Simple Calculations to Reduce Litigation Costs in Personal Injury Cases: Additional Empirical Support for the Offset Rule

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Abstract

This article demonstrates that if the nominal rate of interest equals the growth rate of nominal earnings, then a strong case can be made for calculating lump-sum damage awards by using the offset rule, i.e., by simply multiplying the annual loss by the number of years the loss is expected to continue. An examination of the Canadian data not only supports the offset rule, but also suggests that plaintiffs are being systematically undercompensated by rules currently in use.

SIMPLE CALCULATIONS TO REDUCE LITIGATION COSTS IN PERSONAL INJURY CASES: ADDITIONAL EMPIRICAL SUPPORT FOR THE OFFSET RULE©

BY R.A.L. CARTER AND JOHN P. PALMER[*]

This article demonstrates that if the nominal rate of interest equals the growth rate of nominal earnings, then a strong case can be made for calculating lump-sum damage awards by using the offset rule, *i.e.*, by simply multiplying the annual loss by the number of years the loss is expected to continue. An examination of the Canadian data not only supports the offset rule, but also suggests that plaintiffs are being systematically undercompensated by rules currently in use.

Cet article vérifie l'hypothèse selon laquelle on applique le principe de compensation pour calculer les dommages-intérêts à montant forfaiture lorsque le taux d'intérêt nominal est équivalent au taux de croissance des gains nominaux. La calculation se fait en multipliant la perte annuelle par le numéro d'années dont il est probable que la perte continuera. Outre à souligner le principe de compensation, l'analyse de l'information canadienne suggère que les règles actuellement en force ont l'effet systématique de priver des demandeurs d'une indemnité suffisante.

> The practice of awarding damages for personal injuries in lump sum form makes it
> inappropriate for the courts to simply determine the plaintiff's annual loss and multiply it
> by the number of years during which the loss will be suffered in order to determine the
> amount of the award.
>
> —Denise Reaume[1]

The purpose of this study is to demonstrate two things: first, that the assertion quoted above is wrong; and, second, that the current practice in most Canadian jurisdictions (which is unfortunately based upon the same flawed reasoning underlying the above quotation) is systematically biased against plaintiffs in personal injury cases. To this end, we posit a convincing *a priori* argument for ignoring interest and inflation rates when determining damage awards and then support this argument with Canadian data. Last, we argue that, even if individual cases differ substantially from the average, decisions which ignore this argument typically under-compensate plaintiffs.

I. INTRODUCTION

During the past twenty-five years, Canadians have experienced much wider fluctuations in interest and inflation rates than ever before. As a result, the courts have had few precedents by which to calculate damages for future losses. The courts knew that adequate allowance had to be made for future higher prices, but they did not know how much prices would rise. They also knew that interest rates were changing with expectations about inflation, but they were not sure what interest rate to use in calculating lump-sum awards.

Because of these confusions and uncertainties, lengthy testimony was heard about the appropriate numbers to use in damages

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[1] Ontario Law Reform Commission: Project for Personal Injury and Death, *Compensation for the Loss of Working Capacity* (Toronto, January 1987) [unpublished] at 100.

calculations. In some exceptional instances, the numbers finally accepted seem reasonable with the benefit of hindsight; in others, they seem egregiously unfair.

For over a century, damages awards have taken into consideration that a lump-sum award could be re-invested to earn future income.[2] Only comparatively recently, however, have the courts taken account of the effect of inflation on both the expected future income and on nominal (or stated) interest rates.[3] In these first few attempts to account for inflation, the courts appear to have committed some serious blunders.

In *Andrews* v. *Grand & Toy Alberta Ltd.*,[4] the Court heard testimony about the real rate of interest (defined as the stated or nominal rate of interest minus the expected rate of inflation) and settled on a rate of 7 per cent.[5] After some time, however, the courts began to hear increasing testimony from expert witnesses that the *Andrews* decision relied on an expected rate of inflation that was too low and, consequently, a forecasted real rate of interest that was much too high. As a result, the courts began accepting much lower real rates of interest by which to discount the awards.[6]

During the late 1970s and 1980s, people's expectations changed rapidly to the point of seeming capricious. Actual rates of inflation skyrocketed in comparison with previous North American experiences, and real economic growth rates were highly variable. The courts no longer had firm notions about what to expect for the future and, hence, were at a loss as to the appropriate rate by which to discount plaintiffs' damages awards. The result was that plaintiffs and defendants alike called on copious expert testimony about likely future interest and inflation rates.[7]

The expert testimony on expected interest and inflation rates appears to have had two major results: (1) many people came to accept

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[2] For an excellent summary of the cases and the historical development of the issues, see the unpublished research reports for the Ontario Law Reform Commission: Project for Personal Injury and Death (Toronto, January 1987). In particular, see D. Reaume, *supra* note 1; S.A. Rea, Jr., *An Economic Perspective*; and G. Bale, *Cost of Future Care, Periodic Payments, Structured Settlements, Discounting, Taxation, and Gross-up*.

[3] See *infra* note 4.

[4] [1978] 2 S.C.R. 229 [hereinafter *Andrews*].

[5] *Ibid.* at 259.

[6] See, for example, *Fenn* v. *City of Peterborough* (1979), 25 O.R. (2d) 399. See also Bale, *supra* note 2 at 103; and Reaume, *supra* note 1 at 104-05.

[7] See *Mandzuk* v. *Vieira* (1986), 2 B.C. L.R. (2d) 344 (C.A.), especially at 363.

that, in the long run, the real rate of interest would probably be near 2.5 per cent; and (2) in an attempt to reduce litigation costs, many provinces created rules establishing a real rate of interest of 2.5 per cent.[8]

Our results show that while 2.5 per cent may or may not be an appropriate estimate of the real rate of interest, *it is inappropriate to discount plaintiffs' awards by 2.5 per cent as a first step.*

The empirical evidence we present in the following sections is consistent with adopting what has come to be known as the offset rule.[9] According to the offset rule, one calculates an appropriate lump-sum award simply by multiplying the amount to be awarded for the first year by the number of years for which the loss is expected to continue (the rate of growth of nominal labour income is *offset* by the nominal interest rate). At present, no jurisdiction in Canada uses this rule. The key to our results is that we incorporate additional information about the likelihood that labour productivity will increase—information which is not incorporated into provincial rules that specify the real interest rate to be used for damage awards.

Virtually every writer on the subject of damages awards has recognized that an individual is likely to have increasing earnings over his or her lifetime. These increases generally come from wage inflation and from real increases in labour productivity; but because increases in labour productivity seem to have been perceived as uncertain or more personalized, these increases have frequently been understated, and they have certainly been ignored in the general rules that have been formulated.

Recognition of the importance of overall, economy-wide increases in labour productivity has been slow, but people have finally begun to recognize that even if labour productivity fails to grow in one particular occupation or industry, competition and mobility in labour

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8 Initially, the stated policy in British Columbia was that the courts should use the investment rate of interest minus the expected rate of inflation. Unfortunately, this type of rule provided little assistance to the courts: it did not specify which investment rate should be used, nor did it specify how the expected rate of inflation was to be determined. To solve these issues, the current rate is set at 2.5 per cent in that province. Ontario (Rules of Civil Procedure, r. 53.09) set the real rate at 2.5 per cent, as did Nova Scotia (Civil Procedure Rules, r. 31.10). At about the same time, courts in Australia began using a real discount rate of 3 per cent. See Bale, *supra* note 2 at 104-20.

9 See M.T. Brody, "Inflation, Productivity, and the Total Offset Method of Calculating Damages for Lost Future Earnings" (1982) 49 U. Chicago L. Rev. 1002. For a clear discussion of this rule, see R. Posner, *Economic Analysis of Law*, 4th ed. (Boston: Little, Brown, 1992) at 194-95.

markets tend, over several years, to force wages up throughout the economy to reflect the economy-wide labour productivity changes.[10]

In this study, we find that, in general, one cannot reject the hypothesis that the offset rule is appropriate, *i.e.*, that nominal earnings grow over time, on average, by a rate equal to the nominal rate of interest. Given these results, it is appropriate to adopt the offset rule as a starting point for damage awards.

Current standard practice is for the courts to personalize an award, including a possible allowance for increased labour productivity that is generally less than 2.5 per cent and then to discount the award by the prescribed rate of 2.5 per cent. While this procedure has the beneficial effect of reducing litigation costs considerably (*vis-à-vis* taking testimony about expected inflation and interest rates), it has the drawback that it does not go far enough. Our results imply that this procedure systematically under-compensates plaintiffs,[11] especially those who are expected to suffer a loss over a long period of time. The results also point to a possible additional savings in litigation costs.

In the next section, we present an example of how the offset rule would work. In the subsequent section (and in the Appendix), we review some of the criticisms of this *apparently* naïve rule. In succeeding sections, we present the results of empirical tests of the argument's underlying assumptions; and in the final section, we conclude that, although the rule may be inappropriate if the period of loss is expected to be short, the assumptions on which it is based are fundamentally sound and should provide better guidance for the courts than does the current practice.

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[10] See Reaume, *supra* note 1 at 50: "If these general trends are not taken into account it is likely that the injured plaintiff will end up with a lower standard of living than she would have been able to afford if she had gone on working." See also the Report to the Committee of the Supreme Court of Ontario on Fixing Capitalization Rates in Damage Actions (Toronto, 14 February 1980) at 8: "Other factors which could bear on the ultimate award would have to be dealt with by agreement of the parties or by the adducing of evidence thereon. They would include: ... (2) the probability of increased productivity of the income-producer in question ."

[11] There is no *a priori* reason that this procedure should systematically under-compensate plaintiffs. Instead, we base this assertion on the fact that so many cases rely on the testimony of actuaries and others who assume that increases in labour productivity will average 1 or 2 per cent per year. The result is that the award is discounted by somewhere between .5 per cent and 2 per cent, when, in general, the awards should not be discounted at all.

II. DISCOUNTING: A SIMPLE APPROACH

There are, among other things, two reasons why a plaintiff's annual losses will grow over time: productivity growth and inflation. For example, a $1,000 earnings differential this year would become approximately a $5,427 annual differential in twenty-five years if both earnings (*i.e.*, those that would have been earned before the incident that gave rise to the suit and those that will be earned after the incident) used to calculate this differential grew at the same rate of, for example, 7 per cent each year. How much of this 7 per cent growth is due to inflation and how much is due to productivity growth does not really matter for this example. We would compensate the plaintiff now for the $5,427 loss twenty-five years from now if we award $1,000 today, and if that amount could be invested at 7 per cent, it would grow to become precisely $5,427 after twenty-five years. After twenty-three years, the amount would be $4,741; after fifteen years, $2,759; after fourteen years, $2,579, *et cetera*. In other words, we would award the plaintiff $1,000 today for each year of loss, and the plaintiff could invest it at current interest rates, each year withdrawing an amount equal to what the loss would be in that year.

Regardless of what the amount is in each year, it is important to recognize that if the plaintiff has the opportunity to save at a nominal interest rate equal to the expected rate of growth in the nominal earnings differential, then a current award of $1,000 for each year of future expected loss is appropriate. Given the goal of compensating the plaintiff for lost earnings in each of those years, we should like to make an award today that will allow the plaintiff to receive precisely those amounts in each future year, and for a constant (current dollar) expected loss of $1,000 per year for twenty-five years this amount totals $25,000.[12]

III. REFINEMENTS OF THE BASIC MODEL

The procedure set out in part II has come to be known as "the total offset rule" because of the hypothesis that the various expected inflation, growth, and interest rates all cancel each other out, yielding a simple formula for the award: just multiplying the amount of the loss by the number of years. This formula was used by the Alaska courts in

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[12] The mathematics of the offset rule and its variations are presented in section A of the Appendix.

Beaulieu v. *Elliott.[13]* It was later modified by the rule in *State* v. *Guinn[14]*
which stated, correctly, that age-earnings profiles should also be taken
into consideration in cases involving lost personal earnings.
Refinements to the offset rule are presented in the Appendix, showing
that it is possible to take account of specific deviations from the offset
rule not only because of these life-cycle variations in productivity, but
also because of variations in interest rates over time, differences in life
expectancies, and differences in the likelihood of being employed. The
derivations presented there show that while it may be important to
personalize an award, it is at least as important that the courts begin with
the proper base for the award; and the data confirm that, for awards
involving more than a few years, the offset rule provides this base.

IV. THE ECONOMIC MODEL AND EMPIRICAL RESULTS

The basis of the total offset rule and the hypothesis of interest is
that for any month the ratio of the nominal rate of growth in labour
income, divided by the nominal interest rate, is just as often above one as
it is below one,[15] *i.e.*, it is hypothesized that the median of (the nominal
growth rate in labour income)/(the nominal interest rate) = 1.
Since we typically have no clear expectation about the future
rates of productivity growth in individual occupations or industries (*i.e.*,
the present and post-event occupations or industries), we take as a
starting point the assumption that future productivity growth in all
occupations will equal that for the economy as a whole. Some evidence
in support of this assumption is provided by Anderson and Roberts.[16]
Clear evidence to the contrary would, of course, induce us to alter this
assumption for individual cases. Similarly, we assume that the same

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[13] 434 P.2d 665 (Alaska 1967) [hereinafter *Beaulieu*].

[14] 555 P.2d 530 (Alaska 1976) [hereinafter *Guinn*].

[15] Because the hypotheses will be formed about the multiplicative relationship, it is convenient
for us to test the hypotheses in logarithmic form. Consequently, when we conduct tests concerning
a lognormal distribution, we are making direct inferences about the median of the original form of
the data. In appendices to our more technical papers on this subject ("Real Rates, Expected Rates,
and Damage Awards" University of Western Ontario, 1991 [unpublished]; and "Real Rates,
Expected Rates, and Damage Awards," (1991) 29 J. Legal Stud. 439, we show how inferences about
the median of the logarithmic form lead to inferences about the mean of the original form of the
data.

[16] G.A. Anderson & D.L. Roberts, "Stability in the Present Value Determination of Future
Lost Earnings: An Historical Perspective with Implications for Predictability" (1985) 39 U. Miami
L. Rev. 847.

expected future rates of inflation are relevant to plaintiffs in all occupations.

Of course, the presence of random components in both unobservable expectations and observed data means that schemes such as the offset rule, the Ontario rule (that the real discount rate be 2.5 per cent), and their variants can be expected to hold as base periods only on average over all possible examples, and all possible time spans. *Thus, such schemes can be expected, at best, to yield the correct award, only on average.*

For example, nominal growth rates of earnings and nominal interest rates depend on the date at which the loss occurs. If we imagine various time periods over history with different starting dates, we would find that, for some, the ratio of these two rates would exceed one, while for others, it would be less than one.[17]

Our hypothesis, that the median of the ratio of nominal labour income growth to nominal interest rates equals one, is equivalent to the hypothesis that the mean of the logarithm of the ratio equals zero, which is the hypothesis we tested.[18] For this study, we used average weekly labour earnings, which are reported monthly in *Employment Earnings and Hours*,[19] to calculate the growth rate of labour income. To measure the nominal rate of interest, we used monthly observations on federal government ten-year bonds. We then adjusted the interest-rate data to calculate monthly holding period rates of return on those long-term securities.[20]

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[17] This point is the gist of the criticism of the offset rule by C.J. Lacroix & H.L. Miller, Jr., "Lost Earnings Calculations and Tort Law: Reflections on the *Pfeifer* Decision" (1986) 8 U. Hawaii L. Rev. 31. Lacroix and Miller maintain, at 52, that "this relationship [*i.e.*, the hypothesis that we are testing] has not been empirically verified and may hold, if it does, only over very long periods of time." For a thorough discussion of the problems associated with uncritical applications of the offset rule, see M. Vellrath, "Discounting, Growth, and Interest Rates" (Paper presented at the conference, Damages: Strategies and Calculations, of the Continuing Legal Education Satellite Network series (Ernst and Young), 1989) [unpublished].

[18] Details of some earlier test results are available in Carter & Palmer, *supra* note 15. The tests rely on the additional assumption that expectations are formed rationally, *i.e.*, without any systematic error. See J.F. Muth, "Rational Expectations and the Theory of Price Movements" (1961) 29 Econometrica 315.

[19] (Ottawa: Statistics Canada, Labour Division, December 1947 - November 1985).

[20] See R.J. Shiller, "The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure" (1979) 87 J. Pol. Econ. 1190 [hereinafter "Volatility"]; and section C of the Appendix.

We were unable to reject the hypothesis that the mean of the log of the ratio is equal to zero.[21]   Graphs of the various calculations presented in Figures 1 and 2 show both the short-run variability of the ratio and its lack of a long-term trend. The only difference between the two graphs is that the graph in Figure 2 uses seasonally adjusted data to remove some of the seasonal spikes in the data which occurred regularly each year.

There are also two horizontal lines plotted along the graphs in Figures 1 and 2. The line of short dashes extending from a value of zero shows the assumptions of the offset rules. The second line, of long dashes, shows the effect of using the Ontario rule of discounting 2.5 per cent, but allowing for no growth in labour productivity. Notice that this rule is at great variance with the data. If the current practice of allowing for some growth in labour productivity and using a net discount rate of only 1.5 per cent were used, the long dashed line would be much closer to the observed data, but would still, on average, lie below them.

It is also noticeable that the ratio has much greater variability during periods of greater uncertainty about inflation rates. These results confirm that the assumptions underlying the offset rule are plausible when it is to be used for losses which will continue for a long period of time.   It appears that the assumptions may also hold for short-term losses, though with much less certainty.[22]

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[21] If anything, the data suggest that the growth of nominal labour earnings in Canada has been greater than that of the nominal rate of interest. In this case, if the provincial rules specifying a discount rate of 2.5 per cent are adhered to, then the courts should, on average, be allowing prospective increases in labour earnings of *even more* that 2.5 per cent per year! Using the time period from the end of 1947 until the end of 1985, we had 456 observations of monthly growth in labour income and monthly holding-period rates of return on ten-year government bonds. Unfortunately, the data series on labour income ends in 1986, so we were unable to extend our analysis to more recent years. The average of the difference of the logarithms over these 456 observations is .00139, suggesting that, if anything, labour incomes grew at a rate slightly higher than the nominal interest rate. Taking account of the stationary autocorrelated nature of the series, the standard error is .00108, and the t-statistic is 1.418. These numbers say that we cannot, with any confidence, reject the hypothesis that the two rates are the same. In other words, these statistical tests support what the graphs in Figures 1 and 2 tell us; namely, that the evidence does not contradict the implications of the offset rule. The series plotted in Figures 1 and 2 are d, from equation 22 in the Appendix.

[22] If we subdivide the sample into two time periods, this result becomes clearer. Because the data appear to have been stable from 1947 until 1979, we applied our tests first to that time period and then to the remainder of the data. From the end of 1947 though to the end of 1979, the mean of the differences of the logarithms was .00242, with a standard error, correcting for stationary autocorrection, of .000746 and a resulting t-statistic of 3.24. These results suggest that over that time period, if anything, even the offset rule would have under-compensated plaintiffs because average labour incomes grew at a slightly higher rate than the rate of interest. But for the 72 observations from the end of 1979 through the end of 1985, the mean was -.00410, with a standard

V. COMPARISON OF AWARD RULES

It is apparent from the data in Figures 1 and 2 that the offset rule does a better job of matching awards with actual losses than either a 2.5 per cent rule or even a 1.5 per cent rule. The superiority of the offset rule becomes even clearer in Figures 3 to 6, which show how the different compensation rules would compare with a prescient court.

If a prescient court had been able to make awards precisely based on the actual growth in nominal labour income and the actual nominal interest rate, then the typical plaintiff would, on average, have received exactly 100 per cent of their losses, no more and no less, as indicated by the lines at 100 per cent in each of the figures. The remaining lines plotted in Figures 3 to 6 show the percentage of the *ex post* correct award that plaintiffs would have received had courts not had this perfect foresight, but instead used one of the alternative rules. The problem we address arises because courts do not have perfect foresight and must, therefore, devise criteria and rules for calculating damage awards.

In Figure 3, we consider a series of twenty-year awards, beginning in 1948. For each month, we calculated and plotted the percentage of the perfect-foresight awards that a plaintiff would have received had the offset rule been used to calculate the award for a twenty-year loss. This percentage is shown as a line lying just below the perfect-foresight line at 100 per cent. While the offset rule line is very close to the 100 per cent line, it remains below it, indicating that for twenty-year awards, the offset rule would consistently have under-compensated plaintiffs.

Two other lines in Figure 3 show the percentage of the perfect-foresight awards that plaintiffs would have received under the 1.5 per cent rule and under the 2.5 per cent rule. For twenty-year losses, the 2.5 per cent rule would have under-compensated plaintiffs so severely in each year that the compounded effect would have been to award them generally less than 80 per cent of what they would have received from a prescient court! Even the 1.5 per cent rule would have awarded them only approximately 91 per cent of the perfect-foresight award. In comparison, the offset rule would have under-compensated them by no more than 1 per cent.

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error, correcting for stationary autocorrelation, of .00534 and a corresponding t-statistic of -.769. So once again, using data for this short, turbulent time, we are unable to reject the basic assumptions of the offset rule. Even though these results inspire less confidence in applying the offset rule for the short-term awards than for long-term awards, we wish to emphasize that the offset rule is, on average, still likely to be a better starting point than other rules, even in these instances.

Figures 4 through 6 show the same comparison for fifteen-, ten-, and five-year awards. For shorter time periods, the percentage of undercompensation from the 2.5 per cent rule and the 1.5 per cent rule is less than that shown in Figure 3 because the undercompensation is compounded over fewer years. Nevertheless, in all four Figures, it is clear that the offset rule comes closer to the perfect-foresight award than either alternative rule.

VI. CONCLUSION

Upon careful examination of the available evidence, we cannot find any significant difference between the nominal rate of interest and the growth rate of nominal labour income over spans of up to 38 years in Canada. These results are consistent with the results of our earlier study.[23]

Suppose that, for some reason, it would be easy to establish that a plaintiff's loss in a law suit would be $1,000 this year, and that this loss was going to recur for the next twenty-five years. Then our results present compelling evidence that it is appropriate to award the plaintiff $25,000 as a judgment. Forget about discounting future amounts according to a given interest rate; forget about correcting for the expected rate of inflation; and forget about allowing for overall productivity growth in the economy. The simple solution is just as likely to be correct as any other more complex solution. It is sure to reduce the costs of litigation, and it is clearly a better starting place than that of current practice (which uses net discount rates of between 1 and 2.5 per cent), even if the facts of an individual case deviate from overall averages.

The importance of this result is that it confirms earlier suggestions that there is, on average, no need to try to estimate nominal or real interest rates, inflation rates, or the aggregate growth of labour productivity in cases involving a loss of earning capacity over time: these terms all cancel out in the equation for calculating the present value of the stream of losses (with the proviso that tax effects are taken into account). This result should contribute toward dramatically reducing legal costs to both plaintiffs and defendants who, until now, have been devoting scarce resources to expert testimony on the correct values of these variables.

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[23] See R.A.L. Carter & J.D. Palmer, "Real Rates, Expected Rates, and Damage Awards" (1991) 29 J. Legal Stud. 439.

Even in cases for which the total offset rule is not acceptable, the result will be of some benefit to plaintiffs if, as is the practice in many jurisdictions, the courts implicity rely on assumptions that the nominal rate of interest is larger than the expected growth rate of nominal labour income. The evidence we present here quite clearly shows that, on average, the growth rate of labour income has, if anything, exceeded the nominal interest rate.

Because of the comparative instability of the data from the late 1970s onward, a reasonable question to be asked is whether we expect the assumptions of the offset rule to apply as well in the future as they have in the past. We are not soothsayers. We cannot predict whether changing underlying circumstances will affect these results. We can, however, say with a high degree of confidence that if the underlying conditions do not change, or if they revert to being more like those for the period between 1947 and 1979, then using the offset rule in Canada will, on average, correctly compensate plaintiffs and will reduce litigation costs for both parties. The margin for error may be larger for shorter time periods, but, on average, the offset rule provides a better starting point than the rules currently in use.

**Figure 1: Raw Data on Difference of Logs of:
Rate of Growth of Labour Income Minus Nominal Interest Rate**
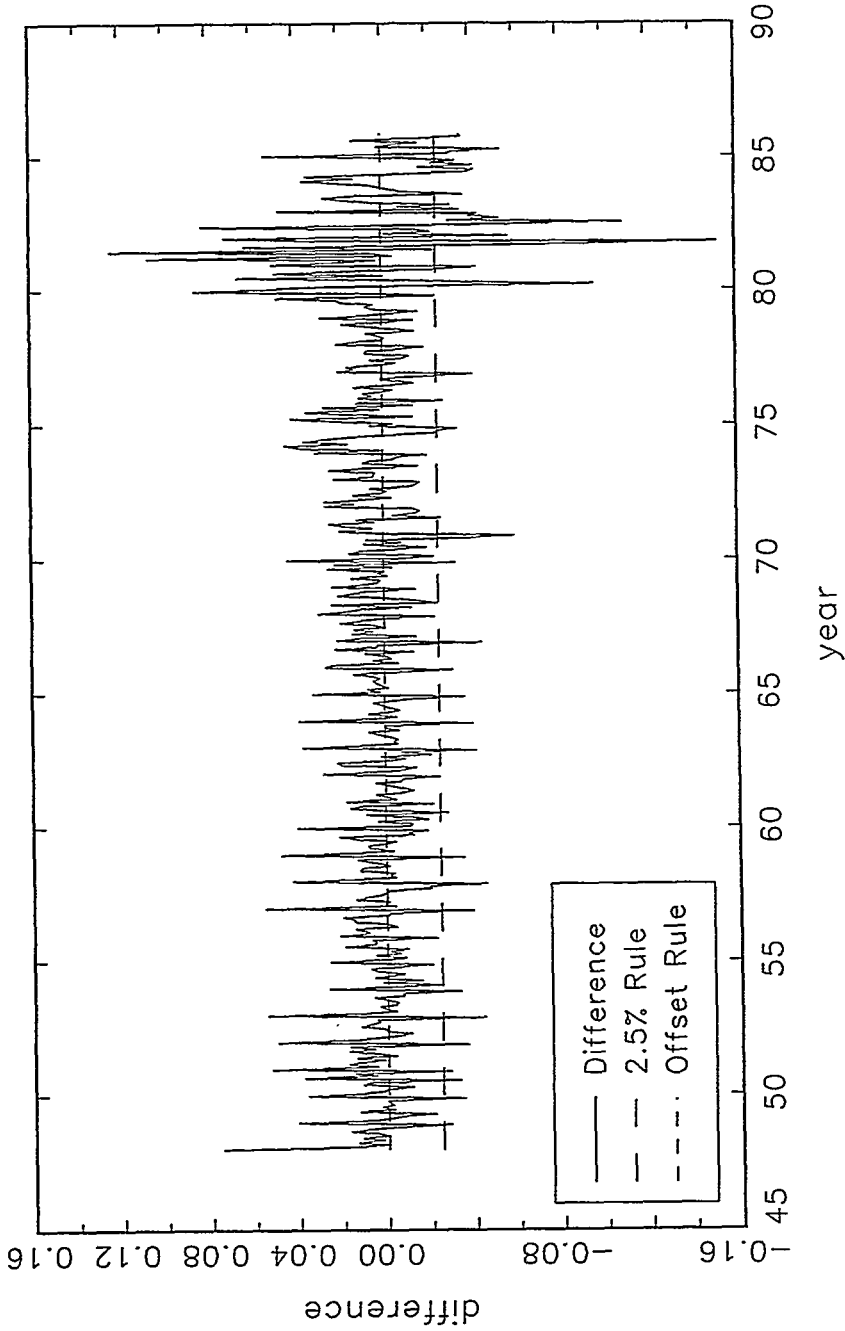
Figure 2: Seasonally Adjusted Data on Difference of Logs of:
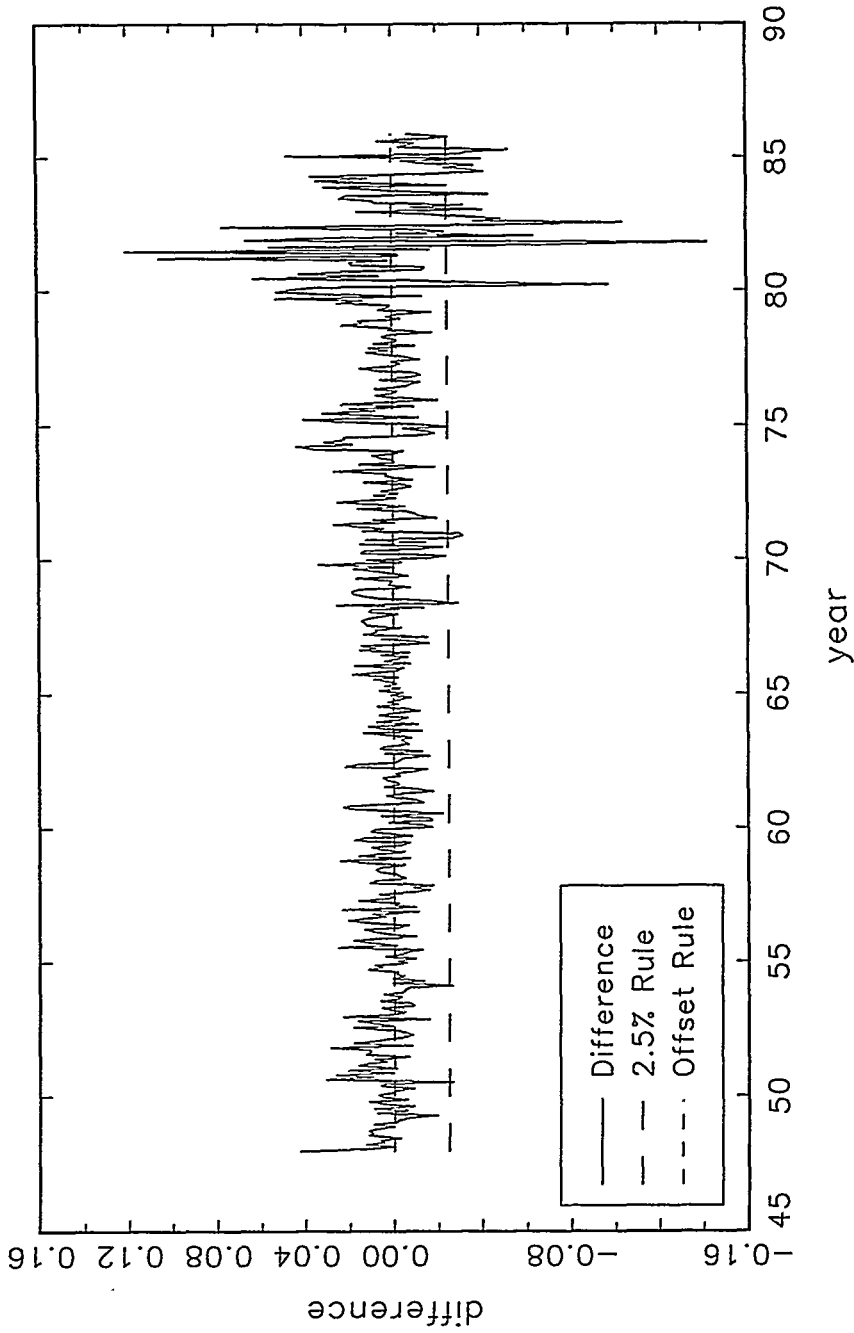Rate of Growth of Labour Income Minus Nominal Interest Rate
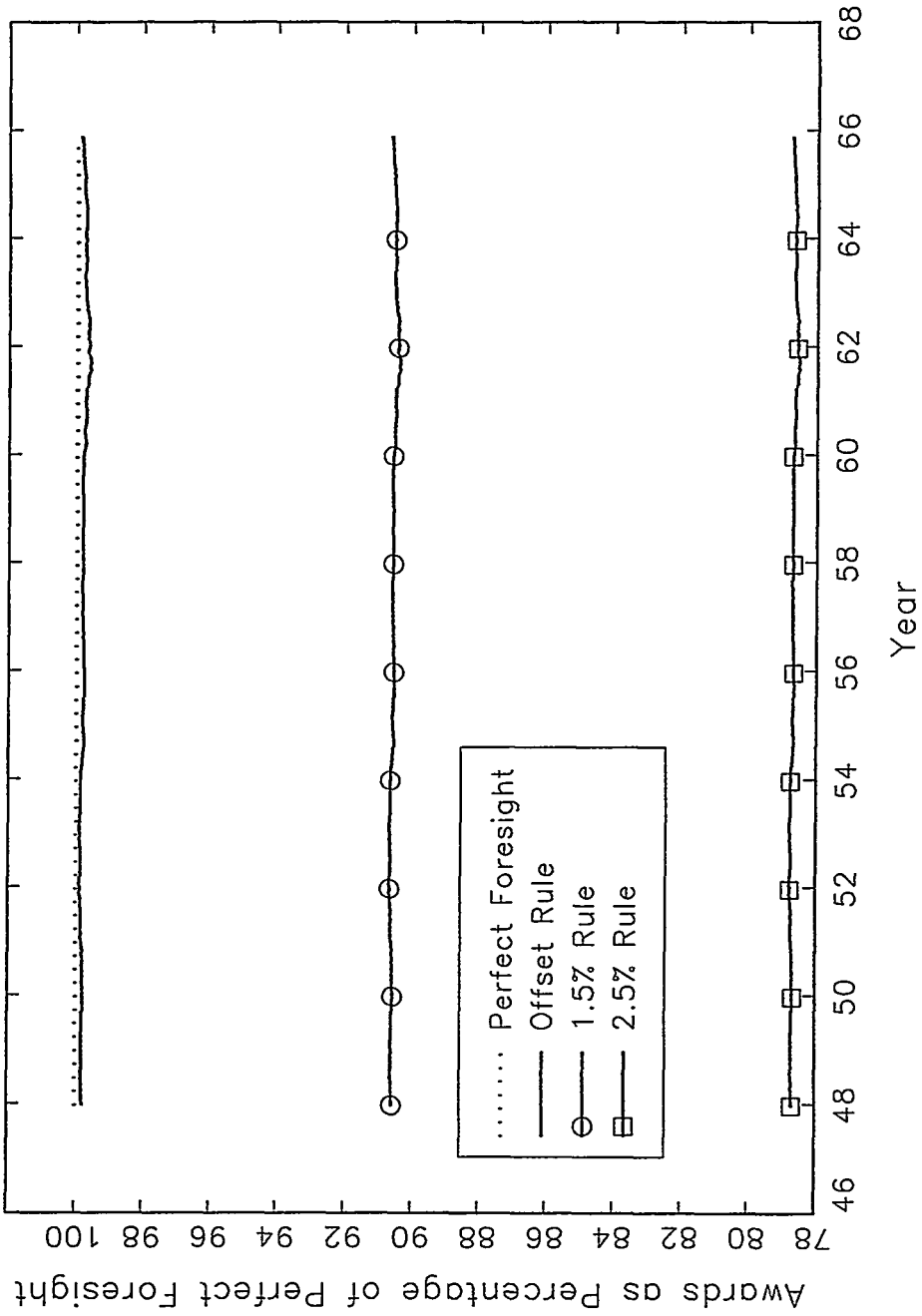
Figure 3: Twenty Year Awards Over Time
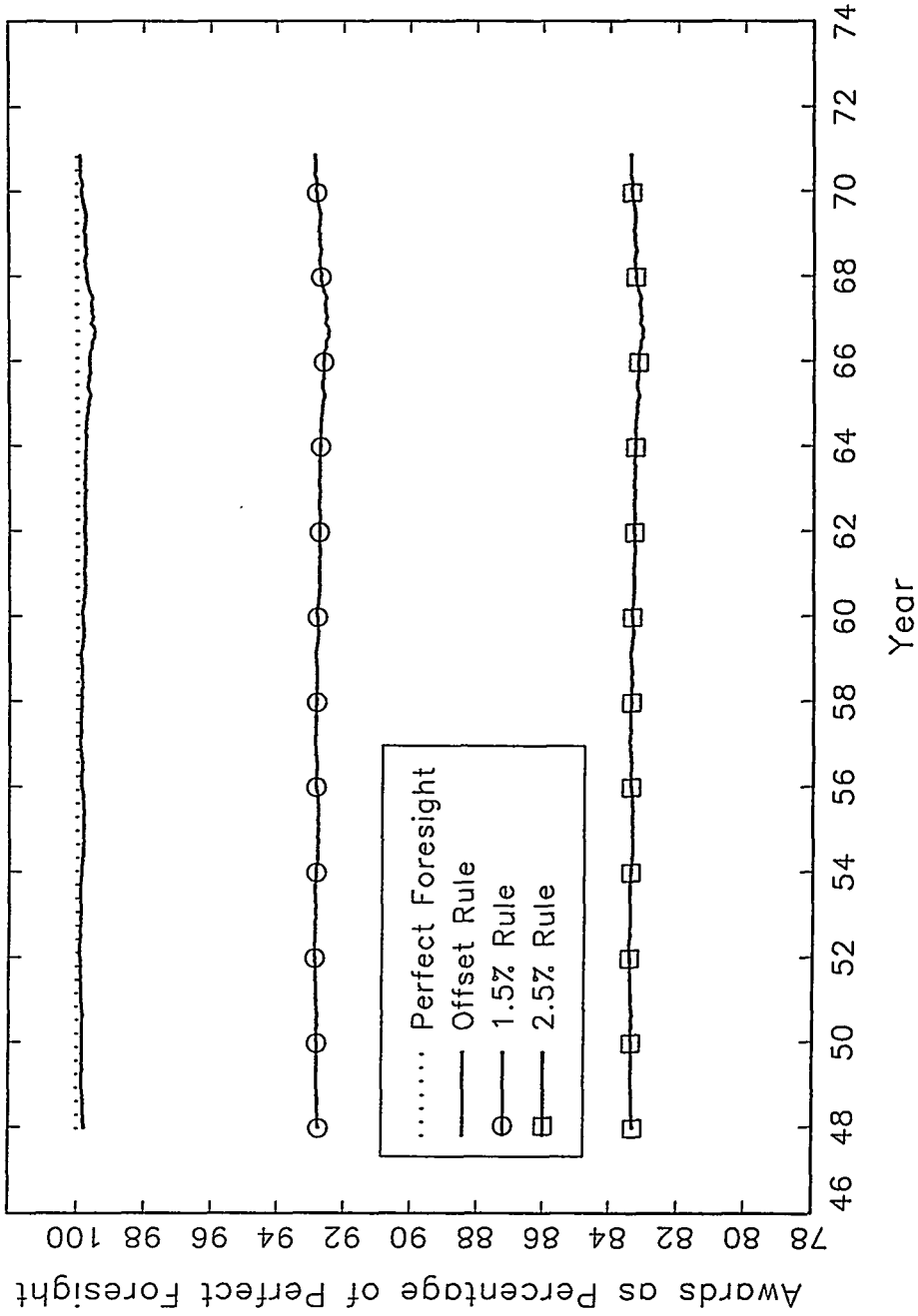
Figure 4: Fifteen Year Awards Over Time

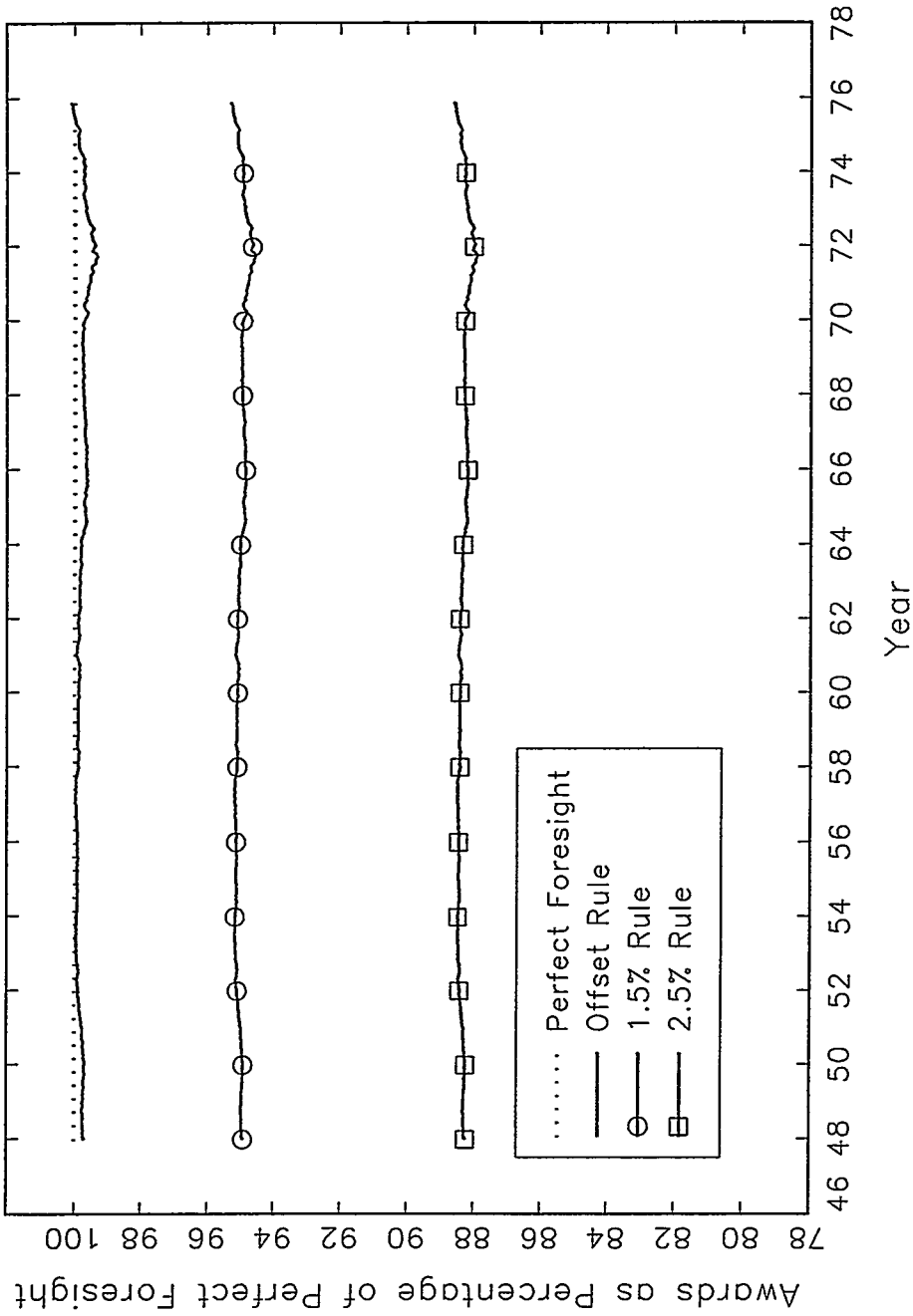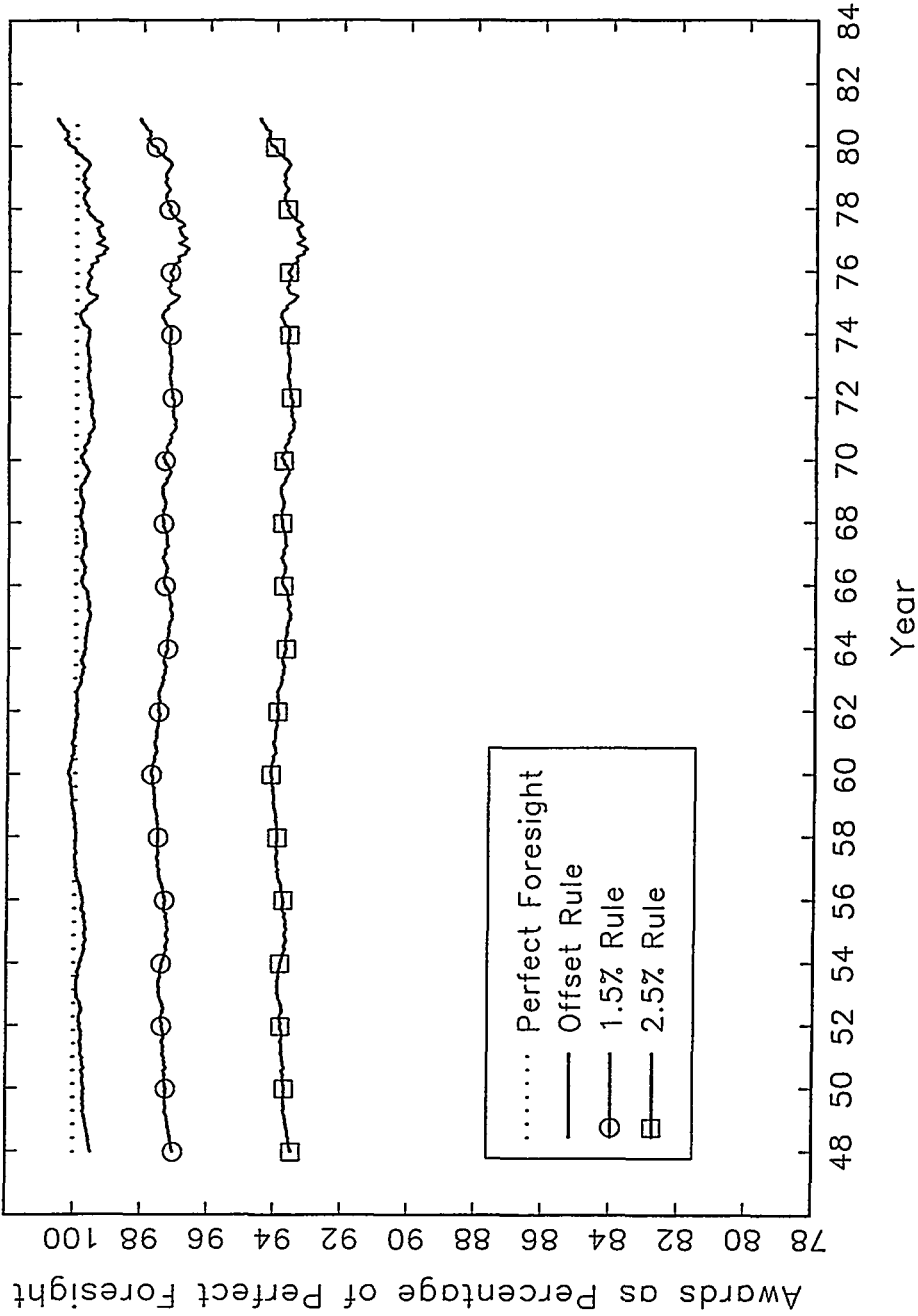Figure 5: Ten Year Awards Over Time

Figure 6: Five Year Awards Over Time

APPENDIX

This appendix discusses some of the technical aspects of the methods used to obtain the results presented in the body of the paper. Some of these points are presented more fully in our earlier papers.[24]

A. *Sufficient Conditions For the Applicability of the Offset Rule*

Assume that in period 0 the plaintiff suffered a loss of $\$D_0$ which is expected to persist for N years. The attractiveness of the offset rule is the simplicity of its formula for setting the size of the award, A_N, the plaintiff should receive as compensation. In this section, we set out the sufficient conditions for the accuracy of this simple formula, based on increasingly realistic assumptions about rates of interest and inflation, life cycle effects, and life expectancies. The body of the paper considers whether Canadian data provide convincing evidence to contradict these conditions.

We begin by assuming that the plaintiff is certain to survive for N years, that rates of interest, growth, and inflation are constant, and that there are no life cycle effects. We want to find the present discounted value of D_0 for a year T years in the future, where $T \le N$. Let g be the real rate of interest, p be the expected rate of labour productivity growth, i be the rate of price inflation, and w be the rate of wage inflation that the plaintiff is expected to experience for each of T years in the future. Then the sum we seek is composed of annual amounts given by

$$(1) \quad PV_T = D_0 \frac{(1 + w)^T (1 + p)^T}{(1 + i)^T (1 + g)^T} \quad .$$

In this simple case, the total award, A_N, which will exactly compensate the plaintiff for his or her loss is the sum of the values given by (1) over the N years for which the loss will persist.

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[24] *Supra* note 15.

$$(2) \quad A_N = \sum_{T=1}^{N} PV_T$$

$$= D_0 \sum_{T=1}^{N} \frac{(1+w)^T(1+p)^T}{(1+i)^T(1+g)^T} \quad .$$

A sufficient condition for this to reduce to

$$(3) \quad A_N = N \times D_0$$

as given by the offset rule, is

$$(4) \quad (1+w)^T (1+p)^T = (1+i)^T(1+g)^T \quad .$$

Of course, it is not necessary to have $w = i$ and $p = g$.

We introduce more realism by allowing $w, p, i,$ and g to vary over the t years between year 0 and year T, which gives

$$(5) \quad PV_T = D_0 \frac{(1+w_1)(1+p_1)(1+w_2)(1+p_2) \dots (1+w_T)(1+p_T)}{(1+i_1)(1+g_1)(1+i_2)(1+g_2) \dots (1+i_T)(1+g_T)}$$

$$= D_0 \prod_{t=1}^{T} \frac{(1+w_t)(1+p_t)}{(1+i_t)(1+g_t)} \quad .$$

Now the award that will correctly compensate the plaintiff for his or her loss is

$$(6) \quad A_N = \sum_{T=1}^{N} PV_T$$

$$= D_0 \sum_{T=1}^{N} \prod_{t=1}^{T} \frac{(1+w_t)(1+p_t)}{(1+i_t)(1+g_t)} \quad .$$

A sufficient condition for this to reduce to (3) is

$$(7) \quad \prod_{t=1}^{T} \frac{(1+w_t)(1+p_t)}{(1+i_t)(1+g_t)} = 1; \ T=1, \dots, N \ .$$

Note that this is weaker than the previous condition (4) in that it allows the w_t, p_t, i_t, and g_t to all be different from one another. A stronger condition than (7) is

$$(8) \quad (1+w_1)(1+p_t)=(1+i_t)(1+g_t); \ t=1, \dots, T$$

which is sufficient for (7) and, hence, for (3).

We now introduce life cycle effects, $l(t)$, which we represent as functions of time which multiply the labour productivity term to give

$$(9) \quad PV_T = D_0 \prod_{t=1}^{T} \frac{(1+w_t)(1+p_t)l(t)}{(1+i_t)(1+g_t)} \ .$$

Now, if (8) holds, we have

$$(10) \quad A_N = \sum_{T=1}^{N} PV_T = D_0 \sum_{T=1}^{N} \prod_{t=1}^{T} l(t)$$

which would reduce to (3) if $\prod_{t=1}^{T} l(t)=1$, which seems credible. Alternatively, if

$$(11) \quad \prod_{t=1}^{T} \frac{(1+w_t)(1+p_t)l(t)}{(1+i_t)(1+g_t)} = 1; \ T=1, \dots, N$$

then (3) would hold. Note that (11) allows the w_t, p_t, $l(t)$, i_t, and g_t to all be different from one another.

Next we let the probability that the plaintiff will be alive in year t be $x(t)$. Now,

$$(12) \quad PV_T = D_0 \prod_{t=1}^{T} \frac{(1+w_t)(1+p_t)l(t)x(t)}{(1+i_t)(1+g_t)}$$

and

$$(13) \quad A_N = D_0 \sum_{T=1}^{N} \prod_{t=1}^{T} \frac{(1+w_t)(1+p_t)l(t)x(t)}{(1+i_t)(1+g_t)} \quad .$$

If (8) holds, (13) reduces to

$$(14) \quad A_N = D_0 \sum_{T=1}^{N} \prod_{t=1}^{T} l(t)x(t)$$

which becomes (3) if $\prod_{t=1}^{T} l(t)x(t) = 1$. Alternatively, a sufficient condition for (13) to reduce to (3) is

$$(15) \quad \prod_{t=1}^{T} \frac{(1+w_t)(1+p_t)l(t)x(t)}{(1+i_t)(1+g_t)} = 1 \quad .$$

Note that (15) allows the w_t, p_t, $l(t)$, $x(t)$, i_t, and g_t to all be different from one another.

B. *The Economic Motivation For Differences of Logs*

If we take the log of both sides of equation (5), we obtain

$$(16) \quad \log(PV_T) =$$

$$\log(D_0) + \sum_{t=1}^{T} [\log(1+w_t) + \log(1+p_t) - \log(1+g_t) - \log(1+i_t)] \quad .$$

The variables w_t, p_t, i_t, and g_t, which appear in (16), and the equations of the previous section are all *ex ante* rates, *i.e.*, they are expectations about the future values of rates which are not directly observable. Their *ex post* realized values will be denoted by epw_t, epp_t, epi_t, and epg_t, respectively. Let epr_t be the *ex post* nominal rate of interest. Then the Fisher[25] equation

$$(17) \quad \log(1 + r_t) = \log(1 + i_t) + \log(1 + g_t)$$

can be extended to relate epr_t to epi_t and epg_t by

$$(18) \quad \log(1 + epr_t) = \log(1 + epi_t) + \log(1 + epg_t) \ .$$

Let r_t be the *ex ante* nominal rate of interest, which is related to the *ex post* rate by

$$(19) \quad \log(1 + epr_t) = [\log(1 + epr_t) - \log(1 + r_t)] + \log(1 + i_t) + \log(1 + g_t)$$

$$= u_t + \log(1 + i_t) + \log(1 + g_t)$$

where $u_t = \log(1 + epr_t) - \log(1 + r_t)$ is a forecast error which shows the extent to which expectations, formed at time 0, about $\log(1 + r_t)$, turn out to be wrong. If expectation like i_t and g_t are formed rationally, then forecast errors like u_t are random variables with zero means which are uncorrelated with anything in the set of information used in making the forecasts.

An equation analogous to (18) is assumed to link the *ex post* nominal rate of growth of labour income, epf_t, to epw_t and epp_t

$$(20) \quad \log(1 + epf_t) = \log(1 + epw_t) + \log(1 + epp_t) \ .$$

Let f_t be the *ex ante* nominal rate of labour income growth. Then an equation analogous to (19) relates it to the *ex post* rate.

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[25] See I. Fisher, "Appreciation and Interest" (1896) 2 *Publications of the American Economic Association* (3d) 341.

(21) $\log(1+epf_t)=[\log(1+epf_t)-\log(1+f_t)]+\log(1+w_t)+\log(1+p_t)$

$\qquad = v_t+\log(1+w_t)+\log(1+p_t)$.

The forecast error v_t has the same properties as u_t if expectations about rates of growth of nominal income are formed rationally.

Using equations (19) and (21) we form the difference

(22) $\quad d_t=\log(1+epf_t)-\log(1+epr_t)$

$\qquad =[\log(1+w_t)+\log(1+p_t)-\log(1+i_t)-\log(1+g_t)]+v_t-u_t$.

If equation (8) holds, equation (22) reduces to $d_t = v_t - u_t$. Since both epf_t and epr_t are *ex post* observable quantities, we are able to examine data to see whether d_t grows over time or behaves like a series with a constant mean.

C. *Holding Period Rates of Return*

The interest rate used in all our calculations is the holding period rate of return on government bonds with maturities of ten years or more. This section explains how these rates were calculated.

Consider a bond which will mature in m years and which carries coupons payable each year. If this bond is held for only one year—a linear approximation to the one year—the *ex post* holding period rate of return at year t is given by Shiller[26] and Shiller *et al.*[27] as:

(23) $\quad eph_{t,m}=epb_{t,m}+(dur_m-1)(epb_{t,m}-epb_{t+1,m-1})$

where $epb_{t,m}$ is the *ex post* nominal rate of interest at time t on a bond of maturity m years; $epb_{t+1,m-1}$ is the *ex post* rate at time $t + 1$ on a bond of maturity $m - 1$; and dur_m is the duration of a bond of maturity m given by $dur_m = (1-h^m)/(1-h)$ and $h = 1/(1+rbar)$ with $rbar$ the rate at which the coupon payment is discounted over one year. Of course, dur_m will be

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[26] See "Volatility," *supra* note 20; and "Alternative Tests of Rational Expectations Models" (1981) 16 J. Econometrics 71.

[27] R.J. Shiller, J.Y. Campbell & K.L. Schoenholtz, "Forward Rates and Future Policy: Interpreting the Term Structure of Interest Rates" (1983) 1 Brookings Pap. Econ. Act. 173.

less than m for any bond carrying coupons. Equation (23) says that $eph_{t,m}$ is the *ex post* rate of interest plus the capital gain obtained by selling the bond after holding it for one year. Since the capital gain could be negative for some period, we could have negative values for $eph_{t,m}$.

Our first step in computing $eph_{t,m}$ was to specify $epb_{t,m}$ as the rate of interest on government bonds with maturities of ten years or longer.[28] Our second step was, following Shiller *et al.*,[29] to set *rbar* equal to the sample average for $epb_{t,m}$. As we did not have data on the rate of interest paid on bonds with maturities of nine years, we approximated $epb_{t+1,m-1}$ with $epb_{t+1,m}$. Equation (23) was then evaluated to obtain 456 values of the (approximate) monthly holding period rate of return from December 1947 to November 1985. The series eph_t was then substituted for epr_t in equation (22).

D. *Testing the Stationarity of* d_t

Perhaps the most important question to ask about the series d_t defined in (22) is whether it grows over time, *i.e.*, whether it is non-stationary or stationary. Box and Jenkins[30] advocated inspecting the sample autocorrelations to decide this question. Dickey, Hasza, and Fuller[31] recommended a more formal procedure for detecting stochastic trends caused by unit roots in the autoregressive polynomial of the series. We used a new procedure by Kwaitkowski, Phillips, Schmidt, and Shin[32] which has two advantages over the earlier methods. First, it takes stationarity as the null hypothesis rather than as the alternative. Second, it is robust to the form of the Auto-Regressive Moving Average (ARMA) process driving d_t under the null, *i.e.*, one need not assume the length of the regular and seasonal autoregressive structures. However, it does

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[28] This series is published monthly (thus, $m = 120$ in equation (23)) in the Bank of Canada Review (Ottawa: Bank of Canada, December 1947 - November 1985) and is available from the CANSIM database with the data bank number B14013.

[29] *Supra* note 27.

[30] G.E.P. Box & G.M. Jenkins, *Time Series Analysis: Forecasting and Control* (San Francisco: Holden-Day, 1976) at c. 2.

[31] D.A. Dickey, D.P. Hasza & W.A. Fuller, "Testing for Unit Roots in Seasonal Times Series" (1984) 79 J. Am. Stat. A. 355.

[32] D. Kwaitkowski *et al.*, "Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root: How Sure Are We That Economic Times Series Have a Unit Root?" (1992) 54 J. Econometrics 159.

require the specification of a lag length for calculation of the long-run variance.

We applied the test to the deviations of d_t about their sample average. The asymptotic right tail 10 per cent critical value is .347 and the 5 per cent value is .463. We computed values of the test statistic for several values of the lag length using both the values of d_t computed from the raw data and the values computed from the seasonally adjusted data. The results are shown in the table below.

TABLE 1

Values of Test Statistics

Lag Length	Raw Data	Seasonally Adjusted
4	.291	.266
12	.258	.217
20	.238	.205
24	.238	.203
36	.228	.195

The lag of 4 was the same as that used to calculate the standard error of the mean in the next section. The lags of 12, 24, and 36 were chosen with the seasonal nature of the data in mind. The lag of 20 is slightly less than the square root of the sample size. None of the values of the test statistic are large enough to cause us to reject the null hypothesis of stationarity, i.e., we conclude that the d_t series is free from both stochastic and deterministic trends. This conclusion justifies the calculations of the next section.

E. *The Standard Error of the Mean*

One of our aims in this paper is to estimate the mean, over time, of d_t. A natural point estimator is the average of the sample points which are plotted in Figure 1. Of course, a point estimate must be accompanied by a measure of precision, the standard error. If the sample observations were independent, the standard error would be simply the sample standard deviation divided by the square root of the

number of observations. However, our sample is a time series whose observations are clearly not independent.

One way to proceed would have been to build a seasonal ARMA model of the series using the techniques of Box and Jenkins[33] and use its parameter estimates to obtain both an estimate of the mean and its standard error. There are two disadvantages to this procedure. First, a single ARMA model could not be expected to fit the whole of the samples because of the obvious change in structure at the end of 1979. Second, the estimate of the standard error of the mean derived from the estimated parameters of an ARMA model would be sensitive to the structure of the model used. The choice of this structure, often referred to as the "identification" of the model, is notorious for being the most difficult aspect of time series analysis and for failing to provide unique answers.

For these reasons, we chose to use an estimate of the standard error which requires only that the process generating the data be stationary but which uses no information about the structure of the ARMA process that generated the data. Although the series d_t appears to be stationary, it contains a large seasonal component, so we first de-seasonalized it by the standard ratio-to-moving-average method. This left the sample average virtually unchanged. The standard error of the samples mean was the square root of[34]

$$(24) \quad (se(\bar{d}))^2 = \sum_{i=0}^{m} \hat{\gamma}(i)[1 - \frac{(i-1)}{(m+1)}]$$

where $\hat{\gamma}(i)$ is the estimated autocovariance at lag i and m is the integer part of $T^{.24}$, which is 4 for $322 \leq T \leq 526$.

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[33] See *supra* note 30 at c. 9.

[34] See P.C.B. Phillips, "Times Series Regressions with a Unit Root" (1987) 55 Econometrica 277 at 286.