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Process Control of Activated Sludge Treatment

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ABSTRACT

General feed forward controllers, conforming to standard control modes, have been derived for an activated sludge process. The analysis indicated that the appropriate controllers are proportional control with measurement of substrate flow rate, and derivative control with measurement of inlet substrate concentration, and manipulation of the rate of return sludge by both controllers.

The performance of these controllers was tested by computer simulation of five dynamic aerator models with and without sludge storage, and with two settling basin models. In all cases significant reduction of the maximum exit substrate concentration was achieved. Additional improvement resulted from the use of sludge storage. As the aerator model became more linear the control results also improved. The first dynamic results were obtained using a perfect steady state settler model, the remainder assumed that the settler dynamics could be represented by a variable time delay. The addition of the settler dynamics caused the control to degrade somewhat.

Finally the generality of the two controllers was proved mathematically for the five biological kinetic models for substrate utilization and bacterial growth.

KEY WORDS: Activated sludge process; Digital simulation; Environmental engineering; Mathematical models; Optimization; Process control; Quality control; Sewage treatment; Settling basins.

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CHAPTER I

INTRODUCTION

Project Objectives

It was the intent of this research to design a feedforward controller for the completely-mixed activated sludge process and to evaluate its performance by computer simulation. The controller desired had to economically maintain maximum removal of organic matter consistent with minimum variation in the pollutional load on the receiving stream.

In order to achieve these ends, the mathematical model from which the controller was to be derived was to include the time-dependent performance of the final clarifier as well as the aeration basin. Moreover, to be practical, the controller had to conform to existing technology, i.e. be a stock item, and use existing or developing sensors to measure perturbations and performances. Furthermore, applicability to existing installations was desired.

Background Information

At the start of this research, a control algorithm was being derived based on the work of Westberg.[39, 40*] This was completed[5] and forms the starting point for the development of process control as described herein.

The research involved process control concepts perculiar to specialized areas of engineering, aeration tank model selection, and formulation of the dynamic performance of the final settling tank. Hence some background information seems appropriate.

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*For all numbered references, see bibliography.

Process Control

Process control deals with changing conditions in the important parameters of process operation, the effect these changes will have on the end product, and the corrective action necessary to maintain the desirable characteristics of the end product. The difference between this approach and conventional design is immediately apparent. The designer assumes operating conditions to be fixed, (often at the least favorable condition or at a time average over the least favorable period) and proceeds to the design assuming a steady-state condition. Hence pure process design cannot predict performance at any but the selected steady-state conditions without recalculation with new numerical values.

The problem of product optimization can be dealt with from many approaches, but two are of great significance, namely, feed back or feed forward control. In one form, a feed back controller accepts a measurement of a characteristic of the output stream, compares it with the desirable quantity, i.e., the setpoint, and delivers an appropriate signal to a manipulation device (switch, valve, pump, etc.) based on the difference between the measurement and the setpoint. Feed forward control involves the measurement of a perturbation to the process, for example, a change in influent concentration, and makes a compensation for it. In theory "perfect" control is possible in feed forward control because no deviation of product quality must occur before corrective action is taken.

In either case, the time dependent performance of the process, i.e., the mathematical model, must be known so that the corrective or compensative action is proper. However, the need for validity of the model for use with pure feed forward control is much more stringent because there is no self-corrective action, hence, combination feed forward and feed back control is often used. Thus process control involves the process proper; the sensor or measuring device; the controller; and the manipulation device. Also to be considered is the form of the perturbation, for example, sinusoidal, step change, etc. In the first approximation, only the process (response function) perturbation (forcing function) and controller (control algorithm) are formulated into time dependent equations. The last is, of course, what is sought, based on the other two. In other words, an equation is obtained, the control algorithm, which accurately describes the performance of a device, a controller. The controller, which may be electronic, pneumatic, or hydraulic, thus solves the equation and is strictly speaking an analog device[31]. That parameter which is varied by the controller to optimize the process is termed the manipulated variable.

To obtain the control algorithm, the time-dependent differential equations muat be solved, and a powerful technique is the use of Laplace transforms. If the differential equations are linear with constant coefficients (or can be made so) the equations may undergo Laplace transformation to algebraic equations in a new "dummy" independent variable. The simultaneous algebraic equations may be solved, and the inverse transformation performed yielding the solution in the original independent variable. Moreover, the Laplace transforms themselves may be manipulated to yield useful formulae. The most basic of these is the transfer function which is defined as the ratio of the transform of the response function to the transform of the forcing function.

Aeration Tank Model Selection

Westberg's model[39] and proposals for control[40] had been innovative and useful for process control development. But some of his concepts were at considerable variance with those of respected authors and were developed analytically without supporting laboratory evidence. Hence the validity of his model was questionable.

For this reason, it was decided to work with a new model or models. Many were available. A listing would include those of Smith and Eilers[36], Lawrence and McCarty[27], Ott and Bogan[32], Schroeder[34], Eckenfelder[18], Busch[6], and McKinney[28]. In general, these had been derived on the basis of pure culture kinetics, a mass balance on the reactor, and considerations of flow regime. Their differences arose from the individual simplifying assumptions, flow regime considered, and ultimate objective.

The first necessity of the research was then to evaluate the models and select an appropriate one to derive the algorithm.

Dynamic Performance of Settling

Except for the inclusion of a factor to reflect compaction of sludge solids, none of these models attempted to consider time dependent performance in the final settling tank.

The performance of the final settling tank is an important consideration as has been pointed out by Dick[12] and by Dick and Javaheri[15]. This is so for a number of reasons. Most important is the fact that it is through the discharge of sludge solids not removed by settling that the activated sludge process may seriously fail.

A second important consideration is the fact that the concentration of active organisms in the recycle stream is determined by the thickening function of the final basin. The characteristics of the sludge and basin may be such that the necessary concentration of recycle sludge cannot be met.

Finally, not only does the operation of settling tank affect the aeration tank performance, but the converse is true. Aeration parameters such as sludge age, BOD loading, and dissolved oxygen contribute to the quality of sludge. These parameters along with the type of raw sewage determine what type of bacteria will be most predominant. Since it is known that certain bacteria settle more readily than others, this greatly affects the relationship between aerator and clarifier.

CHAPTER II

RESEARCH PROCEDURES

In general, this research proceeded as a mathematical analysis of theory and data extant in the literature and did not extend to the gathering of new data. The analysis was two-fold: First, control algorithms were derived from time-dependent model equations descriptive of the process. Then the model equations were programmed into a computer and made to respond to time-variant changes in process input. The controller was likewise programmed. The test of the controller was the degree of improvement in output from the simulated process when under control versus the uncontrolled case.

In brief, the specific steps taken were:

i. The derivation and testing of the controller based on Westberg's model were completed[39, 40, 5].

ii. The controller obtained in step i was tested by applying it to the models of other authors.

iii. Intrigued by the excellent results of step ii, the controller was derived anew for the general case.

iv. The new controller was applied to the aerator models of Eckenfelder[18] and Lawrence and McCarty[27]. The final clarifier was considered perfect and instantaneous.

v. Existing sedimentation theory was critically reviewed to obtain a model predictive of the time-dependent underflow sludge concentration from the final settling basin. This resulted in a settler model which remained perfect but now had a time delay.

vi. The Eckenfelder model was revised to include the model for the settling basin. The controller from step iii was tested in its control of this model.

vii. A new control algorithm which included the time delay from the settler was derived and applied to the complete Eckenfelder model obtained in step vi.

The detailed procedures are more conveniently discussed in Chapter III.

CHAPTER III

CONTROL DEVELOPMENT AND RESULTS

Work on Westberg's Model and Algorithm

The starting point for this research was Westberg's model[39, 40] who introduced a unique mathematical model of the complete-mixing activated sludge process and used it to derive feed-forward control of the process to maintain a constant soluble BOD concentration in the aerator and effluent.

His model was unique in that it made use of a mass balance on dead bacteria as well as the customary balances on limiting substrate and living organisms. He took the growth rate constant to be a true constant, death rate to be inversely proportional to substrate concentration, and the redissolving of cells to be proportional to the product of concentrations of living and dead bacteria. He further assumed that the sedimentation step could be described by an overall material balance of the form:

$$q_1 + q_2 = \beta Q \tag{1}$$

Equation 1 implies no internal substrate utilization or bacteria synthesis in the sedimentation step; and that the settling process produces organism free overflow at a rate $Q - q_2$ and a concentrated underflow at a rate $q_1 + q_2$.

The resulting equations were as follows:

$$\frac{dX}{dt} = X[m - \frac{c}{S} - f(t)] \tag{2}$$

$$\frac{dZ}{dt} = \frac{cX}{S} - bXZ - Z\ f(t) \tag{3}$$

$$\frac{dS}{dt} = \frac{QS_i}{V} - \frac{QS}{V} + bXZ - \frac{m}{y}X \tag{4}$$

$$f(t) = \frac{q_2(Q + q_1)}{V\,(q_1 + q_2)} \tag{5}$$

The disturbances to the steady state performance of the process were considered to be inphase, sinusoidal functions of the inlet flow rate and substrate concentration:

$$Q = \frac{Q_a}{2}\left(2 + \sin\frac{\pi t}{12}\right) \tag{6}$$

$$S_i = \frac{S_{ia}}{2}\left(2 + \sin\frac{\pi t}{12}\right) \tag{7}$$

The sludge recycle stream q_1, was chosen to be the manipulated variable although other possibilities were available. The relationship of the variables is shown in Fig. 1.

By assuming that the term bXZ in equations 2 and 3 could be replaced by bX_aZ, where X_a is the average concentration of X over a cycle, and defining a new variable, $N = X/Z$, Westberg was able to find an analytical solution for an f(t) which would perfectly control S. Equation 7 was his regulation function.

$$f(t) = \frac{mS - c}{S} - \frac{h'(t) - g'(t)S}{h(t) - g(t)S} \tag{8}$$

Where: $$g(t) = \frac{Q}{V}\ ;\ \text{and}\ h(t) = \frac{QS_i}{V} \tag{9}$$

The notatations g'(t) and h'(t) indicate the first derivatives of the functions.

Although this expression could not be solved explicitly for q_1, the flow of return sludge could be determined numerically by imposing

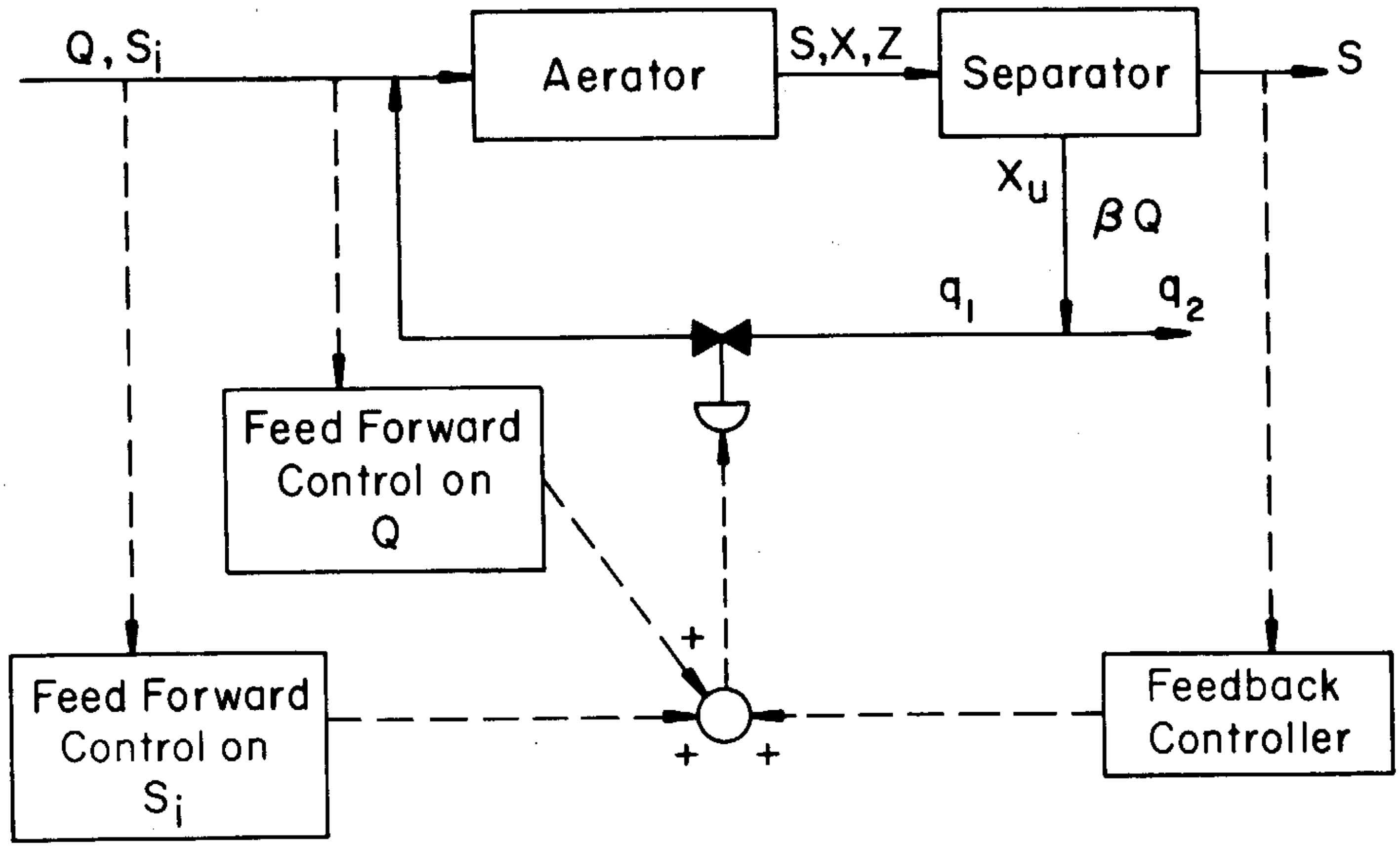

Figure 1. Flow Diagram for Control of a Completely-Mixed Activated Sludge Process

the precept that the sludge stream from the separator, $q_1 + q_2$, take the constant value 0.4Q. He defined p as the ratio $\frac{q_2}{Q}$, the relative flow of excess sludge, and then combined f(t) as defined in equation 4 and the forcing function for Q, equation 5, to obtain:

$$f(t) = \frac{5}{8} p \ (1.4 - p)(2 + \sin \frac{\pi t}{12}).$$

Some observations concerning his algorithm could be made:

1. The differential equations were simplified by assuming an average value for one dependent variable, the concentration of living bacteria, without indicating any test of the effect on the actual control of exit substrate concentration.

2. The controller was only correct (that is, provided zero deviation in outlet BOD) for one function of the form A sin ωt. If the constants A or ω changed, a new control algorithm would have had to be calculated. Hence the control policy was an equation which would require a small computer to implement if control were to be automatic.

3. Other forcing functions such as step changes were not considered.

4. The algorithm required sludge storage to provide make up sludge to maintain the necessary live bacteria concentration during part of the diurnal cycle. This was indicated in his calculations when p became negative.

<u>Objectives</u>

The primary objectives of this study of Westberg's work were an evaluation of the effect of the assumptions made in deriving equation 7 on the controller's effectiveness, and the application of standard linear techniques to select somewhat simpler feed forward controllers for the process. The performance of the new feed forward as well as conventional feed back controllers would then be compared with Westberg's results by computer simulation. It was also expected that some additional information would be gained concerning the dynamics of the process.

Before deriving other controllers, an examination of Westberg's assumptions seemed in order. The two assumptions deemed the most important were the replacement of X by X_a and the need to store sludge. In order to test these assumptions, his forcing functions, equations 6 and 7, and their derivatives were substituted in equation 8. The f(t) which resulted was substituted in the original set of differential equations, i.e. equations 2, 3 and 4. The equations thus manipulated were solved on a digital computer (IBM System/360 Model 50) for the reactor substrate concentration, S, at any time t. The necessary values for the constants in the differential equations are taken directly from Westberg, as follows:

Average influent flow rate, Q_a, m^3/h	10,000
Average dilution rate, $\frac{Q_a}{V}$, h^{-1}	0.5
Average influent substrate concentration, S_{ia}, g/m^3	267
Average concentration of living bacteria, X_a, g/m^3	400
Growth rate constant, m, h^{-1}	0.2
Cell yield, y, g cells/g substrate	0.4
Death rate constant, c, g/m^3 h	4
Redissolving rate constant, b, m^3/g h	5×10^{-4}

Westberg had imposed a value of 22 g/m^3 for S in his numerical solution. Here S is, of course, a dependent variable.

The computer simulation results of four runs are summarized in Table 1 and shown graphically in Figure 2. Run 1 of Table 1 was the uncontrolled case wherein the recycled sludge flow rate, q_1, was kept at a fixed average value. This resulted, as expected, in a very large value for S_{MAX}, i.e., 255 g/m^3. Westberg's results were reproduced in run 2. Here perfect control was achieved by use of equation 8 with the average value X_a taken for the variable X.

Equation 8 was also used as the control algorithm for runs 3 and 4. In run 3, X was allowed to vary and perfect control was no longer possible with S reaching a maximum value of 244 g/m^3. Perfect control was also impossible when sludge storage was prohibited in run 4, that is $p \geq 0$. In this run, X_a was again assigned for X.

Table 1. Results of Computer Simulation of Westberg's Controller

Run	Control Algorithm	Forcing Function	bXZ	Restriction on p	S_{MAX}	S_{MIN}
1	None	Equations 6 & 7	bXZ	None	255	2.1
2	Equation 8	Equations 6 & 7	bX_aZ	None	22	22
3	Equation 8	Equations 6 & 7	bXZ	None	244	0
4	Equation 8	Equations 6 & 7	bX_aZ	$p \geq 0$	237	22

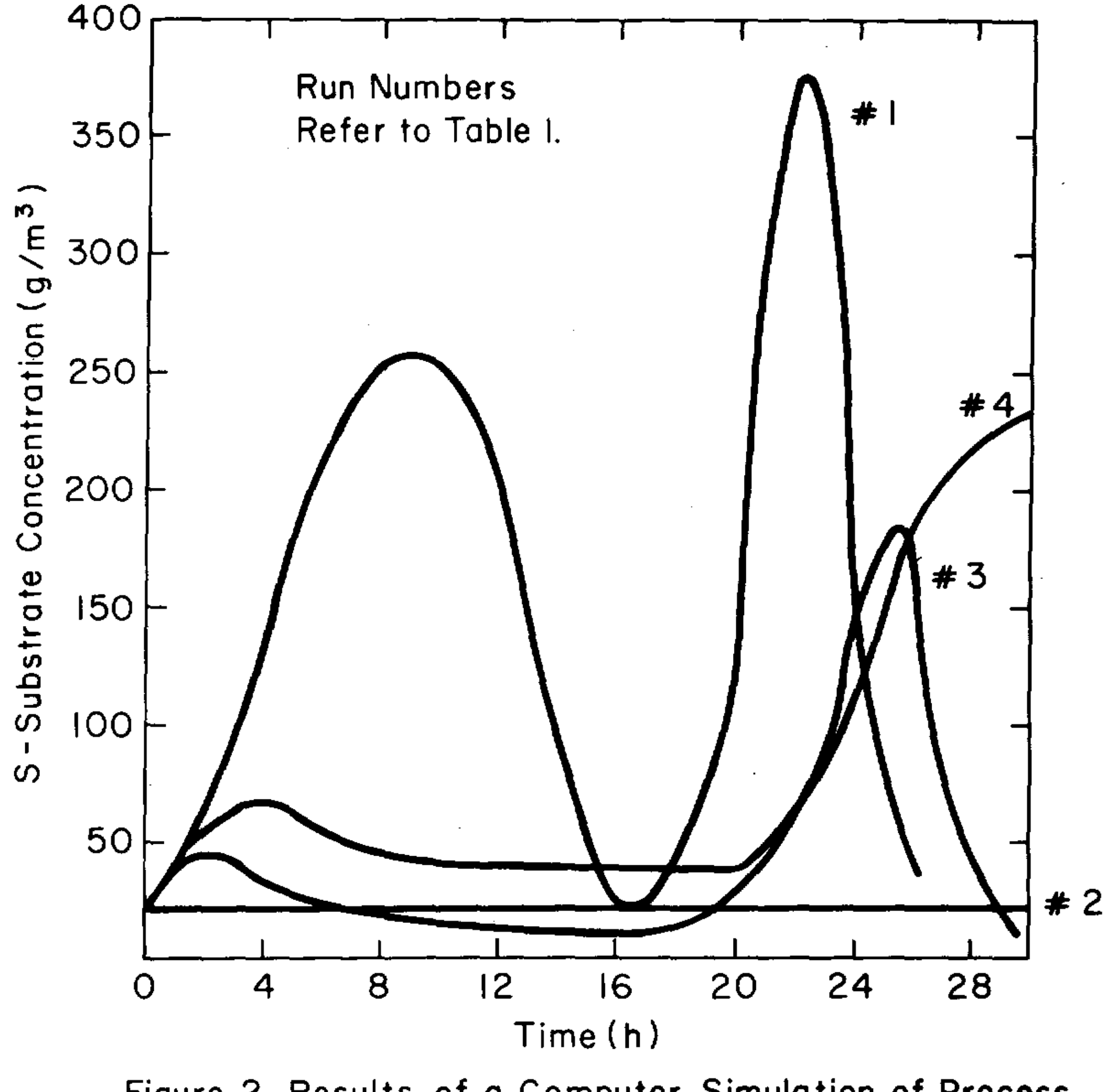

Figure 2. Results of a Computer Simulation of Process Using Westberg's Controller

Feed Forward Controller Design

It can be shown that "perfect" control by a feed forward controller is possible whenever the equations (the model) representing the system are linear[7]. The procedure for finding these controllers is well documented in the literature[4, 7, 24]. Briefly the procedure followed here was:

i. The equations, being non-linear, were linearized, resulting in:

$$\frac{d\bar{X}}{dt} = 3.0247\bar{S} - 0.02524\bar{Q} + 0.06309\bar{q}_1 \qquad (10)$$

$$\frac{d\bar{Z}}{dt} = 0.01631\bar{X} - 0.2012\bar{Z} - 3.0247\bar{S} - 0.02282\bar{Q} + 0.05705\bar{q}_1 \qquad (11)$$

$$\frac{d\bar{S}}{dt} = -\ 0.3345\bar{X} + 0.1830\bar{Z} - 0.5\bar{S} + 0.01225\bar{Q} + 0.5\bar{S}_i \qquad (12)$$

The bar notation is used to indicate a deviation variable, for example

$$\bar{X} = X - X_{SS}$$

where X_{SS} is the steady state value.

In linearizing the equations, it was necessary to impose the condition that $q_1 + q_2 = \beta Q$. This was required to obtain three equations with 4 variables not independently fixed by time (as are Q and S_i through the forcing functions, equations 5 & 6). Further, β was assigned the value 0.4 throughout this study. This constraint on q_1 and q_2 was identical to Westberg's in his numerical solution and for the same reason. His values for constants as given above were also used.

ii. The linearized equations were Laplace transformed and solved simultaneously for $\bar{S}$ as a function of $\bar{Q}$, $\bar{S}_i$, and $\bar{q}_1$, where $\bar{X}$ and $\bar{Z}$ were eliminated algebraically. This resulted in:

$$\overline{S} = \frac{[0.01225s^2 + 0.006730s + 0.001623]\overline{Q}}{[D]} +$$

$$\frac{s[0.5s + 0.1006]}{[D]} \overline{S}_i - \frac{[0.01066s + 0.004057]\overline{q}_1}{[D]} \tag{13}$$

where

$$[D] = s^3 + 0.7012\, s^2 + 1.6658s + 0.1945$$

and

$$P_{11} = \frac{[0.01225s^2 + 0.006730s + 0.001623]}{[D]}$$

$$P_{12} = \frac{[0.5s + 0.1006]}{[D]}$$

$$P_{13} = - \frac{[0.01066s + 0.004057]}{[D]}$$

where s is the Laplace variable defined by

$$M(s) = \int_0^\infty M(t)e^{-st}\, dt,$$

M is any function possessing a Laplace transform.

iii. It can be shown that:

$$F_1 = - \frac{P_{11}}{P_{13}} = 0.400\ \frac{[7.547s^2 + 4.146s + 1]}{[2.627s + 1]} \tag{14}$$

$$F_2 = - \frac{P_{12}}{P_{13}} = \frac{24.79s\ [4.970s + 1]}{[2.627s + 1]} \tag{15}$$

Equations 14 and 15 were the feed forward controllers that would directly give perfect control for all types of disturbances in Q and S_i respectively if the linearized equations were the correct model of the process. This capability exceeded the requirement of this study in that the controller could provide perfect control for step disturbances, sinusoidal disturbances, indeed for any function of time that might be a disturbance. However, disturbances to the process had been limited to sinusoidal variations of Q and S_i, and simplification of the controllers was possible by considering loads like equations 6 and 7 only. It was thus necessary to know only how F_1 and F_2 operate in responding to sine waves of different frequencies.

Because F_1 and F_2 were themselves the ratios of output to input for the controller, an evaluation of the magnitude of the ratios at various frequencies provided the information. Figure 3, termed a Bode plot by control engineers, is a graph of the magnitude ratios of F_1 and F_2, divided by the constants of their equations, plotted against ω. The angular velocity, ω, is the frequency multiplied by 2π and has the units radians per hour.

Examination of the curve for F_1 shows that for all ω up to some critical angular velocity, ω_c, F_1 is approximated by 0.4. Because for this study ω was $\pi/12$ or 0.26 and less than ω_c, 0.4 was a reasonable approximation for F_1. A similar analysis for F_2 showed 24.79s to be a good approximation. Hence the feed forward controller became:

$$\overline{q}_1 = 0.4\overline{Q} + 24.79 \frac{d\overline{S}_i}{dt} \tag{16}$$

Thus proportional feed forward control on $\overline{Q}$ and derivative control on $\overline{S}_i$ were indicated to give improved control. The control would not be perfect because of the linearization of the original equations. Further, the control would be even less nearly perfect for forcing

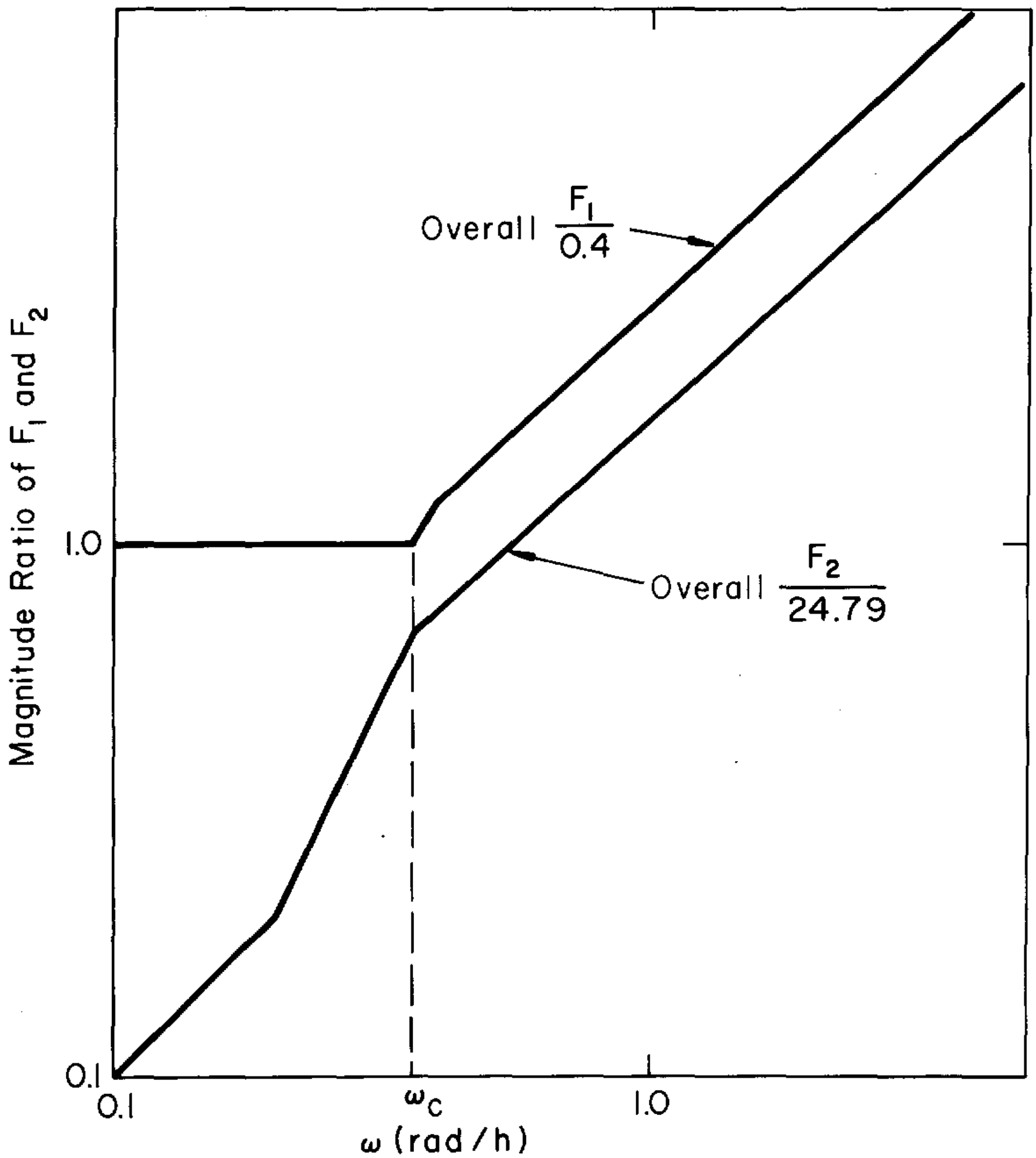

Figure 3. Bode Plot of Controllers F_1 and F_2

functions different from those used in the Bode analysis for the simplification of F_1 and F_2.

Computer Testing of Control

Table 2 is a summary of the various types of control, the step and sinusoidal forcing functions, the form of bXZ terms, any restrictions on the recycled sludge (i.e. what were the restrictions on $p = q_2/Q$) and the value of S_{MAX} and S_{MIN}. In all cases the actual non-linear equations were used as the process. Runs 6, 7, 8 and 9 used the feed forward controller derived in the previous section, equation 14, with the same forcing functions on S_i and Q used by Westberg. His results and run 8 can be directly compared. It was also instructive to include run 5 which was the uncontrolled case. Although perfect feed forward control was not possible, a reduction of S_{MAX} from 255 to 44 was significant and this had been accomplished with conventional proportional and derivative control modes. In run 6 the effect of representing the concentration of living bacteria by bXZ instead of bX_aZ was evaluated. The maximum for S became 153 indicating that the form of this term has a very important effect on the controllability of the system.

In run 7 the added restriction was made that no sludge storage was possible; i.e. $p \geq 0$. Once again there was additional deterioration in the performance of the controller. Inspection of equation 16 reveals the reason for this deterioration. Equation 16 dictates that $\overline{q}_1$ equal 0.4 $\overline{Q}$ when S_i is constant, be less than 0.4 $\overline{Q}$ when S_i is decreasing and exceed 0.4 $\overline{Q}$ when S_i is increasing. However, in deriving equation 16, $q_1 + q_2$ was arbitrarily set equal to 0.4 Q. The only way for q_1 to exceed 0.4 Q is for q_2 to take negative values, i.e. sludge to be

Table 2. Computer Simulation of Process Under Various Control

Run	Control Algorithm	Forcing Function	bXZ	Restriction on p	S_{MAX}	S_{MIN}
5	None	Equations 5 & 6	bXZ	None	255	2.1
6	Equation 14	Equations 5 & 6	bXZ	None	153	10
7	Equation 14	Equations 5 & 6	bXZ	$p \geq 0$	225	10
8	Equation 14	Equations 5 & 6	bX_aZ	None	44	14
9	Equation 14	Equations 5 & 6	bX_aZ	$p \geq 0$	142	12
10	Equation 14 plus proportional feed back	Equations 5 & 6	bX_aZ	$p \geq 0$	152	8.2
11	Equation 14 plus proportional feed back	Equations 5 & 6	bXZ	$p \geq 0$	237	3.6
12	None	Step in Q and S_i	bXZ	None	330	22
13	Proportional feed back on S only	Step in Q and S_i	bXZ	$p \geq 0$	186	21
14	Proportional feed back on S and Proportional feed forward on Q	Step in Q and S_i	bXZ	$p \geq 0$	150	12

returned from storage; hence when p is constrained to positive values, the controller drops the last term and becomes proportional on Q only. This happens at a time when additional control is most necessary.

The dynamic results for runs 5 thru 9 are presented in Figure 4. The dynamic results showed that excellent feed forward control is obtained under a variety of conditions with the exception of about six hours out of twenty four. This maximum always occurred during the first six hours of a new cycle. Examination of equations 6 and 7 indicated the reason; the term $(2 + \sin \frac{\pi t}{12})$ had its maximum value when t was equal to 6 h.

Addition of Feed Back Control

Runs 6 and 7 used the feed forward controllers on $\overline{Q}$ and $\overline{S}_i$ with the linear addition of proportional feed back control on $\overline{S}$. This took the form of $\overline{q}_1 = K_{pfb}\overline{S}$. The complete controller including the feed back addition to equation 16 was thus $\overline{q}_1 = 0.4\ \overline{Q} + 24.79 \frac{d\overline{S}_i}{dt} + K_{pfb}\overline{S}$. The proportionality constant K_{pfb} was assigned several values which were tested for performance improvement by the computer. Only the most favorable result, $K_{pfb} = 25$, is shown. The general shape of the dynamic curves was unchanged. However the S_{max} values were slightly larger indicating that the effect of this added controller was marginal. It should be realized that no standard method such as Ziegler-Nichols' was used to tune the feed back controller. It is therefore possible that a good choice of K_{pfb} has been missed by the somewhat arbitrary choice of test values.

Step Change Forcing Function

Finally, the response of the process to a step change in Q and S_i was tested with and without control. The values for Q and S_i were

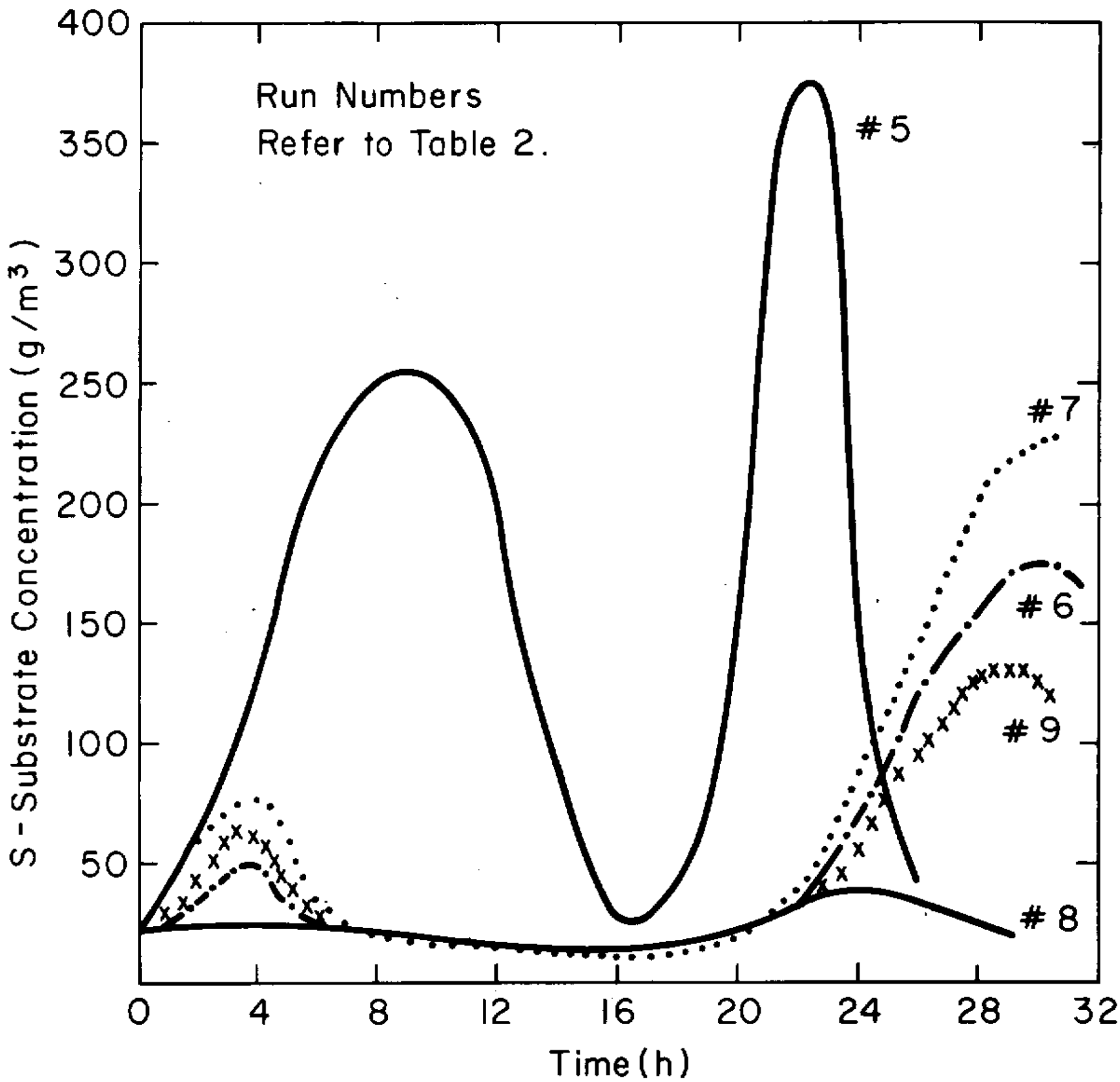

Figure 4. Results of a Computer Simulation of Process Operation Using the Feed Forward Controller Responding to Sinusoidal Forcing Functions.

stepped to the maximums taken by the sinusoidal forcing functions over a day, a fifty per cent increase in Q and S_{ia}. That is, a change which required 6 h sinusoidally was effected instantaneously. The results are shown in Table 2 and Figure 5. The shape of the curve and the value for S_{MAX} were not unexpected.

<u>On-Off Control</u>

It seemed that much could be learned about the controller operation from a plot of p versus time. Figure 6 shows p for run 9 Table 2. Since $p = q_2/Q$, where q_2 is the sludge that is not returned, a zero value for p corresponds to $q_1 = 0.4\ Q$ a value which only occurs with total return of the sludge. Notice that during 60% of a cycle p was zero; the controller was saturated and no longer acted in a linear manner as explained above. This suggested that on-off control would probably have given as good or better control than any continuous linear controller. It also showed that if the requirement of no sludge storage and X instead of X_a were indeed true, that perfect control by simply recycling sludge was impossible.

<u>Value for K_{pff}</u>

It seemed somewhat unusual that the coefficient of Q in the feed forward controller, termed K_{pff}, should equal the assigned value for β, that is 0.4. An attempt was made to show that $K_{pff} = \beta$. Algebraically it can be shown that:

$$K_{pff} = \frac{\beta + (q_1/Q)^2}{1 - \beta + 2(q_1/Q)} \quad \text{and} \quad \frac{q_1}{Q} = -\frac{(1-\beta)}{2} + \sqrt{\frac{(1-\beta)^2 + 4\beta}{4} - \frac{4\beta V(ms-c)}{4QS}}$$

If $\frac{4\beta V(mS-c)}{QS} << (1-\beta)^2 + 4\beta$, this reduces to

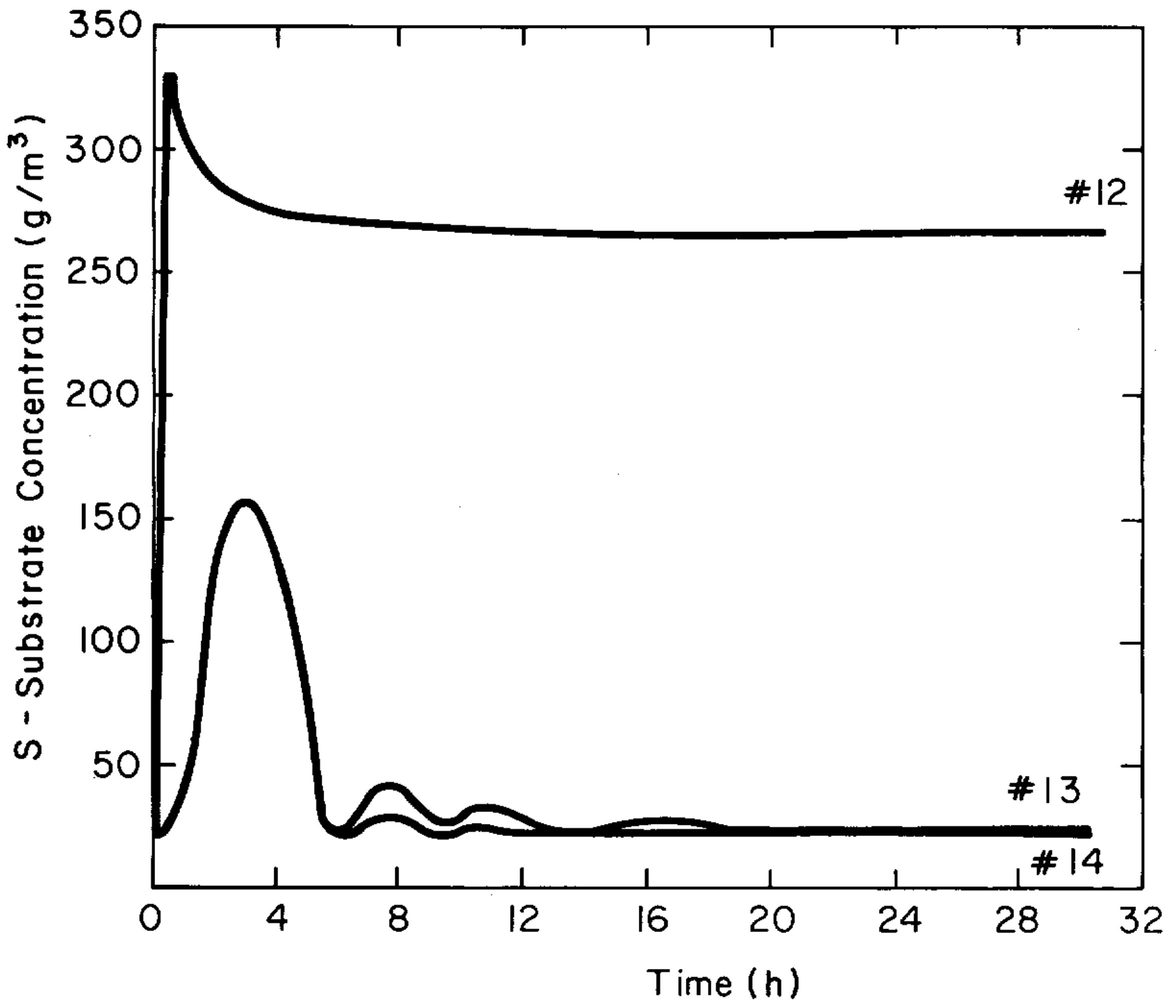

Figure 5 Results of a Computer Simulation of Process Operation Responding to Step Change Forcing Functions.

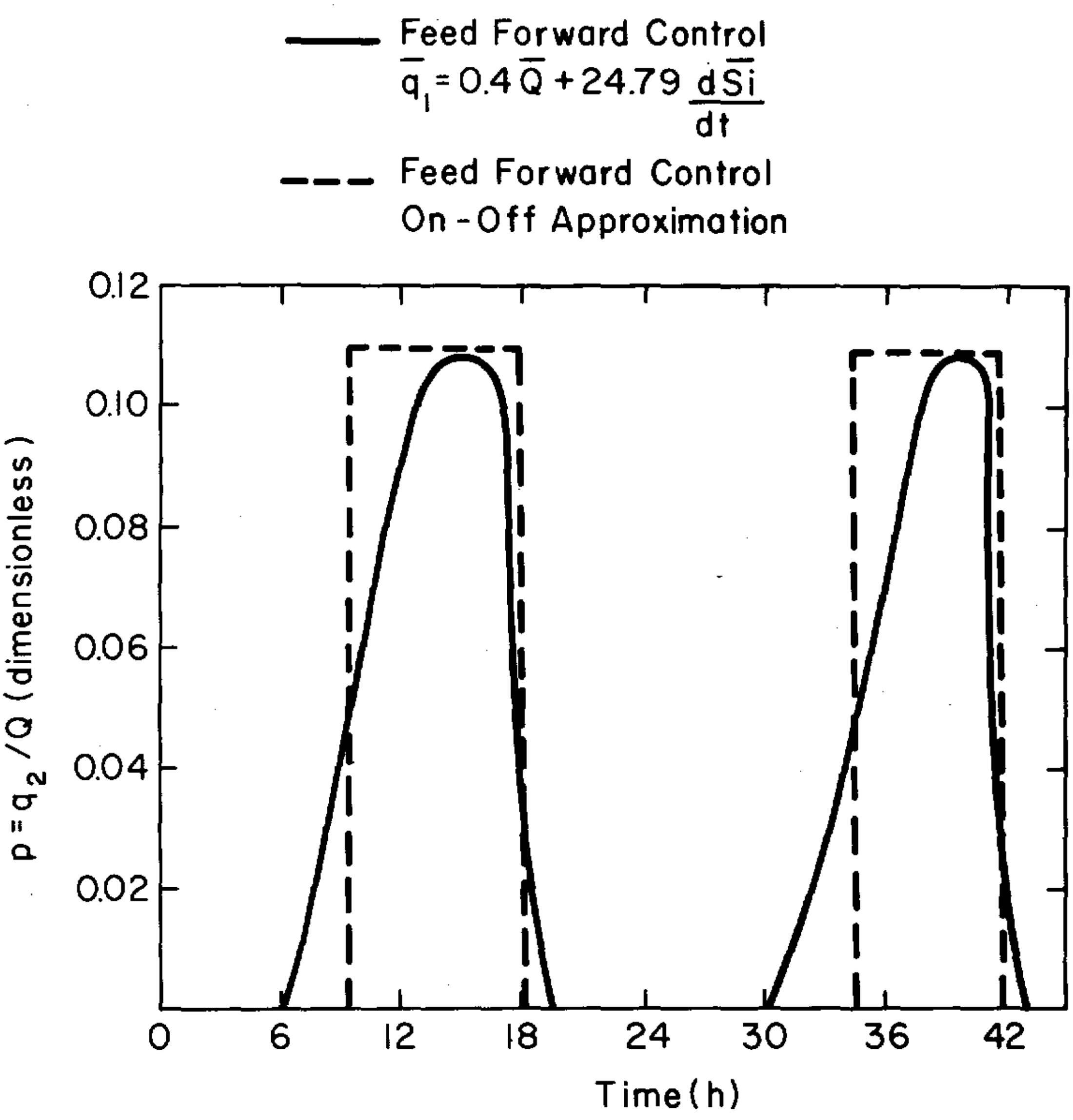

Figure 6 Comparison of the Feed Forward Controller with an on-off Approximation to the Controller

$$K_{pff} = \frac{\beta + \beta^2}{1 - \beta + 2\beta} = \beta.$$

Extention of Control to Other Aerator Models

The application of standard control analysis to Westberg's work made possible the elimination of his major simplifications and improved control. This was sufficiently encouraging to warrant further effort, and control was applied to the more accepted models in the literature.

In applying the feed forward controller, equation 16, to other models, control was even better than with Westberg's model. This was totally unexpected. In hopes of showing general applicability, the controller was derived anew in a slightly different fashion for the general case.

This general controller which neglects the time delay in the return of sludge from the clarifier will be referred to as Davis' controller[11] to distinguish it from the controller later derived to include a clarifier time delay, which will be called the Debelak controller[16].

General Material Balances

The flow diagram remains as shown in Figure 1, and equation 1 (with its simplifying assumptions) describes the settling basin.

Most authors, for example Lawrence and McCarty[27] and Eckenfelder[18] derive material balances on substrate utilization and living bacteria, neglecting Westberg's third equation on dead bacteria. The general differential material balance on the aerator for the substrate concentration is:

Accumulation		Substrate flow in		Substrate flow out		Substrate removal by reaction	
$V\frac{dS}{dt}$	$=$	$QS_i + q_1S$	$-$	$(Q + q_1)S$	$-$	$V\frac{dF}{dt}$	(17)

Where dF/dt is the internal substrate utilization rate per unit volume. While S and F represent the same physical entity, the rates $\frac{dS}{dt}$ and $\frac{dF}{dt}$ are different. To make clear the difference and simplify the analysis, the symbol F was introduced. Equation 2 reduces to:

$$\frac{dS}{dt} = h(t) - S\,g(t) - \frac{dF}{dt} \tag{18}$$

where $h(t) = S_i\,Q/V$, the volumetric substrate loading rate; and $g(t) = Q/V$, the dilution rate as defined by equations 9. Comparing equation 4 shows Westberg's removal rate term to be

$$\frac{dF}{dt} = X\,(\frac{m}{y} - bZ).$$

A similar material balance for the living bacteria is:

$$V\frac{dX}{dt} = q_1\,(\frac{Q + q_1}{q_1 + q_2})\,X - (Q + q_1)\,X + V\frac{dG}{dt} \tag{19}$$

where dG/dt is the internal sludge synthesis rate per unit volume. The symbol G was introduced for the same reason F was used in equation 17. This balance assumes no activated sludge in the sewage itself. Equation 19 can be simplified to:

$$\frac{dX}{dt} = \frac{dG}{dt} - X\,f(t) \tag{20}$$

where

$$f(t) = \frac{q_2\,(Q + q_1)}{V\,(q_1 + q_2)} \tag{5}$$

By combining equations 1 and 5 to eliminate q_2, the regulation function f(t) can be expressed as

$$f(t) = \frac{\beta Q^2 - (1 - \beta)\, Q q_1 - q_1^{\,2}}{V\beta Q} \tag{21}$$

Feed Forward Controller Design

The two material balances were linearized about the steady-state operating point[10] with the following result:

$$\frac{d\bar{X}}{dt} = \left\{ \frac{X \frac{\partial}{\partial X} \left(\frac{dG}{dt}\right) - \frac{dG}{dt}}{X} \right\}_{SS} \bar{X} - \left\{ \frac{X\beta Q^2 + X q_1^{\,2}}{V\beta Q^2} \right\}_{SS} \bar{Q} +$$

$$\left\{ \frac{Q\,(1 - \beta)\, X + 2Xq_1}{V\beta Q} \right\}_{SS} \bar{q}_1 + \left\{ \frac{\partial}{\partial S} \left(\frac{dG}{dt}\right) \right\}_{SS} \bar{S} \tag{22}$$

$$\frac{d\bar{S}}{dt} = - \left\{ \frac{Q}{V} + \frac{\partial}{\partial S} \left(\frac{dF}{dt}\right) \right\}_{SS} \bar{S} + \left\{ \frac{Q}{V} \right\}_{SS} \bar{S}_i + \left\{ \frac{S_i - S}{V} \right\}_{SS} \bar{Q} -$$

$$\left\{ \frac{\partial}{\partial X} \left(\frac{dF}{dt}\right) \right\}_{SS} \bar{X} \tag{23}$$

Once again, the subscript SS denotes steady state, and the variables $\bar{X}$, $\bar{Q}$, $\bar{q}_1$, $\bar{S}$, and $\bar{S}_i$ are all deviations from the steady state values. For example:

$$\bar{X} = X - X_{SS}$$

The terms Q_{SS} and S_{SS} are identical with Q_a and S_{ia} for a 24 hour cycle.

Equations 22 and 23 can be Laplace transformed and solved simultaneously to eliminate $\overline{X}$. This results in:

$$\overline{S} = P_{14}\,\overline{Q} + P_{24}\,\overline{S}_i + P_{34}\,\overline{q}_1 \tag{24}$$

where:

$$P_{14} = \frac{1}{D}\left[\left\{\frac{S_i - S}{V}\right\}_{SS} s + \left\{-\left(\frac{S_i - S}{V}\right)\left(\frac{X\frac{\partial}{\partial X}\left(\frac{dG}{dt}\right) - \frac{dG}{dt}}{X}\right) + \frac{\partial}{\partial X}\left(\frac{dF}{dt}\right)\left(\frac{X\beta Q^2 + Xq_1{}^2}{V\beta Q^2}\right)\right\}_{SS}\right]$$

$$P_{24} = \frac{1}{D}\left[\left\{\frac{Q}{V}\right\}_{SS} s - \left\{\frac{Q}{V}\left(\frac{X\frac{\partial}{\partial X}\left(\frac{dG}{dt}\right) - \left(\frac{dG}{dt}\right)}{X}\right)\right\}_{SS}\right]$$

$$P_{34} = -\frac{1}{D}\left[\left\{\frac{\partial}{\partial X}\left(\frac{dF}{dt}\right)\left(\frac{Q\,(1 - \beta)\,X + 2Xq_1}{V\beta Q}\right)\right\}_{SS}\right]$$

$$D = s^2 + \left\{\frac{Q}{V} + \frac{\partial}{\partial S}\left(\frac{dF}{dt}\right) - \frac{X\frac{\partial}{\partial X}\left(\frac{dG}{dt}\right) - \frac{dG}{dt}}{X}\right\}_{SS} s + \left\{\frac{\partial}{\partial X}\left(\frac{dF}{dt}\right)\frac{\partial}{\partial S}\left(\frac{dG}{dt}\right) - \left(\frac{Q}{V} + \frac{\partial}{\partial S}\left(\frac{dF}{dt}\right)\right)\left(\frac{X\frac{\partial}{\partial X}\left(\frac{dG}{dt}\right) - \frac{dG}{dt}}{X}\right)\right\}_{SS}$$

Equation 24 gives the change in the deviation of the exit substrate concentration $\overline{S}$ as a linear function of inlet flow rate and substrate concentration changes as well as the manipulated variable $\overline{q}_1$. Since the equation is linear and has no interaction terms, the two disturbances can be treated separately. Thus if $\overline{S}$ and $\overline{S}_i$ are zero, equation 24 reduces to:

$$0 = P_{14}\,\overline{Q} + P_{34}\,\overline{q}_1 \tag{25}$$

or

$$\overline{q}_1 = -\frac{P_{14}}{P_{34}}\,\overline{Q} = -F_1\,\overline{Q} \tag{26}$$

Similar reasoning on $\overline{S}$ and $\overline{Q}$ gives:

$$\overline{q}_1 = -\frac{P_{24}}{P_{34}}\,\overline{S}_i = -F_2\,\overline{S}_i \tag{27}$$

where F_1 and F_2 are the feed forward controllers for the respective disturbance variables. Substitution of P_{jk} and simplification leads to somewhat complex expressions for F_1 and F_2. These, however, can be simplified if the sludge synthesis and utilization rate expressions are of the following form.

$$\frac{dG}{dt} = X\,[\phi(S)] \tag{28}$$

$$\frac{dF}{dt} = X\,[\psi(S)] \tag{29}$$

That is, the activated sludge concentration variable can be factored out in both cases. Substitution of equations 28 and 29 into the P_{jk} expressions gives:

$$F_1 = \frac{P_{14}}{P_{34}} = \frac{\left\{\frac{S_i - S}{V}\right\}_{SS} s + \left\{\psi(S)\left(\frac{X\beta Q^2 - Xq_1{}^2}{V\beta Q^2}\right)\right\}_{SS}}{-\left\{\psi(s)\left(\frac{Q(1 - \beta)\ X + 2Xq_1}{V\beta Q}\right)\right\}_{SS}}$$

which when simplified becomes:

$$F_1 = -\left\{\frac{\beta Q^2 + q_1{}^2}{Q(Q - \beta Q + 2q_1)}\right\}_{SS} \left[\left\{\frac{V\beta Q}{\beta Q^2 + q_1{}^2} \quad \frac{1}{X\psi(S)}\ \frac{Q}{V}\,(S_i - S)\right\}_{SS} s + 1\right] \tag{30}$$

At steady state, $\frac{dS}{dt} = 0$, and from equations 9, 18 and 29,

$$\frac{dS}{dt} = \frac{Q}{V}(S_i - S) - X\psi(S) = 0$$

Thus equation 30 becomes:

$$F_1 = -\left\{\frac{\beta Q^2 + q_1{}^2}{Q(Q - \beta Q + 2q_1)}\right\}_{SS} \left[\left\{\frac{V\beta Q}{\beta Q^2 + q_1{}^2}\right\}_{SS} s + 1\right] \tag{31}$$

Similarly,

$$F_2 = -\left\{\frac{V\beta Q}{(S_i - S)\ (Q - \beta Q + 2q_1)}\right\}_{SS} s \tag{32}$$

Equations 31 and 32 show the remarkable feature that, by using equations 28 and 29 for substrate utilization and bacterial growth, the feed forward controllers F_1 and F_2 become independent of the rate expression, i.e., neither $\overline{X}$ nor $\overline{S}$ appear explicitly in equations 31 and 32.

When equations 31 and 32 are placed in standard process control format, utilizing the concepts of gain (K_c) and time constant (τ_D) it becomes apparent that these are common modes of control, F_1 being proportional derivative, and F_2 derivative.

$$F_1 = K_c \, (1 + \tau_D \, s) \tag{33}$$

$$F_2 = K_c' \, \tau_D' \, s \tag{34}$$

with:

$$K_c = -\left\{ \frac{\beta Q^2 + q_1^2}{Q \, (Q - \beta Q + 2q_1} \right\}_{SS} = - \frac{\beta Q_a^2 + q_{1SS}^2}{Q_a \, (Q_a - \beta Q_a + 2q_{1SS})} \tag{35}$$

$$\tau_D = \left\{ \frac{V\beta Q}{\beta Q^2 + q_1^2} \right\}_{SS} = \frac{V\beta Q_a}{\beta Q_a^2 + q_{1SS}^2} \tag{36}$$

$$K_c' = -\left\{ \frac{Q}{S_i - S} \right\}_{SS} = - \frac{Q_a}{S_{ia} - S_{SS}} \tag{37}$$

$$\tau_D' = \left\{ \frac{\beta V}{Q - \beta Q + 2q_1} \right\}_{SS} \quad \frac{\beta V}{Q_a - \beta Q_a + 2q_{1SS}} \tag{38}$$

Overall the equation is:

$$\overline{q}_1 = K_c \, (1 + \tau_D \, s) \, \overline{Q} + K_c' \, \tau_D' \, s \, \overline{S}_i \tag{39}$$

After undergoing inverse Laplace transformation, equation 39 becomes

$$\bar{q}_1 = K_c\, \bar{Q} + K_c\, \tau_D \frac{d\bar{Q}}{dt} + K_c{}'\, \tau_D{}' \frac{d\bar{S}_i}{dt} \quad , \tag{40}$$

and replacing the deviation variables,

$$q_1 = q_{iss} + K_c\, (Q - Q_a) + K_c\, \tau_D \frac{dQ}{dt} + K_c{}'\, \tau_D{}' \frac{dS_i}{dt} \quad . \tag{41}$$

In other words, equation 41 can be expressed in control jargon as "sludge recycle is its steady state value plus proportional control on Q plus derivative control on Q plus derivative control on S_i," in brief: $q_1 = q_{iss}$ + P on Q + D on Q + D on S_i.

All terms of equation 41 have been defined with the exception of the constant q_{iss}. At steady state, equation 20 is equal to zero, and from equation 28

$$f(t)_{SS} = \frac{1}{X}\ \frac{dG}{dt} = \phi(S)_{SS}. \tag{42}$$

Thus from equation 21 and 28

$$\frac{\beta Q_a{}^2 - (1 - \beta)\ Q_a\ q_{1ss} - q_{1ss}{}^2}{V\beta Q_a} = \phi(S)_{SS} \quad , \tag{43}$$

and taking the positive root

$$q_{1ss} = \frac{1}{2} \left\{ Q_a\ (\beta - 1) + [Q_a{}^2\ (1 - \beta)^2 + 4\beta Q_a{}^2 - 4V\beta Q_a\ \phi(S)_{SS}]^{1/2} \right\} . \tag{44}$$

Hence q_{1ss} is seen to be a function of specific growth rate, $\phi(s)$, which in turn is defined by the model selected. All the constants of the control equation thus depend on the model.

<u>Comparison of Controllers</u>

The controller here derived is much more general than the one derived from Westberg's model. First, because no simplification based on frequency analysis was used in its derivation, it is independent of forcing function.

Second, and more important, the controller applies to any model which expresses the internal rate terms as the product of sludge concentration and a function of S as shown in equations 28 and 29. The sanitary engineer will recognize in equation 28 the statement, "Specific growth rate is a function of substrate concentration."

$$\frac{1}{X}\,\frac{dG}{dt} = \phi(S).$$

The classic Monod relationship[30] follows this form with

$$\phi(S) = \mu_{MAX}\,\frac{S}{K_s + S} \quad .$$

All the internal rate terms, substrate utilization $\frac{dF}{dt}$, sludge synthesis $\frac{dG}{dt}$, and bacteria death $\frac{dH}{dt}$, must be of this form. This was true for the four models shown in Table 3. Note that for the Westberg model $\psi(S)$ appears to be a function Z. If, however, the dead bacteria and substrate utilization equations shown are solved simultaneously to eliminate Z, the remaining equation will have a form compatable with equation 29. Thus equations 16 and 17 are perfectly general feed forward

Table 3. Aerator Models

Model's Author	Living Bacteria	Dead Bacteria	Substrate Utilization
Lawrence and McCarty	$\frac{dX}{dt} = \frac{ykSX}{K_s + S} - b'X - Xf(t)$	None	$\frac{dS}{dt} = \frac{Q}{V} S_i - \frac{QS}{V} - \frac{kSX}{K_s + S}$
	$\frac{dG}{dt} = X\left(\frac{yKS}{K_s + S} - b'\right)$	-----	$\frac{dF}{dt} = X\left(\frac{kS}{K_s + S}\right)$
	$\phi(S) = \frac{yks}{K_s + S} - b'$		$\psi(S) = \frac{ks}{K_s + S}$
Eckenfelder (Identical to McKinney[22]).	$\frac{dX}{dt} = K_1 SX - Xf(t)$	None	$\frac{dS}{dt} = \frac{QS_i}{V} - \frac{QS}{V} - \frac{K_1 SX}{a}$
	$\frac{dG}{dt} = X (K_1 S)$	-----	$\frac{dF}{dt} = X\left(\frac{K_1}{a} S\right)$
	$\phi(S) = K_1 S$		$\psi(S) = \frac{K_1 S}{a}$
Westberg	$\frac{dX}{dt} = mX - \frac{cX}{S} - Xf(t)$	$\frac{dZ}{dt} = \frac{cX}{S} - bXZ - Zf(t)$	$\frac{dS}{dt} = \frac{QS_i}{V} - \frac{QS}{V} - bXZ + \frac{m}{y} X$
	$\frac{dG}{dt} = X\left(m - \frac{c}{S}\right)$	$\frac{dH}{dt} = X\left(\frac{c}{S} - bZ\right)$	$\frac{dF}{dt} = X\left(bZ - \frac{m}{y}\right)$
	$\phi(S) = m - \frac{c}{S}$		$\psi(S) = bZ - \frac{m}{y}$

controllers for all the models considered. The generality extends beyond this as any rate expressions proposed in the future, if it is of the required form, will lead to the same controllers. For example, if someone were to propose that substrate utilization were the product of X and a power relationship in S say:

$$\frac{dF}{dt} = X\ (aS^2 + bS + c\ \sqrt{S})$$

this also would have the same feed forward control algorithm.

Controller Testing Procedure

The testing of the controller by computer simulation can be somewhat confusing unless the three parts involved are kept clearly in mind. These are: the forcing functions, the dynamic model, and the controller.

Forcing Functions. The forcing functions dictate what flow rate, Q, and soluble substrate concentration, S_i, are entering the process model as a function of time. Here, the forcing functions were, as before, the sinusoidal equations 6 and 7. It was necessary that the constants, Q_a and S_{ia} be provided. They were, as before, 10,000 m^3/h and 267 g/m^3.

Neither model nor controller have any effect upon the forcing functions. Flow rate and concentration are the independent variables to the process and model. Time is, of course, the independent variable to the forcing functions.

Model. The model accepts three inputs, Q and S_i from the forcing functions, and q_1, the recycle sludge flow rate, from the controller. From these, the model predicts what the substrate and cell mass concentrations, S and X, will be at the outlet of the aeration basin. Equations 18 and 20 comprise the general dynamic model. The specific models require the entry of the internal rate terms from Table 3 with their appropriate constants. This is discussed later with the individual model testing.

Regardless of the rate equation, the volume of the aeration basin must be specified as a constant. It was taken to be 20,000 m^3. Hence mean retention time, $\Theta = \frac{V}{Q_a} = 2$ h.

<u>Controller</u>. The full controller is equation 41 with its terms as defined by the preceding equations. The computor solves the equation yielding q_1 as an input to the model. In addition to the constants previously supplied for the general model and forcing functions, it is now necessary to set the constants β, recycle ratio; and S_{SS}, desired mean soluble effluent concentration. These were taken to be 0.4 and 22 mg/l.

In addition to testing the full control equation, several simplifications of the equation were evaluated for their effect on performance. Simplifications to the controller would result in economies in the implementation of control as will become apparent. These simplifications were applied in four ways as follows:

i. Terms were dropped from equation 41.

ii. Negative values for q_1 were prohibited, that is, sludge storage was made unavailable.

iii. Equation 44 for q_{1SS} was simplified.

iv. Arbitrary constants were applied to the gains K_c and K_c'.

It was permissible to drop one or all of the last three terms of equation 41 in order to simplify the control policy. The terms which were in use for the respective runs are indicated in Tables 4 and 5 under "Control Mode." "No Control" indicates, $q_1 = q_{1SS}$ with no other terms. Under flow, "P" indicates the inclusion of the second term and "D" the third; under concentration, "D" indicates the fourth term. The elimination of the fourth term is of interest because it would obviate the necessity to measure S_i which is a much more difficult and expensive procedure than the measurement of Q.

The availability of sludge storage was a mathematical rather than a physical concept. When sludge storage was available, q_2 could take negative values to supply the needs of the recycle stream when $q_1 > \beta Q$ according to equation 1. In the absence of sludge storage, q_2 was restricted to positive values. No attempt to select a proper reservoir for sludge storage was made. However, if the need for sludge storage could be eliminated an expensive reservoir would not be required.

Equation 44 for q_{1SS} was simplified by setting q_{1SS} equal to βQ_a or a multiple thereof. This eliminated the dependency of control on the specific growth rate function $\phi(S)$. If the performance of the controller did not deteriorate too severely, then control could be applied without postulating any growth model. The selection of βQ_a resulted from the observation that it is the maximum value that q_1 can assume in the absence of sludge storage (equation 1). Further the substitution of βQ_a for q_{1SS} greatly simplifies the controller:

$$K_c = \beta$$

$$\tau_D = \frac{V}{Q_a\ (1 + \beta)}$$

$$K_c' \tau_D' = \frac{\beta V}{(S_{ia} - S_{SS})(1 + \beta)}$$

A value of 1/2 βQ_a was also tested in this fashion. An entry of "Design" under controller constants indicates the full use of equation 41.

The final manipulation was the application, somewhat arbitrarily, of a factor to the gains K_c and K_c' for runs 10 and 11.

Once again, a change, or simplification to the control equations does not imply any change of the model equations. For example, the elimination of $\phi(S)$ from the controller did not alter its use in the model equation for specific sludge growth.

Results of Controller Testing

In order to test the controllers, two models of the activated sludge process were studied by computer simulation. To facilitate the comparison of the models, the values for the following parameters which were used in the study of the Westberg model were used again: $\beta = 0.4$, $\Theta = 2.0$ h, $S_{ia} = 267$ g/m^3, $Q_a = 10{,}000$ m^3/h, $S_{SS} = 22$ g/m^2. Likewise, the forcing functions given by equations 5 and 6 were used.

The first system studied was that of Larwrence and McCarty[27] whose dynamic model is summarized in Table 3. The following choice of kinetic coefficients seemed reasonable based on reported literature values: $y = 0.67$ g/g, $b' = 0.00291$ h^{-1}, $k = 0.233$ g/gh and $K_s = 22.0$ g/m^3. The various conditions studied on the computer are summarized in Table 4, while the more significant dynamic results are summarized in Figures 7, 8, 9 and 11. Figure 7 presents results when settled sludge storage is available. Runs 1 thru 5 are increasingly more complicated control with run 5 using the full proportional-derivative controller on flow rate and a derivative controller on inlet substrate concentration. The improvement in control is quite evident with the run 5 controller which always limits the exit substrate concentration $S \leq S_{SS} = 22.0$. It should be pointed out that run 2 indicates that approximately 75% of the dynamic improvement comes from the addition of proportional feed forward control on flow rate.

Figure 8 shows the very beneficial effect of sludge storage on both the partial controller (P on Q, D on S_i) in reducing the maximum exit concentration by 30 percent, and the full controller in making the control fully effective.

The effect of making the controller constants independent of the aerator model is shown in Figure 9. All runs were made with proportional control on Q and derivative control on S_i, and sludge storage was not available. In run 6, q_{1SS} was obtained through solution of equation 44 and hence was dependent on the model because of the appearance of $\phi(S)$ in that equation. In the other three runs q_{1SS} was set equal to a constant independent of $\phi(S)$: βQ_a in run 8 and 1/2 βQ_a in run 9. For the constants used here q_{1SS} took the numerical values: 3556 m^3/h in run 6; 4000 in run 8; and 2000 in run 9. The significant result was that control was not sensitive to the value of q_{1SS} and hence independent of $\phi(S)$ and the model.

The gains of the controllers were manipulated in runs 10 and 11. Neither an increase nor a decrease in their values improved performance.

The second model simulated on the IBM 360 computer was Eckenfelder's (see Table 3). Eckenfelder proposed a value of 0.39 g/g for a, and a value of $K_1 = 0.00227$ m^3/g h was chosen because it seemed consistent

Table 4. Results for the Feed Forward Control of the McCarty Activated Sludge Process Model

Run Number	Control Mode: Flow	Control Mode: Concentration	Sludge Storage	Controller Constants	S_{MAX}	S_{MIN}	% Of Time Under S_{SS}
1	No Control	No Control	Yes	None	290.6	2.3	30
2	P	None	Yes	Design	87.7	3.1	54
3	None	D	Yes	Design	253.8	3.1	26
4	P	D	Yes	Design	32.7	6.1	55
5	PD	D	Yes	Design	22.0	10.1	100
6	P	D	No	Design	46.6	6.6	46
7	PD	D	No	Design	57.7	14.2	24
8	P	D	No	$q_{1SS}=\beta Qa$	46.7	6.3	48
9	P	D	No	$q_{1SS}=1/2\beta Qa$	46.4	8.5	43
10	P	D	No	$q_{1SS}=\beta Qa$*	85.9	5.3	42
11	P	D	No	$q_{1SS}=\beta Qa$**	47.1	7.4	53
12	PD	None	No	Design	46.7	6.2	48
13	PD	None	Yes	Design	35.4	5.8	54

*K_c & K'_c decreased by 1/4

**K_c & K'_c increased by 1/4

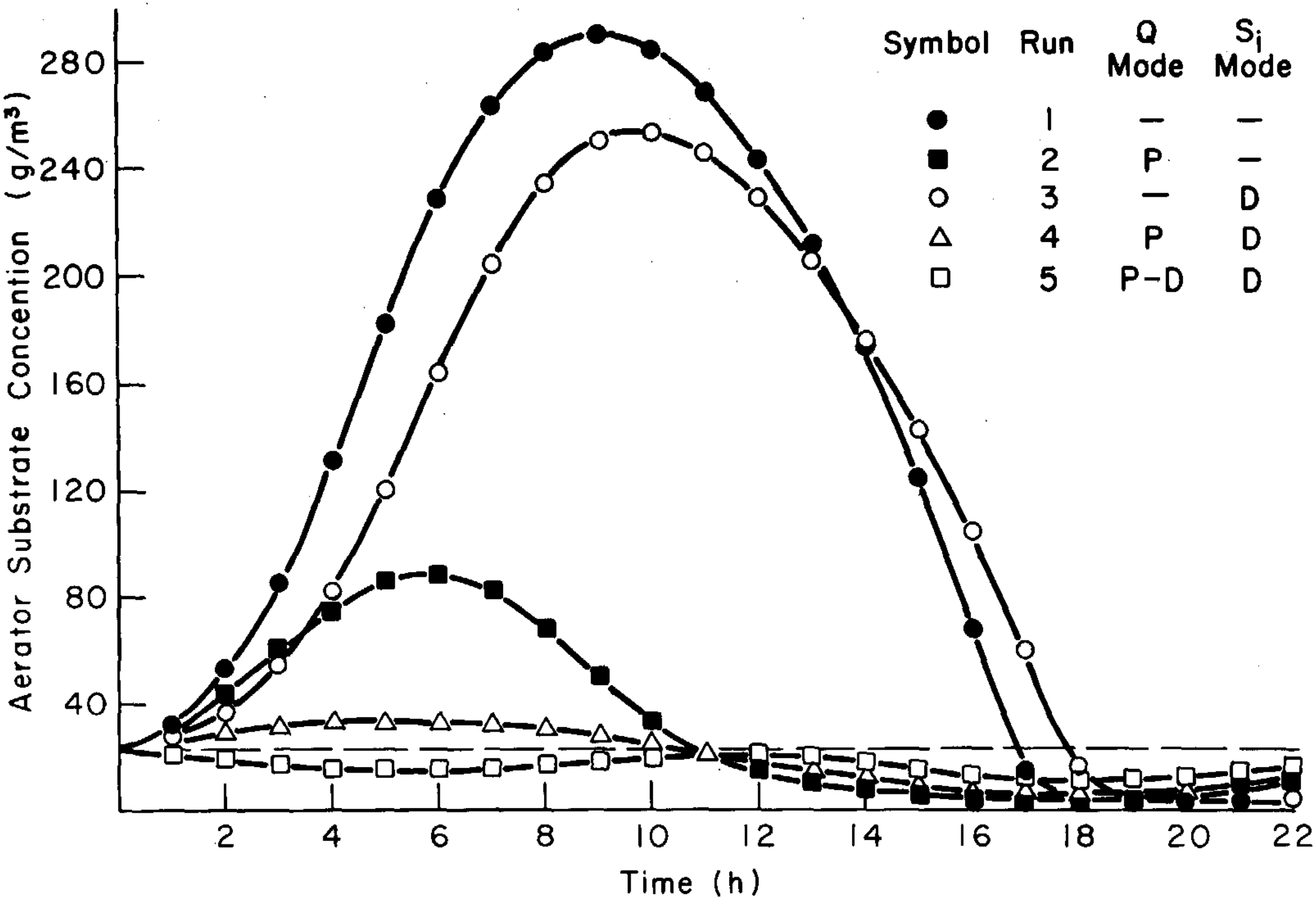

Figure 7. The Effect of Various Control Modes, with Sludge Storge, on the Exit Substrate Concentration Using McCarty's Model.

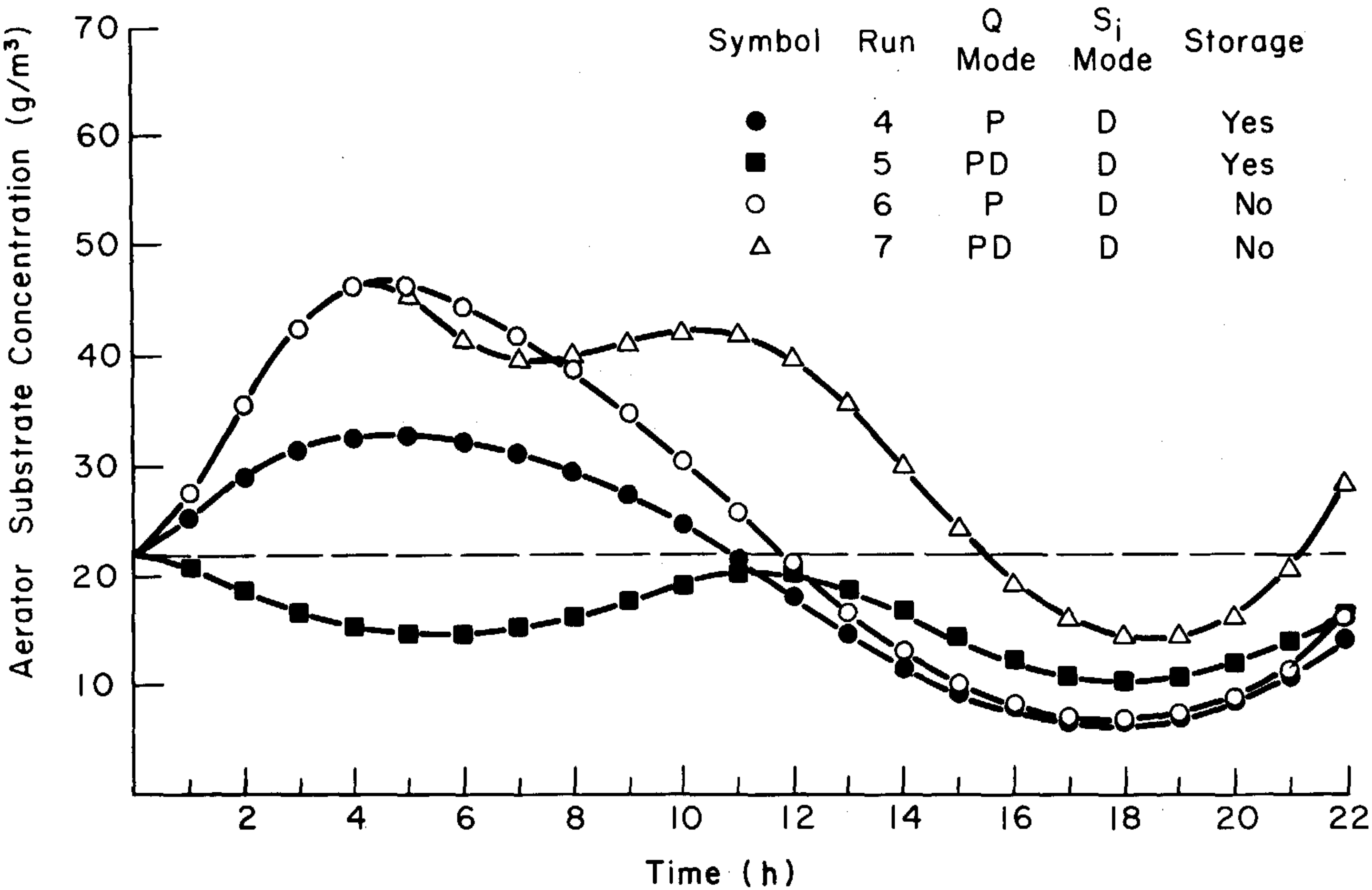

Figure 8. The Influence of Sludge Storage on the Control of the Exit Substrate Concentration from McCarty's Process Model.

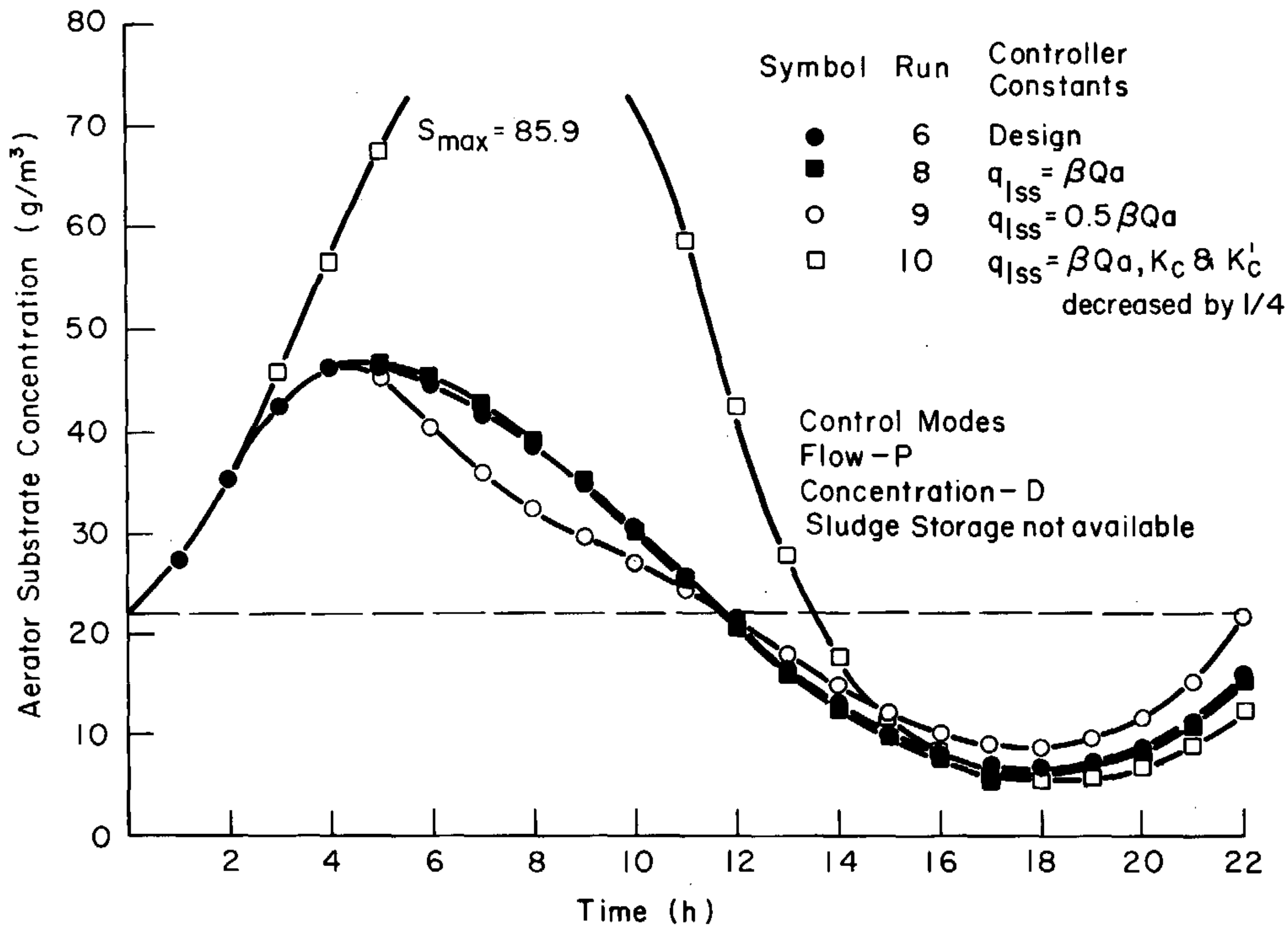

Figure 9. The Effect of Using Controller Constants not Dependent on Aerator Model on the exit Substrate Concentration Using McCarty's Model.

with the constants used in the McCarty model. Table 5 summarizes the computer runs and Figure 10 is a plot of the dynamic results.

In all cases the maximum peak heights are less than the corresponding cases using the Lawrence and McCarty model. Once again, using proportional control, measuring inlet flow rate, and manipulating the recycle sludge accounts for 60% of the reduction in S_{MAX}.

The last comparison was based on the premise that both sludge storage and measurement of S_i would be difficult and expensive to implement, especially in existing plants. Figure 11 is a display of the results using McCarty's model. It is apparent that the provision of sludge storate is more important than monitoring influent substrate concentration for control. Also a comparison of run 12 with run 2, Figure 7, indicates the salutary effect of adding derivative control on flow in that maximum exit concentration was almost halved.

Clarifier Model

A complete settler model would be one that could predict the performance of both the clarifying and thickening operations of the final settling basin as functions of the influent sludge solids concentration which in turn could be predicted from the aerator model. Prediction of performance in the clarifying mode would include the motion of the top of the sludge blanket and if possible the escape of discrete sludge solids from the sludge blanket to the overflow. On the other hand, a model of the thickening mode would foretell the underflow concentration.

The most serious problem is the escape of solids from the sludge blanket. Unfortunately, only the beginings of a quantitative analysis of this problem have been made[2]. The phenomenon was neglected

Table 5. Results for the Feed Forward Control of the Eckenfelder Activated Sludge Process Model

Run Number	Control Mode Flow	Control Mode Concentration	Sludge Storage	Controller Constants	S_{MAX}	S_{MIN}	% Of Time Under S_{SS}
14	No Control	No Control	Yes	None	102.5	2.3	39
15	P	None	Yes	Design	41.3	5.1	55
16	P	D	Yes	Design	27.1	9.1	54
17	P	D	No	Deisgn	34.5	10.4	42
18	P	D	No	$q_{1SS}=\beta Qa$	34.5	10.1	44

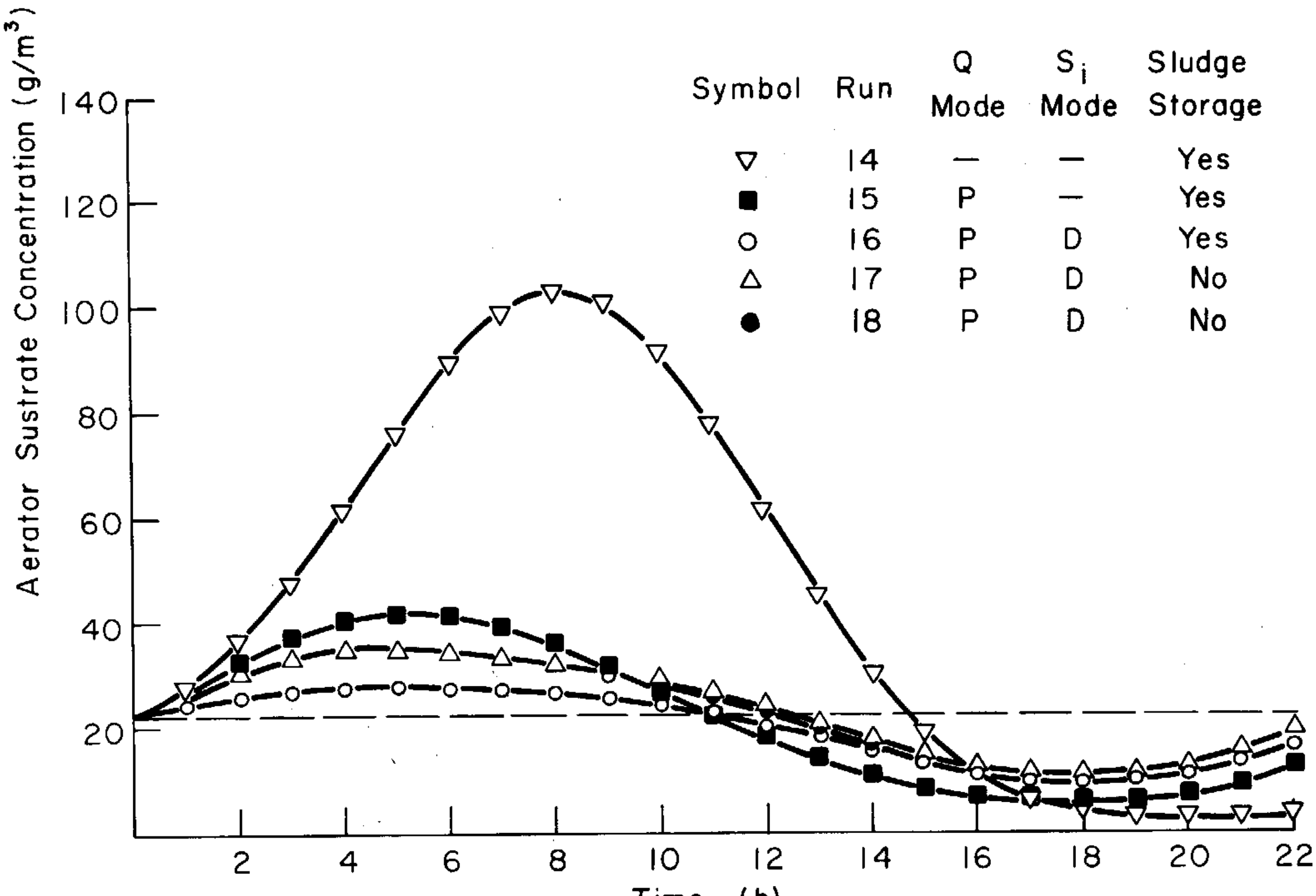

Figure 10. The Effect of Various Control Modes on the Exit Substrate Concentration Using Eckenfelder's Model

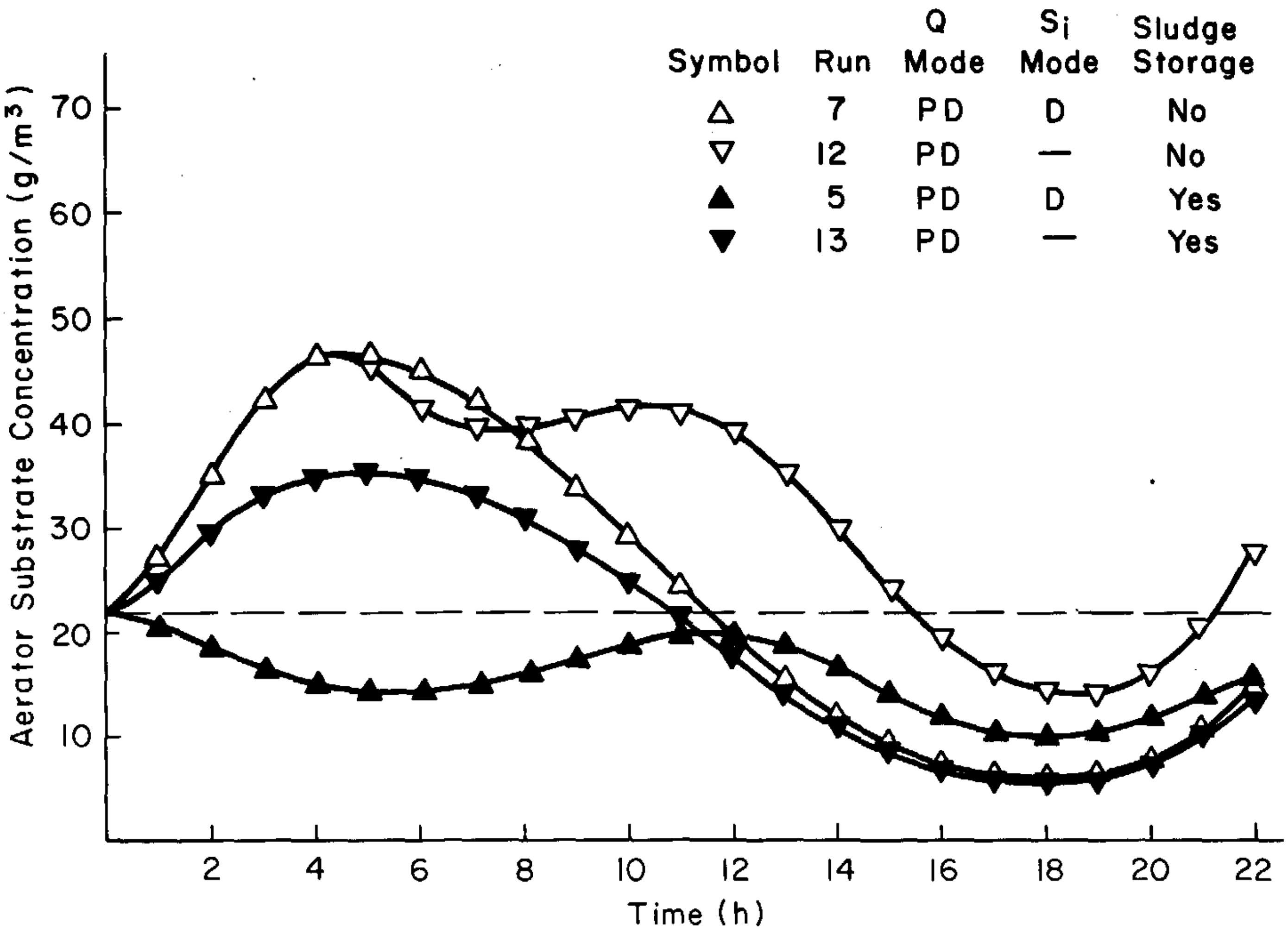

Figure 11. The Influence of Control Based on Influent Concentration on the Substrate Concentration with and without Sludge Storage Using McCarty's Model

in the model finally developed herein.

The quantitative approach to settler performance has been to develop criteria for design of a basin at steady-state rather than to predict temporal variations. A notable exception was a study by Rex Chainbelt, Inc., funded by the Environmental Protection Agency to develop a mathematical model of a final clarifier predictive of return sludge concentration and effluent suspended solids[1]. The research resulted in empirical equations based on data obtained at three sewage treatment plants. These equations did not provide the model desired.

The clarification function of a final clarifier is served if the area of the clarifier is sufficient so that the rise rate of the supernatant does not exceed the settling velocity of the slowest settling solids.[8] This velocity is usually considered to be the zone settling velocity of the mixed liquor suspended solids issuing from the aerator[19].

It was decided to neglect all functions of the final clarifier except that of thickening. What was essential to this research was the prediction of sludge concentration in the underflow from the clarifier as an input to the aerator so that the two could be coupled.

The coupling of aeration and settling has been done by Berthouex and Polkowski[3] in a statistical analysis of the optimum steady-state design. Unfortunately they did not derive relationships useful in the prediction of time-dependent performance for the system.

Thickening Theory

As pointed out by Edde and Eckenfelder,[20] thickening is affected by laboratory test vessel diameter, initial height of the test suspension, raking action, rheological properties of the sludge, and sludge blanket depth. However, as was first pointed out by Coe and Clevenger[9] and much later proven mathematically by Kynch[26], the basic property governing settling is solids concentration. Unfortunately, thickening performance predicted solely on the basis of concentration must be considered an idealization especially for activated sludge because flocculation effects are ignored. However, for the purposes of this paper, and as a first approximation, sludge concentration was taken as governing.

Thickening theory has been reviewed by Debelak[16], Dick[12], and Dick and Ewing[13] among others. The basic approach used herein was to use the Kynch theory as developed by Hasset[23].

Thickener Model

Following Hasset's procedure and the notation adopted herein, the theory is developed as follows: The mixed liquor at concentration X is fed at flow rate $Q + q_1$ into a cylindrical clarifier in an infinitely thin layer over the whole cross-section at the top of the volume of concentrating solids. At this layer, the suspension divides into an upflow having an overflow discharge rate $Q - q_2$, and a downflow with an underflow discharge rate: $Q_u = q_1 + q_2 = \beta Q$. If the settling basin is of uniform cross-section, the downflow linear velocity (or volumetric flow rate per unit area) will be $U = \frac{Q_u}{A}$. The particles of the suspension are also moving downwards relative to the liquid with a velocity taken to be equivalent to that of batch settling, u. Hence the downwards velocity induced by the underflow pumping rate is augmented by the particle settling rate, and the total solids flux becomes:

$$E_T = X_s\ (U + u). \tag{45}$$

(Note: Hasset used volumetric concentration vol/vol throughout; we have proceeded directly to mass concentration mass/vol.).

Because the bulk suspension is proceeding downwards at U, it is possible to define a "kinetic solids concentration" which is the ratio of solids mass flux to total volumetric flux $(UX_s + uX_s)/U = X_s + (u/U)X_s$. At steady state, this kinetic concentration does not change with depth below the feed and also must be equal to the discharge concentration X_u. Hence the relationship between solids concentration X_s at any level and the underflow concentration is: $X_s = X_u/(1 + u/U)$. This reasoning predicts that if sludge is withdrawn uniformly from a cross-section at the bottom of the settler, there will be a step increase in the concentration at the moment of withdrawal from some end concentration X_u' to the discharge concentration X_u.

As shown by Shannon and Tory[35] a single batch settling curve (wherein the height of the interface between supernatant and sludge is observed with time) can be used to predict a sludge settling flux curve. Alternately and preferably[12], several batch settling tests are performed at differing initial concentrations and the initial settling velocities are taken to calculate the flux curve. By either method, a batch flux curve would be similar to Figure 12A.

Figure 12B shows the sum of: the batch flux curve which gives the flux due to settling, E_U; and the induced flux curve $E_U = X_sU$ to yield the total potential solids flux for a given underflow volumetric flow rate, Q_u, and area, A. The minimum on the total flux curve is the maximum solids handling rate of the settler, E_M; and the limiting concentration is X_{LM}.

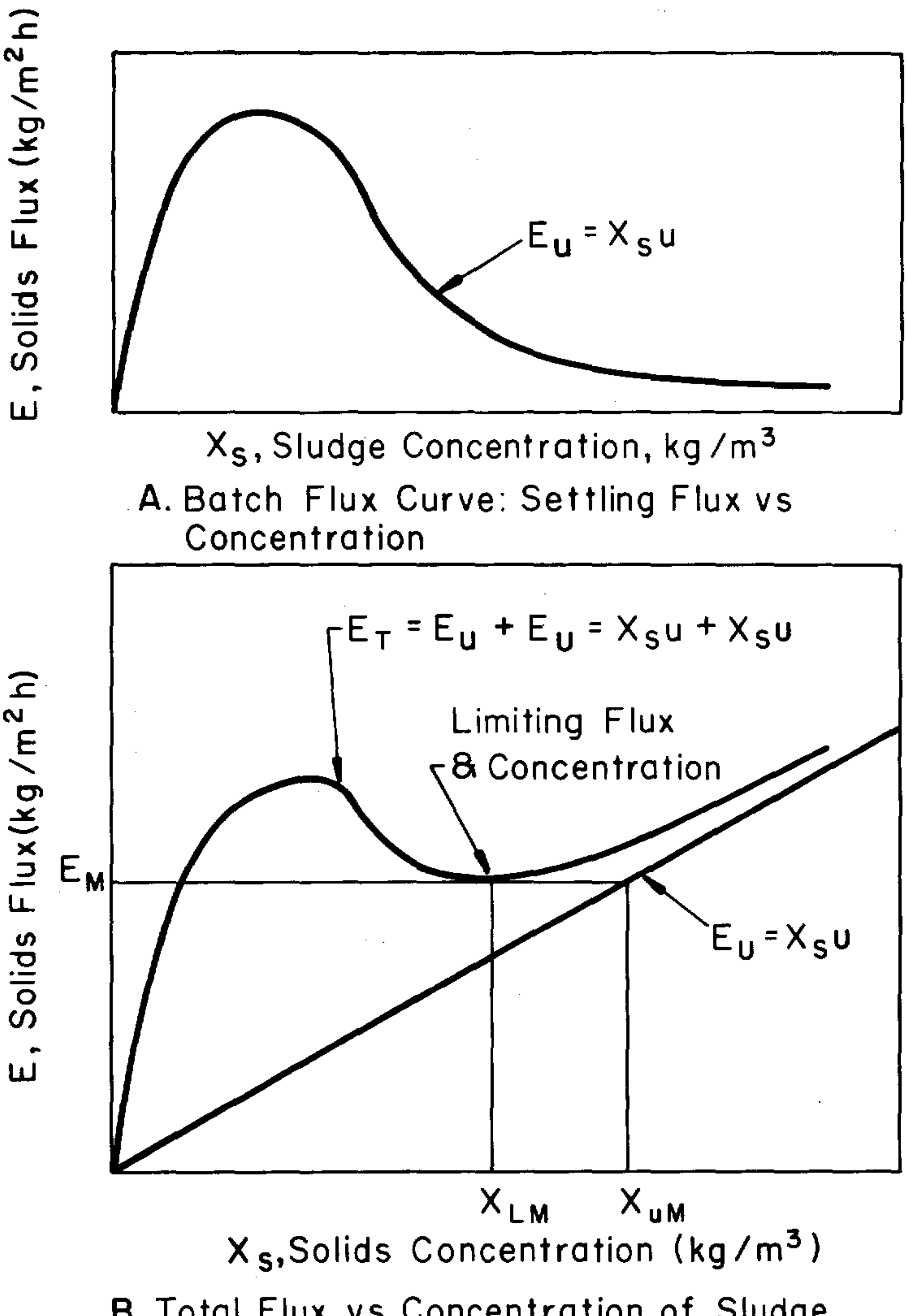

Figure 12. Total Flux from Batch Flux Curve, Limiting Flux in a Cylindrical Settler

Thickening in a Conical Section. Thus far the discussion has dealt with a cylindrical settling basin. If the basin is considered to have a cylindrical portion of area A_M above an inverted cone, the maximum solids handling capacity will remain E_M as set by the area of the cylinder. However, the settler operates normally at less than maximum capacity. The extension of the model to this is best explained by considering Figure 13, where the flux curve of Figure 12B is reproduced for further manipulation.

At less than maximum capacity, the flux in the cylinder will fall from E_M to E_F, the operating flux, and the underflow concentration will likewise fall to X_u from X_{uM} where $E_F = (Q + q_1)\, X_s/A_M = (Q + q_1)\, X_s/(Q_u/U)$. In the conical section, cross-sectional area is no longer A_M, but some other value A depending upon position in the cone, and induced velocity at A becomes:

$$U_A = U \frac{A_M}{A} \quad .$$

There will likewise be an induced solids flux dependent on A:

$$E_{UA} = UX_s \ \frac{A_M}{A} = E_U \frac{A_M}{A} \ ,$$

and the formulation for total flux in the cone becomes

$$E_T = X_s \ U \frac{A_M}{A} + X_s \ u \ . \tag{46}$$

On the flux curve, the result will be a counter-clockwise rotation of the induced flux line. As a consequence, the total flux curve will also be displaced upwards as will the horizontal line represented by E_F. The horizontal and the induced flux lines are displaced by the same ratio, $\frac{A_M}{A}$, and will always intersect at X_u. The limiting flux will be found when the horizontal line once again has a point of tangency with the total flux curve.

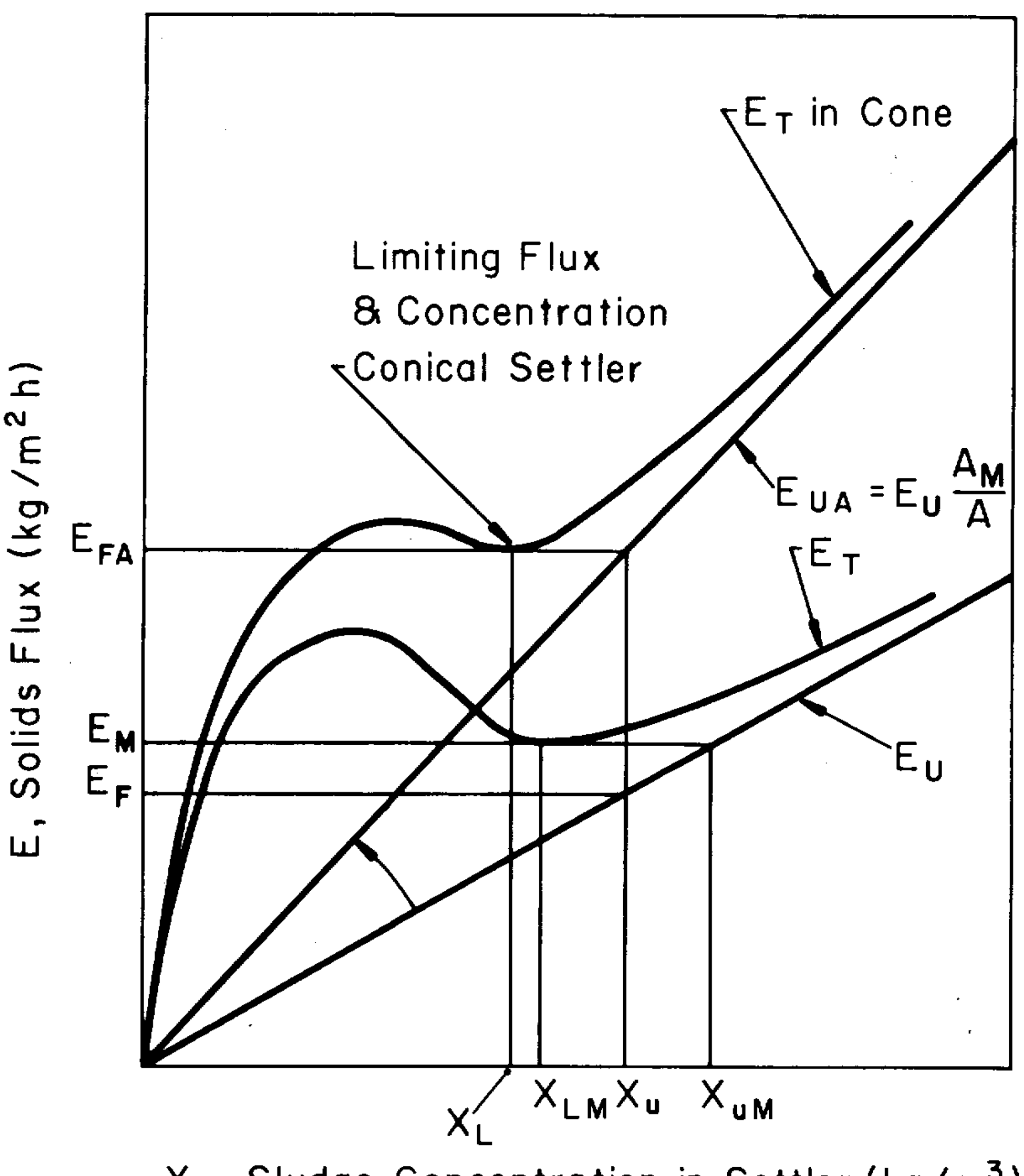

Figure 13. Determination of Limiting Flux in a Conical Settler

Stated another way, at a particular value of A, corresponding to some particular height in the conical section, the lines will touch and an abrupt change in concentration will occur. This can be imagined as the liquid solids interface in a batch settling. However, unlike a batch, there is a low concentration of solids passing through the liquid. When they reach the height of the sludge layer there is an abrupt increase in concentration at that level. The concentration at the top of the sludge layer is given by X_L. Down through the sludge depth the concentration increases according to

$$X_s = \frac{X_u}{1 + u/U_A}$$

where u corresponds to the particular value of X_s at the level. At the discharge opening the concentration becomes

$$X_u' = \frac{X_u}{1 + \frac{A_D}{A_M}\frac{u}{U}}$$

and if $A_M >> A_D$ then $X_u \cong X_u'$.

Under normal operating conditions, the height of the sludge layer would be somewhere in the conical region. At maximum loading, the sludge layer would rise into the cylindrical part of the thickener. The concentration at the top of the sludge layer wouldbe X_{LM} and the underflow concentration X_{uM}.

Should the loading exceed the solids handling capacity of the layer at concentration X_{LM}, the sludge layer will rise to the feed level with a constant concentration zone of X_{LM}. Above the feed level, the concentration will be that of a suspension having a settling velocity equal to the rise rate $\frac{Q - q_2}{A_M}$.

Hasset proposed that this analysis was applicable to the general case and not only to settlers of conical structure. He stated:

> In any operating thickener with a small discharge opening, which is the usual case, the flow pattern in the lower region will approximate to the form of an inverted cone, so that a zone of increasing concentration towards the discharge can always be expected[23].

Volume Requirement. Procedures to calculate a volume requirement for compaction are presented by several authors[21, 29, 33] but the calculations yield little information because the calculated depths are usually around three feet. In practice then, the depth is customarily taken to be three feet.

Formulation of the Batch Flux Curve. In order to make use of Hasset's analysis by computer simulation, it was necessary to have a formula approximating the batch settling curve. Dick and Ewing in the closure[14] to their paper[13] evaluated the formulae relating settling velocities to sludge concentration which had been put forward by the discussants. These are presented in Table 6 with notation changed to conform to that adopted herein.

All three are empirical and have shortcomings, but that of Vesilind[37] was used for the following reasons. It yielded a curve intermediate between the other two, had successfully been used by Berthouex and Polkowski[3], was further justified by Vesilind[38], and finally had the benefit of simplicity. However, no physical significance should probably be ascribed to u_0; it should be considered merely a constant for a given sludge.

Table 6. Empirical Equations for Sludge Settling Rate as a Function of Concentartion

Equation Presented by	Equation	Variables
Krone[25]	$u = u_o (1 - KX_s)^5$	u = settling velocity of sludge at concentration X_s.
Duncan and Kawata[17]	$u = aX_s^b$	u = settling velocity of the individual aggregate particles of sludge
Vesilind[37]	$u = u_o e^{-k'X_s}$	a,b,k,K = constants for a particular sludge

Development of Working Model

Using the information presented, the dynamic model for the final clarifier was developed as follows.

For a thickener with a conical bottom as in Figure 14, γ is the angle between the sloping side of the tank and the vertical. The radius R, at any height is then equal to

$$R = H \text{ Tan } \gamma$$

where H is the height from the apex. The cross sectional area is then

$$A = \pi H^2 (\text{Tan } \gamma)^2 . \tag{46}$$

In a thickener there must be a discharge opening with a cross sectional area A_D. This will cut the cone at the bottom at a height z above the apex. The volume above the discharge will be

$$V_c = \frac{\pi(H^3 - z^2)(\text{Tan } \gamma)^2}{3} .$$

By putting a mass balance around the thickener of Figure 1, and assuming that no solids are carried over the weirs.

$$(Q + q_1) X = Q_u X_u$$

$$X_u = \frac{(Q + q_1)X}{Q_u} \tag{47}$$

Solids are transported to the bottom of the thickener through the cone by settling and convective flow given by

$$E_T = X_s U \frac{A_M}{A} + X_s u . \tag{48}$$

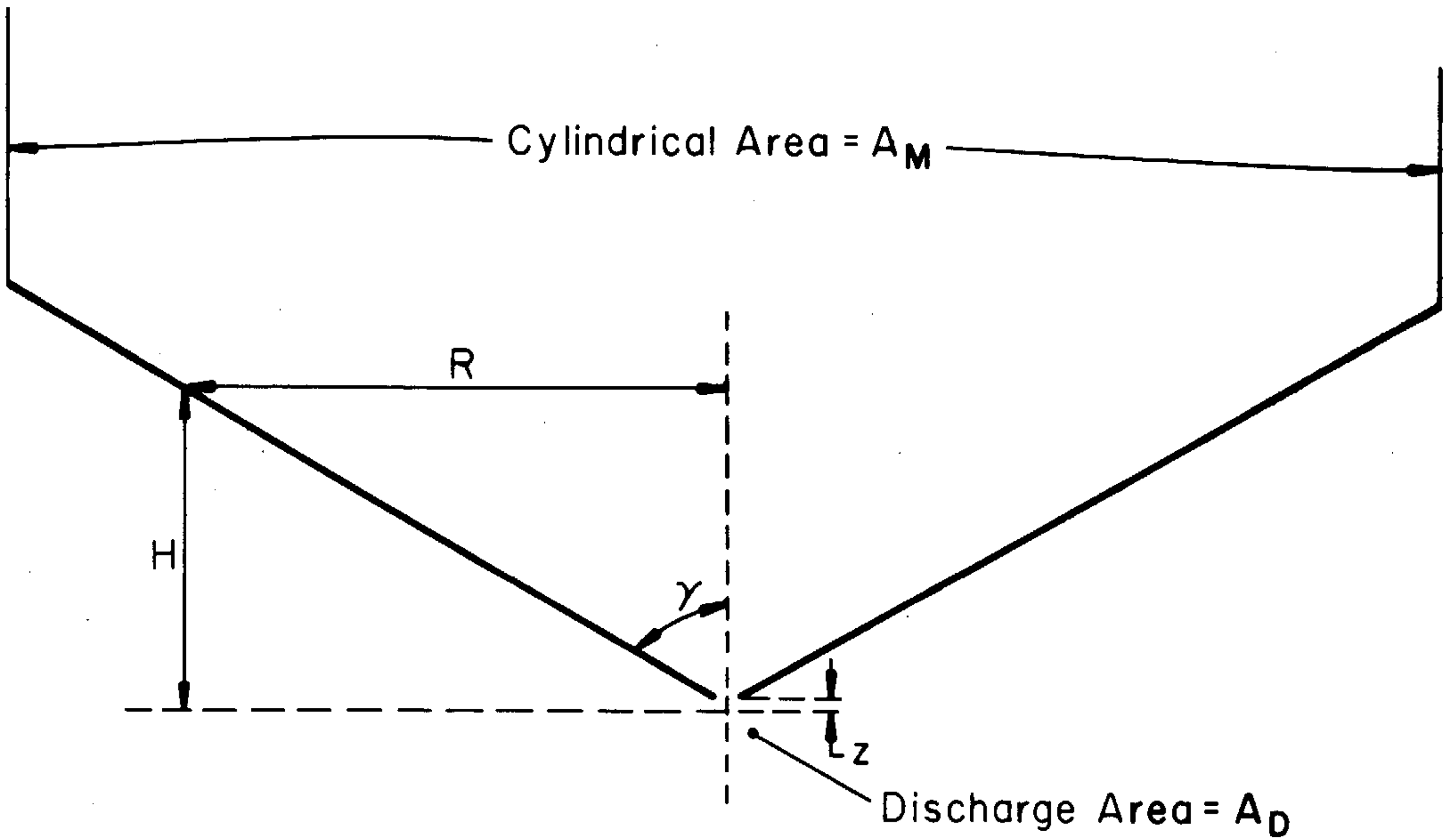

Figure 14. Diagram of a Clarifier with a Conical Lower Portion

where U is the downward velocity of the bulk liquid phase in the cylinder below feed level. Taking Vesilind's equation[37] for settling velocity as a function of concentration

$$u = u_o \, e^{-k'X_s} \tag{49}$$

and substituting into equation 48 gives:

$$E_T = X_s \, U \frac{A_M}{A} + X_s \, u_o \, e^{-k'X_s}. \tag{50}$$

As was pointed out, the minimum of the total flux curve in the cone gives the maximum solids handling capacity of the settler. Because the slope of the tangent to the curve at this point is zero, differentiating equation 50 with respect to X_s and setting the differential equation equal to zero will yield the limiting value of X_s, which is termed X_L. This is the concentration of the layer transmitting the maximum flux.

$$\frac{dE_T}{dX_s} = U \frac{A_M}{A} + (1 - k'X_L) \, u_o \, e^{-k'X_L} = 0 \tag{51}$$

From Figure 13 and equation 49

$$E_{FA} = X_u \, U \frac{A_M}{A} = X_L \, U \frac{A_M}{A} + X_L \, u_o \, e^{-k'X_L} \quad . \tag{52}$$

Solving for U in equation 51 and substituting in equation 52

$$k'X_uX_L - X_u = k'X_L^2 \; .$$

In turn, solving for X_L and taking the larger root

$$X_L = \frac{X_u}{2} + \left(\frac{X_u^{\,2}}{4} - \frac{X_u}{k}\right)^{1/2} . \tag{53}$$

Rearranging equation 52,

$$A = \frac{A_M U}{(kX_L - 1)\, u_o\, e^{-k'X_L}} = \frac{Q_u}{(k'X_L - 1)\, u_o\, e^{-k'X_L}} . \tag{52a}$$

The cross-sectional area A where the concentration is X_L can therefore be determined. Knowing A, H can be determined from rearrangement of equation 46.

$$H = \frac{1}{\mathrm{Tan}\ \gamma} \left(\frac{A}{\pi}\right)^{1/2} \tag{46a}$$

This development follows from steady state considerations. To summarize, the mass balance (equation 47) yields the value of X_u corresponding to X issuing from the aerator. The concentration at the top of the sludge blanket, X_L, is in turn calculated from equation 53. Using this value for X_L, the area, and hence the height, of the sludge blanket interface are calculated from equations 52a and 46a.

Coupling of Aerator and Settler Model

To make use of steady state values in a regime where Q and X are varying with time, as in the aerator model, the concept of time delay was used. At a given instant, the mixed liquor solids concentration coming from the aerator will be X for which there is a corresponding steady state value of underflow concentration, X_u, that will appear in the underflow some variable time interval L later. Mathematically, this is expressed

$$X_u(t) = X_u(t - L). \tag{54}$$

Essentially, the final clarifier remains completely efficient but a variable time delay L in its return of sludge to the aerator is introduced.

The substrate balance around the aerator continues to be

$$\frac{ds}{dt} = h\ (t) - S\ g(t) - \frac{dF}{dt}\ . \tag{18}$$

But the organism balance becomes

$$\frac{VdX}{dt} = q_1 X_u(t - L) - (Q + q_1)\ X + \frac{VdG}{dt}\ . \tag{55}$$

Because no functional relationship to predict L was derived, the time delay was realized in the computer simulation by an iterative technique. A one-dimensional matrix was set up in the computer representing incremental layers in the cone of the clarifier. At the time X issued from the aerator the calculations yielding $X_u(t)$, A, and H were performed. This input at time t was considered to be parcel that moved downward through the matrix at an induced sludge velocity U_A;

$$U_A = \frac{Q_u}{A_h}\ . \tag{56}$$

The area A_h is that of the incremental layer in which the parcel is found at the time of the iteration. When the parcel was found to have entered the lowest layer, X_u of the parcel was released to the aerator.

There was a compromise in this technique in that the velocity of the particles relative to the bulk fluid was disregarded.

Two variations on this model were run. In the first, Q_u was set equal to βQ_a, hence it was a constant and induced velocity U_A, equation 56, was a function of the area of the conical increment only. In the second variation, $Q_u = \beta Q$, and Q_u depended on the forcing function as well as the area.

Control of Coupled Process - Davis' Controller

The first attempt to control the process thus coupled was with Davis' controller, equation 41. The results are presented in Table 7 and Figures 15 and 16. The forcing functions were once again equations 6 and 7. The following constants, steady state values, and kinetic coefficients based on literature values for the Eckenfelder model were used for the aeration portion of the model: $\beta = 0.4$; $S_{ia} = 0.267$ kg/m^3; $Q_a = 250$ m^3/hr; $S_{SS} = 0.022$ kg/m^3; $a = 0.39$ kg/kg; and $K_1 = 1.13$ m^3/kg hr. The first three runs were made with $Q_u = \beta Q_a$.

For the final clarifier portion of the model, settling data from the literature[13] were obtained, and values of u_o and k' were determined for equation 49

$$u = u_o \, e^{-k' X_s} \tag{49}$$

which relates settling velocity as a function of concentration. These values were $u_o = 7.514$ m/hr and $k' = 1.0$ kg^{-1}. It should be noted that

Table 7. Results for Feed Forward Control of the Eckenfelder Aerator Model Coupled to the Settling Model - Davis Controller

Run Number	Model Q_u	Control Mode Flow	Control Mode Concentration	Sludge Storage	S_{MAX}	S_{MIN}	H_{MAX}	H_{MIN}
1	βQ_a	No Control	No Control	Yes	0.056	0.0044	1.78	0.819
2	βQ_a	P	None	Yes	N. O.	N. O.	8.42	0.541
3	βQ_a	P	None	No	0.088	0.0055	1.98	0.492
4	βQ	No Control	No Control	Yes	0.099	0.0058	4.05	0.371
5	βQ	No Control	No Control	No	0.085	0.0089	1.27	0.368
6	βQ	P	None	Yes But Not Needed	0.047	0.0057	1.72	0.891
7	βQ	P	D	Yes	0.048	0.0060	2.55	0.710
8	βQ	P	D	No	0.057	0.0082	1.93	0.411

N. O. - Not Obtainable

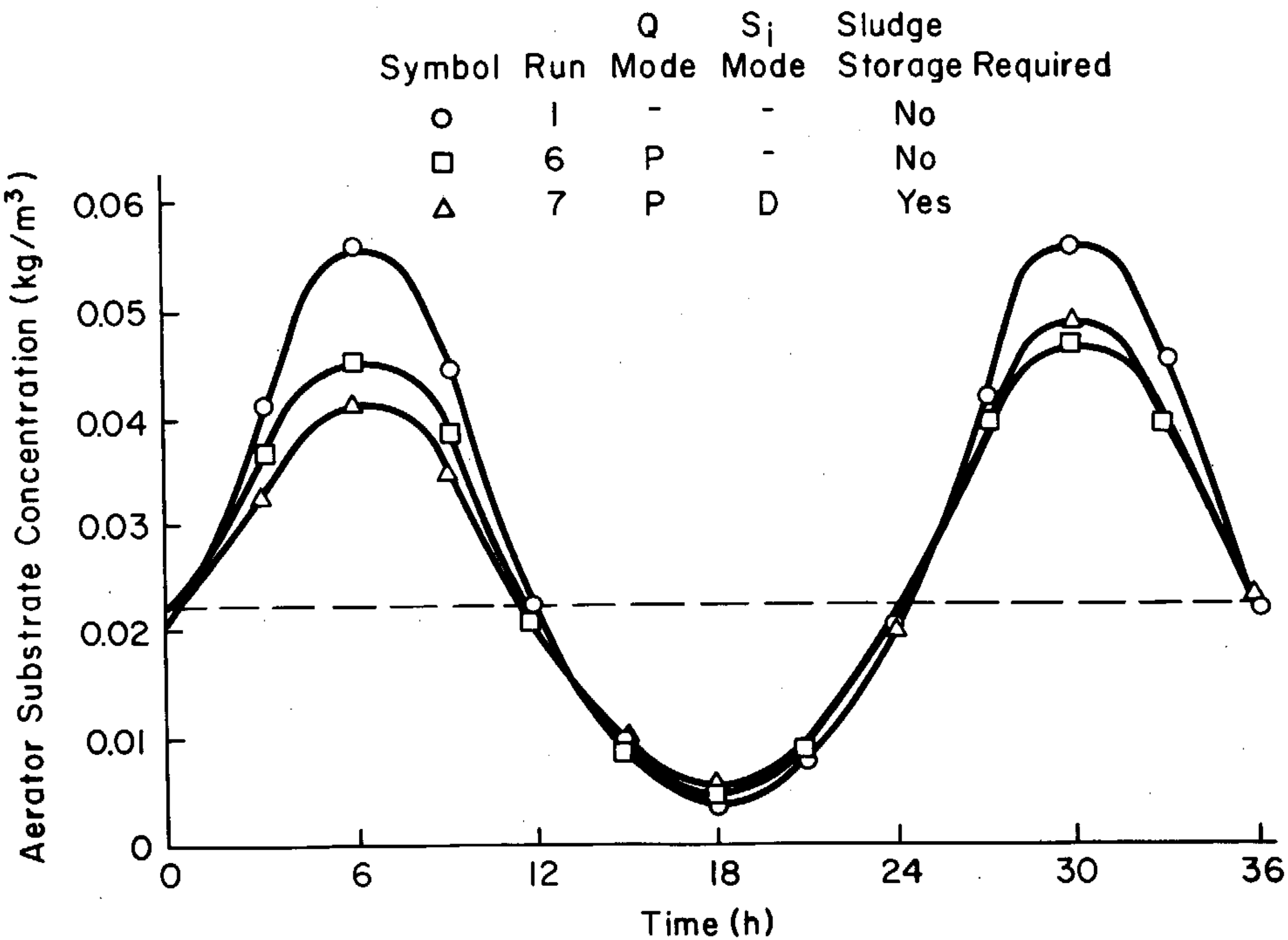

Figure 15. Effect of Various Control Mode on Exit Substrate Concentration Using Davis Controller with the Coupled Process Model

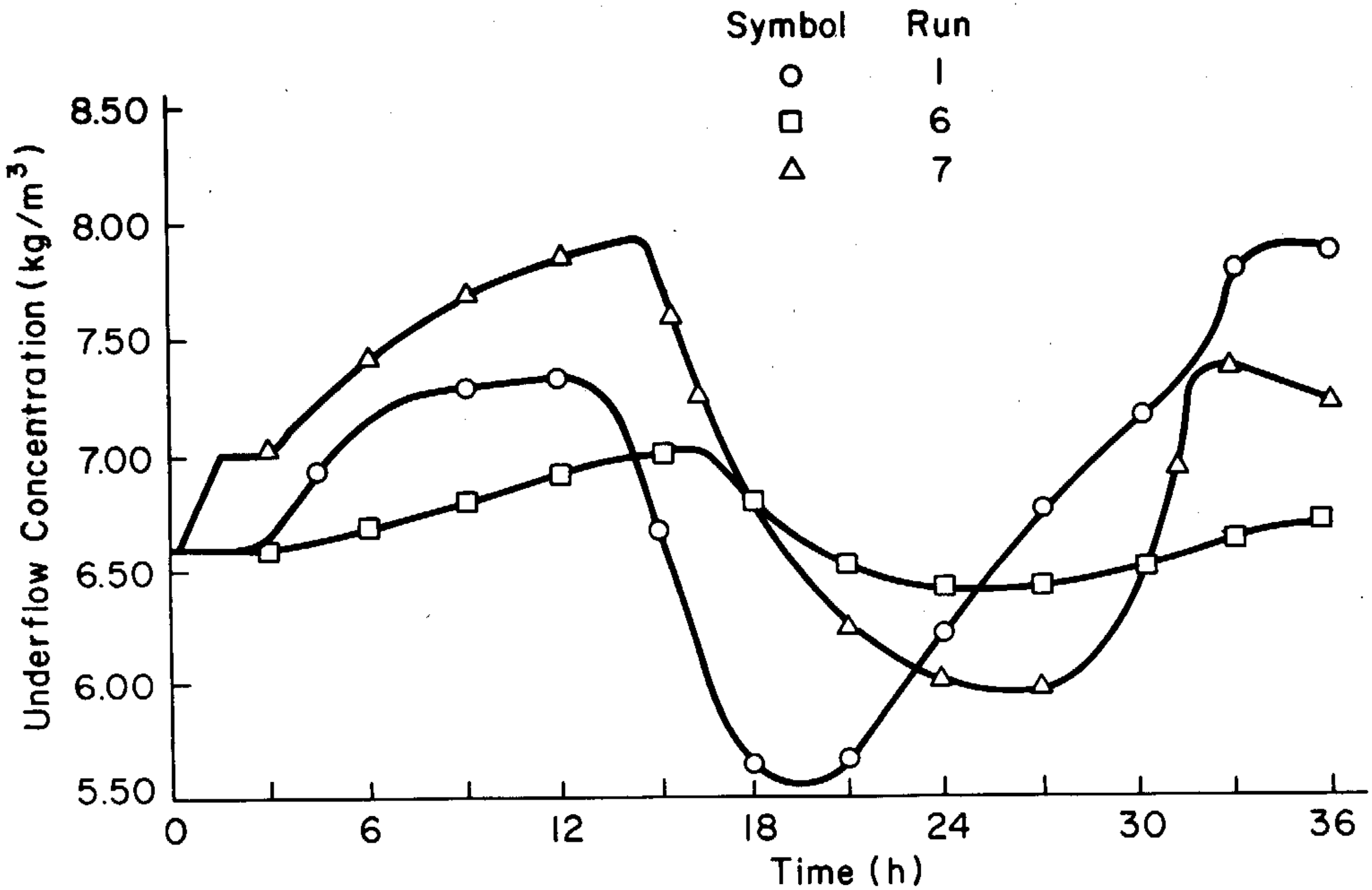

Figure 16. Effect of Various Control Modes on Underflow Solids Concentration Using Davis's Controller with the Coupled Process Model

these constants were obtained from one settling curve and apply only to that particular sludge; however, a case typical of activated sludge was chosen for determining k' and u_o.

Operation of the system as proposed in runs 2 and 4 would not be desirable based on the large depth necessary to handle the sludge in the final clarifier. Operation of the system as proposed in runs 3, 4, and 5 would not be desirable based on the large values of S_{MAX} and S_{MIN} as compared with run 1 which is the uncontrolled case. The reason for these large values of S_{MAX} and S_{MIN} is that the final clarifier was unable to return to the aerator a recycle stream of sufficient concentration. Hence, the importance of the interaction between the aerator and the final clarifier is demonstrated.

The dynamic results of runs 1, 6, and 7 are shown in Figures 15 and 16. It is obvious from Figure 15 that proportional control on flow results in the best control over the 36 hour period. It decreases S_{MAX} by approximately 23%, and has the additional feature that no sludge storage is required. It should be pointed out also that it is easier to measure changes in flow in an actual secondary treatment process than it is to measure changes in inlet substrate concentration.

From Figure 15 it can be seen that proportional flow and derivative substrate control does give a lower value of S_{MAX} over the first 24 hour period. However, it only improves performance by 7% over the proportional flow control, and creates the added problems of sludge storage, a deeper final clarifier, and a sophisticated system to detect changes in inlet substrate concentration.

Derivation of Controller with Time Delay

Davis' controller was derived on the premise that recycle sludge was instantaneously available. An improvement should result if the controller were derived to include the time delay. Debelak proceeded to the derivation thus.

Once again the material balances were linearized about the steady state operating point[10]. Equation 23 for substrate remains the same as for Davis' controller.

$$\frac{d\bar{S}}{dt} = \left[\frac{Q}{V}\right]_{SS} \bar{S}_i - \left[\frac{Q}{V} + \left(\frac{\partial}{\partial S}\ \frac{dF}{dt}\right)\right]_{SS} \bar{S} + \left[\frac{S_i - S}{V}\right]_{SS} \bar{Q} - \left[\frac{\partial}{\partial X}\ \frac{dF}{dt}\right]_{SS} \bar{X} \qquad (23)$$

But that for sludge solids becomes:

$$\frac{d\bar{X}}{dt} = \left[\frac{\partial}{\partial S}\left(\frac{dG}{dt}\right)\right]_{SS} \bar{S} + \left[\frac{q_1}{(q_1 + q_2)V}\ X\ (t - L) - \frac{X}{V}\right]_{SS} \bar{Q} + \left[\frac{q_1}{V}\left(\frac{q_1 + Q}{q_1 + q_2}\right)\right]_{SS} \bar{X}\ (t - L) + \left[\left[\frac{2q_1 + Q}{V(q_1 + q_2)} - \frac{q_1(q_1 + Q)}{V(q_1 + q_2)^2}\right] X\ (t - L) - \frac{X}{V}\right]_{SS} \bar{q}_1 - \left[\frac{q_1(q_1 + Q)}{V(q_1 + q_2)^2}\ X\ (t - L)\right]_{SS} \bar{q}_2 + \left[\frac{\partial}{\partial X}\left(\frac{dG}{dt}\right) - \frac{Q + q_1}{V}\right]_{SS} \bar{X} \qquad (57)$$

The bar and SS subscript notations are used as before for deviation and steady state variables. Also equation 1 holds for the deviation variables

$$\bar{q}_1 + \bar{q}_2 = \beta Q_a \tag{58}$$

The procedure remains as before: Laplace transformation and simultaneous solution to eliminate $\bar{X}$. Substitution of $\bar{q}_2$ in the resulting equation gives

$$\bar{S} = P_{11}\bar{Q} + P_{12}\bar{S}_i + P_{13}\bar{q}_1 \tag{59}$$

$$P_{11} = \frac{1}{D} \left[\alpha_{23} + \alpha_{24}\frac{\alpha_{13}}{\alpha_{14}} + \alpha_{25}\frac{\alpha_{13}}{\alpha_{14}} e^{-Ls} - \alpha_{27}\beta - \frac{\alpha_{13}}{\alpha_{14}} s\right]$$

$$P_{12} = \frac{1}{D} \left[\alpha_{24}\frac{\alpha_{11}}{\alpha_{14}} + \alpha_{25}\frac{\alpha_{11}}{\alpha_{14}} e^{-Ls} - \frac{\alpha_{11}}{\alpha_{14}} s\right]$$

$$P_{13} = \frac{1}{D} \left[\alpha_{26} + \alpha_{27}\right]$$

$$D = -\frac{\alpha_{12} + s}{\alpha_{14}} s - \alpha_{22} + \alpha_{24}\frac{\alpha_{12} + s}{\alpha_{14}} + \alpha_{25}\frac{\alpha_{12} + s}{\alpha_{14}} e^{-Ls}$$

The listing of the relationships for the α's is in Table 8.

Equation 59 shows the deviation of exit substrate concentration from the steady state as a linear function of inlet flow, inlet concentration, and recycle stream. The objective of feed forward control is to keep $\bar{S}$ at zero. The two disturbances can be separated, and treated separately since the equation has no interaction terms, and is therefore linear. If

Table 8. Listing of Coefficients α

$$\alpha_{11} = \left[\frac{Q}{V}\right]_{SS}$$

$$\alpha_{12} = \left[\frac{Q}{V} + \frac{\partial}{\partial S}\left(\frac{dF}{dt}\right)\right]_{SS} = \left[\frac{Q}{V} + \frac{K_1\ X}{a}\right]_{SS}$$

$$\alpha_{13} = \left[\frac{S_i - S}{V}\right]_{SS}$$

$$\alpha_{14} = \left[\frac{\partial}{\partial X}\left(\frac{dF}{dt}\right)\right]_{SS} = \left[\frac{K_1 S}{a}\right]_{SS}$$

$$\alpha_{21} = 0$$

$$\alpha_{22} = \left[\frac{\partial}{\partial S}\left(\frac{dG}{dt}\right)\right]_{SS} = \left[K_1 X\right]_{SS}$$

$$\alpha_{23} = \left[\frac{q_1}{(q_1 + q_2)V}\quad X_u(t - L) - \frac{X}{V}\right]_{SS}$$

$$\alpha_{24} = \left[\frac{\partial}{\partial X}\left(\frac{dG}{dt}\right) - \frac{Q + q_1}{V}\right]_{SS} = \left[K_1 S - \frac{Q + q_1}{V}\right]_{SS}$$

$$\alpha_{25} = \left[\frac{q_1}{V}\left(\frac{q_1 + Q}{q_1 + q_2}\right)\right]_{SS}$$

$$\alpha_{26} = \left[\left(\frac{2q_1 + Q}{(q_1 + q_2)V} - \frac{q_1(q_1 + Q)}{V(q_1 + q_2)^2}\right)\quad X_u(t - L) - \frac{X}{V}\right]_{SS}$$

$$\alpha_{27} = \left[\frac{q_1(q_1 + Q)}{V(q_1 + q_2)^2}\quad X_u(t - L)\right]_{SS}$$

$\overline{S} = 0$ and $\overline{S}_i = 0$, equation 38 becomes

$$0 = P_{11}\overline{Q} + P_{13}\overline{q}_1$$

$$\overline{q}_1 = -\frac{P_{11}}{P_{13}}\overline{Q} = F_1\overline{Q}$$

If $\overline{S} = 0$ and $\overline{Q} = 0$

$$\overline{q}_1 = -\frac{P_{12}}{P_{13}} = F_2\overline{S}_i$$

$$F_1 = \frac{-\alpha_{23} - \alpha_{24}\dfrac{\alpha_{13}}{\alpha_{14}} + \alpha_{27}\beta + \dfrac{\alpha_{13}}{\alpha_{14}}s - \alpha_{25}\dfrac{\alpha_{13}}{\alpha_{14}}e^{-Ls}}{\alpha_{26} + \alpha_{27}} \tag{60}$$

$$F_2 = \frac{-\alpha_{24}\dfrac{\alpha_{11}}{\alpha_{14}} + \dfrac{\alpha_{11}}{\alpha_{14}}s - \alpha_{25}\dfrac{\alpha_{11}}{\alpha_{14}}e^{-Ls}}{\alpha_{26} + \alpha_{27}} \tag{61}$$

$$F_1 = K_c\,(1 + \tau_D s - K_d e^{-Ls}) \tag{62}$$

$$F_2 = K_c'\,(1 + \tau_D' s - K_d' e^{-Ls}) \tag{63}$$

$$\overline{q}_1 = K_c\,(1 + \tau_D s - K_d e^{-Ls})\,\overline{Q} + K_c'\,(1 + \tau_D' s - K_d' e^{-Ls})\,\overline{S}_i \tag{64}$$

Testing of the Debelak Time Delay Controller

Debelak's controller thus takes into account the time delay in the underflow concentration of the final clarifier. The results of computer runs using these feed forward controllers are summarized in Table 9, and Figures 17 and 18 are plots of the dynamic results. It should be noted that the final clarifier was operated with $Q_u = \beta Q$; sludge storage was available; and L = 2 h. The other constants and steady state values were as in the test of Davis' controller.

Operation as proposed in runs 9, 11, and 12 would not be desirable based on the large depth requirements in the final clarifier. The value of S_{MAX} and S_{MIN} were also too high to warrant operation of the system in these modes. Operation as in run 10 is the only control mode worth considering. Referring to Figure 17, the value of S during the first 24 hour period was substantially reduced. However, operation was not that much better than proportional flow control only as in run 6 .

It was obvious that operation with the feed forward controllers designed to account for the time delay in the clarifier, sludge storage was always necessary. The controllers were such that they required a larger recycle stream, q_1, than was available from the clarifier. Although this is not a serious handicap, these do not offer any improvement in performance. In fact, performance deteriorated under some of these control modes. These controllers were designed to minimize the variation in outlet substrate concentration, S. However, their effect on the final clarifier could not be anticipated. At times their action created extremely large sludge depths at one extreme, and insufficient underflow concentrations at the other extreme.

Table 9. Results for Feed Forward Control Eckenfelder Aerator Model Coupled to the Settling Model - Debelak Controller and On-off Control

Run Number	Model Q_u	Control Mode Flow	Control Mode Concentration	Sludge Storage	S_{MAX}	S_{MIN}	H_{MAX}	H_{MIN}
9	βQ	P D Delay	P D Delay	Yes	N. O.	N. O.	9.96	0.381
10	βQ	P Delay	None	Yes	0.048	0.0069	2.47	0.493
11	βQ	P Delay	D	Yes	0.050	0.0083	3.95	0.345
12	βQ	None	P Delay	Yes	0.074	0.0087	3.71	0.346
13	βQ	On-Off Control*		Not required	0.045	0.0045	2.09	1.04

Note: P = Proportional
D = Derivative
N. O.= Not Obtainable

*See Text

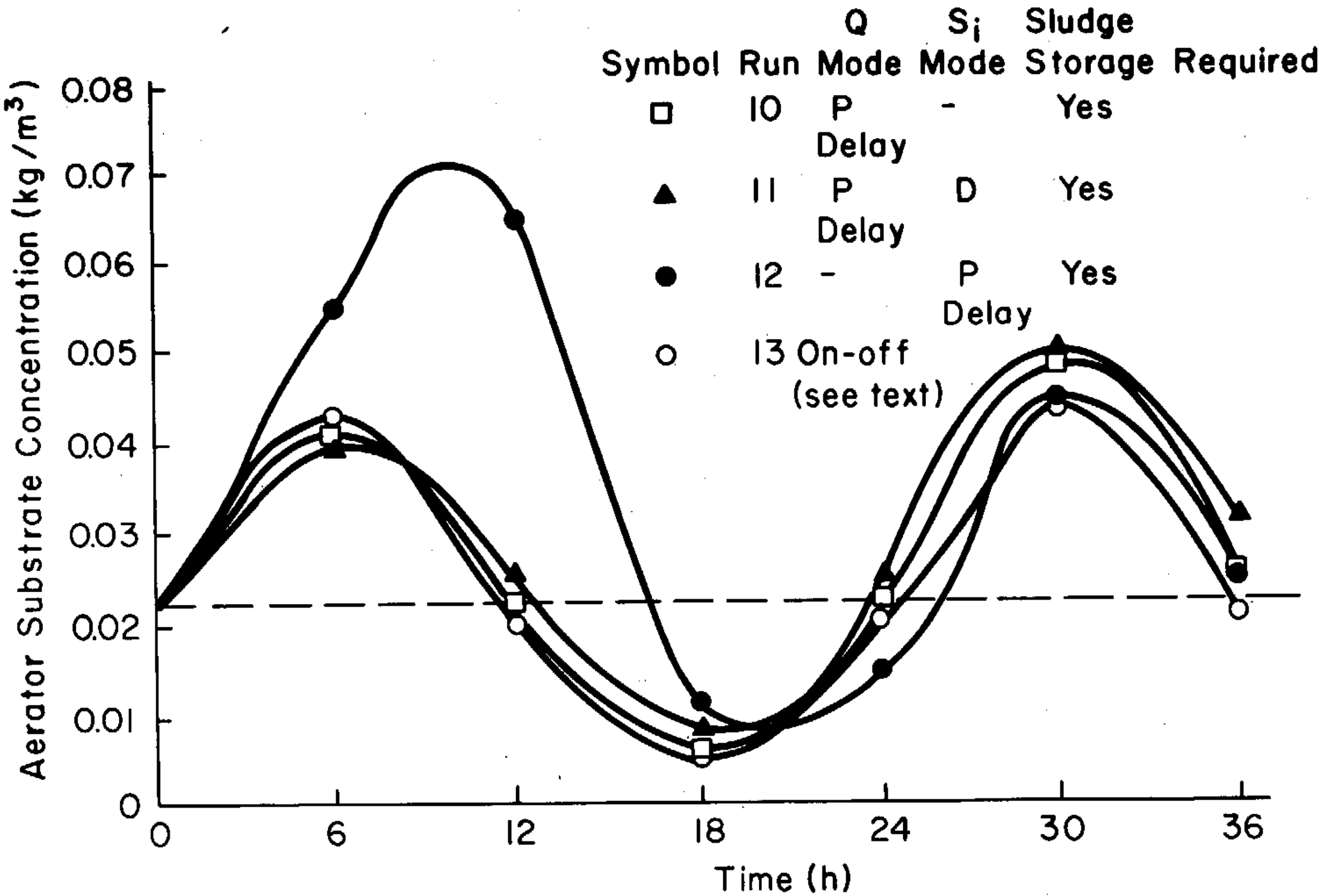

Figure 17 Effect of Various Control Modes on Exit Substrate Concentration Using Debelak's Controller and On-off Control with the Coupled Process Model

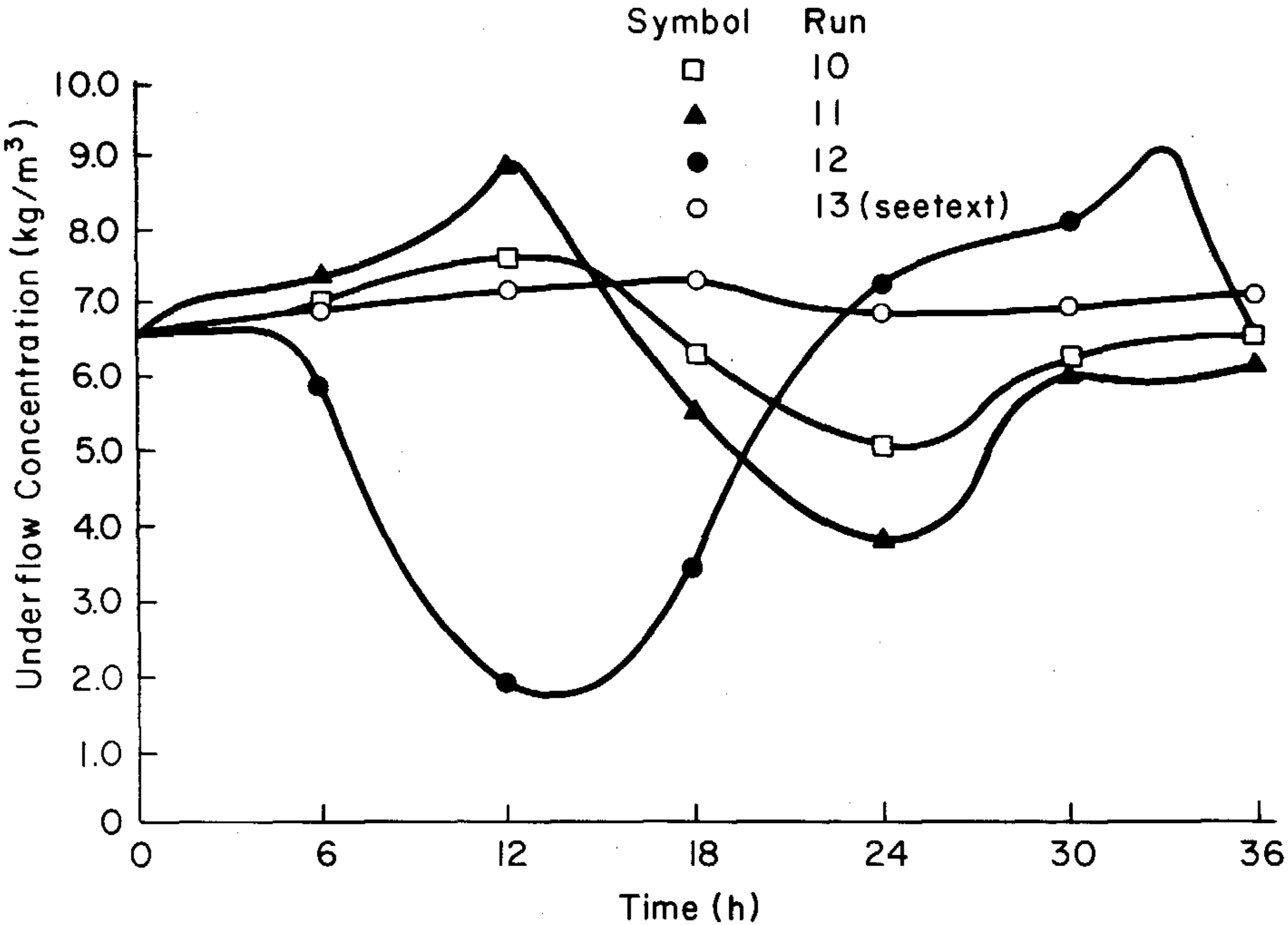

Figure 18. Effect of Various Control Modes on Underflow Solids Concentration Using Debelak's Controller and on-off Control with the Coupled Process Model

Brett et. al.[53] indicated that on-off control coupled with feed forward control could possibly give better control than any continuous control. A run was made to test this possibility. The final clarifier was operated with $Q_u = \beta Q$. The on-off control was simulated as follows: if $Q > Q_A$ the control stream q_1 was made equal to βQ; in other words, all the material from the clarifier was recycled to the aerator. If $Q < Q_A$, control was the same as in run 6, proportional control on flow only using the Davis' Controller. The dynamic results are plotted on Figures 17 and 18 in order to compare them with the other modes of control.

Comparison of the on-off controller to all the other control schemes shows very favorable results. Indeed, S_{MAX} was less than for any other controller tested against the settler model. Further, this was accomplished with the simplest measurement, Q, and sludge storage was not required.

It is interesting to note that this model could be used in the design of a final clarifier. If settling data for the particular sludge were available, and the proper model for the aerator was chosen, one could determine the required area and depth of a final clarifier. Referring to the theory section of this paper, the area and depth for the particular loading are calculated through the diurnal variation of flow and inlet substrate concentration. It would appear possible, therefore, to simulate an activated sludge process prior to construction and with the proper control modes and operation modes of the aerator and clarifier, optimize the system in regards to capital costs for physical plant and effluent quality.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

1. The ability to provide make-up sludge (allow q_2 to take negative values) when required greatly enhances process control; perfect control is impossible without it.

2. For the completely mixed process with an instantaneous clarifier, the control algorithm is independent of the aerator model provided the internal growth rate term of the model is of the form

$$\frac{dG}{dt} = X\,\phi(S)$$

The function $\phi(S)$ enters the controller through the constants of the algorithm.

3. The dependence of the controller constants on $\phi(S)$ can be neglected with little loss of control effectiveness.

4. Including a time-delay term in the sludge mass balance to reflect the action of the final clarifier complicates the controller derived therefrom. However, good control can be obtained from on-off proportional control on Q, and in this case sludge storage would not be required.

5. The claim of many authors that the performance of aerator and final clarifier of the activated sludge process are interdependent has been demonstrated. This interaction must be considered in any control scheme for the process.

Recommendations

In further research, the following should be included:

1. In the research just completed, the underflow as a proportion of sewage flow ($\beta = \frac{q_1 + q_2}{Q}$ was held constant and the recycle flow rate (q_1) was varied. This made q_2, flow of excess sludge, a key control parameter. If possible β should be made a control parameter with q_2 to be set by total sludge mass considerations.

2. Mixing regimes other than complete - mixing should be studied.

3. The rudimentary final settling model should be improved.

4. A real reservoir for sludge storage should be studied. This should have three good results. First it would allow q_2 to take negative values. Second, the performance of aerator and final settler could be de-coupled. Sludge would be withdrawn from the clarifier at a rate optimum to its performance, and delivered to the aerator when required by the controller. Lastly, q_1 and q_2 would also be decoupled making realization of recommendation 1. easier.

5. Control should be evaluated against more realistic forcing functions.

6. When funds are available for the purchase of equipment the control algorithm developed should be tested on a physical treatment plant.

NOTATION

Upper Case:

A - Cross sectional area of clarifier, m^2.

A_D - Area of discharge opening of clarifier, m^2.

A_m - Maximum cross sectional area of clarifier, m^2.

E - Settling flux, kg/m^2h.

E_F - Operating solids flux, kg/m^2h.

E_{FA} - Solids flux at cross sectional area A, kg/m^2h.

E_M - Maximum solids flux, kg/m^2h.

E_T - Total solids flux, kg/m^2h.

E_U - Induced solids flux, kg/m^2h.

E_u - Solids flux due to settling, kg/m^2h.

E_UA - Induced solids flux at cross sectional area A, kg/m^2h.

F_i - Feed forward controller for index i.

H - Height of sludge in clarifier, m.

K_1 - Pseudo first order growth rate constant for Eckenfelder model, m^3/kgh.

K_c - Gain of the proportional-derivative feed forward controller for the sewage influent flow, dimensionless.

K_c' - Gain of the derivative feed forward controller for the sewage influent concentration, m^6/gh.

K_d - Delay time constant of proportional-derivative-delay feed forward controller for sewage influent flow, dimensionless.

K_d' - Delay time constant of proportional-derivative-delay feed forward controller for sewage influent substrate concentration, dimensionless.

K_{pfb}- Proportional controller constant, feed-back mode, $\frac{m^3/h}{g/m^3}$

K_{pff}- Proportional controller constant, feed-forward mode, dimensionless.

K_s - Half velocity coefficient in the McCarty model, g/m^3.

L - Variable time delay in return of sludge to recycle, h.

N - Westberg's simplifying variable, $\frac{X}{Z}$, dimensionless

P_{jk} - Any transfer function, with indices j and k

Q - Sewage flow rate at time t, m^3/h.

Q_a - Average sewage flow rate over a day, m^3/h.

Q_u - Flow rate out of bottom of clarifier, m^3/h, $Q_u = q_1 + q_2$.

R - Radius of clarifier, m.

S - Concentration of substrate in reactor and effluent at time t, g/m^3.

S_i - Influent substrate concentration at time t, g/m^3

S_{ia} - Average influent substrate concentration over a day, g/m^3.

U - Induced downward velocity in clarifier, m/h.

U_A - Induced downward velocity in clarifier at cross sectional area A, m/h.

V - Aerator volume, m^3.

X - Concentration of activated sludge in the aerator, g/m^3.

X_a - Average concentration of living bacteria in the aerator over a day, g/m^3.

X_L - Limiting solids concentration in clarifier, kg/m^3.

X_{LM} - Limiting concentration at maximum solids flux, kg/m^3.

X_s - Solids concentration in clarifier at any level, kg/m^3.

X_u - Underflow concentration of sludge from clarifier, kg/m^3.

X_u' - Concentration of sludge in clarifier immediately prior to withdrawal, kg/m^3.

X_{uM} - Maximum concentration of underflow at maximum solids flux, kg/m^3.

Z - Concentration of dead bacteria in the reactor at time t, g/m^3.

Lower Case:

a - Activated sludge synthesis per removal of substrate for the Eckenfelder model, dimensionless.

b - Redissolving rate constant Westberg model, m^3/g h.

b' - Bacteria decay coefficient for the McCarty model, h^{-1}.

c - Death rate constant Westberg model, g/m^3 h.

$\frac{dF}{dt}$ - Internal substrate utilization rate per unit volume, g/m^3 h.

$\frac{dG}{dt}$ - Internal activated sludge synthesis rate per unit volume, g/m^3 h.

f(t)- Regulation function, $\frac{q_2\ (Q+q_1)}{V\ (q_1+q_2)}$, h^{-1}

g(t)- Dilution rate, $\frac{Q}{V}$, h^{-1}.

h(t)- Substrate loading rate, $\frac{Q\ S_i}{V}$, g/m^3 h.

k - Maximum rate of substrate utilization per unit weight of activated sludge, Lawrence and McCarty model, h^{-1}.

k' - Sludge settling constant, kg^{-1}.

m - Growth rate constant, Westberg's model, h^{-1}.

p - Flow of excess sludge as proportion of sewage flow, q_2/Q, dimensionless.

q_1 - Flow of return sludge at time t, m^3/h.

q_2 - Flow of excess sludge at time t, m^3/h.

s - Laplace variable.

t - Time, h.

u - Settling rate of particles relative to bulk liquid, m/h.

u_o - Settling velocity of the individual aggregate particles of sludge.

y - Growth yield coefficient for the McCarty model, dimensionless.

z - Height of discharge above apex of cone; settler, m.

Greek:

α - Coefficients in Debelak's control model.

β - Underflow from separator as proportion of sewage flow, $\frac{q_1+q_2}{Q}$, dimensionless.

γ - Angle between vertical and side of conical settler, rad.

θ - Average hydraulic retention time, h.

τ_D - Derivative time constant of the proportional-derivative feed forward controller for the sewage influent flow, h.

$\phi(S)$- Internal kinetic mechanism for sludge synthesis, a function of substrate concentration only.

$\psi(S)$- Internal kinetic mechanism for substrate utilization, a function of substrate concentration only.

ω - Angular velocity, rad/h.

<u>Subscripts:</u>

MAX - Maximum value.

MIN - Minimum value.

SS - Steady state value.

a - Average value over a day.

<u>Symbol:</u>

$\overline{}$ - Bar notation indicates deviation variable,

eg $\overline{X} = X - X_{SS}$.

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