Numerical Methods for Variable Order Differential Equations

Harihar Khanal

Department of Mathematics, Embry-Riddle Aeronautical University
1 Aerospace Blvd., Daytona Beach, FL 32114
harihar.khanal@erau.edu

Abstract

We introduce an initial value problem of the variable order differential equation

$$\frac{d^{\alpha(t)}y}{dt^{\alpha(t)}} = f\left(t, y\right), \quad m - 1 < \alpha(t) < m, \ m \in \mathbb{Z}^{+}; \quad y(0) = y_0,$$

where the order of differentiation $\alpha(t)$, a smooth function of the independent variable, is defined as the generalization of fractional derivatives and review some physical situations of interest in which such equations can model a complex phenomenon. Such variable order differential equations can be approached by using the formalism for Volterra integral equations of second kind with singular integrable kernel. We present numerical methods and solutions for particular cases, and discuss the asymptotic approach of the solutions towards the limiting classical integer order differential equation.

Keywords: Variable order differential equation; dynamical order derivative; fractional differential equation; Voltera equation; singular integrable kernel; collcation method