. The *utility preferences* (see *Appendix B*) are key to choosing which actions the consumers will play in response to observing a product's price. It should be noted that we are only interested in the final payoffs for the firms. However, we must look at how the consumers will purchase a good to find how much each type of firm will sell and what profits will follow from those decisions.

The utility a consumer gets from an action is composed of two basic inputs - quality and price. Conditional on the prices of each good being the same, a consumer will prefer to have a high quality good over a low quality good. Similarly, if two goods were of the same quality, a consumer will choose to purchase the good at the lowest price. These preferences allow us to make a few observations about how the signaling game will be played.

To decide what action each firm should take, each firm will attempt to maximize their profit. Given that a firm is unsure of the price charged by other firms, it can only charge a price that will help to maximize the *expected* profit. Therefore a firm's expected profit is important in deciding to mimic or not, which is then used in determining what price to charge. The expected profits depend on consumer beliefs along with the unit costs and prices used. Per the model outlined above, consumers' beliefs are $\mu_1^H = \alpha$ and $\mu_1^L = 1 - \alpha$ that a firm is of high or low quality, respectively.

A firm of high quality will only sell in the case that the other firm is of high quality.[1] If the other firm is low quality, the lower price will be chosen by all consumers because they cannot differentiate the two types. Therefore, the low quality firm is able to undercut the other firm and take all consumers, so the high quality firm can expect zero profit. If the second firm is of high quality, they will both charge the same price and will evenly split the profits. Therefore, similarly to Janssen and Roy, we find that the expected profit for a high quality firm is

$$E(\pi^{H_1*}) = \frac{\alpha}{2}(p^H - c^H). \tag{4.1}$$

A low quality firm has the ability to sell in any market, but will not always be able to. If the other firm is of high quality as discussed above, the low quality firm will be able to undercut its competitor and have a per-unit profit of $p^L - c^L$. This gives a low quality firm an expected profit of

$$E(\pi^{L_1*}) = \alpha(p^L - c^L). \tag{4.2}$$

in accordance to Janssen and Roy's work. Note that both expected profits are dependent on α, which is the probability that all other firms (in our case, just one) are

[1] This stems from the assumption of homogeneity in consumer valuations. By assuming consumer valuations are constant for each type of firm, we don't allow for consumers to be partial to high priced goods as opposed to low priced goods.

of high quality. The ability to sell a good for either type of firm relies on the type of the other firm.

If both firms are of low quality, neither firm is guaranteed to sell a non-zero quantity. Due to the mixed strategy of choosing a price (low quality firms will randomize their choices of p^L over the interval $[\underline{p}^L, \bar{p}^L]$), a low quality firm can expect to be undercut by the other firm and will therefore not expect any profit in this case. In the case that both firms randomly select to charge the same price, they will split the profits just as if both were high quality.

Now that preferences and expected profits are set out for consumers and firms respectively, we can begin analyzing the Bayesian game. The game begins with each firm's quality being assigned, via the probability α from the common distribution, for both markets. We can assign the qualities for each firm for both markets at the beginning, or assign qualities at the beginning of each market. The type profiles (quality combinations over the two markets) come from the type space mentioned in the previous section.

For the separating equilibria, we will only consider those where the low quality firm uses a mixed strategy of $p^L \in [\underline{p}^L, \bar{p}^L]$ and the high quality firm uses a deterministic price $p^H > p^L$ for $p^H \in [c^H, V^H]$.[2] Therefore, we consider the following Bayesian Nash Equilibrium.

[2] This is because according to Janssen and Roy, "Bertrand price competition between (symmetric) firms eliminates the rent for low quality types that is needed to sustain a fully revealing equilibrium. Therefore, full revelation necessarily involves randomization over prices (price dispersion)."

Proposition. *With price supports as action spaces for each type of firm, the deterministic high quality price, p^H, and low quality price, $p^L \in [\underline{p}^L, \bar{p}^L]$, selected via mixed strategy constitutes a Bayesian Nash Equilibrium (BNE).*

Refer to [10] for a proof of this proposition. Hence we will only need to consider fully revealing Bayesian equilibria which involve the high quality firms charging a strictly higher price than the low quality firms that may lead to separating equilibria. The high quality price will be deterministic, meaning that any high quality firms will charge a common price p^H in equilibrium. The low quality price is chosen via randomization over the price support $[\underline{p}^L, \bar{p}^L]$ because no pure strategy (choosing the same, specific price independent of the other firm's price) will be sustainable. Hence, the low quality firms will randomize their prices to give themselves the best opportunity to earn a positive profit in equilibrium.

Given that we are only looking at two firms, we have three possibilities for the type distribution within the market that are described below.

4.1 Both Firms are of High Quality

In the case where both firms are of high quality, a common deterministic price p^H leads to an even share of sales in the market. Because of this, each receives a profit of exactly

$$\pi^{H_1*} = \frac{p^H - c^H}{2} > 0. \tag{4.3}$$

4.2 Both Firms are of Low Quality

In the scenario where both firms are of low quality, a firm will sell at price $p \in [\underline{p}^L, \bar{p}^L]$ as long as the other firm does not offer a lower price. The expected profit at price p is then $\big(1-(1-\alpha)(1-F(p))\big)[p-c^L]$ where $F(p)$ is a cumulative probability distribution function that gives the probability of selling at any price $\underline{p}^L < \hat{p} < p$ where both firms are low quality.[3] In the case where both firms are of low quality we have the following two sub-cases.

4.2.1 Firms Select Different Prices

Suppose, without loss of generality, that for the two firms we have $p^A < p^B$, where p^A and p^B correspond to the prices set by the first and second firms respectively. Then firm A will undercut firm B and be able to gain all of the sales because consumers will choose the lower price given that they are unaware of their types. The equilibrium profits for each firm are then

$$\pi^{A_1*} = p^A - c^L \geq 0 \text{ and } \pi^{B_1*} = 0. \tag{4.4}$$

4.2.2 Both Firms Select the Same Price $p^L \in [\underline{p}^L, \bar{p}^L]$

In this case, both firms will sell to the consumers similarly to the case for high quality firms. As in the scenario for two high quality firms, they will both split the profits evenly. This yields an equilibrium profit of

$$\pi^{L_1*} = \frac{p^L - c^L}{2} \geq 0 \tag{4.5}$$

[3]To see how $F(p)$ was derived, see *Appendix C*.

for both firms. In this case $\alpha = 0$, so actual profits can exceed the expected profit as given by (4.2). It should be noted, however, that this case will happen with a probability that is essentially zero given the randomization of prices. This is because there are large number of discrete prices that can be chosen by each low quality firm within the price support $[\underline{p}^L, \bar{p}^L]$. Even with a price support of length 1, there are 100 different prices to choose from. Randomly having each firm pick the same price within a \$1 range is only expected to occur with probability $.01^2 = .0001$. Therefore, we can expect this to occur rarely, if at all, within any price support which is likely to be more than \$1 wide.

4.3 Firms Differ in Quality

When the two firms are of opposite qualities, a firm's ability to signal it is high quality (or to fool the consumer into thinking it is high quality) is especially important. As stated previously, a high quality firm relies on being able to separate from low quality firms in order to sell its products. If they cannot signal their true quality to consumers, all products will be seen as homogeneous in quality and the low quality good will be chosen because of its lower price.

It should be noted that a low quality firm charges its highest possible price, $\bar{p}^L$ in the scenario where it is the only low quality firm in the market. When there are multiple low quality firms, the lowest price of all will take the entire market share and all others will not be able to sell. As the number of firms increases, the low quality price will approach the low quality marginal cost.[4] Because the low quality firm is the lone low quality firm in the market, it will earn a positive profit of

$$\pi^{L_1*} = \bar{p}^L - c^L$$

In this case, the high quality firm earns zero profit.

For either type of firm to mimic each other, there must be some economic incentive. A low quality firm will imitate a high quality firm by setting a high quality price if its profits increase above non-imitation levels. Likewise, the high quality firm will want to sell some quantity at a low quality price that gives a positive profit.

If a low quality firm is able to successfully mimic the high quality firm, it can sell at the deterministic price p^H with a profit of $\pi^L = \frac{1}{2}(p^H - c^L)$. The high quality firm would earn a profit of $\pi^H = \frac{1}{2}(p^H - c^H)$.

Consider the case where a low quality firm wishes to mimic a high quality firm by charging price $p^H > \bar{p}^L$. This deviation is not gainful if, and only if,

$$\begin{aligned} \frac{\alpha}{2}(p^H - c^L) &\leq \alpha(\bar{p}^L - c^L) = E(\pi^{L*}) \\ \implies p^H &\geq 2(V^H - V^L) + c^L. \end{aligned} \tag{4.6}$$

Another possibility is that a high quality firm will attempt to mimic a low quality firm and undercut all other firms. If the second firm was of low quality and was unable to act as high quality, this would lead to a positive profit which would

[4]If N is the number of low quality firms, then according to Janssen and Roy $\underline{p}^L = \alpha^{N-1}\bar{p}^L + c^L(1 - \alpha^{N-1})$. Then $N \to \infty \implies \underline{p}^L \to c^L$ because $\alpha < 1$.

make the firm better off. However, the high quality firm must be careful to charge a price which is still above its costs while being within the low quality price support.

We consider the high quality firm wishing to mimic the low quality firm by choosing a price $\hat{p} \in [\underline{p}^L, \bar{p}^L]$. For this to be gainful, the expected profits for the firm at price $\hat{p}$ must be higher than at price p^H. Let $q(p)$ be the quantity sold at price p in equilibrium.[5] Then, the expected net profit for the high quality firm, as was found in [10], is

$$\begin{aligned}
(\hat{p} - c^H)q(\hat{p}) &= (\hat{p} - c^L)q(\hat{p}) - (c^H - c^L)q(\hat{p}) \\
&= \pi_{net}^{L*} - (c^H - c^L)q(\hat{p}) \\
&\leq \pi_{net}^{L*} - (c^H - c^L)q(\bar{p}^L) \\
&= \alpha(\bar{p}^L - c^H).
\end{aligned}$$

Hence, a high quality firm's best mimicking strategy would be to charge $p^H = \bar{p}^L$. This deviation to a price of $\bar{p}^L$ is not gainful if, and only if, the profit from deviating is smaller than the expected profit from the price p^H. This implies that p^H must be such that

$$\begin{aligned}
\alpha(\bar{p}^L - c^H) &\leq \frac{\alpha}{2}(p^H - c^H) = \pi^{H*} \\
\Longrightarrow p^H &\leq 2(V^H - V^L) + c^H \qquad (4.7)
\end{aligned}$$

for the deviation to price $\bar{p}^L$ from price p^H to not be gainful for the high quality firm.

[5]Through correspondence with Dr. Santanu Roy, it was explained that α is identical to $q(\bar{p}^L)$ in the case where only two firms exist. Here, α can be thought of as equaling the expected quantity at price $\bar{p}^L$. This gives us the final result in the inequality below.

Using the results from (4.6) and (4.7), we can guarantee that neither type of firm will attempt to mimic another type if the high quality firm sets a price p^H such that

$$2(V^H - V^L) + c^L \leq \quad p^H \quad \leq 2(V^H - V^L) + c^H \tag{4.8}$$

which can also be written as

$$\frac{V^H - V^L}{p^H - c^L} \leq \quad \frac{1}{2} \quad \leq \frac{V^H - V^L}{p^H - c^H}.$$

Hence if a high quality firm chooses its price in such a way that satisfies (4.8), neither type of firm will wish to mimic each other and the high quality firm has the ability to signal its type to consumers. Equilibrium beliefs then imply that any equilibria will not involve deviation by the firms in an attempt to deceive the consumers into believing they are of a different quality.

In addition to the above analysis, we will need to discuss the out-of-equilibrium beliefs. The out-of-equilibrium beliefs are used when a consumer observes a price that is not what is expected. This means that they observe a price p which is neither equal to p^H nor in the low quality price support $[\underline{p}^L, \bar{p}^L]$.

If $p \neq p^H, p \notin [\underline{p}^L, \bar{p}^L]$, then consumers will believe the firm is of low quality with probability one. This is because the price observed is not conducive for the high quality firm, due to the price not matching the deterministic price p^H. Hence, the firm charging this price is assumed to be for a low quality firm with reasonable certainty.

Given that a high quality firm will charge a common deterministic price of p^H, any price outside of the low quality price support will be the result of attempted mimicry by the low quality firm. Therefore, consumers will choose not to purchase a good at a price $p \in (\bar{p}^L, p^H)$ and will be better off by purchasing from another firm. Firms will not want to deviate to such prices because of this. Also, if $p > p^H$ the firm will not be able to sell any product. All consumers will strictly prefer to purchase from other firms even if they are certain the product is of high quality. Similarly, choosing $p < \underline{p}^L$ will not be beneficial to the firm by (4.7).

If these pricing strategies hold as shown in Janssen and Roy, we will have a separating equilibrium if, and only if, p^H satisfies

$$\theta_0 \equiv \max\{c^H, 2(V^H - V^L) + c^L\} \leq p^H \leq \min\{2(V^H - V^L) + c^H, V^H\} \equiv \theta_1. \quad (4.9)$$

For this separating equilibrium to exist, it is a necessary and sufficient condition[6] to require only that

$$\frac{V^L - c^L}{V^H - c^L} \geq \frac{1}{2}.$$

This inequality and (4.9) come directly from the work of [10]. A summary of this section is done in the following lemma, similarly to Lemma 1 of Janssen and Roy.

Lemma. *A symmetric fully revealing equilibrium exists in which the high quality firm charges p^H and all consumers purchase one good if, and only if,*

$$\frac{V^L - c^L}{V^H - c^L} \geq \frac{1}{2}. \quad (4.10)$$

[6]For a derivation of this condition, refer to *Appendix C*.

The set of possible prices for the high quality firm is $[\theta_0, \theta_1]$. *In this equilibrium, the low quality firm will randomize over the interval* $[\underline{p}^L, \bar{p}^L], \bar{p}^L < p^H$. *The equilibrium expected unit-profits of each firm are defined as in* (4.1) *and* (4.2).

CHAPTER V

EQUILIBRIA IN A SECOND MARKET

In this section, we will discuss the portion of the game which takes place in a second market after consumers are able to observe the first products' qualities and following a separating equilibrium in the first market. In other words, the high quality firm was able to separate from the low quality firm through setting p^H in such a way that it deterred any mimicking by the other firm. The results and analysis of the second market are our contributions to price signaling research concerning multiple markets.

The analysis of this scenario begins with adjusting the beliefs used by the consumers when trying to decide the probability that the new product is of high quality. At the beginning of the second market, the consumer and both firms have learned the qualities of both goods in the first market. For consumers, the beliefs that a second product is of high quality is either $\mu_2^H = \alpha + \rho(1 - \alpha)$ or $\mu_2^H = \alpha - \rho\alpha$ according to the description of μ_2^H in Chapter 3. Firms use these same probabilities to compute their expected profits, which are then used in determining their strategies.

The adjustment of beliefs at the beginning of the second market causes a change in some variables that affect the payoff structure and decisions of the firms. Per one of our initial assumptions, the marginal costs and consumer valuations (c^τ and V^τ) remain the same in the second market. The upper boundary of the low quality price support, $\bar{p}^L$ will also remain the same amount below p^H. The lower end of the low quality price support, $\underline{p}^L$, will change along with of the adjustment in beliefs. In the second market, the magnitude of this change depends on both the initial belief α as well as the correlation coefficient ρ. The specific adjustments of $\underline{p}^L$ will be discussed within each scenario below.

In an attempt to find equilibria conditions in a second market which follows a separating equilibrium in the first market, there are two scenarios of interest. In the first scenario, neither firm changes its type. Therefore, a firm which was of low quality in the first market will remain low quality and a high quality firm will remain of high quality in the second market. The second scenario involves both firms switching types, so a low quality firm will become high quality in the second market and vice versa.[1] We will look at these cases for the rest of the chapter.

5.1 Firms Remain the Same Types

While both firms remain the same type as in the first market, neither is aware that the other firm has remained the same quality. As expressed earlier, the high quality firm believes that the low quality firm is of high quality with probability $\mu_2^H(L_1) = \alpha - \rho\alpha$. The low quality firm believes that the high quality firm is high quality with probability $\mu_2^H(H_1) = \alpha + \rho(1 - \alpha)$. It should be noted that these beliefs match those of the consumers at the beginning of the second stage.

[1] The scenarios where only one firm changes its type are of little interest, given that they would be similar to the single market cases where both firms are of the same type.

These beliefs are the adjusted probabilities using the correlation coefficient ρ. Using the correlation coefficient allows for a spillover effect to be identified if it exists. The expected profits for the second market are

$$E(\pi^{H_2*}) = \frac{\alpha - \rho\alpha}{2}(p^H - c^H) \tag{5.1}$$

and

$$E(\pi^{L_2*}) = [\alpha + \rho(1-\alpha)](\bar{p}^L - c^L) \tag{5.2}$$

for the high and low quality firms respectively. The beliefs are mixed compared to the first market due to the expected profits depending on the probability the *other* firm is of high quality. This relates to the firms updating their beliefs, along with consumers, about the probability that the other firm is high quality. For example, the high quality firm in the second market was of high quality in the first market. Therefore, the other firm was of low quality in the previous market and the belief $\mu_2^H(H_1) = \alpha - \rho\alpha$ is used when calculating the expected profit for the high quality firm. A similar analysis provides us with the probability used for the low quality firm's expected profit.

While many of the results remain constant, the lower end of the price support for low quality firms $\underline{p}^L$ will change. Let $\underline{p}_S^L$ be the lower bound with the firm types remain the same in the second market. Given the new beliefs and substituting them into (3.3), we find that

$$\begin{aligned} \underline{p}_S^L &= \bar{p}^L(\alpha - \rho\alpha + \rho) + c^L(1 - (\alpha - \rho\alpha + \rho)) \\ &= \bar{p}^L(\alpha - \rho\alpha + \rho) + c^L(1 - \alpha + \rho\alpha - \rho). \end{aligned} \tag{5.3}$$

Therefore, the lower bound will be more heavily weighted toward $\bar{p}^L$, and will cause an increase in $\underline{p}^L_S$. Hence, the price support for the low quality firm actually shrinks in size. With the low quality price support in place, we can continue toward finding a revealing equilibrium.

The analysis of mimicking is similar to that of the single market. The low quality firm only wishes to mimic the high quality firm if the deviation to a price p^H is profitable over charging $\bar{p}^L$. This deviation is not profitable if, and only if,

$$\begin{aligned} [\alpha - \rho(1-\alpha)](\bar{p}^L - c^L) &\geq \frac{\alpha - \rho\alpha}{2}(p^H - c^L) \\ \Longrightarrow p^H &\geq 2\beta_S(V^H - V^L) + c^L, \end{aligned} \tag{5.4}$$

where $\beta_S = \left(\dfrac{\alpha - \alpha\rho + \rho}{\alpha - \alpha\rho + 2\rho}\right)$. Therefore if p^H satisfies the above inequality, a low quality firm will choose not to mimic in the second market.

It should be noted that a restriction on the coefficient β_S must be positive for the inequality to not be reversed. This holds if either the numerator is positive, or if the denominator is negative. Setting the numerator greater than zero yields

$$\begin{aligned} \alpha - \alpha\rho + \rho &> 0 \\ \Longrightarrow \rho &> \frac{\alpha}{\alpha - 1} \end{aligned}$$

which is always satisfied, given that $\rho \in [0,1]$ and $\alpha \in (0,1)$. Therefore the inequality above holds regardless of the value of ρ. Hence, we do not need to place any restrictions on product quality correlation or consumer beliefs for a separating equilibrium in this market.

The high quality firm will decide whether or not to mimic in a similar way. The deviation to a price $\bar{p}^L$ from a price p^H is not beneficial if, and only if,

$$\begin{aligned} [\alpha - \rho(1-\alpha)](\bar{p}^L - c^H) &\leq \frac{\alpha - \rho\alpha}{2}(p^H - c^H) \\ \implies p^H &\leq 2\beta_S(V^H - V^L) + c^H. \end{aligned} \tag{5.5}$$

Note inequality (5.5) holds for any values of α and ρ.

Assuming the out of equilibrium beliefs are the same as in the first market, we can create an interval of high quality prices that give the opportunity for separation in the second market. Combining inequalities (5.4) and (5.5) gives a result similar to that of the single market defined in the following way. A separating equilibrium exists in a second market where firms do not change types if, and only if, p^H satisfies

$$\begin{aligned} \theta_{0,S} &= \max\{c^H, 2\beta_S(V^H - V^L) + c^L\} \\ &\leq p^H \\ &\leq \min\{2\beta_S(V^H - V^L) + c^H, V^H\} = \theta_{1,S}. \end{aligned} \tag{5.6}$$

For this separating equilibrium to exist, it is a necessary and sufficient condition to require only that

$$\frac{2\beta_S - 1}{2\beta_S} \leq \frac{V^L - c^L}{V^H - c^L}.$$

It should be noted that since $0 \leq \beta_S < 1$ for any possible ρ, the price support for a fully revealing p^H will contain lower prices than in the single market case. Therefore, the high quality firm will be able to separate from the low quality firm with a lower price than previously. This is indicative of spillover effects being present in the second market when neither firm switches its type. These spillover effects are present regardless of the correlation between firm types in different markets, as long as the first market resulted in a revealing equilibrium. The results from this scenario are summarized in the following lemma.

Lemma. *A symmetric fully revealing equilibrium for the second market where firm types are held constant exists in which the high quality firm charges p^H and all consumers purchase one good if, and only if,*

$$\frac{2\beta_S - 1}{2\beta_S} \leq \frac{V^L - c^L}{V^H - c^L} \tag{5.7}$$

The set of possible prices for the high quality firm is $[\theta_{0,S}, \theta_{1,S}]$. There is no additional restriction on the correlation coefficient ρ. In this equilibrium, the low quality firm will randomize over the interval $[\underline{p}^L_S, \bar{p}^L], \bar{p}^L < p^H$. The equilibrium expected unit-profits of each firm are defined as in (5.1) *and* (5.2).

5.2 Firms Change Types

This indicates a firm that was high quality in the first market is now low quality and vice versa. Again we assume that neither firm is aware of the changes to the other firm's type, but only of its own change. Each firm updates their beliefs as in the previous case, yielding the expected profits

$$E(\pi^{H_2*}) = \frac{\alpha + \rho(1-\alpha)}{2}(p^H - c^H) \tag{5.8}$$

and

$$E(\pi^{L_2*}) = [\alpha - \rho\alpha](\bar{p}^L - c^L) \tag{5.9}$$

for the high and low quality firms respectively. Note that the probabilities associated with each expected profits are reversed from the previous case.

As with the previous case, the lower bound of the low quality price support will change. Let $\underline{p}_C^L$ be the lower bound with the firms change types in the second market. After using the new beliefs in (3.3), it follows that

$$\begin{aligned}\underline{p}_C^L &= \bar{p}^L(\alpha - \rho\alpha) + c^L(1 - (\alpha - \rho\alpha)) \\ &= \bar{p}^L(\alpha - \rho\alpha) + c^L(1 - \alpha + \rho\alpha). \qquad (5.10)\end{aligned}$$

Therefore the lower bound, $\underline{p}_C^L$, will be more heavily weighted toward the marginal cost c^L, and will decrease. Hence, the price support for the low quality firm actually expands to include prices below the *original* $\underline{p}^L$. With the low quality price support in place, we can once again continue toward finding a revealing equilibrium.

For a low quality firm to deviate to a price p^H, it must be more profitable than charging $\bar{p}^L$. Therefore, the low quality firm will not deviate if p^H is chosen in a way that

$$\begin{aligned}[\alpha - \rho\alpha](\bar{p}^L - c^L) &\geq \frac{\alpha + \rho(1-\alpha)}{2}(p^H - c^L) \\ \Longrightarrow p^H &\geq 2\beta_C(V^H - V^L) + c^L, \qquad (5.11)\end{aligned}$$

where $\beta_C = \left(\dfrac{\alpha - \rho\alpha}{\alpha - \rho\alpha - \rho}\right)$, is satisfied. The direction of the inequality is dependent on the requirement we place on ρ. If the correct restriction is not chosen, the direction of inequality (5.11) will be reversed. This will be discussed after we look at the mimicking strategy of the high quality firm.

The high quality firm will deviate to a price $\bar{p}^L$ if, and only if, p^H satisfies

$$\begin{aligned} [\alpha - \rho\alpha](\bar{p}^L - c^H) &\leq \frac{\alpha + \rho(1-\alpha)}{2}(p^H - c^H) \\ \implies p^H &\leq 2\beta_C(V^H - V^L) + c^H. \end{aligned} \tag{5.12}$$

Once again, the direction of inequality (5.12) depends on the value of ρ.

To ensure that inequalities (5.11) and (5.12) do not change direction, we place a restriction on ρ that guarantees

$$2\beta_C(V^H - V^L) + c^L \leq p^H \leq 2\beta_C(V^H - V^L) + c^H. \tag{5.13}$$

Therefore, we need to restrict ρ such that

$$\begin{aligned} 0 &< \frac{\alpha - \rho\alpha}{\alpha - \rho\alpha - \rho} \\ \implies \rho &< \frac{\alpha}{1+\alpha} \end{aligned} \tag{5.14}$$

is satisfied. Note that ρ is positive and bounded above by $\frac{1}{2}$ regardless of α. Hence, this restriction on ρ is consistent with the definition of ρ being between 0 and 1. With ρ being less than $\frac{1}{2}$, consumers will view firms in the second market as being the opposite type of the first market more times than not; the lower correlation coefficient indicates that firms are more likely to change types between markets.

With the restriction on ρ above, we can once again find an interval for p^H where we can guarantee the existance of a separating equilibrium for the second market. A separating equilibrium exists if $\rho < \dfrac{\alpha}{1+\alpha}$, and p^H satisfies

$$\begin{aligned}\theta_{0,C} &= \max\{c^H, 2\beta_C(V^H - V^L) + c^L\} \\ &\leq p^H \\ &\leq \min\{2\beta_C(V^H - V^L) + c^H, V^H\} = \theta_{1,C}.\end{aligned} \tag{5.15}$$

From this inequality, it is a necessary and sufficient condition to require only that

$$\frac{2\beta_C - 1}{2\beta_C} \leq \frac{V^L - c^L}{V^H - c^L}$$

for the separating equilibrium to exist in the second market. Keeping in mind the restriction on ρ and noting that $\beta_C \geq 1$ allows us to make some observations about the high quality price support.

Given that $\beta_C \geq 1$, it should be clear that the lower and upper bounds of the high quality price support should increase in the second market. This is reflective of the additional cost of signaling brought on by the low correlation coefficient. Consumers are unable to use information from the first market on which to base their buying preferences. Firms are unable to use any of the results from the first market to benefit themselves and it becomes harder for consumers to believe a firm is high quality. Therefore, in the second market, the high quality firm necessarily has to charge a higher price p^H than in the first market to reveal its type. We see no indication that spillover effects are present in the second market when firms change types.

The results from this scenario are similar to that of the previous market. The final inequality that is sufficient in guaranteeing a separating equilibrium is nearly the same, but we must also set a requirement on ρ when the firms change types. These results are summarized in the following lemma.

Lemma. *A symmetric fully revealing equilibrium for the second market where firms change types exists in which the high quality firm charges p^H and all consumers purchase one good if, and only if,*

$$\rho < \frac{\alpha}{1+\alpha} \qquad \text{AND} \qquad \frac{2\beta_{\mathrm{C}} - 1}{2\beta_{\mathrm{C}}} \leq \frac{\mathrm{V^L} - \mathrm{c^L}}{\mathrm{V^H} - \mathrm{c^L}} \tag{5.16}$$

The set of possible prices for the high quality firm is $[\theta_{0,C}, \theta_{1,C}]$. In this equilibrium, the low quality firm will randomize over the interval $[\underline{p}^L_C, \bar{p}^L], \bar{p}^L < p^H$. The equilibrium expected unit-profits of each firm are defined as in (5.8) and (5.9).

CHAPTER VI

NUMERICAL EXAMPLES

Following the theoretical derivation of prices which constitute a separating equilibrium, we will use numerical examples to further explain and demonstrate the above results. By selecting multiple levels of α and ρ and choosing values of c^H, c^L, V^H, and V^L, we hope to give realistic examples of how the above research could apply to real life markets. We will consider the following scenarios: when firm costs and consumer valuations are nearly identical and when firms have large cost and valuation differences.

To provide the following numerical examples, a MATLAB code[1] was written and used. The levels of α and ρ were varied, using $\rho \in \{0.1, 0.4, 0.7, 1\}$ and $\alpha \in \{0.1, 0.25, 0.4, 0.55, 0.7, 0.85, 1\}$. To calculate expected profits and the price supports, a price for p^H is needed to be selected. It doesn't matter which value it takes, as long as it lies within the possible separating price support $[\theta_0, \theta_1]$ prescribed in the market as in (4.9), (5.6) and (5.15) . For simplicity, the price p^H was selected as the midpoint of this range. In the case that the chosen $p^H \leq \bar{p}^L$, p^H was increased until $p^H > \bar{p}^L$ to avoid a possible pooling equilibrium scenario.

[1]The MATLAB code used has been provided in *Appendix D*

6.1 Similar Costs and Valuations

Consider an industry where two firms (one of each quality) produce two similar products (one of each quality) in each of two markets. Additionally we assume that consumers value the low and high quality goods similarly, with the high quality good being valued slightly higher than the low quality good for each market. Also assume that the high quality firm has a slightly higher marginal cost in each market.

For context suppose that Apple plays the role of the high quality firm in the first market, while Microsoft is the low quality firm. Think of the two goods as two of the original MP3 players that were released in the early 2000's - Apple's iPod versus the Microsoft Zune. Upon their initial release, consumers were unaware of their qualities given that MP3 players were in their infancy. Both companies had made their mark in the computer industry, but were new to the MP3 market. Therefore, being able to convince consumers they had a high quality product would prove beneficial for either firm.

To differentiate its product and show that it is of higher quality, the price of the iPod will necessarily need to be high enough to keep Microsoft from imitating it. This is handled in the first market, which can be thought of as the beginning of the MP3 player battle that occured in the early 2000's. We assume without loss of generality that Apple is the high quality firm in the first market. It is assumed that the marginal costs were comparable to each other, with Apple's being slightly higher. Additionally, we can assume that consumers would value the two products similarly because of how new the MP3 industry was at the time, but favor the iPod over the Zune.

For this example, we chose to use $c^H = 6, V^H = 20, c^L = 5$, and $V^L = 19$ for Apple and Microsoft respectively. Note that these choices for costs and consumer valuations satisfy inequality (4.10). Therefore, we will always have a separating equilibrium in the first market.

To demonstrate a specific example, we will choose $\alpha = 0.4$ and $\rho = 0.1$ and explain the results. By choosing a high quality price of \$7.50, we find the set of prices that can be sustained as the high quality price p^H in any separating equilibrium is the interval from \$7.00 to \$8.00. The price supports for the first market of our specific example are found in Figure 6.1. In such an equilibrium, the low quality firms follow a mixed strategy with support $[\$5.60, \$6.50]$. The equilibrium per-unit profits of the high and low quality firms are \$0.30 and \$0.60, respectively.

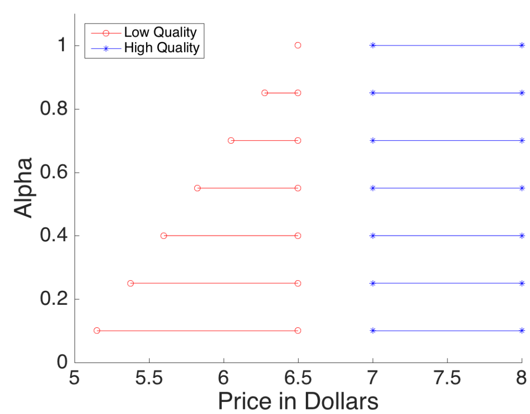
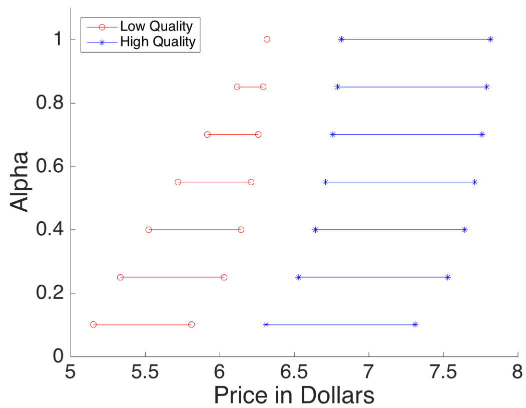


Figure 6.1: Similar Costs and Valuations - Price supports in 1st market.

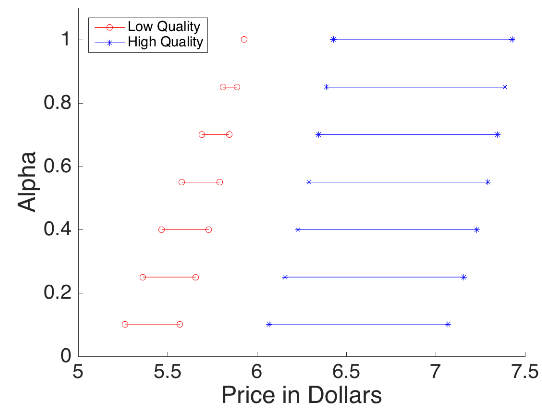
In the second market, with the above parameters, we must look at the cases where both firms maintain their qualities and where both change their quality. When firms do not change qualities, a high quality price p^H exists that produces a fully revealing equilibrium. We use $p^H = \$7.14$ for the rest of the calculations. The set of prices that can be sustained as the high quality price p^H in any such equilibrium is the interval from \$6.64 to \$7.64. In such an equilibrium, the low quality firms follow a mixed strategy with support $[\$5.53, \$6.14]$. These price supports (along with those for other values of ρ and α) can be seen in Figure 6.2. The equilibrium per-unit profits of the high and low quality firms are \$0.21 and \$0.53, respectively.

When firms change their qualities, our choices of $\rho = 0.1$ and $\alpha = 0.4$ satisfy (5.16). Therefore we will have a separating equilibrium in the second market. However, $\rho = 0.4$ only provided separation for α close to 1. Any value of $\rho \geq 0.5$ will not allow for separation by (5.16). We use a high quality price of \$8.27, as determined through calculations of the high quality price support. The set of prices that can be sustained as the high quality price p^H in any such equilibrium is the interval from \$7.77 to \$8.77. In such an equilibrium, the low quality firms follow a mixed strategy with support $[\$6.04, \$7.27]$. The price supports for both qualities are shown in Figure 6.3. The equilibrium per-unit profits of the high and low quality firms are \$0.41 and \$1.04, respectively.

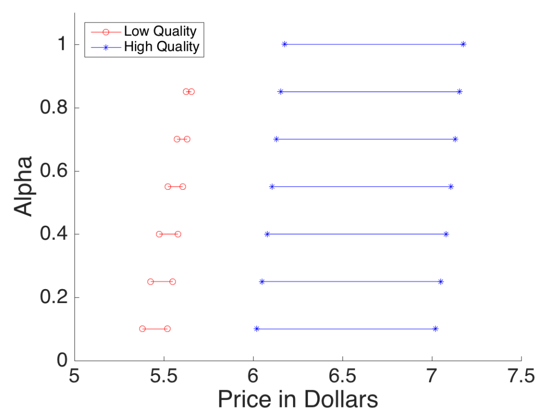
In general, we can determine if a separating equilibrium can be achieved based on our parameter choices, as well as how the price supports will change. The parameters chosen are arbitrary, but are meant to demonstrate how the analysis done



(a) Low correlation: $\rho = 0.1$



(b) Medium correlation: $\rho = 0.4$



(c) High correlation: $\rho = 0.7$

Figure 6.2: Similar Costs and Valuations - Price supports in 2nd market where firms do not change quality. As $\rho \to 1$, the high quality price support shifts toward c^H. As $\alpha \to 1$, the low quality price support shrinks toward $\bar{p}^L$.

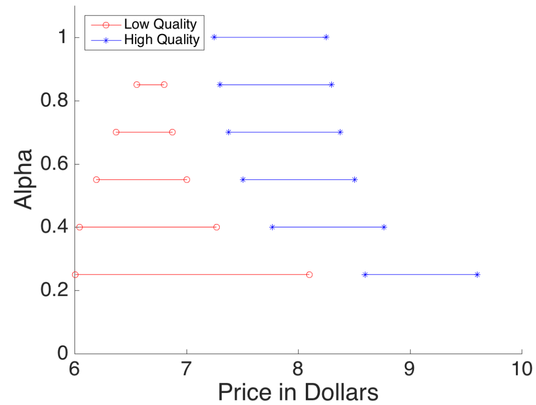


Figure 6.3: Similar Costs and Valuations - Price supports in 2nd market where firms change types, with $\rho = 0.1$. As $\alpha \to 1$, we see the high quality price support shifts toward c^H and the low quality price support shrinks. Both price supports are now at higher levels than in the first market.

in Chapters 4 and 5 can be applied. An overall analysis is provided below for the scenario when the two firms have similar costs and consumer valuations. The results are then related back to the specific scenario posed at the beginning of the case.

1. When firms do not change their quality in the second market, all values of ρ and α allow for a separating equilibrium. The high quality price support is larger than in the first market. Additionally as $\rho \to 1$, we see the high quality price support shift toward the marginal cost c^H. As $\alpha \to 1$, the high quality price support slightly increases while the low quality price support shrinks in size. The lower bound $\underline{p}^L$ approaches the upper bound $\bar{p}^L$, giving the low quality firm fewer choices for its product's price. These results can be seen in Figure 6.2.

2. When both firms change their qualities, we only see a separating equilibrium for $\rho = 0.1$ and $\alpha \geq 0.25$. This is as expected, given that ρ must satisfy the result from (5.16). These are the only combinations of ρ and α which give a separating equilibrium in the second market where both firms have changed qualities (for the chosen parameters). As $\alpha \to 1$, the high quality price support shifts downward, and the low quality price support shrinks. Additionally, we should note that both price supports are higher than in the first market, because they are unable to take advantage of spillover effects due to changing qualities. These results are shown in Figure 6.3.

Therefore in the case of Apple and Microsoft, Apple can take advantage of a spillover effect in the second market (the release of the iPod 2nd generation) and be able to charge a lower price while maintaining its signaling power. Not changing types would allow Apple to take advantage of low levels of α and ρ and separate again in the second market. If the firms change types and α and ρ are too high, then there won't be a second separating equilibrium.

In actuality, the first iPod debuted at \$399.99, while the Zune started at \$249.99. The higher price charged by Apple likely was a signal to demonstrate their true quality and it seemed to have worked. Also, the initial prices of each basic iPod release before the video iPod or iPod Touch indicate that Apple was able to take advantage of spillover effects. The prices began to fall with each basic model, which is in line with our model's expectations and prior research.

6.2 High Differences in Costs and Valuations

Now we consider an industry with considerable cost and valuation differences between a high quality and low quality firm. Again, we have two firms (one of each quality) which produce two products (one of by each firm) in each of two markets. For simplicity, we assume the goods being produced are common items that vary in both cost and valuation: jewelry.

Larger jewelry chains like Kay Jewelers or Jared claim to produce high end diamonds with real silver or gold settings. With all the special collections of diamonds and settings, Jared offers what seem to be high quality variations on basic jewelry. Smaller jewelers may offer alternatives made of cubic zirconia or low quality metals, without the 'premium' styles or collections. Therefore, the jewelry industry could certainly be analyzed within this framework.

Just as in the previous case, we assume that the costs for Jared, our high quality firm, are much higher than the small business jeweler, our low quality firm. Also, assume that consumers would much rather purchase jewelry from Jared than the small jeweler - consumer valuation for the high quality jewelry is much higher than for the low quality jewelry.

With this in mind, we used the parameters $c^H = 20, V^H = 40, c^L = 10$, and $V^L = 30$ for Jared and its small business competitor respectively. These parameters satisfy inequality (4.10), which implies that a separating equilibrium will occur in the first market regardless of the value of α.

To demonstrate a specific example, we will choose specific levels of $\alpha = 0.7$ and $\rho = 0.1$ and explain the results once again. Choosing p^H to be the midpoint of the high quality price support gives us $p^H = \$35.00$. The set of prices that can be sustained as the high quality price p^H in any such equilibrium is the interval from \$30.00 to \$40.00. In such an equilibrium, the low quality firms follow a mixed strategy with support $[\$20.50, \$25.00]$. The price supports from our example can be seen in Figure 6.4. The equilibrium per-unit profits of the high and low quality firms are \$5.25 and \$10.50, respectively.

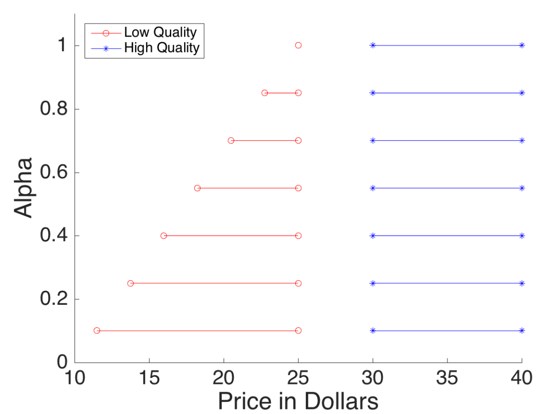


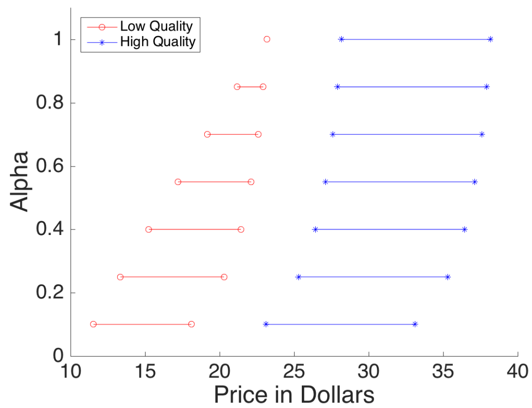
Figure 6.4: High Cost and Valuation Differences - Price supports in 1st market.

In the second market involving the chosen parameters, we must once again look at the two scenarios described previously. When firms do not change their qualities, we use the high quality price $p^H = \$32.59$. The set of prices that can be sustained as the high quality price p^H in any such equilibrium is the interval from \$27.59 to \$37.59. In such an equilibrium, the low quality firms follow a mixed strategy with support [\$19.19, \$22.59]. These price supports are shown in Figure 6.5. The equilibrium per-unit profits of the high and low quality firms are \$3.97 and \$9.19, respectively.

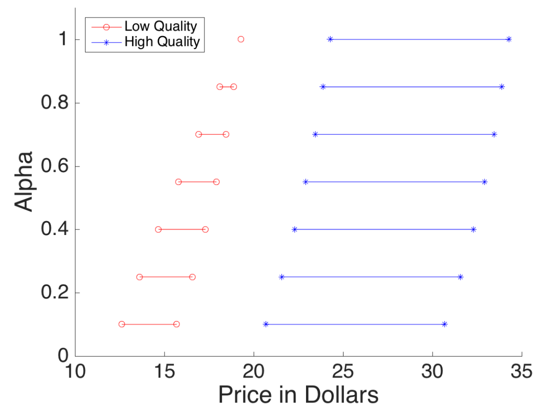
When firms change their qualities, our choices of $\rho = 0.1$ and $\alpha = 0.7$ satisfy (5.16). Just as above, we are able to achieve a separating equilibrium. We use a price $p^H = 36.89$ which lies at the middle of the high quality price support which was calculated as the interval from \$33.77 to \$40.00. In such an equilibrium, the low quality firms follow a mixed strategy with support [\$22.33, \$26.89]. These price supports can be seen in Figure 6.6. The equilibrium per-unit profits of the high and low quality firms are \$5.32 and \$12.33, respectively.

Again, we can analyze the effects different levels of ρ and α will have on the shifting and size of the price supports. We summarize the findings below, and relate them back to the example of Jared versus the small jewelry store.

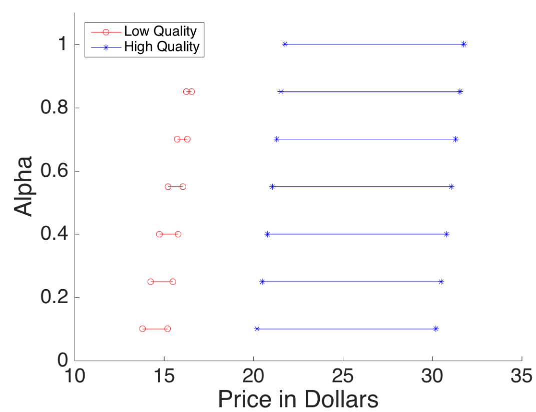
1. When firms do not change their qualities in the second market, we see similar results as when the firms have similar costs and consumer valuations. All values of ρ and α provide a separating equilibrium, while also indicating spillover effects in the second market. The high quality price support is at a lower level than in the first market equilibrium, and shifts slightly upward as α increases. The low quality price support also shrinks in size and shifts upward as $\alpha \to 1$. Therefore, Jared can charge a lower price in the second market and take advantage of the first market separation. These results can be seen in Figure 6.5.



(a) Low correlation: $\rho = 0.1$



(b) Medium correlation: $\rho = 0.4$



(c) High correlation: $\rho = 0.7$

Figure 6.5: High Cost and Valuation Differences - Price supports in 2nd market where firms do not change quality. As $\rho \to 1$, the high quality price support shifts downward. As $\alpha \to 1$, the low quality firm's price support shrinks and shifts upward.

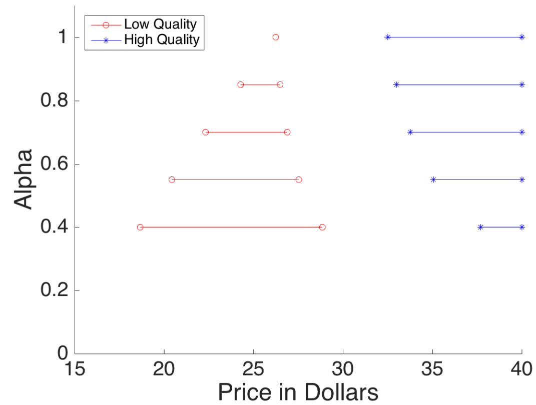


Figure 6.6: High Cost and Valuation Differences - Price supports in 2nd market where firms change types, with $\rho = 0.1$. As $\alpha \to 1$, we see the high quality price support grows and shifts toward c^H. The low quality price support again shrinks as $\alpha \to 1$. Both price supports are also at higher levels than in the previous market.

2. When firms change types, ρ must again satisfy (5.14) to achieve a separating equilibrium. We see that a separating equilibrium occurs for the same values of α and ρ as when firms change types. There would have to be a very small probability that a firm was low quality in the previous market to even allow for them to effectively signal any change to high quality in a second market. Therefore if Jared produces a low quality product in the second market, while the small business jeweler has a high quality product, the small business jeweler will have to charge a higher price to separate than was needed in the first market. They will have to prove that they have a high end product by charging a higher price p^H than in the first market, as seen in Figure 6.6.

Given these parameters, Jared can separate in both markets as long as both firms remain the same quality in both markets. Spillover effects are present, as expected. Even in the case that firms change quality after the first market, a separating equilibrium is unlikely to occur. Comparing the real-world prices of these two firms, we typically see that the prices charged by Jared are much higher. But is it indicative of their true quality? We'll leave that to the consumers.

6.3 Numerical Conclusions

There are many more scenarios that can be evaluated with the use of the MATLAB code provided in *Appendix D*. These should lead to more numerical results and situations that would be applicable to many real world markets. We have chosen to include only a couple of the examples to demonstrate how the code works within the theoretical model discussed in the previous two chapters. The numerical results agree with the theoretical predictions included in the report above.

As expected, when firms stay the same type and are able to separate in the first market, we see the high quality price support shift downward toward the high quality marginal cost c^H. This is indicative of the spillover effects from a separating equilibrium in the first market. The high quality firm can now separate with a lower price than in the first market. When the firms change types, the price ranges are initially higher than those in the first market when ρ is small. If ρ is too large, we will not have a separating equilibrium. We see the high quality separating price range expand and shift downward as α increases toward 1. At the same time, the low quality range shrinks as $\underline{p}^L$ increases toward $\bar{p}^L$. These findings agree with the conclusion of the theoretical work done above.

CHAPTER VII

CONCLUSION

By combining the ideas and models from previous literature, we have created a model to analyze pricing decisions within a two-period duopoly framework. By comparing the expected profit functions for high and low quality firms, we were able to identify when mimicking was or was not beneficial to each type (quality) of firm. Under certain assumptions regarding the costs, valuations, and beliefs of the firms and consumers, we were able to find price supports that would allow the high quality firm to effectively signal its type to consumers.

In the first market case, we were able to replicate and confirm the findings of Janssen and Roy [10]. The high quality firm is able to separate from the low quality firm as long as (4.10) is satisfied. In short, if the difference between consumer valuation of the low quality good and the low quality marginal cost, $V^L - c^L$, is at least half of this difference for the high quality valuation, $V^H - c^L$, then a separating equilibrium can be achieved in the market. This was as expected, given that Janssen and Roy's model was the basis for our research of extending the results to multiple-period markets by introducing the ideas used by Pires and Catalão-Lopes [17].

From there, we created the adapted model to be used in the second market. We assumed that the first market ended in a separating equilibrium, allowing all consumers and both firms to observe the true qualities of each first market product. Holding costs and consumer valuations constant from the previous market, we updated beliefs and expected profits at the beginning of the second market. In this market, we accounted for the correlation of the products. If a firm was identified as high quality in the first market, it receives a 'bump' in consumer beliefs - they now believe it is high quality with probability $\alpha + \rho(1 - \alpha)$. If the firm was identified as low quality, they are penalized and believed to be high quality with probability $\alpha - \rho\alpha$.

We also accounted for two scenarios for when the firms do or do not switch their types between markets. The cases where only one firm changes its type produces two firms of the same quality, with results similar to those at the beginning of Chapter 4. For this reason, we did not include them in our analysis. Using a similar analysis as in the single market, via the updated expected profits, we found high quality price supports that would also allow the firm to separate in the second market for each case. The existence of these price supports is confirmed as long as (5.7) and (5.16) are satisfied. As in the first market, we can look at the ratio of differences in valuations and the low quality marginal cost to determine whether separation is feasible. When firms change types, we also require that the product qualities are not highly correlated according to the first inequality in (5.16).

In the second market, our price supports are slightly different from those in the single market case. We observe that the price supports can shift in a second market following a separating equilibrium in the previous market. In the case where firms do not change types, the price support for separating high quality prices is shifted downward to contain lower prices. This implies that the cost of signaling is lower in the second market, indicating that spillover effects are present in this case. When firms switch types, we see an upward shift of the price supports. Hence, the high quality firm will now have to charge a higher price to compensate for being low quality in the previous market. To convince consumers the firm is now high quality, they must pay a higher premium and charge a higher price. This is what causes the set of viable prices to shift upward, where spillover effects do not help the high quality firm.

Our findings are somewhat limited given the assumptions we made concerning number of firms, costs, and valuations. Future research could be done to adapt this model to multiple subsequent markets, as well as markets involving more than two firms. In addition, varying costs and consumer valuations across markets would allow for a more realistic simulation of real businesses.

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APPENDICES

APPENDIX A

VARIABLE LIST

Variable	Description
μ_t^τ	Consumer beliefs at the beginning of the t^{th} market for a good of type τ, $\mu_t^\tau \in [0,1]$
$F(p)$	Probability distribution of selling a good by charging price p
τ	True quality of the firm, $\tau \in \{H, L\}$
α	The probability that a firm is of high quality, $\alpha \in [0,1]$. Taken from consumer beliefs about the distribution of firm qualities.
ρ	Degree of correlation between the first and second products, $\rho \in [0,1]$
c^τ	Marginal (unit) cost of production for firm of quality τ, $c^\tau \geq 0$
V^τ	Consumer valuation for 1 unit of good of quality τ, $V^\tau \geq 0$
p_t^τ	Market price chosen by firm in market i, $p_t^\tau \in [0, V^H]$
π^{τ_t}	Equilibrium profit for firm of type τ in market t
$\underline{p}^L$	Lower bound on the low quality price support.
$\bar{p}^L$	Upper bound on the low quality price support.
β_C, β_S	Coefficients from adjustment in beliefs in the second market. The size of the coefficient determines shifting of equilibrium price supports.

APPENDIX B

GLOSSARY

Action Set: The group of actions a player can take during the course of the game.

Bayes' Rule: Used to calculate conditional probabilities. Bayes' Rule relates current probability to a prior probability. It is written as:

$$\Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\Pr(B)}$$

Bayesian Equilibrium: The strategy profiles and beliefs specified for each player about the types of the other players that maximizes the expected payoff for each player, given their beliefs about the other players' types and given the strategies played by the other players.

Belief System: A belief system is an assignment of probabilities to every node in the game such that the sum of probabilities in any information set is 1.

Bertrand Oligopoly: A market with a small number of firms which set prices, which consumers then use to decide how much of a good to purchase.

Consumer Valuations: The highest value given to a product by consumers, which varies depending on the product's type. It can be thought of as the highest price a consumer is willing to pay for a good of a certain quality.

Deterministic Price: A price which is consistent among all firms, independent of input costs. Therefore, differing inputs will always lead to firms charging the same price.

Domination Criterion: No player will choose an action which leads to a worse payoff. If action A leads to a higher payoff than action B, the player will not choose action B.

Expected Profit: The profit a firm expects to gain based on the likelihood that it can sell in the market with another firm of an unknown type.

Hybrid Equilibrium: An equilibrium in which one or both types (qualities) of player randomizes its strategy. The firm will randomly choose to mimic or not mimic based on a given probability.

Information Set: A set, for the particular player, which establishes all the possible moves that could have taken place to that point given what the player has observed.

Information Spillovers: Extra effects which occur due to results from a previous stage. These can be thought of as carryover effects, which cause adjustments in a player's beliefs in subsequent stages.

Intuitive Criterion: Also known as the Cho Kreps Intuitive Criterion. If the receiver observes a deviation from the equilibrium, then the receiver should not believe that the sender is of type B if both of the following are true:

1. The deviation would result in type B being worse off then if he has stuck to the equilibrium for any beliefs.
2. There is some type A who is better off by playing the deviation than by sticking to the equilibrium for some belief other than B.

For a formal definition, refer to the Harvard University lecture notes [**?**].

Marginal Cost: The additional cost of producing one unit of output. Here, it is the cost associated with the unit of output sold by a firm.

Mixed Strategy: A probability distribution over the player's pure strategies. It is often thought of as randomizing which action is chosen to be played by a player.

Nash Equilibrium: A stable state reached when no strategy would yield a player a higher payoff, given all strategies played by the other players. This is also referred to as a player's strategy being the best response to all other strategies.

Perfect Bayesian Equilibrium (PBE): The strategy profile and a belief system such that the strategies are sequentially rational given the belief system and the belief system is consistent, wherever possible, given the strategy profile.

Pooling Equilibrium: An equilibrium in which the types of the players are not revealed through their actions.

Posterior Belief: The belief of a player which has been modified conditional on an observation.

Priori: The initial belief of the player, which is independent of any observations.

Pure Strategy: A function that assigns an action in the player's action set to each of the player's information set.

Out-of-Equilibrium Beliefs: These beliefs are tied to equilibria that are not Nash Equilibria. A player's out-of-equilibrium beliefs are used to determine if any equilibria exist where a deviation by all players could increase their payoffs, but would not occur because of player rationality.

Rent: The additional income brought on by use of a resource, less the costs associated with bringing the resource into the production process.

Revealing or Separating Equilibrium: An equilibrium in which player types are revealed; they are able to separate from other players by demonstrating their true types (qualities) to the players.

Strategy Profile: The set of strategies for all players which specifies all actions that are taken in a game.

Spillover Effect: The effects in a secondary market that stem from the actions taken in a previous market. An equilibrium in the first market may loosen the restrictions on a similar equilibrium in the following market if spillover effects are present.

Symmetric Equilibrium: An equilibrium in which all players choose the same action. In our game, this would occur if both firms choose not to mimic each other.

Type Space: Set of all possible types, or combinations of them in multi-stage games, that a player can be during the game.

Utility Preferences: Preferences which rank the utility, essentially the 'happiness', gained by choosing an action. These give a list which gives possible actions in order of preference.

APPENDIX C

PROOFS AND MATHEMATICAL DERIVATIONS

Theorem. *Suppose that all firms know their own type, but this information is not observable by the other firms and consumers. Then for any number of profit maximizing firms, all of whom are of high quality, each firm's profit maximizing price is some common deterministic Nash Equilibrium price* p^H*.*

Proof. Suppose there are two firms, firm 1 and firm 2, which are both of high quality. Assume both firms have identical unit costs so that $c_1 = c_2 = c^H$. Let $q(p)$ be the quantity sold at price p. Suppose that firm 1 charges price $p_1 = p^H \in [c^H, V^H]$, the deterministic Nash equilibrium price, which maximizes its profit $\pi_1 = (p^H - c^H)q(p^H)$. Now assume that firm 2 charges a price $p_2 \neq p^H$ which maximizes its profit $\pi_2 = (p_2 - c^H)q(p_2)$. The following cases arise:

1. Suppose firm 2 charges a price $p_2 < p_1 = p^H$. In this case, because consumers cannot differentiate the types of the two firms they will choose to buy for the lower price. Therefore firm 2 will be able to sell $q(p_2)$ units and have a profit of $\pi(p_2)$. However, they can still increase their profit by charging a higher price $p_2 < p_2^* < p_1$. Hence, they are not maximizing profit and will increase the price to p^H to maximize their profit.

2. Suppose firm 2 charges a price $p_1 = p^H < p_2$. With a lower price, firm 1 is able to undercut firm 2. Therefore, firm 2 is unable to sell and will therefore receive a profit equal to 0 ($\pi_2 = 0$). To be able to sell any positive quantity, the second firm will necessarily have to adjust the price downward to p^H and match the first firm's price. Therefore, p_2 will fall from its original value to p^H in order to make a positive profit.

Given these two cases, a price different from the deterministic equilibrium price p^H will necessarily converge to a common price charged by all firms $p^H \in [c^H, V^H]$. Hence if even one firm charges differently from p^H which is common for all firms, they will eventually begin charging p^H themselves. Therefore, all high quality firms will charge a common deterministic equilibrium price of p^H. $\square$

Derivation of $F(p)$

Taken from the work done by Janssen and Roy, we can find π^{L*} for every price $p \in [\underline{p}^L, \bar{p}]$ if

$$\begin{aligned}\big(\alpha + (1-\alpha)(1-F(p))\big)[p - c^L] &= \alpha(p^H - (V^H - V^L) - c^L) \\ \implies F(p) &= \frac{1}{\alpha - 1}\left(\frac{\alpha(p^H - (V^H - V^L) - c^L)}{p - c^L} - 1\right)\end{aligned}$$

which is continuous on $[\underline{p}^L, \bar{p}]$, $F(\bar{p}^L) = 1$, and $F(\underline{p}^L) = 0$.

Results for Inequality (4.10)

To find such a p^H exists to satisfy (4.10), we require that the below inequality

$$\theta_0 \equiv \max\{c^H, 2(V^H - V^L) + c^L\} \leq p^H \leq \min\{2(V^H - V^L) + c^H, V^H\} \equiv \theta_1$$

is satisfied. This gives us four possible combinations to check for conditions, the strongest of which will be used to find when such a p^H exists. Consider the following four cases:

1. Assume $c^H > (c^L + 2(V^H - V^L))$ and $(c^H + 2(V^H - V^L)) > V^H$. We have that $c^H \leq V^H$, which is an initial restriction on p^H. Therefore, $p^H \in [c^H, V^H]$ is plausible given the initial conditions on prices in such a case.

2. Assume $c^H > (c^L + 2(V^H - V^L))$ and $(c^H + 2(V^H - V^L)) < V^H$. Then we require that

$$\begin{aligned} c^H &\leq c^H + 2(V^H - V^L) \\ 0 &\leq V^H - V^L \\ V^L &\leq V^H \end{aligned}$$

which is an initial assumption in our model. Therefore, finding a p^H to satisfy the inequality above is plausible in this case.

3. Assume $c^H < (c^L + 2(V^H - V^L))$ and $(c^H + 2(V^H - V^L)) < V^H$. We want that

$$\begin{aligned} c^L + 2(V^H - V^L) &\leq c^H + 2(V^H - V^L) \\ c^L &\leq c^H \end{aligned}$$

which is satisfied by our initial assumptions about unit costs. Once again, it is reasonable to be able to find a p^H to satisfy the inequality in this scenario.

4. Assume $c^H < (c^L + 2(V^H - V^L))$ and $(c^H + 2(V^H - V^L)) > V^H$. We want to show that $c^L + 2(V^H - V^L) \leq V^H$. This implies that

$$\begin{aligned} c^L + V^H &\leq 2V^L \\ \frac{V^H + c^L}{2} &\leq V^L \\ \frac{V^H - c^L}{2} &\leq V^L - c^L \\ \frac{1}{2} &\leq \frac{V^L - c^L}{V^H - c^L}. \end{aligned}$$

Therefore, if the condition above is satisfied we can find a p^H that satisfies the original inequality. This is the result that is used for (4.10). $\square$

APPENDIX D

MATLAB CODE

```
% 'Price Signaling in a Two-Market Duopoly'
% Numerical Code      Version 4.3.0
% Matthew Hughes      3/21/2016
% This code is meant as a numerical device to provide examples of the
% situations discussed in the accompanying thesis.
% This code is broken down into several sections, as is the paper.
% It will be used only as a tool to implement the ideas discussed
% after Section 2 of the paper. The checks and assumptions made in
% the paper are used in the development and use of this code to
% ensure results are plausible and valid in calculation.
% By using selected values for variables, it provides numerical
% examples of possible scenarios as well as associated outputs
% and conclusions.
clear all
clc
%%
%=====================================================================
%                        Setting up the Model
%=====================================================================
```

```
% VARIABLE DECLARATION/INITIALIZATION
%====================================================================
cH = 20;                 % Unit cost of production for high quality
cL = 10;                 % Unit cost of production for low quality
vH = 40;                 % Consumer valuation of high quality good
vL = 30;                 % Consumer valuation of low quality good
alpha = 0;               % Probability of firm being high quality
rho = 0;                 % Correlation coefficient, in (0,1)
for rho = .1:.3:1           % Loop to change values of rho
    for alpha = .1:.15:1    % Loop to change values of alpha
        id = num2str(alpha);    % Strings created for naming files
        id2 = num2str(rho);
        startfile = ['alpha=' id ' rho=' id2];
        fid = fopen(startfile, 'w');
        % Initializing Variables to 0 for use later.
        pLUnder = 0;     % Lowest price charged by low quality firm
        pLBar = 0;       % Highest price charged by low quality firm
        HEprofit = 0;    % Equilibrium profit, for high quality
        LEprofit = 0;    % Equilibrium profit, for low quality
        %===========================================================
        % RESTRICTIONS ON VARIABLES
        % These are checks that are in place to make sure the
        % calculations work out for all the variables.
        % They make sure that there will not be paradoxes or invalid
        % scenarios within the calculations.
        %===========================================================
```

```
% Alpha, eta, lambda must be within [0,1]
if (0 > alpha || 1 < alpha || 0 > rho || 1 < rho )
    fprintf('WARNING: One of alpha or rho is outside the')
    fprintf('allowed range of [0,1].\n')
    return
end
% cH > cL ≥ 0
if (cH ≤ cL || cL ≤ 0 || cH ≤ 0)
    fprintf('\nWARNING: One of cL or cH is invalid.\n')
    fprintf('They must both be ≥ 0 and cH > cL.\n')
    return
end
% vH > vL and vL > cL, vH > cH
if (vH ≤ vL || vL ≤ cL || vH ≤ cH)
    fprintf('\nWARNING: One of vH or vL is invalid.\n')
    fprintf('Make sure vH > vL, vH > cH, and vL > cL.\n')
    return
end
%%
%============================================================
%                      Decisions in First Market
%============================================================
fprintf(fid,'===========================================\n');
fprintf(fid,'            FIRST MARKET\n');
fprintf(fid,'===========================================\n');
% Declaring parameters
```

```
fprintf(fid,'\nWe have the following parameters:\n');
fprintf(fid,'alpha = %3.2f \t',alpha);
fprintf(fid,'cH = $%4.2f \t',cH);
fprintf(fid,'vH = $%4.2f \n',vH);
fprintf(fid,'rho = %3.2f \t\t',rho);
fprintf(fid,'cL = $%4.2f \t',cL);
fprintf(fid,'vL = $%4.2f \n',vL);
fprintf(fid,'============================================');
% Check if a separating equilibrium exists
if (vL - cL)/(vH - cL) ≥ 1/2
    fprintf(fid,['\nA high quality price pH exists that' ...
             ' produces a fully revealing equilibrium.\n']);
    % Calculate range for pH, make sure it is valid
    theta0 = max(cH,2*(vH-vL)+cL);
    theta1 = min(2*(vH-vL)+cH,vH);
    if theta0 > theta1
        fprintf('\nWARNING: theta0 > theta1.');
        fprintf(['\nThere is no valid range for pH for a'...
                 ' separating equilibrium.\n']);
    end
    % Pick a pH in the existent range, maybe midway
    pH = (theta0 + theta1)/2;
    % Calculate price support bounds for low quality
    pLBar = pH - (vH - vL);
    pLUnder = pLBar*alpha + cL*(1-alpha);
    % Make sure pLBar < pH
```

```
        while pLBar ≥ pH
            pH = (pH +theta1)/2;
        end
        % Equilibrium Profits
        LEprofit = (pLBar - cL)*alpha;
        HEprofit = (pH - cH)*(alpha/2);
        % Conclusion for First Market and Plot
        fprintf(fid,['\nBy choosing a high quality price of'...
                     '$%4.2f, we find the following:'],pH);
        fprintf(fid,['\nThere exists a symmetric fully'...
                     ' revealing equilibria where the high' ...
                     ' quality firm charges a deterministic' ...
                     ' price of $%4.2f and all buyers buy' ...
                     ' with \nprobability 1.\n The set of'...
                     ' prices that can be sustained as' ...
                     ' the high quality price pH in any' ...
                     ' such equilibrium is the interval' ...
                     ' from $%4.2f to $%4.2f.']...
                     ,pH,theta0,theta1);
        fprintf(fid,['\nIn such an equilibrium, the low'...
                     ' quality firms follow a mixed' ...
                     ' strategy with support[$%4.2f,'...
                     ' $%4.2f]. \n The equilibrium per-unit'...
                     ' profits of the high and low quality' ...
                     ' firms are \n$%4.2f and $%4.2f,'...
                     'respectively.'] ...
```

```
                        ,pLUnder, pLBar, HEprofit, LEprofit);
            % Create plot of price supports
            ALPHA = [alpha, alpha];
            LSup = [pLUnder, pLBar];
            HSup = [theta0, theta1];
            figure(1)
            hold on
            plot(LSup,ALPHA,'-o','color','red');
            plot(HSup,ALPHA,'-*','color','blue');
            set(gca,'fontsize',20)
            ylim([0,1.1]);
            legend('Low Quality','High Quality','Location',...
                    'NorthWest');
            set(legend,'FontSize',14);
            ylabel('Alpha', 'FontSize', 24)
            xlabel('Price in Dollars','FontSize', 24)
        else
            fprintf(fid,['\nThere is no high quality price pH'...
                        'which guarantees the existence of a'...
                        'separating equilibrium.\n']);
        end
        %%
        %==========================================================
        %                 Decisions in Second Market
        %==========================================================
        fprintf(fid,'\n\n');
```

```
% When firms stay the same types
fprintf(fid,'==========================================\n');
fprintf(fid,'      SECOND MARKET - Staying the Same Type\n');
fprintf(fid,'==========================================\n');
% Calculate new probability ratio
Beta_stay =(alpha-rho*alpha+rho)/(alpha - rho*alpha + 2*rho);
% Check if a separating equilibrium exists
if (vL - cL)/(vH - cL) ≥ (2*Beta_stay - 1)/(2*Beta_stay)
     fprintf(fid,['\nA high quality price pH exists that'...
                  ' produces a fully revealing' ...
                  ' equilibrium.\n']);
    % Calculate range for pH
    theta0_stay = max(cH,2*Beta_stay*(vH-vL)+cL);
    theta1_stay = min(2*Beta_stay*(vH-vL)+cH,vH);
    if theta0_stay > theta1_stay
        fprintf('\nWARNING: theta0 > theta1.');
        fprintf(['\nThere is no valid range for pH for'...
                  ' a separating equilibrium.\n']);
    end
    % Pick a pH in the existent range, maybe midway
    pH_stay = (theta0_stay + theta1_stay)/2;
    % Adjust pLUnder and pLBar
    pLBar_stay = pH_stay - (vH - vL);
    pLUnder_stay = pLBar_stay*(alpha - rho*alpha + rho)...
                   + cL*(1 - alpha + rho*alpha - rho);
    % Make sure pLBar < pH
```

```
while pLBar_stay ≥ pH_stay
    pH_stay = (pH_stay +theta1_stay)/2;
end
% Expected unit-profits
HEprofit_stay = ((alpha - rho*alpha)/2)*(pH_stay - cH);
LEprofit_stay =(alpha+rho*(1-alpha))*(pLBar_stay - cL);
% Conclusion for 2nd Market and Plot
fprintf(fid,['\nBy choosing a high quality price of'...
             ' $%4.2f, we find the following:']...
             ,pH_stay);
fprintf(fid,['\nThere exists a symmetric fully'...
             'revealing equilibria where the high'...
             ' quality firm charges a deterministic'...
             ' price of $%4.2f and all buyers buy'...
             ' with probability 1.\n The set of'...
             ' prices that can be sustained as' ...
             ' the high quality price pH in any'...
             ' such equilibrium is the interval'...
             ' from $%4.2f to $%4.2f.']...
             ,pH_stay,theta0_stay,theta1_stay);
fprintf(fid,['\nIn such an equilibrium, the low'...
             ' quality firms follow a mixed'...
             ' strategy with support' ...
             ' [$%4.2f, $%4.2f]. \nThe equilibrium'...
             ' per-unit profits of the high and'...
             ' low quality firms are \n$%4.2f' ...
```

```
                 ' and $%4.2f, respectively.']...
                 ,pLUnder_stay, pLBar_stay, ...
                 HEprofit_stay, LEprofit_stay);
    % Plot 2nd market 'stay' case results
    LSup_stay = [pLUnder_stay, pLBar_stay];
    HSup_stay = [theta0_stay, theta1_stay];
    figure(2)
    hold on
    plot(LSup_stay,ALPHA,'-o','color','red');
    plot(HSup_stay,ALPHA,'-*','color','blue');
    set(gca,'fontsize',20)
    ylim([0,1.1]);
    legend('Low Quality','High Quality','Location',...
           'NorthWest');
    set(legend,'FontSize',14);
    ylabel('Alpha', 'FontSize', 24)
    xlabel('Price in Dollars', 'FontSize', 24)
else fprintf(fid,['\nThere is no high quality price'...
                  'pH which guarantees the existence'...
                  'of a \nseparating equilibrium.\n']);
end
%%
fprintf(fid,'\n\n');
fprintf(fid,'========================================\n');
fprintf(fid,'  SECOND MARKET - Both Firms Change Type\n');
fprintf(fid,'========================================\n');
```

```
    % Calculate new probability ratio
    Beta_change = (alpha-rho*alpha)/(alpha - rho*alpha - rho);
    if ((vL - cL)/(vH - cL) ≥ ...
            (2*Beta_change-1)/(2*Beta_change) ...
            && rho < alpha/(1+alpha))
        fprintf(fid,['\nA high quality price pH exists'...
                     ' that produces a fully revealing'...
                     'equilibrium.\n']);
        % Calculate range for pH
        theta0_change = max(cH,2*Beta_change*(vH-vL)+cL);
        theta1_change = min(2*Beta_change*(vH-vL)+cH,vH);
        if theta0_change > theta1_change
            fprintf('\nWARNING: theta0 > theta1.');
            fprintf(['\nThere is no valid range for pH'...
                     ' for a separating equilibrium.\n']);
        end
        % Pick a pH in the existent range, maybe midway
        pH_change = (theta0_change + theta1_change)/2;
        % Adjust pLUnder and pLBar
        pLBar_change = pH_change - (vH - vL);
        pLUnder_change = ...
                       pLBar_change*(alpha-rho*alpha+rho)...
                       + cL*(1 - alpha + rho*alpha - rho);
        % Make sure pLBar < pH
        while pLBar_change ≥ pH_change
            pH_change = (pH_change +theta1_change)/2;
```

```
            end
            % Expected unit-profits
            HEprofit_change =((alpha-rho*alpha)/2)*(pH_change-cH);
            LEprofit_change =(alpha+rho*(1-alpha))*(pLBar_change-cL);
            % Conclusion for 2nd Market and Plot
            fprintf(fid,['\nBy choosing a high quality price of'...
                         ' $%4.2f, we find the following:'] ...
                         ,pH_change);
            fprintf(fid,['\nThere exists a symmetric fully' ...
                         ' revealing equilibria where the high'...
                         ' quality firm charges a deterministic'...
                         ' price of $%4.2f and all buyers buy'...
                         ' with probability 1.\nThe set of prices'...
                         ' that can be sustained as the high'...
                         ' quality price pH in any such'...
                         ' equilibrium is the interval from' ...
                         ' $%4.2f to $%4.2f.'],...
                         pH_change,theta0_change,theta1_change);
            fprintf(fid,['\nIn such an equilibrium, the low'...
                         ' quality firms follow a mixed strategy'...
                         ' with support[$%4.2f, $%4.2f].'...
                         '\nThe equilibrium per-unit profits of'...
                         ' the high and low quality firms are'...
                         '\n$%4.2f and $%4.2f, respectively.']...
                         ,pLUnder_change,pLBar_change,...
                         HEprofit_change, LEprofit_change);
```

```
            % Plot 2nd market 'change' case results
            LSup_change = [pLUnder_change, pLBar_change];
            HSup_change = [theta0_change, theta1_change];
            figure(3);
            hold on
            plot(LSup_change,ALPHA,'-o','color','red');
            plot(HSup_change,ALPHA,'-*','color','blue');
            set(gca,'fontsize',20)
            ylim([0,1.1]);
            legend('Low Quality','High Quality','Location',...
                   'NorthWest');
            set(legend,'FontSize',14);
            ylabel('Alpha', 'FontSize', 24)
            xlabel('Price in Dollars', 'FontSize', 24);

        else fprintf(fid,['\nThere is no high quality price'...
                          ' pH which guarantees the existence'...
                          ' of a \nseparating equilibrium.\n']);
        end
        fclose(fid);
    end
    if rho == .1         % Create labeling for 2nd market based
        r = 'Low';       % on value of rho. These are those attained
    elseif rho == .4     % in line 41 ('for' loop of rho).
        r = 'Med';
```

```
    elseif rho == .7
        r = 'High';
    else
        r = 'Perfect';
    end
    if (vL - cL)/(vH - cL) ≥ (2*Beta_stay - 1)/(2*Beta_stay)
        figure(2)
        name = ['Market2Stay ' r ' corr'];
        saveas(gcf,name,'png')          % Save results, clear figure
        clf(2)
    end
    if ((vL - cL)/(vH - cL) ≥ (2*Beta_change-1)/(2*Beta_change) ...
        && rho < alpha/(1+alpha))
        figure(3)
        name = ['Market2Change ' r ' corr'];
        saveas(gcf,name,'png')          % Save results, clear figure
        clf(3)
    end
end
figure(1)
saveas(gcf,'Market1','png')     % Save results, clear figure
clf(1)
```