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# Analysis of Short Crack Growth in Particulate Composites

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## ABSTRACT

This paper presents the analysis of the short-crack propagation and growth in particulate composite materials by using the image correlation techniques and fracture mechanics theory. The fracture mechanics model for a long crack does not work very well with short-crack propagation when the initial crack length is less than 5.1 mm (0.2 inch). In order to investigate the short crack effect, a series of tests of particulate composite specimens with long and short cracks have been performed and the results were recorded on a video tape. These test data were analyzed to determine the strain fields near the crack tip, crack growth rate, and fracture parameters. Two initial crack lengths, 2.5 mm (0.1 inches) and 7.6 mm (0.3 inches) were used in the crack propagation tests. Images of particulate composite specimens with initial short crack in unloaded state and under loading state were digitized from the videotape. Image correlation techniques were employed to obtain the crack-tip propagation data - 2D displacements as well as in-plane strains. By analyzing these test data, the crack growth rate  $da/dt$  and the stress intensity factor  $K_I$  were calculated. Log-log charts of  $da/dt$  vs.  $K_I$  for both 0.1-inch and 0.3-inch initial crack test data were generated and the results were compared.

**Keywords:** short-crack propagation, particulate composites, fracture mechanics, image correlation.

## 1 INTRODUCTION

The particulate composite materials are used in the solid propellant rocket structures. The initiation and propagation of cracks have crucial effects on the structural integrity of the rocket which can lead to a premature failure of a component. How to ensure the structural integrity and reliability in the structure design is an important engineering problem [1]. Recently, many fracture experiments and researches have been performed to characterize the failure mechanisms of the particulate composites. Some studies stem from the fact that damage, expressed in terms of attenuation of the acoustic energy, can significantly affect the constitutive and the crack growth behavior in the particulate composites. A study shows that the unstable crack growth is consistent with the need of decreasing energy dissipation for higher crack tip speed [2]. Experimental findings revealed that the damage rate is relatively small during the early stage of the crack development. However, the damage rate increases rapidly when the crack is about to be

formed. Mode I (tensile) fracture tests under inert (non-burning) conditions concluded that the fracture toughness increases with decreasing temperature and loading rates and the presence of three distinct stages in the failure process [3]. The crack increases with increasing strain rate and the critical damage is relatively insensitive to the strain rate. The damage state near the tip of a stationary crack is dependent on the loading history.

Digital image correlation of speckle patterns has been used extensively to measure surface displacements and deformations, which assists to investigate the strain fields of the desired region near the crack tip. Two speckle pattern images were taken shown in figure 1, one being taken before loading (undeformed state) and another during loading (deformed state). The aim of the method is to find the displacements and strains of small subsets from the second image relative to the first one. This is accomplished by comparing the intensity levels (0-255) of the subsets in the images [4].

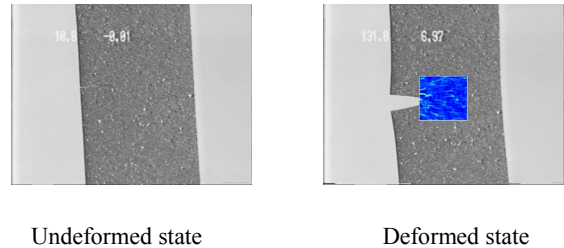


Figure 1. Image Correlation for the Area of Interest

Usually method for comparing two subsets is commonly given by the use of the cross-correlation coefficient,  $C$  [4]:

$$C\left(\xi(u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}), \eta(v, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y})\right) = \frac{\int_{\Delta M^*} f(x, y) * (x + \xi, y + \eta) dA}{\left[ \int_{\Delta M} [f(x, y)]^2 dA \int_{\Delta M^*} [f^*(x + \xi, y + \eta)]^2 dA \right]^{1/2}}$$

where  $\Delta M$  is the subset in the undeformed image,  $\Delta M^*$  is the subset in the deformed image,  $u$  and  $v$  are the displacement in  $X$  and  $Y$  directions.

$$\xi = u + \frac{\partial u}{\partial x} \Delta x + \frac{\partial u}{\partial y} \Delta y$$

$$\eta = v + \frac{\partial v}{\partial x} \Delta x + \frac{\partial v}{\partial y} \Delta y$$

The values of  $u$ ,  $v$ ,  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$  and  $\frac{\partial v}{\partial y}$  which maximize  $C$  are the local deformation gradients for the selected subset (Figure 2), the best correlation value is defined as -1.

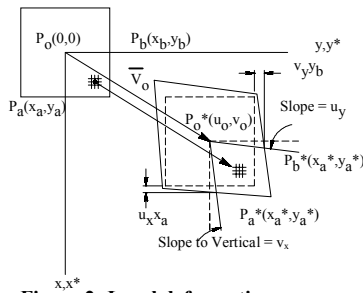


Figure 2: Local deformation

There are two alternative approaches to fracture analysis: the stress intensity and the energy criterion approach. In the elastic stress analysis for opening mode I lead to *stress intensity factor*  $K_I$ , which is employed to describe the elastic stress field surrounding the crack tip [5]. To illustrate elastic-plastic deformation the  $J$  contour integral, *an energy release rate*, has enjoyed great success as a fracture characterizing parameter for nonlinear materials [6]. Due to the fatigue behavior of short crack is often very different from that of longer cracks and the growth rate is relatively insensitive to the microstructure and tensile properties, the short-crack growth model will be built and developed based on the above analytical approaches.

## 2 THEORETICAL BACKGROUND

For a single edge notched tension (SENT) specimen in plane stress the stress intensity factor  $K_I$  can be calculated as [6]:

$$K_I = \sigma \sqrt{\pi a} f(a/W)$$

$$= \frac{P}{B} \sqrt{\pi a} f(a/W)$$

where  $\sigma$  is the extension stress,  $W$  is the specimen width,  $a$  is the crack length.  $f(a/W)$  is the correction factor,  $P$  is the extension load, and  $B$  is the specimen thickness.

A linear elastic finite analysis illustrates the finite width corrections for the SENT particulate composite specimen as shown in figure 3. The average Young's modulus ( $E_0$ ) and Poisson ratio ( $\nu$ ) of the material are 725 psi and 0.449. Thus, the correlation factor  $f(a/W)$  and  $a/W$  are related as:

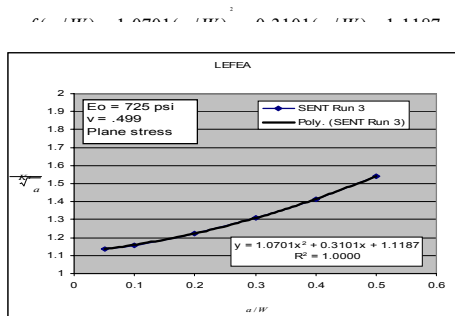


Figure 3. Finite Width Corrections for the SENT Particulate Composite Specimen in a Linear Elastic Finite Analysis

For SENT specimen in plane stress, the energy released rate  $J$  is the sum of the integral fully plastic  $J_{pl}$  and the elastic  $J_{el}$ . The general solutions to fully plastic  $J_{pl}$ :

$$J_{pl} = \alpha \epsilon_0 \sigma_0 \frac{ba}{W} h_1(a/W, n) \left( \frac{P}{P_0} \right)^{n+1}$$

$$P_0 = 1.072 \eta B b \sigma_0$$

$$\eta = \sqrt{1 + \left( \frac{a}{b} \right)^2} - \frac{a}{b}$$

and the elastic  $J_{el}$  is:

$$J_{el} = \frac{K_I^2(a_{eff})}{E'}$$

where  $\sigma_0$  is a reference stress value that is usually equal to the yield strength,  $\epsilon_0 = \sigma_0 / E$ ,  $\alpha$  is a dimensionless constant,  $n$  is the strain hardening exponent,  $h$  is a dimensionless function of geometry,  $P$  is the remote load,  $P_0$  is a reference load,  $a_{eff}$  is the effect of plastic zone correction factor,  $E' = E / (1 - \nu^2)$ .

## 3 EXPERIMENTAL SETUP

Two kinds of single edge notched tension (SENT) specimens were built. The initialed edge crack lengths are 2.5 mm (0.1 inches) – short crack and 7.6 mm (0.3 inches) – long crack. The width and the length of composite panel are 25 mm (1 inch) and 75mm (3 inches), the thickness is 5mm (0.2 inches). Two crack growth and propagation experiments have been performed under the opening mode I conditions with an applied tensile loading. Because the load was controlled on equal strain, so it kept increasing to a peak value and then decreased until the specimen failure. The whole process from the onset of the crack propagation to the failure of the specimen has been recorded on a video tape.

Some images are digitized from the video of the experiments as shown in figure 4. The particulate composite specimens have natural white-black speckle pattern. The digitized images were used to calculate the strain fields near the crack tip. The strain fields of the initialed short-crack specimen when the crack has propagated to 7.6 mm will be compared to that of the long-crack specimen at the onset of crack propagation.

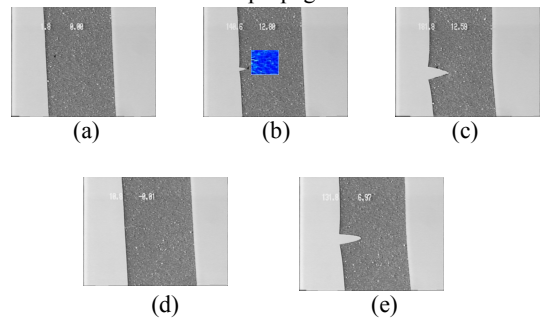


Figure 4. Digitized images of specimen with a 2.5 mm (a) at unloading state (b) at onset of crack propagation (c) when crack propagated to 7.6 mm, and digitized images of specimen 7.6 mm initial crack length (d) at unloading state (e) at onset of crack propagation

## 4 RESULTS AND DISCUSSION

The deformation measurements with image correlation technique are able to obtain the surface displacements and stains distribution

near the crack tip area. This information will be used to construct the J-integral at different steps of deformation during the crack propagation process.

The undeformed image shown on figure 4 – (a) is the referenced image which was extracted in unloading state. Figure 4 – (b) shows the deformed image at onset of crack propagation. The investigated area near the crack-tip is 12.7×12.7 mm (0.5×0.5 inches, 148×148 pixels). The resolution is 11.6535 pixels/mm (0.08581 mm/pixel). The correlation was performed for 21 by 21 pixels in 2 pixels intervals and the displacements and strains were obtained for every 2 pixels (0.17162 mm). The correlation results shown in figure 5 describe the displacements  $u$  – in X direction and  $v$  – in Y direction, correlation value  $r$ , gradients  $u_x$ ,  $v_y$  and normal strain fields in 2D dimensions.

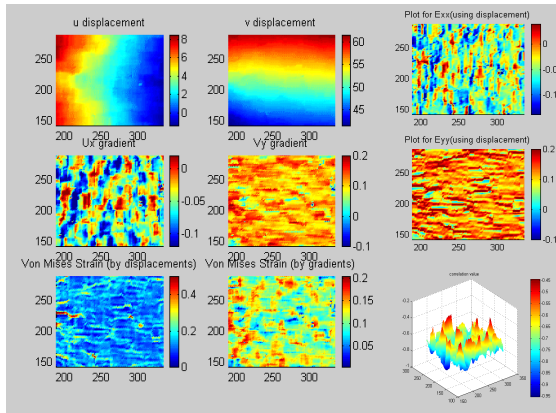


Figure 5. Finite Width Corrections for the SENT Particulate Composite Specimen in a Linear Elastic Finite Analysis

The displacement  $u$  is from 2 to 8 pixels, and  $v$  is from 40 to 60 pixels. Since the tensile load is applied in the Y direction, the normal strain  $\epsilon_{xx}$  is very small but  $\epsilon_{yy}$  is getting as high as 20%.

To obtain the crack growth and propagation data six crack growth measurements which include a series of tests for short-crack (0.1-inch) and for long-crack (0.3-inch) SENT specimen recorded on the video tape have been performed. The amplified projection assists to provide the higher resolution images and to get more measurement accuracy. The measurements represent the entire process from onset of the crack propagation to the failure of the specimen.

In each test point the loading condition, the crack length and spent time are recorded. This is useful in studying the crack growth rate and generating  $K_I$ - $t$  (the stress intensity factor vs. time) as well as the  $da/dt$ -  $K_I$  (crack growth rate vs.  $K_I$ ) curves. These curves created with the spreadsheet Excel file can be used to develop future short-crack growth in particulate composite materials.

To determine the crack growth rate, the relationship of crack length and time during the entire propagation period for short crack (0.1 inch) specimen was created as shown in figure 6. We notice that at around 28s after the onset of propagation the crack length tends to grow linearly. In this period the crack always has once sudden expending. It illustrates that this instant increasing could always occur at the moment close to the peak load. When the load

increases over 12 lb the crack is possible to start propagation at any time.

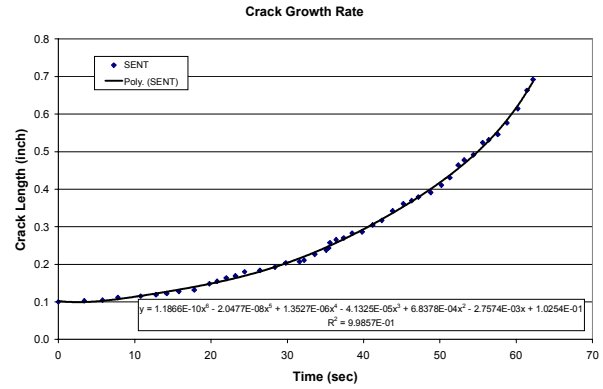


Figure 6. Relationship of Crack Length and Time during the Entire Propagation Period for 0.1 Inch SENT Specimen

Figure 7 shows the relationship of  $K_I$  and time. With time increasing,  $K_I$  linearly increases in the first 28s after the onset of crack propagation, and then keeps increasing nonlinearly to a peak value. The peak  $K_I$  and corresponding load are 91.29  $psi \sqrt{in}$  and 12.13 lb. The average critical stress intensity  $K_{IC}$  was calculated to be 41.05  $psi \sqrt{in}$ .

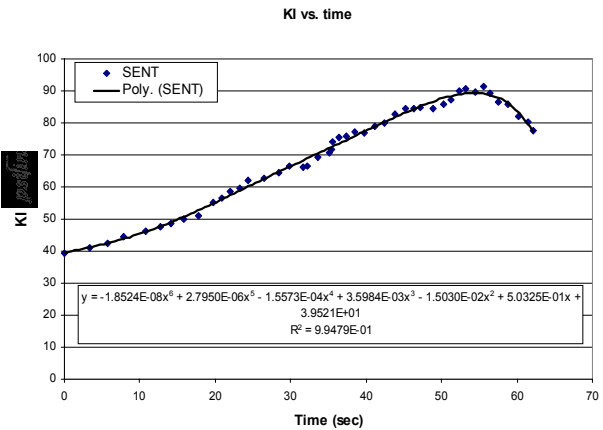


Figure 7. Relationship between  $K_I$  and Time during the Entire Propagation Period for 0.1 Inch SENT Specimen

The relation between crack growth rate ( $da/dt$ ) and  $K_I$  is also very important for short-crack model establishment. By using the 2<sup>nd</sup> order polynomial fit over the crack propagation data ( $a$ - $t$  curve), the crack propagation rate ( $da/dt$ ) for all test data is generated. The linear portions of the Log  $da/dt$  vs. Log  $K_I$  data for all 0.1 and 0.3 inch initial crack specimens are presented in Figure 8. This period can be regarded as stable state and can be expressed by the following relations:

For 0.1 inch crack:  $\text{Log}(da/dt) = 2.4326\text{Log}(K_I) - 6.5347$

$$\text{or } da/dt = 2.9194 \times 10^{-7} (K_I)^{2.4326}$$

For 0.3 inch crack:  $\text{Log}(da/dt) = 2.7891\text{Log}(K_I) - 7.3316$

$$\text{or } da/dt = 4.6602 \times 10^{-8} (K_I)^{2.7891}$$

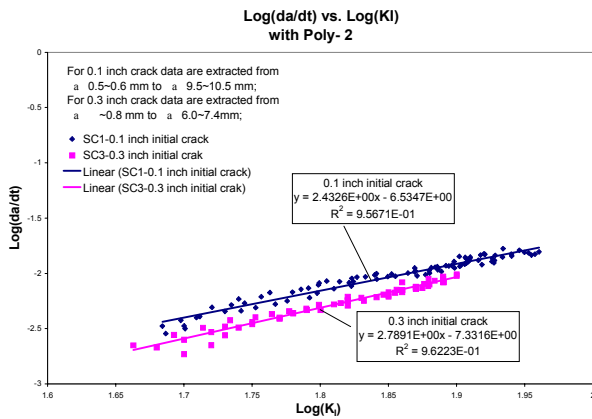


Figure 8. Relationship between Crack Growth Rate (2 Order Polynomial fit) and  $K_I$  during the Linear Crack Growth Period

## 5 CONCLUSION AND RECOMMENDATION

Through the analysis in short crack growth fields for the updated particulate composites the fracture parameters such as  $K_I$  were determined. Experiments were employed to investigate the whole failure process and the crack growth rate. The current results involve in finding out the relationship of  $da/dt$  and  $K_I$ .

The main observations to characterize the behavior of this kind of composite are based on shape, size, loading condition and material properties. It demonstrated that there is no obvious unstable period on the onset of crack propagation: the crack length relative smoothly propagates;  $K_I$  and time have more linearity. It also seems that crack growth rate ( $da/dt$ ) increases with  $K_I$  linearly. A few abrupt growth or cease may cause some data scattered.

Several recommendations are listed below upon the completion of the previous tasks. They are the main working focus and the important replenishments for building the short crack model.

- Based on regional strain fields near the crack tip, construct the J-integral at the expected moments during the crack propagation process, such as at the propagation started and ended as well as the points when the load or  $K_I$  reaches to the peak.
- The same analysis methods are used for 0.3-inch initialed crack specimen. The results from long and short crack growth patterns shall be compared.
- The calculated J-integral will be reversed to derive a new  $K_I$ . The new  $K_I$  will be compared with the present one to verify the accuracy of the analytical results.

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