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CHARACTERIZATION OF WATER MOVEMENT INTO AND THROUGH  
SOILS DURING AND IMMEDIATELY AFTER RAINSTORMS

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Principal Investigator

Project Number A-025-KY (Completion Report)  
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University of Kentucky Water Resources Institute  
Lexington, Kentucky

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December, 1972

## PREFACE

This report is the technical completion report for a research project entitled "Characterization of Water Movement Into and Through Soils During and Immediately After Rainstorms". The project was supported in part by funds provided by the United States Department of Interior to the University of Kentucky Water Resources Institute as authorized by the Water Resources Act of 1964, Public Law 88-379, as office of Water Resources Research Project No. A-025-KY. Partial funding was also provided by the Kentucky Agricultural Experiment Station as a contribution to Southern Regional Research Project S-53 "Factors Affecting Water Yields from Small Watersheds and Shallow Ground Aquifers".

## ABSTRACT

The movement of water into and through soils in the unsaturated state is basic to many water resources problems including rainfall-runoff models, ground water recharge, irrigation, drainage, evapotranspiration and the movement of pollutants in soils. This study was conducted in an effort to determine if the flow equation based on Darcy's Law and the continuity equation could be used to describe watershed infiltration and thus be incorporated into hydrologic models.

The results of the study indicate that even on apparently uniform soils there is a great deal of variability in soil water properties. Handling this variability plus the difficulty of solving the flow equation led to the conclusion that a simpler approach to modelling watershed infiltration is needed.

A simple infiltration model was developed and included in a rainfall-runoff model. Tests with the model indicate that it produces satisfactory estimates of monthly runoff from small, rural watersheds.

Keywords: infiltration, diffusion, soil water, hydrology, water yield.

## ACKNOWLEDGMENTS

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## Chapter I

### INTRODUCTION

In recent years, there has been a tremendous increase in research related to modeling the hydrologic cycle. Many of these models are synthesized from components that represent the various phases of the hydrologic cycle such as rainfall, overland flow, channel flow, infiltration, evapotranspiration, and water movement in the soil (Huggins and Monke, 1968; Crawford and Linsley, 1966; Haan and Johnson, 1968a; Dawdy and O'Donnel, 1965; Wooding, 1965; Amorocho and Orlob, 1961). Most of the models are constructed so that as knowledge is gained about a particular model component, this knowledge can be incorporated into the model.

It goes without saying that hydrologic models are no better than their weakest component. All of the present hydrologic models employ empirical relationships for predicting the infiltration component of the hydrologic cycle and many of them ignore the redistribution of moisture in the soil between rainstorms. In many areas of the U.S., 60 to 75 percent of the annual rainfall is infiltrated into the soil. From these figures, it is apparent that unless a hydrologic model adequately describes the infiltration and redistribution of moisture in the soil, the model is not an accurate representation of what is occurring in nature. Empirical equations such as the ones proposed by Horton (1939) and Holtan (1961) provide estimates for infiltration but do not describe the redistribution of the moisture and thus cannot be used to estimate the antecedent moisture content at the beginning of subsequent storms. The antecedent moisture content is one of the factors that governs the initial infiltration rates during a storm.

There have been several "theoretical" approaches at describing moisture movement through soils. The theoretical or general flow equation has become generally accepted as describing moisture movement in soils. Thus far, the validity of the diffusivity equation has been established for some laboratory soils but complete evaluations of the equation under field conditions have not been made.

The objectives of this proposed research are:

- a. To evaluate the validity of the diffusivity equation for describing infiltration and moisture movement under field conditions.
- b. To use the knowledge gained from objective (a) to predict infiltration on a watershed scale.

Chapter II  
PRINCIPLES OF UNSATURATED WATER FLOW

The Diffusivity Equation

In a soil mass, there exists a water potential field defined by

$$\phi = \psi + Z \quad (1)$$

where  $\phi$  is the total potential,  $Z$  is the gravitational potential, and  $\psi$  is the pressure potential. The movement of water through soils is in response to a potential gradient and follows Darcy's law

$$\begin{aligned} V_x &= -K_x(x, y, z, \theta) \partial\phi/\partial x \\ V_y &= -K_y(x, y, z, \theta) \partial\phi/\partial y \\ V_z &= -K_z(x, y, z, \theta) \partial\phi/\partial z \end{aligned} \quad (2)$$

where  $V$  is the velocity;  $K$  is the hydraulic conductivity;  $x$ ,  $y$ , and  $z$  are the coordinate directions and  $\theta$  is the volumetric water content. The positive  $z$  direction is taken as upward.

The continuity equation for water flow in soil can be written

$$-\left(\frac{\partial \rho V_x}{\partial x} + \frac{\partial \rho V_y}{\partial y} + \frac{\partial \rho V_z}{\partial z}\right) = \frac{\partial \rho \theta}{\partial t} \quad (3)$$

Substituting equations (2) into (3), we have

$$\begin{aligned} \frac{\partial}{\partial x}[\rho K_x(x, y, z, \theta) \frac{\partial \phi}{\partial x}] + \frac{\partial}{\partial y}[\rho K_y(x, y, z, \theta) \frac{\partial \phi}{\partial y}] + \\ \frac{\partial}{\partial z}[\rho K_z(x, y, z, \theta) \frac{\partial \phi}{\partial z}] = \frac{\partial \rho \theta}{\partial t} \end{aligned} \quad (4)$$

In most infiltration studies, the assumption is made that all water movement is in the vertical direction or that both  $V_x$  and  $V_y$  are zero. Since water is relatively incompressible, equation (4) becomes

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} K_z(x, y, z, \theta) \frac{\partial \phi}{\partial z} \quad (5)$$

A further assumption that is commonly made is that  $K_z$  is not a function of  $x$ ,  $y$  or  $z$  and varies only with  $\theta$ . Therefore, equation (5) becomes

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} K(\theta) \frac{\partial \phi}{\partial z} \quad (6)$$

Since  $\phi = \psi + z$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} K(\theta) \frac{\partial (\psi + z)}{\partial z}$$

or

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} K(\theta) \frac{\partial \psi}{\partial z} + \frac{\partial K(\theta)}{\partial z} \quad (7)$$

Defining the diffusivity,  $D(\theta)$  as

$$D(\theta) = K(\theta) \partial \psi / \partial \theta \quad (8)$$

and equation (7) can be written

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} D(\theta) \frac{\partial \theta}{\partial z} + \frac{\partial K(\theta)}{\partial z} \quad (9)$$

and equation (2) in the  $z$  direction becomes

$$V_z = -[D(\theta) \frac{\partial \theta}{\partial z} + K(\theta)] \quad (10)$$

Equation (9) is known as the diffusivity equation for the movement of water in unsaturated soil.

### Soil-Water Properties

Both the hydraulic conductivity,  $K$ , and the diffusivity,  $D$ , are indicated in equation (1) to be functions of the soil-water content,  $\theta$ . In normal situations,  $K$  may vary by 6 or 7 orders of magnitude and  $D$  by 3 or 4 orders of magnitude as the soil-water content ranges from very dry to saturation. Thus, a small change in the soil-water content produces a large change in  $D$  and an even larger change in  $K$ . Except possibly for the very dry range, the diffusivity and conductivity increase with increasing soil-water content until saturation is reached.

Studies by many investigators have shown that there is a definite relationship between the soil-water content and the pressure potential. For unsaturated soils, the pressure potential is less than zero and is sometimes referred to as a tension. Typically, the drier a soil is, the lower will be the pressure potential or the higher will be the soil water tension. Unfortunately, the relationship between the pressure potential and the water content is not unique but exhibits a characteristic known as hysteresis (figure 1). Thus, a soil that is drying may have a higher water content than a soil that is wetting at the same pressure potential.

The most common way of measuring the relationship between soil-water content and pressure potential is with a pressure plate extractor as described by Black (1965).

Several methods have been advanced for measuring the relationship between the diffusivity and the water content. One common method is known as the one-step method of Doering (1965) based on the following equation developed by Gardner (1962)

$$D(\theta) = \frac{4L^2/\pi^2}{\theta - \theta_f} \frac{d\theta}{dt} \quad (11)$$

where  $L$  is the length of the soil sample and  $\theta_f$  is the final moisture content in equilibrium with the pressure applied to the sample. In using this procedure, a soil sample is placed

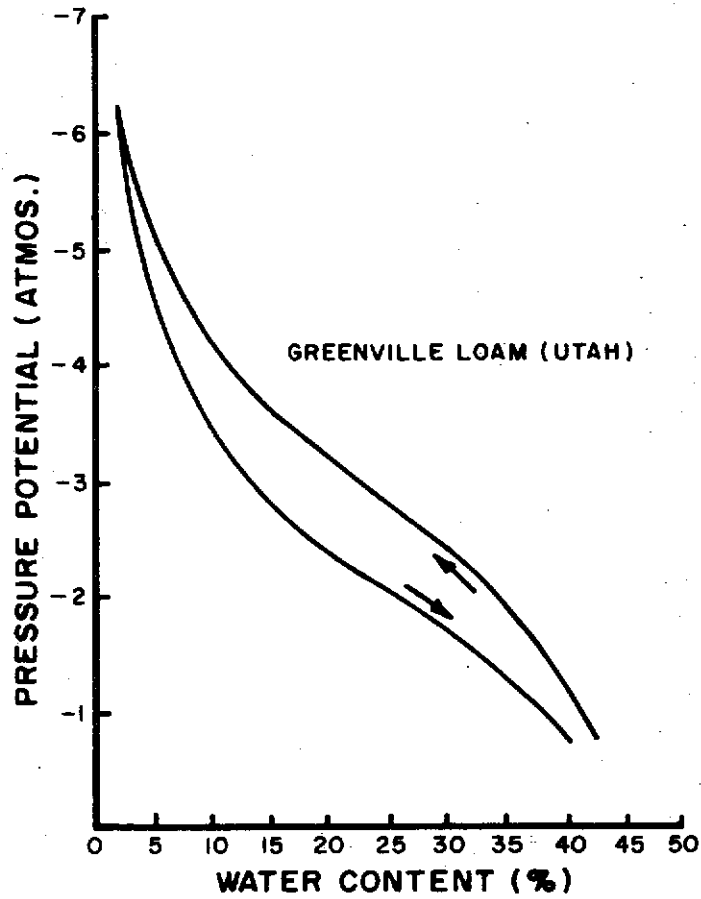


Figure 1. A typical relationship between pressure potential and water content (Schwab et al., 1966).

in a cell to which a constant pressure is applied. This pressure forces water out of the originally wet sample. The outflow of water is collected from which both  $\theta$  and  $d\theta/dt$  can be determined. The hydraulic conductivity can then be determined from

$$K(\theta) = D(\theta) \frac{d\theta}{d\psi} \quad (12)$$

where  $d\theta/d\psi$  is the slope of the curve relating water content to pressure potential.

A major problem in trying to use equations 9 and 10 for estimating watershed infiltration is the non-homogeneity that characterizes field soils (Nielsen, et al., 1967). Some studies have shown that for soils that are apparently uniform, the relationship between  $K$  and  $\theta$  still shows considerable variability. For instance, Melvin et al. (1969) reported "Values of conductivity at a given moisture content varied by a factor of five between replicates from the same depth in the same soil".

The affect of this type of variability on estimating infiltration rates can be seen from equation 10. As the infiltration process goes on during a rainstorm, the upper part of the soil profile approaches a constant water content. A constant water content means that  $\partial\theta/\partial z$  is zero and that  $V_z$  is equal to  $-K(\theta)$ . Thus, the downward rate of movement is equal to  $K(\theta)$ . A change in  $K(\theta)$  by a factor of 5 would mean a change in  $V_z$  by a factor of 5 or a change in the infiltration rate by this amount. This would indicate that even for a "uniform" soil, a single measurement of the relationships between  $K$ ,  $\theta$ , and  $\psi$  would not provide a reliable estimate of infiltration. Many measurements would have to be taken and somehow combined to give a single "representative" set of relationships.

The first part of this study was devoted to investigating the variability in the relationships between  $\theta$ ,  $\psi$ ,  $K$  and  $D$  on

an apparently uniform soil and to study several ways of combining measurements on these properties from several samples to get a single, representative set of relationships.



Chapter III  
STUDIES ON A SMALL WATERSHED

The objective of this part of the study was to illustrate the variability present in the  $\theta$ - $\psi$ -K relationship when sampling from a "uniform" soil and to discuss several methods of combining  $\theta$ - $\psi$ -K data from samples to get average relationships representative of a field plot or small watershed. When using average relationships, one should realize that the single  $\theta$ - $\psi$ -K relationship derived from several samples does not represent the flow situation at any particular point but is an average flow situation over many points.

Experimental Procedures

Four sampling locations were selected on a 4 1/2 acre experimental watershed located near Lexington, Kentucky. The soil on the watershed has all been mapped as Maury Silt Loam (Sims et al., 1968). Visually, the watershed is very uniform in its properties. The slope is about 7 percent and the cover is bluegrass sod. At each location, 6 samples were taken, all within one foot of each other. The samples were taken by peeling back the sod and using a core sampler to obtain an "undisturbed" sample 3 cm long by 5.35 cm in diameter. This procedure resulted in the samples being taken beginning at a depth of about 2 cm. The samples were carefully trimmed and placed in plastic bags for transporting to the laboratory.

The relationship between the water content and pressure head for each sample were determined by standard techniques using a pressure plate extractor. The water content-diffusivity relationship was determined by the one-step method of Doering (1965) explained earlier. The hydraulic conductivity was then determined from equation (12).

There is no completely satisfactory method of measuring the  $\theta$ - $\psi$ -K or  $\theta$ - $\psi$ -D relationships on undisturbed soil samples. It was felt that the method used is as appropriate as the other methods presently available. Nielsen et al. (1967) predicted within experimental error the amount of water drained from a Panoche clay loam profile after irrigation, using diffusivity values calculated by the one-step method of Doering (1965) and using a value of  $d\psi/d\theta$  averaged over the depth L of the soil.

#### Averaging Methods

The information available as a result of the tests on the pressure plate extractor and the outflow method is a curve relating  $\theta$  to  $\psi$  and a curve relating D to  $\theta$  for each soil sample. To get an average curve relating D to  $\theta$ , values of D from the sample curves at a constant  $\theta$  were averaged. A similar procedure was used to get an average relationship between  $\theta$  and  $\psi$ . Figure (2) shows schematically this averaging procedure for  $\theta = \theta_c$ . The curves labelled with an "a" are the average curves. Hysteresis was not considered in this work.

In evaluating K from equation (12), the values of D and of  $d\theta/d\psi$  were taken at the same value of  $\psi$ . In other words,  $\psi$  was taken as the controlling variable. The information is presented in this way since many problems in unsaturated flow have the boundary conditions specified in terms of the pressure head. The problem under consideration in this report is determining an average hydraulic conductivity through the use of equation (12) from information such as shown schematically in figure (2).

In the discussion that follows, the subscript i refers to the sample number and an overbar indicates a value taken from an average curve.

In this work, three methods of obtaining an average hydraulic conductivity were investigated. The three methods were:

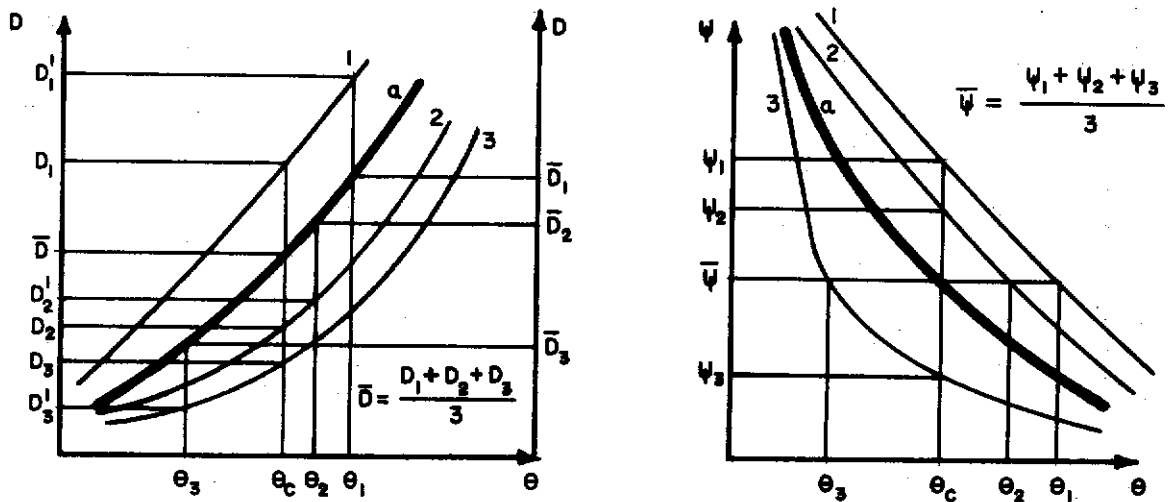


Figure 2. Definition of symbols. The curve labelled "a" is the average curve. The numbered curves represent different samples.

- (a) Using the  $\theta$ - $\psi$  and  $\theta$ -D relationship for each individual sample.

$$K_a = \frac{1}{n} \sum_{i=1}^n D_i \left. \frac{d\theta}{d\psi} \right|_i \quad (13)$$

- (b) Using the  $\theta$ -D relationship for individual samples and an average  $\theta$ - $\psi$  relationship.

$$K_b = \frac{1}{n} \sum_{i=1}^n D_i \overline{\frac{d\theta}{d\psi}} \quad (14)$$

- (c) Using the  $\theta$ - $\psi$  relationship for individual samples and an average  $\theta$ -D relationship.

$$K_c = \frac{1}{n} \sum_{i=1}^n \overline{D}_i \left. \frac{d\theta}{d\psi} \right|_i \quad (15)$$

### Results

A total of 22 samples were analyzed -- 4 from location 1 and 6 from locations 2, 3 and 4. As expected, a great deal of variability was found to exist in the  $\theta$ - $\psi$ -D and  $\theta$ - $\psi$ -K relationships among samples at a given location and among locations. Figures 3, 4 and 5 show the variability in the  $\theta$ - $\psi$ ,  $\theta$ -D and  $\theta$ -K relationships, respectively, at location 2. The variability at the other three locations was similar to that at location 2. Although the variation in these relationships from sample to sample is large, it is in agreement with that shown by other investigators (Melvin, et al., 1969).

The average  $\theta$ - $\psi$  and  $\theta$ -D curves are shown in figures 6 and 7. Again, the importance of a large number of sampling locations is evident from these figures.

Results such as this indicate that to describe the  $\theta$ - $\psi$ -K or  $\theta$ - $\psi$ -D relationship for a field location will require a large

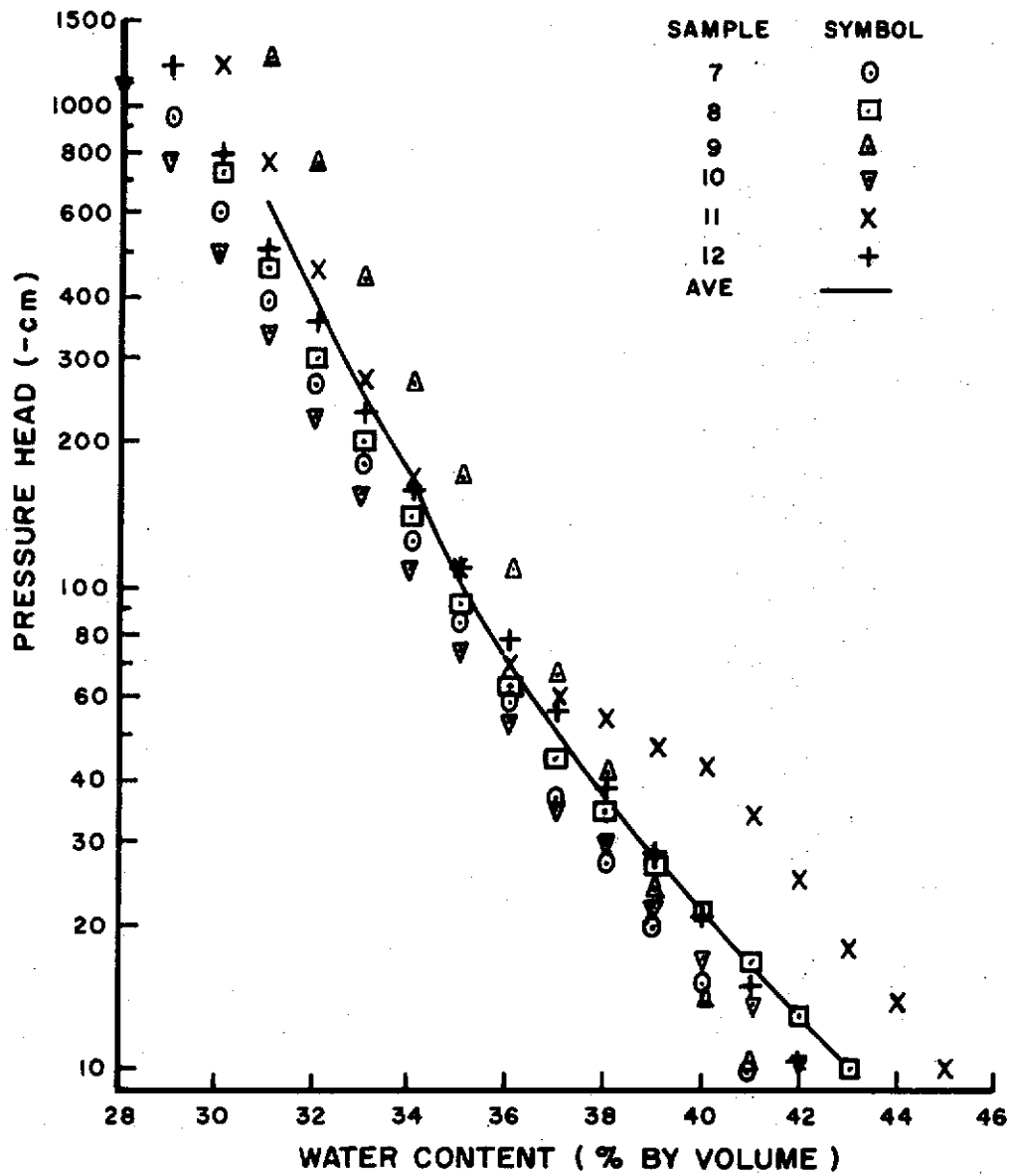


Figure 3.  $\theta$ - $\psi$  relationship for location 2.

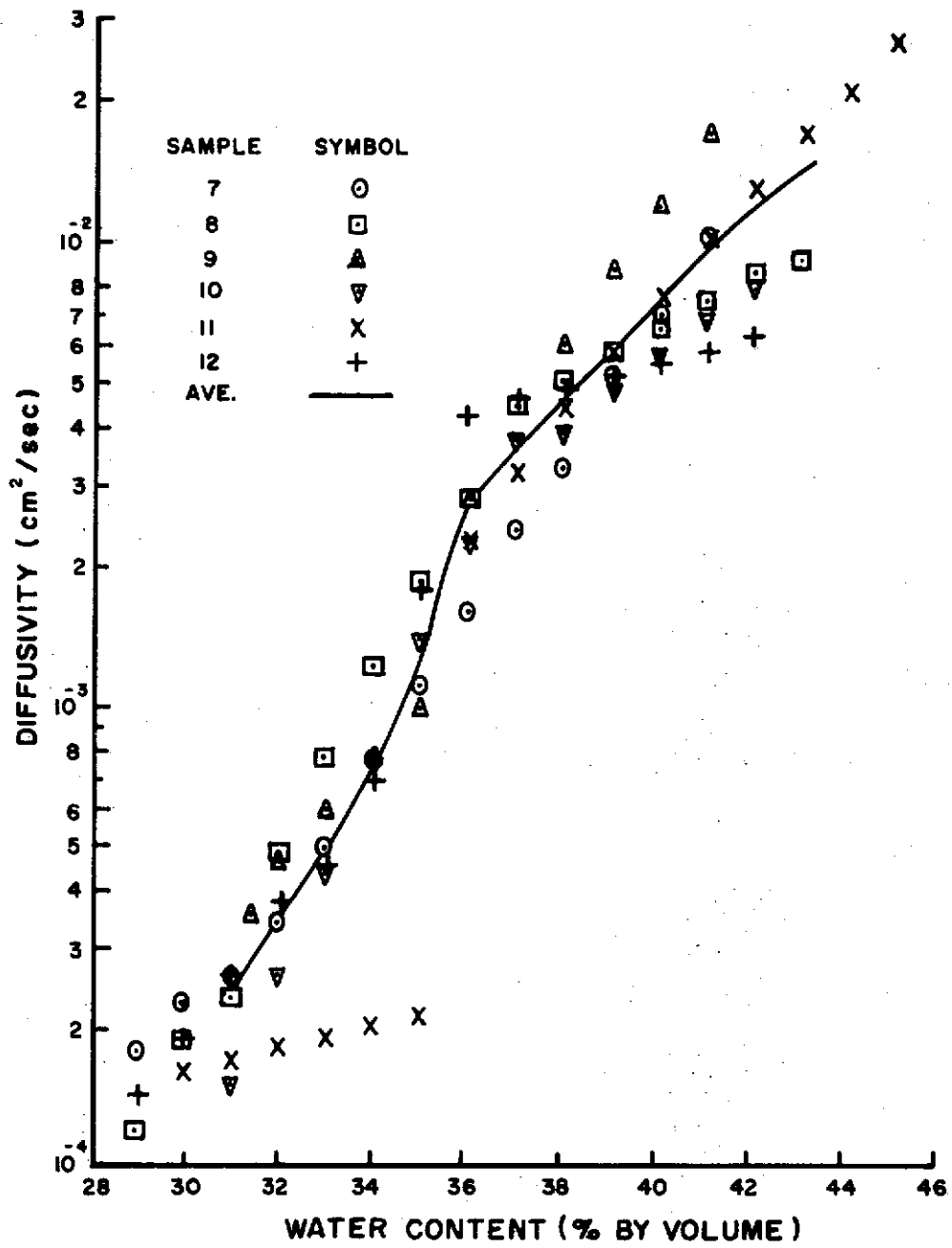


Figure 4.  $\theta$ -D relationship for location 2.

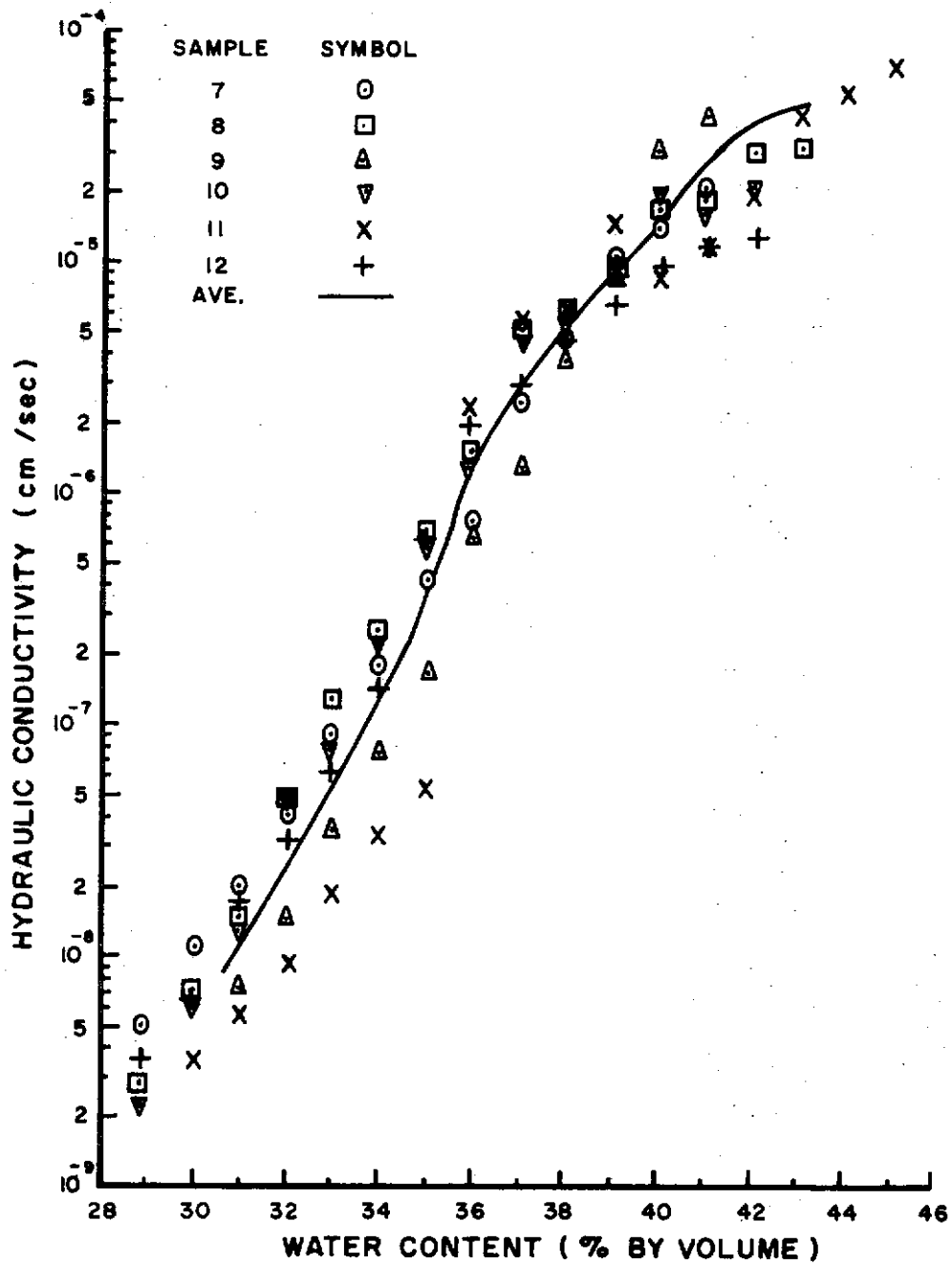


Figure 5.  $\theta$ -K relationship for location 2.

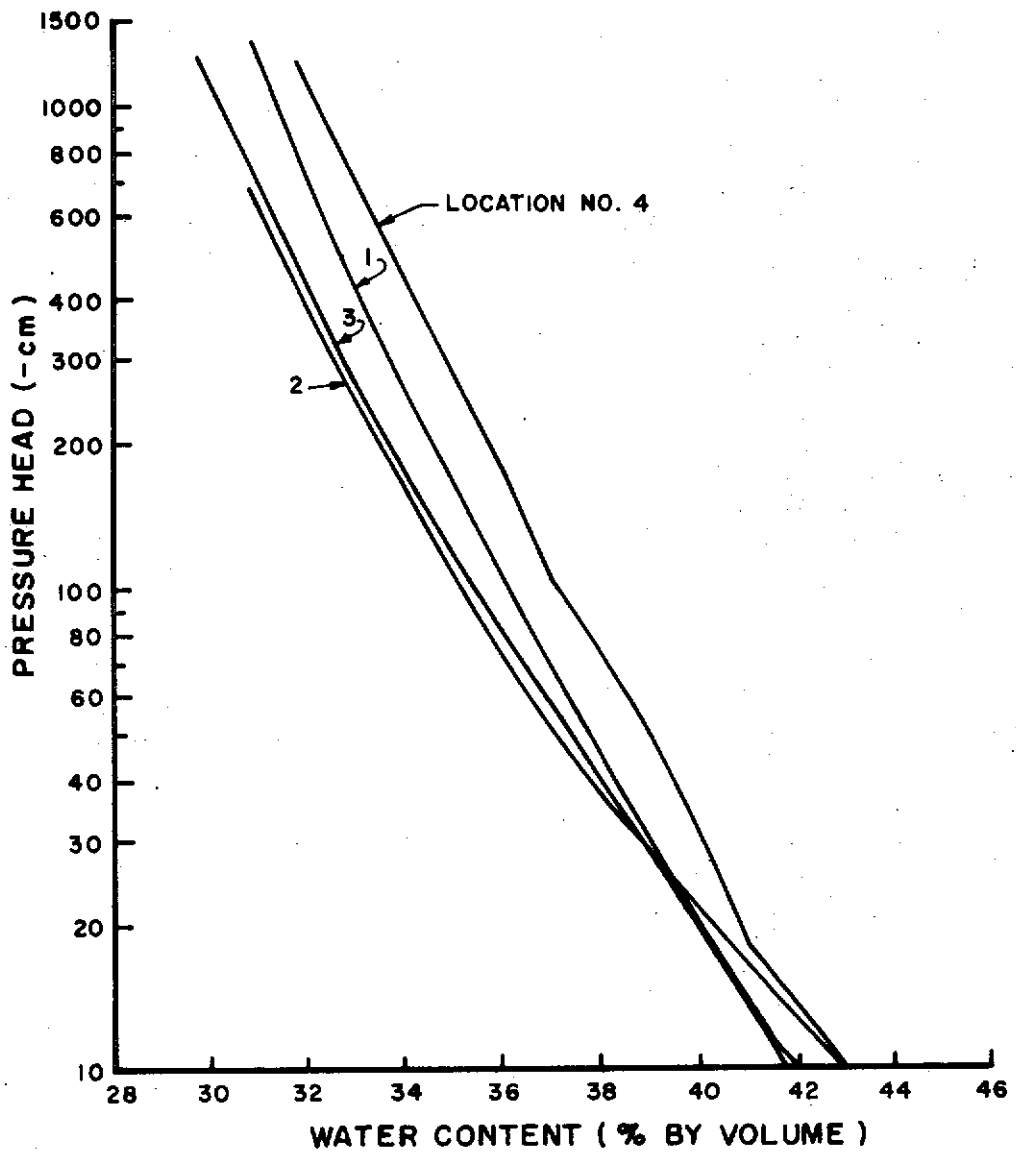


Figure 6. Average  $\theta$ - $\psi$  relationships for 4 locations.



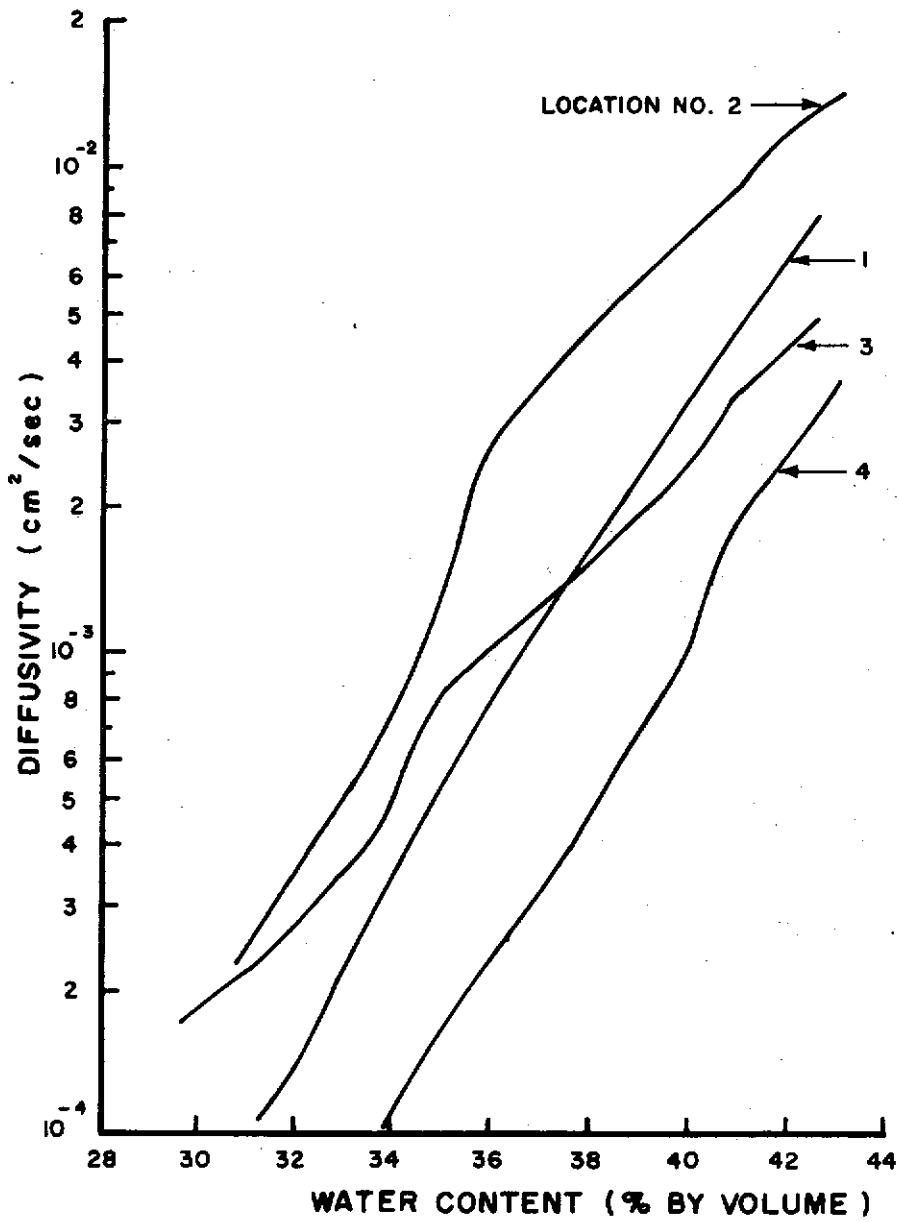


Figure 7. Average  $\theta$ -D relationships for 4 locations.

number of samples even if the field has soil that is apparently uniform in its physical properties.

To illustrate the 3 methods of combining data from several samples to get a single estimate of unsaturated conductivity, the data measured on the 22 samples were combined according to averaging methods a, b and c. The results of this analysis are shown in Table 1 for values of  $\psi$  equal to -10, -100 and -500 centimeters.

Table 1 shows that the 3 averaging methods produce different estimates for the average unsaturated conductivity. If method a is taken as the base, method b overpredicts K by an average of 40 percent and method c by 7 percent. Because of the uncertainty in the data,  $K_a$  and  $K_c$  are not significantly different at the 95% confidence level as shown by a paired "t" test while  $K_a$  and  $K_b$  are significantly different.

Which of the methods a, b and c is the best averaging method could not be determined from this study. It is thought that method a when used in conjunction with equations to predict unsaturated movement of water would more nearly describe the actual situation in the field. If this proves to be the case, then using method b to determine an average  $\theta$ - $\psi$ -K relationships could result in erroneous estimates of soil water movement.

One reason for the difference in  $K_a$ ,  $K_b$ , and  $K_c$  is that the average of products does not generally equal the product of averages. Method c seems to be a better approximation to method a than does method b.

Method c is attractive from the standpoint that the  $\theta$ - $\psi$  relationship is easily determined on an individual sample basis with the pressure plate extractor. It may be possible to determine an average  $\theta$ -D curve by modifying a pressure plate extractor so that the outflow from many samples can be collected simultaneously and used to calculate an average  $\theta$  for all the samples as a function of time. This average  $\theta$  and equation (11) could then be used to determine an average  $\theta$ -D relationship.

Table 1. Unsaturated conductivity ( $\text{cm sec}^{-1} \times 10^{+6}$ ) of Maury silt loam, 2 to 5 cm depth, at pressure potentials of -10, -100 and -500 cm of water as calculated by three averaging methods at four locations.

Pressure potential ( $\psi$ ), cm of water	Averaging method*	Location			
		1	2	3	4
-10	$K_a$	7.94	32.31	10.57	6.03
	$K_b$	14.96	46.67	13.89	10.29
	$K_c$	7.96	32.30	10.03	5.34
-100	$K_a$	0.168	0.316	0.168	0.110
	$K_b$	0.171	0.651	0.197	0.092
	$K_c$	0.173	0.338	0.188	0.095
-500	$K_a$	0.009	0.008	0.011	0.004
	$K_b$	0.014	0.007	0.014	0.004
	$K_c$	0.011	0.008	0.013	0.005

\* Averaging methods,  $K_a$ ,  $K_b$ ,  $K_c$ , are defined in text.

It can be shown that using an average  $\theta$ -D relationship and an average  $\theta$ - $\psi$  relationship is equivalent to averaging method b. Using average relationships between  $\theta$ -D and  $\theta$ - $\psi$  is probably the most common method presently being employed. Table 1 shows that this procedure produces poor estimates for the average conductivity if method a is taken as the best procedure.

Chapter IV  
ESTIMATING WATERSHED INFILTRATION WITH THE  
DIFFUSIVITY EQUATION

The first objective of this research was to evaluate the validity of the diffusivity equation for describing infiltration and moisture movement under field conditions. A complete evaluation of the diffusivity equation or more properly the flow equation would require that equation (4) be considered. Generally this is not done but the flow equation is considered in the form of equation (7) or (9). The reason for using equations (7) or (9) is that it is greatly simplified. Of course, these simplifications require several assumptions. This chapter will be devoted largely to considering some of these assumptions when the problem of estimating watershed infiltration is being considered.

The discussion contained in this chapter is concerned with watersheds a few square miles or more in area. Several investigators have successfully used the flow equation in limited situations on small, experimental plots.

The two main assumptions in going from equation (4) to equation (7) are that the flow is one dimensional (water moves either up or down but not sideways) and that the hydraulic conductivity does not change with location or depth (uniform, homogeneous soil).

These two assumptions are related. If a uniform soil of infinite extent is considered, if the slope of the soil surface is zero, if a uniform boundary condition (not necessarily constant with time) is imposed at the soil surface and at some constant depth, and if these conditions have existed for an infinitely long time, then the flow would be one dimensional. Obviously, all of the "if's" do not apply on a watershed.

Natural soils are certainly not uniform in their hydraulic properties over an entire watershed. As shown in the previous chapter, even for soils that are apparently uniform, there is considerable variation in the soil hydraulic properties. This variation exists not only in the horizontal direction but in the vertical direction in the soil profile as well. Numerical methods have been advanced for solving the flow equation in the presence of horizontal layers of soil having different hydraulic properties (Hanks and Bowers, 1962). These numerical procedures assume the soil has definite layers with sharp boundaries. Many soils gradually change in these properties with depth exhibiting no definite boundaries. Even those soils that are layered vary in the thickness of the layers from place to place in a watershed.

It is possible to define the soil hydraulic properties over a large area including all of the variability and to solve the flow equation for each of the many different soil profiles that would be encountered. In view of the computer time required to simulate only a few hours of moisture movement at a single point, the continuous simulation of a large number of points would require prohibitive amounts of computer time. Future generations of computers may reduce this time requirement. Another approach to handling the soil variability might be through fitting probability distributions to the soil hydraulic properties and from this deduce watershed infiltration. This approach has not been successful so far.

In view of the variation present in soil hydraulic properties, one must question the assumption of unidimensional, vertical soil-water flow. If the soil hydraulic properties vary from point to point, then the water uptake by the soil and the subsequent redistribution of this water will proceed at different rates. This will in turn cause cross water potential gradients to be established and thus produce a horizontal

component to the flow. To handle this situation, a two or three dimensional analysis such as the one by Jeppson and Schreiber (1972) would be required. The procedures would have to be more complex than those of Jeppson and Schreiber; however, since they considered steady-state flow. The fact that subsurface water movement is two and three dimensional has long been recognized (Meinzer, 1942; Betson, 1964; Amerman, 1965). The treatment of soil water flow on a watershed scale as a one dimensional process appears to be a poor approximation of actual conditions.

Additional problems that must be considered in using the flow equations to predict watershed infiltration are the changes in hydraulic properties with time, the sealing of bare soils during rainstorms, the cracking of certain soils when dry, the extremely important aspect of plant roots and plant water uptake, and the difficulty of defining the boundary conditions to be applied to the flow equation.

In view of the many problems to be overcome and the assumptions that must be made, it seems clear that considerable research is needed before the flow equations can be used to describe infiltration and water movement under field conditions.

Chapter V  
AN EMPIRICAL APPROACH

Because of the difficulties of applying the theoretical equations of Chapter II to the problem of predicting watershed infiltration, several empirical approaches have been advanced (Horton, 1939; Holtan, 1961). In this chapter, a simple empirical model will be discussed that has been successfully tested in a rainfall-runoff model (Haan, 1972). Figure 8 is a schematic of the model.

The model is developed around the idea that the moisture-holding and moisture-transmitting characteristics of the soil and underlying strata, along with the rainfall intensities, are the most important factors governing the runoff volumes from small watersheds.

The moisture-holding capacity of the soil is divided into a volume  $M_r$ , which is readily available for evapotranspiration, and a volume  $M_1$ , which is less readily available for evapotranspiration. The maximum capacity of  $M_r$  is 1 inch of water. The maximum capacity of  $M_1$  is  $C$ .

Precipitation is divided into infiltration and surface runoff. The infiltration rate  $f$  is determined from

$$\begin{aligned}
 f &= f_{\max} && \text{for } P \geq f_{\max} \\
 &&& M_r < 1 \text{ or } M_1 < C \\
 f &= P && \text{for } P \leq f_{\max} \\
 &&& M_r < 1 \text{ or } M_1 < C \\
 f &= 0 && M_r = 1 \text{ and } M_1 = C
 \end{aligned}
 \tag{13}$$

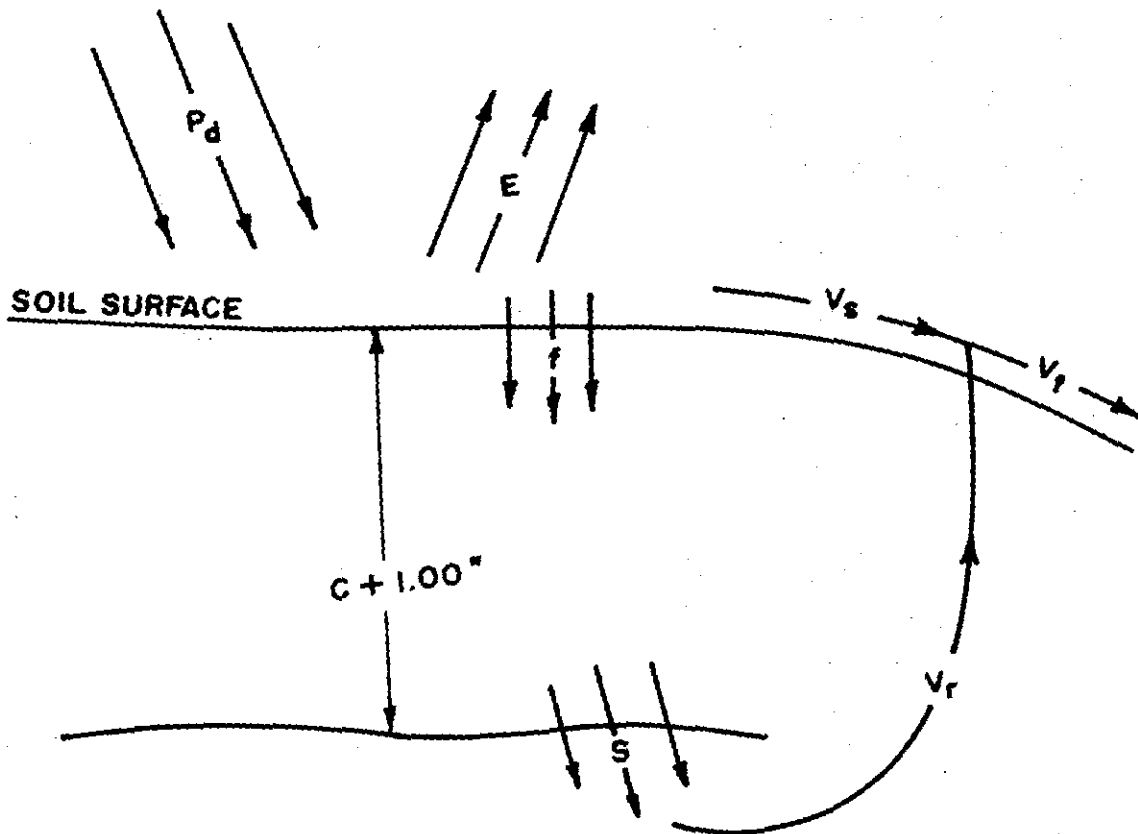


Figure 8. Schematic of water yield model.



where  $f_{\max}$  is the maximum possible infiltration rate and  $P$  is the precipitation rate. All infiltrated water is stored in  $M_r$  until the entire 1-inch capacity is filled, at which point any additional infiltrated water is transferred directly to  $M_1$ . When both storages are filled to their capacity, all precipitation is assumed to be runoff.

The surface runoff volume  $V_s$  is determined from

$$V_s = (P - f)t \quad P > f \quad (14)$$

$$V_s = 0 \quad P \leq f$$

where  $t$  is the time increment involved.

The daily evapotranspiration  $E$  is determined from

$$E = E_p \quad P_d = 0 \quad 0 < M_r \leq 1.00$$

$$E = E_p \left( \frac{M_1}{C} \right) \quad P_d = 0 \quad M_r = 0$$

$$E = \frac{1}{2} E_p \quad P_d \geq 0.01 \quad 0 < M_r \leq 1.00$$

$$E = \frac{1}{2} E_p \left( \frac{M_1}{C} \right) \quad P_d \geq 0.01 \quad M_r = 0$$

(15)

where  $E_p$  is the potential daily evapotranspiration and is an input to the model.  $P_d$  is the depth of rainfall (inches) that occurred on the day in question.

Evapotranspiration is equal to potential evapotranspiration as long as water is readily available and then is reduced by the ratio of  $M_1$  to  $C$ . On days when precipitation occurs, the evapotranspiration rate is reduced by a factor of 2 to account for cloudy conditions and low solar radiation.

Deep seepage  $S$  or water that does not appear as stream-flow within the watershed is determined from

$$S = S_{\max}(M_1/C) \quad (16)$$

where  $S_{\max}$  is the maximum possible seepage rate in inches per day. A certain amount of return flow  $V_r$  is allowed within the watershed and is calculated from

$$V_r = F \times S \quad (17)$$

where  $F$  is a constant defining the fraction of seepage that becomes runoff.

The total runoff  $V_t$  is then equal to the sum of the surface runoff and the return flow

$$V_t = V_s + V_r \quad (18)$$

#### Parameter Estimation

This model contains four parameters that must be estimated. The model is termed self-calibrating because it is capable of estimating the values for the parameters when some observed monthly flow data are available. The parameters that must be determined are  $f_{\max}$ ,  $S_{\max}$ ,  $C$ , and  $F$ . The best set of parameter values is the set that minimizes the sum of the squares of deviations between the observed and simulated monthly runoff volumes.

The procedure used is a simple univariate technique. The program requires initial estimates for the parameters and the increment size to be used in changing the value of each of the parameters. The process starts by calculating the value of the objective function at the initial parameter estimates. Next the value of  $f_{\max}$  is changed by one increment, all other parameter values remaining constant. The objective function is

recomputed, and if it is improved, the process of incrementing only  $f_{\max}$  is continued until the minimum value of the objective function is found. If, after the first step the objective function does not improve, a step in the opposite direction is tried. Steps in this direction are continued until the minimum value of the objective function is found.

This procedure is repeated,  $S_{\max}$ , C, and F being varied one at a time. After all four variables have been operated on, the entire process can be repeated for as many iterations as desired. Two or three iterations are normally all that are required.

The use of this self-calibration procedure is optional and may be bypassed if the parameters are already known or if the program operator wants to estimate the parameters himself.

If a model were able to duplicate exactly what is actually happening on a watershed, the years used to obtain the parameters for the model would not matter, since every year would yield the same parameters. Unfortunately, there are presently no models with this capability, and so the estimated parameters are to some extent a function of the years used in estimating them. The more years of record used to find the optimum parameter values, the better the estimates for these parameters will be; however, the computer time required is also increased.

For this model about 3 minutes of IBM 360/50 time are required to estimate the optimum values of the four parameters when 24 months of observed flows are used. The run time is proportional to the number of years of record being used to optimize the parameters. Since the optimization procedure is a univariate procedure with a fixed step size, the run time is also influenced by the initial estimates and the step size used on each of the four parameters. If reasonable initial estimates for the parameters are not available, large step sizes can be used to obtain preliminary estimates of the parameters. These estimates and a small step size can then be used to refine the final parameter estimates.

When several years of record are available for estimating the parameters, satisfactory results can be obtained by finding the optimum parameters for the first year of record, simulating the entire period, and then selecting the two years with the poorest fit. These two years are then used in the optimization scheme. The final parameter values are obtained by averaging the two optimum sets weighted according to the sum of the deviations of the observed and predicted values. In many cases, the parameters found from the first two years of record are satisfactory.

As discussed by Ross (1970), self-calibrating models are somewhat 'self-healing' in that they attempt to overcome their deficiencies by adjusting their parameters. This characteristic is advantageous in some situations but can lead to errors if the model is optimized during a non-representative period of record. This characteristic also means that components of the hydrologic cycle that are missing from the model or poorly represented by it will be represented to some extent by other parameters contained in the model. For instance, true infiltration is a time-varying process and not simply a function of rainfall as used in this model. Thus, the parameter  $f_{\max}$  should not be termed a maximum infiltration rate, since the actual maximum possible infiltration rate is a function of the soil moisture conditions. We call  $f_{\max}$  the maximum infiltration simply to aid in visualizing the operation of the model. The optimum value of  $f_{\max}$  is a function of the other parameter values. These same comments can be made about the other parameters.

This model has been evaluated by Haan (1972) and by Jarboe and Haan (1973). Basically, the model is performing satisfactorily as tested on 27 watersheds. The model is designed to produce monthly runoff volumes from daily rainfall. These runoff values can then be used in the design of water supply reservoirs.

The model demonstrates that simple infiltration and water movement submodels can be employed in special purpose rainfall-runoff models.

## Chapter VI

### CONCLUSIONS

From this study and the work of many others, it has been concluded that the diffusivity equation or any other form of the soil water flow equations cannot at this time be effectively used to estimate watershed infiltration especially in conjunction with a rainfall-runoff model. This conclusion is based on the non-homogeneity of watershed soils, the three dimensionality of the soil water flow system, the complexity of the three dimensional flow equation, the computer time required to solve the flow equation, and the difficulty of defining the applicable field boundary conditions.

It is further concluded that simpler empirical soil water models can be successfully used in rainfall-runoff models. This last conclusion has been demonstrated by considering Haan's (1972) water yield model.

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## PUBLICATIONS

The following two reports resulted from this study:

Haan, C. T., B. J. Barfield and R. E. Phillips. 1970. Influence of moisture content-potential-diffusivity relations on steady state infiltration. Paper No. 70-235 presented at 1970 Annual Meeting of American Society of Agricultural Engineers, Leamington Hotel, Minneapolis, Minnesota, July 7-10.

Haan, C. T. 1972. A water yield model for small watersheds. Water Resources Research 8(1):58-69.