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Research Report 92

SUPPLY AND DEMAND IN WATER PLANNING:

STREAMFLOW ESTIMATION AND CONSERVATIONAL WATER PRICING

By

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January, 1976

Preface

This report is the technical completion report for a research project entitled "Optimal Sizing of Water Supply Reservoirs Under Alternative Demand and Management Strategies". The project was supported in part by funds provided by the United States Department of Interior to the University of Kentucky Water Resources Institute as authorized by the Water Resources Act of 1964, Public Law 88-379, and the Office of Water Research and Technology Project A-052-KY. Partial funding was also provided by the Kentucky Agricultural Experiment Station as a contribution to Southern Regional Research Project S-53 "Factors Affecting Water Yields from Small Watersheds and Shallow Ground Aquifers".

ABSTRACT

HYDROLOGIC AND ECONOMIC MODELS
IN RESERVOIR DESIGN

Recent studies indicated the need for development of surface water supplies in Kentucky. Rising resource costs make economically efficient reservoir designs increasingly important. This study was undertaken to provide methods in water supply reservoir design that increase system benefits.

Two major factors influencing reservoir design were studied: estimated future streamflow into the reservoir and demands placed on the reservoir. Standard reservoir sizing methods rely on historical streamflow data. This data is frequently limited and uncertainty in required storage estimates may result. To assess the reliability of a design, the use of mathematical models in simulation studies was proposed. Existing stochastic and parametric models of streamflow were reviewed and their limitations discussed. Parameters for the stochastic models must be estimated from historical streamflow data, and limited data produces unreliable estimates of the true values for these parameters. A streamflow record extended by a parametric model through simulation

may provide more reliable estimates of the parameters in the stochastic streamflow model than the short historical record. A methodology was presented to evaluate the ability of a parametric model to improve the stochastic model parameter estimates in this manner. It was found that the parameter estimates of a stochastic model might be significantly improved by this process. A long historical record of rainfall may not be available to provide the necessary inputs to a parametric model. One method for providing these inputs is to model the daily rainfall process at the potential site. A modified Markov Chain model was proposed which used continuous distributions, rather than discrete transition probabilities, to represent the process when rainfall actually occurred. A two-parameter gamma distribution fit the Kentucky data. The model provided a good representation of the daily point rainfall process. 15-20 years of historical daily rainfall data were required to produce stable estimates of model parameters.

The role of the demand function in reservoir design was examined. Projected demand is commonly assumed not to depend on the concurrent water rates. Data on rural residential water demand in Kentucky has indicated that a price-demand relationship does exist for this sector. The second part of the study was undertaken to see if benefits to a hypothetical community from water supply could be increased by utilizing price-demand information in reservoir design studies. Three pricing policies were examined and their effect on reservoir design determined. The first policy assumed no price-demand relationship, and demand was

based on existing community usage with a low water rate. A price-demand relationship was assumed in the second policy, and the water rate was constant. The third policy assumed the price-demand relationship, and the price charged for water during each billing period was a non-linear function which increased as the amount of water in storage at the beginning of the period decreased.

It was found that the use of the conservation pricing policies substantially reduced storage requirements while providing increased, demonstrable net benefits to the community. The conservation pricing policies substantially lowered the average price paid for water. The effect of uncertainty in consumer response to changes in price was studied by using a probabilitistic price-demand relationship. This uncertainty did not significantly reduce the effectiveness of the conservation policy. It was concluded that demand management by the use of a proper pricing policy could significantly increase water supply benefits to a community.

Descriptors: Pricing*, Water Rates, Water Demand*, Reservoir Storage

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CHAPTER I

INTRODUCTION

Recent investigations revealed that in many areas of Kentucky there exists, or will exist in the near future, a need for the development of water supply sources. For most areas it was felt that these needs could best be met by the impoundment of streamflow. According to Beattie and Haan (1973),

> A recent report of the Kentucky Division of Water (1971) indicates that there are 37 municipalities in Kentucky with existing or projected water supply problems. A water supply problem exists at 28 of the municipalities at the present time. The U.S. Department of Agriculture (1970) states that individual water supplies are inadequate for present and future domestic water needs and require project action in 315 rural Kentucky communities. In addition, water supplies are inadequate for present and future needs in 169 Kentucky towns, villages, and communities. The U.S. Department of Agriculture (1970) further states that the majority of the needs can be met through project action by impoundment of surface runoff.
>
> The two sources of public water are surface supplies and groundwater supplies. Groundwater supplies amount to only 14 per cent of the total water use in Kentucky excluding hydropower and fuel-electric production (Kentucky Department of Natural Resources, 1965). The importance of groundwater supplies in proportion to surface water supplies is decreasing as more communities develop water supply systems. A report by the Kentucky Department of Natural Resources (1965) shows that in the period 1953 to 1965 the number of surface water systems increased by 39 while the number of groundwater systems decreased by 2.

If these Kentucky water requirements are to be met, considerable investment in water supply systems will be required. In 1965, L. R. Howson (1965) indicated, "It now costs almost ten times as much to build facilities to supply water to 1000 people as it did 50 years ago, but the volume of water required has scarcely doubled." The high level of investment required to provide water supply dictates that the components of the system be defined as precisely as possible in order that economic (and extra-economic) efficiency be maximized.

The development of the water supply source (reservoir) represents a major expenditure in the system. The net benefits derived from the reservoir will be a function of many factors including the seasonal flow into the reservoir (hydrologic factors) and the seasonal demand requirements on the reservoir (social and economic factors). In this study the demand function of interest is the rural residential water demand.

In order to assess the probable net benefits obtainable from the construction and/or (in the case of an existing reservoir) operation of a proposed reservoir, it is necessary that future inflows and demands on the reservoir be estimated as reliably as possible. This entails a maximum utilization of all information pertaining to those factors influencing inflows and demands. Primary information on inflow factors would include historical data of streamflow, rainfall, evaporation, and land-use changes, for instance. Primary information on residential demand factors could consist of historical data on water use as a function of household type, price, pressures maintained, season of the year, economic and population growth, etc.

One method which has been used to determine the required capacity of a water supply reservoir using the historical information has been to require a storage capacity which would have supplied the projected demands throughout the period of historical streamflow record. Projected demands are based on prevalent and anticipated growth in demand in the various sectors of the community. A standard procedure for projecting demand is to estimate population growth over the design life of the project and then multiply this figure by an assumed per capita demand rate. Designs based on this type of analysis may be satisfactory in the sense that demands for water may be met; however, the method has limitations.

A primary difficulty which may be encountered with this method results from the reliance on the historical streamflow record for the determination of the required storage capacity. Often there may be little, if any, historical streamflow data at the potential site. Data from nearby streams may be used to approximate the streamflow at the site; however, except for a few major streams in Kentucky, historical streamflow records are usually less than twenty years in length. The design life of a proposed reservoir may be 40, 50, or often 100 years. An estimate of the storage capacity needed to meet the demands during the design life must thus be made from streamflow data which may poorly approximate the true streamflow and which represents a time interval which may be only a small fraction of the design period. A very uncertain estimate of required capacity may result. If the uncertainty as to the true storage requirement results in overdevelopment of supply, then costly premature investment will occur. If the estimated storage requirement is too small, then estimated project net benefits will be inflated. Methods which extend or improve the reliability of short historic streamflow records and provide better estimates of required storage will thus produce more economically efficient designs.

This design procedure might also be improved by a more critical analysis of the projected demands. An examination of the present method reveals the implicit assumption that the future demands will not depend on the concurrent water rates. If the demand for water is related to the price charged, then this relationship should play an important part in assessing water requirements and determining storage needs and project benefits.

A limitation of all methods which utilize only the historical record to evaluate a reservoir design is that no assessment of the risks involved is available (Fiering and Jackson, 1971). If storage is designed to meet demands throughout the design life, it would be desirable to know the likelihood that the reservoir would run dry during this period. The stochastic nature of streamflow implies that the sequence of benefits and receipts from the project during the design life will also be stochastic, and a knowledge of the probability distributions of these quantities would be desirable for project evaluation.

In the following chapters methods for reducing these difficulties and limitations and improving present methods of water supply reservoir design based on economic efficiency will be presented.

CHAPTER II

OBJECTIVES AND SCOPE

The objective of this study was to develop methods for water supply reservoir design and operation that increase the net benefits of the system to the community. It was assumed that the water supply system is publicly owned.

The two major components of a water supply reservoir system are the streamflow into the reservoir and the demands (outflow) placed on the reservoir. The estimation of the required storage for a proposed water supply system depends strongly upon the estimated streamflow into the reservoir during the design life. Reliable streamflow estimates will reduce the risk of wasteful investment in water supply. Except possibly for a few extremely large watersheds, the length of most historical streamflow records in Kentucky is quite short. Mathematical models of streamflow might be used to improve the reliability of streamflow estimates based on these short records. These models can be used in simulation studies in order to examine the reliability of proposed systems. The first part of this study examined currently available streamflow models and determined their suitability for use in water supply reservoir simulation studies. New methods and models were developed as necessary in order to obtain the most reliable streamflow data. The second part of the study concerned the demand factor in system design. The role of price in water demand was examined. Price-demand information obtained by Grunewald et al. (1975) for rural residential water demand in Kentucky was used in order to evaluate the economic consequences of various water pricing policies in system design. In particular, simulation studies were performed to determine if the net benefits of a water supply reservoir could be increased through the use of a conservation water pricing policy designed to promote maximum utilization of the reservoir capacity.

CHAPTER III

REVIEW OF LITERATURE

HYDROLOGIC CONSIDERATIONS

The water supply planner must insure that reliable quantities of water are supplied by a reservoir during its design life in an economical manner. To this end the pattern of inflows (evaporation and seepage losses from the reservoir are considered negative inflows) to the reservoir must be known. The large number of variables affecting future reservoir inflows precludes, at this time, exact attainment of this knowledge for any significant future time intervals. Hence the planner must rely on knowledge of the future inflow pattern in some probabilistic sense. The assumption must be made that probabilistically the future inflows will behave as past flows within a range of predictable changes (e.g. effects of reservoir, watershed development, etc.).

One of the first steps in reservoir planning is an examination of relevant historical watershed data. The major component of inflow to the reservoir will be streamflow. For the sizing and operation of a water supply reservoir, reliable information on monthly streamflow is generally considered adequate. Historical streamflow records on a watershed are generally shorter than the design life of the reservoir and the probability that the historical streamflows will be reproduced during the design life is extremely small. Therefore, sole reliance on these records in design and operation studies will yield only one point from an infinite spectrum of possibilities. It is now generally accepted that reservoir studies must recognize the probabilistic aspects of future streamflow estimates. Historical developments of this idea are given in Fiering (1967). Mathematical models of streamflow must be used to examine the broader range of possible streamflow sequences that may occur during the life of the reservoir.

Two broad classes of models which are presently available for this examination are the lumped parameter models and the stochastic models.

Stochastic Models of Streamflow

Stochastic models of streamflow attempt to model streamflow behavior by preserving stochastic characteristics of historical streamflow. A good discussion of stochastic models and their use in water planning is given by Jackson (1975a). A number of models have been formulated. Rather complex synthetic flow generators have been proposed by Mandelbrot and Wallis (1969) and Mandelbrot (1971) to account for long-term, low frequency dependencies (the so-called Hurst phenomenon) of streamflows. The Broken Line Process suggested by Rodriguez-Iturbe, et al. (1972), Mejia, et al. (1972), Garcia, et al. (1972), and Mejia, et al. (1974) is also capable of generating synthetic streamflow data with long-term dependence. ARIMA (autoregressive integrated moving average) models described by Box and Jenkins (1970) are recommended by McKerchar and Delleur (1974) as being economical in terms of number of parameters required for cases in which long term dependencies are not important. Markov models are perhaps the simplest type of streamflow models which have been studied. Thomas and Fiering (1962) proposed the first such model for streamflow behavior. For normally distributed annual flows their first-order Markov model has the form

$$Q_{i+1} = \mu + \rho\ (Q_i - \mu) + \sigma\ (1 - \rho^2)^{1/2}\ t_i$$

where Q_i is the annual flow in year i; μ, σ, ρ are the estimates of the annual mean, standard deviation, and first-order serial correlation coefficient, respectively; and t_i is an independent random variable with a standard normal distribution. The model can be used to generate traces of annual streamflow values which preserve the values of μ, σ, and ρ. The model can be modified to produce monthly flows, flows with dependence greater than first-order, and flows with non-normal distributions. Markov-mixture models have been proposed by Jackson (1975b) to generate synthetic streamflow records with long and severe droughts for use in water supply studies.

Thus a wide variety of stochastic models are available from which to choose. A model should be selected only if it contributes to the decision making process. A model which accurately reproduces low flows but is unreliable for high flows is not going to make a useful contribution to flood studies. Enough streamflow information should be available so that the model parameter estimates will be stable. If a number of models contribute substantially the same information, then the model should be selected which is most economical in terms of number of parameters, computation costs, and ease of parameter adjustment for sensitivity analyses.

Parametric Models of Streamflow

The lumped parameter models attempt to model the outputs (streamflow) from a watershed by performing deterministic operations on the inputs (rainfall, evapotranspiration, etc.) to the watershed. The operations on the inputs are controlled by parameters. The optimum values for these parameters for a given watershed are determined (usually) by adjusting the parameter values until the model outputs agree in some sense with the measured values for a period of historical record. Once these optimum parameter values have been determined, the model can be used to predict streamflow from any given set of watershed inputs.

Many parametric models have been developed. Probably the most comprehensive and well known model of this type is the Stanford Watershed Model IV developed by Crawford and Linsley (1966). The model uses climatological data and watershed characteristics as inputs. A variety of outputs may be selected including soil moisture conditions, monthly interflow and actual evapotranspiration, complete hydrographs for all storms that produced flows greater than some preselected base flow, and mean daily flows for designated flowpoints on the watershed. Another example of a comprehensive parametric watershed model is the USDAHL-70 Model of Watershed Hydrology proposed by the Agricultural Research Service (Holtan and Lopez, 1971). These models are capable of providing a great deal of information; however, as noted by Tennessee Valley Authority (1972) researchers, "One significant drawback that most of the present watershed models have in common is that they rival the real hydrologic system in complexity....... the number of parameters to be estimated is large and data management so involved that adjusting the model to data is a time-consuming and expensive procedure." Fortunately, less detailed models can be used for water supply studies. Two such models which have been developed in the Kentucky-Tennessee area are the Continuous Daily Streamflow Model proposed by the Tennessee Valley Authority (1972) and a water yield model developed by Haan (1972b) for small watersheds.

Inputs required by the TVA model are daily rainfall and monthly estimates of evapotranspiration. Five parameters must be optimized in the model. These are a volumetric parameter used to preserve mass balance of runoff, two surface runoff parameters associated with winter and summer storms, a parameter representing water stored in the groundwater reservoir, and a parameter representing the fraction of surface runoff on the i'th day which results from rainfall on day i. Five additional parameters may be optimized at the user's option. The model is relatively insensitive to these parameters, however, and the model developers recommend that general estimates of the optimal values will suffice. These parameters represent a surface runoff recession constant, a threshold constant used in allocating groundwater for routing, soil B horizon permeability, an interflow or winter recession constant, and a summer groundwater routing constant. Six additional constants must be supplied to the model. These are drainage area, winter and summer interception capacity, moisture capacity in the soil A horizon, a groundwater reservoir allocation constant, and the day of the year for the beginning of each of the four seasons. Outputs from the model include daily streamflow, storm precipitation excess, and sediment transport.

Inputs to the Haan model are daily rainfall and estimated average monthly potential evapotranspiration. The model has four parameters that must be optimized. These are maximum infiltration rate, maximum daily seepage rate, moisture holding capacity of less readily available groundwater storage, and fraction of seepage that becomes runoff. (As noted by Haan (1972b), nominal designations of parameters in lumped parameter models should not be interpreted too literally.) The Haan model simulates monthly runoff from small watersheds.

Both models require at least one year of observed streamflows (TVA model-daily flows, Haan model-monthly flows) for parameter optimization and 3 to 5 years of observed flows would be recommended.

ECONOMIC CONSIDERATIONS

In order to perform a reservoir evaluation, a water supply planner must not only obtain information concerning possible future inflows, he (or she) must also obtain information concerning the pattern of future outflows (demands) from the reservoir. The demand and supply functions play equal roles in determining the system behavior. The outflow from a reservoir is determined by societal demands for water.

Water Demand

The development of water supply sources has traditionally been the domain of engineers and hydrologists, whose primary concern has been with the study of water availability. The lack of emphasis on the demand side of the problem in more humid areas has been due in part to the relative abundance of water and the general notion that water is a necessity to life and must be supplied at any cost. These two factors created an inertia in the water supply industry toward overdevelopment of supply. The initial relative abundance of water insured that large amounts of water could be made available at a relatively low cost.

Water usage habits were engendered in society which promoted the use of water for every possible purpose. When additional sources of supply were required the "necessity to life" idea required that all "needs" be met through any anticipated drought period. These "needs", of course, were based on the demands then prevalent (i.e. demands for low cost water). The price of water was set to recover total costs of supplying this projected "need", and the marginal cost of abundant supply during drought periods of varying severity was obscured.

Today, increasing demands on water resources and increasing developmental costs require a closer examination of past policies. The idea that water is a "necessity to life", although true in the extreme (as with many other resources), is no longer a valid reason for ignoring the demand function when more and more water is being supplied beyond the basic needs of society. Water supplied above the level of health requirements cannot be considered more necessary for life than any other resource. At this point the question to be answered is whether the delivered water is worth the cost of delivery. As stated by Judith Rees (1969, p. 28), "There appears to be no rational grounds for allowing water supplies to be extended to meet all foreseeable 'needs', when the supply of most other commodities is only increased by foregoing alternative goods. It is possible that the construction of additional water supply capacity is diverting resources away from uses valued more highly on the margin by consumers." Most efficient use of water supplies will assure that resources are not misused. Hence demands for water should be evaluated within the economic realm if possible. The quantity of water demanded at a given price indicates a willingness-to-pay, or user value, and the price provides a guideline for efficient use of resources.

Rural Residential Water Demand in Kentucky

A recent study of rural residential water demand (Grunewald et al., 1975) in Kentucky showed that the price paid for water was the significant factor in determining demand. Using least-squares regression techniques to evaluate the model

$$Q = f(P,I,V,E,N,u)$$

where

Q = quantity of water used in thousands of gallons/year/dwelling unit,

P = average water bill in dollars/1000 gallons,

I = mean income in thousands of dollars/year/dwelling unit,

V = value of dwelling unit in thousands of dollars,

E = evaporation in inches for June-September,

N = number of persons/dwelling unit, and

u = stochastic error term, normally distributed with mean 0 and variance S^2.

They derived the model

$$Q = \exp(4.5)/P^{.915} \exp(u) \qquad (1)$$

significant at the .01 level, with a correlation coefficient of 0.82.

The price elasticity of this rural residential demand is

$$(dQ/Q)/(dP/P) = -.915.$$

That is, a 1% increase in price results in a .915% decrease in demand. Evidently then, the price charged for water will strongly affect the demand. The value of water to these consumers is reflected in the price and any "needs" derived independently of price will not be realistic. Thus the price-demand function should be an integral part of any reservoir study for rural residential water supply.

Water Supply Benefits

The price-demand function makes water an economic commodity. Using the price-demand function, it is possible to determine the value, in explicit monetary units, of an increase in supply in a given time period (Howe, 1971). The decreasing demand function derived by Grunewald (figure 1) shows that the incremental value (price the consumer is willing to pay) of each unit of water decreases with each additional unit supplied. If a project increased total supply from Q_{wo}, supply without the project, to Q_w, supply with the project, in a given time period, then the increase in benefits (value) to the community is equal to the sum of the values of each additional unit supplied or,

$$\text{Added benefits} = \int_{Q_{wo}}^{Q_w} P(Q)dQ$$

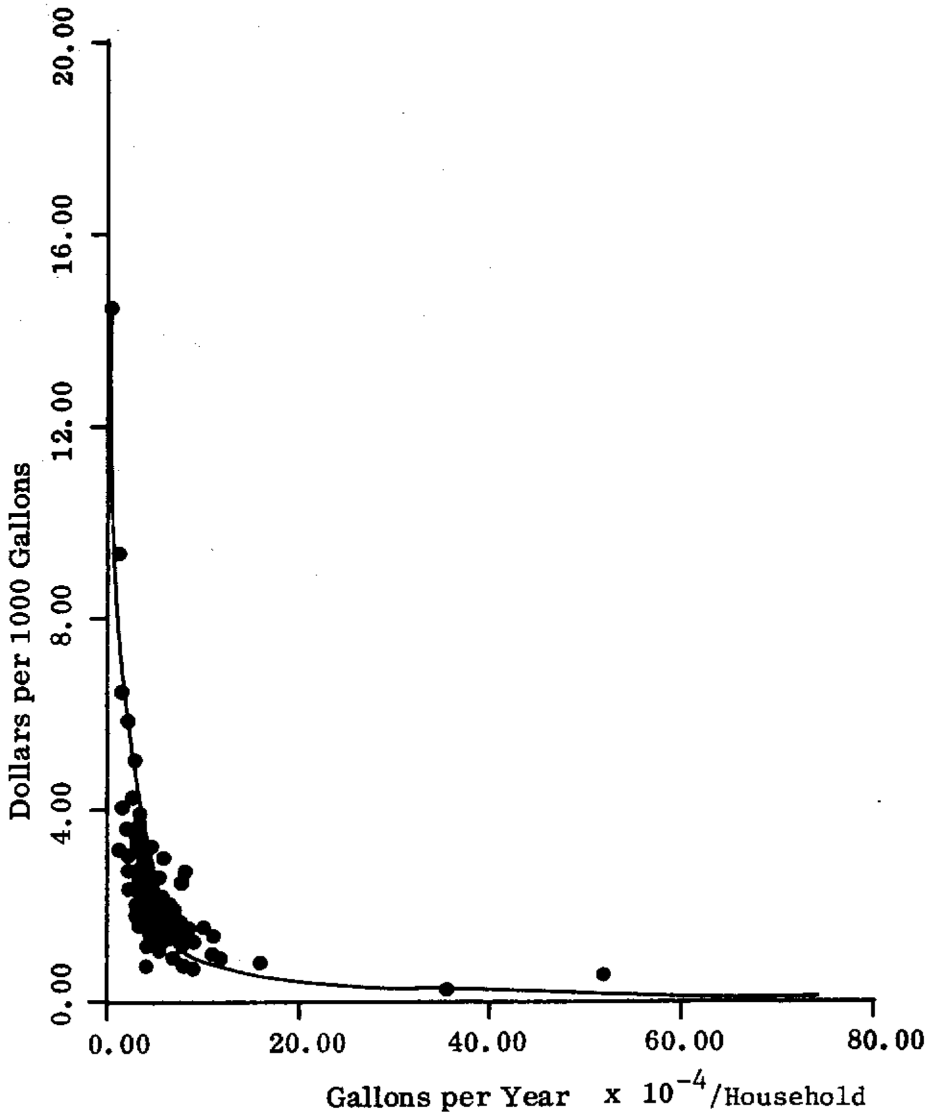

Fig. 1.--Demand function for water, 150 Kentucky water districts, 1972.
(after Grunewald et al., 1975)

where P(Q) is the incremental value (price) when an amount of water, Q, has been supplied.

The benefits produced by a project can be compared to the project costs to determine the net benefits of the project. Since the flow of benefits is distributed over the design life of the project and the bulk of the project costs may occur at the beginning of the project life, it is necessary to evaluate the benefits and costs in terms of present worth (or equivalent annual values) by discounting future monetary values to a common time reference using the discount rate (James and Lee, 1971, ch. 2). The proper choice of the discount rate which should be used for public works is widely debated (cf. James and Lee, 1971, ch. 6) and will not be examined in this study. By comparing the net benefits produced by various proposed systems, it is then possible to select an optimal system. If all project costs are expressible in monetary units, then the optimal project is that which provides the maximum net benefits. Practically, it may be necessary to choose a sub-optimal system if the optimal system does not satisfy the constraint of financial feasibility. If the costs include non-quantifiable components, such as the loss of aesthetic benefits suffered through the inundation of a wilderness area, then the decision becomes more difficult. In this study it will be assumed that all costs have been converted to monetary units.

Water Demand and Water Price

The demand function obtained by Grunewald et al. (equation 1) was based on cross-sectional data and does not claim to represent the response of an individual community to changes in price over time. The plausibility of its use to represent the price-demand function for the water demand of a single community is enhanced, however, by the results of studies based on time-series analysis of water usage for individual communities subject to rate changes. Using data from Boulder, Colorado, representing residential water use from 1955-1968, Hanke and Boland (1971) studied the effect on demand of a shift in 1962-63 from a flat rate to a universal metered rate of $.35/1000 gallons. The authors found that the average domestic usage fell 36 percent and "... consumers reacted immediately to an incremental commodity charge..." when the metered rate was effected. The change in usage was persistent and no significant recovery was noted. It was concluded that a pricing policy, by affecting the quantity of water demanded, is an effective tool that could be used to satisfy the varied goals of the water enterprise. Chiogioji and Chiogioji (1973) give the results of a number of similar studies which also indicated that introducing a fluctuating (meter with unit pricing) rate leads to a substantial decrease in per capita water consumption. Thus both cross-sectional and time-series data on water usage indicated that the price of water might be a significant factor in water demand and that water pricing might be a useful device for improving the efficiency of a water supply system.

Demand Management of Water Supply

Hanke and Davis (1971) suggested that uniform water rates mask significant differences in the marginal costs of serving customers during different periods and that in an economic sense water supplied during summer low flow periods was significantly different from water supplied during winter high flow periods. The additional facilities required to provide water during low flow periods make this water high cost water. The authors used data from the Washington D.C. area and a two-season price model to shift seasonal demand patterns by charging a higher price for water in the summer than in the winter. The result was a reduction in summer demand, and a slight rise in winter demand, which made possible a 10 year postponement of investment in additional sources. The seasonal pricing policy also yielded a more efficient use of the resources employed to provide and distribute water. The authors concluded that by not varying water rates to reflect the cost differences, investments are larger than economically justified.

In a general proposal for public utilities, including water utilities, Vickery (1971) suggested that utility prices which are made to respond appropriately to adventitious variations in demand or supply can produce substantial improvement in the efficiency of utilization of utility facilities. Gysi (1972) studied the effect of what he termed dynamic pricing policies on the probability of reservoir failure in a water supply system. In a simulation study, the price charged for water during a given season (he assumed two seasons per year) was made a function of the level of storage in the reservoir at the beginning of the season. The household daily demand functions used were for U.S. western cities from a study by Howe and Linaweaver (1967). By increasing the price of water linearly as the amount of water in storage decreased, Gysi found that the probability of reservoir failure was smaller when the dynamic-price policy was used than when the fixed-price policy was used, for comparable average prices. He concluded that varying the price of water in accordance with the water supply situation in order to reflect its relative value could greatly reduce the risk of shortages.

CHAPTER IV

THE SIMULATION OF RELIABLE STREAMFLOW DATA

LIMITATIONS OF STREAMFLOW MODELS

The review of literature indicated that for the purpose of data generation, two distinct types of streamflow models are available to the planner. The stochastic models utilize directly the historic streamflow data to probabilistically simulate streamflow behavior. Stochastic models are advantageous in simulation studies because a large number of possible future inflows can be generated inexpensively (usually) on a computer. A major drawback to the stochastic models is their reliance on long historical streamflow records for estimates of parameters defining the stochastic process. The serial correlation of streamflows, particularly, can be a significant factor in the streamflow behavior and reasonably accurate estimates of serial correlation require a long historical record. In Kentucky the majority of watersheds have historical records less than twenty years in length. Records of this length are inadequate for estimating parameters of most stochastic models (Haan, 1972a).

The parametric models, on the other hand, are not rigidly bound by the availability of long historical streamflow records. Historical daily rainfall data, the primary input to these models, is usually more readily available than the flow data. In addition, rainfall data may be more reliably transfered between nearby locations than streamflow data which is highly dependent upon the particular watershed. Unfortunately, the historical rainfall data on which the parametric models rely is not sufficiently long for simulation studies, nor is it computationally feasible to use parametric models (of those known to the author) in simulation studies.

In this chapter it will be shown how the symbiotic use of these two model types can produce useful results for reservoir simulation studies.

RELIABILITY OF PARAMETER ESTIMATES FOR STOCHASTIC STREAMFLOW MODELS

As noted earlier, the use of stochastic models of streamflow is constrained by the availability of historical streamflow data.

The problem of estimating stochastic model parameters from limited streamflow data has concerned modelers for several years. The parameter estimates will vary depending on the historical record length and the degree to which the record represents the actual, long-term streamflow pattern.

Short streamflow records present inherent difficulties in parameter estimation. Wallis and O'Connell (1972) examined the problem of estimating the lag-one serial correlation coefficient, ρ, for normal Markov models. Using Monte Carlo methods they examined the bias and variance of ρ when estimated by different methods from record lengths of 20 to 100 time periods. They found that the usual algorithms for estimating ρ yielded estimates which were biased toward zero and that the biases were rather large for the shorter records. They then examined bias correction methods by repeated Monte Carlo trials. They found that unbiased estimates resulted, although the corrected estimates for ρ tended to have larger variance than the uncorrected estimates. One procedure for obtaining unbiased short sample estimates of ρ was to use the estimator suggested by Box and Jenkins (1970),

$$\hat{\rho} = \sum_{i=1}^{n-1} [x_i-(1/n) \sum_{j=1}^{n} x_j] [x_{i+1}-(1/n) \sum_{k=1}^{n} x_k]/ \sum_{i=1}^{n} [x_i-(1/n) \sum_{j=1}^{n} x_j]^2$$

where n is the number of observed values, x_i, and correct $\hat{\rho}$ for bias by using

$$\hat{\hat{\rho}} = ((\hat{\rho} + (1/n))/(1-4/n).$$

The variance amplification factor resulting from this correction is $(n/n-4)^2$.

The fact that an unbiased sample estimate of ρ can be obtained for short samples gives no justification for assuming that this estimate is a reliable estimate for the true value, however. If $\hat{\rho}$ is the sample estimate of the first-order serial correlation coefficient then the expected value of $\hat{\rho}$, $E(\hat{\rho})$, is approximately

$$E(\hat{\rho}) = \rho-(1+4\rho)/n$$

where ρ is the true value. The variance of $\hat{\rho}$ is approximately

$$var(\hat{\rho}) = (1-\rho^2)/n$$

(Kendall and Stuart, 1966). If r is the corrected estimate of ρ, as above, then

$$E(r) = \rho$$

$$var(r) = n(1-\rho^2)/(n-4)^2$$

approximately.

To get a rough idea of the data requirements necessary for reliable estimates of ρ, a Monte Carlo experiment was performed. Synthetic standardized annual streamflow values were generated using

$$Q_{i+1} = \rho Q_i + (1-\rho^2)^{1/2}\ t$$

where t is an independent standard normal random variable. A value of .25 was used for ρ. Unbiased estimates of ρ were made from each sample. The distribution of the estimates was approximately normal (of course the distribution of ρ cannot be normal since $-1 < \rho < 1$, furthermore as moves closer to ± 1 the distribution of the estimate must become more skewed). For small values of ρ the distribution of r was assumed to be approximately normal. Hence

$$(r-\rho)/(n(1-\rho^2)/(n-4)^2)^{1/2} = z$$

has the standard normal distribution and

$$P(|z| < 1.96) = .95.$$

Rearranging gives

$$\rho-1.96(n(1-\rho^2)^{1/2}/(n-4)^2)^{1/2} < r < \rho+1.96(n(1-\rho^2)^{1/2}/(n-4)^2)^{1/2}$$

or,

$$.25 - 1.90(n^{1/2}/(n-4)) < r < .25 + 1.90(n^{1/2}/(n-4))$$

with probability .95.

To be 95% sure that the sample estimate is within ±10% of the true value of .25 requires $n \cong 5763$.

To obtain estimates of data requirements for the monthly lag-one correlation coefficients the quantity

$$(\text{arctanh}(r)-\text{arctanh}(\rho))(n-3)^{1/2} = z$$

which has approximately the standard normal distribution (Cramer, 1951, p. 399ff.) was used (for $n > 25$).

$$P(|z| < 1.96) = .95$$

or, rearranging,

$$\tanh(\text{arctanh}(\rho)-1.96/(n-3)^{1/2}) < r < \tanh(\text{arctanh}(\rho)+1.96/(n-3)^{1/2})$$

with probability .95. Setting the left and right side equal to $\rho-\Delta\rho$ and $\rho+\Delta p$, respectively, and solving for n for various values of ρ and Δp will give a rough idea of the length of record required to achieve the given accuracy with 95% reliability. For $\Delta\rho = .05$ and ρ ranging from 0.0 to 0.5, the value of n required ranged from approximately 1500 to approximately 800.

Although it may be argued that the preceding arguments were crude, they at least yield some appreciation of the data requirements and point out the need for sensitivity analysis if the value of this parameter is important in the decision process.

Kirby (1974) observed that some sample statistics, e.g. skew coefficient, have algebraic bounds which are independent of the process generating the samples. These bounds are a function of the sample size and are properties of the algebraic formulas defining the statistics.

The required accuracy of parameter estimates for stochastic models is dependent on the sensitivity of the decision function to that parameter. At some stage in stochastic modelling, assumptions must be made and the experience of the planner plays an important role in establishing these assumptions. Some general procedures to aid the planner at this stage have been developed. For example, Haan (1972a) presented a simulation procedure to determine the distribution of a design variable as a function of the number of years of historical data used to estimate the parameters of a stochastic model.

IMPROVING STOCHASTIC STREAMFLOW MODEL PARAMETER ESTIMATES

When parameter estimates are deemed unacceptable, a water resource planner must either forego the use of the stochastic model or improve his parameter estimates so that they fall into an acceptable range.

Several methods of improving parameter estimates for stochastic models can be found in the literature. One method suggested by Benson and Matalas (1967) is to derive parameters from generalized multiple regression relations with physical and climatic characteristics of the drainage basin. They recommended the use of generalized statistical parameters to reduce the spatial and temporal errors in the original record. The method also provides a means of developing a synthetic series for an ungauged basin. Lenton, et al. (1974) used sample and non-sample information in a Bayesian approach to improve estimates of the first-order serial correlation coefficient.

Many methods of improving parameter estimates are based upon the extension of the runoff record, primarily through the use of available precipitation data. Usually, the length of a precipitation record in an area is greater than the length of the streamflow record. This is particularly true of small watersheds, where streamflow records are most often either short or non-existent.

For small (less than 104 km^2), ungauged watersheds, Jarboe and Haan (1974) described a procedure for simulating monthly runoff using the four-parameter water yield model of Haan (1972b) and watershed characteristics. This method provided a close approximation to the annual streamflow values and yielded a simulated streamflow record equal in length to the length of the available rainfall record.

On watersheds where short runoff records exist, stochastic model parameter estimates might be improved by using a parametric model to extend the runoff record and then estimating the stochastic model parameters from the extended record (Burges, 1971). Since data scarcity is the problem, it is desirable that the parametric model require as few measured inputs as possible. To apply a parametric model to the extension of short runoff records for improvement of stochastic model parameter estimates, it is necessary to first test the parametric model's behavior on a similar watershed from which long historical records are available. The behavior of the parametric model is assumed to be consistent on similar watersheds.

EVALUATING A PARAMETRIC MODEL

Given a stochastic model and its parameters, it must first be determined if the parametric model has the ability to preserve the parameters of the stochastic model. In this determination a test watershed as defined earlier is required.

First the parameters, p_i, i=1, ..., n, for the stochastic model are estimated from the entire record. Confidence, or acceptance regions for the parameters should also be estimated, either by a statistical method or, preferably, by a sensitivity analysis. Optimum parameters for the parametric model should then be determined using the complete historical record. These optimum parameters are then used in the parametric model to simulate runoff for the entire period of record. The stochastic model parameters, p_i', i=1, ..., n, are then estimated from the simulated runoff. If a parameter estimate p_k' falls outside the acceptance region around p_k, then it may be assumed that the parametric model does not adequately preserve this parameter. In this case there would be no basis for assuming that the parametric model could adequately improve a short record estimate of this parameter. At this point other parametric models might be evaluated for their ability to preserve this parameter. It may be found that one parametric model would be best for improving estimates of one parameter, while another parametric model would be best for improving estimates of a different parameter. If the estimated parameters, p_j', j=1, ..., m, fall in the acceptance regions around the corresponding p_j, j=1, ..., m, then the parametric model might improve short record estimates of these parameters and further investigation is suggested.

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It must then be determined whether the record extended by the parametric model will reduce the variance and bias of short-record estimates of the stochastic model parameters. For this test the historical record is divided into a number, S_r, of sub-records. The length of the sub-records (i.e. short records) is presumably that which will exist on the watershed for which streamflow records are to be ultimately simulated. For each runoff sub-record, the stochastic model parameters, $\hat{p}_{jr}$, j=1, ..., m; r=1, ..., S_r, are estimated. The optimum parameters for the parametric model are determined for each sub-record and used in the parametric model to extend, by simulation, each sub-record to a runoff record equal in length to the length of the historical record.

From these extended records the stochastic model parameters, $\hat{p}'_{jr}$, j=1, ..., m; r=1, ..., S_r, are estimated. If the variance of the parameter estimate $\hat{p}'_j$ around the best estimate p_j is not less than that of $\hat{p}_j$, then it may be assumed that an extended runoff record obtained from the parametric model will not improve short-record estimates of p_j. If the variance of $\hat{p}'_j$ is less than that of $\hat{p}_j$, then the parametric model should improve short-record estimates of p_j. However, if $\hat{p}'_{jr}$ is not in the acceptance region of p_j, then the parametric model is not improving the estimates to a satisfactory level for this length of sub-record indicating that a longer record would be required for adequate improvement.

It may often be the case that a planner is not interested specifically in the improvement of individual parameters in a stochastic model, but rather in the improvement of the whole set of parameters in the context of his planning situation. The use of a parametric model to improve parameter estimates of a stochastic model may be evaluated within the hypothesis that the stochastic model is correct. For example, the effect of variance in parameter estimates on a test variable of interest (e.g. storage requirements) might be studied. Using the stochastic model parameters p_j, j=1, ..., n, obtained from the entire record, synthetic sequences can be generated and the statistics of the test variable determined. Acceptance regions should be determined for each of these statistics. Using the optimum parameter values in the parametric model, as before, runoff is simulated and the stochastic model parameters $\hat{p}_i$, i=1, ..., n estimated. These estimates are used in the stochastic model to generate synthetic sequences to evaluate the test variable. If the statistics of the test variable thus obtained fall into the acceptance regions determined earlier, then the parametric model may be capable of improving short-record estimates of the stochastic model parameters in this context. In this event, the procedure is repeated for the parameter sets $\hat{p}'_{ir}$, i=1, ..., n; r=1, ..., S_r, and $\hat{p}_{ir}$, i=1, ..., n; r=1, ..., S_r derived as earlier. If the variance of the test variable statistics around their best estimates is reduced by using the extended data, and the test variable statistics thus obtained fall into an acceptable region, then the use of the parametric model to improve short-record estimates of the parameters in the stochastic model is recommended.

It should be noted that all methods for improving stochastic model parameters are not necessarily mutually exclusive, and advantage should be taken of every available method or combination of methods.

A Particular Parametric Model

The water yield model developed by Haan (1972b) was tested for its ability to improve parameter estimates for stochastic models of the Markov type. These parameters are the monthly and annual means, $\bar{x}$, standard deviations, s, and lag-one serial correlation coefficients, r (cf. Fiering and Jackson, 1971).

The test watershed was the South Fork of the Little Barren River, Metcalfe County, Kentucky, with a watershed area of 47.4 km^2. Thirty-one years (10/41-9/72) of historical data in the form of daily rainfall and monthly runoff (cm) were available.

Estimates of the stochastic model parameters were determined from the historical runoff record (Table 1). These values were considered best estimates of the respective parameters. Acceptance regions for monthly parameter estimates were defined by statistical approximations.

The monthly flow for most months in this example is approximately normally distributed. The departure from normality is the greatest for the drier months such as October. For the purpose of this example it was concluded that confidence intervals based on normality and/or large sample assumptions would approximate acceptable ranges for monthly parameter estimates.

Monthly means were assumed to be normally distributed by appealing to the central limit theorem, and 95% confidence intervals were computed. The 95% confidence intervals were computed for monthly standard deviation estimates by assuming that $(n-1)s^2/\sigma^2$ followed a χ^2 distribution. For lag-one monthly serial correlation coefficient estimates, 95% confidence intervals were computed using the statistic $(\text{arctanh}(r) - \text{arctanh}(\rho))\ (n-3)^{1/2}$, which approximately follows a standard normal distribution, where ρ is the expected value of r, and n is the number of observations ($n > 25$). Upper and lower bounds defining the acceptance regions for the monthly parameters are taken as the computed confidence limits as shown in Table 1.

TABLE 1

BEST ESTIMATES AND BEST PARAMETRIC MODEL ESTIMATES

Month	$\bar{x}_o^*$ (cm)	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound	$\bar{x}_s$ (cm)	s_o (cm)	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound	s_s (cm)	r_o	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound	r_s
Oct	0.38	0.18	0.58	0.89	0.61	0.48	0.81	0.71	0.66	0.40	0.83	0.46
Nov	2.21	1.22	3.23	2.03	2.84	2.26	3.78	2.82	0.63	0.36	0.81	0.58
Dec	5.72	4.04	7.42	5.49	4.83	3.86	6.45	5.13	0.64	0.38	0.81	0.64
Jan	8.18	5.97	10.36	8.31	6.25	5.00	8.36	7.06	0.13	-0.24	0.46	0.05
Feb	8.74	7.09	10.39	8.23	4.72	3.78	6.30	5.72	0.16	-0.20	0.49	0.16
Mar	9.27	7.77	10.77	8.94	4.27	3.40	5.69	5.54	0.14	-0.22	0.47	-0.01
Apr	6.48	5.38	7.54	5.36	3.05	2.44	4.09	2.90	-0.17	-0.49	0.20	-0.27
May	3.71	2.67	4.72	3.63	2.90	2.31	3.89	2.13	-0.08	-0.42	0.28	0.24
Jun	1.90	1.14	2.69	2.64	2.21	1.78	2.95	2.49	-0.03	-0.38	0.32	-0.03
Jul	1.83	0.69	2.97	1.85	3.28	2.62	4.37	2.08	0.18	-0.18	0.50	0.15
Aug	0.74	0.33	1.14	1.17	1.14	0.91	1.52	1.32	0.49	0.17	0.72	0.16
Sep	0.61	0.23	1.02	1.07	1.14	0.91	1.52	1.63	0.59	0.30	0.78	0.44
Ann.	49.78	42.32	57.25	49.56	14.83	12.60	17.07	14.63	0.24	0.00	0.60	0.10

* Subscript o refers to estimate from observed record.

Subscript s refers to estimate from simulated record.

n -- underlined quantity n falls outside acceptance region.

Acceptance regions for the annual mean and standard deviation estimates were defined as including those values within ±15% of the best estimate. An acceptance region for the annual serial correlation coefficient estimate was not defined; however, any values outside the interval (0.0, 0.6) would be considered highly suspect for this watershed.

Optimum parameter values were found for the parametric model using the entire historical record. The parametric model with these parameters was called the best parametric model. Runoff was simulated for the entire period, and the stochastic model parameters estimated from this simulated record. These estimates are shown in Table 1. Underlined values were not acceptable by the acceptance region criterion. For example, the mean flow in October predicted by the parametric model was well beyond the upper 95% confidence limit. Thus, for this required accuracy, the parametric model would not preserve the parameters associated with the underlined values. At this point another parametric model could have been tested to determine if it would more accurately preserve these parameters, including the annual serial correlation coefficient estimate which was lower for the simulated record than for the historical record. However for the purpose of this example all parameter estimates were further studied.

The historical record was then divided into the 5 sub-records, each of 4 years' length. Sub-records 1 (1942-45), 4 (1962-65), and 5 (1969-72) were selected so that possible variations in time over the entire record would be represented. Sub-records 2 (1949-52) and 3 (1953-56) represented periods of high and low average flows, respectively, and were selected in order that the parametric model be evaluated under extreme conditions. Stochastic model parameter estimates for each of these periods were computed and are shown in Tables 2-5. The variance of these estimates around their respective best estimates was computed for each of the stochastic model parameters. For example, the variance of the sub-record estimates of the mean flow in October around the best estimate, 0.38 cm (Table 1), was computed as

$$\text{Var } \bar{X}_{Oct,obs} = (1/5) \sum_{S_r=1}^{5} (\bar{X}_{Oct,S_r} - 0.38)^2 = 0.134 \text{ cm}^2.$$

Variances of the parameter estimates are shown in Table 6.

Optimum parameter values from each sub-record were used in the parametric model to extend the corresponding sub-record into a runoff record 31 years in length. The stochastic model parameters (Tables 2-5) were estimated from each of these extended records. The variances of these estimates around their respective best estimates was computed (Table 6).

The reduction in the variance of estimates for each parameter, p_i, which occurred from the use of the extended records was computed as

$$100\ (\text{Var}(p_{i,obs}) - \text{Var}(p_{i,ext}))/\text{Var}(p_{i,obs}).$$

These values are shown in Table 6.

TABLE 2

MONTHLY MEANS--OBSERVED SUB-RECORDS AND EXTENDED SUB-RECORDS

		SUB-RECORD									
		1		2		3		4		5	
Month	Best Est.	$\bar{x}_o^*$	$\bar{x}_e$	$\bar{x}_o$	$\bar{x}_e$	$\bar{x}_o$	$\bar{x}_e$	$\bar{x}_o$	$\bar{x}_e$	$\bar{x}_o$	$\bar{x}_e$
Oct	.38	.28	.81	1.22	.86	.05	.38	.23	.94	.53	.84
Nov	2.21	1.42	1.73	6.05	1.96	.25	1.47	1.27	2.16	1.35	2.44
Dec	5.72	7.37	4.72	10.21	5.13	1.88	4.57	5.79	5.44	5.59	6.27
Jan	8.18	5.33	7.24	16.84	7.75	4.88	7.49	5.89	8.51	7.90	8.92
Feb	8.74	9.83	7.62	10.31	8.48	10.16	7.92	8.33	8.38	9.35	8.64
Mar	9.27	9.98	8.36	11.84	8.69	9.42	8.51	14.63	9.42	5.54	9.17
Apr	6.48	5.87	5.05	4.93	5.38	6.65	5.11	6.91	5.79	7.98	5.61
May	3.71	2.79	3.35	2.41	3.58	3.30	2.82	1.14	3.76	3.66	3.68
Jun	1.90	1.17	2.39	4.22	2.59	1.19	2.08	.86	2.72	2.18	2.59
Jul	1.83	.25	1.80	3.76	2.08	.41	1.30	2.06	2.06	2.51	1.96
Aug	.74	1.12	1.07	.89	1.12	.10	.61	.41	1.24	1.63	1.22
Sep	.61	1.37	1.02	1.27	1.14	.36	.69	.20	1.24	.61	1.17

All values in centimeters.

* Subscript o indicates estimates from sub-record, subscript e indicates estimates from extended records.

n -- underlined quantity lies outside acceptance region.

TABLE 3

MONTHLY STANDARD DEVIATION ESTIMATES--OBSERVED

SUB-RECORDS AND EXTENDED SUB-RECORDS

		SUB-RECORD									
		1		2		3		4		5	
Month	Best Est.	s_o^*	s_e	s_o	s_e	s_o	s_e	s_o	s_e	s_o	s_e
Oct	.61	<u>.41</u>	.74	<u>1.17</u>	.71	<u>.08</u>	<u>.30</u>	<u>.43</u>	<u>.81</u>	.64	.74
Nov	2.84	<u>1.70</u>	2.31	<u>4.83</u>	2.69	<u>.36</u>	2.84	<u>.91</u>	2.72	<u>1.24</u>	3.23
Dec	4.83	<u>7.44</u>	4.72	6.10	4.72	<u>3.10</u>	5.26	4.98	5.00	<u>2.72</u>	5.44
Jan	6.25	<u>3.58</u>	6.81	7.42	5.99	<u>1.60</u>	7.37	<u>4.24</u>	7.06	6.02	7.11
Feb	4.72	4.70	5.26	<u>3.71</u>	5.72	<u>8.28</u>	5.66	6.07	5.11	<u>3.10</u>	5.82
Mar	4.27	2.90	5.03	3.68	4.70	<u>6.20</u>	5.64	<u>2.18</u>	5.59	<u>2.44</u>	5.61
Apr	3.05	<u>1.57</u>	2.64	<u>2.18</u>	2.84	<u>2.31</u>	2.92	<u>4.90</u>	3.15	3.10	3.00
May	2.90	<u>1.73</u>	<u>1.96</u>	<u>2.26</u>	<u>2.18</u>	<u>2.11</u>	2.41	<u>.20</u>	<u>2.24</u>	<u>2.16</u>	2.36
Jun	2.21	<u>1.12</u>	2.13	<u>3.73</u>	1.98	<u>1.35</u>	2.69	<u>.30</u>	2.49	<u>3.51</u>	2.74
Jul	3.28	<u>.30</u>	<u>1.93</u>	<u>4.57</u>	<u>2.44</u>	.18	<u>1.90</u>	<u>1.98</u>	<u>2.08</u>	3.96	<u>2.57</u>
Aug	1.14	<u>1.70</u>	1.02	<u>1.55</u>	1.19	<u>.10</u>	<u>.79</u>	<u>.58</u>	<u>1.42</u>	<u>1.98</u>	<u>1.73</u>
Sep	1.14	<u>2.57</u>	1.35	<u>1.73</u>	<u>1.63</u>	<u>.43</u>	1.14	<u>.13</u>	<u>1.70</u>	<u>.74</u>	<u>1.85</u>

All values in centimeters.

* Subscripts: o - estimated from observed sub-record, e - estimated from extended sub-record.

<u>n</u> -- quantity n is outside acceptance region.

TABLE 4

MONTHLY CORRELATION ESTIMATES--OBSERVED SUB-RECORDS

AND EXTENDED SUB-RECORDS

		SUB-RECORD									
		1		2		3		4		5	
Month	Best Est.	r_o^*	r_e	r_o	r_e	r_o	r_e	r_o	r_e	r_o	r_e
Oct	.66	.84	.49	.71	.60	.76	.42	.63	.53	.94	.42
Nov	.63	.30	.60	.58	.54	.71	.62	.76	.60	.84	.53
Dec	.64	.98	.78	.54	.58	.96	.61	.34	.67	-.13	.59
Jan	.13	.11	.13	-.45	.09	-.89	.05	.84	.07	-.69	.02
Feb	.16	.64	.13	.53	.31	-.20	.18	.86	.18	.82	.17
Mar	.14	-.24	.05	-.78	.08	.66	-.03	-.71	.05	.73	-.04
Apr	-.17	.85	-.20	-.41	-.20	-.73	-.23	-.96	-.18	.67	-.31
May	-.08	.65	.20	-.34	.17	-.39	.23	-.06	.13	-.74	.20
Jun	-.03	.03	.03	.37	-.02	-.73	-.14	.39	.10	-.47	-.12
Jul	.18	.14	.20	.84	.42	-.77	.05	.48	.16	-.34	.02
Aug	.49	-.43	.25	.89	.28	.64	.29	-.40	.13	.98	.14
Sep	.59	.99	.43	.94	.50	-.16	.43	-.32	.40	-.12	.34

* Subscripts: o - estimated from observed sub-record, e - estimated from extended sub-record.

n -- underlined quantity n is outside acceptance region.

TABLE 5

ANNUAL PARAMETER ESTIMATES--OBSERVED SUB-RECORDS AND EXTENDED SUB-RECORDS

Sub-record Number	$\bar{x}_o$*	$\bar{x}_e$	s_o	s_e	r_o	r_e
1	46.76	45.19	13.39	14.07	-.21	.06
2	<u>73.96</u>	48.77	11.40	14.83	-.52	.33
3	<u>38.66</u>	42.95	13.84	14.94	-.05	.11
4	47.73	51.66	15.67	14.63	-.21	.11
5	48.82	52.55	<u>9.04</u>	14.83	.27	.11

Means and standard deviations in centimeters.

* Subscripts: o - estimated from sub-record, e - estimated from extended sub-record.

<u>n</u> -- underlined quantity falls outside acceptance region.

TABLE 6

VARIANCE OF MONTHLY ESTIMATES AROUND BEST ESTIMATES

Month	Var* $\bar{x}_o$	Var $\bar{x}_e$	Var Red. %	Var s_o	Var s_e	Var Red. %	Var r_o	Var r_e	Var Red. %
Oct	0.17	0.19	-8	0.13	0.04	73	0.025	0.033	-32
Nov	4.15	0.18	96	3.54	0.10	97	0.036	0.004	89
Dec	7.54	0.61	92	3.18	0.12	96	0.182	0.006	97
Jan	19.86	0.44	98	6.83	0.61	91	0.511	0.005	99
Feb	1.25	0.42	66	3.63	0.70	81	0.284	0.005	98
Mar	9.95	0.36	96	2.73	1.24	55	0.466	0.016	97
Apr	1.96	1.26	36	1.38	0.05	97	0.548	0.005	99
May	1.85	0.19	90	2.04	0.47	77	0.227	0.072	58
Jun	1.51	0.37	75	1.92	0.13	93	0.205	0.008	96
Jul	1.75	0.08	95	4.45	1.26	72	0.340	0.020	94
Aug	0.30	0.16	49	0.52	0.11	78	0.412	0.079	61
Sep	0.25	0.23	7	0.81	0.22	73	0.435	0.032	93
Ann.	144.58	16.00	89	9.81	0.13	99	1.380	0.116	91

Values for variance of mean and standard deviation estimates are in cm^2.

* Subscripts: o - estimates from sub-record, e - estimates from extended record.

Discussion of Results

The use of the extended record reduced the parameter estimate variance significantly for all annual model parameters (Table 6). The extended record estimates of the annual mean and standard deviation were within the hypothetical acceptance regions (Table 5), indicating that the parameter estimates were being adequately improved. The majority of the extended record estimates of the serial correlation coefficient, r, were low but were much more reliable than those obtained directly from the sub-record. This was somewhat expected since the sub-records were quite short.

The variance around the best estimate for monthly mean flows was reduced by using extended-record estimates in all months except October (Table 6). Poor results for October were expected, since the best parametric model would not preserve the mean for this month. Extended-record estimates of the mean for the remaining months were checked to determine if they had been adequately improved. Underlined values of extended-record estimates in Table 2 indicate those months for which parameter improvement was not adequate by the 95% confidence interval criterion. In addition to October, extended-record estimates of the mean in April, August, September, and June were not adequate. This was expected for April, August, and September, since the best parametric model could not preserve these parameters (cf. Table 1). The best parametric model estimate of the mean flow in June was also very near the upper 95% confidence limit (Table 1). For the remaining months the extended-record estimates of the mean were found to be adequate, and the average reduction in variance, around the best estimate, obtained by using the extended record was 90%.

Using the extended-record estimates of monthly flow standard deviations reduced the variance around the best estimate for all months (Table 6). Checking for adequacy of improvement revealed that extended-record parameter estimates for the months of October, August, May, July, and September were not always within the acceptance regions (Table 3). This result was expected for May, July, and September (cf. Table 1). However, the best parametric model estimates of the standard deviation for flows in October and August were well within the respective acceptance regions (Table 1). For these two months an historical record longer than the 4-year sub-records would be required for this parametric model to yield extended-record estimates of the standard deviations which were within the given acceptance regions. For the remaining months, in which extended-record estimates of the standard deviation were found adequate, the average reduction in variance around the best estimate was 87%.

A reduction in the variance around the best estimate of the lag-one monthly serial correlation coefficient, r, was produced by using the extended-record estimates in all months but October (Table 6). It seemed that for October extended-record estimates for r would be consistently low (cf. Table 4) using this parametric model. The value obtained by the best parametric model (Table 1) appeared to verify this. Extended-record estimates of r were adequate for all remaining months except August (Table 4). The value of r for August produced by the best parametric model (Table 1) indicated that August might be troublesome. In the remaining months the average reduction in variance resulting from using extended records was 92%.

Hence, in this example, estimates of annual stochastic model parameters were found to be significantly and, tentatively accepting the serial correlation coefficient estimates, adequately improved by extending a four year record with the parametric model to a record 31 years in length. The reduction in variance of parameter estimates was about 90%.

Except for the month of October, the variance around the best estimates for monthly parameter estimates was reduced by using extended records. The only months in which all three parameter estimates were adequately improved (i.e. always fell in acceptance region) for this example, however, were November, December, January, February, and March.

PARAMETRIC MODEL EVALUATION IN THE CONTEXT OF WATER SUPPLY RESERVOIR SIZING

At this point the Haan model was tested for its ability to improve stochastic model parameter estimates in a water supply context. The test variable was S, the storage required to meet assumed demands from a reservoir on the watershed with 99% reliability.

The stochastic monthly Markov streamflow model (Fiering and Jackson, 1971) was used in the form

$$q_j = \bar{q}_j + \frac{r_j s_j}{s_{j-1}} (q_{j-1} - \bar{q}_{j-1}) + s_j(1 - r^2)^{1/2} t_j \qquad (2)$$

where

q_j = flow in month j, inches;

$\bar{q}_j$ = estimated mean flow in month j, inches;

s_j = estimated standard deviation of flow in month j, inches;

r_j = estimated first-order serial correlation coefficient between flows in months j and j-1; and

t_j = independent normally distributed random variable with mean 0 and variance 1.

Equation 2 was used to generate synthetic streamflow sequences for input to the reservoir, with a 40 year design life, from which an outflow demand of D = .833 inches/month was required.

The parametric model evaluation in this context proceeded as described earlier. The monthly values of the stochastic model parameter estimates obtained previously were used. One hundred possible 40-year monthly streamflow sequences were generated, using equation 2, for each set of parameter estimates. Each of these sequences was routed through an initially full, large, hypothetical reservoir under the assumed demands using

$$S_{j+1} = \min (S_{max}, S_j + X_j - D) \qquad j = 1, 2, \ldots, 480$$

$$S_1 = S_{max}$$

where

S_j = storage beginning month j, inches;

S_{max} = maximum storage available, inches;

X_j = inflow month j, inches; and

D = demand month j = .833 inches.

The hypothetical reservoir was large enough so that $S_j < 0$ could not occur for any j. The maximum deficit which occurred in this reservoir corresponded to the capacity required to meet demands during this period. The 100 estimates of required storage thus obtained were fit to a probability distribution (Gumbel's Extreme Value distribution, Appendix A) in order to find the storage which would meet demands with 99% reliability. For each set of monthly stochastic model parameter estimates an associated value of reservoir required storage capacity was found. These values are shown in Table 7.

The storage associated with the stochastic model parameter set estimated from the 31 year historical record was taken as the best estimate, S*. The storage associated with the set of parameter estimates obtained from the simulated streamflow of the best parametric model was compared with S* to determine if the parametric model had the potential to improve stochastic model parameter estimates in this context. In this case the values differed by about 4.5% which indicated that the parametric model might be useful.

TABLE 7

STORAGE REQUIREMENTS

Stochastic Model Parameter Estimates Based on	Storage Required (inches)
31 year historical record	8.60
Simulated streamflow from parametric model	8.23
Historical sub-record 1	9.02
Historical sub-record 2	7.29
Historical sub-record 3	9.78
Historical sub-record 4	8.59
Historical sub-record 5	5.78
Extended sub-record 1	9.42
Extended sub-record 2	8.82
Extended sub-record 3	10.90
Extended sub-record 4	8.09
Extended sub-record 5	7.57

Five values of required storage were found using the sets of parameter estimates obtained from the 5 sub-records. Five values of required storage were also found using the sets of parameter estimates derived from the records which had been extended using the parametric model. The variance of each set of 5 values around the best estimate was computed as

$$\mathrm{Var} = \sum_{i=1}^{5} (S_i - S^*)^2/5.$$

The variance, around the best estimate, of the estimated storage requirements based on the original sub-records was 2.23. The variance, around the best estimate, of the estimated storage requirements based on the extended sub-records was 1.44. The reduction in the variance of estimated storage requirements obtained by using the parametric model to extend the sub-records was 35%. Thus it appeared that the parametric model could be used to improve the reliability of short-record estimates of the stochastic model parameters in the context of water supply reservoir sizing.

RAINFALL SIMULATION

It was shown in the previous section how a parametric model might be employed to improve estimates of parameters in stochastic models by utilizing historical rainfall records. This procedure may yield adequate results in areas where long rainfall records are available.

Frequently, however, these long rainfall records are not available. Rainfall data may be transferred from a location which is not near the watershed, but the errors introduced may be significant. An alternative to the transfer of data is to model the daily rainfall.

MARKOV MODELS OF DAILY RAINFALL

Studies have indicated that a first order Markov process may give a reasonably accurate representation of daily rainfall amounts at a point (Adamowski and Smith, 1972). Markov chains have been used to synthesize daily rainfall records (Khanal and Hamrick, 1971; Allen and Haan, 1975). These Markov chain models are constructed from transition probabilities which represent the probability of an amount of rain X_{n+1} on day n+1 given that the rainfall on day n was X_n. To reduce the number of transition probabilities which must be estimated, daily rainfall is divided into a number of states, e.g. state 1 = no rain, state 2 = (.01, .05), ..., state m-1 = (.45, .75), state m = (.75, ∞), (values in inches, say). Assuming that the transition probabilities are stationary within a season of the year and that there are no over-year periodicities or trends, the model then consists of N transition matrices of order m (assuming an equal number of states in each season), where N = number of seasons in the year and m = number of rainfall states. The number of transition probabilities which must be estimated is then Nm(m-1). If, for example, N = 12 (monthly seasons) and m = 7, 12 x 7 x 6 =

504 parameters, at a minimum, must be estimated from the data. Additional parameters must be estimated depending upon the choice of rainfall distribution within states. Allen and Haan (1975) used uniform distributions for all states but state m for which a shifted exponential was used, requiring the estimation of an additional parameter. Thus a long historical record will be required to yield stable parameter estimates. In an effort to reduce the historical record requirements while retaining the Markovian structure and comparable accuracy, the present model modification was examined. First the general structure of the proposed model will be discussed. Then the model will be fit to data from the seven weather stations used in the Allen and Haan (1975) study to determine if the modified model is in fact an improvement.

GENERAL MODEL STRUCTURE

The basic assumptions of this model, and the previously mentioned models, are:

(1) The probable amount of rain on day i+1 depends only on the amount of rain on day i;

(2) For a given season within the year, the stochastic structure of daily rain is the same for each day and does not change from year to year.

Like the previous models, the present model requires that daily rainfall amounts be divided into convenient "states", state 1 = no rain, etc. In the earlier models the probability density function of the amount of rain on day n+1, given the state of the system on day n, has essentially the form of a histogram determined by the transition probabilities. The present model smooths this function by fitting a continuous density function to that portion of the histogram where rainfall actually occurred. (A similar approach was used by Jones, et al. (1970) with states consisting of "dry" and "wet".) That is, given that the system is in state i on day n in season k, then the probability distribution of the amount of rain on day n+1 is given by

$$P(X_{n+1} \leq X | X_n \varepsilon\ i,\ \text{season } k) = p_{i1}^k + (1 - p_{i1}^k)F(X|i,k) \qquad (3)$$

where

X_n = rainfall on day n;

p_{i1}^k = probability of no rain on day n+1 given X_n in state i, season k; and

$F(X|i,k)$ = distribution of rainfall values given rain occurs on day n+1, $X_n \varepsilon$ i, season k.

Hence to each state in each season there is a corresponding distribution function of the form (3). The parameters p_{i1}^k are estimated from the historical data as follows. Let f_{i1}^k equal the historical frequency of transitions from state i to state 1 (no rain) in season k and let f_i^k be the total number of occurrences in state i, season k. Then $p_{i1}^k = f_{i1}^k/f_i^k$ is the maximum likelihood estimator. The parameters for each distribution $F(\cdot|i,k)$ are determined from the historical data using the set of observations $[X_{n+1}|X_{n+1} > 0,\ X_n \varepsilon\ i,\ \text{season } k]$. In the application described later, the method of maximum likelihood was used to estimate these parameters.

Synthetic traces of daily rainfall are generated in the following manner, given that the system is in state i in season k:

(1) Generate a uniform random number, x, from (0, 1).

(2) If $x \leq p_{i1}^{k}$, then $X_{n+1} = 0$.

(3) If $x > p_{i1}^{k}$, generate a random observation, u, from $F(\cdot|i,k)$. Set $X_{n+1} = u$ and determine state of X_{n+1}.

(4) Repeat 1-3 changing seasons when required.

A MODEL FOR KENTUCKY

The model was fit to daily rainfall records from weather station #0254 in Ashland, Ky. Forty years, 1932-1971, of daily rainfall data were available, with rainfall measured to the nearest hundredth of an inch. For model fitting a trace of rain was considered equivalent to no rain.

Each month of the year was considered a season satisfying the assumptions of the model. Daily rain was divided into three states, state 1 = (0.0, .005) = no rain, state 2 = (.005, .145), state 3 = (.145, ∞), values in inches. This classification was chosen since approximately equal numbers of rainfall events fell in class 2 and 3 for this station. The two-parameter gamma distribution given by

$$F(x|i,k) = \int_0^x \frac{\lambda_{i,k}^{\eta_{i,k}}}{\Gamma(\eta_{i,k})} U^{(\eta_{i,k} - 1)} \exp(-\lambda_{i,k} U)\, dU$$

where $\eta_{i,k}$ and $\lambda_{i,k}$ are the shape and scale parameters, respectively, was investigated as a distribution of rainfall amounts out of state i, season k. The parameters p_{i1}^k, $\eta_{i,k}$, $\lambda_{i,k}$ were estimated from the data by the method of maximum likelihood. The maximum likelihood estimators for $\eta_{i,k}$ and $\lambda_{i,k}$ were approximated by a method due to Greenwood and Durand (1960)

$$\eta^* = (.5000876 + .1648852y - .0544274y^2)/y$$

$$\text{if } 0 \leq y \leq 0.5772$$

$$\eta^* = \frac{8.898919 + 9.05995y + .9775373y^2}{y(17.79728 + 11.968477y + y^2)}$$

$$\text{if } 0.5772 \leq y \leq 17.0$$

where,

$$y = \ln\left(\sum_{i=1}^{n} x_i/n\right) - \sum_{i=1}^{n} \ln(x_i)/n$$

x_i = i^{th} sample observation, from a sample of n observations.

The estimate η^* was corrected for small-sample bias as shown in Shenton and Bowman (1970, p. 61)

$$\eta = (n-3)\eta^*/n.$$

The estimate for λ is

$$\lambda = \eta/\left(\sum_{i=1}^{n} x_i/n\right).$$

For the months of June-October a single density function was used for wet events from states 2 and 3 since it was felt that the number of observations was not sufficient to reliably estimate the parameters of a density function for each state. Parameter estimates obtained are shown in Tables 8 and 9.

The theoretical distributions were then checked for goodness of fit to the historical data by using χ^2 tests. To compute the value of χ^2, rainfall values were partitioned into 10 intervals using the interval boundaries .005, .025, .045, .075, .105, .145, .205, .325, .565, 1.255, and UPPER BOUND. The last two intervals were combined for state 3 in November. The value selected for UPPER BOUND depended on the estimated maximum rainfall value for the season. These values ranged from 1.955 in November to 3.405 in July. Observed frequencies in each class were determined from the historical data. Expected frequencies in each class were determined from the theoretical gamma distribution by numerically integrating between the interval boundaries to obtain the expected probability and then multiplying the expected probability times the total number of events. Since two parameters were estimated from the data, the χ^2 obtained had 6 degrees of freedom for state 3 in November and 7 degrees of freedom for the remaining months. The computed χ^2 values are shown in Table 10a and seemed to justify the use of the gamma distribution.

At this point the model consisted of 12 seasons and 3 states per season with a different distribution for each state. The probabilities

TABLE 8

ESTIMATED PROBABILITY OF NO RAIN ON DAY n+1 WHEN RAIN ON DAY n IS IN STATE i, ASHLAND, KY.

	State, day n		
Month	1	2	3
Jan	.68	.56	.41
Feb	.65	.56	.49
Mar	.63	.54	.43
Apr	.69	.50	.45
May	.73	.47	.39
Jun	.72	.57	.55
Jul	.71	.53	.51
Aug	.78	.55	.53
Sep	.81	.64	.50
Oct	.80	.64	.57
Nov	.73	.58	.45
Dec	.68	.61	.43

TABLE 9

ESTIMATED PARAMETERS FOR GAMMA DISTRIBUTED

RAINFALL VALUES, ASHLAND, KY.

		Initial Model	
Month	State	η	λ
Jan	1	.79	3.2
	2	.65	2.9
	3	.80	2.2
Feb	1	.84	3.4
	2	.74	2.8
	3	.68	2.4
Mar	1	.86	2.8
	2	.63	1.9
	3	.74	2.5
Apr	1	.86	3.1
	2	.77	2.6
	3	.77	2.5
May	1	.91	3.1
	2	.83	2.8
	3	.89	2.6
Jun	1	.81	2.1
	2	.75	2.1
Jul	1	.88	2.3
	2	.75	1.9
Aug	1	.71	1.8
	2	.74	2.0
Sep	1	.71	2.1
	2	.72	1.7
Oct	1	.90	3.5
	2	.73	2.6
Nov	1	.74	3.0
	2	.80	3.3
	3	.65	2.5
Dec	1	.90	3.9
	2	.70	2.3
	3	.73	2.5

TABLE 10a

GOODNESS OF FIT OF STATE RAINFALL DISTRIBUTIONS

	Computed χ^2, 7 degrees of freedom		
		State	
Month	1	2	3
Jan	10.3	10.9	6.1
Feb	7.1	7.0	8.3
Mar	9.6	16.5	18.8*
Apr	5.0	5.4	11.0
May	5.4	13.2	8.5
Jun	10.2	10.5	-
Jul	2.1	7.2	-
Aug	41.4*	20.5*	-
Sep	11.7	13.5	-
Oct	11.2	14.2	-
Nov	29.3*	13.0	9.2[1/]
Dec	10.7	7.7	7.8

1 - 6 degrees of freedom.

* - significant at the .01 level.

p_{i1}^k, i = 1, 2, 3; k fixed, were distinctly not equal and appeared to be monotonically decreasing for increasing i (cf. Table 8). In other words, as the amount of rainfall on day n increases, the probability of no rain on day n+1 decreases. The possibility of making this probability a continuous function of the rainfall amount was considered but not pursued. Visual examination indicated that the $F(\cdot|i,k)$ were very similar in most cases for a particular season, k. Rainfall values from each of the states were pooled and parameters η_k and λ_k for a common distribution were estimated. The hypothesis that the rainfall values out of each of the states came from this common distribution could not be rejected in most seasons based on computed χ^2 values (cf. Table 10b). Thus for each of the 12 seasons only one distribution, $F(\cdot|k)$, of rainfall values was used and the number of parameters needed to be estimated was reduced by 38. Some modelling detail was lost by assuming one distribution per month; however, since a primary goal was to reduce historical data requirements and thus extend the usefulness of the model it was hoped (and subsequently was the case) that the loss of detail would not be significant. The final model consisted of 12 seasons with 3 states and one rainfall distribution per season. The estimation of p_{i1}^k, i = 1, 2, 3 and 2 parameters to fit $F(\cdot|k)$ in each season was required, or a total of 12 x (3 + 2) = 60 parameters. The parameters η_k and λ_k for the final model for Ashland, Ky. are shown in Table 11.

The method of generating synthetic rainfall traces was described earlier. The process is initialized as follows, assuming the first day

TABLE 10b

COMPUTED χ^2 FOR TEST THAT OBSERVATIONS FROM INDIVIDUAL STATES CAME FROM COMMON DISTRIBUTION

	State			
Month	1	2	3	Pooled Data[2/]
Jan	12.0	24.9*	12.5	18.1
Feb	5.8	8.3	10.6	12.5
Mar	8.2	24.4*	20.1*	25.4*
Apr	5.3	5.9	10.7	8.8
May	7.3	14.3	7.8	12.6
Jun	9.3	12.5	-	16.0
Jul	2.0	9.4	-	7.5
Aug	41.2*	21.6*	-	42.5*
Sep	15.0	13.5	-	17.1
Oct	10.6	16.6	-	17.0
Nov	29.3*	11.9	11.8[1/]	34.0*
Dec	11.3	6.0	9.9	12.4

1 - 6 degrees of freedom.

2 - values of χ^2 in this column indicate goodness-of-fit of the gamma distribution to all rainfall observations in the season.

* - significant at the .01 level.

TABLE 11

ESTIMATED PARAMETERS FOR GAMMA DISTRIBUTED RAINFALL VALUES

ASHLAND, KY.

	Final Model	
Month	η	λ
Jan	.75	2.7
Feb	.78	3.0
Mar	.77	2.5
Apr	.82	2.8
May	.89	2.9
Jun	.79	2.1
Jul	.82	2.1
Aug	.73	1.9
Sep	.72	1.9
Oct	.83	3.1
Nov	.74	2.9
Dec	.81	3.1

generated is January 1:

(1) Determine the absolute probabilities p_i^{12}, i = 1, 2, 3 of being in state i on December 31 from the historical data.

(2) Generate a uniform random number, x, on (0, 1).

(3) If $0 < x \le p_1^{12}$, initial state = 1
$p_1^{12} < x \le p_1^{12} + p_2^{12}$, initial state = 2
$p_1^{12} + p_2^{12} < x \le 1$, initial state = 3.

The generation of random observations from a gamma distribution with non-integral shape parameter was performed using a method suggested by Whittaker (1973). Because of closure under addition, a gamma random variable with any shape parameter can be constructed if one can be constructed for shape parameter p, $0 < p < 1$. If $0 < \eta < 1$ and U_1, U_2, U_3 are three independent uniform random numbers from (0, 1), let $S_1 = U_1^{1/\eta}$ and $S_2 = U_2^{1/(1-\eta)}$. If $S_1 + S_2 \le 1$, define $Y = S_1/(S_1 + S_2)$ and $X_1 = -(Y/\lambda) \log_e U_3$. Then X_1 has the gamma distribution with shape parameter η and scale factor λ. If $S_1 + S_2 > 1$, U_1 and U_2 are rejected and two more uniform random numbers on (0, 1) are generated. The probability that $S_1 + S_2 \le 1$ depends on the value of η and has a minimum of 0.785 for $\eta = .5$.

Evaluating the Model

The model form obtained using the Ashland, Ky. data was used with 6 other Kentucky weather stations shown in Table 12. Each of these

TABLE 12

WEATHER STATIONS USED IN STUDY

Station Number	Location
0254	Ashland
0909	Bowling Green
1345	Carrollton Lock
3762	Henderson
3994	Hopkinsville
4825	Little Hickman
6353	Pikeville

stations had 40 years of historical data from which model parameters were estimated. In order to evaluate the model performance, a number of descriptive statistics were tabulated for historical and simulated rainfall:

(a) Mean monthly rainfall (Table 21, Appendix B),

(b) Standard deviation of monthly rainfall (Table 22, Appendix B),

(c) Maximum runs of wet and dry days (Table 23, Appendix B),

(d) Maximum daily rainfall by months (Table 24, Appendix B),

(e) Average annual number of wet days (Table 25, Appendix B),

(f) Maximum and minimum total annual rainfall (Table 26, Appendix B).

For each station, 6 simulated traces of 40 years length were generated.

Table 21, Appendix B, shows that the model produced values for monthly and annual mean rainfalls which agreed well with historical values. The mean annual rainfall based on the average of six simulations was within 0.45 inches or 1.1% of the historical values for all stations.

In Table 22, Appendix B the observed and simulated standard deviations are shown to be similar. The simulated rainfalls appeared to have standard deviations which were slightly smaller than those from observed rainfall, particularly for the months of January and February. The smaller standard deviations might be attributable to the decision to use a single rainfall distribution in each season for model simplicity and reduction of parameters.

Maximum runs of wet and dry days are shown in Table 23, Appendix B. The model seemed to be able to produce runs similar to those historically observed. Table 24, Appendix B, shows maximum daily rainfalls by month. The model appeared to produce large rainfalls with less frequency than historically encountered. There was usually one simulation with a larger maximum than the historical maximum, but the average over all simulations tended to be smaller.

Table 25, Appendix B, shows the average annual number of wet days, which the model reproduced very well. Maximum and minimum total annual rainfalls are shown in Table 26, Appendix B. The historic values were approached or exceeded by the simulated values for most stations. The average minimum simulated values were slightly larger than the historic minimums for all stations, however.

In Tables 23 and 25 it was seen that the Markov model gave a good representation of sequences of wet and dry days. The fitted distributions (or perhaps the stationarity assumptions) appeared to produce extreme values less frequently than historically observed.

Since the rainfall modeling was performed to provide inputs to a watershed model, a test of the rainfall model for this purpose would be to compare streamflow values produced by inputs of simulated and historical rainfall into a watershed model. The four-parameter monthly water yield model developed by Haan (1972b) was used. Table 13 shows the values of the four parameters used in the Haan model at each of the seven weather stations. Simulated and historical rain was routed

TABLE 13

MODEL PARAMETERS USED IN RUNOFF GENERATION

Rainfall Station	f_{max}	s_{max}	c	f
254	.67	.054	3.64	.65
909	.53	.080	4.85	.70
1345	.50	.017	4.11	.36
3762	.53	.068	5.33	.23
3994	.58	.080	5.65	.14
4825	.55	.058	3.80	.52
6353	.53	.032	7.50	.53

f_{max} = maximum infiltration rate, inches/hr.

s_{max} = maximum daily seepage rate, inches/day.

c = moisture holding capacity of less readily available groundwater storages, inches.

f = fraction of seepage that becomes runoff.

through the watershed model to produce simulated monthly runoff at each station. Mean monthly and annual runoff values are shown in Table 27, Appendix B. Runoff from simulated rain agreed well with runoff from historical rain. Mean annual runoff averaged over the six sets produced by simulated rainfall was slightly lower than the mean annual runoff from historical rain at each station. The maximum difference never exceeded 0.71 inches, however.

For each 40 year runoff record, from simulated and historical rain, the most severe 1-12 month low flows were computed, based on a March-February year, and are shown in Table 28, Appendix B. The average values of depth-duration low flows produced by the simulated rain approximated corresponding values produced by historical rain except for the longer durations of 10, 11, and 12 months. The low flows for these longer durations were usually larger for the runoff produced from simulated rain than the low flows produced from the historical rain.

Each 40 year record of runoff was routed through a monthly reservoir storage model to compare required storages based on runoff produced from simulated rain with storage required based on runoff produced from historical rain. Two constant annual water demand levels (reservoir outflows) were used, 5 inches and 10 inches. Results are shown in Table 14. With the exception of station #1345, the 5 inch demand storage requirements were similar for runoff produced by historical rain and runoff produced by simulated rain. For the 10 inch annual demand, storage requirements from simulated rain tended to be

TABLE 14

RESERVOIR CAPACITY (INCHES)

5 inch demand

Simulation

Sta	1	2	3	4	5	6	Ave.	Hist
254	2.0	2.0	2.0	2.2	1.7	1.8	1.95	1.8
909	1.5	1.2	1.2	1.9	1.2	1.6	1.43	1.5
1345	2.9	2.5	2.8	3.1	3.2	2.9	2.90	4.0
3762	2.8	2.8	3.8	3.4	3.3	2.4	3.08	3.3
3994	5.1	5.6	3.9	4.7	4.1	4.1	4.58	4.3
4825	2.1	1.6	2.4	1.8	1.9	1.8	1.93	2.0
6353	1.6	2.1	2.0	2.5	1.5	1.9	1.93	2.0

10 inch demand

Simulation

Sta	1	2	3	4	5	6	Ave.	Hist
254	7.3	4.8	9.8	5.2	5.2	4.7	6.17	9.0
909	3.4	3.6	4.2	4.6	3.0	4.4	3.87	4.0
1345	7.1	7.0	11.4	11.0	9.4	7.0	8.82	13.0
3762	20.3	12.0	12.5	11.6	14.8	10.2	13.57	12.5
3994	16.7	17.5	16.0	16.8	20.5	14.2	16.95	19.0
4825	4.7	4.4	7.1	4.9	7.7	4.7	5.58	5.8
6353	6.6	9.3	6.5	8.7	6.5	6.4	7.33	11.0

smaller than those for the historical rain. For all stations except #1345 and #6353, however, at least one simulation required a reservoir size which was greater than that required by the historical rainfall. Thus it appeared that although simulated runoff produced from synthetic rainfall might give an estimate for the storage required to be 99% sure of no reservoir failure during the 40-year design life of the reservoir which was less than the true 99% value, the simulated storage requirement would still offer a better estimate of this storage than the single historical record.

Data Requirements

The final step in the model evaluation was to determine how much historical data would be required to yield stable estimates of the model parameters. This determination was made within the hypothesis that the model was correct. The first 15 years of historical data from station #0254 were used to estimate the model parameters. Forty years of simulated rainfall were then generated and model parameters were estimated from the initial 5, 10, 15, 20, 25, 30, 35 years of the simulated record. Visual inspection of these parameter estimates indicated that about 15-20 years, or 150 rainfall events per season, were required to yield stable estimates. Figures 2 and 3 show an example of parameter estimate stability vs. years of record used. Thus about 15 years of historical data is necessary for a monthly seasonal model in the Kentucky area. If fewer than 15 years of data were available it would be

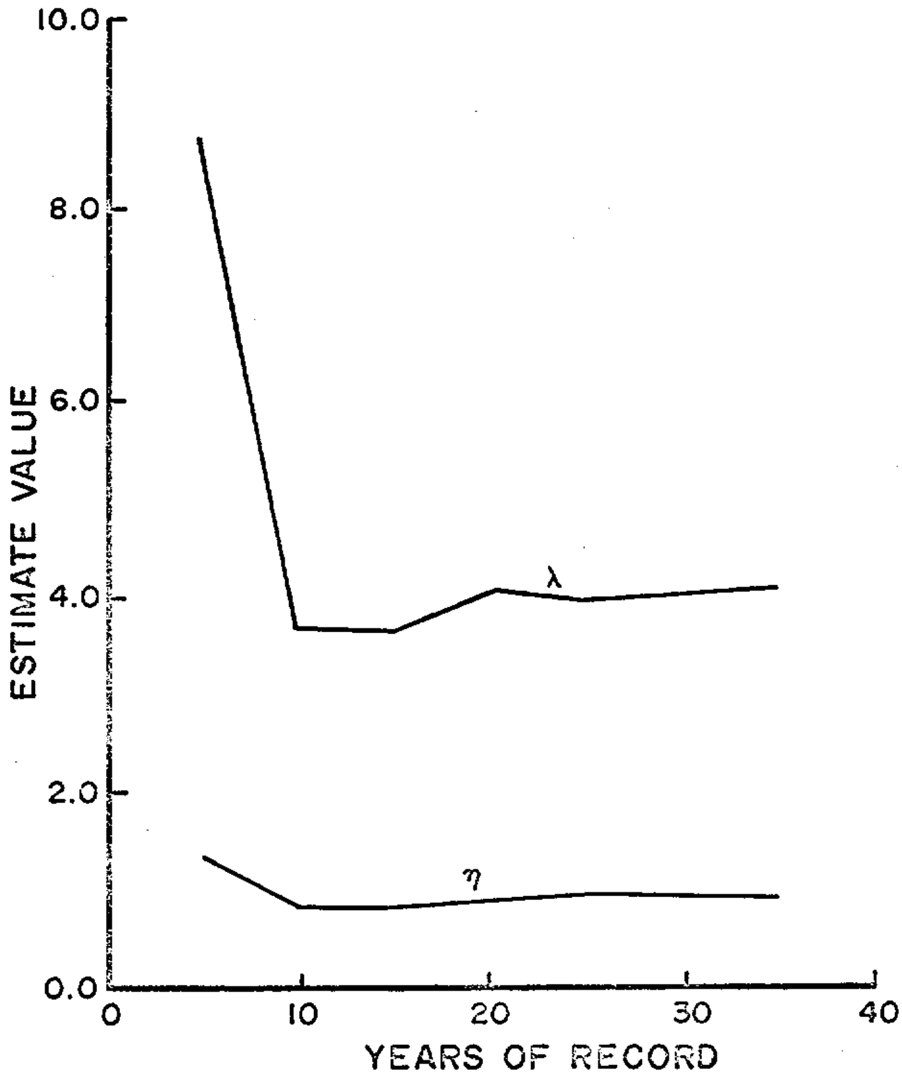

Fig. 2. Behavior of η and λ as a function of record length used in estimation. October.

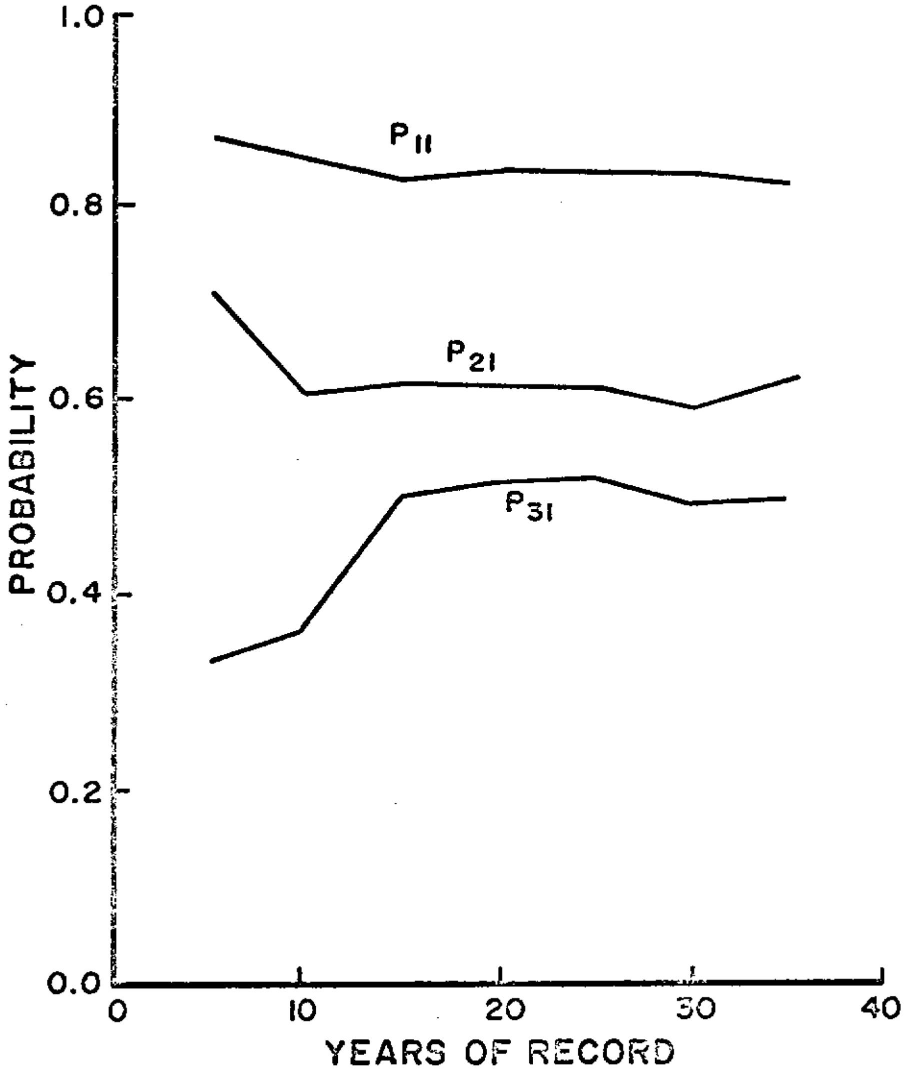

Fig. 3. Behavior of estimates for transitions to dry state as a function of the number of years of record used in estimation. October.

necessary to use a model with seasons longer than a month.

The modified model was then compared with the Markov chain model developed by Allen and Haan (1975) to determine if the modified model provided any improvement. The statistics generated by the modified model were compared with those obtained using the same seven weather stations and the Markov Chain model. The modified model was better able to reproduce monthly and annual mean rainfall than the Markov Chain model which was biased toward higher values. The modified model reduced the absolute error in annual rainfall from 2.5% to 0.5%. The remaining rainfall statistics were of comparable accuracy. Simulated runoff produced using each rainfall model with the Haan water yield model (1972) was compared. Synthetic rain from the modified model produced significantly better values of mean monthly and annual runoff than did rainfall obtained using the Markov Chain model. Annual runoff values with the Markov Chain model averaged about one inch high for all stations. The annual runoff from the modified model was slightly less than that from historical rainfall for all stations. Comparing the reservoir storage requirements using each of the rainfall models showed that the modified model produced better results on the average. The historical data requirements of the modified model were found to be considerably less than those of the Markov Chain model, which appeared to require about 40 or more years of record.

SUMMARY OF CHAPTER IV

A methodology was presented for evaluating parametric streamflow models for use in improving estimates of parameters in stochastic models of streamflow. In an example, a particular parametric streamflow model was studied (Haan, 1972). This parametric model was found to significantly improve annual stochastic model parameter estimates based on 4 years of runoff data, when 31 years, including those years of runoff record, of daily rainfall data were available. The parametric model improved estimates for 34 of 36 monthly stochastic model parameters (cf. Table 6), as compared with estimates based on four year historical records of monthly flow. The parametric model was evaluated for its ability to improve estimates of parameters in a monthly Markov streamflow model in the context of water supply reservoir sizing. It was found that the reliability of short-record estimates of these parameters could be improved in this context by using the parametric model.

The type of parametric model used in improving stochastic model parameters depends on the inputs required and the available data. A parametric model for which few inputs are needed is less restricted by lack of available data but will generally give results which are less accurate than those of a model which requires a greater number of inputs. Lack of data usually forces the use of the simpler parametric model with its inherent limitations. Due to these limitations there will necessarily exist a runoff record length beyond which the historical data will give better estimates of stochastic model parameters than any extended record obtained from the parametric model. An example of this ($\bar{X}$ and r for October) was seen here. The parametric model predicted values of runoff in October which were too high. As a result, estimates of the October mean derived from 4 years of historical data had less variance about the best estimates than that of estimates derived from the extended record. Once the capabilities of a parametric model are determined to be sufficient for a given problem, it may aid in reducing the uncertainty of stochastic model parameter estimates, when adequate rainfall data is available.

For the case when adequate rainfall data is not available, a daily point rainfall model was developed to provide the necessary inputs to the parametric model. The model used to generate synthetic traces of daily rainfall amounts at a point is a modification of the Markov Chain models. The present model requires discrete probabilities only for transitions into the state corresponding to no rain. Transitions to wet states are determined from a continuous probability distribution. The use of the continuous probability distribution, rather than discrete transition probabilities, greatly reduces the number of parameters which must be estimated from the data and hence the historical data requirements. A two-parameter gamma distribution was used in the model at seven rainfall stations in Kentucky. The model was able to more accurately model the daily rainfall process than the standard Markov Chain model and appeared to offer a promising approach to the problem of synthesizing records of daily rainfall at a point.

The methods and models presented in this chapter make it possible to simulate reliable streamflow data for simulation studies in reservoir design and operation when minimal amounts of historical streamflow and rainfall data are available.

CHAPTER V

WATER PRICE AND RESERVOIR DESIGN

It was noted earlier that reservoir sizing studies require not only a knowledge of the probable inflow pattern to the reservoir, but also the probable outflow (demand) requirements on the reservoir. In the previous chapter methods for improving the reliability of estimates of possible future inflows to a reservoir when historical data was limited were discussed. In the present chapter it will be assumed that reliable estimates of probable future inflows are available, and the role of the demand function in reservoir design will be examined.

The findings of the review of literature pertaining to residential water demand indicated that the price of water could be a significant factor in determining household demand. In the present study the economic efficiencies of various reservoir designs based on different assumptions concerning a possible price-demand relationship for a hypothetical community were evaluated. The purpose of these evaluations was to determine if an appropriate water pricing policy could be used to create greater community benefits from a water supply system than could be realized with conventional methods. Pricing policies based on three different basic assumptions were evaluated. A description of these policies will first be presented and then a hypothetical example will be given to illustrate the economic consequences of their application in reservoir design.

PRICING POLICIES FOR WATER SUPPLY

The assumption of the first policy (really a non-policy) was that demand was not a function of price. This corresponded to the approach which views the demand for water as an intrinsic societal need which must be supplied. With this approach the projected demands on a proposed reservoir are based on present usage. The reservoir capacity was determined which would satisfy the projected demands with some degree of reliability. The price of water was then set to recover system costs.

The assumption of the second policy was that a valid price-demand relationship did exist. A low price for water could cause high usage and benefits, but would require a large reservoir capacity at high cost. A higher price for water would cause lower usage and benefits, but would require less storage capacity at lower cost. That price for water, and the associated reservoir capacity, which yielded maximum net benefits (cf. James and Lee, 1971) was determined for the given level of reliability.

The third policy assumed the price-demand relationship of policy 2 and made the additional assumption that consumers would respond to changes in price so that the price charged might be used to effect water conservation. This permitted the use of flexible pricing policies similar to the short time scale peak-load pricing schemes. The motivation of a variable pricing policy was that if some degree of demand management were possible, increased system benefits might be achieved through a proper pricing policy. When water was plentiful and the reservoir was near full, a lower price for water would increase usage and utilize water which would otherwise have been lost to spillage. Conversely, the required storage might be reduced and net benefits increased if, during periods when reservoir storage was low, conservation were achieved by increasing the price of water. The optimal variable-price policy was determined as that policy which yielded maximum net benefits to the community.

Specific variable-price policies were studied in the example to be given. The policies were first evaluated using a deterministic price-demand function to determine if system benefits could be increased. Since there might be a great deal of uncertainty associated with actual consumer demands at various price levels, the variable-price policies were also evaluated using a random price-demand function. By incorporating the probabilistic price-demand relationship into the design study, the effectiveness of the variable-price policies could be studied when only partial knowledge of consumer response to price changes was available.

Although the following example was based on Kentucky data, the general procedures are valid for any area where a price-demand relationship exists.

Example

A rural Kentucky community of 750 households with a non-residential water demand equivalent to that of 3250 households was assumed. The total community demand for water was thus equivalent to that of 4000 households.

Water to the community was supplied by run-of-the-stream drainage from a four square mile basin. That is, the community was able to utilize available streamflow for water supply, but no facilities were available for monthly carryover storage. During summer and early fall the streamflow was often quite low and the water supply was augmented by pumped groundwater. Excessive groundwater use was undesirable, and during the months of low streamflow enough groundwater was supplied to insure that a minimum of 7 million gallons per month was available to the community but no more. The cost of the water supply was $.40/1000 gallons and this was the price charged for water with the existing system.

It was desired to see if water supply benefits to the community could be increased by building a water supply reservoir on the drainage basin. The design life of the project was taken as 50 years in economic evaluations, with a nominal annual interest rate of 7 percent. The demands for water were assumed to be relatively stable during this period. The probability that the reservoir fail to meet demands during the design life was set to be less than .01. That is, the reservoir was designed to meet demands throughout the design life for at least 99% of all possible future 50-year sequences of monthly inflows and design outflows. A reservoir size-cost curve was assumed. This curve, labelled "cost curve" in figure 4, is given by: present value cost = $.005S^2 + 0.175S - 0.27$ where cost is in \$ million and storage S in inches.

The 31-year (10/41-9/72) monthly streamflow record from the South Fork of the Little Barren River in Metcalfe County, Kentucky, was used as the historical record at the dam site. Since the most severe drought in the 31-year record could be quite different than that of the future 50-year design period, and in any event would not suffice to assess the system reliability, a stochastic streamflow model was used in simulation studies to evaluate the consequences of a number of different possible future 50-year streamflow sequences. A stochastic model which would preserve the estimated values of the first-order serial correlation coefficient, r, mean, $\bar{x}$, and variance, s^2, of monthly streamflow was considered satisfactory. The use of a model requiring a long historical record for parameter estimation was precluded by the amount of available data. Even with 31 years of record, the estimated values of the monthly first-order serial correlation coefficients carried a great deal of

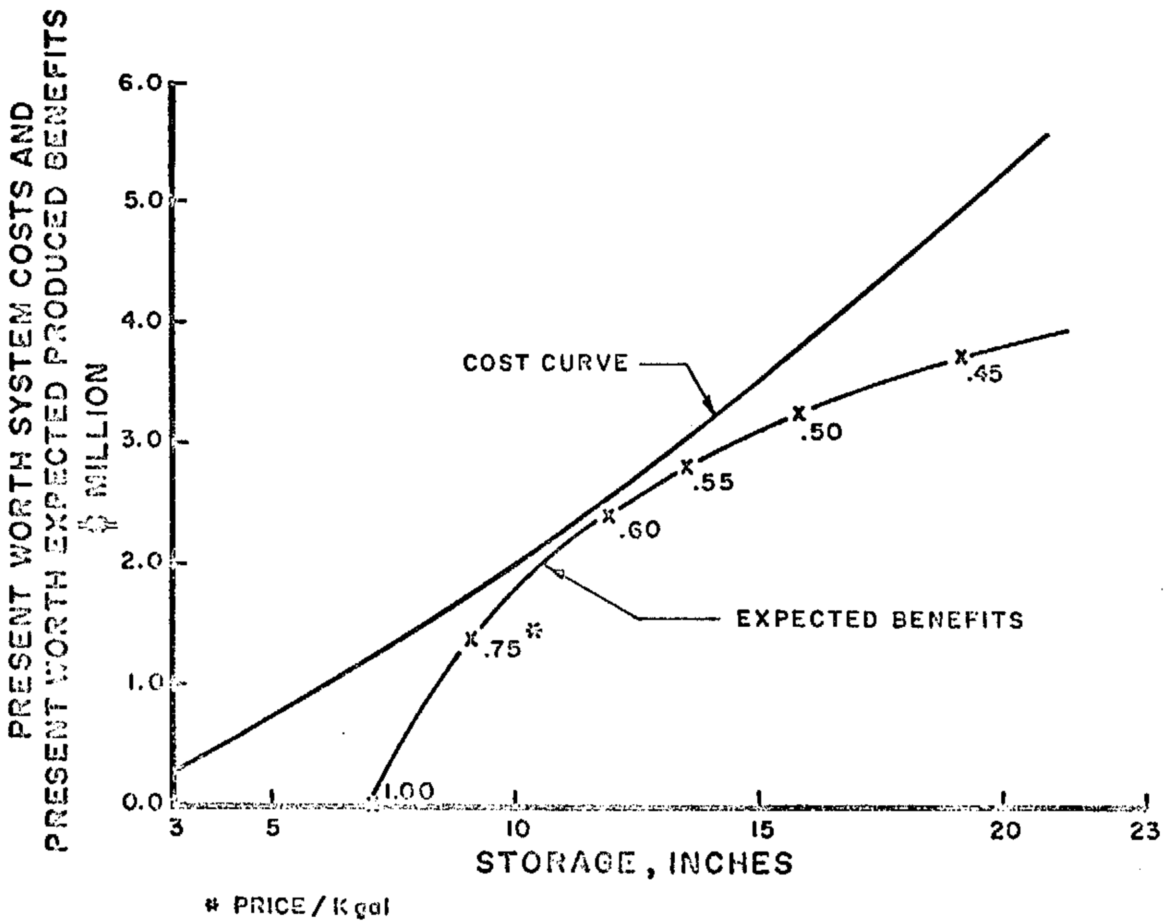

Fig. 4. Costs and present worth of expected produced benefits vs. storage requirements, constant price.

uncertainty. For simplicity it was assumed that all parameter estimates for the streamflow model in this example were correct. The values of the three parameters were estimated for each month of the year and are given in Table 15.

A Thomas-Fiering type streamflow model (Thomas and Fiering, 1962) was used in the form

$$q_j = \bar{q}_j + \frac{r_j\, s_j}{s_{j-1}} (q_{j-1} - \bar{q}_{j-1}) + s_j (1 - r_j^2)^{1/2}\, t \qquad (4)$$

where

q_j = flow in month j, inches;

$\bar{q}_j$ = mean flow in month j, inches;

s_j = standard deviation of flow in month j, inches;

r_j = first-order serial correlation coefficient of flows in months j and j-1; and

t = independent normally distributed random variable with mean 0 and variance 1.

If a value of $q_j < 0$ was generated, it was used to generate q_{j+1} and then q_j was set to 0. It was found that the probability that $q_j = 0$, $p(q_j = 0)$, for a given month j using equation (4) agreed well with $p(q_j = 0)$ derived from the historical data and that equation 4 provided a good representation of the streamflow process, while preserving the estimated values of the parameters.

TABLE 15

STREAMFLOW STATISTICS

Month	r	s*	$\bar{x}^*$
Jan	0.13	2.46	3.22
Feb	0.16	1.86	3.44
Mar	0.14	1.68	3.65
Apr	-0.17	1.21	2.55
May	-0.08	1.14	1.46
Jun	-0.03	0.87	0.75
Jul	0.18	1.29	0.72
Aug	0.49	0.45	0.29
Sep	0.59	0.45	0.24
Oct	0.66	0.24	0.15
Nov	0.63	1.12	0.87
Dec	0.64	1.90	2.25

r - first-order serial correlation coefficient.

s - standard deviation of monthly flows.

$\bar{x}$ - mean monthly flow.

* - values in inches.

A design based on the assumption that the demand for water was not a function of price was first evaluated. The community's "true' annual demand, D, for water was assumed to be 11.98 inches (2560 ac-ft). This assumed annual demand was determined by using the Grunewald et al. relationship, equation 1, with u = 0 and the assumed cost=price of \$.40/1000 gallons to find the annual household demand and then multiplying this value by the assumed 4000 household equivalent community demand. The monthly fractions of annual demand, f_j, j=1, 2, ..., 12, were assumed using information obtained by Dowell (1967) from Lexington, Ky., and are shown in Table 16. "True" demand in each month was then computed as $D_j = D \times f_j$. An additional outflow of .283 inches/month was required to satisfy low flow requirements, evaporation and seepage losses, etc. from the proposed reservoir.

Equation (4) was used to generate synthetic streamflow data for input into a simulation model to determine how much reservoir storage would be required to meet the outflow demands at the given level of reliability, assuming a full reservoir at the beginning of operation. One hundred 50-year sequences of possible future monthly inflows were generated for this purpose. A hypothetical reservoir of sufficient capacity such that no failure (storage < 0) could occur was assumed initially full in the simulation study. Each 50-year sequence of synthetic flows was routed through this hypothetical large reservoir, subject to the given demands, using,

TABLE 16

MONTHLY FRACTION OF ANNUAL DEMAND

Month	Annual Fraction
Jan	0.071
Feb	0.073
Mar	0.079
Apr	0.077
May	0.080
Jun	0.101
Jul	0.099
Aug	0.095
Sep	0.095
Oct	0.081
Nov	0.073
Dec	0.076

Lexington, Ky. data

$$S_{j+1} = \min(S_{max}, S_j + X_j - D_j - Y_j) \qquad j = 1, 2, \ldots, 600 \quad (5)$$

$$S_1 = S_{max}$$

where

S_j = storage at beginning of month j;

S_{max} = maximum storage available in hypothetical reservoir;

X_j = inflow during month j;

D_j = demand during month j; and

Y_j = low flow requirement, losses during month j.

The maximum deficit which occurred in the reservoir for a given 50-year sequence corresponded to the storage capacity which would have sufficed to meet the demands during that period. The 100 values of required storage, S, corresponding to the 100 possible future inflows, were fit to a probability distribution in order to determine the storage capacity, S*, which would insure that the probability of failure during any given future 50-year period was less than .01. It was found that Gumbel's Extreme Value distribution (Type-I maximum) fit the values well (Appendix A). The required storage was found to be 20.3 inches (4330 ac-ft). The estimated cost of the project was assumed to be $5.34 million (cf. hypothetical cost curve, fig. 4). The present value of the system costs without the reservoir over the 50-year design life was assumed to be $3.25 million (based on average supply at the $.40/1000 gallons cost), so that total system costs were $8.59 million. (The added costs of the reservoir system were assumed to be completely separable and additive to the costs of the original system.) The price of water was set so that receipts from the sale of water would equal system costs. Solving

$$\sum_{i=0}^{49} \sum_{j=1}^{12} PQf_j(1/(1+r/12))^{12i+j} = 8,590,000$$

for P, where

P = price, \$/1000 gallons;

Q = annual water use, 1000s gallons;

f_j = fraction of annual use in month j; and

r = nominal annual interest rate

gave P = \$.75/1000 gallons.

The second method of design, which assumed a valid price-demand relationship, was then evaluated. Using the relationship derived by Grunewald et al. (1975), the expected demand at a given price P was determined as:

$$Q = \exp(4.5)P^{-.915}\ \overline{\exp(u)} = 1.05\ \exp(4.5)P^{-.915} \qquad (6)$$

where

Q = household demand in thousands of gallons per year,

u = normally distributed random variable with mean 0 and variance 0.096,

P = price of water in \$/1000 gallons, and

$\overline{\exp(u)}$ = expected value of exp(u).

Using the assumed monthly breakdown of annual use (Table 16) gave the expected community monthly demand at price P,

$$Q_j = 1.05\ f_j\ H \exp(4.5)\ P^{-.915} \tag{7}$$

where

H = number of households and

f_j = monthly fraction of annual demand.

Equation 7 was used to find the community demands for prices ranging from \$.40/1000 gallons to \$4/1000 gallons.

Using these demand levels and the streamflow sequences generated earlier, the reservoir routing procedure was repeated as before, for each price level, in order to determine the price for water, and corresponding reservoir size, which would yield maximum benefits to the community. The Gumbel distribution fit the values of required storage at each price level, and the 99% reliable required storages were found for each corresponding price. The present worth of the benefits produced by each system for each 50-year inflow sequence was computed using the nominal annual interest rate of 7%. The benefits produced were the benefits received from additional water provided which would not have been available without the reservoir. For a given system and 50-year inflow sequence, the produced benefits were computed for each month from,

$$B_j = \int_{Q_{wo}}^{Q_w} P(Q_j)\ dQ_j \qquad j = 1, 2, \ldots, 600 \tag{8}$$

where

B_j = added benefits in month j, \$;

$P(Q_j)$ = marginal value (price) of water, \$/1000 gallons;

Q_j = quantity of water supplied in month j, 1000s gallons;

w = with reservoir; and

wo = without reservoir.

Solving (7) for P gave the marginal value (price) function,

$$P(Q_j) = (1.05\ f_j H \exp(4.5)/Q_j)^{1/.915} \qquad (9)$$

Equation 9 was used in (8) and the integration performed between the appropriate limits to find the monthly added benefits, B_j. In months of large streamflow, usage without the reservoir, Q_{wo}, could be higher than usage with the reservoir, Q_w, since with the reservoir a portion of this streamflow might be stored for later use. Benefits added by the reservoir for these months are negative. That is, benefits from water supply to the community are less in these months with the reservoir than without the reservoir. The present worth of the produced benefits for a given system and 50-year inflow sequence was computed as

$$B_n = \sum_{j=1}^{600} B_j/(1+r/12)^j$$

where

B_n = present worth of produced benefits if n^{th} 50-year inflow sequence occurs;

r = nominal annual interest rate; and

j = monthly index.

The expected present worth of produced benefits for a given system was computed as

$$B = \sum_{n=1}^{100} B_n/100.$$

A plot of the present worth of expected produced benefits vs. required storage is shown in figure 4. Present worth of net benefits for a given reservoir are represented by the difference between the values of the corresponding points on the benefit and cost curves. With the reservoir cost curve used, expected benefits produced were never greater than costs for any level of development. Thus, in this case, the construction of a reservoir of any size would yield negative expected net benefits.

Assuming the community would be price-responsive on a seasonal basis, the third method of design was investigated. The following variable-price strategy was evaluated.

The price charged for water during month j was made a function of the amount of water in storage at the beginning of month j,

$$P_j = P_{max} - (P_{max} - P_{min})((S_j - S_{con})/(S_{max} - S_{con}))^{1/n} \quad \text{if } S_{con} \leq S_j \leq S_{max} \tag{10}$$

$$= P_{max} \quad \text{if } S_j \leq S_{con}$$

where

P_j = price charged during month j, \$/1000 gallons;

P_{max} = maximum charge allowed, \$/1000 gallons;

P_{min} = minimum charge, \$/1000 gallons;

S_j = storage level of reservoir at beginning of month j;

S_{con} = storage level below which rigid conservation is practiced and the price of water is set at P_{max};

S_{max} = maximum storage available; and

n = a variable whose value is to be determined.

The quantity $S_{max} - S_{con}$ represents the total storage available during normal operation when rigid conservation is not required. The quantity $(S_j - S_{con})/(S_{max} - S_{con})$ represents the fraction of this storage remaining at the beginning of month j. From (10) it is seen that when the reservoir is full, $P_j = P_{min}$ and when the reservoir level drops below S_{con}, $P_j = P_{max}$. For P_{max} = \$4/1000 gallons and P_{min} = \$.40/1000 gallons the pricing curves represented by (10) for various values of n are shown in figure 5. It can be seen that for small values of n the price increases rather rapidly as the storage level drops. For large values of n the price is relatively low until the storage level approaches the conservation level in which region the price increases

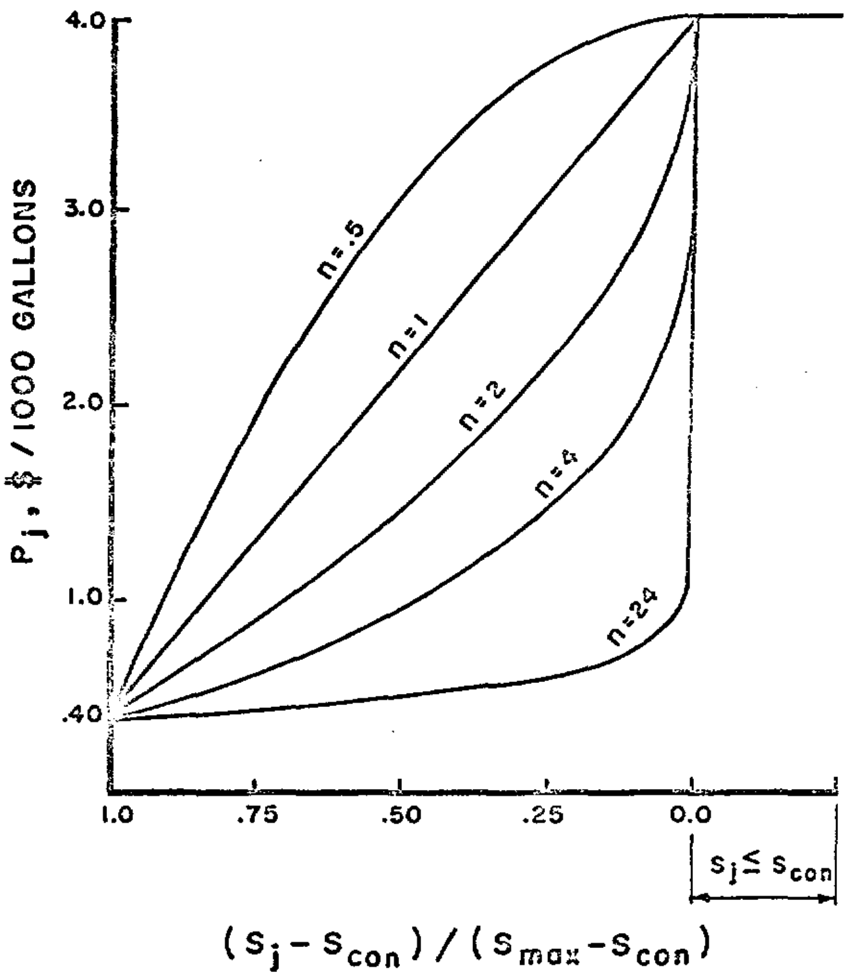

Fig. 5. Variable pricing policy curves.

rapidly to P_{max}. As $n \to 0$, $P_j \to P_{max}$ for all j. As $n \to \infty$, $P_j \to P_{min}$ for $S_{con} \leq S_j \leq S_{max}$, with $P_j = P_{max}$ for $S_j \leq S_{con}$. By varying n from 0 to ∞ a broad spectrum of pricing policies can be obtained for selected P_{max}, P_{min}, and S_{con}.

The reservoir routing procedure was used with the one hundred 50-year monthly inflow sequences and different pricing policies to determine the storage required and benefits obtained from each pricing policy. The storage level in the reservoir at the beginning of month j was determined from equation 5. This value was used in equation 10 to determine the price to be charged in month j. The price in month j was used in equation 7 to find the corresponding demand for month j. This value was used in equation 5 to determine the initial storage in month j+1 and the process was repeated. The procedure was first performed using P_{max} = \$4/1000 gallons and P_{min} = \$.40/1000 gallons. Pricing policies associated with $S_{max} - S_{con}$ = 4, 6, 8, 10 inches and n = 8, 24, 64, and 128 were evaluated. Results are shown in Table 17. The distribution of required storage values was found to be less skewed for the variable-price policies than for the constant price policies. In the former case it was found that the normal distribution fit the data better than the Gumbel distribution. Estimated curves of the present worth of expected net benefits for various policies are shown in figure 6a. It was found that maximum expected net benefits were produced by the price curves given by n = 24 with $S_{max} - S_{con}$ = 6 and by n = 64 with $S_{max} - S_{con}$ = 8. The present worth of the expected net benefits for

TABLE 17

SIMULATION RESULTS
VARIABLE PRICE STRATEGY
P_{max} = $4/1000 GALLONS

n	S_{max} - S_{con} in.	$\bar{P}$ $/kgal	Capacity ac-ft	Present Worth Net Benefits $ million	Water Usage gpcd
8	4	.58	1300	1.02	154
8	6	.53	1540	1.52	166
8	8	.50	1810	1.53	173
24	4	.54	1410	1.05	165
24	6	.48	1700	1.74*	181
24	8	.45	2060	1.70	190
64	4	.52	1450	.98	171
64	6	.46	1810	1.69	189
64	8	.43	2180	1.74*	199
64	10	.42	2620	1.32	203
128	8	.42	2260	1.68	202
128	10	.41	2690	1.30	206

* - optimal policy by expected net benefits criterion

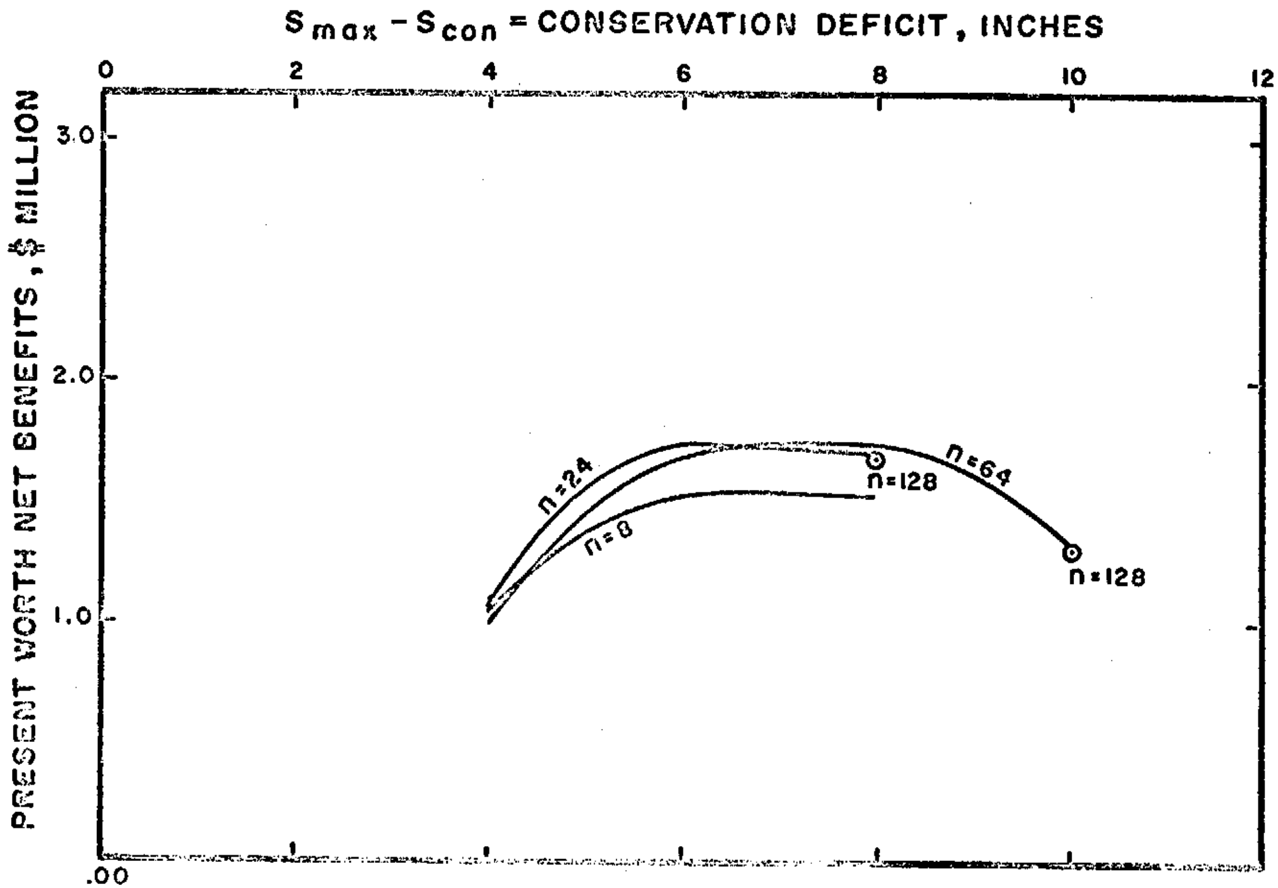

Fig. 6a. Present worth net benefits -- variable pricing policies -- P_{max} = $4/1000 gallons.

these policies was about \$1,740,000. For the former policy, required storage was 7.97 inches (1700 ac-ft) and the average price charged was \$.48/1000 gallons with an average usage of 181 gpcd. For the latter policy required storage was 10.24 inches (2180 ac-ft) and the average price charged was \$.43/1000 gallons with an average usage of 199 gpcd. The pricing policy associated with n = 24 and $S_{max} - S_{con} = 6$ is shown in figure 7.

The pricing policy evaluation was repeated with P_{max} = \$10/1000 gallons. Results are shown in Table 18 and net benefit curves seen in figure 6b. Maximum expected net benefits were produced by the policy corresponding to n = 128 and $S_{max} - S_{con} = 8$ shown in figure 7. The present worth of the expected net benefits for this policy was \$1,810,000. Required storage was 11.57 inches (2100 ac-ft). The average price charged was \$.45/1000 gallons with an average usage of 197 gpcd.

In order to reflect the uncertainty in the prediction of consumer demand for water when a variable-price strategy is used, the stochastic form of the demand model developed by Grunewald et al. (1975) was used, and is repeated here for convenience,

$$Q = \exp(4.5)P^{-.915} \exp(u) \tag{11}$$

where

TABLE 18

SIMULATION RESULTS
VARIABLE PRICE STRATEGY
P_{max} = $10/1000 GALLONS

n	S_{max} - S_{con} in.	$\bar{P}$ $/kgal	Capacity ac-ft	Present Worth Net Benefits $ million	Water Usage gpcd
24	4	.63	1240	1.06	155
24	6	.53	1510	1.59	167
24	8	.50	1830	1.56	175
64	4	.63	1340	1.06	165
64	6	.50	1640	1.79	181
64	8	.46	2020	1.76	189
128	4	.63	1360	1.00	170
128	6	.50	1710	1.77	187
128	8	.45	2100	1.81*	197
128	10	.43	2570	1.36	201
10000	8	.43	2260	1.70	207
10000	10	.41	2700	1.32	210

* - optimal policy by expected net benefit criterion

Q = annual household demand, 1000s gallons;

P = price of water, \$/1000 gallons; and

u = independent random variable, normally distributed with mean, $\bar{u} = 0$, and variance, $s^2 = 0.096$.

The function for the community stochastic demand at price P in month j is then given by

$$Q_j = f_j \; H \exp(4.5) P^{-.915} \exp(u) \tag{12}$$

where

f_j = fraction of annual demand in month j and

H = number of households.

The variable-price policies with P_{max} = \$4/1000 gallons were re-evaluated using equation 12 to generate community demand at a given price. This was done so that the optimal system produced in this manner could then be compared to that produced by using the deterministic equation 7 and the effect of the uncertainty in demand could be evaluated.

The simulation procedure used with stochastic demands differed from the earlier procedure only in that it was necessary to generate a random observation u from a normal distribution with mean 0 and variance 0.096 at the beginning of each month j in order to derive the monthly demand, Q_j, at a given price P. Also, in the computation of monthly benefits produced by the reservoir, equation 12 was used rather than equation 7 to obtain the marginal value function, $P(Q_j)$, used in equation 8. Curves representing the upper and lower bounds which include 95% of all realizations of equation 11 are shown in figure 8. The probability density function of the amount of water, Q, demanded by the community at a price of \$.50/1000 gallons in the month of August is shown in figure 9. It can be seen that the true demand may vary quite a bit from the expected demand in this case.

The simulation results obtained using a variable-price strategy with P_{max} = \$4/1000 gallons and stochastic demands are shown in Table 19.

Discussion of Results

Four possible pricing policies were studied. A description of the resulting optimal reservoir systems is given in Table 20. In system 1 it was assumed that demand was not a function of price. The required reservoir storage was about twice that required by the variable-price systems. The price charged for water was about 67% higher than the average charge with the variable-price systems. If the assumption that no significant price-demand relationship existed were true, then the average usage would be 204 gpcd and net benefits assumed at least equal to costs.

In systems 2, 3, and 4 the price-demand relationship given by equation (6) was assumed true. In system 2 the price charged for water was the same for each time period. Systems 3 and 4 were based on variable-price schemes developed to promote maximum utilization of

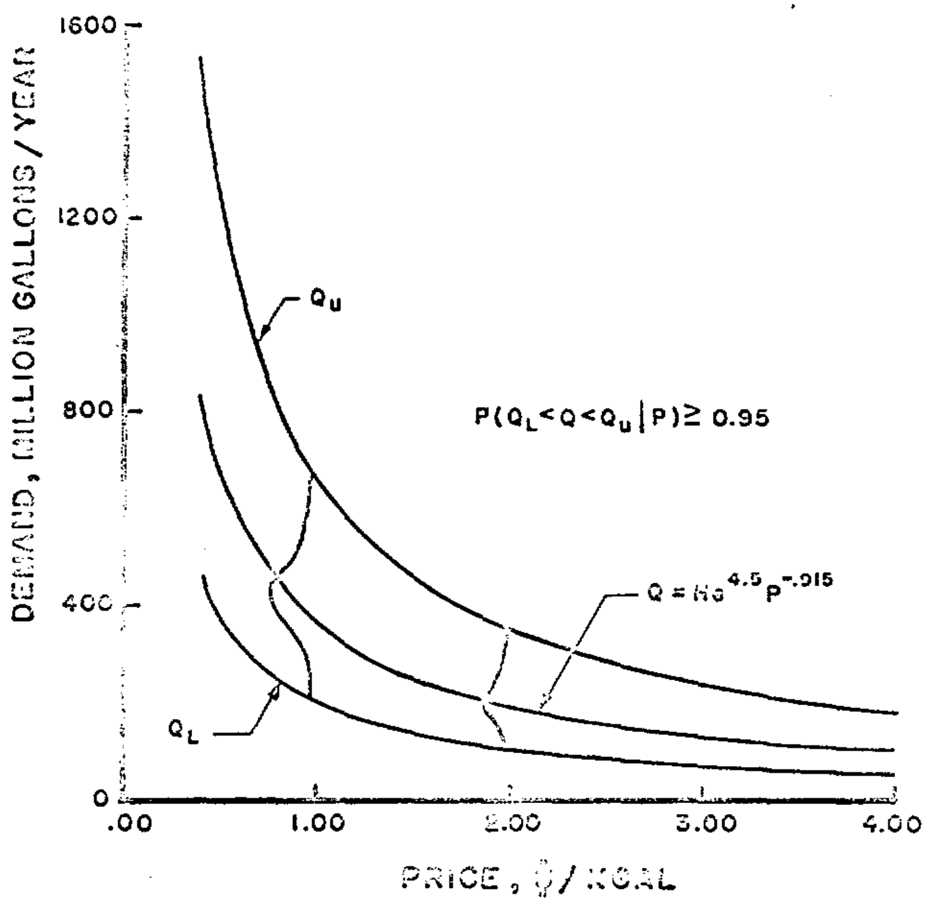

Fig. 8. Probabilistic function of community annual demand.

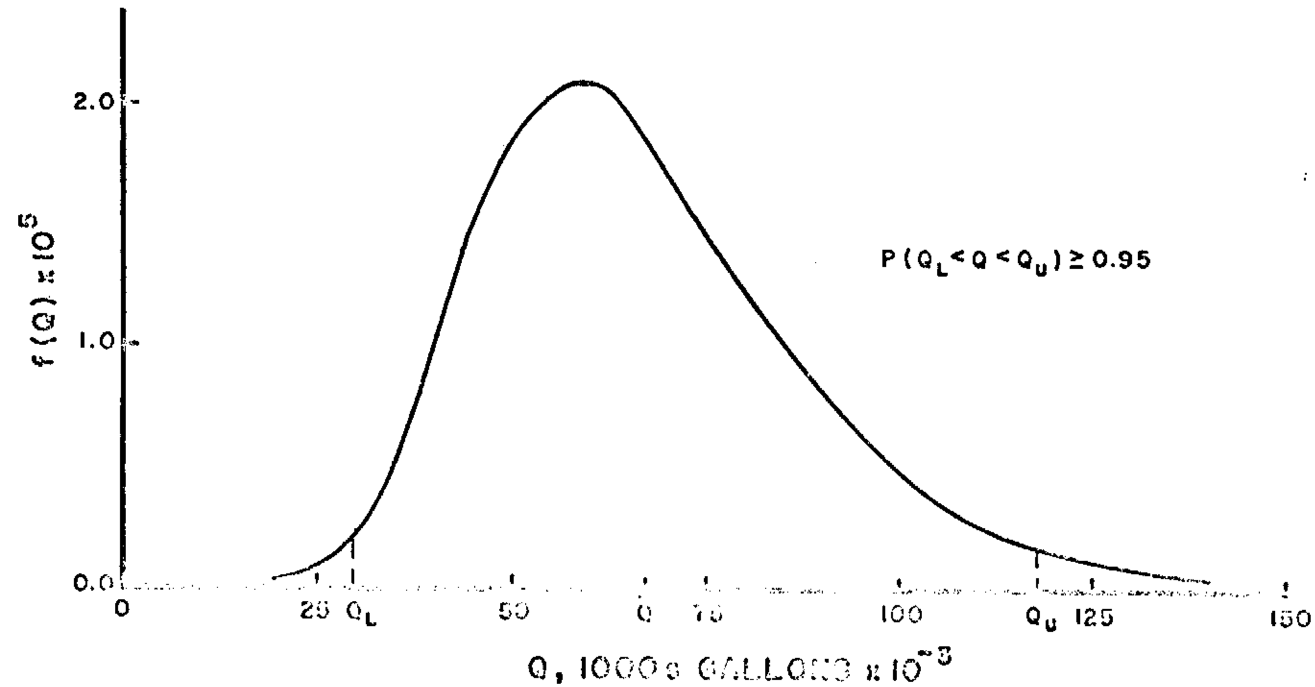

Fig. 9. Probability density function for demand in August at a price of \$.50/1000 gallons.

TABLE 19

SIMULATION RESULTS
VARIABLE PRICE STRATEGY
--STOCHASTIC DEMAND--
P_{max} = $4/1000 GALLONS

n	S_{max} - S_{con} in.	$\bar{\bar{P}}$ $/kgal	Capacity ac-ft	Present Worth Net Benefits $ million	Water Usage gpcd
8	4	.58	1340	1.22	153
8	6	.53	1590	1.71	165
8	8	.50	1880	1.71	173
8	10	.49	2090	1.63	178
24	4	.54	1460	1.26	164
24	6	.48	1780	1.88	180
24	8	.46	2110	1.90*	189
24	10	.44	2520	1.54	194
64	4	.53	1560	1.14	170
64	6	.46	1890	1.83	188
64	8	.43	2250	1.89	198
64	10	.42	2680	1.52	203

* - optimal policy by expected net benefits criterion

TABLE 20

OPTIMAL SYSTEMS

System	Capacity ac-ft	Average Price $/kgal	Average Usage gpcd	Present Worth Expected Net Benefits $ million
1	4330	.75	204*	not relevant
2	DO NOT BUILD RESERVOIR			
3	2180	.43	199	1.74
3	1700	.48	181	1.74
3a	2110	.46	197	1.90
4	2100	.45	189	1.81

Reservoir System Assumptions

1 - Reservoir design based on assumed "true" demands and price for water set to recover system costs.

2 - Reservoir design based on constant price-demand derived from price-demand function.

3 - Reservoir design based on variable price-demand with price a function of reservoir level. Maximum permitted charge is $4/1000 gallons.

3a - Same as system 3, but stochastic demand function used.

4 - Same as system 3, but maximum permitted charge is $10/1000 gallons.

reservoir capacity. With system 2 the construction of a reservoir could not increase the expected net benefits from water supply when the given reservoir cost curve was assumed.

Table 20 shows that system 3 produced two optimal systems based on the expected net benefits criterion. One policy required a higher average charge with lower average usage, but needed 380 ac-ft less storage. System 4 produced results which were roughly equivalent to the policy of system 3 associated with the pricing curve with n = 128 and $S_{max} - S_{con} = 8$.

The assumptions of system 3a were identical to those of system 3 except that uncertainty in consumer demand response was introduced by using the random price-demand function. The simulation results indicated that the ability of the conservation pricing policies to increase system benefits was not significantly altered when this type of uncertainty was present. Since the effectiveness of the conservation pricing was not significantly reduced by the demand uncertainty, the net benefits of system 3a were actually slightly larger than those of system 3. This result was felt to be attributable to the stochastic nature of the operating policies.

The choice of system 1 or either of systems 3 and 4 would depend upon the assumptions which were accepted. If it were believed that no price-demand relationship existed, then system 1 would be selected. If a price-demand relationship did exist, the use of the conservational pricing policies of systems 3 and 4 would produce substantially reduced storage requirements and demonstrable expected net benefits.

It was seen that the conservation pricing strategy remained effective when the stochastic price-demand function was used to reflect uncertainty in demand response to changes in price.

It should be noted that if it were assumed that demand was not a function of price, and system 1 were built, when in fact a price-demand relationship did exist, as in equation (6), then expected net benefits from the project would be negative, as was seen in the examination of the constant-price policies.

SUMMARY AND CONCLUSIONS

A hypothetical community was assumed in order to examine the effects of water pricing policies on water supply reservoir design. When it was assumed that the water rates did not influence the demand for water, a large storage capacity was required. When a price-demand relationship derived by Grunewald et al. (1975) for rural residential water demand was used in the design studies with the assumed reservoir cost function, it was determined that a reservoir should not be constructed if a non-varying pricing policy was to be used. Conservation pricing policies designed to promote efficient use of available storage and reflect the higher marginal costs of supplying water during low-flow periods were examined. A low price for water was charged when storage levels were high in order to reduce losses due to spillage. A higher price was charged for water when storage levels were low in order to conserve water. Reservoirs designed with these policies were found to provide a large average supply at low average prices and to produce substantial expected net benefits to the community. The effect of uncertainty in consumer response to changes in price was studied by using a probabilistic, rather than deterministic, price-demand function in the design studies. It was found that the effectiveness of the conservation pricing policies was not reduced when this type of uncertainty in demand was present.

Since any conservation pricing policy would probably be a new experience to water users, it is doubtful that the policy could be implemented without an educational campaign to stimulate consumer awareness. This consumer awareness could be achieved; however, as stated by Gysi (1972), "... the water utility must be a very public oriented organization that advertises clearly and repeatedly the changing price schedules and the reasons behind them." Of course in any actual operating system the price-demand relationship must be continually reevaluated in order to achieve maximum utilization of the reservoir capacity.

The simplifications used and the results obtained in this example serve only to outline the general consequences of the different pricing policies. It is strongly felt however that the results obtained do indicate that even in a more realistic framework the potential benefits to be achieved are of such significance that demand management of water supply by rational conservation pricing policies should not be ignored.

CHAPTER VI

SUMMARY OF DISSERTATION

The purpose of this study was to develop methods for water supply reservoir design which would increase the net benefits of the system to the community. To achieve this purpose, two major components of the reservoir system were studied, the estimated streamflow into the proposed reservoir and the demands placed on the reservoir.

Uncertainty in the estimation of future streamflow into a proposed reservoir results in uncertainty as to the true storage needed to meet demand throughout the design life. Overdesign results in unnecessary investment and underdesign produces an inflated estimate of project benefits. For the majority of watersheds in Kentucky, historical streamflow data is limited. Capacity requirements for water supply which were based only on historical streamflow events could vary significantly from the true requirements for these watersheds. In order to overcome the difficulties of limited data and provide for the assessment of system reliability, the use of mathematical models of streamflow in simulation studies was examined.

Two broad classes of streamflow models were reviewed; the stochastic models which preserve selected streamflow statistics, and the parametric models which attempt to model watershed behavior. The stochastic models were found to be more suitable for simulation studies. The parameters required by these models cannot be reliably estimated from short historical streamflow records however. Two possible situations of historical data inadequacy which might confront a water supply planner were envisioned.

In the first case, only a few years of streamflow data is available, but a relatively long record of daily rainfall exists in the area. By using the longer record of daily rainfall as input to a parametric runoff model, the historical streamflow record could be extended to a record whose length equals that of the rainfall record. Parameter estimates for a stochastic streamflow model obtained from an extended record might be more reliable than estimates based on a short historical record. A methodology was presented for evaluating the ability of a parametric runoff model to improve stochastic model parameter estimates in this manner. A water yield model (Haan, 1972b) was evaluated and was found to markedly improve the estimates of monthly and annual mean, variance, and serial correlation of streamflow when 31 years of daily rainfall was available and a 4 year historical streamflow record was assumed. In the water supply context, the parametric model was used to improve the parameter estimates for a Markov streamflow model. The use of the extended streamflow data for the estimation of the parameters of the Markov model reduced the variance of required capacity estimates around the best estimate by 35%, when compared with capacity estimates based on the 4 year historical records. This method of improving streamflow estimates for design studies could be used to improve designs when no streamflow data were available at a proposed site. By installing a streamgage at the proposed site, several years of data could be collected while preliminary investigations took place and project approval was being obtained. These few years of streamflow data could then be used to obtain the optimum parameters for a water yield model. The water yield model could then be used with existing rainfall data to produce a streamflow record equal in length to the existing rainfall record. One procedure presently used when no streamflow data is available at a site is to extrapolate data from a nearby watershed which is similar to the project watershed. The use of the water yield model would eliminate the need for this extrapolation and provide a more reliable storage estimate, if the water yield model represented the streamflow process with sufficient accuracy.

The second case envisioned occurs when the rainfall record near a site is too short to adequately extend the streamflow record by the above method. The necessary inputs to the water yield model might be obtained from rain data transferred from a location away from the site. More reliable data might be obtained, however, by modelling the rainfall process at the watershed. An existing Markov chain model of daily point rainfall (Allen and Haan, 1975) was examined. The model represented the process reasonably well but was not completely satisfactory. The model tended to produce rainfall values which were too large, and when used in conjunction with a water yield model, produced values of runoff which were also too large. It appeared that at least 40 years of historical data was required to reliably estimate the parameters of the model, and a model requiring less historical data was desired. A modification of the Markov chain model was proposed. Continuous distributions, rather than discrete transition probabilities, were used to represent the process when rainfall actually occurred. A two-parameter gamma distribution was found to provide a good representation for daily point rainfall events in Kentucky. Daily rainfall data from seven weather stations across the state was used to test and compare the modified model with the original model. The modified model was found to more accurately represent the daily point rainfall process. Using the modified model with the water yield model produced runoff values which did not differ significantly from runoff produced by using historical rainfall in the water yield model. It was found that 15-20 years of historical data was necessary to obtain reliable estimates for the parameters of the modified model. Runoff produced by the water yield model using historical and simulated rainfall was used to determine reservoir storage requirements subject to given demands. It was concluded that the storage required for a given level of reliability could be more accurately estimated by examining a large number of runoff sequences derived from simulated rainfall than could be obtained using the single historical rainfall record.

By using the methods discussed, streamflow data for water supply reservoir design could be made available at a potential site where no historical streamflow data and as little as 15 years of daily rainfall data existed. The position taken in this study was that the streamflow data thus obtained would be used to estimate the parameters of a stochastic model of streamflow so that the probabilistic nature of the design could be studied. If it were felt that the streamflow could not be adequately represented by a stochastic model, the data obtained by these methods could be used directly, as in the traditional approach, in order to determine capacity requirements. In either case, improved estimates of future streamflow would produce better estimates of required capacity for water supply and increase the economic efficiency of design procedures.

The second part of the study examined the effect of water price on system design and behavior. The price to be charged for water is usually not considered when projected demands on a proposed reservoir are made. Recent studies indicated that the price charged for water may influence the demand however. A study by Grunewald et al. (1975) indicated that a price-demand relationship did exist for rural residential water demand in Kentucky. The information obtained by Grunewald et al. was used to derive three different water pricing policies which were used in a water supply reservoir design study for a hypothetical community. Without the proposed reservoir, water was supplied by unreliable streamflow. The first policy assumed that no price-demand relationship existed and projected demands were based on community usage when adequate streamflow was available. Required reservoir capacity was determined based on these demands and the price of water was set to recover system costs. The second pricing policy assumed a valid price-demand relationship, which enabled the benefits of added water supply to be evaluated in monetary terms. The price for water, and corresponding reservoir capacity, which provided maximum net benefits to the community was determined. The third pricing policy assumed the price-demand relationship and allowed a variable price to be charged for water in order to promote maximum utilization of reservoir capacity and reflect the higher marginal costs required to supply water during low-flow periods. Nonlinear monthly pricing policies based on available water in storage were used. The optimal system was given by that variable price policy which yielded maximum expected net benefits to the community.

It was found that the first policy required a large amount of reservoir storage. Since no price-demand relationship was used in this policy, it could only be assumed that expected benefits would equal or exceed costs. The evaluation of the second (constant) pricing policy indicated that no reservoir should be built, since the present worth of expected net benefits was never greater than zero using the assumed cost function. By using optimal conservational variable-price policies, however, the construction of a reservoir would yield expected net benefits whose present worth was about \$1.8 million. The average price paid for water was reduced by about 40% and the required storage was reduced by about 50%, when compared with the policy which assumed no price-demand relationship.

The variable-price policy was evaluated using a probabilistic price-demand function to represent uncertainty in demand response when the price of water was changed. This uncertainty in demand response did not significantly reduce the effectiveness of the conservational variable-price policy. It was concluded that the demand management of water supply by the use of a proper conservational pricing policy might increase water supply benefits to a community.

The methods and models proposed in this study do not pretend to represent final solutions to the problem of water supply reservoir design. There are many areas where further research is required. For example, a parametric water yield model which was computationally feasible for simulation studies would eliminate the need for stochastic models and possibly provide the ability to study the effect of anticipated land-use changes on streamflow from the watershed. Demand functions for the various sectors of municipal water use could provide for better estimates of projected demands at different price levels. The rising value of all resources insures that the study of methods which will reduce unnecessary expenditures for water supply will be of more than academic interest. The methods proposed in this study provide at least a tentative step in the direction of more economical water supply reservoir design.

APPENDIX A

GUMBEL'S EXTREME VALUE DISTRIBUTION

$$P(Y) = \exp(-\exp(-Y))$$

$$Y = (X - b)/a$$

$$a = s/1.283$$

$$b = \bar{x} - 0.45s$$

where,

X = random variable with Gumbel's Extreme Value distribution

$\bar{x}$ = estimated mean of X

s = estimated standard deviation of X

Required storage for 99% reliability is found by solving

$$P(S \leq S^*) = .99$$

as

$$S^* = b - a \ln(-\ln(.99)).$$

APPENDIX B

STATISTICS OF SIMULATED AND HISTORICAL

RAINFALL AND SIMULATED RUNOFF

TABLE 21

MONTHLY MEAN RAINFALL

Rain Generated for Station 254

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	2.94	3.09	3.16	2.97	3.62	3.63	3.24	3.43
Feb	3.13	2.78	2.98	2.88	2.81	3.06	2.94	2.94
Mar	4.05	3.63	3.84	4.10	4.45	3.94	4.00	4.16
Apr	3.36	3.52	3.64	3.64	3.35	3.56	3.51	3.39
May	3.92	4.60	3.84	3.84	4.34	3.48	4.00	3.91
Jun	3.71	3.73	3.47	3.85	4.03	4.04	3.80	3.65
Jul	4.23	3.94	4.52	4.65	4.72	4.19	4.38	4.37
Aug	3.63	3.73	3.30	3.25	3.19	3.68	3.46	3.43
Sep	3.44	3.39	2.72	2.92	2.64	3.09	3.03	2.76
Oct	2.15	2.00	2.17	2.29	1.90	2.15	2.11	2.02
Nov	2.47	3.05	2.21	2.59	2.52	2.89	2.62	2.76
Dec	3.01	3.07	3.19	2.96	3.02	2.98	3.04	2.86
Ann.	40.02	40.54	39.04	39.94	40.58	40.69	40.13	39.68

TABLE 21 - CONTINUED

Rain Generated for Station 909

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	5.13	5.29	4.85	4.41	4.58	5.02	4.88	5.14
Feb	4.54	3.94	3.53	4.03	4.07	3.74	3.98	4.07
Mar	5.50	5.83	5.70	4.92	5.69	5.01	5.44	5.32
Apr	4.60	4.20	4.45	4.38	4.66	4.20	4.42	4.23
May	4.12	4.15	3.85	4.28	3.91	3.76	4.01	3.97
Jun	4.44	4.02	4.14	4.29	4.38	4.20	4.24	4.23
Jul	3.78	4.27	4.43	4.26	4.17	3.87	4.13	4.24
Aug	3.67	3.40	3.70	4.06	4.34	3.89	3.84	3.52
Sep	3.06	3.13	3.00	3.21	3.19	2.91	3.08	2.94
Oct	2.56	2.58	2.21	2.32	2.62	2.52	2.47	2.42
Nov	3.02	2.94	2.82	2.87	3.44	3.46	3.09	3.50
Dec	4.33	4.24	4.37	4.17	4.52	4.49	4.35	4.22
Ann.	48.76	47.99	47.04	47.20	49.57	47.08	47.94	47.78

TABLE 21 - CONTINUED

Rain Generated for Station 1345

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	3.14	3.45	3.15	3.24	2.88	3.35	3.20	3.45
Feb	3.20	2.74	3.01	2.84	2.91	2.95	2.94	3.04
Mar	4.14	4.12	4.99	4.11	4.81	4.17	4.39	4.45
Apr	3.49	3.92	3.50	3.30	3.88	3.80	3.65	3.71
May	3.32	3.37	3.63	3.38	3.45	3.93	3.51	3.67
Jun	4.21	4.55	4.12	4.06	3.92	4.28	4.19	3.94
Jul	4.18	3.59	4.06	3.70	3.24	3.73	3.75	3.75
Aug	3.87	3.11	3.79	4.32	3.51	3.09	3.62	3.25
Sep	2.68	2.85	2.25	2.18	2.91	2.27	2.52	2.69
Oct	2.31	2.61	2.41	2.48	2.63	2.49	2.49	2.25
Nov	2.91	2.49	2.77	2.85	2.43	3.12	2.76	2.94
Dec	2.71	2.91	2.53	2.87	3.39	2.93	2.89	2.67
Ann.	40.16	39.72	40.20	39.32	39.96	40.11	39.91	39.81

TABLE 21 - CONTINUED

Rain Generated for Station 3762

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	4.49	4.75	3.74	3.84	3.83	5.31	4.33	4.35
Feb	3.19	3.17	3.10	3.38	3.15	3.55	3.26	3.46
Mar	5.32	6.12	4.69	5.25	5.10	5.29	5.30	5.22
Apr	4.19	3.89	4.83	4.93	4.42	4.57	4.47	4.40
May	4.17	4.55	4.04	4.73	4.24	4.76	4.42	4.25
Jun	3.52	3.11	3.34	3.44	3.53	3.14	3.35	3.65
Jul	3.75	4.07	4.19	3.76	3.71	4.01	3.92	4.00
Aug	3.11	3.49	3.56	3.45	3.67	3.06	3.39	3.15
Sep	3.46	2.83	2.79	2.68	3.34	3.09	3.03	3.19
Oct	2.60	2.97	3.11	2.64	2.83	2.73	2.81	2.70
Nov	3.15	2.89	3.06	3.17	3.14	3.33	3.12	3.33
Dec	3.03	3.50	3.95	3.91	3.70	3.49	3.60	3.50
Ann.	43.98	45.34	44.40	45.20	44.66	46.32	44.98	45.20

TABLE 21 - CONTINUED

Rain Generated for Station 3994

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	4.70	4.46	4.60	4.83	4.96	4.79	4.72	4.85
Feb	4.07	4.46	4.59	3.70	3.86	4.33	4.17	4.01
Mar	5.69	5.15	5.53	5.31	4.81	5.04	5.26	5.21
Apr	4.47	4.66	4.57	5.00	4.58	4.34	4.60	4.26
May	3.88	3.96	4.25	4.00	4.49	4.12	4.12	4.31
Jun	4.33	4.08	4.11	4.08	4.16	3.97	4.12	4.17
Jul	4.06	4.04	3.39	3.95	4.35	4.53	4.05	4.02
Aug	4.00	3.74	3.12	3.29	3.10	3.77	3.50	3.43
Sep	2.92	2.94	3.44	2.92	2.80	2.50	2.92	2.97
Oct	2.84	2.78	2.66	2.60	2.25	2.70	2.64	2.55
Nov	3.24	3.67	3.49	3.95	3.42	2.79	3.43	3.79
Dec	4.23	3.65	3.95	4.44	4.31	4.46	4.17	3.94
Ann.	48.42	47.58	47.70	48.07	47.08	47.36	47.70	47.52

TABLE 21 - CONTINUED

Rain Generated for Station 4825

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	4.78	4.26	4.08	4.13	4.28	4.38	4.32	4.37
Feb	3.60	3.51	4.10	3.54	3.49	3.47	3.62	3.67
Mar	4.43	4.75	4.70	4.39	5.01	4.42	4.62	4.71
Apr	3.53	3.94	3.78	3.93	3.64	3.55	3.73	3.87
May	4.01	4.26	4.38	3.69	3.99	3.95	4.05	4.07
Jun	3.92	4.20	4.84	4.64	4.43	4.39	4.40	4.35
Jul	5.07	4.94	4.97	4.65	5.14	5.10	4.98	4.81
Aug	3.94	4.35	3.82	3.51	4.07	3.61	3.88	3.81
Sep	2.82	2.60	2.67	2.62	2.78	2.47	2.66	2.96
Oct	2.18	2.20	1.78	2.13	1.96	2.34	2.10	1.94
Nov	2.65	3.24	2.69	3.17	3.02	2.93	2.95	3.17
Dec	3.67	4.02q	3.48	3.52	3.93	3.65	3.71	3.40
Ann.	44.60	46.28	45.30	43.93	45.74	44.26	45.02	45.12

TABLE 21 - CONTINUED

Rain Generated for Station 6353

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	3.84	3.66	3.11	3.38	3.31	3.49	3.46	3.56
Feb	3.26	3.78	3.31	3.68	3.48	3.59	3.52	3.70
Mar	4.69	4.45	4.17	4.49	4.61	4.26	4.44	4.34
Apr	3.76	3.60	3.41	3.60	3.60	3.51	3.58	3.59
May	4.09	3.89	4.03	4.22	4.15	4.50	4.15	3.91
Jun	4.43	4.20	4.39	3.98	4.64	3.69	4.22	4.15
Jul	5.91	5.26	5.24	5.38	4.62	5.05	5.24	5.16
Aug	3.90	3.64	3.53	3.68	3.35	3.90	3.67	3.60
Sep	3.54	3.22	3.14	2.92	2.79	3.22	3.14	3.24
Oct	2.44	2.56	2.49	2.28	2.20	2.26	2.37	2.16
Nov	2.72	2.77	2.25	2.40	2.52	2.46	2.52	2.79
Dec	3.02	3.03	3.59	3.09	3.30	3.47	3.25	3.12
Ann.	45.60	44.06	42.66	43.10	42.57	43.40	43.56	43.32

TABLE 22

STANDARD DEVIATION OF MONTHLY RAIN

Standard Deviation of Monthly Rain for Station 254

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	1.52	1.47	1.75	1.26	1.73	1.49	1.54	2.10
Feb	1.04	1.17	1.19	1.31	1.28	1.02	1.17	1.77
Mar	1.92	1.50	1.35	1.46	1.80	1.62	1.61	1.76
Apr	1.52	1.22	1.42	1.40	1.07	1.47	1.35	1.46
May	1.65	1.55	1.52	1.48	1.92	1.38	1.58	1.88
Jun	1.50	1.88	1.60	1.93	1.77	1.64	1.72	1.84
Jul	1.56	2.04	1.58	1.98	2.30	2.42	1.98	1.96
Aug	2.24	1.97	1.63	2.04	2.02	1.54	1.91	1.72
Sep	1.96	1.56	1.47	1.51	1.57	2.44	1.75	1.41
Oct	1.42	1.06	1.12	1.08	1.20	1.24	1.19	1.06
Nov	1.26	1.62	0.95	1.09	1.54	1.09	1.26	1.34
Dec	1.30	1.19	1.32	1.08	1.17	1.23	1.22	1.41

TABLE 22 - CONTINUED

Standard Deviation of Monthly Rain for Station 909

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	2.29	2.84	1.87	2.23	2.28	2.11	2.27	3.95
Feb	1.51	1.65	1.51	1.94	2.01	1.58	1.70	2.39
Mar	1.86	2.55	2.87	2.40	2.34	2.02	2.34	2.31
Apr	2.54	2.11	2.11	2.26	1.94	1.72	2.11	1.59
May	1.90	1.78	1.55	1.94	1.96	2.06	1.86	2.23
Jun	2.31	2.02	2.03	2.16	2.52	2.18	2.20	2.57
Jul	1.51	2.06	2.41	1.67	2.20	1.78	1.94	2.32
Aug	1.90	1.39	1.78	1.90	2.94	1.85	1.96	1.65
Sep	2.11	1.77	1.67	2.14	1.94	1.86	1.92	1.66
Oct	1.58	1.77	1.42	1.31	1.65	1.88	1.60	1.28
Nov	1.48	1.48	1.59	1.52	1.65	1.39	1.52	1.71
Dec	3.08	1.98	2.03	1.78	2.57	1.98	2.24	2.13

TABLE 22 - CONTINUED

Standard Deviation of Monthly Rain for Station 1345

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	1.70	1.70	1.70	1.51	1.31	1.55	1.58	2.74
Feb	1.86	1.48	1.59	1.46	1.40	1.47	1.54	2.07
Mar	2.02	1.88	2.13	1.91	1.94	1.94	1.97	2.88
Apr	1.52	1.49	1.61	1.80	1.64	1.98	1.67	1.87
May	1.28	1.75	1.63	1.49	1.78	1.78	1.62	1.87
Jun	2.01	2.51	2.31	1.73	2.01	1.65	2.04	2.40
Jul	2.47	2.00	1.91	1.91	1.47	2.24	2.00	1.56
Aug	2.40	1.69	2.40	2.62	2.27	1.98	2.23	1.79
Sep	1.71	1.65	1.64	1.74	1.49	1.33	1.59	1.56
Oct	1.56	1.42	1.43	1.40	1.68	1.55	1.51	1.34
Nov	1.24	1.09	1.70	1.37	1.40	1.72	1.42	1.60
Dec	1.48	1.27	1.41	1.43	1.66	1.44	1.45	1.37

TABLE 22 - CONTINUED

Standard Deviation of Monthly Rain for Station 3762

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	2.38	2.71	2.00	2.51	1.93	2.78	2.38	4.05
Feb	1.64	1.57	1.66	1.63	1.47	1.63	1.60	2.15
Mar	3.06	2.58	2.09	2.59	2.18	2.16	2.44	3.50
Apr	1.76	1.79	1.61	2.30	2.09	2.17	1.95	1.91
May	1.99	1.89	1.79	2.27	2.01	1.97	1.99	2.08
Jun	1.72	1.62	1.80	1.56	1.74	1.69	1.69	1.90
Jul	1.60	2.47	2.01	1.59	1.99	2.37	2.00	2.60
Aug	1.91	2.22	2.48	1.82	2.23	1.88	2.09	2.05
Sep	2.21	1.82	1.63	1.84	2.04	2.13	1.94	2.26
Oct	1.43	2.02	1.60	1.32	1.64	1.01	1.50	1.69
Nov	1.99	1.43	1.81	1.88	1.69	1.86	1.78	2.02
Dec	1.54	1.79	1.72	2.04	2.00	2.00	1.85	1.93

TABLE 22 - CONTINUED

Standard Deviation of Monthly Rain for Station 3994

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	2.42	2.29	2.49	2.20	3.20	2.30	2.48	3.96
Feb	2.08	1.89	2.00	1.81	1.90	1.91	1.93	2.32
Mar	2.74	2.01	2.41	2.39	2.02	2.47	2.34	2.39
Apr	1.69	2.19	1.81	1.98	2.10	2.17	1.99	1.68
May	2.30	2.08	2.35	2.04	1.99	2.23	2.16	2.54
Jun	2.19	2.32	2.21	2.66	2.11	2.23	2.29	2.40
Jul	1.64	1.66	1.66	1.91	2.32	2.12	1.88	1.88
Aug	2.25	2.05	1.64	1.74	1.95	2.16	1.96	1.82
Sep	1.77	1.55	1.80	1.68	1.45	1.54	1.63	1.85
Oct	1.18	1.49	1.30	1.67	1.40	1.83	1.48	1.63
Nov	1.92	2.45	2.04	2.65	2.30	1.76	2.19	2.33
Dec	1.83	2.22	2.12	1.86	1.96	2.14	2.02	1.89

TABLE 22 - CONTINUED

Standard Deviation of Monthly Rain for Station 4825

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	2.07	1.51	1.96	1.96	1.92	1.80	1.87	2.98
Feb	1.44	1.93	1.94	1.69	1.42	1.38	1.63	2.33
Mar	2.00	2.06	1.58	2.15	1.98	1.59	1.89	2.15
Apr	1.49	1.56	1.66	1.76	1.60	1.58	1.61	1.81
May	1.53	1.66	2.25	2.35	1.83	1.88	1.92	2.02
Jun	1.83	2.12	2.22	2.30	1.87	1.89	2.04	2.13
Jul	2.18	2.10	2.19	2.18	1.95	1.98	2.10	1.96
Aug	1.74	2.33	1.60	1.78	2.29	1.75	1.92	1.90
Sep	1.69	1.45	1.42	1.55	1.87	1.56	1.59	1.50
Oct	1.31	1.26	1.04	1.08	0.92	1.39	1.17	0.93
Nov	1.33	1.62	1.49	1.58	1.42	1.28	1.45	1.66
Dec	1.94	1.73	1.47	1.68	1.92	1.56	1.72	1.76

TABLE 22 - CONTINUED

Standard Deviation of Monthly Rain for Station 6353

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	1.30	1.51	1.28	1.28	1.44	1.30	1.35	1.67
Feb	1.49	1.30	1.18	1.43	1.44	1.44	1.38	1.85
Mar	1.61	1.70	1.51	1.71	1.60	1.54	1.61	2.12
Apr	1.41	1.39	1.40	1.38	1.43	1.49	1.42	1.42
May	1.64	1.67	1.74	1.36	1.71	1.77	1.65	2.05
Jun	2.20	1.81	2.08	1.65	1.96	2.00	1.95	1.71
Jul	1.99	2.23	1.68	2.23	2.06	1.82	2.00	2.29
Aug	2.16	1.59	1.41	1.96	1.74	1.82	1.78	2.21
Sep	2.38	1.71	1.42	1.76	1.79	1.79	1.81	1.75
Oct	1.26	1.22	1.64	1.23	1.19	1.08	1.27	1.27
Nov	1.25	1.38	1.00	0.83	1.02	1.03	1.08	1.32
Dec	1.55	1.28	1.77	1.68	1.24	1.32	1.47	1.38

TABLE 23

MAXIMUM RUNS OF WET DAYS AND MAXIMUM RUNS OF DRY DAYS

Station 254

Maximum Run Wet Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	7	7	11	12	9	11	9.50	11
Feb	7	7	7	9	11	5	7.67	9
Mar	8	6	7	10	8	9	8.00	9
Apr	11	9	8	8	6	12	9.00	6
May	8	13	9	10	16	9	10.8	9
Jun	6	8	8	10	7	7	7.67	10
Jul	8	7	10	9	6	8	8.00	7
Aug	7	7	6	7	7	6	6.67	7
Sep	6	6	6	6	6	7	6.17	8
Oct	7	6	6	7	6	6	6.33	5
Nov	7	13	9	9	7	7	8.67	7
Dec	9	7	8	8	7	8	7.83	8

TABLE 23 - CONTINUED

Station 254

Maximum Run Dry Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	20	13	16	19	14	13	15.8	14
Feb	12	13	14	12	20	11	13.7	12
Mar	15	11	15	13	9	13	12.7	12
Apr	18	13	11	14	14	24	15.7	20
May	16	17	18	15	19	12	16.2	12
Jun	16	18	17	14	17	15	16.2	19
Jul	16	13	11	13	16	17	14.3	12
Aug	22	19	16	17	16	20	18.3	22
Sep	20	30	24	23	30	26	25.5	24
Oct	29	21	21	19	20	27	22.8	27
Nov	20	14	15	12	22	15	16.3	21
Dec	15	12	21	14	16	13	15.2	14

TABLE 23 - CONTINUED

Station 909

Maximum Run Wet Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	10	10	7	9	8	8	8.7	8
Feb	7	7	7	6	6	10	7.2	6
Mar	7	7	9	12	10	6	8.5	8
Apr	7	7	8	6	9	8	7.5	8
May	6	7	6	7	10	9	7.5	8
Jun	7	7	7	8	6	7	7.0	7
Jul	6	5	8	8	10	6	7.2	8
Aug	7	7	9	5	7	6	6.8	8
Sep	7	6	6	8	9	9	7.5	9
Oct	7	6	6	6	6	5	6.0	4
Nov	5	7	9	6	6	6	6.5	7
Dec	11	9	7	7	7	9	8.3	6

TABLE 23 - CONTINUED

Station 909

Maximum Run Dry Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	17	15	14	18	16	15	15.8	16
Feb	14	13	13	11	13	13	12.8	12
Mar	18	13	15	14	19	13	15.3	13
Apr	21	18	17	21	14	15	17.7	16
May	26	16	29	21	17	23	22.0	13
Jun	22	16	20	19	20	20	19.5	18
Jul	18	13	21	21	15	18	17.7	16
Aug	18	26	30	19	25	17	22.5	21
Sep	23	27	26	22	23	22	23.8	25
Oct	27	28	20	21	28	24	24.7	26
Nov	15	17	19	23	19	17	18.3	19
Dec	19	18	12	14	14	14	15.2	19

TABLE 23 - CONTINUED

Station 1345

Maximum Run Wet Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	6	11	9	7	8	9	8.3	7
Feb	9	6	9	8	5	6	7.2	5
Mar	6	10	8	7	7	6	7.3	7
Apr	7	9	6	7	9	9	7.8	5
May	7	7	6	9	8	10	7.8	7
Jun	8	9	9	6	9	7	8.0	8
Jul	8	6	4	7	4	5	5.7	7
Aug	7	4	8	5	7	5	6.0	4
Sep	5	6	7	6	8	4	6.0	8
Oct	8	7	7	6	7	6	6.8	5
Nov	9	8	10	11	8	6	8.7	5
Dec	6	8	8	9	6	7	7.3	7

TABLE 23 - CONTINUED

Station 1345

Maximum Run Dry Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	21	25	18	17	18	21	20.0	20
Feb	18	17	22	22	16	20	19.2	21
Mar	16	18	19	18	18	16	17.5	13
Apr	23	16	17	17	19	20	18.7	15
May	19	18	14	24	19	14	18.0	15
Jun	16	23	20	18	23	15	19.2	28
Jul	22	20	19	26	22	18	21.2	15
Aug	21	24	31	19	26	29	25.0	31
Sep	30	30	30	24	22	30	27.7	23
Oct	22	24	28	27	25	30	26.0	27
Nov	19	20	18	20	30	17	20.7	25
Dec	18	18	16	20	20	20	18.7	25

TABLE 23 - CONTINUED

Station 3762

Maximum Run Wet Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	7	8	5	7	8	7	7.0	6
Feb	7	7	6	6	9	6	6.8	6
Mar	9	7	6	8	7	8	7.5	6
Apr	7	8	7	6	8	9	7.5	6
May	7	7	7	13	7	7	8.0	6
Jun	6	5	6	6	7	8	6.3	8
Jul	6	6	5	6	8	5	6.0	7
Aug	4	5	6	6	6	5	5.3	4
Sep	7	5	6	6	6	7	6.2	10
Oct	6	8	4	6	6	6	6.0	5
Nov	5	7	5	6	5	8	6.0	5
Dec	7	7	6	8	8	7	7.2	6

TABLE 23 - CONTINUED

Station 3762

Maximum Run Dry Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	18	15	20	18	19	25	19.2	23
Feb	16	13	15	17	13	14	14.7	19
Mar	20	15	14	13	14	18	15.7	15
Apr	18	25	10	14	13	16	16.0	15
May	19	20	16	14	23	15	17.8	16
Jun	18	20	26	21	18	17	20.0	26
Jul	16	19	24	20	16	18	18.8	19
Aug	24	29	20	25	29	19	24.3	26
Sep	22	26	28	30	24	29	26.5	23
Oct	21	31	25	27	21	19	24.0	31
Nov	22	19	20	21	18	22	20.3	24
Dec	21	20	23	21	25	22	22.0	19

TABLE 23 - CONTINUED

Station 3994

Maximum Run Wet Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	9	6	8	9	8	8	8.0	7
Feb	7	5	8	6	10	9	7.5	6
Mar	8	10	7	14	8	6	8.8	6
Apr	10	7	11	6	8	7	8.2	6
May	8	7	10	8	9	9	8.5	8
Jun	8	11	6	5	8	8	7.7	8
Jul	7	5	6	5	6	7	6.0	6
Aug	6	6	6	7	8	7	6.7	7
Sep	6	9	8	9	5	6	7.2	6
Oct	6	7	6	6	7	5	6.2	8
Nov	7	8	8	6	10	5	7.3	7
Dec	9	9	7	6	9	6	7.7	8

TABLE 23 - CONTINUED

Station 3994

Maximum Run Dry Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	15	19	20	17	22	18	18.5	27
Feb	18	14	11	15	16	16	15.0	15
Mar	18	18	16	17	17	20	17.7	14
Apr	13	11	11	12	16	13	12.7	13
May	26	14	21	20	14	20	20.7	18
Jun	20	20	20	21	19	24	20.7	20
Jul	21	18	23	18	28	19	21.2	20
Aug	18	22	22	26	23	22	22.2	23
Sep	23	24	30	24	16	24	23.5	28
Oct	26	26	26	20	31	24	25.5	27
Nov	20	20	25	25	20	27	22.8	21
Dec	17	18	21	19	17	24	19.3	20

TABLE 23 - CONTINUED

Station 4825

Maximum Run Wet Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	9	6	10	6	8	8	7.8	7
Feb	7	5	8	7	5	8	6.7	6
Mar	6	6	7	6	7	11	7.2	5
Apr	7	7	10	7	7	9	7.8	7
May	6	5	6	7	7	8	6.5	7
Jun	6	5	8	6	5	7	6.2	6
Jul	7	8	7	5	12	7	7.7	9
Aug	5	7	6	4	6	5	5.5	6
Sep	5	6	8	5	6	6	6.0	7
Oct	4	6	5	5	5	4	4.8	4
Nov	6	7	8	6	5	5	6.2	7
Dec	8	6	7	6	8	9	7.3	7

TABLE 23 - CONTINUED

Station 4825

Maximum Run Dry Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	15	16	15	17	13	13	14.8	19
Feb	19	17	13	17	15	13	15.7	18
Mar	14	15	16	17	13	12	14.5	13
Apr	17	16	16	26	19	21	19.2	21
May	15	17	16	27	18	19	18.7	17
Jun	22	20	18	17	17	17	18.5	23
Jul	18	14	15	17	17	21	17.0	19
Aug	22	20	18	19	26	21	21.0	16
Sep	29	30	29	18	23	29	26.3	28
Oct	30	27	31	28	25	20	26.8	27
Nov	18	23	24	18	20	18	20.2	21
Dec	20	25	20	19	17	19	20.0	15

TABLE 23 - CONTINUED

Station 6353

Maximum Run Wet Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	12	9	8	8	12	8	9.5	11
Feb	6	9	9	7	8	11	8.3	9
Mar	7	10	8	7	8	9	8.2	7
Apr	9	11	8	6	8	7	8.2	9
May	9	7	10	9	10	10	9.2	9
Jun	8	8	8	7	11	11	8.8	8
Jul	12	7	12	9	9	11	10.0	9
Aug	8	9	11	8	8	8	8.7	7
Sep	6	6	8	7	5	9	6.8	5
Oct	8	7	6	7	8	7	7.2	7
Nov	8	7	7	8	8	8	7.7	7
Dec	9	9	10	9	11	8	9.3	6

TABLE 23 - CONTINUED

Station 6353

Maximum Run Dry Days

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	12	14	17	11	17	11	13.7	11
Feb	13	11	10	10	15	12	11.8	17
Mar	12	12	12	10	11	13	11.7	31*
Apr	12	13	12	11	15	11	12.3	16
May	12	14	14	14	16	17	14.5	13
Jun	17	18	17	20	18	15	17.5	17
Jul	15	15	15	16	13	12	14.3	13
Aug	22	17	22	17	18	20	19.3	26
Sep	19	22	26	20	24	21	22.0	18
Oct	22	24	24	21	21	18	21.7	27
Nov	21	22	16	16	19	17	18.5	20
Dec	14	13	15	15	17	15	14.8	15

* - Investigation revealed that no reports were given from this station in March, 1971. The symbol '-' on the weather bureau tape was read into the computer as '0'. The true value should be about 15.

TABLE 24

MAXIMUM DAILY RAINFALL

Maximum Daily Rainfall for Station 254

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	1.98	2.44	1.88	1.69	2.77	1.86	2.10	2.31
Feb	1.49	1.40	1.66	1.92	1.95	2.48	1.82	2.40
Mar	2.74	2.08	1.61	2.24	3.31	1.70	2.28	2.63
Apr	2.17	1.69	2.64	2.44	1.49	2.75	2.20	2.67
May	2.61	2.36	2.04	2.40	2.28	1.76	2.24	3.21
Jun	2.27	2.50	2.11	3.33	3.39	2.34	2.66	4.09
Jul	3.86	2.50	2.87	2.92	3.89	2.83	3.14	3.38
Aug	3.22	3.94	3.30	2.43	3.20	2.88	3.16	3.97
Sep	3.09	2.65	2.50	1.81	2.50	2.96	2.58	2.91
Oct	2.09	1.97	2.09	1.85	1.85	2.11	1.99	1.62
Nov	2.00	2.01	2.05	2.06	1.89	1.83	1.97	2.35
Dec	2.00	1.93	1.64	2.57	1.41	1.46	1.84	2.09
Ann.	3.86	3.94	3.30	3.33	3.89	2.96	3.55	4.00

TABLE 24 - CONTINUED

Maximum Daily Rainfall for Station 909

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	3.30	4.79	3.82	3.08	3.47	4.70	3.86	4.75
Feb	3.14	2.98	2.42	2.79	3.79	3.01	3.02	3.01
Mar	3.72	4.01	4.50	3.76	3.27	2.72	3.66	4.48
Apr	3.66	3.01	2.92	2.50	3.70	2.17	2.99	4.61
May	2.88	2.37	2.29	2.70	2.39	4.05	2.78	2.98
Jun	3.34	2.96	4.11	3.53	4.35	3.20	3.58	5.69
Jul	2.26	3.00	3.72	3.29	2.95	2.76	3.00	3.20
Aug	4.01	3.09	2.69	3.45	4.77	4.56	3.76	2.87
Sep	3.61	3.85	2.84	3.26	2.77	3.29	3.27	3.91
Oct	2.62	2.96	2.37	2.85	2.75	2.67	2.70	2.70
Nov	2.50	2.35	2.66	2.06	3.19	2.90	2.61	3.54
Dec	4.03	2.24	2.97	4.72	3.03	2.49	3.25	4.47
Ann.	4.03	4.79	4.50	4.72	4.77	4.70	4.58	5.69

TABLE 24 - CONTINUED

Maximum Daily Rainfall for Station 1345

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	2.36	2.19	2.44	2.44	2.32	3.54	2.55	2.93
Feb	2.28	2.65	2.67	2.83	2.21	2.49	2.52	3.14
Mar	3.12	3.06	3.07	2.33	3.03	2.25	2.81	3.66
Apr	3.30	2.74	2.82	2.16	2.22	2.87	2.68	2.45
May	2.25	2.10	2.41	2.42	2.63	2.29	2.35	3.85
Jun	2.56	2.68	4.81	3.40	4.12	2.20	3.30	4.25
Jul	4.19	2.90	2.98	3.17	2.26	3.92	3.24	2.88
Aug	2.59	3.40	4.18	4.38	2.97	2.65	3.36	5.05
Sep	4.01	4.43	2.29	2.50	3.52	2.43	3.20	3.41
Oct	2.20	2.08	2.76	2.26	1.95	2.73	2.33	3.80
Nov	2.09	1.79	1.64	1.49	2.11	1.96	1.85	3.20
Dec	2.16	1.99	1.77	2.57	1.77	1.92	2.03	2.90
Ann.	4.19	4.43	4.81	4.38	4.12	3.92	4.31	5.05

TABLE 24 - CONTINUED

Maximum Daily Rainfall for Station 3762

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	4.87	5.42	2.89	3.88	3.46	5.72	4.37	4.15
Feb	3.95	4.95	2.25	2.35	3.94	2.93	3.40	3.57
Mar	3.25	3.72	3.85	3.52	3.54	3.69	3.60	6.33
Apr	2.95	2.90	3.17	3.95	3.66	3.55	3.36	3.91
May	3.29	2.43	2.43	3.36	3.20	2.84	2.92	3.25
Jun	2.53	2.57	3.54	3.40	3.71	2.88	3.10	3.06
Jul	3.34	3.70	4.60	2.89	4.57	3.44	3.76	5.02
Aug	2.67	2.64	5.50	3.48	4.80	2.53	3.60	4.32
Sep	3.26	2.35	2.76	2.62	5.37	3.22	3.26	4.10
Oct	2.40	2.66	2.67	3.00	2.64	2.52	2.65	2.35
Nov	2.60	2.04	2.90	2.63	3.15	2.90	2.70	2.83
Dec	2.24	2.82	3.14	3.07	5.32	3.24	3.30	4.28
Ann.	4.87	5.42	5.50	3.95	5.37	5.72	5.14	6.33

TABLE 24 - CONTINUED

Maximum Daily Rainfall for Station 3994

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	2.91	4.40	3.42	3.14	6.15	2.95	3.83	4.47
Feb	5.22	2.79	3.12	3.01	2.83	3.43	3.40	4.75
Mar	4.21	2.77	4.48	3.45	2.49	3.00	3.40	3.44
Apr	3.04	3.62	4.42	2.44	2.34	3.17	2.84	3.37
May	3.54	3.83	2.60	4.93	2.76	2.80	3.08	3.86
Jun	2.58	2.84	5.00	5.08	3.74	3.93	3.86	4.22
Jul	3.91	3.89	2.25	3.68	3.08	3.04	3.31	4.13
Aug	2.87	3.53	3.14	3.13	4.60	5.06	3.72	2.93
Sep	3.05	2.36	2.41	3.37	2.89	2.99	2.84	4.15
Oct	2.14	2.52	2.04	2.27	2.47	2.59	2.34	2.55
Nov	2.96	2.44	2.52	3.35	4.46	3.36	3.18	4.07
Dec	2.63	4.45	3.06	2.49	3.12	2.66	3.07	2.75
Ann.	5.22	4.45	5.00	5.08	6.15	5.06	5.16	4.75

TABLE 24 - CONTINUED

Maximum Daily Rainfall for Station 4825

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	2.88	2.53	2.23	3.06	2.85	2.69	2.71	3.10
Feb	1.99	2.44	3.99	2.15	1.99	2.34	2.48	2.75
Mar	2.24	2.67	2.69	2.60	3.92	2.76	2.81	3.00
Apr	2.13	2.53	3.39	2.14	2.68	1.87	2.46	2.80
May	3.00	3.68	2.77	2.70	2.47	2.31	1.90	2.65
Jun	2.76	3.13	3.55	3.62	2.70	4.34	3.35	2.75
Jul	2.24	2.46	2.65	2.63	3.84	2.55	2.73	3.50
Aug	2.96	4.04	2.73	3.69	2.74	3.81	3.33	2.95
Sep	2.51	2.54	3.96	2.09	2.67	2.93	2.78	2.44
Oct	2.18	1.78	1.84	1.52	2.41	1.43	1.86	1.99
Nov	1.95	2.49	1.88	2.51	1.72	1.93	2.08	2.15
Dec	2.02	2.37	2.25	2.32	2.60	2.78	2.39	2.45
Ann.	3.00	4.04	3.99	3.69	3.92	4.34	3.83	3.50

TABLE 24 - CONTINUED

Maximum Daily Rainfall for Station 6353

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	2.59	1.87	1.41	1.74	1.97	1.98	1.93	2.05
Feb	2.92	1.67	2.07	2.93	2.59	2.49	2.44	2.67
Mar	3.05	4.20	3.05	2.16	2.58	2.46	2.92	2.76
Apr	2.21	2.05	1.91	2.26	2.16	2.33	2.15	2.94
May	2.23	1.82	2.44	1.88	2.54	2.11	2.17	3.02
Jun	2.92	3.39	2.70	2.29	3.33	3.01	2.94	3.12
Jul	3.76	2.16	2.47	2.58	2.86	2.75	2.76	3.41
Aug	2.96	2.60	2.75	2.29	2.21	3.05	2.64	2.43
Sep	3.75	3.40	2.72	2.58	2.16	3.80	3.07	2.83
Oct	1.48	1.88	2.68	1.90	2.10	1.76	1.97	2.20
Nov	1.71	1.91	1.91	1.56	1.95	1.72	1.79	2.20
Dec	2.10	1.58	2.47	2.73	2.00	1.98	2.14	2.76
Ann.	3.76	4.20	3.05	2.93	3.33	3.80	3.51	3.41

TABLE 25

AVERAGE ANNUAL WET DAYS

Simulation

Sta	1	2	3	4	5	6	Ave	Hist
254	128	131	130	129	131	131	130	126
909	115	115	113	110	117	114	114	113
1345	102	99	100	101	102	103	101	102
3762	102	102	102	102	101	105	102	102
3994	109	107	110	109	106	108	108	108
4825	101	105	103	100	105	106	103	103
6353	137	136	131	133	133	133	134	133

TABLE 26

MAXIMUM AND MINIMUM TOTAL ANNUAL RAIN

Maximum

Sta	Simulation 1	2	3	4	5	6	Ave	Hist
254	53.49	50.98	54.65	50.77	54.99	51.04	52.65	53.32
909	69.72	64.97	68.72	68.01	68.96	57.24	66.27	63.72
1345	53.43	55.08	51.61	52.99	52.66	60.53	54.38	53.26
3762	58.42	63.82	63.90	55.93	59.83	56.79	59.78	71.01
3994	61.57	64.86	63.59	62.41	60.82	69.57	63.80	71.17
4825	58.28	57.21	60.98	59.16	56.13	59.31	58.51	64.75
6353	61.46	59.84	56.68	54.24	59.32	56.64	58.03	56.83

Minimum

Sta	Simulation 1	2	3	4	5	6	Ave	Hist
254	26.80	29.88	30.79	31.23	27.31	29.78	29.30	28.26
909	33.81	34.63	33.82	33.66	37.78	32.16	34.31	31.33
1345	25.48	27.05	25.38	25.01	28.88	26.50	26.38	23.78
3762	32.03	25.87	29.92	29.99	30.61	30.80	29.87	28.25
3994	34.01	28.97	34.32	33.46	31.64	29.14	31.92	28.47
4825	34.44	33.91	25.69	31.90	32.84	35.88	32.44	31.04
6353	34.03	31.78	29.74	27.81	31.59	33.12	31.34	30.69

TABLE 27

MEAN MONTHLY RUNOFF FROM HAAN MODEL

Mean Runoff for Station 254

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	1.88	1.92	2.05	2.02	2.39	2.44	2.12	2.21
Feb	2.06	1.85	2.02	1.93	1.97	2.21	2.01	2.11
Mar	2.97	2.41	2.66	2.89	3.12	2.77	2.80	2.90
Apr	1.65	1.84	1.82	1.92	1.72	1.81	1.79	1.82
May	1.20	1.63	1.27	1.14	1.52	1.13	1.32	1.33
Jun	0.88	0.96	0.75	0.91	0.90	0.79	0.86	0.79
Jul	0.57	0.67	0.57	0.80	0.99	0.87	0.74	0.76
Aug	0.52	0.54	0.46	0.49	0.49	0.50	0.50	0.54
Sep	0.58	0.44	0.33	0.40	0.30	0.50	0.42	0.36
Oct	0.48	0.43	0.35	0.32	0.33	0.36	0.38	0.31
Nov	0.49	0.65	0.35	0.48	0.44	0.57	0.50	0.50
Dec	1.24	1.53	1.14	1.09	1.09	1.22	1.22	1.28
Ann.	14.51	14.85	13.77	14.40	15.27	15.18	14.66	14.92

TABLE 27 - CONTINUED

Mean Runoff for Station 909

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	3.60	3.89	3.38	3.12	3.33	3.70	3.50	3.81
Feb	3.43	3.03	2.65	3.03	3.15	2.84	3.02	3.18
Mar	4.13	4.49	4.35	3.63	4.32	3.82	4.12	3.92
Apr	2.91	2.62	2.74	2.64	2.83	2.31	2.68	2.48
May	1.69	1.56	1.46	1.76	1.56	1.72	1.62	1.76
Jun	1.41	1.20	1.20	1.31	1.31	1.23	1.28	1.42
Jul	0.84	0.86	1.00	0.86	0.94	0.81	0.88	0.94
Aug	0.66	0.62	0.68	0.78	1.07	0.67	0.75	0.71
Sep	0.56	0.57	0.57	0.67	0.68	0.56	0.60	0.58
Oct	0.53	0.59	0.50	0.61	0.70	0.60	0.59	0.48
Nov	0.76	0.74	0.61	0.70	0.94	0.75	0.75	0.85
Dec	2.19	1.79	1.91	1.79	2.37	2.25	2.05	2.17
Ann.	22.71	21.97	21.05	20.91	23.20	21.27	21.85	22.30

TABLE 27 - CONTINUED

Mean Runoff for Station 1345

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	2.29	2.70	2.25	2.39	2.07	2.47	2.36	2.61
Feb	2.68	2.19	2.35	2.26	2.17	2.36	2.34	2.37
Mar	3.22	3.19	4.02	3.17	3.90	3.25	3.46	3.44
Apr	1.69	2.14	1.88	1.62	2.23	2.18	1.96	2.02
May	0.64	0.89	0.97	0.70	0.95	0.94	0.85	1.15
Jun	0.93	1.04	0.97	0.71	0.62	0.89	0.86	0.87
Jul	0.69	0.45	0.56	0.54	0.27	0.53	0.51	0.47
Aug	0.58	0.26	0.57	0.77	0.47	0.30	0.49	0.41
Sep	0.37	0.20	0.21	0.27	0.25	0.21	0.25	0.24
Oct	0.21	0.30	0.22	0.20	0.28	0.24	0.24	0.19
Nov	0.62	0.40	0.73	0.63	0.60	0.71	0.62	0.56
Dec	1.26	1.37	1.07	1.51	1.93	1.63	1.46	1.35
Ann.	15.19	15.14	15.81	14.78	15.74	15.70	15.39	15.68

TABLE 27 - CONTINUED

Mean Runoff for Station 3762

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	2.13	2.52	1.85	1.94	1.76	2.93	2.19	2.34
Feb	1.56	1.47	1.47	1.65	1.47	1.86	1.58	1.72
Mar	3.02	3.77	2.47	2.94	3.00	3.20	3.07	2.99
Apr	1.55	1.44	1.80	2.09	1.65	1.82	1.72	1.86
May	1.02	1.11	0.94	1.36	1.08	1.14	1.11	1.12
Jun	0.40	0.40	0.50	0.42	0.50	0.50	0.45	0.45
Jul	0.36	0.44	0.48	0.34	0.40	0.48	0.42	0.59
Aug	0.33	0.34	0.44	0.29	0.38	0.24	0.34	0.39
Sep	0.27	0.22	0.23	0.20	0.36	0.28	0.26	0.32
Oct	0.22	0.26	0.28	0.24	0.30	0.23	0.26	0.25
Nov	0.43	0.34	0.42	0.51	0.51	0.40	0.44	0.49
Dec	0.75	0.98	1.04	1.11	1.06	1.09	1.00	1.02
Ann.	12.04	13.30	11.93	13.09	12.48	14.16	12.83	13.54

TABLE 27 - CONTINUED

Mean Runoff for Station 3994

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	1.93	1.69	1.98	2.26	2.34	2.13	2.06	2.30
Feb	2.04	2.18	2.46	1.70	1.85	2.21	2.07	1.95
Mar	3.15	2.51	2.83	2.73	2.18	2.54	2.66	2.50
Apr	1.42	1.67	1.46	1.97	1.37	1.34	1.54	1.38
May	0.81	0.75	0.99	0.73	0.97	0.91	0.86	1.09
Jun	0.49	0.50	0.45	0.68	0.50	0.51	0.52	0.50
Jul	0.39	0.26	0.21	0.31	0.32	0.43	0.32	0.37
Aug	0.26	0.23	0.19	0.19	0.21	0.30	0.23	0.28
Sep	0.19	0.16	0.16	0.17	0.17	0.14	0.16	0.28
Oct	0.15	0.17	0.13	0.12	0.12	0.17	0.14	0.15
Nov	0.43	0.47	0.44	0.65	0.48	0.22	0.45	0.51
Dec	0.97	1.10	0.96	1.19	0.82	0.96	1.00	1.00
Ann.	12.22	11.68	12.28	12.69	11.33	11.84	12.01	12.32

TABLE 27 - CONTINUED

Mean Runoff for Station 4825

Simulation

Month	1	2	3	4	5	6	Ave	Hist
Jan	3.42	3.04	2.55	2.75	2.93	3.09	2.96	2.96
Feb	2.58	2.61	3.14	2.55	2.51	2.50	2.65	2.66
Mar	3.06	3.31	3.29	3.07	3.61	2.98	3.22	3.26
Apr	1.61	1.97	1.85	1.90	1.73	1.65	1.78	1.93
May	1.12	1.23	1.66	1.35	1.25	1.21	1.30	1.36
Jun	0.76	0.94	1.15	1.06	0.89	0.84	0.94	0.99
Jul	0.82	0.91	0.95	0.75	0.82	0.88	0.86	0.92
Aug	0.66	0.90	0.60	0.56	0.70	0.50	0.65	0.52
Sep	0.42	0.32	0.37	0.36	0.46	0.34	0.38	0.35
Oct	0.33	0.35	0.27	0.24	0.29	0.32	0.30	0.32
Nov	0.51	0.64	0.44	0.62	0.48	0.55	0.54	0.62
Dec	1.46	2.02	1.32	1.62	1.82	1.53	1.63	1.56
Ann.	16.75	18.24	17.59	16.83	17.50	16.40	17.22	17.44

TABLE 27 - CONTINUED

Mean Runoff for Station 6353

Simulaton

Month	1	2	3	4	5	6	Ave	Hist
Jan	2.32	2.29	1.61	1.96	1.81	2.15	2.02	2.09
Feb	2.34	2.68	2.26	2.69	2.57	2.53	2.51	2.60
Mar	3.52	3.30	3.10	3.34	3.44	3.12	3.30	3.20
Apr	1.94	1.81	1.54	1.69	1.73	1.57	1.71	1.82
May	1.30	1.17	1.34	1.37	1.50	1.58	1.38	1.36
Jun	1.14	0.97	1.11	0.84	1.19	0.91	1.03	0.95
Jul	1.27	1.06	0.84	1.01	0.82	0.76	0.96	1.00
Aug	0.81	0.62	0.49	0.62	0.47	0.57	0.60	0.68
Sep	0.75	0.50	0.39	0.51	0.37	0.56	0.51	0.47
Oct	0.51	0.37	0.44	0.35	0.30	0.32	0.38	0.31
Nov	0.58	0.45	0.53	0.36	0.36	0.39	0.44	0.45
Dec	1.24	1.16	1.19	0.90	0.84	1.13	1.08	1.02
Ann.	17.71	16.37	14.86	15.65	15.41	15.59	15.93	15.96

TABLE 28

MOST SEVERE LOW FLOWS DURING PERIOD

(MARCH-FEBRUARY WATER YEAR)

Low Flows for Station 254

Simulation

Mon Dur	1	2	3	4	5	6	Ave	Hist
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
3	0.04	0.00	0.00	0.01	0.01	0.01	0.01	0.06
4	0.07	0.03	0.05	0.03	0.15	0.10	0.07	0.16
5	0.29	0.22	0.13	0.24	0.43	0.28	0.26	0.58
6	0.53	0.48	0.60	0.41	0.98	0.68	0.61	0.91
7	0.96	1.28	1.08	0.76	1.65	1.24	1.16	1.58
8	1.52	1.94	1.69	1.52	2.08	2.00	1.79	2.19
9	2.43	2.73	2.77	2.37	2.83	2.76	2.65	2.89
10	3.45	3.69	3.62	3.20	3.63	4.34	3.66	3.55
11	4.95	6.09	4.46	3.96	6.53	6.98	5.50	4.25
12	7.11	7.82	7.50	8.77	10.09	9.20	8.42	4.92

TABLE 28 - CONTINUED

Low Flows for Station 909

Simulation

Mon Dur	1	2	3	4	5	6	Ave	Hist
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.01	0.02	0.04	0.01	0.00	0.01	0.01
3	0.09	0.16	0.17	0.12	0.17	0.00	0.12	0.08
4	0.21	0.45	0.51	0.24	0.49	0.14	0.34	0.32
5	0.83	0.89	1.22	0.34	1.15	0.50	0.82	0.79
6	1.56	1.44	1.98	0.57	2.19	0.88	1.44	1.27
7	2.41	2.23	2.49	1.26	3.36	1.43	2.20	2.08
8	3.69	3.29	3.32	2.24	4.93	2.41	3.31	2.98
9	5.03	4.67	4.26	3.35	6.44	3.79	4.59	4.01
10	6.97	6.69	5.59	4.59	8.33	5.43	6.27	5.26
11	8.61	8.54	7.72	8.47	10.22	7.75	8.55	7.10
12	12.18	14.26	9.85	10.44	14.33	11.15	12.04	11.84

TABLE 28 - CONTINUED

Low Flows for Station 1345

Simulation

Mon Dur	1	2	3	4	5	6	Ave	Hist
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
3	0.01	0.03	0.03	0.02	0.01	0.00	0.02	0.00
4	0.04	0.08	0.07	0.08	0.01	0.01	0.05	0.01
5	0.09	0.13	0.14	0.15	0.01	0.03	0.09	0.04
6	0.16	0.24	0.21	0.25	0.04	0.10	0.17	0.08
7	0.27	0.40	0.34	0.35	0.13	0.19	0.28	0.16
8	0.46	0.86	0.49	0.57	0.22	0.57	0.53	0.34
9	1.12	2.17	1.15	1.42	0.82	0.82	1.25	0.63
10	3.03	3.59	1.58	2.21	0.93	1.38	2.12	1.48
11	4.26	5.99	2.41	3.05	2.08	2.26	3.34	1.78
12	6.35	8.38	2.96	5.35	3.91	4.04	5.16	3.03

TABLE 28 - CONTINUED

Low Flows for Station 3762

Simulation

Mon Dur	1	2	3	4	5	6	Ave	Hist
1	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00
3	0.01	0.00	0.03	0.00	0.06	0.07	0.03	0.02
4	0.09	0.03	0.12	0.04	0.16	0.14	0.10	0.08
5	0.220	0.10	0.34	0.14	0.27	0.29	0.23	0.21
6	0.45	0.20	0.52	0.30	0.53	0.69	0.45	0.37
7	0.74	0.35	0.80	0,59	0.73	0.92	0.69	0.68
8	0.97	0.56	1.02	0.95	0.94	1.38	0.97	0.81
9	1.29	0.91	1.37	1.40	1.26	1.65	1.31	0.98
10	1.60	1.55	2.03	1.88	2.72	1.94	1.95	1.21
11	1.95	2.51	2.37	3.16	3.41	2.62	2.67	1.59
12	2.35	4.33	4.74	3.94	3.76	4.16	3.88	3.83

TABLE 28 - CONTINUED

Low Flows for Station 3994

Simulation

Mon Dur	1	2	3	4	5	6	Ave	Hist
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.01	0.00	0.00	0.02	0.00	0.00	0.01
3	0.03	0.04	0.03	0.02	0.03	0.01	0.03	0.04
4	0.08	0.08	0.08	0.07	0.05	0.04	0.07	0.08
5	0.19	0.28	0.14	0.13	0.14	0.21	0.18	0.13
6	0.35	0.34	0.26	0.23	0.32	0.28	0.30	0.25
7	0.54	0.45	0.42	0.44	0.52	0.41	0.46	0.52
8	0.92	0.69	0.65	0.70	0.79	0.59	0.72	0.79
9	1.11	1.07	0.89	0.97	1.04	0.81	0.98	1.00
10	1.39	1.29	1.39	1.18	1.52	1.11	1.31	1.20
11	2.63	1.58	1.67	2.86	2.97	2.12	2.30	1.49
12	2.95	3.58	3.39	4.26	3.97	6.01	4.03	3.64

TABLE 28 - CONTINUED

Low Flows for Station 4825

Simulation

Mon Dur	1	2	3	4	5	6	Ave	Hist
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.04	0.01	0.00	0.03	0.00	0.04	0.02	0.00
3	0.04	0.04	0.03	0.10	0.03	0.14	0.06	0.02
4	0.15	0.15	0.03	0.36	0.11	0.21	0.17	0.06
5	0.33	0.55	0.08	0.57	0.27	0.45	0.38	0.28
6	0.43	0.87	0.23	0.74	0.61	0.73	0.60	0.79
7	1.17	1.45	0.50	1.20	1.13	1.20	1.11	1.36
8	2.07	2.62	1.03	1.82	1.94	1.98	1.91	1.74
9	3.02	3.94	2.33	2.65	2.47	3.55	2.99	2.21
10	5.13	4.40	3.64	3.43	4.68	5.16	4.41	2.81
11	7.09	5.79	5.46	4.30	7.39	5.90	5.99	3.99
12	8.53	8.75	7.50	7.60	9.83	8.28	8.42	6.03

TABLE 28 - CONTINUED

Low Flows for Station 6353

Simulation

Mon Dur	1	2	3	4	5	6	Ave	Hist
1	0.09	0.04	0.08	0.07	0.06	0.09	0.07	0.06
2	0.21	0.09	0.17	0.17	0.18	0.24	0.18	0.14
3	0.35	0.18	0.26	0.31	0.35	0.36	0.30	0.22
4	0.55	0.29	0.43	0.44	0.56	0.59	0.48	0.40
5	0.79	0.43	0.61	0.63	0.82	0.74	0.67	0.58
6	1.07	0.61	0.83	0.84	1.04	1.00	0.90	0.79
7	1.32	0.92	1.12	1.10	1.54	1.27	1.21	1.15
8	1.69	1.23	1.48	1.42	2.06	1.55	1.57	1.69
9	2.12	1.69	1.80	1.80	2.63	1.83	1.98	2.09
10	2.59	2.09	2.17	2.14	4.02	2.33	2.56	3.29
11	5.69	2.90	2.66	2.83	5.41	2.87	3.73	5.02
12	8.05	4.93	5.74	5.35	7.51	6.30	6.31	6.58

APPENDIX C

SELECTED NOTATION

p_i	stochastic model parameter
p'_i	stochastic model parameter estimated from runoff simulated by the parametric runoff model
$\hat{p}_{jr}$	stochastic model parameter estimated from sub-record r
$\hat{p}'_{jr}$	stochastic model parameter estimated from the parametric model extension of sub-record r
$p_{i,obs}$	sub-record estimate of a stochastic model parameter
$p_{i,ext}$	extended-record estimate of a stochastic model parameter
p^k_{i1}	probability of no rain on day n+1 given rainfall on day n in state i, season k
r	estimate of the lag-one serial correlation coefficient for a given period of flow
s	estimate of the standard deviation of flow for a given period
S_r	number of sub-record
$\bar{x}$	estimate of the mean flow for a given period
$\Gamma(x)$	the Gamma function

$\eta_{i,k}$	shape factor for gamma distributed rainfall from state i in season k
$\lambda_{i,k}$	scale factor for gamma distributed rainfall from state i in season k
μ	mean flow for a given period
ρ	lag-one serial correlation coefficient
σ^2	variance of flow for a given period

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