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# A Multiobjective Branch-and-Bound Method for Planning Wastewater and Residual Management Systems

Digital Object Identifier: <https://doi.org/10.13023/kwrri.rr.130>

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A MULTIOBJECTIVE BRANCH-AND-BOUND METHOD FOR  
PLANNING WASTEWATER AND RESIDUAL MANAGEMENT SYSTEMS

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Project Number: A-083-KY (Completion Report)  
Agreement Numbers: 14-34-0001-0119 (FY 1980)  
14-34-0001-1119 (FY 1981)  
Period of Project: October 1979 - December 1981

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The work upon which this report is based was supported in part by funds provided by the Office of Water Research and Technology, United States Department of the Interior, Washington, D.C., as authorized by the Water Research and Development Act of 1978. Public Law 95-467.

January 1982

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#### ACKNOWLEDGEMENTS

The authors wish to acknowledge valuable suggestions provided by Dr E.D. Brill, Jr., and Dr J.C. Liebman, both in the Department of Civil Engineering at the University of Illinois. Earlier versions of the paper were presented at the Joint National Meeting of the Operations Research Society of America and the Institute of Management Sciences, Milwaukee, Wisconsin, October 15 - 17, 1979 and at the Speciality Conference of the Water Resources Planning and Management Division, American Society of Civil Engineers, Greenbay, Wisconsin, July 30 - August 1, 1980.

## ABSTRACT

A multiobjective branch-and-bound algorithm is proposed for use in analysing multiobjective fixed-charge network-flow problems which are found commonly in water resources planning situations. Also proposed is a multiobjective imputed value analysis which makes use of the branch-and-bound tree structure and allows the comparison of the importance of facilities in the network as represented by individual arcs or sets of arcs. The mathematical formulation and the analysis procedure of the method are described, and the potential usefulness of the method is demonstrated using two hypothetical example problems dealing with regional wastewater treatment and residual management systems. A FORTRAN program for implementing the algorithm is available from the first author.

Descriptors: Regional Wastewater Planning, Mathematical Models, Multiobjective Analysis, Network-flow Analysis.

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## I. INTRODUCTION

### A. General Discussion of Regionalization Problems

Regional management of wastewater collection, treatment and residual sludge disposal has become a matter of concern in many population centers throughout the world. For example, in the U.S.A. the congress passed the Federal Water Pollution Control Act Amendment of 1972 (PL92-500) and, subsequently, the Clean Water Act of 1977 (PL 95-217) which require stringent water quality management practices by municipalities and industries by 1984. Section 208 of the PL 92-500 requires regional facility planning, principally through phasing out and integration of existing facilities. In Japan, on the other hand, the national plan for wastewater treatment was drawn up almost from a scratch in the late 1960's. A network of basin-wide wastewater management systems is to cover the entire nation by the end of the century and a very ambitious construction programme has been carried out since early 1970's. In many major metropolitan areas of the developing countries regional wastewater systems are also being planned or implemented to various degrees.

Under the right circumstances, regional wastewater management is one of the most effective means of coping with water pollution problems (e.g., Canham, et al., 1971, and Lyon, 1967). The kind of regionalization scheme most suitable to an area depends not only on the size and physical constraints of the region under consideration, but also on the socio-economic and cultural background of the nation in which it is to be implemented. The planning issues associated with regionalization of wastewater and residual management system in different nations are, therefore, very different and difficult to understand without thorough knowledge of the nation itself.

The existing literature reveals the complexities of the planning issues involved in regionalization attempts in different nations. Brill and Nakamura (1978-b), for example, have presented a review of the issues raised in the process of regionalization in Japan. Recent experiences

with regionalization in Britain are discussed by Ardill (1974), Buckley (1975), and Okun (1975). U.S. Advisory Commission on Intergovernmental Relations (1972) described the planning issues commonly identified in the regionalization attempts in the United States of America. There are a number of other publications on the subject of wastewater regionalization in the U.S.A. (see, for example, Environmental Protection Agency, 1975-d, Texas Advisory Commission on Inter-governmental Relations, 1974, National Science Foundation, 1976, Kentucky-Indiana Planning and Developing Agency, 1978, Whipple, 1978).

Regardless, the planning of regional wastewater systems is an exceedingly difficult public sector problem. The issues involved are often very complex, diverse and interdependent. Since there are many conflicting objectives and incommensurate criteria, it is often impossible to generate a plan that is satisfactory to all parties involved.

Some of the major issues related to planning regional wastewater systems are:

- 1) Economies of scale: Regional wastewater systems generally include joint facilities for treating wastewater piped from several sources. The major advantages of regionalization are the potential economies of scale in capital and operation and maintenance costs associated with joint facilities (e.g. Classen, et al., and 1970 Linzing, 1972)
- 2) Plant performance: Large plants are generally considered more reliable than small plants if efficiently managed and operated. Simplified administration, concentration of skilled personnel, automation of auxillary equipment, and reduction in the variability of wastewater quality and quantity may be possible in a regionalized system. On the other hand, effluent flows from a small number of large plants may pose serious threats to the natural purification capacities of the receiving streams and the breakdowns of a large system may result in catastrophic environmental damage (see, for example, Adams and Gemmell, 1973).

- 3) Compatibility with existing systems: A regional wastewater system involving a small number of large facilities may appreciably alter the existing condition of a region. For example, a large regional treatment plant requires a large piece of land and may disturb the local environmental conditions. Large interceptor pipes, once constructed, may promote unplanned growth or urban sprawl of the immediate neighbourhood and surrounding areas.
  
- 4) Residual Management Systems: Since all wastewater treatment systems on the regional level generate large quantities of sludge, planning efforts should also be concerned with the ways of disposing sludge or reclaiming the reusable portion of sludge. Many of the existing methods for disposal of sludge concern land usage such as landfills or agricultural applications, and these disposal sites are rapidly being exhausted. The growing awareness on energy conservation is also making resources recovery and reclamation more and more attractive.

There are a number of other issues which are also vital to the planning of regional wastewater treatment systems. For example, the institutional and financial arrangements, which include the ownership and administration of the system, as well as the cost allocation among participating municipalities and industries, are very important. Also, legal constraints such as treatment regulations and water quality standards should be carefully examined in planning a regional system. The planning of regional wastewater systems, therefore, is an exceedingly difficult problem. What is more, the process of reaching decisions about any large-scale technological projects with social consequences involves a highly complex human interaction.

Mathematical methods have been used frequently at the screening stage of the planning process of large-scale public sector planning problems. Because of the easy access to prepackaged computer programs and the extensive literature available on the application of mathematical analysis methods to problems possessing seemingly similar problem structure, the

single-objective optimization techniques have been popularly accepted as the standard "tools" of analysis for regional wastewater system problems. On the subject of planning regional wastewater, there is a large body of literature as described in Chapter II. Although they are often very useful, the traditional single-objective mathematical models (e.g. cost minimization with or without water quality constraints) are sometimes grossly inadequate and/or inappropriate because of the inherent multi-objective nature of planning wastewater systems. Consequently, the need for research on practical methods of multiobjective analysis has been strongly urged in recent years. One good example may be the heavy emphasis placed through the federal guidelines in the U.S.A. on the pursuit of more comprehensive and innovative planning strategies, including more effective public participation (United States General Accounting Office, 1978).

#### B. Research orientation

The orientation of this research is based on the premise that it is very difficult to define, much more to find by mathematical means, the optimal solution to such a complex public sector problem as planning regional wastewater treatment systems (Brill and Nakamura 1978-a). Difficulties arise because many planning issues are involved and they are all closely interrelated, as stated in the previous section.

For example, in the past several years, many population centers in the U.S. have become subject to facilities planning for wastewater treatment and residual sludge management under the Federal Water Pollution Control Act (PL 92-500) of 1972. Cost minimization mathematical models have been used frequently at the screening stage, along with engineering judgements, to select a small number of alternative plans for further evaluation. The engineering judgement is used to weigh the relative importance of more than two incommensurate planning criteria such as cost and water quality. A major weakness of such an approach, however, is that cost may be emphasized too heavily, and that the process of choosing "desirable" alternative plans using the engineering judgement is not very explicit.

This research deals with an application of a multiobjective programming method called multiobjective branch-and-bound method to the analysis of alternative wastewater treatment and residual management system. The example application deals with minimizing objectives such as treatment plant and interceptor construction costs, sludge handling costs for land-filling or landspreading, water quality impacts on receiving streams, and land impacts from regionalization.

The main emphasis of the proposed method is to generate alternative plans, while paying attention to several major planning criteria in such a way:

- 1) to integrate major planning objectives other than cost in the multiobjective method proposed;
- 2) to identify efficiently dominant or dominated alternatives with respect to a given set of decision criteria;
- 3) to identify trade-off values between objectives, and
- 4) to select a manageable number of "good" alternative plans.

The use of appropriate technique for quantitative expressing various objectives is essential for applying any mathematical methods of multi-objective analysis. No research effort was made in this research, however, to justify the use of existing quantification techniques or to develop new methods of quantification. This research brings its focus on the mathematical properties and computational aspects of the multiobjective branch-and-bound method proposed.

## II. REVIEW OF MATHEMATICAL METHODS AND BACKGROUND FOR BRANCH-AND-BOUND TECHNIQUES

### A. Review of Mathematical Methods of Analysis

There is a rather large body of literature dealing with analytical methods pertaining to water resources management, water pollution control and other public sector planning problems. No attempt was made in this paper to review all of these methods. The reader is referred to Dracup (1970), Pentland, et al., (1972), and Bundgaard - Nielsen and Hwang (1976), for a comprehensive literature review and discussion of the numerous techniques. There are, however, certain methods which deal specifically with regional water quality management and regional wastewater facility planning, and these methods will be reviewed presently.

One group of these models emphasizes the water quality aspect of the regionalization problem. The principal objective of these models is to find the least-cost layout for regional wastewater treatment plants and the associated interceptors while satisfying the water quality constraints. For example, Klemetson and Grenney (1976) have developed a dynamic programming model which analyses the staging of regional facilities. Graves, et al, (1970) suggested a nonlinear formulation that allows at-source treatment, joint treatment at candidate sites, and bypass piping of water in order to meet explicit water quality constraints. <sup>and Liebman</sup> Rossman/(1974) used nonlinear programming and dynamic programming methods, and Whitlatch (1975) suggested a heuristic method for solving this problem. However, each of these models deal basically with regions where wastewater sources were located along a river.

More attention has been directed in the past several years towards mathematical methods for a network rather than linear configurations. For example, Meier (1971) has presented a branch-and-bound procedure to solve for the least-costly regional system. Deininger (1972) described an extreme point ranking algorithm and Converse (1972) suggested a dynamic programming method for solving for the least-costly system. Wanielista



and Joeres, et al.(1974), and Lauria (1975) suggested mixed-integer programming approaches, and Jarvis, et al., (1975) presented a network formulation and a group theoretic solution approach to the same problem. A heuristic procedure offered by McConagha and Converse (1973) includes an evaluation of cost savings and cost allocation among participating municipalities. Nakamura and Brill (1977) suggested a branch-and-bound algorithm that focuses on generating alternative physical plans efficiently and systematically based on cost and facility location. The branch-and-bound tree is transformed into a matrix for efficient retrieval of cost trade-offs (Nakamura and Brill 1977). The above work also appear in Brill and Nakamura (1978-a) and in Nakamura and Brill (1979).

Some attempts have been made also to consider several planning periods. Nakamura, Brill and Liebman (1981) expanded their branch-and-bound algorithm with imputed value matrix into a multiperiod analysis. A heuristic method developed for general facility location problems was proposed for application to wastewater regionalization problems by Bahlla and Rikker (1971). Lauria (1975) demonstrated that mixed integer programming can be applied to multiperiod analysis. Also, Rossman (1977) applied the Weeter and Belarde algorithm and dynamic programming method for a multiperiod solution.

The primary emphasis of their works cited above, with the exception of those of Nakamura and Brill<sup>(1977)</sup>, and of Nakamura, Brill and Liebman (1981), has been to achieve computational efficiency and/or mathematical optimality in solving for the economically most favourable solution. All of these methods deal only with a single objective, cost.

Mathematical methods which deal with problems involving more than one objective have drawn much attention in recent years. The methods proposed by Hill (1968), Major (1969), Freeman and Havenman (1970), Hockman (1977), Nijkamp and Vos (1977), and Keeney and Wood (1977) are some of the examples of the methods which integrate a subjective weighting system to compare a small number of discrete alternatives. Numerous attempts to apply these methods to water resources planning problems appear in the literature. Each of these methods has its unique features, and the applicability of

these methods depends on the nature of the problem under consideration. A comparison of several of these multicriteria analysis methods, was attempted by McAriff (1980) for a river basin planning problem.

There is a group of methods which are designed to generate and compare a large number of potential alternative plans. For the sake of dealing with numerous alternatives, they resort to some type of mathematical optimization. The most notable among these is the use of single objective optimization techniques, such as linear programming, for generating non-inferior solutions one at a time. Brill, et al., (1976), for example, presented trade-off relationships between economic efficiency and equity for various regional water quality management schemes using linear programming solutions. There are also mathematical programming methods which are specifically/as multi-objective programming methods. Multiobjective linear programming methods, (e.g. Zeleney, 1974, and Steuer, 1976) goal programming method (Charnes and Cooper, 1961) and Surrogate Worth Trade-off method (Haines and Hall (1974), and Haims, et al., (1977), are notable examples of these multiobjective programming methods, although they may be vastly different in orientation and scope of application. The noninferior set estimation (NISE) proposed by Cohon, et al., (described in Cohon, 1978) also belongs to this category. Some examples of application of these methods to water resources and environmental planning problems include Lindsey (1976) on the application of the Surrogate Worth Trade-off method to the analysis of sewage sludge disposal alternatives, Rossimiller (1979), and Lohani and Adulbhan (1979), on the application of goal programming to water resources planning problems.

There are a number of other analytical methods proposed to deal with multiobjective (or multicriteria) public sector planning problems which are directly or indirectly related to regional water resources and environmental management systems. For example, McAvoy (1973) proposed an affinity coefficient matrix method for analysing the potential for regionalizing separate political entities. Neering, et al., (1971), used a weighting procedure and a viewpoint triangle method for determining land requirements and/or restrictions in a regional land use scheme. Bammi and Bammi (1979)

attempted to integrate multiple objective analysis of land use planning into a linear programming model. Some of the objectives considered in their model include minimization of local conflicts, minimization of travel distance, minimization of air pollution, maximization of fiscal soundness. These methods are, however, empirical, in that the evaluation of multiple criteria and the synthesis of values associated with individual criteria is quite arbitrary. Some interesting results of a research on the use of mathematical methods to generate alternative plans in the public sector planning problems was presented recently by Chiang, et al., (1980), making use of the example problem presented by Nakamura and Brill (1977). The significance of such research efforts rests on the premise that the human articulation of preference relationship is rather fragile and that the human intuition has to be reenforced by a repeated generation of very different alternatives.

#### B. Background For Branch-and-Bound Method

The ability to generate alternatives mathematically depends on the properties of the particular modeling technique and on the type of problem to be solved. Generally, in order for a mathematical model to be a useful tool for generating and comparing alternative plans, the model should be capable of generating many alternatives efficiently and systematically. Although many mathematical models may be efficient and systematic in generating alternatives, they may not be applicable to the wastewater regionalization problem because the problem has a network-flow structure. The branch-and-bound techniques, as illustrated later, appears to be quite satisfactory in generating alternative network-flow configurations efficiently and systematically.

Branch-and-bound algorithms have been extensively used in the past for solving a variety of combinatorial problems. Efraymson and Ray (1965) suggested the use of a branch-and-bound algorithm in solving plant location problems. Liebman (1967) presented a branch-and-bound algorithm to minimize the cost of wastewater treatment under equity constraints. Sá (1968) treated the capacitated plant location problem using an approximation method and a

branch-and-bound algorithm. Marks and Liebman (1970) suggested a combination of a network algorithm and a branch-and-bound technique for solving a problem of locating solid waste management facilities.

The conceptual similarity between the branch-and-bound process and the general process of planning was suggested by Harris (1970). In either process, alternatives are generated systematically and trade-off information is evaluated for groups of alternatives generated. It was this conceptual similarity which Brill and Nakamura (1978<sup>a</sup>) expanded to a practical analytical method in the planning of regional wastewater treatment systems.

The basic concept for the branch-and-bound procedure is readily available in the literature, e.g. Agin (1966), Lowler and Wood (1966), Mitten (1960), Hiller and Lieberman (1974). A summary of the branch-and-bound procedure described by Hiller and Lieberman (1974) is briefly presented below.

Since any bounded programming problem has only a finite number of feasible solutions, it is natural to consider an enumeration procedure for generating alternatives and possibly finding an optimal solution. Because this finite number is usually very large, exhaustive enumeration would be prohibitively time-consuming. For example, if there are 10 variables with each one having 10 feasible values, there can be as many as  $10^{10}$  feasible solutions which would require extensive computational time even with the high speed digital computers of today. Therefore, it is imperative that any enumeration procedure be structured so that only a tiny fraction of the feasible solutions need be examined.

A brief description of the branch-and-bound method is presented next by taking, as an example, a problem in which the objective function is to be minimized. First, assume that an upper bound on the optimal value of the objective function is available, i.e., the value of the objective function for the best feasible solution identified thus far. This step involves in general a simple heuristic computation. The next step is to partition the set of all feasible solutions into several subsets, and, for each one, a

lower bound is obtained for the value of the objective function of the solutions within the subset. This step involves in general an appropriate optimization technique as applied to a relaxed version of the original problem. Then, those subsets whose lower bounds are found to exceed the current upper bound on the objective function value (already identified) are then excluded from further consideration. This exclusion of a subset is said to be fathoming. One of the remaining subsets is then partitioned further into several subsets, e.g. the subset with the smallest lower bound is further partitioned. The lower bounds of the new partitioned subset are in turn obtained and used as before to exclude some of these subsets from further consideration. From all the remaining subsets, another one is selected for further partitioning and the elimination process is continued. This process is repeated until a feasible solution or set of solution is found such that the corresponding value of the objective function is no greater than the lower bound for any subset. This procedure, resulting in a tree structure call the branch-and-bound tree, terminates when there are no remaining unfathomed subsets and the current incumbent solution is considered optimal. If the objective is to maximize rather than minimize the objective function, the procedure is unchanged except that the roles of the upper and lower bounds are reversed.

As stated earlier, Nakamura and Brill (1977) expanded this optimality concept by considering a dichotomy of alternatives in a wastewater regionalization problem. They suggested grouping of the potential alternatives into two distinct sets, those which contain a treatment facility and those which do not. This is an attractive dichotomy from a planning point of view since, as mentioned earlier, many of the issues to be considered in planning regional systems are directly related to the physical configuration of the network of regional facilities. By considering the economic feasibility, it was possible to compare the cost of the least-cost alternative with a specific facility with the cost of the least-cost alternative without the facility. From this an imputed value associated with a specific facility was defined as the difference between the costs of including or not including the facility. Further, the information obtained from the branch-and-bound tree could be transformed into a matrix called the imputed value incidence

matrix, which can be used to compare alternatives, selecting trade-offs, and giving insight into selecting the most feasible regionalization plan. For a detailed discussion of the imputed value analysis the reader is referred to Nakamura and Brill (1979).

### C. Multiobjective Branch-and-Bound Method

The concept of branch-and-bound process can be expanded to multiple-objective analysis. This is done by associating the nodes of the branch-and-bound tree with a vector rather than a scalar as in the case of single-objective analysis. Although the term "multiobjective branch-and-bound method" does not appear in the literature to the authors' knowledge, there have been attempts in the past to make use of this vector branch-and-bound process as an analytical tool. For example, Bitran (1977) proposed a linear multiple objective programmes with zero-one variables in which he used a multiobjective branch-and-bound process to resolve non-integrality for solutions. Although in small scale, Bitran and Lawrence (1979) applied this method to an insurance service office location problem. Villarreal, et al., (no date) used basically the same approach to solve multicriterion (multiobjective) integer programming problems. They called the method interactive branch-and-bound method as the branch-and-bound tree was grown interactively using a time-sharing computing system. Also, Marcotte and Soland (1980) proposed an interactive branch-and-bound algorithm for multiple criteria optimization which is applicable to both discrete and convex problems.

The multiobjective branch-and-bound method proposed herein is an extension of the single objective branch-and-bound method proposed by Nakamura and Brill (1977) for generating and evaluating alternative network flow solutions. The principal objective of the method is to identify a set of alternatives which are noninferior to each other from a large number of multiobjective alternatives generated on the branch-and-bound tree. The concept of noninferiority, therefore, is applied only to the alternatives generated based on a set of criteria for growing and fathoming a particular branch-and-bound tree. Presented below is a brief conceptual sketch of the general multiobjective branch-and-bound algorithm.

An illustrative multiobjective branch-and-bound tree for a two-objective minimization problem is shown in Figure 2.1. The basic structure of the tree is similar to a single-objective branch-and-bound tree. Associated with each node of the tree, however, is a vector consisting of two objective function values such as  $\vec{z}_1 = (z_1^1, z_1^2)$  or  $\vec{z}_5 = (z_5^1, z_5^2)$  (we use  $z$  and  $Z$  to indicate infeasible lower bounds and feasible alternative solutions, respectively, and superscripts and subscripts indicate the objective and node numbers, respectively). Because of these vectors the selection of a node from which to branch next and the bounding of tree limbs, including the termination of the entire branch-and-bound process, are not as straightforward as in the single objective case.

Referring to Figure 2.1 for illustration, the branching process can be continued from lower bound nodes 3 or 4. A branching rule such as "branching from the lowest lower bound" (Hiller and Lieberman, 1979) is not applicable unless there exists a clear dominance relationship between the two objective vectors. For example, if the relationship  $\vec{z}_3 \leq \vec{z}_4$  (i.e.,  $z_3^1 \leq z_4^1$  and  $z_3^2 \leq z_4^2$ ) holds, node 3 is the logical choice for the next node from which to branch based on the above rule. If there is no such relationship between the two objective vectors, some additional provisions must be made for a systematic branching process.

One faces basically the same difficulty in the bounding of the tree limbs and in the termination of the branch-and-bound process. The branch-and-bound process illustrated in Figure 2.1 may be terminated altogether if the relationships  $\vec{z}_5 \leq \vec{z}_3$  and  $\vec{z}_5 \leq \vec{z}_4$  hold, because none of the feasible alternatives which can be identified under node 3 or under node 4 may have lower objective function values than those associated with node 5 either with respect to objective 1 or objective 2. On the other hand, if, for example, the second relationship,  $\vec{z}_5 \leq \vec{z}_4$ , does not hold, the branching process must be continued from node 4. The second relationship can be violated in one of two ways. First, the infeasible lower bound vector  $\vec{z}_4$  may dominate the feasible alternative solution vector  $\vec{z}_5$ , i.e.  $\vec{z}_4 < \vec{z}_5$ .

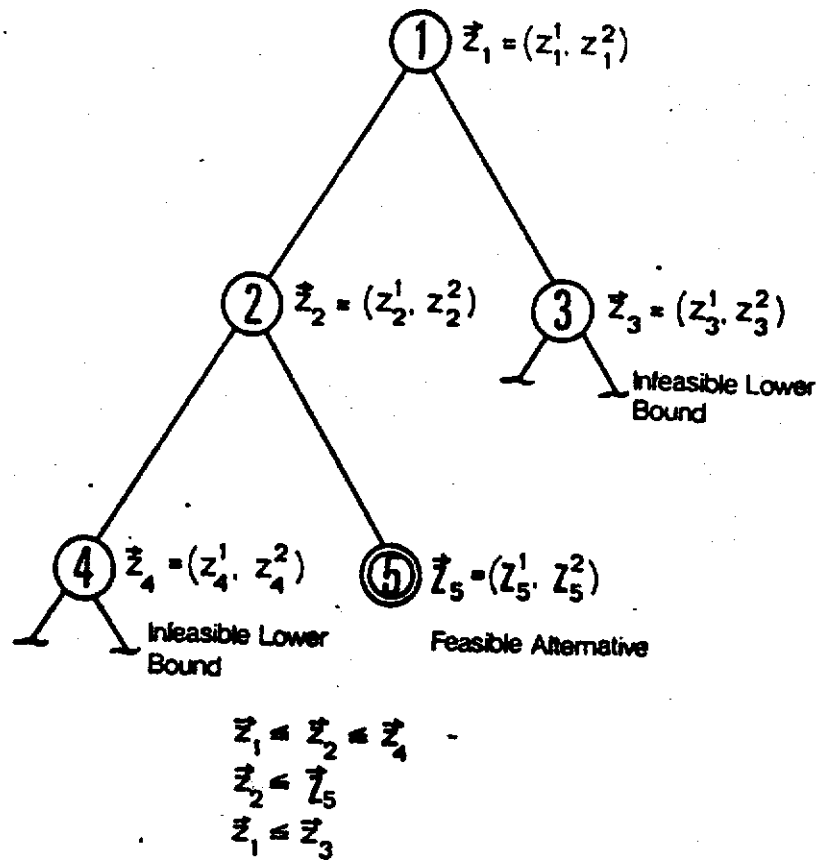


Figure 2.1 A Conceptual Sketch of Multiobjective Branch-and-Bound Tree for a Two-Objective Minimization Problem



In this case, there is a possibility that one or more of the feasible alternatives under node 4 may be dominant over the one associated with node 5. As soon as such an alternative is identified, the one associated with node 5 may be eliminated from the multiobjective analysis, at least with respect to the two objectives studied. Second, if the relationships,  $z_4^1 \leq z_5^1$  and  $z_4^2 > z_5^2$ , hold, there is no possibility that any of the feasible alternatives which can be generated under node 4 would dominate the alternative associated with node 5. The possibility exists, however, that one or more feasible alternatives which satisfy the relationship such as  $z_\psi^1 < z_5^1$  and  $z_\psi^2 > z_5^2$  could be found in the portion of the tree under node  $\psi$ . The alternative associated with node  $\psi$ , therefore, would be non-inferior to the one associated with node 5.

The branch-and-bound process can be terminated in one of three ways. The first of which is equivalent to the identification of the optimal solution in the single objective branch-and-bound method. This situation occurs when, for example,  $\vec{z}_5 < \vec{z}_4$  and  $\vec{z}_5 < \vec{z}_3$  in Figure 2.1 and  $\vec{z}_5$  is found to be the dominant solution over any other alternative solutions to be generated on the branch-and-bound tree. This situation rarely occurs in practice. The second method is to use some arbitrary cut-off vector. When an infeasible lower bound vector associated with a node is greater than the cut-off vector, the tree is fathomed at that node. The third method is to use some arbitrary weighting vector to combine the objective function values. This approach, of course, reduces the multiobjective branch-and-bound process to, at least at the time of termination, a single objective process and the tree is fathomed when the optimal combined objective value is identified.

As illustrated above, the multiobjective branch-and-bound process is conceptually more complex than the single objective process. The increase in computational burden of using this approach for multiobjective network flow analysis is, however, only moderate, due to the special properties of the formulation. The details of the multiobjective branch-and-bound method as applied to network-flow formulation will be discussed in Chapter IV.

### III. REVIEW OF SINGLE-OBJECTIVE BRANCH-AND-BOUND METHOD

#### A. Single-Objective Fixed-Charge Network-Flow Formulation

The wastewater facility regionalization problem introduced here belongs to a class of facility location problems represented by a fixed-charge network-flow formulation. The formulation was originally proposed by Nakamura and Brill (1977). The formulation involves an objective function and five types of constraints. The objective function is taken to be cost that is to be minimized, and this cost function is a concave function of wastewater flow. The concavity (economics of scale) can be approximated by a fixed charge and one or more piecewise linear segments.

The five sets of constraints are:

- 1) physical continuity constraint set;
- 2) slack introduction constraint set;
- 3) proper sequencing constraint set (nonlinear binary constraint set);
- 4) lower and upper bound constraint set; and
- 5) nonnegativity constraint set.

The physical continuity constraint set insures that the flow conservation is met at each regional facility and for the entire regional system. The slack introduction constraint set together with the proper sequencing constraint set insures that flow variables associated with linearized cost functions will assume values in proper sequence; that is, the fixed charge will be accounted for before the variable associated with the first linear piece will assume a non-zero value, etc. The lower and upper bound constraint set along with the slack introduction constraint set insures that activity flow variables will satisfy the physical limits imposed at each regional facility. The nonnegativity constraint set requires that all the variables are either positive or zero.

The entire formulation, involving cost approximations consisting of a fixed charge and one linear segment (Figure 3.1); is presented below. For the formulation involving more than one piecewise linear segment, refer to Brill and Nakamura (1978-a):

Objective Function:

Minimize:

$$z = \sum_{ij} C_{ij}^P \cdot f_{ij} + \sum_{ij} x_{ij} + \sum_j C_j^T \cdot q_j + \sum_j y_j \quad (3.1)$$

where the constants (upper case) and the variables (lower case) are:

$C_{ij}^P$  = unit cost of the linear approximation of the cost function for constructing the interceptor from location i to j (dollars/year/MGD),

$C_j^T$  = unit cost of the linear approximation of the cost function for constructing a plant at site j (dollars/year/m<sup>3</sup>/day),

$f_{ij}$  = linear piecewise variable for interceptor plant capacity from location i to location j (m<sup>3</sup>/day)

$q_j$  = linear piecewise variable for plant capacity at site j (m<sup>3</sup>/day),

$x_{ij}$  = fixed cost variable for constructing an interceptor from i location j (either 0 or  $FC_{ij}^P$ ) (dollars/year),

$y_j$  = fixed cost variable for constructing a plant at site j (either 0 or  $FC_j^T$ ) (dollars/year),

$FC_{ij}^P$  = fixed cost associated with constructing an interceptor from location i to location j (dollars/year), and

$FC_j^T$  = fixed cost associated with constructing a plant at site j (dollars/year).

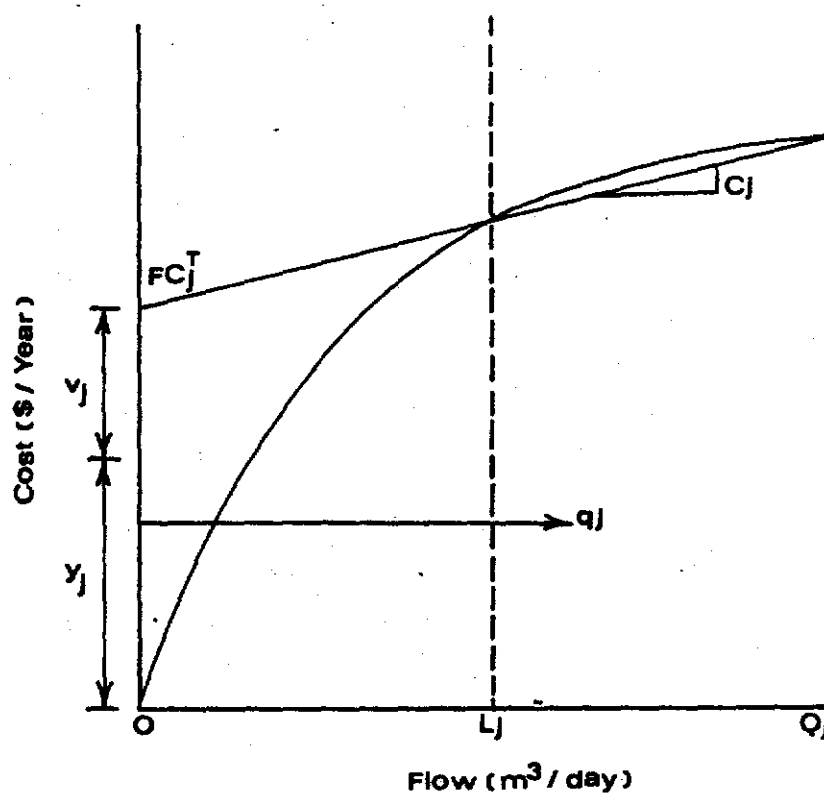


Figure 3.1 Piecewise Approximation of a Concave Cost Function with a Fixed-charge Component

Constraints Sets:

(1) Continuity:

$$\sum_{i \neq j} f_{ji} - \sum_{j \neq i} f_{ij} + \sum_{j \neq i} q_j = W_j \quad \forall j \quad (3.2)$$

where  $W_j$  is the waste flow generated at source  $j$  ( $m^3/day$ )

(2) Slack Introduction:

$$x_{ij} + u_{ij} = FC_{ij}^P \quad \forall i, j \quad (3.3)$$

$$y_j + v_j = FC_{ij}^P \quad \forall i, j \quad (3.4)$$

$u_{ij}$  = slack variable associated with  $x_{ij}$  (dollars/year),

$v_j$  = slack variable associated with  $y_j$  (dollars/year),

(3) Proper Sequencing:

$$f_{ij} \cdot u_{ij} = 0 \quad \forall i, j \quad (3.5)$$

$$q_j \cdot v_j = 0 \quad \forall j \quad (3.6)$$

(4) Lower and Upper Bound:

$$f_{ij} \leq F_{ij} = 0 \quad \forall i, j \quad (3.7)$$

$$q_j \leq Q_j = 0 \quad \forall j \quad (3.8)$$

Where:

$F_{ij}$  = upper limit of variable  $f_{ij}$

$Q_j$  = upper limit of variable  $q_j$

(5) Nonnegativity:

$$f_{ij}, q_j, u_{ij}, v_j, x_{ij}, y_j \geq 0 \quad (3.9)$$

Additional constraints may also be added to prevent split flows and two way flows. For a rigorous discussion of the additional constraints that may be included in this formulation, refer to Nakamura and Brill (1977).

If a problem involves a regionalization of residual sludge management as well as wastewater treatment, the notations used for interceptors can also represent sludge transport routes, and those used for plants can represent sludge management facilities as described in the example application of Chapter V.

Other formulations of the single-objective fixed-charge network-flow problem have been proposed. For example, Converse (1972), Joeres, et al., (1974), and Lauria (1975) all proposed mixed-integer programming formulations and the use of a mixed-integer programming solution method. Jarvis, et al., (1975), also suggested a mixed-integer formulation but proposed a fixed-charge network-flow solution method. Although the fixed-charge network-flow formulation and the mixed-integer formulation can be considered mathematically equivalent, the former appears to be particularly attractive in that it can explain the mathematical logic of generating and comparing alternative plans based on the branch-and-bound concept. Also, as discussed in Nakamura and Brill (1977), the general mixed-integer solution method is not designed to provide many feasible alternative solutions.

The analysis procedure of the above formulation is described in the next section.

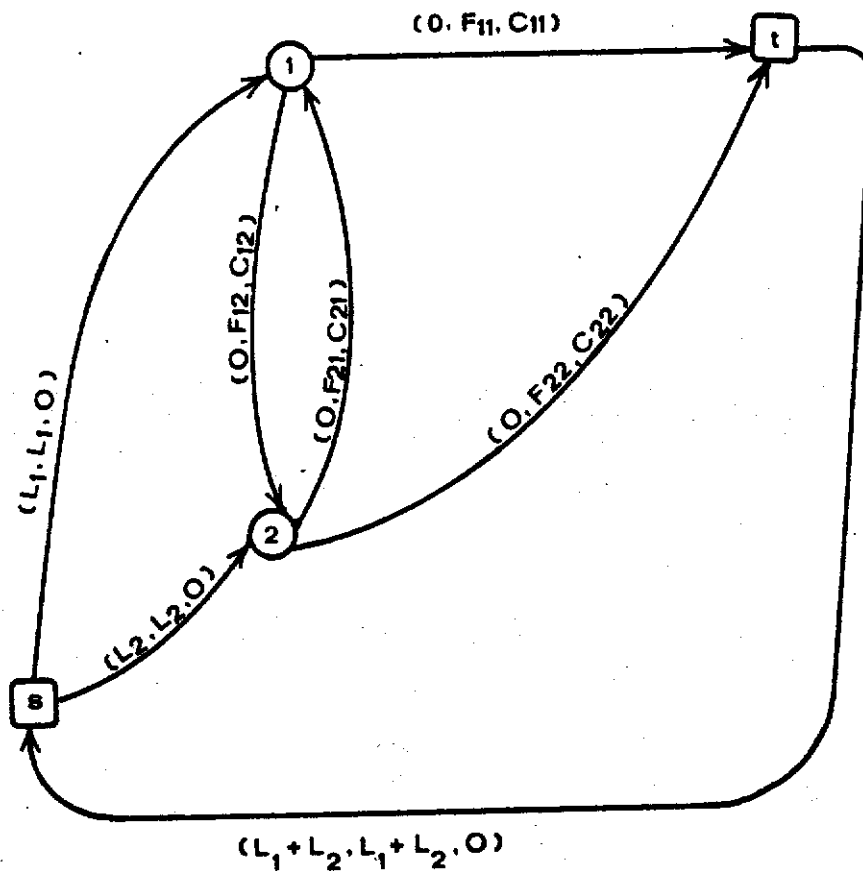
#### B. Analysis Procedure of Single-Objective Fixed-Charge Network-Flow Problems

By definition, the solution to the mathematical formulation presented in the previous section ought to be the set of values assigned to the activity variables which gives the least overall objective function value, i.e., the minimum cost. Since the intent here is not to solve the formulation to identify the least cost solution but to exploit the formulation to generate and compare alternative plans, the term analysis procedure is used rather than solution procedure.

The mathematical logic which must be satisfied for the proper sequencing of activity variables, which was described in the previous section, suggests the branch-and-bound process as an analytical procedure. The objective function (3.1) and the constraint sets, (3.2) through (3.4), and (3.7) through (3.9), form a linear programming formulation, and can be solved using a version of a network-flow algorithm or a linear programming code. If this portion of the problem is solved alone, it is quite likely that the nonlinear sequencing constraints, (3.5) and (3.6), would be violated. If so, this solution is mathematically infeasible to the original formulation, and the branching process on one of the violated nonlinear constraints would follow. This branching process generates a configuration termed the branch-and-bound tree.

The network representation of a two-source regionalization system is shown in Figure 3.2. For a discussion pertaining to the use of an Out-of-Kilter algorithm for solving a class of linear programming problems represented by the network-flow structure, readers may refer to Phillips and Jensen, (1971).

Before the details of the branch-and-bound computational procedure are described, an important observation can be made with regard to obtaining the costs of feasible alternative plans, which can be used in the bounding of the branch-and-bound tree. In the process of branch-and-bound computation the linear constraints of the forms  $(q_j = 0, v_j \geq 0)$  or  $(q_j \geq 0, v_j = 0)$  will be added sequentially to the original linear program to grow the tree down toward its base. If the solution to any of the new linear programming (i.e., the ones with a set of branching constraints added to the original linear programming problem) contain no violations of the nonlinear constraints (3.6), or equivalently, if the fixed charge associated with each of the fixed-charge linear cost approximations (Figure 3.1) is properly accounted for, then the solution is feasible, though not necessarily the least-cost. The total cost, or the sum of the objective function value of the original linear programming problem and the appropriate fixed charges, is used for bounding the branch-and-bound tree.



Arc Data = ( Lower Limit, Upper Limit, Objective Function Approximation )

○ = Point Sources and Candidate Sites

□ **s** = Dummy Sink

$F_{11} \leq L_1 + L_2$

$F_{12} \leq L_1 + L_2$

□ **t** = Dummy Source

Figure 3.2 Network Representation of a Two-Source System Using One-Piece Linearization



The above observation is extremely important not only because the feasible costs are available for bounding the tree from the early part of the branch-and-bound process, but also because each linear program solution represents a feasible alternative plan which can be evaluated for its merits other than cost in an imputed value analysis (Nakamura and Brill, 1979) and in the multiobjective branch-and-bound method described in Chapter IV.

The details of the computational procedure are now described next. First, the initial linear programming problem which consists of the objective function (3.1), and is solved with no branching constraints being included. The solution to this subprogram provides the objective function  $z_1$  (see Figure 3.3), a lower bound on the least-cost solution. The solution to the linear program problem would have many violations of the nonlinear binary constraints, (3.5) or (3.6), as described above. Assume that the variable  $q_j$  turned out to be nonzero and yet the associated fixed charge  $FC_j$  was not accounted for in the objective function. The violated nonlinear constraint is, therefore,  $q_j \cdot v_j = 0$ . On one branch a new linear programming problem, which consists of the same objective function and the same constraint set as the initial programming problem plus  $q_j = 0$ , is defined. The solution to this problem would give the objective function value associated with the new branch node. On the other branch another linear programming problem, which includes the new constraint  $v_j = 0$ , must be defined. Since the constraint  $v_j = 0$  is equivalent to adding the fixed charge  $FC_j$  to the objective function ( $q_j > 0$  is implied, but it is a redundant constraint since the initial linear programming solution already had a nonzero value for  $q_j$ ), the solution to this new linear programming problem can be given by simple correction of the objective function value of the parent node. This step is called an inspection step. As a matter of fact, the above reasoning applies to the branching of the form  $v_j = 0$  (or  $u_{ij} = 0$ ), and as shown in Figure 3.3, the entire string of nodes, 2, 3, ..., L, can be generated by inspection along the limb of the tree originating from the branch-one side of node 1. It is always possible to evaluate one of the two branches from each node by inspection.

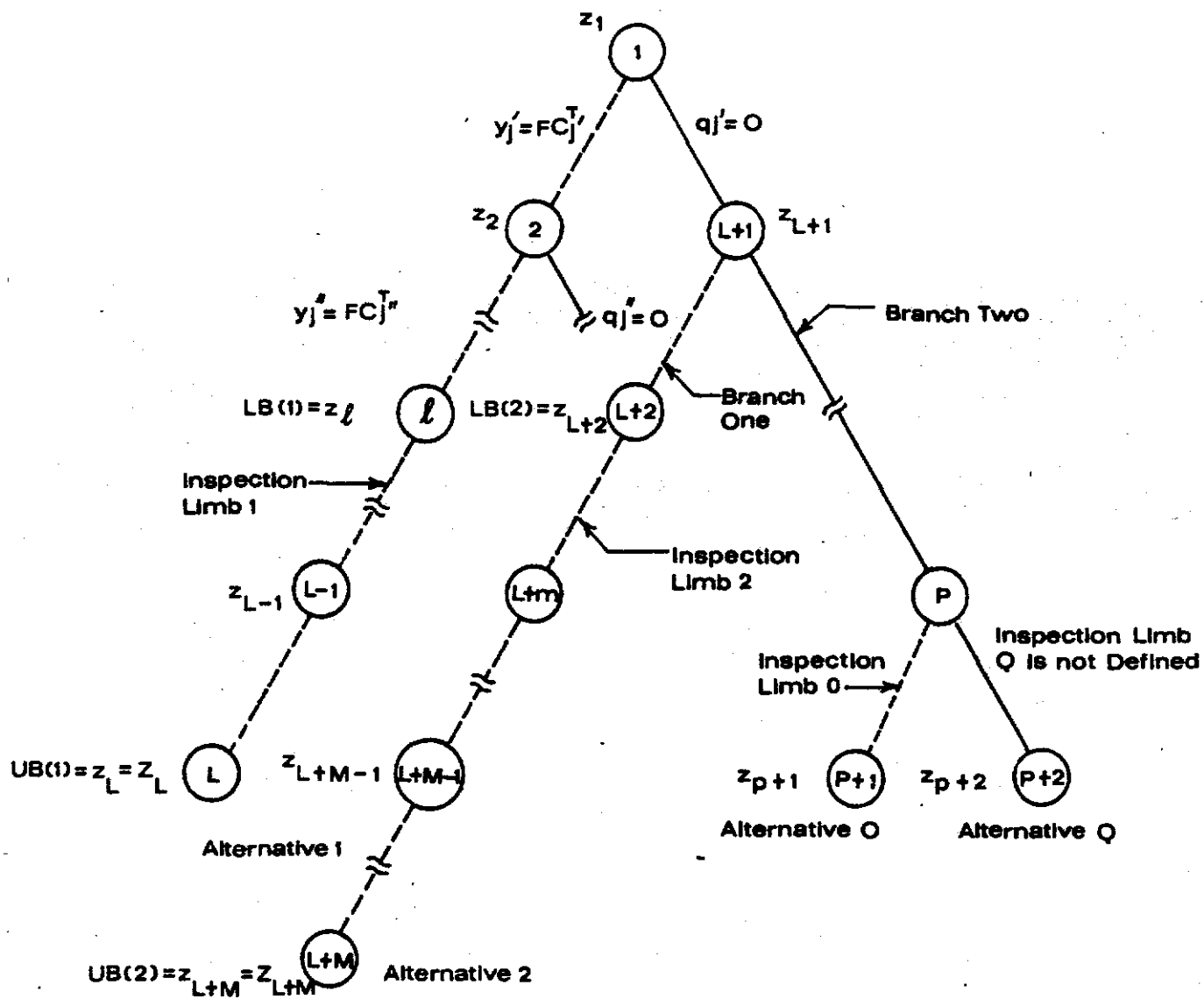


Figure 3.3 Schematic Representation of a Single-Objective Branch-and-Bound Tree

Branch one from the left in Figure 3.3 will always yield  $y_j = FC_j^T$  (or  $x_{ij} = FC_{ij}^P$ ), and the value of  $q_j$  (or  $f_{ij}$ ) will be unchanged. Therefore, this limb indicates that the solution to the initial subprogram contains L-1 violation of the nonlinear sequencing constraints. The fixed-charge components corresponding to each of the violated constraints are added to  $z_1$ , one at a time, to determine the objective function values,  $z_2, z_3, \dots, z_L$ . The terminal node, L, provides a feasible alternative plan as long as none of the  $Q_j$ 's or  $F_{ij}$ 's are limiting. The node value,  $z_L$ , will assume  $Z_L$  (i.e.,  $z_L = Z_L$ ) to indicate the objective function value for the feasible alternative generated at this terminal node. Note that this feasible plan identified at the bottom of the limb is, in fact, the same feasible alternative plan obtained by converting directly the initial linear programming solution by adding all of the fixed charges at once.

The sequence of the branching variables along a limb of a tree can be based on the magnitude of the fixed charge. Adding fixed charges in descending order may help in pruning the branches closer to node 1 (vertex), since fewer branches may be needed before an intermediate node cost exceeds any cost limit specified in the branch-and-bound process. However, the generation of nodes beyond those necessary for the completion of the branch-and-bound process does not decrease the computational efficiency significantly, as the necessary fixed charges are simply added in the inspection steps.

The Figure 3.3 shows also the branching to the right with a constraint of the form  $q_j = 0$ . Note that a string of nodes,  $L + 1, L + 2, \dots, L + M$ , is generated along the limb of the tree originating from branch one side to the left of node  $L + 1$ . A feasible alternative is identified at the terminal node,  $L + M$  ( $z_{L+M} = Z_{L+M}$ ). The branching procedure follows the rule of "branching from the lowest infeasible lower bound", and the branching continues as shown in Figure 3.3. The series of solving one subprogram by an optimization algorithm, carrying out a string of inspection steps, and identifying a feasible alternative can be repeated until a given stopping rule is satisfied.

The stopping rule depends on the purpose of the application of the method. If the objective is to obtain the least cost solution, then the branch-and-bound process may be terminated when all of the infeasible node costs exceed the cost of a feasible alternative; i.e., when the lowest upper bound generated in the branching process is exceeded by all of the infeasible lower bounds of the branch-and-bound tree. If the objective is to generate alternatives with a given cost limit, then the branching process can be continued until all of the infeasible-node costs exceed the cost limit.

At any point in the branch-and-bound process there is at the most as many inspection limbs as the number of feasible alternatives generated (some linear programming subproblem solutions may happen to satisfy all of the nonlinear binary constraints, and, thus, themselves become feasible alternatives without providing an inspection limb). Each inspection limb has two nodes of special significance. One is the node which is closest to the vertex and has not yet extended the branch to the right (for example, node  $L + 2$  of inspection limb 2 in Figure 3.3), and the other is the node at the bottom of the tree providing a feasible alternative (for example, node  $L + M$  of inspection limb 2 in Figure 3.3). The objective function associated with the former node is denoted as  $LB(r)$  and the one associated with the latter node is denoted as  $UB(r)$  for a given inspection limb  $r$  (for example,  $LB(2) = z_{L+2}$  and  $UB(2) = z_{L+M} = Z_{L+M}$  in Figure 3.3). The case in which a linear program subproblem solution happened to satisfy all of the nonlinear constraints (node  $P + 2$  in Figure 3.3) may be considered to be a special case above where a dummy inspection limb is defined so that the relationship,  $LB(r) = UB(r)$ , holds (for example,  $LB(Q) = UB(Q) = z_{P+2} = Z_{P+2}$ ). Notation  $r$  may now represent a given feasible alternative among the total of  $R$  feasible alternatives.

Let the minimum of  $LB(r)$ ,  $r = 1, 2, \dots, R$  be denoted as  $z_{\min}$  and the minimum of  $UB(r)$ ,  $r = 1, 2, \dots, R$  be denoted as  $Z_{\min}$ . Then the following relationship holds:

$$z_{\min} \leq z^* \leq Z_{\min} \leq UB(r) \quad \forall r \quad (3.10)$$

$$z_{\min} \leq z^{**} \quad (3.11)$$

where:

$Z^*$  = the least-cost solution, and

$Z^{**}$  = cost limit or cut-off value

If the objective is to find the least-cost solution, the branch-and-bound process terminates when:

$$Z^* = Z_{\min} = Z_{\min} \leq LB(r) \leq UB(r) \quad \forall r \quad (3.12)$$

If alternative solutions are generated, the process terminates when:

$$Z^{**} \leq Z_{\min} \leq LB(r) \quad \forall r \quad (3.13)$$

Since the concern here is to generate feasible alternatives for further consideration, the branch-and-bound process is terminated by the stopping rule dictated by equation (3.13).

In general, the branch-and-bound tree contains many inspection limbs as shown in Figure 3.3 at any given stage of the procedure. Many of the nodes, e.g., nodes  $l + 1$  through  $L$  in the figure, have not become candidates for branching since complete branching has not yet been performed on the preceding node, i.e. node  $l$ . Since these nodes are not actively involved in the branch-and-bound process, they are called inactive nodes. Also, the corresponding part of the inspection limb is called an inactive portion of the tree, and extra inspection steps are called inactive inspection steps. Other nodes obtained by inspection, such as node 2, are in the active portion of the tree. The importance of inactive nodes lies in their potential for becoming active and leading to additional growth of the tree to generate more alternative solutions.

The branch-and-bound procedure, therefore, is readily described by the flow chart as shown in Figure 3.4. The structure of the tree conveniently enables information associated with each feasible alternative to be readily retrievable. An illustrative example of this single-objective branch-and-bound method is presented in Brill and Nakamura (1978-a).

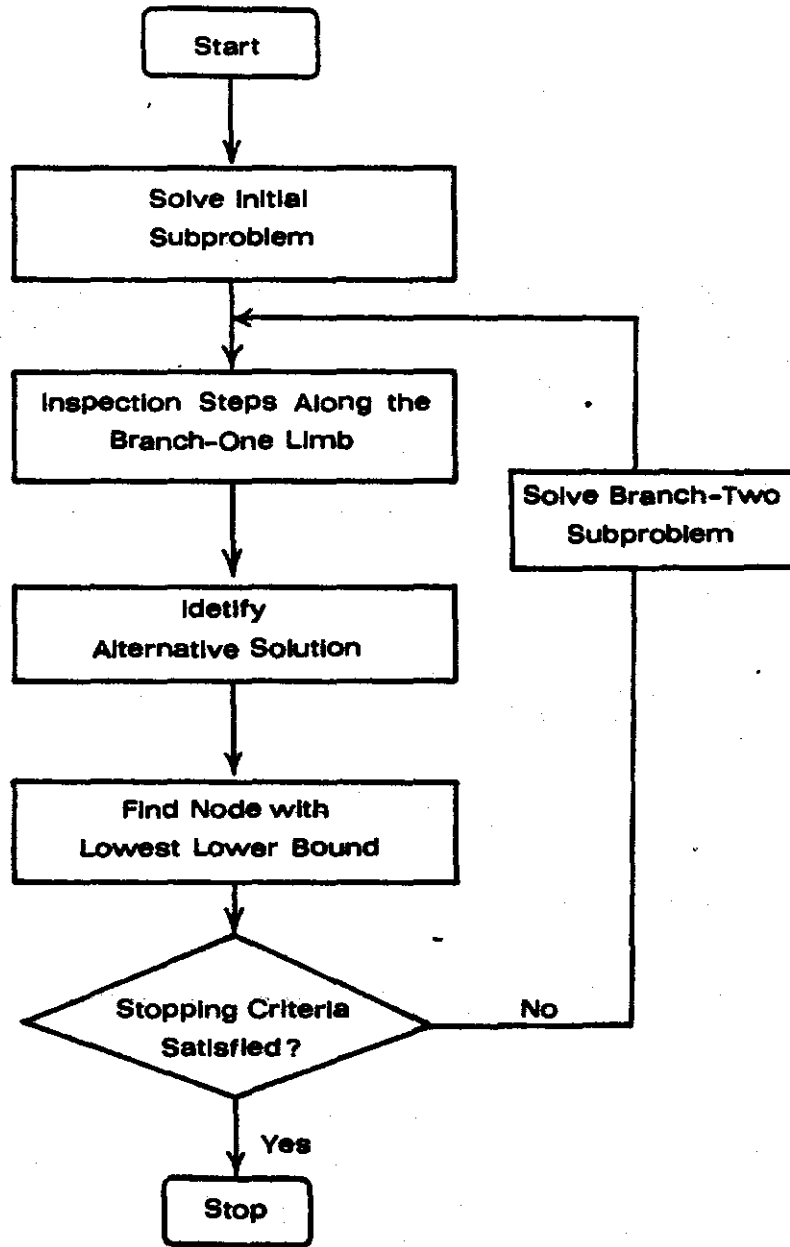


Figure 3.4: Flow Chart for the Single-Objective Branch-and-Bound Method

#### IV. MULTIOBJECTIVE BRANCH-AND-BOUND METHOD

##### A. Multiobjective Fixed-Charge Network-Flow Formulation

The single-objective fixed-charge network-flow formulation can be expanded to include multiple objectives. In the single-objective formulation described in Section III-A, it was cost which has to be minimized. Consider a situation where, besides cost, some other minimizing objective function are to be included in the mathematical formulation as functions of flow through the same network as the one for the single-objective analysis. The set of activity variables, representing the flows assigned to the arcs in the network, remains unaffected (i.e., they are the unknowns to be determined in the multiobjective programming method). The physical continuity constraint set, therefore, remains unchanged. The nonnegativity of flow variables must also be maintained just as in the single-objective formulation.

Since there is more than one objective to consider, there will be as many sets of slack introduction constraints and proper sequencing constraint sets as the number of objective functions introduced in the formulation. The number of constraints in neither the slack introduction constraint sets nor the proper sequencing constraint sets need to be constant. This is so because those objective functions associated with an arc in the network which are independent of the amount of flow assigned to them, may be assigned the value of zero and, therefore, there is no need to introduce slack introduction and proper sequencing constraints. The number of constraints in a particular slack introduction constraint set (pertaining to a particular objective function) must, however, be the same as the number of constraints in the corresponding proper sequencing constraint set (pertaining to the same objective function). This is so because a slack introduction constraint and the corresponding proper sequencing constraint work as a pair, as described for the single-objective formulation.

Therefore, for each objective to be considered for a given network-flow problem the formulation will consist of an objective function and five constraint sets (physical continuity, slack introduction, proper sequencing, lower and upper bound, and nonnegativity). The generalized mathematical formulation is presented below and the analysis procedure is presented in the next section.

Objective Functions:

Minimize:

$$\begin{aligned}
 z^1 &= \sum_{ij} c_{ij}^1 \cdot f_{ij} + \sum_{ij} x_{ij}^1 & \forall i, j \in S^1 \\
 z^2 &= \sum_{ij} c_{ij}^2 \cdot f_{ij} + \sum_{ij} x_{ij}^2 & \forall i, j \in S^2 \\
 &\vdots \\
 z^N &= \sum_{ij} c_{ij}^N \cdot f_{ij} + \sum_{ij} x_{ij}^N & \forall i, j \in S^N \quad (4.1)
 \end{aligned}$$

where:

$c_{ij}^n$  = unit value of the linear approximation for objection function n, each associated with the arc (i,j) connecting node i to node j in the network,

$f_{ij}$  = linear piecewise capacity variable associated with the arc (i,j) connecting node i to node j in the network,

$x_{ij}^n$  = fixed charge variable associated with objective function n, each associated with the arc (i,j) connecting node i to node j in the network,

N = the number of objective functions

$S^N$  = the set of arcs for which the n<sup>th</sup> objective function is defined.



Constraint Sets:

(1) Continuity:

$$\sum_{j \neq i} f_{ji} - \sum_{i \neq j} f_{ij} = W_j \quad \forall j \quad (4.2)$$

(2) Slack Introduction:

$$\begin{aligned} x_{ij}^1 + u_{ij}^1 &= FC_{ij}^1 & \forall i, j \in S^1 \\ x_{ij}^2 + u_{ij}^2 &= FC_{ij}^2 & \forall i, j \in S^2 \\ &\vdots & \\ x_{ij}^N + u_{ij}^N &= FC_{ij}^N & \forall i, j \in S^N \end{aligned} \quad (4.3)$$

where:

$W_j$  = flow generated at source  $j$ ,

$u_{ij}^n$  = slack variable associated with  $x_{ij}^n$ , if objective  $n$ , and

$FC_{ij}^n$  = fixed charge associated with variable  $x_{ij}^n$  of objective  $n$

(3) Proper Sequencing:

$$\begin{aligned} f_{ij} \cdot u_{ij}^1 &= 0 & \forall i, j \in S^1 \\ f_{ij} \cdot u_{ij}^2 &= 0 & \forall i, j \in S^2 \\ &\vdots & \\ f_{ij} \cdot u_{ij}^N &= 0 & \forall i, j \in S^N \end{aligned} \quad (4.4)$$

(4) Upper Bound:

$$f_{ij} \leq F_{ij} \quad \forall i, j \quad (4.5)$$

where:

$F_{ij}$  = the upper limit of variable  $f_{ij}$ .

(5) Nonnegativity:

$$f_{ij}, x_{ij}^1, \dots, x_{ij}^N, u_{ij}^1, \dots, u_{ij}^N \geq 0 \quad \forall i, j \quad (4.6)$$

Each objective may be expressed using one or more piecewise linear segments, with or without a fixed charge. The value of the  $n^{\text{th}}$  objective function associated with arc  $(i, j)$ , therefore, could be expressed using one of the following approximation methods.

$$z_{ij}^n = c_{ij}^n \cdot f_{ij} \quad (\text{strictly linear approximation}) \quad (4.7)$$

$$z_{ij}^n = c_{ij}^n \cdot f_{ij} + x_{ij}^n \quad (\text{linear fixed-charge approximation}) \quad (4.8)$$

$$z_{ij}^n = \sum_{ijk} c_{ijk}^n \cdot f_{ijk} + \sum_{ijk} x_{ijk}^n \quad (\text{multiple linear fixed-charge approximation with } k \text{ components}) \quad (4.9)$$

The overall objective function value  $Z^n$  equals  $\sum_{ij} z_{ij}^n$ .

The formulation presented above and the analysis procedure described below is for a case involving minimization of multiple objective functions, each of which consists of one linear segment and the associated fixed charge (Equation 4.8 above). If some of the objective functions are strictly linear (Equation 4.7 above), then the corresponding fixed charges are zero and it is a special situation of the first case. The third case (Equation 4.9) is actually a combination of the first two cases and the basic analysis procedure presented here applies equally well but with increased computational burden.

## B. Analysis Procedure of Multiobjective Fixed-Charge Network-Flow Problems

The analysis procedure described below is for a situation in which  $S^1 = S^2 = \dots = S^N = S$ , or all of the objective functions are defined for the same set of arcs,  $S$ . When  $S^1, S^2, \dots, S^N$  are not the same set  $S$ , one can redefine the set  $S$  such that  $S = S^1 \cup S^2 \cup \dots \cup S^N$ . One must then include additional dummy variables with their coefficients having a value of zero to allow  $S$  to be the common set.

In single-objective branch-and-bound analysis it was possible to generate the optimum solution as well as many feasible alternatives whose objective function values are above (or below) a given cut-off value. Basically the same approach is taken in the multiobjective case, except that the optimal solution cannot be determined in the multiobjective analysis when preference information is not a priori available. It is possible to generate, however, a set of alternative solutions within a given cut-off vector which approximates the complete noninferior set (Cohon, 1978, p.69).

The details of the analysis procedure are presented in the following four subsections.

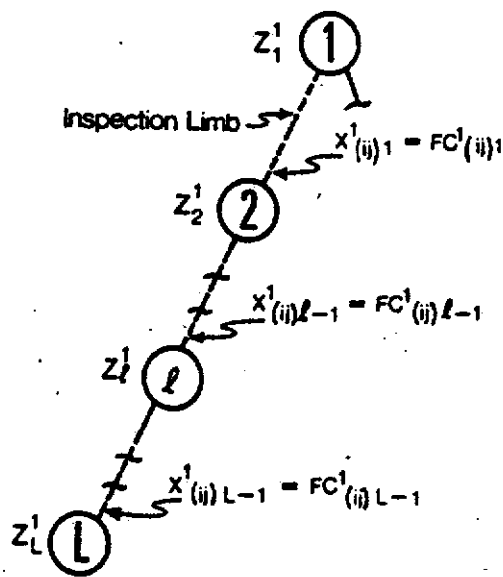
### 1) Constructing the Branch-and-Bound Tree for Objective 1

The analysis procedure begins with selecting arbitrarily one of the objectives and solving its initial linear programming problem. Let this objective be labeled objective 1. The mathematical formulation for this decomposed problem DCP-1, is defined by equations(4.1)through(4.6), except that the part of the formulation which relates to the remaining  $N-1$  objectives is ignored momentarily. The analysis procedure for DCP-1 is identical to that proposed for the single-objective problem described in the previous chapter.

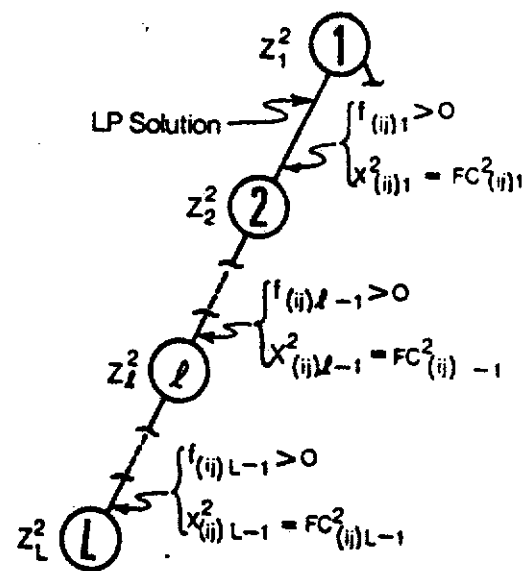
The objective function and the continuity, upper bound, and non-negativity constraint sets form a linear programming formulation, which can be solved using a network-flow algorithm or a linear programming code. If this portion of the problem is solved alone, it is quite likely that the nonlinear sequencing constraint set (4.4) would be violated. If so, this solution is mathematically infeasible to the original formulation, and the branching process on one of the violated nonlinear constraints would follow. This branching process generates a branch-and-bound tree.

As described previously, the computational burden to grow the tree is reduced significantly since it is possible to identify by inspection all of the node values associated with one of the tree limbs stemming from the vertex of the tree at which the original linear programming problem is solved.

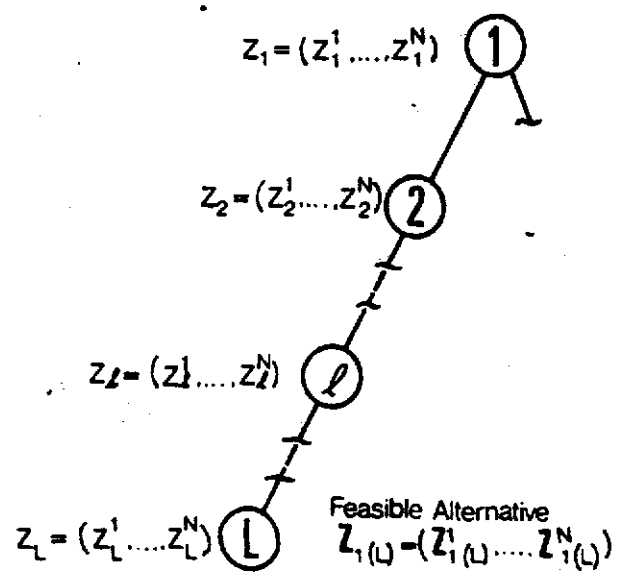
For example, suppose variable  $f_{(ij)1}$  was the first branching variable chosen out of  $(L-1)$  variables which assumed a positive value in the solution to the original linear programming problem (The number of nodes created along the limb of the tree in this case is  $L$  as shown in Figure 4.1). The branching from the vertex, node 1 in Figure 4.1-(a), is initiated based on the binary constraint,  $f_{(ij)1} \cdot u_{(ij)1}^1 = 0$ , or the first of the  $N$  constraints in equation constraint set (4.4). On one branch (Branch-one) the constraint  $(f_{(ij)1} \geq 0, u_{(ij)1}^1 = 0)$ , is imposed, and on the other branch (Branch-two) the constraint set,  $(f_{(ij)1} = 0, u_{(ij)1}^1 \geq 0)$ , is imposed. In the former case, however,  $f_{(ij)1} \geq 0$  is a redundant constraint since it was already positive in the solution to the original linear programming problem. Therefore, the second constraint  $u_{(ij)1}^1$  is set to zero, or simply  $x_{(ij)1}^1$  is set to  $FC_{(ij)1}^1$ . The new objective function value,  $z_2^1 = z_1^1 + FC_{(ij)1}^1$ , associated with node 2 of the branch-and-bound tree is thus obtained by inspection. The flow variables in the solution to the revised linear programming, had it been actually solved, would have exactly the same values as the ones in the original linear programming problem solution.



(a) First Inspection Limb of DCP-1 Branch-and-Bound Tree



(b) First Limb of DCP-2 Constraint Tree



(c) First Limb of Coupled Branch-and-Bound Tree

Figure 4.1: First Limbs of DCP-1, DCP-2 and Coupled Trees

Only the objective function value would have increased by  $FC_{(ij)1}^1$ .

At node 2, (L-2) nonlinear binary constraints are still in violation. Variable  $f_{(ij)2}$  is taken to be the next branching variable and the inspection step proceeds in exactly the same way as in the previous case to reach node 3. The objective function value associated with node 3, then,

$$z_3^1 = z_2^1 + FC_{(ij)2}^1 \quad (4.10)$$

The entire limb of the tree can be constructed simply by adding, one at a time, fixed charges associated with flow variables violating the nonlinear binary constraint set (4.4) in the original linear programming problem. There will be as many inspection nodes as the number of nonlinear constraints violated, and a feasible alternative plan is identified at the bottom of the inspection limb. Note at this point that the feasible alternative identified at the bottom of an inspection limb has its flow assignment completely specified. The objective function values associated with this feasible alternative, or the components of the feasible upper bound vector, are expressed as:

$$z_1^1(L) = (z_{1(L)}^1, z_{1(L)}^2, \dots, z_{1(L)}^N) \quad (4.11)$$

The subscript,  $1(L)$ , denotes that it is the first alternative plan identified on the branch-and-bound tree and it is located at the  $L^{\text{th}}$  node of the branch-and-bound tree. Recall at this point that although the lower bound on the objective function 1 is given  $(z_1^1)$ , there is no information on the lower bounds of other objective functions.

The branching from the vertex with the other constraint set,  $(f_{(ij)1} = 0, u_{(ij)1}^1 \geq 0)$ , cannot be performed by inspection. The new linear programming problem which consists of the original linear programming plus a constraint,  $f_{(ij)1} = 0$  (note that  $u_{(ij)1}^1 \geq 0$  is a redundant constraint to the original linear programming problem), must be actually solved using a network-flow algorithm to identify the new node value. The solution to this

new linear programming problem may contain a different set of flow variables in violation of nonlinear binary constraints, and, then, inspection steps may be carried out from the node just the same way as described above.

## 2) Integration of Remaining Objectives into the Branch-and-Bound Tree

Now a second objective is selected and labeled as objective 2. This objective function can be superimposed on the branch-and-bound tree using the coupling procedure described below. The mathematical formulation involving the second objective and the associated constraint set is also imbedded in the multiobjective formulation presented in the previous section. In this case, the objectives 1, 3, ..., N are ignored in the formulation involving equations (4.1) through (4.6). The solution to the linear programming portion of this problem, DCP-2, gives a lower bound on the optimal solution with respect to objective 2, and it is denoted  $z_1^2$ , as shown in Figure 4.1-(b). The inspection limb associated with objective 2, however, will not be constructed. Instead, attention is directed to the slack introduction constraint set (4.3), and the nonlinear binary constraint set (4.4).

The nonlinear binary constraint for objective 1 for a particular arc (i,j) is  $f_{ij} \cdot u_{ij}^1 = 0$ , while the corresponding constraint for objective 2 for the same arc is  $f_{ij} \cdot u_{ij}^2 = 0$ . Note here that variable  $f_{ij}$  appears in both equations. The two constraints imply that if  $f_{ij}$  assumes a non-zero value, then  $u_{ij}^1$  and  $u_{ij}^2$  both must be simultaneously zero. This implies that  $x_{ij}^1 = FC_{ij}^1$  and  $x_{ij}^2 = FC_{ij}^2$  simultaneously. If, on the other hand,  $f_{ij}$  is zero, then  $u_{ij}^1 = FC_{ij}^1$  (i.e.,  $x_{ij}^1 = 0$  because it is a minimization problem) and similarly  $u_{ij}^2 = FC_{ij}^2$  (i.e.,  $x_{ij}^2 = 0$ ). In other words, the variable,  $f_{ij}$  works as a coupling variable for the two decomposed problems involving objective functions 1 and 2.

The above observation leads to the following procedure for constructing the coupled tree limbs for the multiobjective branch-and-bound process. The inspection limb shown in Figure 2-(a) implies that there were  $L-1$  non-zero flow variables ( $f_{(ij)1}, f_{(ij)2}, \dots, f_{(ij)L-1}$ ) in the solution to the original linear programming problem. There are, therefore,  $L-1$  coupling computations to be performed for the complete coupling of the two limbs. Since the limb of the tree associated with the DCP-2 is created by the constraints of the form  $f_{ij} \cdot u_{ij}^2 = 0$  for the same  $L-1$  arcs, it is named a constraint tree (see Figure 4.1-(b)). The additional constraint set associated with the first branch along this limb would be  $f_{(ij)1} > 0$ , and  $x_{(ij)1}^2 = FC_{(ij)1}^2$ , the constraint associated with the second branch along this limb would be  $f_{(ij)2} > 0$  and  $x_{(ij)2}^2 = FC_{(ij)2}^2$ , and so on. The linear programming problem (of DCP-2) associated with the first branch, therefore, consists of the original linear programming problem plus the first set of branching constraints above. The one associated with the second branch consists of the linear programming problem associated with the first and the second set of branching constraints above.

Now, the coupling of the tree limbs for the first two decomposed problems is completed. Exactly the same procedure follows for the third, fourth, ..., and  $N^{\text{th}}$  decomposed problems involving the third, fourth, ..., and  $N^{\text{th}}$  objectives, respectively. There are  $N$  values of  $N$  decomposed problems associated with each of the  $L$  nodes which make up the limb of the coupled branch-and-bound tree (Figure 4.1-(c)). Note that the relationships  $z_{1(L)}^1 = z_L^1, z_{1(L)}^2 \geq z_L^2, \dots, \text{ and } z_{1(L)}^N \geq z_L^N$  hold.

The second branch must now be extended from node 1 of the branch-and-bound tree associated with DCP-1. The new linear programming problem consists of the original linear programming problem plus  $f_{(ij)1} = 0$ . Also, the second branch must be extended from node 1 of each of the  $N - 1$  constraint trees associated with DCP-2, DCP-3, ..., DCP- $N$ , respectively.



For example, the new DCP-2 linear programming problem associated with the second branch must consist of its original linear programming problem plus  $f_{(ij)1} = 0$ . Similar operations must be performed for DCP-3, ..., DCP-N.

If the solution to the new linear programming problem of DCP-1 contains  $M - 1$  violations of nonlinear binary constraints, then there will be  $M - 1$  inspection steps associated with the second inspection limb of DCP-1. Construction of the second limb of the coupled branch-and-bound tree (extending from node  $L + 1$  to node  $L + M$  in Figure 4.2) proceeds in just the same way as construction of the first limb (extending from node 1 to node  $L$  in Figure 4.2). There will be  $N$  objective function values associated with each of the  $M$  nodes along the second limb of the coupled branch-and-bound tree. The tree thus constructed is henceforth referred to as the multiobjective branch-and-bound tree. If the branch-and-bound process were to continue from this tree, the branching must be performed from either node 2 or from node  $L + 1$ . The choice may be made, for example, based on one of the objective function values (e.g. take  $L + 1$  as the next branching node if objective 1 is chosen and if the relationship,  $z_{L+1}^1 < z_2^1$ , holds). These two nodes are called the infeasible lower bound nodes, and  $z_2^1$  and  $z_{L+1}^1$  are called the infeasible lower bounds. Note at this point if the relationship,  $\vec{z}_{2(L+M)} < z_2$  holds, then alternative 2 would dominate any potential alternatives which could be generated in the portion of the coupled branch-and-bound tree under node 2. Similarly, if the relationship,  $\vec{z}_{1(L)} < \vec{z}_{L+1}$ , holds, then none of the alternatives which could be generated in the portion of the tree under node  $L + 1$ , including the already identified alternative 2, would dominate alternative 1. In this case, the branch-and-bound process would continue from node 2, creating node  $L + M + 1$  at the edge of the branch extending to the right of node 2.

The multiobjective branch-and-bound process continues until one of three stopping rules is satisfied. The details of each of the three rules will be presented later.

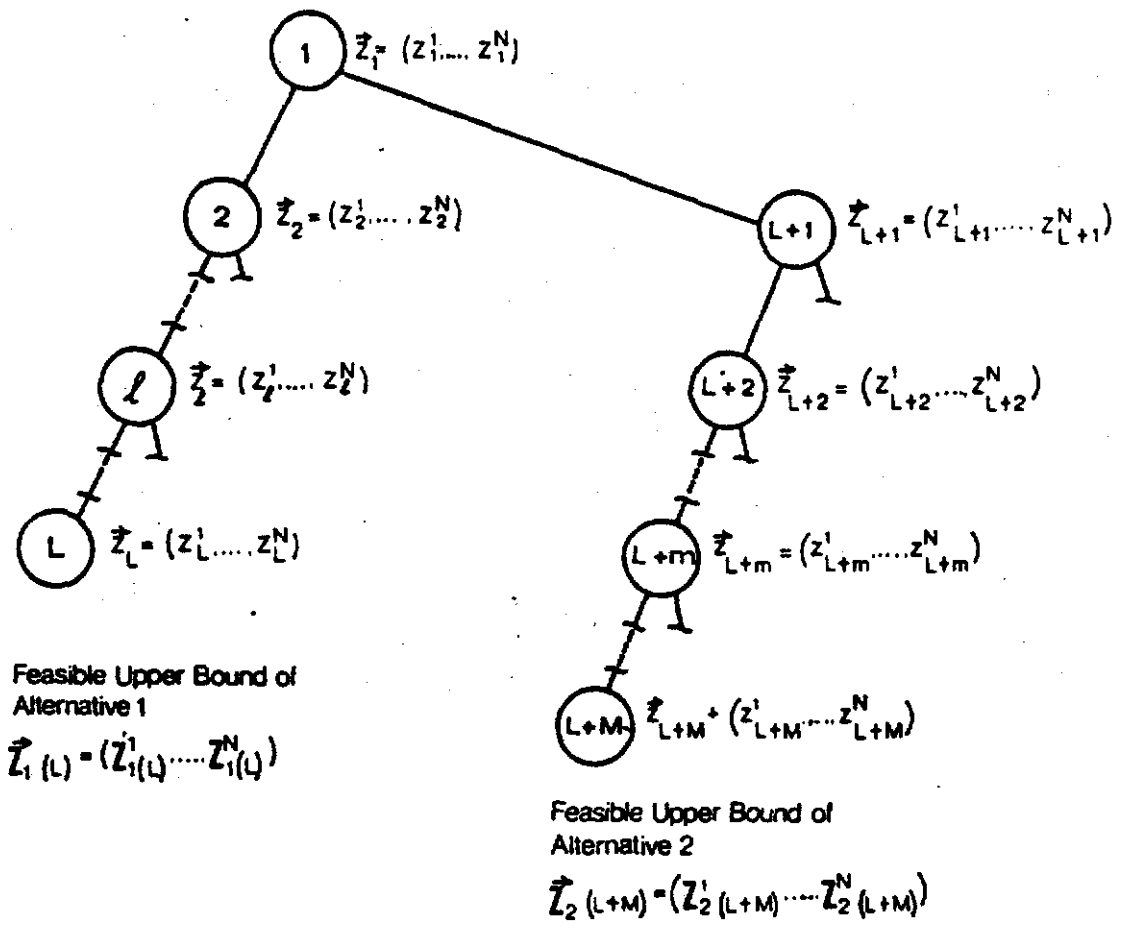


Figure 4.2: First Two Limbs of Coupled Branch-and-Bound Tree

### 3. Relaxation of Coupling Constraints and the Multiobjective Branch-and-Bound Tree

For the complete coupling of respective limbs of the DPC-1 branch-and-bound tree and the DCP-2, DPC-3, ..., DPC-N constraint trees, many linear programming subproblems need to be solved. If no inspection steps were used in solving DCP-2, DCP-3, ..., and DPC-N, the total number of linear programming subproblems (including the initial linear programming problem of DCP-1) to be solved for constructing the first limb of the coupled branch-and-bound tree (extending from node 1 to node L in Figure 4.2) would be  $1 + (N - 1) \cdot L$ , in addition to  $(L - 1)$  inspection steps needed for the DCP-1.

The computational burden for the coupling of the entire tree limbs, however, would be trivial if the objective function values associated with nodes 2, 3, ..., L of each of the N-1 constraint trees (DCP-2 tree, DCP-3 tree, ..., DCP-N tree) could be determined by inspection steps just as the objective function values of the original branch-and-bound tree (DCP-1 tree). In this case, the number of linear programming subproblems to be solved for constructing the first limb of the coupled tree (extending from 1 to node L in Figure 4.2) would be only N. In other words, one linear programming subproblem is required for each of the N objectives. The remaining  $N \cdot (L-1)$  computations would be simple additions of appropriate fixed charges. The relaxation of coupling constraints in the original formulation enables the use of inspection steps which can provide the lower bounds instead of the exact solutions of linear programming subproblems.

Consider first the two objective cases as shown in Figure 4.1. If the solution to the original DCP-2 linear programming problems had  $f_{(ij)1} > 0$ , then the first of the two constraints in the branching constraint set

$(f_{(ij)1} > 0, u_{(ij)1} = 0)$  becomes redundant. The fixed charge  $FC_{(ij)1}$  is added to  $z_1^2$ , to obtain  $z_2^2$  associated with node 2 of the constraint tree

shown in Figure 4.1-(b). If, however, the variable  $f_{(ij)1}$  was zero in the original DCP-2 linear programming problem, then the objective function value associated with node 2 would have to be obtained by solving the DCP-2 problem augmented with the above branching constraint set. Note that if the constraint  $f_{(ij)1} > 0$  is dropped from this constraint set, then the resulting DCP-2 problem is of the same form as that of the inspection step  $(FC_{(ij)1}^2)$  is added directly to  $z_1^2$  to obtain a lower bound of the lower bound  $z_2^2$ , defined as  $\underline{z}_2^2$ . This is equivalent to the relaxation of the coupling constraint. Similarly, the remaining nodes of the first limb of DCP-2 constraint tree may be assigned the objective function values  $(\underline{z}_3^2, \underline{z}_4^2, \dots, \underline{z}_N^2)$ , instead of  $(z_3^2, z_4^2, \dots, z_N^2)$ .

Exactly the same procedure can be applied to the remaining (N-2) decomposed problems, resulting in the construction of the first limb of the coupled branch-and-bound tree. The same procedure can be repeated for other limbs as well. For example, the first two limbs of the relaxed coupled branch-and-bound tree appear exactly like the ones shown in Figure 4.2 except that  $\vec{z}_k = (z_k^1, z_k^2, \dots, z_k^N)$  must be replaced by  $\vec{z}_k = (\underline{z}_k^1, \underline{z}_k^2, \dots, \underline{z}_k^N)$  for  $k = 1, 2, \dots, L + M$ . Fathoming of branching is performed in the same way as in the previous case. The relationship,  $\vec{z}_k \leq \vec{z}_k$ , holds for all  $k$  in the entire coupled branch-and-bound tree. Denoting a feasible alternative identified at node  $\pi$  as  $\vec{z}_{\pi}(\pi)$ , if the relationship  $\vec{z}_{\pi}(\pi) < \vec{z}_k$  holds, then the relationship  $\vec{z}_{\pi}(\pi) \leq z_k$ , must be hold, and the relaxed coupled branch-and-bound tree can be fathomed at node  $k$ . The tree is henceforth referred to as the relaxed multiobjective branch-and-bound tree. It is important to note that each of N linear subproblems to be solved to construct a multiobjective inspection limb provides, unless the flow assignment is infeasible, one feasible flow assignment which can be used to identify arithematically a feasible multiobjective alternative plan. (In actuality, it is possible to take advantage of the information obtained in the process of network-flow computations to identify many

feasible alternative plans. This will be described in Section V-B, Case 2 of the example applications of the proposed multiobjective analysis method.) In other words, associated with each inspection limb, there will be  $N$  feasible upper bounds rather than one as in the case of the single-objective branch-and-bound method, any one of which can be used to fathom the tree based on one of the three rules described in the next section.

#### 4. Fathoming the Multiobjective Branch-and-Bound Tree

The multiobjective branch-and-bound process continues until one of three stopping rules is satisfied. A detailed discussion on each of the methods is presented next.

##### Using Optimal Objective Function Values

Although the decomposed problems DCP-1, DCP-2, ..., DCP- $N$  can be solved individually (without coupling) to identify  $z^{1*}$ ,  $z^{2*}$ , ..., and  $z^{N*}$ , the multiobjective branch-and-bound process can identify them just as well. The  $n^{\text{th}}$  objective ought to have attained optimally in the multiobjective branch-and-bound process when the lowest of the feasible ( $n^{\text{th}}$ ) objective function value exceeds, for the first time, all of the infeasible ( $n^{\text{th}}$ ) objective function values on the tree regardless of the states of other objective function values. When such an optimum objective function value is found for each of all  $N$  objectives, one at a time but not necessarily sequentially, it is equivalent to fathoming the multiobjective branch-and-bound tree with  $\vec{z}^{**} = \vec{z}^* = (z^{1*}, z^{2*}, \dots, z^{N*})$ .

If all  $N$  objectives attain optimally simultaneously, then the feasible alternative must be the only noninferior solution to the problem. Such a situation is quite unlikely to occur.

In general, fathoming the tree with  $\vec{z}^{**} = \vec{z}^*$  guarantees the identification (by inspection) of all of the alternatives with at least one of the N objective function values optimal. There is a good chance, however, that this method of fathoming the multiobjective branch-and-bound tree would leave out a significant portion of the noninferior set uncovered since there may be many noninferior alternatives, none of whose objective function values is equal to its individually identified optimal value. These alternatives may never be explored as the multiobjective branch-and-bound tree might have been fathomed before they were generated.

Using Cut-off Vector,  $\vec{z}^{**} \neq \vec{z}^*$

To insure that a sufficient number of noninferior alternatives are generated, the cut-off value of each of the N objective functions must be raised adequately above its individual optimal value. Let  $\alpha^n \geq 1.0$  be a multiplier associated with the individual optimal value of the  $n^{\text{th}}$  objective function. The new cut-off value of the  $n^{\text{th}}$  objective is:

$$z^{n**} = \alpha^n \cdot z^{n*} \quad \text{for } n = 1, 2, \dots, N. \quad (4.12)$$

Then, the vector  $\vec{z}^{**}$  guarantees the identification of all of the noninferior solutions whose objective function values for  $n = 1, 2, \dots, N$ , are simultaneously less than or equal to  $\alpha^1 \cdot z^{1*}, \alpha^2 \cdot z^{2*}, \dots, \alpha^N \cdot z^{N*}$ , respectively.

If the components of the cut-off multiplier vector,  $\vec{\alpha}$ , are not restricted to be greater than unity, some unique situations can be considered. For example, consider the case where  $\alpha_\eta = 1$ , and  $\alpha_n = 0$ , for all  $n \neq \eta$ . In this case, the termination rule of the multi-objective branch-and-bound process depends only on the  $\eta^{\text{th}}$  objective, similar to the single objective analysis. This particular  $\vec{z}^{**}$  vector, therefore, guarantees to provide at least one feasible but quite likely to be nonoptimal. Again, in the process of identifying this particular

alternative, the multiobjective branch-and-bound tree is likely to provide other feasible alternatives by inspection steps which have to  $\eta^{\text{th}}$  objective greater than its optimum.

When  $0 < \alpha^n \leq 1.0$  is used and  $Z^{n*}$  is not a priori identified, it is convenient to use  $Z^{n**} = \alpha^n \underline{Z}^{n*}$  instead of  $Z^{n**} = \alpha^n Z^{n*}$ , where  $\underline{Z}^{n*}$  is the current best (least) feasible upperbound associated with the  $n^{\text{th}}$  objective.

### Using Weights Between Objectives

An arbitrary commensurate weighting vector,  $\vec{\beta} = (\beta^1, \beta^2, \dots, \beta^N)$ , such that  $\sum_{n=1}^N \beta^n = 1.0$ , and  $0 \leq \beta^n \leq 1.0$  for all  $n \in N$ , may be used for the purpose of fathoming branches of a multiobjective branch-and-bound tree. Sensitivity analysis of the weights on the ranking of alternatives is, however, an essential part of the evaluation process. As described later, once a multiobjective branch-and-bound tree is constructed, several different weight sets can be tested repeatedly on the same tree with little computational burden.

### 5. Selecting Infeasible Lower Bound Nodes for Branching

The last two approaches described in the previous section for fathoming the branch-and-bound tree may be employed also for selecting the infeasible lower bound nodes from which to branch next. The first method (cut-off vector of individual optimal objective function values) is equivalent to finding the dominance relationship among infeasible lower bounds with respect to a subset of  $N$  objectives. If the subset consists of only one objective, say, objective  $\eta$ , then node  $\emptyset$  is selected over node  $\lambda$  only if  $z_{\emptyset}^{\eta} \leq z_{\lambda}^{\eta}$  holds, regardless of the relationships among the other  $N - 1$  objectives. The same principle applies when more than one objective out of  $N$  objectives are selected for comparison, but as the number of objectives increases, fewer and fewer alternatives tend to exhibit a clear dominance relationship and the principle becomes more difficult to implement.

The second method (trade-off values between objectives) is to compare the weighted sums of objectives associated with all of the infeasible lower bound nodes. For example, if there are two infeasible nodes,  $\theta$  and  $\lambda$ , and if the vector,  $\vec{\beta} = (\beta^1, \beta^2, \dots, \beta^n)$ , is identified such that  $0 \leq \beta^n \leq 1.0$  and  $\sum_{n=1}^N \beta^n = 1.0$ , then choose node  $\theta$  as the branching node of the relationship,  $\vec{z}_1 \cdot \vec{\beta}^T \leq \vec{z}_2 \cdot \vec{\beta}^T$ , holds. Exactly the same qualification previously discussed holds for the role of the weighting vector. The vector is chosen simply to give a certain guideline, and the alternatives generated using this method are subject to further elaboration particularly with respect to the trade-off relationships among objective functions.

The method of evaluating generated alternatives are described in the next section.

### C. Comparing Generated Multiobjective Alternatives

The multiobjective branch-and-bound method described herein can be classified as a technique for generating an approximate noninferior set. It is developed as a problem specific technique, and the applicability of the method to problems which do not possess the network flow structure remains to be investigated. The problem specificity allows, however, some unique ways of comparing generated alternatives (Nakamura, 1979).

A potentially very powerful technique is an imputed value analysis method. It is developed for the single-objective branch-and-bound analysis, and described in detail in Brill and Nakamura (1978-a). In essence, it provides information efficiently and systematically on the imputed value of an individual facility or a group of facilities from an imputed value incidence matrix, or a matrix transformation of the branch-and-bound tree. For example, consider the situation, in which the best alternative with respect to objective X, which includes facility A and the one which does not include facility A, are identified in the process of the branch-and-bound analysis. Then the imputed value (IV) of facility A with respect to objective X is defined as follows:



$$IV(A) = Z^X * (A) - Z^{X*} * (A) \quad (4.13)$$

Where:

$Z^X * (A)$  = optimal objective function value with respect to objective X, among those of the feasible alternatives which include facility A, and

$Z^{X*} (A)$  = optimal objective function value with respect to objective X, among those of the feasible alternatives which do not include facility A.

Note that the basic approach described above for expressing an imputed value can be applied to any individual facility (or any group of facilities) as long as it is (they are) included in the branch-and-bound analysis. Also, the imputed value analysis can be performed with respect to any objectives included in the multiobjective branch-and-bound tree. For the details of a mathematical treatment on the transformation of a branch-and-bound tree into a matrix form and the computational procedures of imputed values, readers may refer to Nakamura and Brill (1977).

The matrix transformation of branch-and-bound tree (imputed value incidence matrix) can be used to perform other analysis on the alternatives generated on the multiobjective branch-and-bound tree. For example, the matrix, which contains information on all of the generated alternatives, may be reduced to include only those which are noninferior to each other so as to examine the sensitivity of the weighted sum of objective function values by the change in the distribution of weights in the trade-off vector, . Mathematically, this process can be accomplished as follows:

$$\begin{matrix} 1 \\ 2 \\ \cdot \\ \cdot \\ \cdot \\ M \end{matrix} \begin{bmatrix} 1 & 2 & \dots & N \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \text{Matrix A} \quad X \quad \begin{matrix} 1 \\ 2 \\ \cdot \\ \cdot \\ \cdot \\ N \end{matrix} \begin{bmatrix} 1 & 2 & \dots & T \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \text{Matrix B} = \begin{matrix} 1 \\ 2 \\ \cdot \\ \cdot \\ \cdot \\ M \end{matrix} \begin{bmatrix} 1 & 2 & \dots & T \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \text{Matrix C}$$

(4.14)

where:

**Matrix A** = Noninferior Alternative Matrix which consists of M alternatives generated so far and N objectives considered. The cell (m, n) contains the  $n^{\text{th}}$  objective function value of the  $m^{\text{th}}$  alternative.

**Matrix B** = Commensurate Trade-Off Matrix which consists of N objectives and T sets of trade-off vector,  $\vec{\gamma}$ . the cell (n, t) contains the trade-off value  $\gamma_n^t$  associated with the  $n^{\text{th}}$  objective in the  $t^{\text{th}}$  trade-off set.

**Matrix C** = Sensitivity Matrix which contains the normalized weighted sum of N objective functions associated with alternative M for trade-off set T.

The Matrix C reflects the changes in preference relationships due to changes in the values of trade-offs in each trade-off set. Note at this point the normalized weight vector used to fathom the multiobjective branch-and-bound tree is considered here simply as a tool to terminate the process. The analyst is not bound to the vector identified in the fathoming step except that the vector provided a unique set of noninferior alternatives based on unique weighting values. The crux of trade-off analysis lies in this sensitivity step.

## V. EXAMPLE APPLICATIONS OF THE MULTIOBJECTIVE BRANCH-AND-BOUND METHOD

In this chapter, the application of the multiobjective branch-and-bound method described in previous chapters will be studied, taking as examples two hypothetical regional wastewater and residual management systems.

### A. Example Application - Case 1

#### 1) Description of Hypothetical Problem

To illustrate the potential application of the multiobjective branch-and-bound method described above, a revised version of the example problem on regional wastewater system planning presented in Brill and Nakamura (1978-a) was used. The regional network of the revised problem includes the potential plants and interceptor routes included in the original problem plus the potential transportation routes and disposal sites of waste sludge. The regional network, consisting of seven waste sources, eleven interceptor sewer routes, three landfill sites, two land spreading sites and nine sludge transportation routes, is shown in Figure 5.1. Of the seven waste sources all but two (sites 2 and 6) are allowed to be potential treatment plant sites, each discharging its effluent to its nearby receiving stream.

The three objectives of cost, water quality impact and land use impact were to be minimized in the multiobjective analysis. The cost objective expressed in dollar/year includes construction and operation and maintenance costs for treatment plants and pipes (Deiningger and Su, 1971) plus sludge transportation costs (EPA, 1977). The water quality impact, in dimensionless units, reflects the degradation in water quality at the discharge points using BOD as an index (Dee, et al., 1972). The land use impact, in dimensionless units, reflects the extent to which various land sites are adversely affected by sludge disposal (Dee, et al., 1972). The

cost and the water quality objectives associated with facilities (plants and pipes) were represented using linear as well as fixed-charge and linear approximations. The land use impact objective was represented using only linear approximations. The data used for analysis are presented in Figure 5.1 and Table 5.1. The validity of these functions themselves was not examined in this study. For the details of the problem formulation the reader may refer to Riley (1979).

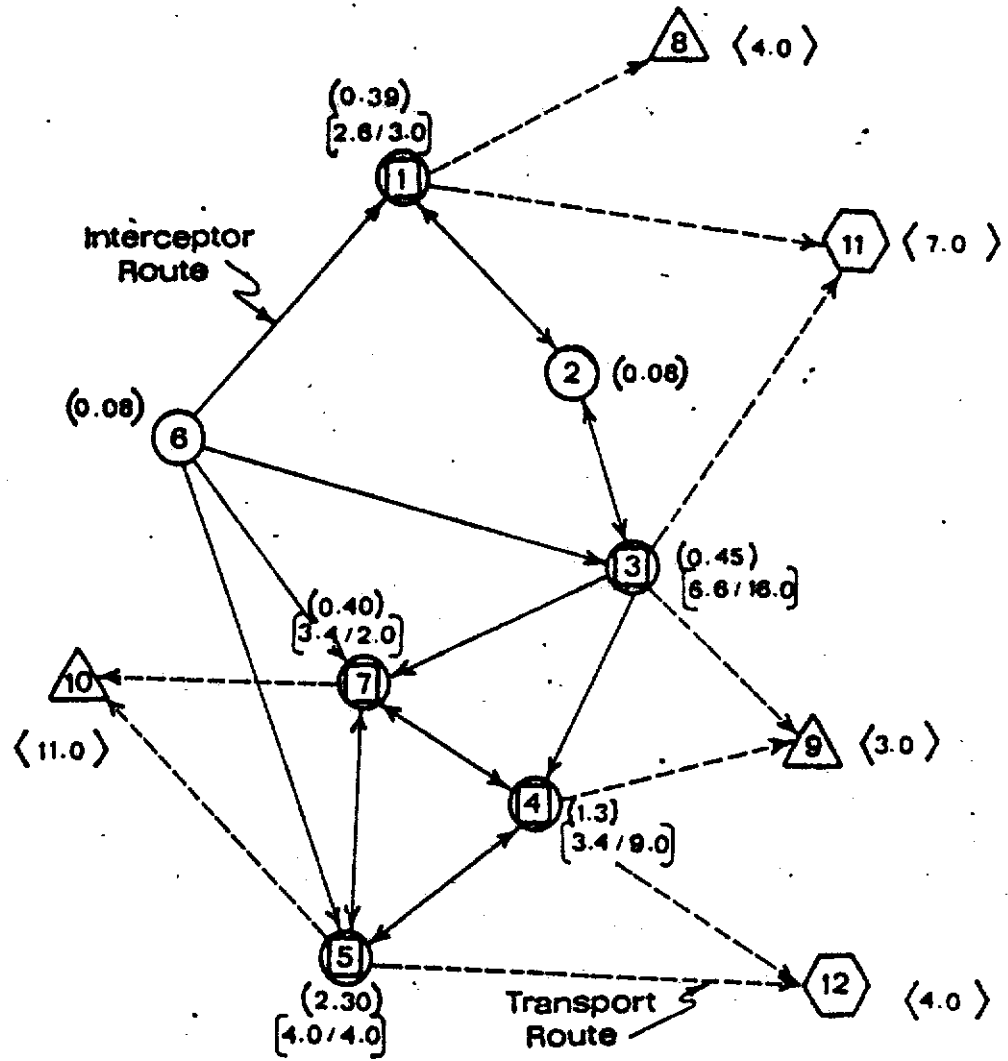
## 2) Computational Results

### Generating Three-Objective Alternatives

Several multiobjective branch-and-bound trees were constructed using the previously mentioned methods of identifying branching nodes and of fathoming the tree. In all cases the parent tree was constructed using cost as the first (primary) minimizing objective. The water quality and the land use objectives were taken to be the second and the third objectives, respectively, and the associated relaxed constraint trees were superimposed on the parent tree to form a relaxed multiobjective branch-and-bound tree. The general shape of the tree changed somewhat as different methods of branching and fathoming the tree were used.

## 3) Fathoming the Tree Using $\alpha$ Vectors

First, seven multiplier vectors each of which consisted of a multiplier  $\alpha^C$  for the cost objective,  $\alpha^W$  for the water quality objective, and  $\alpha^L$  for the land use objective, were examined. The seven sets of vector components examined are shown in Table 5.2. Each set of vector components forms a unique cut-off vector based on the relationship,  $\vec{z}^{**} = (\alpha^C z^C, \alpha^W z^W, \alpha^L z^L)$ .



- : Waste Source      ( ): Flow Generated (MGD)
- : Potential Plant Site      < >: Landuse Impact (  $\times 10^2$  unit / MGD)
- △: Landfill Site      [a/b]: Water Quality Impact  
a (  $\times 10^2$  unit / MGD)
- ⬡: Landspreading Site      b (  $\times 10^2$  unit / location)

Figure 5.1: Regional Network for the Hypothetical Example Problem

Table 5.1

## Cost Data for Case 1 Example Problem

	Facility	Fixed Charge (\$10 <sup>4</sup> /year)	Unit Cost (\$10 <sup>4</sup> /yr/MGD)
1	Plant 7 together Route 7-10	9.51	4.04
2	Route 5-10	7.00	1.80
3	Route 4-9	6.70	1.81
4	Route 3-9	6.40	1.82
5	Route 1-8	6.20	1.81
6	Plant 5	2.21	2.35
7	Route 3-11	2.00	5.10
8	Plant 4	1.69	2.45
9	Route 1-11	1.30	4.25
10	Route 5-12	1.30	4.71
11	Route 4-12	1.25	4.00
12	Route 4-5	0.94	0.50
13	Route 7-5	0.87	0.84
14	Plant 1		3.97
15	Plant 3		3.97
16	Route 1-2		0.74
17	Route 2-1		0.65
18	Route 2-3		1.27
19	Route 3-2		1.30
20	Route 3-4		1.12
21	Route 3-7		1.38
22	Route 4-7		0.81
23	Route 5-4		0.93
24	Route 5-7		1.26
25	Route 6-1		4.50
26	Route 6-3		6.52
27	Route 6-5		7.75
28	Route 6-7		4.11
29	Route 7-4		1.33

Table 5.2

Fathoming The Multiobjective Tree (a)

$$\text{WITH } Z^{**} = (C \cdot Z^C, W \cdot Z^W, L \cdot Z^L)$$

Case	Multiplier Vector	No. of OKA Computations	No. of Feasible Alternatives	No. of Noninferior Alternatives	CPU Time (b) (sec)
1	(1, 0, 0)	36 (12)(c)	36 (12)(d)	5 (4)(e)	2.32
2	(0, 1, 0)	42 (14)	42 (14)	7 (4)	2.65
3	(0, 0, 1)	3 (0)	3 (1)	2 (0)	0.62
4	(0.5, 0.5, 0.5)	3 (0)	3 (1)	2 (0)	0.52
5	(0.8, 0.8, 0.8)	49 (4)	45 (15)	6 (6)	2.81
6	(1, 1, 1)	110 (23)	87 (29)	8 (7)	5.29
7	(1.1, 1.1, 1.1)	345 (174)	171 (57)	8 (8)	12.45

- (a) The trees were grown using cost as the primary objective and using  $\lambda = (1, 0, 0)$  for selecting branching nodes.
- (b) DEC-10 system at the University of Louisville.
- (c) No. of infeasible network flow solutions in parenthesis.
- (d) No. of inspection limbs in parenthesis.
- (e) No. of noninferior alternatives which are also in the noninferior set in Case 7 in parenthesis.

In the first three cases presented, the branch-and-bound trees were grown and fathomed based only on the value of one of the three objectives as the multipliers associated with the remaining two were set to zero. For example, in Case 1, cost was used as a sole criterion for growing and fathoming the entire multiobjective tree since  $\alpha_1 = (1, 0, 0)$  was used. Similarly, water quality and land use impacts alone were used in Cases 2 and 3, respectively.

In Case 1, out of the 36 alternatives generated, 5 turned out to be noninferior. One of the 5, however, was not in the noninferior set identified in Case 7 in which a larger number of alternatives was generated. In Case 2, the number of noninferior alternatives was 7 out of 42, but again 4 out of 7 were in the noninferior set identified in Case 7. In Case 3, only 1 inspection limb was constructed and 3 feasible alternatives were found. Two of the 3 alternatives turned out to be noninferior, neither of which was in the noninferior set in Case 7.

In Cases 4 through 7, four sets of cut-off vectors, each with identical vector components, were used to fathom the multiobjective tree. In Cases 4 and 5, the termination of the branch-and-bound process was based on the currently lower feasible upper bounds (i.e.  $\underline{z}^{C*}$  ( $\leq z^{C*}$ ) rather than  $z^{C*}$  in the case of cost objective) multiplied by the corresponding  $\alpha$  vector components ( $\alpha^C$  in the case of the cost objective), since the individual optimal solutions may not have been identified prior to the termination of the branch-and-bound process. Note that the number of feasible alternatives increased as the values of the cut-off vector components were increased. The number of noninferior alternatives increased from case 4 through 6 as the  $\alpha$  values were raised, but there were eight noninferior alternatives in both Cases 6 and 7. One of the noninferior alternatives in Case 6 was replaced subsequently by a dominant alternative generated in Case 7. The individual optimal solutions identified on the Multiobjective tree were,  $z^{C*} = 3.47$  ( $10^5$  dollars/year),  $z^{W*} = 1.9$  units, and  $z^{L*} = 1.5$  units, respectively.



#### 4. Fathoming the Tree with $\vec{\beta}$ Vectors

Four weighting vectors,  $\vec{\beta}_1$ ,  $\vec{\beta}_2$ ,  $\vec{\beta}_3$ , and  $\vec{\beta}_4$ , were used to examine how they affect the growth of the multiobjective branch-and-bound tree. Each vector consists of an arbitrary set of weighting factors,  $\beta^C$ ,  $\beta^W$ , and  $\beta^L$ , which can combine the three objectives to a single measure of the total worth. The computational results are given in Table 5.3. The first case involves the assignment of equal weights to cost (in  $10^5$  dollar/year), the third, and the fourth cases involves the assignment of weights in the ratios, (1, 2, 1), (1, 2, 2) and (1, 3, 1) for costs, water quality and land use, respectively. The weighting vectors affect, as expected, the size of the tree and the number of feasible alternatives generated. Although each of these cases was solved separately, one can retrace a posteriori a given multiobjective branch-and-bound tree using different weighting vector for combining the three objectives. Note also that it is computationally quite burdensome to identify noninferior solutions on the branch-and-bound tree by solving the aggregate single objective problem in which the objective function is defined a priori as the weighted sum of the three different objectives.

As for the sensitivity of ranking order of the alternative generated, the choice of weight vectors affected the generation pattern significantly. For example, among the three sets of ten best (least combined objective) alternatives independently generated using  $\vec{\beta}_1$ ,  $\vec{\beta}_2$  and  $\vec{\beta}_3$ , there were 9 common alternatives between the first and the third sets. In general, the choice of the weighting vector becomes quite important as it affects the order of generation of noninferior alternatives.

#### 5) Imputed Values and Tradeoff Sensitivity

The multiobjective imputed value incidence matrix obtained for the tree constructed using  $\vec{\beta} = (0.33, 0.33, 0.33)$  is shown in part in Table 5.4. Note that the number of rows in the matrix corresponds to the number of inspection limbs rather than the number of feasible alternatives generated which was 42. Associated with the incidence matrix are three pairs of lower and upper bounds at the termination of the branch-and-bound process for cost, water quality and land use objectives, respectively.

Table 5.3

## Fathoming The Multiobjective Tree With Weighting Vectors (a)

Case	Multiplier Vector	No. of OKA Computations	No. of Feasible Alternatives	No. of Noninferior Alternatives	CPU Time (b) (sec)
8	(0.33, 0.33, 0.33)	45 (3)(c)	42 (14) (d)	5 (4) (e)	2.62
9	(0.25, 0.50, 0.25)	67 (10)	57 (19)	5 (4)	3.45
10	(0.20, 0.40, 0.40)	53 (5)	48 (16)	5 (4)	2.92
11	(0.20, 0.60, 0.20)	62 (5)	57 (19)	5 (4)	3.51

(a) The trees were grown using cost as the primary objective.

(b) DEC-10 system at the University of Louisville.

(c) No. of infeasible network flow solutions in parenthesis.

(d) No. of inspection limbs in parenthesis.

(e) No. of noninferior alternatives which are also in the noninferior set in case 7 in Table 5.2 in parenthesis

Table 5.4

Imputed Value Incidence Matrix and Upper and Lower Bounds of Objectives  
For Case 1 in Table 5.3

Inspection Limb	Variable No.								Cost (\$10 <sup>5</sup> /year)		W. Q. Impact (Impact Unit)		L. U. Impact (Impact Unit)	
	1	2	3	4	5	.	.	18	LB	UB	LB	UB	LB	UB
1	1	1	-1	-1	-1			-2	3.91	5.59	2.22	3.60	1.50	3.77
2	2	1	1	-2	-1			-2	3.67	4.77	2.92	3.43	1.50	3.77
3	2	2	1	-2	1			-2	3.86	4.03	2.82	2.86	1.50	1.55
4	2	2	2	1	-1			-1	4.03	5.30	3.22	5.14	1.90	1.96
5	2	1	2	-2	-1			-2	3.07	4.09	2.16	2.63	3.51	5.17
6	1	2	1	-2	-1			-2	4.11	4.90	2.72	3.06	1.82	1.93
7	2	2	2	2	1			-1	4.09	4.74	1.92	3.39	2.00	2.00
8	1	2	2	-2	1			-2	4.17	4.17	2.12	2.16	2.18	5.17
9	2	2	2	2	2			-1	3.99	4.38	1.62	3.41	2.00	2.12
10	2	2	2	2	2			-1	3.97	4.10	1.92	2.87	2.00	2.12
11	2	2	1	-2	2			-2	3.47	3.47	2.52	2.60	1.50	1.50
12	2	2	2	2	2			-1	4.02	4.46	1.86	2.63	2.00	2.14
13	2	2	2	2	2			-1	3.90	4.02	1.70	2.60	2.00	2.00
14	1	2	2	-2	2			-2	3.61	3.61	1.82	1.90	2.14	5.50

Five sets of imputed values, respectively associated with the first five variables in the matrix, are shown in Table 5.5. Just as in the case of the single objective analysis, the multiobjective imputed values associated with individual facilities or sets of facilities in the network are extremely useful for gaining insights into the underlying characteristics of the problem.

For example, the imputed values associated with the first variable (representing the planning option of constructing a plant at site 7, discharging effluent at the site and transporting sludge to landfill site 10) reveal the following:

- a. For cost it is better not to have a plant at site 7 and the sludge transport route 7-10, as indicated by the negative imputed value range.
- b) For water quality it is better to have a plant at site 7, as indicated by the positive imputed value range obtained from the augmented branch-and-bound tree.
- c) For land impact it is at least not detrimental to do away with a landfill at site 10, as indicated by a negative or zero imputed value.
- d) From the aspect of the aggregate objective for the weight vector  $\beta = (0.33, 0.33, 0.33)$ , it is probably better not to have a plant at site 7 and the sludge transport route to landfill site 10.

The degree to which these imputed values are significant depends on the magnitude of the values as compared with the respective overall objective function values. For example, the cost imputed value associated with variable 3, 0.04 ( $\$10^5/\text{yr.}$ ) is only 1% of  $Z^C*$ , and it is not very significant. On the other hand, the water quality and the land use imputed values associated with the same variable are, respectively, at least 0.62

Table 5.5

Multiobjective Imputed Values of Selected Planning Options<sup>(a)</sup>

Variable	Planning Option	Aggregate Objective	Cost (\$10 <sup>5</sup> /year)	Water Quality Impacts (Impact Unit)	Land Use Impacts (Impact Unit)
1	Plant 7				
	-- Route 7-10	-0.79~ -0.01	-0.55~ -0.14	-0.28~ -0.78	-0.43~ 0
	-- Landfill 10	(-0.51~ - 0.01)(b)	(-0.14)	(0.22 ~ 0.34)	(-0.43~0)
2	Plant 5				
	-- Route 5-10	-1.45~ -0.02	-0.62~ 0.4	-1.01~ -0.26	-2.27~ 0
	-- Landfill 10	(-1.14~ -0.54)	(0.04)	(-0.58~ -0.26)	(-1.89~ 0)
3	Plant 4				
	-- Route 4-9	-0.01~ 0.37	-0.40~ 0.14	-0.98~ -0.32	0~ 0.46
	-- Landfill 9	(0.014~ 0.37)	(0.04)	(-0.78~ -0.62)	(0.40)
4	Plant 3				
	-- Route 3-9	-1.62~ -0.26	-2.23~ 0.40	-3.52~ 0.08	-0.46~ 0
	-- Landfill 9	(-1.13~ 0.03)	(-0.78~ 0)	(-2.3~ 0.08)	(-0.40~ 0)
5	Plant 1				
	-- Route 1-8	-0.31~ -0.02	-0.96~ 0.40	-0.54~ -0.02	-0.05~ 0
	-- Landfill 8	(-0.31~ -0.26)	(-0.56~ -0.22)	(-0.34~ -0.22)	(-0.05~ 0)

(a) For Case 1 in Table 5.3. The tree is fathomed when an optimal aggregate (weighted sum) objective is identified.

(b) From the tree grown to the point where all of the infeasible aggregate objective node value exceed 1.25 times the optimal aggregate objective.

unit (33% of  $Z^{W*}$ ) and exactly 0.40 unit (27% of  $Z^{L*}$ ), and they are certainly very significant. The implication here is that the noninferior solution containing a plant at site 4 and sludge transport route 4-10 with the lowest overall land use impact (achieving  $Z^{L*} = 1.50$  unit), and the noninferior solution containing neither a plant at site 4 nor sludge route 4-10 with the lowest overall water quality impact ( $Z^{**} = 1.90$ ) are in significant conflict.

#### B. Example Application - Case 2

A considerably larger example was analysed also using a hypothetical regional wastewater and residual management system. The network configuration of this example, however, is much more complex and realistic than that of Case 1, since it was based on a map of the geographic layout of the regional wastewater system in Lexington, Kentucky and its surrounding areas (United State Army Corps of Engineers, 1978).

As in Case 1, the basic objective of the application study was not to examine the regionalization scheme itself, but to examine the process of generating and evaluating alternative plans, using the multiobjective method proposed. No effort was made, therefore, to construct authentic objective functions based on the existing data pertaining to the region.

##### 1) Description of Hypothetical Problem

Figure 5.2 shows the geographic layout of the system including the wastewater sources, potential sludge application sites, interceptor routes and sludge transport routes. A simplified network representation of this geographical layout is shown in Figure 5.3. In this network, there are 13 wastewater sources (communities), 8 wastewater treatment plant candidate sites, 6 potential land application sites, 13 possible interceptor routes, and 12 possible sludge transport routes. Of the 13 interceptor routes, 6 are two-way routes.

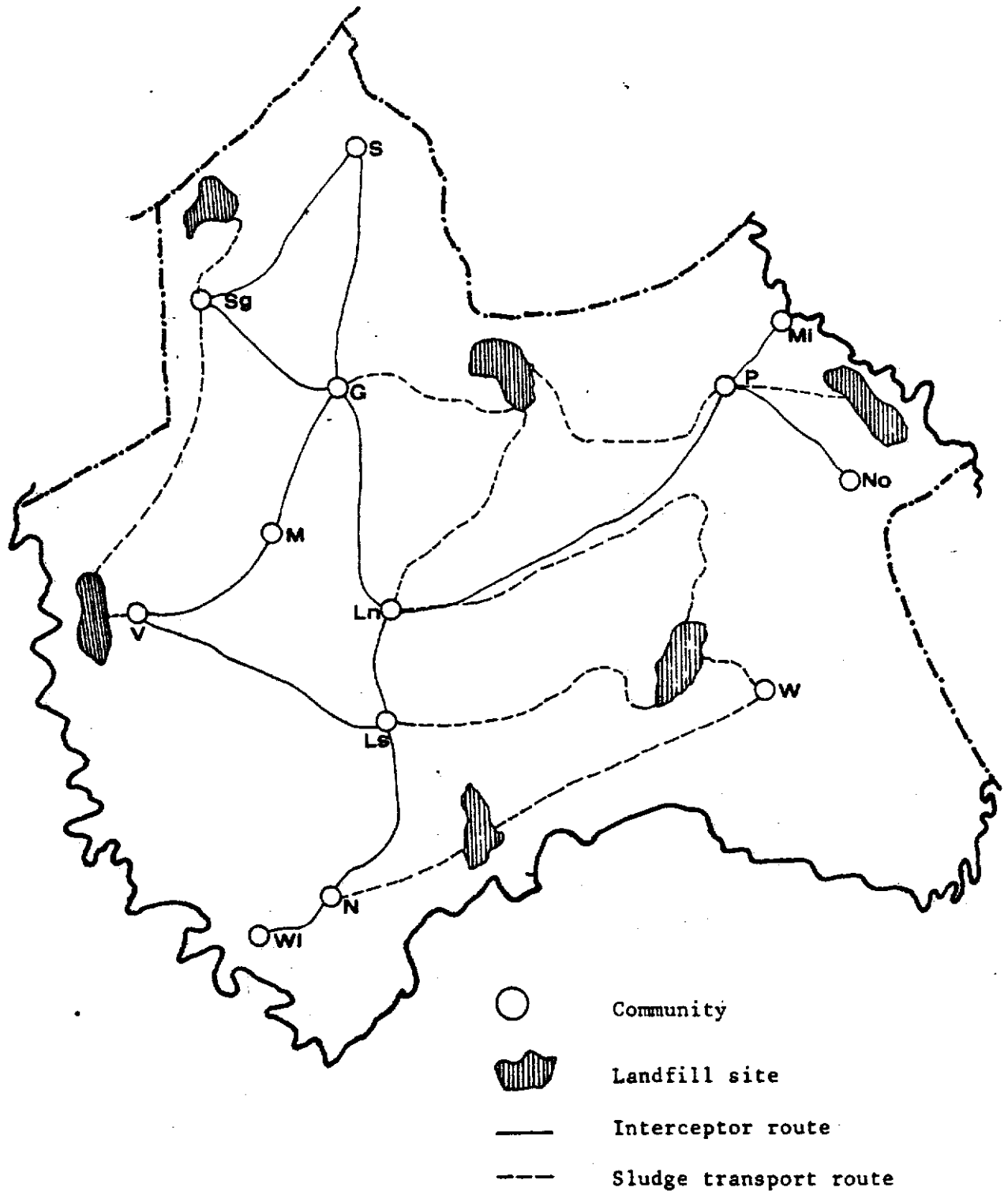


Figure 5.2: Hypothetical Wastewater and Residual Management System for Example Problem 2

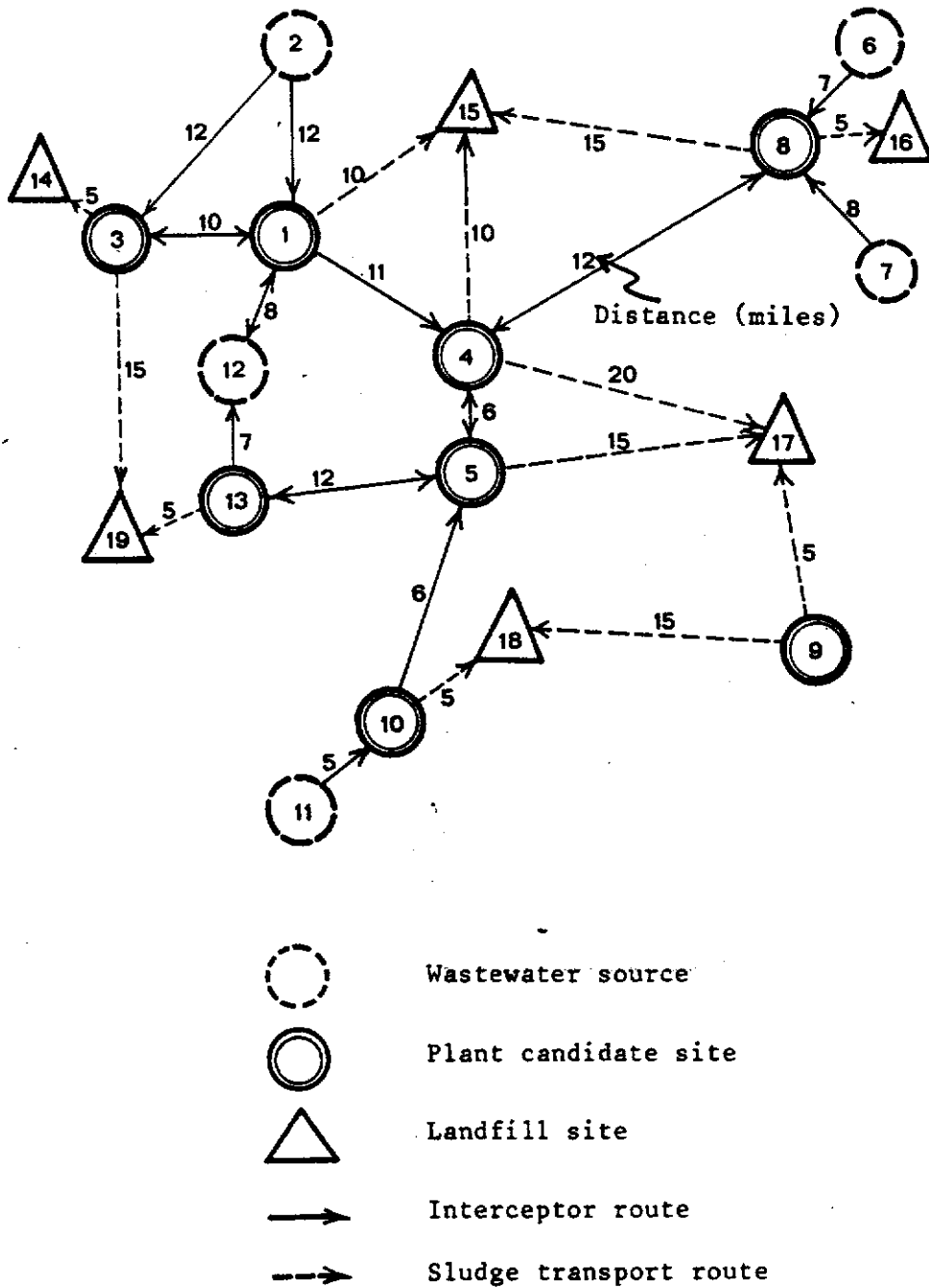


Figure 5.3: Network Representation of Example Problem 2



The maximum capacities of plants and interceptors were determined based on the direction of the network areas. Similarly to Case 1 example, the maximum capacities of sludge produced per unit volume of wastewater was assumed to be constant (0.15%). Therefore, the amount of sludge transported through the transport routes and disposed at the landfills could be represented by the equivalent amount of wastewater. This makes it possible for the entire network to deal with only a single commodity, wastewater. In turn the unit costs associated with the network arcs representing transport of sludge and landfilling were a priori adjusted by multiplying 0.0015. The resulting capacity bounds of the network arcs are shown in Table A.1 through A.4 in Appendix A.

## 2) Objective Functions

As described in Chapter I, construction of a meaningful objective function involves on a variety of factors including the correct interpretation of the problem and the synthesis of right information. In effect, it depends on whether an abstraction of the reality in the form of an objective can be achieved without gross misrepresentation. In the analysis of the above hypothetical problem, three simple objective functions were constructed. The basic premise stated above on the construction of objective functions, however, was not explored rigorously in this study.

The first objective is cost (COST) which is to be minimized. Two types of cost are considered. They are: (1) cost for the construction of treatment plants and interceptors (wastewater-related cost); and, (2) cost for site preparation for land application of sludge and hauling of sludge (sludge-related cost). The second objective is water quality impact (WQI) which is to be minimized. A very simple function was constructed for each plant site where wastewater treatment effluent is to be discharged to a nearby receiving stream with specific water quality. As in Case 1 the degradation of water quality was reflected upon the water quality impact unit proposed by Dee, et al., (1972)., to develop the objective function in final form. The third objective is land contamination potential (LCP) by

sludge which is to be minimized. A set of linear functions were prepared based on some rather arbitrary assumptions applied to the study by Garrigan (1977). A brief summary of the background data and the procedure for constructing these four functions (two cost functions, water quality impact function and land contamination potential function) presented in Appendix A.

Beside these three objective functions, two additional objectives were also included in the analysis for the purpose of examining the computational efficiency of the proposed multiobjective branch-and-bound method. They were arbitrarily constructed simply to make the problem more complex. These two functions are also presented in Tables A.1 and A.2 in Appendix A.

### 3) Computational Results

Several different example problems were analysed. Presented below are the computational results including the network configuration of various alternatives generated, the plots of the computed cut-off value  $\alpha$  versus the growth of the branch-and-bound tree, the noninferior sets identified at various stages of the growth of the tree.

#### Minimization of Cost

Minimization of combined cost of wastewater treatment and sludge disposal is presented first. The network feature of low-cost alternatives was identified so that it can be compared with the network configurations of various noninferior alternatives generated in other example problems.

The branch-and-bound process of the cost-minimization problem was terminated when the least-cost alternative was identified. The branch-and-bound tree generated 3649 nodes of which about one-third was active (see Section III-B for definition). The total number of alternatives generated were 324. The way in which  $\alpha$  increases to 1.0 (identification of the least-cost solution) is described in Figure 5.4. The least cost was 3.31 million

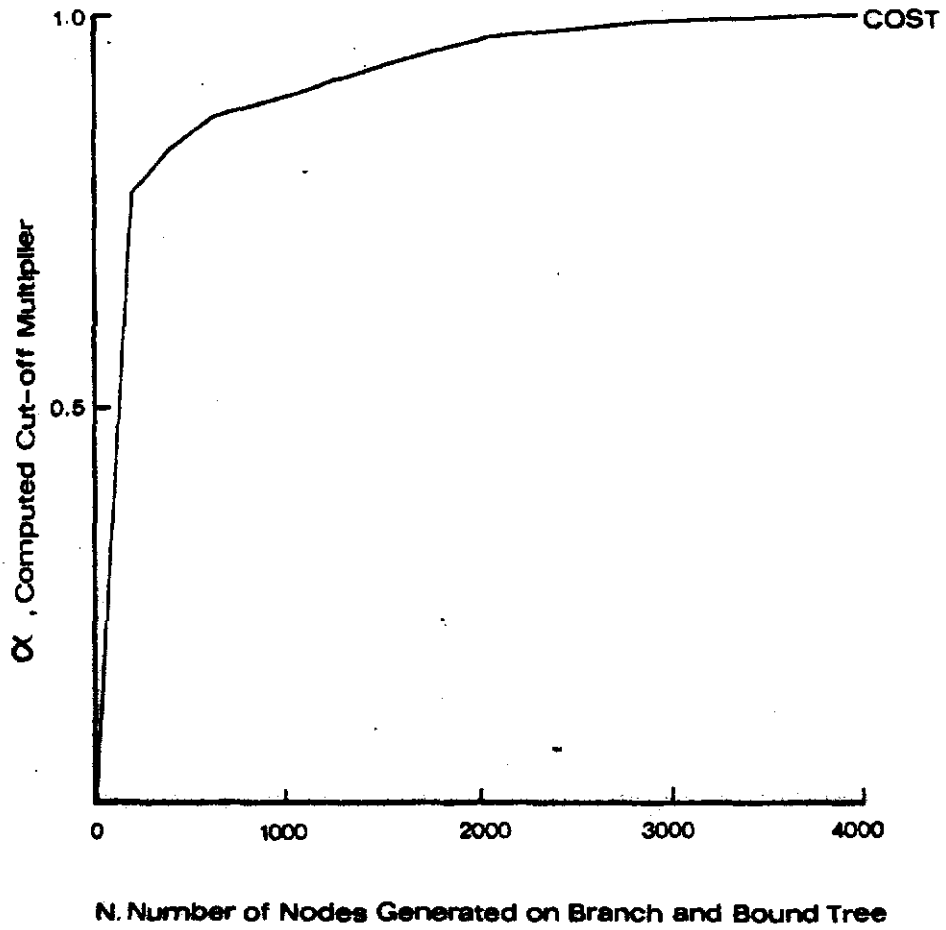


Figure 5.4: Plot of  $N$  versus  $\alpha$  for Single-Objective Problems Involving Only COST

dollars per year while the seventh least cost was 3.34 million dollars. The cost difference amounts to only 1% of least cost, an insignificant amount. This was due to the fact that the same unit cost were assigned to the facilities of the same kind (e.g., interceptors) with a similar maximum capacity.

The network configuration of these first seven least-cost alternatives turned out to be quite similar and the network representation of the common feature is shown in Figure 5.5. Interestingly, the difference in network configuration depends only on three candidate sites; that is (a) wastewater collected at site 1 ( $W_1 + W_2 + W_3$ ) is either treated there and the sludge hauled to landfill 15 or transhipped to wastewater source 12 to be sent together with  $W_{12}$  to plant candidate site 3, to be treated there with  $W_{13}$ ; (b) wastewater collected at site 4 ( $W_6 + W_7 + W_8 + W_9$ ) is either treated there and the sludge hauled to landfill 15, or transhipped to plant candidate site 5 to be treated there, together with  $W_5$ ; (c) wastewater at site 5 ( $W_{11} + W_{10} + W_5$ ) is either treated there and sludge hauled to landfill 17, or transhipped to plant candidate site 4 to be treated there, together with  $W_4$ .

Although the cost figures are somewhat unrealistic due to rough approximations of cost functions, the identification of the common feature of the least-cost network configurations is quite important and useful for examining noninferior alternatives generated in the multiobjective analysis as described in the following sections.

#### Multiobjective Analysis of Cost

In this section the cost associated with transport and treatment of wastewater and the cost associated with transport and disposal of sludge were considered separately and a multiobjective branch-and-bound analysis was performed to generate noninferior solutions. The significance of such an analysis lies in the fact that the factors which affect wastewater-related cost may be quite different from those which affect the sludge-

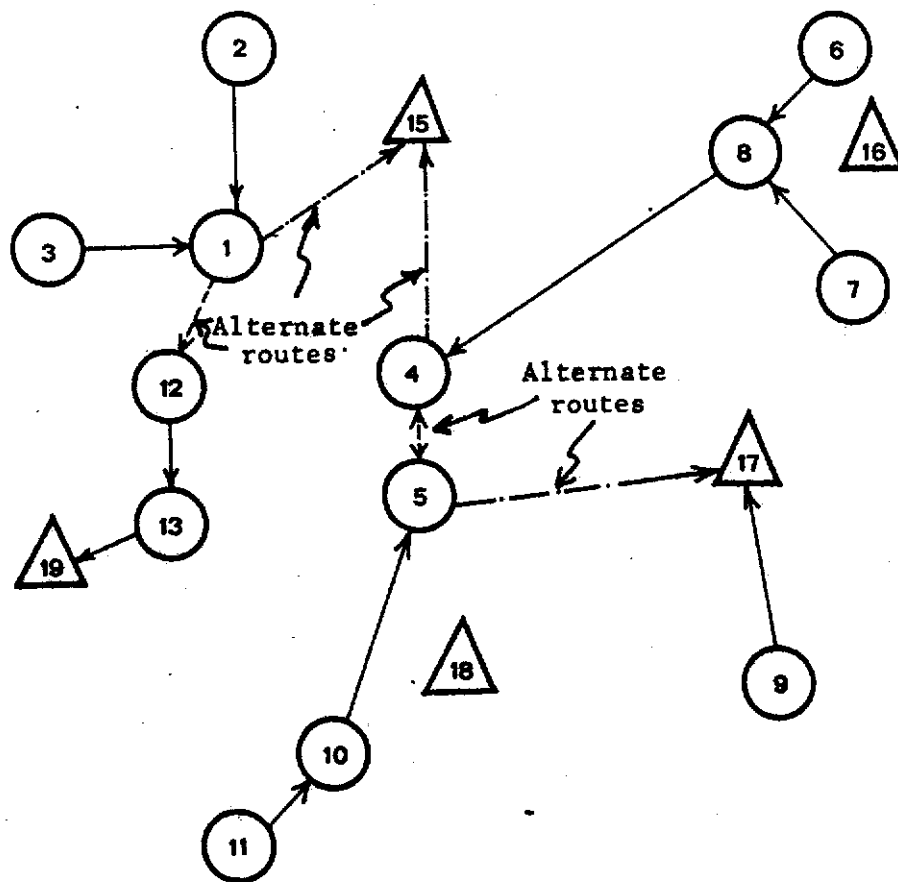


Figure 5.5: Common Feature of Network Configuration in Eight Least-Cost Alternatives

related cost. For example, sludge-related cost is likely to be heavily dependent on the short-term price of energy since it involves hauling vehicles and site preparation vehicles, while wastewater-related cost is likely to be less dependent and yet it involves heavy initial capital investment.

The branch-and-bound process was based on  $\alpha = (1, 1)$ , or the straight sum of the two cost components was used, for the selection of the infeasible lower bound node from which to branch-off next. The number of noninferior alternatives generated on the multiobjective branch-and-bound tree was 7 when the tree was grown to  $N$  (number of nodes) = 1015 and  $\vec{\alpha} = (0.97, 0.64)$  and 11 when it was grown to  $N = 2000$  and  $\vec{\alpha} = (1.00, 0.64)$ . Of the 7 noninferior alternatives identified at  $N = 1015$ , 6 still remained as noninferior at  $N = 2000$  (see Figure 5.6) one alternative dropped out of the noninferior set as at least one dominant alternative was generated in the process of growing the multiobjective branch-and-bound tree. The branch-and-bound process could be continued to generate more alternatives and the computed values may increase. The change in the content of the noninferior set was considered marginal at this point.

Two important points must be noted in this figure. First, as mentioned in section II-C and IV-B, the set of alternatives generated are noninferior only with respect to the alternatives generated so far. The branch-and-bound process could be continued to generate more alternatives some of which may dominate or be noninferior to the currently noninferior 11 alternatives. Second, of the 11 alternatives shown in Figure 5.6, only 4 (alternatives 1, 2, 6 and 7) span the linear noninferior frontier as indicated with dotted line, implying that, for any linear indifference relationship, the remaining 7 alternatives will never be preferred to these 4 alternative. Some elaboration is required on these two points.

The first point has a direct reference to the criteria for the termination of the multiobjective branch-and-bound process. As discussed in Section IV-D, if  $\alpha$  is greater than 1, say 1.1 in the current example, it

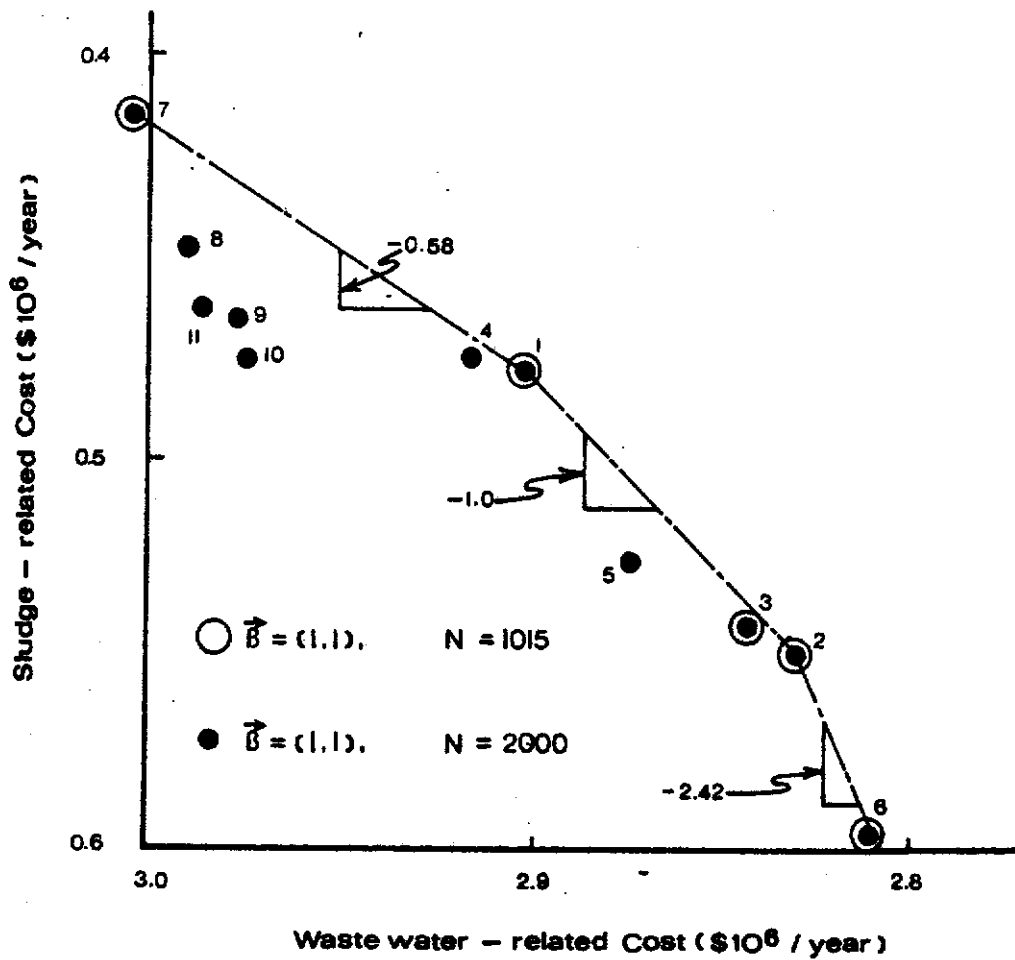


Figure 5.6: Noninferior Alternatives based on Wastewater-related Cost and Sludge-related Cost

may be sufficient for the termination of the branch-and-bound process since all of the noninferior solutions which include at least one of the objective function values being less than 110 percent of the corresponding optimal objective function value should have already been generated. As is the case with the current problem, however, one or more components of  $\vec{\alpha}$  may "strike a snag" or "stagnate" at a certain value. The criteria for the termination of the branch-and-bound process do become less straight forward. In the example analysis above and in the analyses to follow, the branch-and-bound process was terminated when the values of  $\vec{\alpha}$  components reached a plateau and at the same time the change in the noninferior set was marginal. The stagnation of the values of  $\vec{\alpha}$  components and criteria for the termination of the multiobjective branch-and-bound process remain to be an important area of future research.

As for the second point, the nonlinear (fixed-charge/linear in this case) network structure is likely to result in nonconvex noninferior set (see for example, Cohon 1978, p. 290). For a highly nonlinear indifference curve any of the noninferior solutions which lie below the linear line segments may be preferred over those which span the linear noninferior frontier. For all practical purposes, however, the latter noninferior solutions are of primary significance in the multiobjective analysis, since the definition of indifference can rarely be obtained in the form other than linear trade-off relationship between objectives. It is interesting to note in this context, that all 4 alternatives were among the noninferior set identified at  $N = 1015$ .

Figure 5.6 indicates that for the indifference slope of -1.0, or  $\vec{\beta} = (1, 1)$ , alternative 1 and alternative 2 are the most attractive, that for the indifference slope greater than -0.58 (milder slope) alternative 7 is most attractive and that for the indifference slope less than -2.42 (steeper slope) alternative 6 is most attractive. In other words, when the sludge-related cost is to be weighted more than the wastewater-related cost, then alternative 7 would be more attractive than the rest of the noninferior alternatives, and when the wastewater-related cost is to be weighted more than sludge-related cost, then alternative 6 would be more attractive.



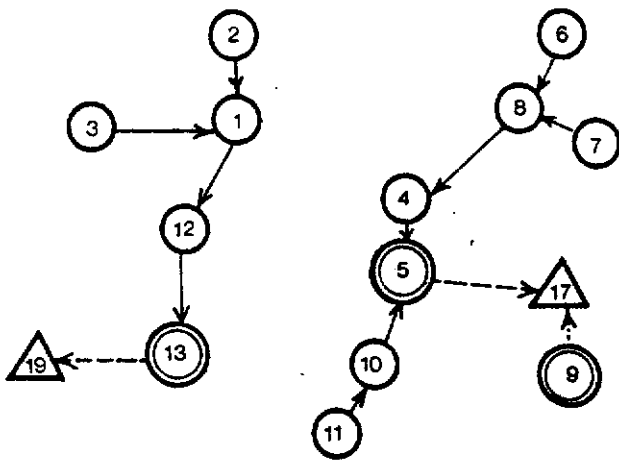
The network configuration of these four alternatives are shown in Figure 5.7. It is to be noted that alternatives 1 and 2 are only slightly different from each other while the remaining two are quite different from each other and from the first two. Alternatives 1 and 2 do conform to the network configuration identified in the least-cost analysis shown in Figure 5.5.

### Analysis of Cost and Environmental Objectives

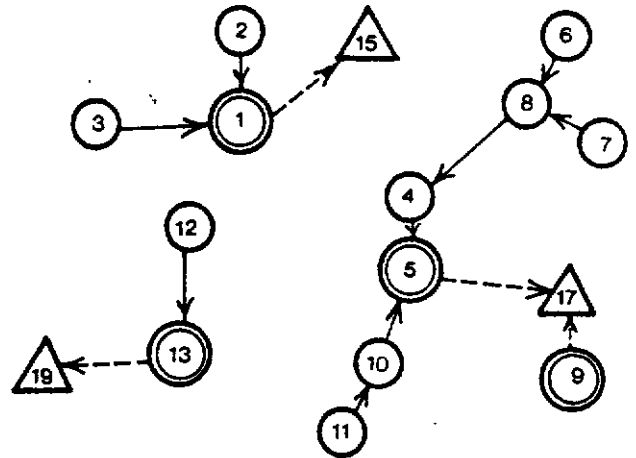
Two minimizing environmental objectives, water quality impact (WQI) and land contamination potential (LCP) were included in the multiobjective analysis along with minimization of cost. The significance of such multiobjective analysis, as mentioned in chapter I, lies in the fact that, if appropriately dealt with, additional environmental considerations will broaden the scope of decision-making by allowing the quantitative evaluation of the trade-off between cost and environmental impacts with respect to various network configurations. The example analysis presented in this section are yet quite simplistic and preliminary due to the fact that the environmental objectives used in the analysis are hypothetical and not based on the analysis of real data. The computational results, nonetheless, seem to provide some very useful insight into the use of the multiobjective network-flow method to regional wastewater and residual management systems. Two cases were analysed. The first case involves minimization of COST and WQI and the second case involves COST, WQI and LCP.

#### a) COST and WQI

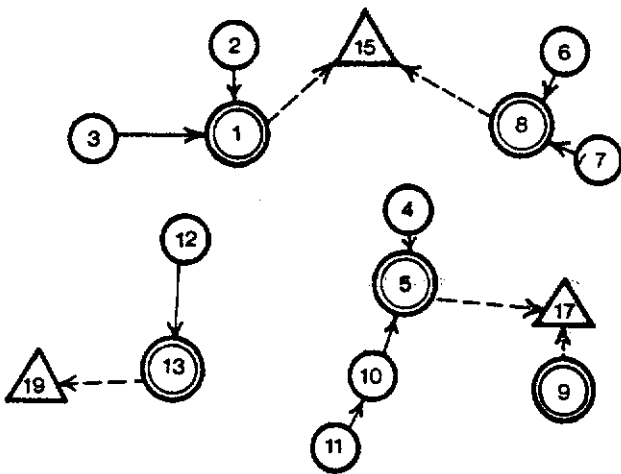
Two arbitrary weighting vectors,  $\vec{\beta} = (1, 1)$  and  $\vec{\beta} = (1, 100)$  were used to combine COST and WQI objectives for growing the branch-and-bound tree. Two noninferior sets generated are shown in Figure 5.8. It turned out that, for the multiobjective trees of about the same size ( $N = 1003$  and  $N = 1006$ , respectively), the noninferior sets generated using  $\vec{\beta} = (1, 1)$  and  $\vec{\beta} = (1, 100)$  were identical. They both contained 7 noninferior alternatives, 4 of which spanned the noninferior front as shown in the figure.



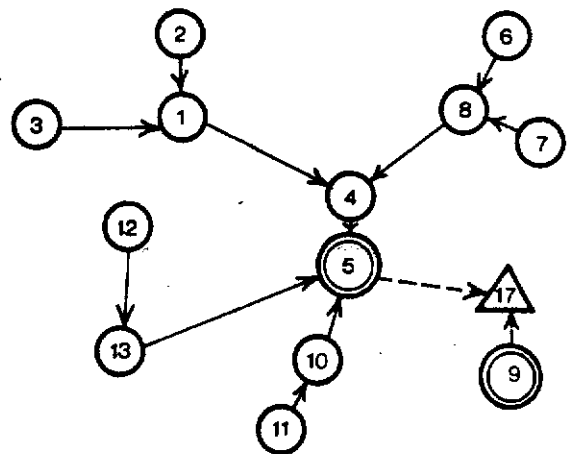
(a) Alternative 1  
 $\$3.378 \times 10^6/\text{year}$



(b) Alternative 2  
 $\$3.378 \times 10^6/\text{year}$



(c) Alternative 6  
 $\$3.405 \times 10^6/\text{year}$



(d) Alternative 7  
 $\$3.424 \times 10^6/\text{year}$

Figure 5.7: Configurations of Noninferior Alternatives Based on Two-Cost Analysis

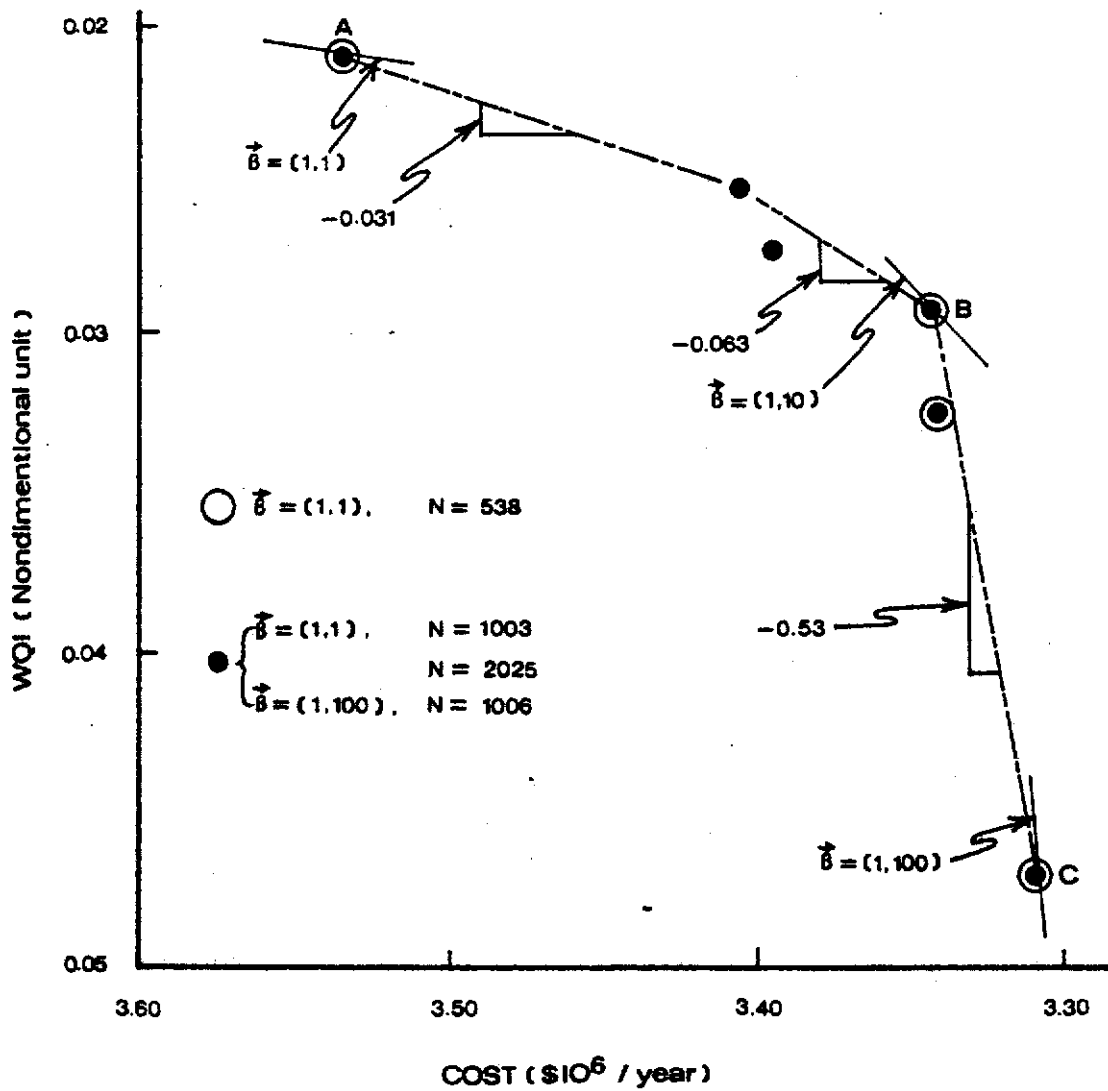


Figure 5.8: Noninferior Alternatives Based on Cost and Water Quality Impact

Taking the case at  $\vec{\beta} = (1, 1)$ , the relationship between the tree size and the noninferior sets generated was examined at  $N = 538$ ,  $N = 1003$  and  $N = 2025$ , or for the tree size ratio of 1, 2 and 4. At  $N = 538$ , there were five noninferior alternatives, four of which were also noninferior at  $N = 1003$ . They are marked with a circle in Figure 5.8. Of the four, three spanned the noninferior set described above. When the tree was grown further to  $N = 2025$ , the noninferior set remained identical to that at  $N = 1003$ . No new noninferior alternative was identified despite the fact that the tree was grown twice as large.

It is clear from Figure 5.8, Alternative B is a very attractive alternative since it is the best compromise solution to a wide range of indifference slope between  $-0.063$  and  $-0.53$ . This alternative, incidentally, happened to be the same alternative as Alternative 1 in Figure 5.7. (Theoretically, Alternative 1 in Figure 5.7 should be identical to the least-cost solution identified in section V-C.1, and, therefore, it should have matched with alternative C in Figure 5.6. This did not happen because of the error due to different cost approximation in the analysis performed in Section V-C.2. Nonetheless, the low cost alternative did turn out to be attractive multiobjective alternatives with respect to cost as well as water quality impact). In retrospect, cost and water quality impact were quite compatible due to the fact that plant sites 4, 5 and 13 are attractive with respect to cost as well as water quality impact as shown in Tables A.1 and A.2 in Appendix A.

Although the set of noninferior alternatives generated using  $\vec{\beta} = (1, 1)$  turned out to be identical to that generated using  $\vec{\beta} = (1, 100)$ , the two multiobjective trees were grown quite differently. Figures 5.9 and 5.10 show, respectively for  $\vec{\beta} = (1, 1)$  and  $\vec{\beta} = (1, 100)$ , the plots of the number of tree nodes versus  $\alpha$  associated with COST and WQI. As apparent from Figure 5.9, when  $\vec{\beta} = (1, 1)$  is used to grow the branch-and-bound tree, the  $\alpha$  value associated with COST approaches 1.0 much faster than that associated with WQI, indicating the relatively heavier weight placed on COST in the original data set. When  $\vec{\beta} = (1, 100)$  is used,  $\alpha$  associated with WQI reached optimality very quickly and yet  $\alpha$  associated with COST reached to a plateau at 0.73 and never improved as shown in Figure 5.10.

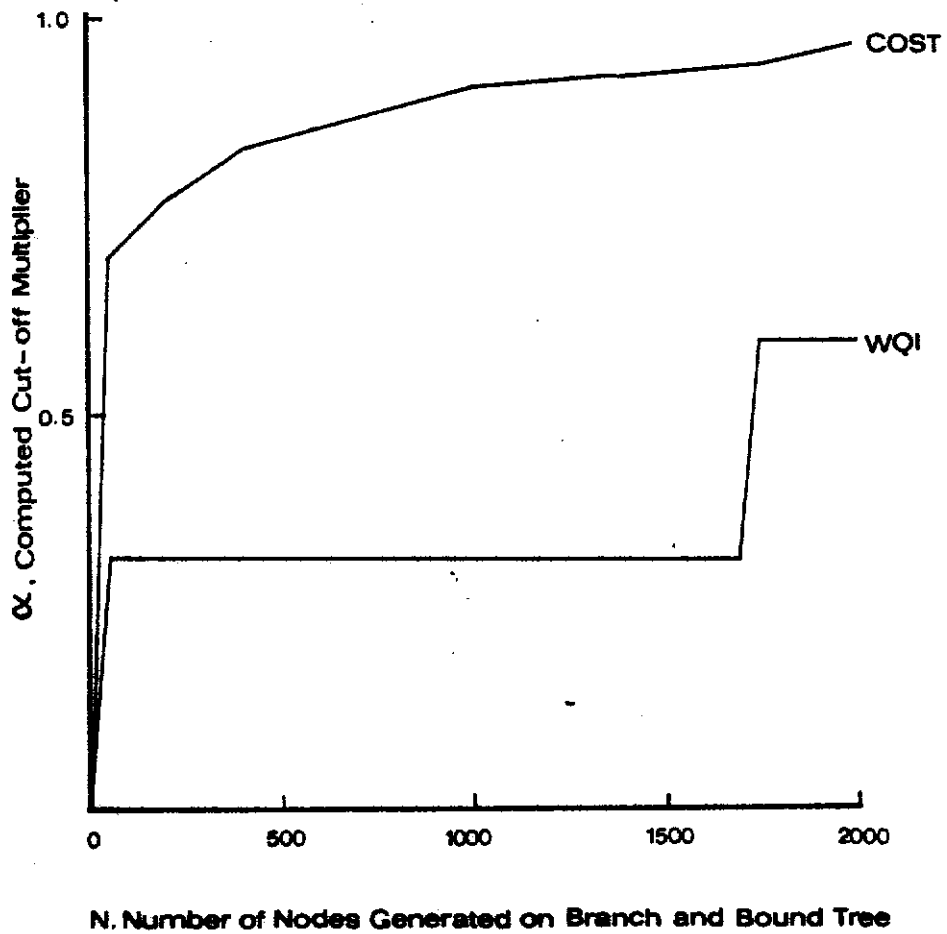


Figure 5.9: Plot of  $N$  versus  $\alpha$  for Two-Objective Problem,  $\beta = (1, 1)$  for COST and WQI

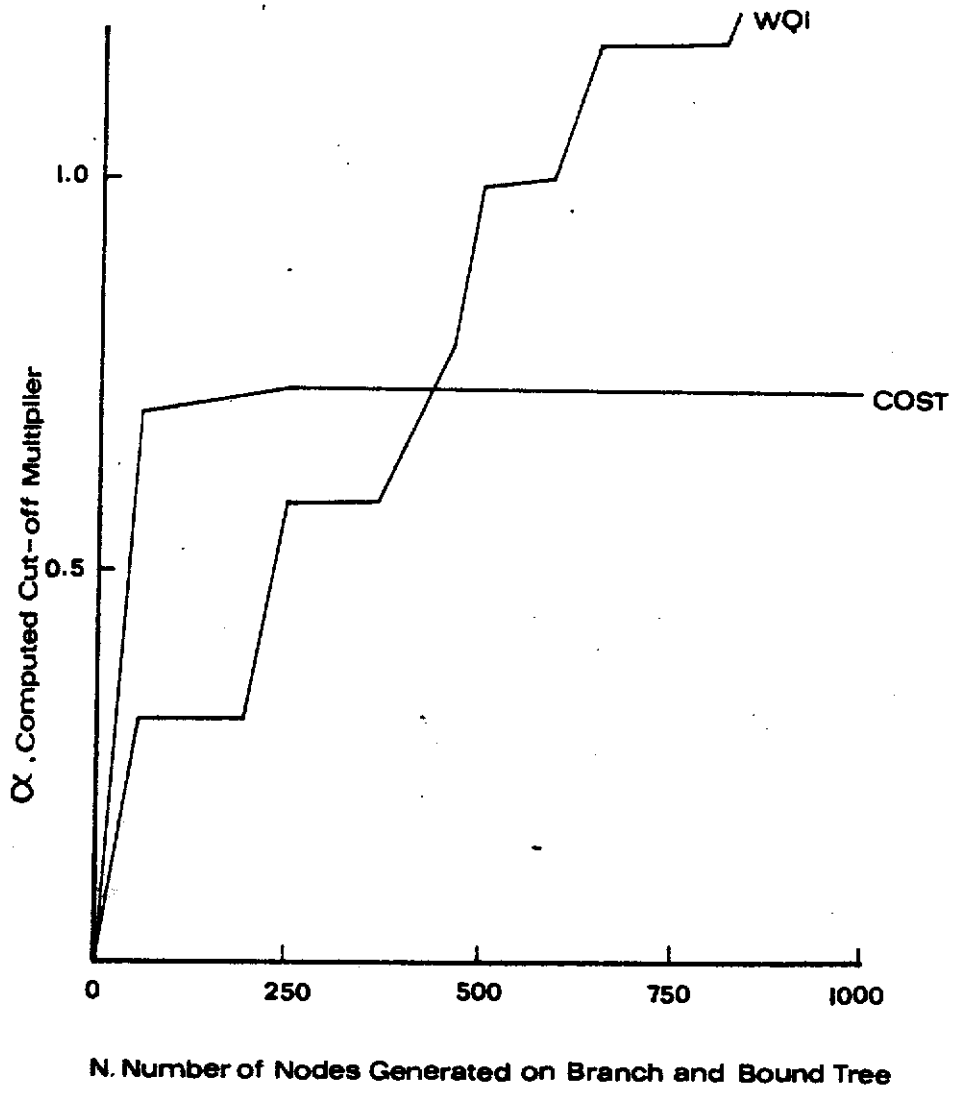


Figure 5.10: Plot of N versus  $\alpha$  for Two-Objective Problem,  $\beta = (1, 100)$  for COST and WQI

As mentioned in Section III-B,  $\alpha$  associated with a particular objective is an indicator of how close or how far the least of the infeasible lower bounds is to the optimal value of that objective. From Figure 5.9, for example, it is not certain whether or not the alternative whose WQI objective is optimal has already been generated on the tree by the time the tree was grown to  $N = 2000$ . If it has been, then  $\alpha$  associated with WQI should have reached 1.0. This implies also that the tree may have to be grown much larger before such an alternative is guaranteed to have been generated. (Of course there is a very good chance that such an alternative has already been identified among a large number of alternatives generated heuristically in the Out-of-Kilter computational step. - See Appendix B). As the number of fixed-charge variables increases,  $\alpha$  will be more likely to "strike a snag" or remain at a certain value below 1.0.

b) COST, WQI and LCP

The addition of land contamination potential (LCP) objective to be minimized changed the noninferior set profile significantly. As shown in Table A.4 in Appendix A, the linear coefficient of LCP is lowest at site 17, make the site most attractive for landfilling. Sites 14, 15, 19, 18 and 16 are successively less attractive in that order. Using an arbitrary weighting vector  $\vec{\beta} = (1, 100, 1)$ , 15 noninferior alternatives were generated after the tree was grown to  $N = 1527$  and  $\vec{\alpha}$  was computed as (0.75, 10.2, 1.00). Many attractive alternatives had site 17 as the central regional landfill site.

Table 5.6 compares noninferior alternatives generated in the two-objective problem (involving COST and WQI) with those generated in the three-objective problem (involving COST, WQI and LCP). The comparison is based on the amount of sludge (as expressed in terms of wastewater flow) sent to six landfill sites. Noninferior alternatives generated are ranked according to the weighted sum using the same  $\vec{\beta}$  used to grow the multi-objective branch-and-bound tree, or  $\vec{\beta} = (1, 100)$  and  $\vec{\beta} = (1, 100, 1)$ . The best 7 alternatives are shown for each of the two problems.

Table 5.6

Sludge Disposal at Landfill sites for  
Two-Objective and Three-Objective Problems

Site	Two-objective problems							Three-objective Problem						
	Alternative Ranking with $\beta = (1, 100)$							Alternative Ranking with $\beta = (1, 100, 1)$						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	48	0	40	48	40	0	0	0	0	0	0	48
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	2	50	2	35	2	2	2	50	43	42	35	43	42	2
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	48	0	0	15	8	0	8	0	7	8	15	7	8	0

Amount of Sludge as Equivalent Wastewater Flow (MGD)



As apparent from the table, in the two-objective problems, 4 alternatives have site 15 as the regional landfill site, while, in the three-objectives problem, 6 alternatives have site 17 as the regional landfill site. The former is due principally to the attractiveness of plant site 4 as a central plant location site with respect to COST as well as WQI, the latter is due principally to the attractiveness of landfill site 17 as a regional landfill site with regard to LCP. It is interesting to note, however, that alternative 2 in the two-objective problem and alternative 1 in the three objective problem are identical (which is also the alternative at point B in Figure 5.8, the network configuration of which is shown in Figure 5.11), and that alternative 4 in the former and alternative 4 in the latter are also identical.

Figure 5.12 shows the normalized objective function values of selected noninferior alternatives in the three objective problem. The definition of normalized objective function value is given as:

$$\frac{Z(n) - Z_{\min}}{Z_{\max} - Z_{\min}}$$

for each of the three objectives, where  $Z(n)$  is the objective function value of the  $n^{\text{th}}$  alternative, and  $Z_{\min}$  and  $Z_{\max}$  are the minimum and maximum objective function values among the 15 alternatives. The figure indicates that alternatives 1, 8 and 15 are optimal with respect to LCP, WQI and COST, respectively. Alternative 8, however, has the highest (worst) objective value with respect to COST and LCP. Alternative 15, on the other hand, has the highest objective value with respect to WQI. Alternatives 2, 3 and 4 show nearly the same pattern with regard to the magnitude of each of the three objective function values. From this figure alone it is not possible to identify the noninferior alternatives which span the noninferior surface. However, it is clear at least alternatives 1, 8 and 15 are among those alternatives which span the noninferior surface in the three dimensional objective space.

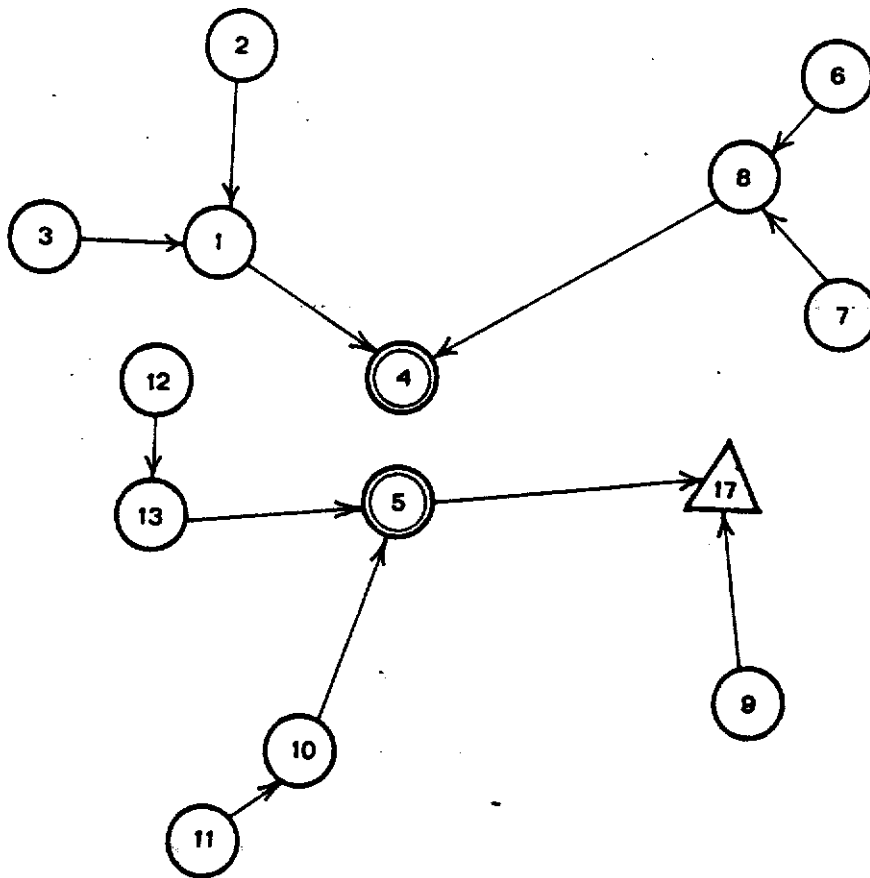


Figure 5.11: Best Alternative for Three-Objective Problem,  
 $\beta = (1, 100, 1)$  for COST, WQI and LCP

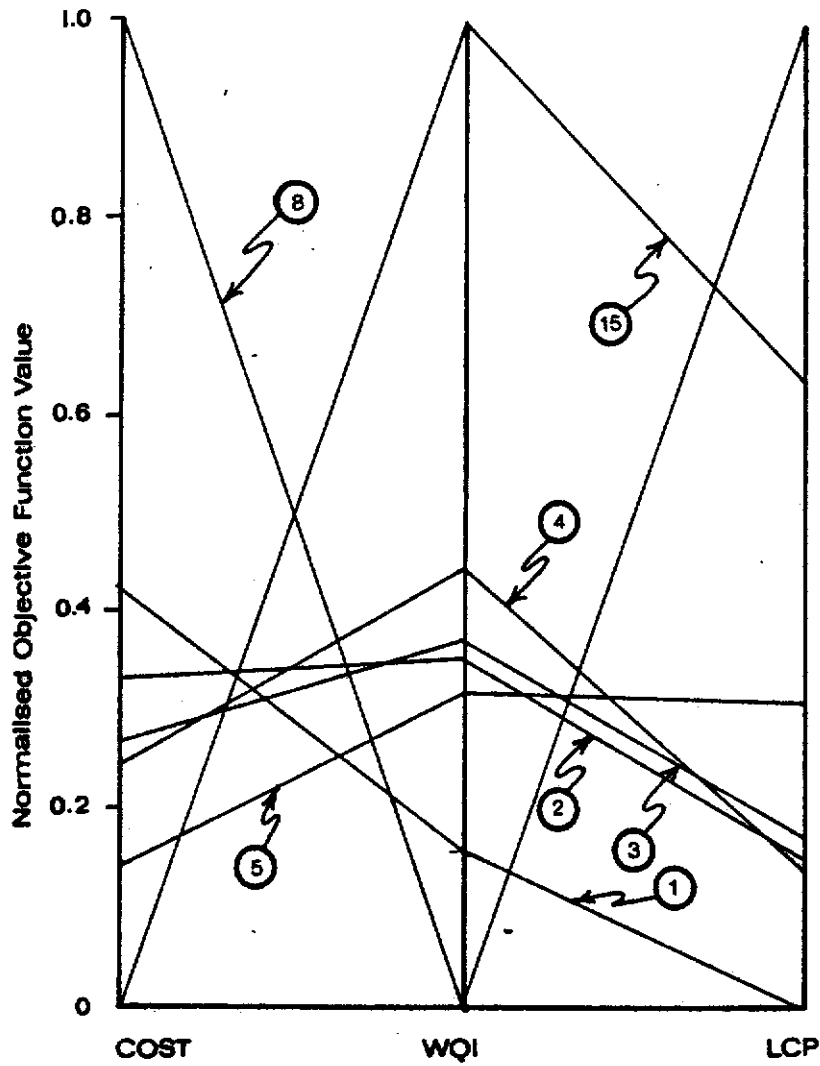


Figure 5.12: Selected Noninferior Alternatives for Three-Objective Problem,  $\vec{p} = (1, 100, 1)$  for COST, WQI and LCP

Figure 5.13 shows the plot of  $\alpha$  values versus the growth of branch-and-bound tree for the three objective problem described above. It is seen that  $\alpha$  associated with LCP was 1.0 on the first node of the tree and remained such throughout the branch-and-bound process. This is because the LCP objective functions are all linear as shown in Table A.4. Alpha associated with WQI reached 1.0 around  $N = 750$  and remained such up to  $N = 1500$  when the branch-and-bound process was stopped. Just as the case with the two-objective problems,  $\alpha$  associated with COST reached 0.73 and remained such except towards the very end of the branch-and-bound process. A comparison of Figures 5.10 and 5.13 reveals that the additional objective LCP suppressed the growth of the tree in the direction where alternatives can be generated with WQI objective function value greater than its optimal value.

When an approximate trade-off relationship between two objectives is known and can be used to combine the two objectives into a single objective, the branch-and-bound process may proceed differently and  $\alpha$  associated with the combined objective may reach 1.0 much earlier than when two objectives were dealt with separately. Figure 5.14 shows a plot of  $\alpha$  versus  $N$  for a modified version of the three objective problem in which WQI and LCP are a priori combined using  $\vec{\beta} = (100, 1)$ . The figure indicates that  $\alpha$  associated with the combined objective (100 WQI + LCP) reach 1.0 around  $N = 700$  and it continue to increase almost linearly before the computation was terminated around  $N = 1600$ . It shows also that  $\alpha$  associated with COST objective remained nearly identical to the original three-objective case shown in Figure 5.13. As for the combined objective of (100 WQI + LCP), all of the noninferior alternatives with the combined objective function value less than 1.3 times the optimal objective value ought to have been generated by the time the branch-and-bound process was stopped.

#### 4) Computational Statistics

As in the case with example case 1, all of the computations described for example case 2 were carried out using a computer programme called MOBNET, an interactive programme written in FORTRAN for the DEC10 system at the University of Louisville. Although there is much room for improvements the

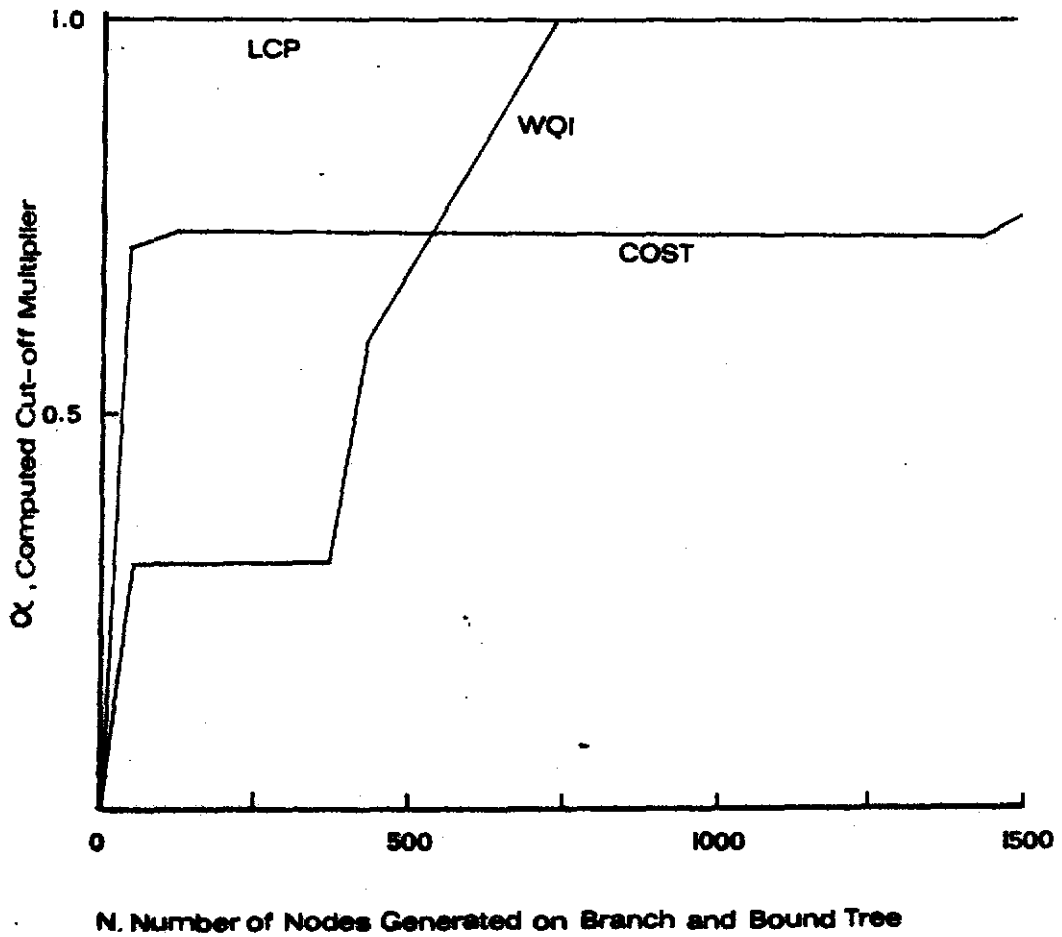


Figure 5.13: Plot of  $N$  versus  $\alpha$  for Three-Objective Problem,  $\beta = (1, 100, 1)$  for COST, WQI and LCP

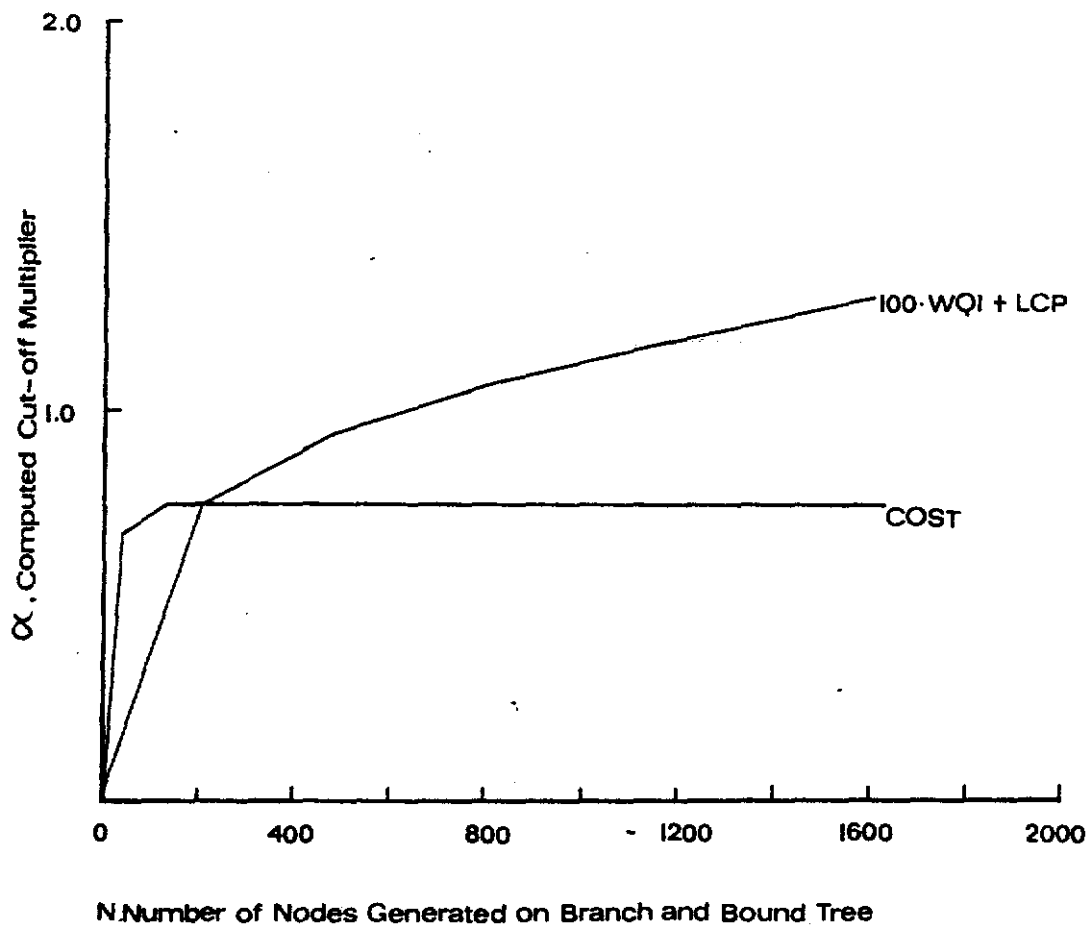


Figure 5.14: Plot of  $N$  versus  $\alpha$  for Three-Objective Problem Modified to Two-Objective Problem,  $\beta = (1, 1)$  for COST, and  $(100 \text{ WQI} + \text{LCP})$

programme, the following discussion on the computational statistics is based on the experience of using the most MOBNET FORTRAN programme. The table 5.7 shows 6 cases, 5 of which have already been discussed in detail in this chapter. Case 6 involves 5 objectives three of which are COST, WQI and LCP while the remaining two (shown in Tables A.1 and A.2 in Appendix A) are contrived objectives added simply to test the computational efficiency of the programme.

There are several rather obvious observations to be made on the table. First, although CPU time required appears to be related to the number of OKA computations, the relationship is not necessarily consistent. The type of objective functions (linear or fixed-charge linear appropriations) included in the analysis, the growth pattern of the branch-and-bound tree, and, most of all,  $\alpha$  value used to fathom the tree (or the maximum number of nodes at which the growth of the tree was stopped), affect the CPU times required to identify an appropriate set of noninferior alternatives.

Second, as described in chapter IV, the multiobjective network-flow method illustrated in this report takes advantage of the information obtained in the computational process for identifying feasible network-flow solutions, i.e, an Out-of-Kilter break-through computation. The number of break-throughs in each OKA computation varies depending on the problem defined. In general, however, several break-throughs are observed to occur. Therefore, the actual number of feasible alternatives examined for noninferiority is perhaps several times as large as the number of OKA computations shown in the table (see Appendix B for detailed discussion).

Third, strictly speaking, the termination of the multiobjective branch-and-bound process was not based either on  $\alpha$  vector or  $\beta$  vector as suggested in chapter IV but rather based on the number of tree nodes generated. The table, therefore, is not useful for examining the computational efficiency of the multiobjective method as against any other multiobjective network-flow methods which terminate the multiobjective computational process based on some mathematically rigorous criteria. Additional research and computational experience is required to thoroughly test this method.

Table 5.7  
Computational Statistics for Case 2 Example Problem

Case	No. Objectives	B-B Nodes (Active Nodes)	Inspections (Active Inspections)	OKA Computations	Feasible <sup>d</sup> Alternatives	CPU Time <sup>e</sup> (sec.)	Reference in Chapter 5
1	1	3649 (1193)	3052 (596)	597	324	49.87	C.1 COST
2	2	2038 (501)	1787 (250)	502 (251 x 1)	328 (164 x 2)	34.93	C.3-a COST, WQI; $\beta = (1, 1)$
3	2	1006 (563)	724 (281)	562 (281 x 2)	564 (238 x 2)	24.29	C.3-b COST, WQI; $\beta = (1, 100)$
4	3	1527 (1017)	1018 (508)	1527 (509 x 3)	552 (184 x 3)	54.97	C.3-c COST, WQI, LCP; $\beta = (1, 100, 1)$
5	2	2053 (1579)	1263 (789)	1580 (790 x 2)	528 (264 x 2)	1:11.69	C.3-c COST, (100 WQI + LCP); $\beta = (1, 1)$
6	5	3009 (1441)	2288 (720)	3605 (721 x 5)	1770 (354 x 5)	2:49.26	

- <sup>a</sup> Active Nodes: Those node which contributed directly to the generation of alternatives. More alternatives can be generated from the currently inactive nodes by growing the tree further making inactive nodes active or creating new nodes. (See Section 3.B for detailed definition.)
- <sup>b</sup> Active Inspection Nodes: Number of inspections to identify active nodes.
- <sup>c</sup> OKA Computations: For each inspection limb, the number of OKA computations required is the same as the number of objectives included in the analysis. (Number of OKA breakthroughs is not included.)
- <sup>d</sup> Feasible Alternatives: One feasible alternative generated per OKA computation. (Number of OKA breakthroughs is not included.)
- <sup>e</sup> CPU Time: Using DEC10 Time-sharing system at the University of Louisville.



## VI. SUMMARY AND DISCUSSION

A multiobjective branch-and-bound method is proposed for analysing problems involving a network-flow structure which are commonly found in water resources planning field. The method is capable of identifying systematically and efficiently a set of good planning alternatives. The method was applied to two example regional wastewater planning problems.

Although the method proposed is designed to deal with the network-flow structured problems, the application of the branch-and-bound concept to general multiobjective planning analysis itself is quite appealing. By its very structure, the branch-and-bound tree can be extended to any desired set of objective cut-off values, and a set of noninferior solutions can be identified among the generated alternatives. In fact, in many planning problems it is impractical as well as unnecessary to generate all of the noninferior solutions for further elaboration. The multiplier vector  $\vec{\alpha}$  may be used when it is desired to identify a set of noninferior solutions whose objective function values are simultaneously less than or equal to the respective cut-off values. On the other hand, when there is an a priori indication of preferences, the weighting vector  $\vec{w}$  may be used to generate an appropriate set of noninferior solutions. These flexibilities imbedded in the multiobjective branch-and-bound method may prove it worthwhile in the analysis of problems not possessing a network structure.

The multiobjective imputed value analysis is also a potentially useful planning tool. Computationally, it requires no additional mathematical steps other than the transformation of a branch-and-bound tree into a matrix and a search through the matrix. The imputed values obtained for a given problem provide the information which is difficult to obtain efficiently using the conventional mathematical optimization techniques. The imputed value analysis method may be applicable also to problems not possessing a network-flow structure.

A FORTRAN computer programme called MOBNET was developed and used for analysing the example problems. It is an interactive programme which consists of approximately 700 executable statements and provides information on the growth of multiobjective branch-and-bound tree and the generation of noninferior alternatives in the process of computation. The analyst can also control the growth of the tree and the pattern of generation of noninferior alternatives by adjusting  $\alpha$  or  $\beta$  vectors at any time during the execution of the program.

The proposed method has been tested for practical application to regional wastewater and residual management systems. Although the method is found to be capable of providing very useful information on multiobjective network-flow planning problems, there is room for improvements both with regard to the computational aspects of the proposed method and to the refinement of the application procedure. First, the computational efficiency of the method may be improved by refining the proposed multiobjective branch-and-bound algorithm. Emphasis in further research should be placed on dealing with problems having a large number of fixed-charge variables. Second, the application procedure of multiobjective methods in general need further refinements particularly with respect to constructing appropriate objective functions. This aspect was left untouched in this particular research.

A programme listing and user guide of MOBNET is prepared for those who are interested in applying the method to practical problems as well as for carrying out additional research on the subject.

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## Appendix A

The objective functions used in Case 2 Example Application were prepared using the following data:

### 1. Capacity Bounds (Tables A.1, A.2 and A.3).

The capacity bounds of network arcs representing treatment plant, interceptor pipes, and sludge transport routes are shown in tables A.1, A.2 and A.3, respectively.

### 2. Cost Functions

The cost functions used are taken out from the pertinent literature. No rigorous attempt was made to refine the functions or to correct logical inconsistencies (e.g., while plant and interceptor costs exclude operations and maintenance costs, sludge hauling and landfilling include them).

#### a) Treatment Plant Cost (Table A.1)

The cost function obtained from Klemetson and Grenney (1976) was amortized over 25 years using 7% discount rate. The function was adjusted to June 1974.

$$C^T = (1,010 \times 10^3 \cdot Q^{0.78}) \cdot 0.08581 \text{ (dollar/year)}$$

where Q is the plant capacity in MGD.

The piecewise linear approximation used is:

$$CT = 52 (10^3 \text{ dollars/year}) \text{ for } 0 - 10 \text{ MGD}$$

$$CT = 167 + 35 \cdot Q (10^3 \text{ dollars/year}) \text{ for } 10 - 50 \text{ MGD.}$$

b) Interceptor Cost (Table A.2)

The cost function obtained from Klemetson and Grenney (1976) was amortized over 50 years using 7% discount rate. The function was adjusted to June 1974.

$$C_p = (127 \times 10^3 \cdot Q^{0.39}) \cdot 0.07246 \text{ (dollars/year/mile)}$$

the piecewise linear approximation used is

$$C_p = 2.26 \cdot Q (10^3 \text{ dollars/year/mile}) \text{ for } 0-10 \text{ MGD}$$

$$C_p = 16.55 + 0.605 \cdot Q (10^3 \text{ dollars/year/mile}) \text{ for } 10-50 \text{ MGD}$$

c) Sludge Hauling Cost (Table A.3)

The cost function presented in U.S. EPA (Figure 9-78, P9-20, 1978) was adjusted for 1974 using the ENR cost index (i.e., 227 for July 1978 and 176 for the 1974 average).

d) Sludge Landfill Cost (Table A.4)

The cost function presented in U.S. EPA (Figure 9-8, P9-23, 1978) for 1978 was adjusted for 1974 using the ENR cost index (i.e., 227 for July 1978 and 176 for the 1974 average).

3. Water Quality Impact Function (Table A.1)

An attempt was made to include in the analysis a minimizing objective of water quality impact on the receiving stream as a simple function of the amount of treatment effluent discharged from regional plants. The flow and BOD were arbitrarily assigned to streams to which effluent from each plant was to be discharged. The degradation of water quality in terms of BOD was calculated for the effluent flow up to 50 MGD based on the assumption that the effluent BODs from the potential treatment plants were uniformly 20 ppm. The degradation in BOD was translated into the degradation in water quality impact unit using the convention proposed by Dee, et al (1972).



#### 4. Land Contamination Potential (Table A.4)

Referring to the study by Garrigan (1977), an attempt was made also to include in the analysis a minimizing objective of potential contamination of land application site by heavy metals contained in sludge. Based on the hypothetical values of the organic content of land and the maximum permissible metal equivalent at each land application site, the maximum sludge application rate at each site was computed. The land contamination potential was arbitrarily defined to be the inverse of the maximum application rate normalized to a scale of 0 to 1.0. The function obtained for each site is, therefore, a linear function without a fixed-charge.

#### 5. Two Additional Hypothetical Impact Functions (Tables A.1 and A.2)

Two other functions were arbitrarily constructed and used to make the problem a little more complex. The first of these two functions pertains only to potential treatment plant (e.g., water reuse potential) and the second of the two pertains to interceptor routes (e.g., land use impact).

Table A.1  
Capacity Bounds and Objective Functions  
Associated with Treatment Plants

No.	Wastewater Source	Capacity(a)		Plant Cost(b)		Water Quality Impact(c)		Additional Objective 1(c)	
		Wastewater Generated	Minimum	Maximum	Fixed charge	Linear coefficient	Fixed charge	Linear coefficient	Linear coefficient
1	1	2	2	11	167	35	0.110	0.0060	0.010
2	2	4	-	-	-	-	-	-	-
3	3	1	1	11	167	35	0.430	0.0070	0.010
4	4	9	9	50	167	35	0.070	0.0023	0.0033
5	5	14	14	50	167	35	0.050	0.0018	0.0033
6	6	2	-	-	-	-	-	-	-
7	7	1	-	-	-	-	-	-	-
8	8	3	3	50	167	35	0.225	0.0105	0.010
9	9	2	2	2	-	52	0.085	0.0075	0.005
10	10	3	3	4	-	52	0.050	0.0022	0.005
11	11	1	-	-	-	-	-	-	-
12	12	1	-	-	-	-	-	-	-
13	13	7	7	50	167	35	0.060	0.0003	0.010

- (a) Capacity in million gallons per day (MGD) =  $3.785 \times 10^3 \text{ m}^3/\text{day}$ .  
Minimum Capacity = minimum capacity of a plant once it was to be constructed.
- (b) Fixed-charge in  $10^3$  dollars/year, linear coefficient in  $10^3$  dollars/year/MGD.
- (c) In dimensionless unit.

Table A.2  
Capacity Bounds and Objective Functions Associated with Interceptors

No.	Route	Distance (miles)	Capacity(a)		Interceptor Cost (b)		Additional Objective 2(c)	
			Minimum	Maximum	Fixed charge	Linear coefficient	Fixed charge	Linear coefficient
1	1-3	10	2	50	166	6	-	0.001
2	1-4	11	2	50	182	7	0.4	0.003
3	1-12	8	2	7	-	18	0.1	0.003
4	2-1	12	0	4	-	27	-	0.001
5	2-3	12	0	4	-	27	-	0.001
6	3-1	10	0	1	-	23	-	0.001
7	4-5	6	9	50	97	4	0.6	0.003
8	4-8	12	9	50	199	7	-	0.010
9	5-4	6	14	50	99	4	0.6	0.003
10	5-13	12	14	50	199	7	-	0.007
11	6-8	7	2	2	-	2	-	0.001
12	7-8	8	1	1	-	18	-	0.001
13	8-4	12	0	6	-	27	-	0.010
14	10-5	6	1	4	4	11	0.3	0.007
15	11-10	5	1	1	-	11	-	0.001
16	12-1	8	1	8	-	18	0.1	0.003
17	12-13	7	1	8	-	16	0.1	0.003
18	13-5	12	7	50	199	7	-	0.007
19	13-12	7	7	7	-	16	0.1	0.003

- (a) Capacity in million gallons day (MGD) =  $3.785 \times 10^3 \text{ m}^3/\text{day}$ .  
Minimum Capacity = minimum capacity of a pipe once it were to be constructed.
- (b) Fixed-charge in  $10^3$  dollars/year, linear coefficient in  $10^3$  dollars/yer/MGD.
- (c) Fixed-charge and linear coefficient in dimensionless units.

Table A.3  
Capacity Bounds and Objective Functions  
Associated with Sludge Transport Routes

No.	Route	Distance (Miles)	Capacity(a)		Hauling Cost(b)	
			Minimum	Maximum	Fixed Charge	Linear Cost
1	1 - 15	10	1	50	37	2.32
2	3 - 14	5	1	50	37	2.14
3	3 - 19	15	1	50	37	2.50
4	4 - 15	10	9	50	37	2.32
5	4 - 17	20	9	50	37	2.67
6	5 - 17	15	14	50	37	2.50
7	8 - 15	5	9	50	37	2.50
8	8 - 16	15	3	50	37	2.14
9	9 - 17	5	2	2	37	2.14
10	9 - 18	15	2	2	37	2.50
11	10 - 18	5	4	4	37	2.14
12	13 - 19	5	7	50	37	2.14

(a) Capacity in million Gallons per day (MGD) =  $3.785 \times 10^3 \text{ m}^3/\text{day}$ .  
Minimum Capacity = minimum sludge hauling capacity in equivalent amount of wastewater (sludge assumed to be 0.15% of wastewater treated) once the sludge is to be transported using the indicated route.

(b) Fixed-charge in  $10^3$  dollars/year, linear coefficient in  $10^3$  dollars/year/MGD

Table A.4  
Capacity Bounds and Objective Functions  
Associated with Landfill Sites

No.	Landfill site	Capacity(a)		Landfill Cost(b)		Land Contamination(c)
		Minimum	Maximum	Fixed Charge	Linear Cost	Linear Coefficient
1	14	1	50	230	28	0.046
2	15	2	50	230	28	0.054
3	16	6	50	230	28	0.010
4	17	2	50	230	28	0.038
5	18	0	6	-	6	0.081
6	19	7	50	230	28	0.067

(a) Capacity in million gallons per day (MGD) =  $3.785 \times 10^3 \text{ m}^3/\text{day}$ .  
Minimum Capacity = minimum capacity of landfill site if it were to be constructed.

(b) Fixed charge is  $10^3$  dollars/year, linear coefficient in  $10^3$  dollars/year/MGD.

(c) In dimensionless unit.

## Appendix B

### Generating Multiobjective Alternatives Using OKA Breakthrough Information

As mentioned in Section IV-B, multiobjective branch-and-bound trees are grown either by inspection (to the left in Figure 4.2) or by solving linear programming subproblems (to the right in Figure 4.2). For an  $N$ -objective problem there are  $N$  subproblems involved for each extension of a branch to the right, resulting in  $N$  feasible network-flow configurations, each of which can be used as a candidate for a noninferior alternative. In effect then, the total number of feasible alternatives generated is  $N$  times the number of feasible nodes created on the tree, or simply the number of inspection limbs. This number is generally not very large. The number of feasible alternatives generated in the branch-and-bound process, however, can be increased significantly by taking advantage of the network-flow computational process, i.e., the Out-of-Kilter algorithm.

The Out-of-Kilter algorithm makes use of the complimentary slackness condition of linear programming, and, in the process of identifying the optimal network-flow solution to each of the branch-and-bound subproblems, systematically generates a large number of flow patterns. When a flow pattern is feasible, but not necessarily optimal, it is called a breakthrough. In general, a significant number of breakthroughs occurs before an optimal network-flow pattern is identified. If multiple objectives are associated with the network arcs, each breakthrough flow pattern can be regarded as a multiobjective alternative. Suppose, there is an average of  $M$  breakthroughs per network-flow subproblem computation, then the number of feasible alternatives which can be identified per multiobjective-inspection limb is  $M \times N$  at the maximum (some breakthrough network flow patterns may be redundant).

## Appendix C

### Multiobjective Network-Flow Analysis of a Production-Distribution System.

Multiobjective network-flow problems are commonly found in the fields other than water resources and environmental engineering field. Most public sector planning problems which involve network-flow structure are by nature multiobjective, and the proposed method may be successfully applied. On the other hand, profit-orientated private sector planning problems are generally more inclined to be single-objective, and efficient single objective network-flow analysis methods have found extensive application possibilities. Even what seems to be ordinary single-objective problems, however, may find multiobjective analysis quite useful. For example, just as in the case of one of the example problems presented in Chapter V, cost minimization may be achieved taking several cost components as separate minimizing objectives since the factors affecting each cost component may be quite different, and the trade-off information between the cost components may serve a useful purpose.

An attempt was made to apply the proposed method to the analysis of typical commodity production-distribution system (Bloemer, 1981). The example problem analysed involved minimization of production cost, distribution cost and warehousing cost while preserving the integrity of the network involving the flow of a commodity. The analysis results indicate the potential usefulness of the method particular in view of the fact that the method is capable of generating efficiently a large number of alternatives, some of which are likely to be noninferior and that various marketing strategies may be examined based on the analysis of different cost compounds.

