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Hydraulic Analysis of Surcharged Storm Sewer Systems

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HYDRAULIC ANALYSIS OF SURCHARGED STORM SEWER SYSTEMS

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ABSTRACT

HYDRAULIC ANALYSIS OF SURCHARGED STORM SEWER SYSTEMS

Surcharge in a storm sewer system is the condition in which an entire sewer section is submerged and the pipe is flowing full under pressure. Flow in a surcharged storm sewer is essentially slowly varying unsteady pipe flow and methods for analyzing this type of flow are investigated. In this report the governing equations for unsteady fluid flow in pressurized storm sewers are presented. From these governing equations three numerical models are developed using various assumptions and simplifications. These flow models are applied to several example storm sewer systems under surcharge conditions. Plots of hydraulic grade and flow throughout the sewer network are presented in order to evaluate the ability of each model to accurately analyze surcharged storm sewer systems. Computer programs are developed for each of the models considered and these programs are presented and documented in the Appendix of this report.

Descriptors: storm sewer, surcharged, pressurized, unsteady, transient

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INTRODUCTION

A storm sewer system is characterized by a series of manholes or junctions (nodes) which are connected by sewer pipes (links) to form a network. Manholes serve two main purposes in drainage systems. They provide access to the sewer system for maintenance and repair and they act as a junction box for the connection of vertical drop inlets and sewer lines. Most storm systems are of the branched or tree type since looped systems are difficult to analyze. The fluid flow in storm sewers is classified as transient or unsteady since the flow source, the rainstorm, is a time varying phenomenon.

Many flow conditions are possible in a storm sewer during a storm event and a typical storm flow cycle may be as follows: At the onset of a rainstorm most storm sewers begin with dry bed or small base flow conditions. As the storm intensifies with time, runoff accumulates and eventually enters the sewer system by way of manholes or other vertical inlets. Sewer flow at this point is small and is classified as open channel in which gravity flow prevails. This type of flow condition is most common in storm sewers under typical rainstorm events. If the storm and runoff increase further in magnitude a change from open channel-gravity flow to pressurized-closed conduit flow is likely to occur. This is known as a two-phase flow transition and is one of the most complicated and largely unsolved problems in storm sewer analysis (50). Additional storm loading may eventually force the complete system to behave under pressurized or surcharged flow conditions. Surcharge in a storm sewer system is defined as the condition in which the sewer pipe is flowing full under pressure. With severe storm events advanced stages of surcharge may cause surface flooding. This is the most extreme flow condition which can exist in a storm sewer network.

Traditionally, storm sewer systems are designed assuming open channel flow due to the complexity and cost of a two phase flow analysis which includes the transition from gravity flow to surcharge flow. Pipeline sizes and manhole locations are often determined from simplified hydraulic nomographs which insure open channel flow for a given 'design storm'. Any storm event with equal or less intensity than that of the design storm will be safely contained in the sewer system. This design procedure is popular because of its low cost and simplicity. However, due to subsequent development of the watershed or additions or alterations to the storm sewer system, it is possible that the assumption that open channel flow always exists is not valid. Also, a certain degree of surcharging may be perfectly acceptable and the design which does not allow this is conservative and may result in excessive costs. Therefore, it may be desirable or necessary to consider the storm sewer system operating in a surcharged condition.

Several situations which lead to surcharged flow conditions are as follows:

(a) Underdesigned systems as a result of using simplified flow equations or hydraulic nomographs when sizing hydraulic structures (piping, manholes, etc.)

(b) System overloading in the upper segments while the lower segments may be flowing well below the design capacity.

(c) System overloading due to alterations and/or extensions of existing storm sewer systems.

(d) Construction errors and/or material defects in the storm sewer system.

(e) A hydrologic risk due to the possibility that the design flow of the storm sewer system will be exceeded during its service life.

(f) Surface drainage basin changes which may increase runoff into the storm sewer system.

(g) Failure of in-line pumping facilities

Hence, to be able to properly judge the performance of a storm sewer system the design engineer must be able to properly evaluate surcharge flow conditions.

Presently, several advanced computer models are available which route storm sewer flow using various forms of the full dynamic equations for unsteady open channel and pressurized flow. Typically, however, these routing models are extremely complex and require considerable computer time on large computers. These models are discussed in Chapter 2.

As a feasible alternative to these complex unsteady flow models it is proposed that a single-phase surcharge flow model be developed to aid in the design of storm sewer networks. Such a model would accurately predict pressure and flow under the most extreme conditions, that of surcharge and flooding. Consequently, the model would not route low flow open channel conditions. One of the most important design considerations in storm sewer analysis is the proper handling of storm water under peak flow conditions. Therefore, primary consideration is given to storm events which fully load and overload the systems. Any storm of minor intensity (less than the design storm) would be contained in the sewers and cause no problems.

The principle objective of this research is to carry out a preliminary investigation for developing a hydraulic flow model for analysis of storm sewer systems at peak flows. In this thesis the governing partial differential equations of unsteady (transient) flow are formulated for specific application to surcharged storm sewer systems. From these governing equations three numerical models are developed using various assumptions and simplifications. These include: a) an implicit finite element unsteady distributed parameter flow model b) a explicit dynamic lumped parameter flow model using Euler forward differencing and c) a kinematic (steady state with storage) flow model. These models vary greatly in complexity and amount of computations required and it is essential to evaluate the ability of the models to analyze surcharged storm sewer flow.

Five examples are presented to illustrate the ability of each of the three models to accurately predict peak flow conditions in storm sewers. Based on these results and the author's familiarity with the various models, a recommendation will be presented for further investigation and eventual development of a workable, well documented computer flow model for the analysis of storm sewers operating at peak flow capacity.

REVIEW OF EXISTING STORM SEWER WORK

In the past decade several storm sewer flow routing models have been developed, ranging from the popular rational method (1) to the complex computer based Storm Water Management Model (SWMM) (26). The majority of these methods are open channel flow and/or pressurized flow models. The primary research herein is concerned with developing a model for analyzing surcharge in storm sewer systems which is a pressurized flow phenomena. For an in depth review of existing open-channel flow models the reader is referred to several published references: Chow and Yen (8,49); Brandstetter (4); and Cloyer and Pethick (9); Burke and Gray (5).

2.1 Pressurized Flow Models.

The majority of pressurized flow models are steady flow models developed for the analysis of water distribution systems and not for the specific application to storm sewer analysis. These models handle both looped and branching networks in using one of several methods: those which utilize the Hardy Cross method of flow adjustment (Hardy Cross (12), Dillingham (13)); those methods using simultaneous flow adjustment (Epp and Fowler (14), Martin and Peters (22), Jeppson (17), Lemieux (20)); and those using linearization techniques (Wood and Charles (44)). Of these steady flow models only Wood, using the linear theory, has addressed surcharged storm sewer analysis (45).

Transient flow models have been developed by Wylie and Streeter (48), and Chaundhry (6) (method of characteristics) and Wood (42) (wave plan method) but are concerned primarily with surge or water hammer analysis. These methods, however, can be readily modified for the analysis of surcharged flow in storm sewers.

2.2 Surcharge Storm Sewer Flow Models

Recently several flow routing models have been developed to handle surcharge in storm sewer systems. Most of these models use the Manning or Darcy-Weisbach formulas coupled with steady flow theory to approximate surcharge flow.

The TRRL (41), Chicago Hydrograph Method (37) and ILLUDAS (35) are steady flow hydrograph routing models which consider the effects of in-line storage. In these models the sewer flow is routed pipe by pipe from upstream to downstream in a cascading manner. Hydrograph inflow and junction pressure heads are related to the steady flow equations through a junction storage continuity equation.

The popular Storm Water Management Model (26) routes the stormwater using the Saint Venant Equations for unsteady spatially varied open channel flow in a computer model called EXTRAN. Whenever surcharge occurs, a modified continuity relationship is satisfied at each junction to predict the manhole pressure heads. If flooding occurs the excess surface water is assumed lost and not recoverable.

Several storm sewer flow models handle surcharge by using the so-called Preissmann slot technique. These include the French model CAREDAS (7); the Danish Hydraulic Institute model, System 11 Sewer (15); and DAGVL-A and DAGVL-DIFF (28) developed at Chalmers

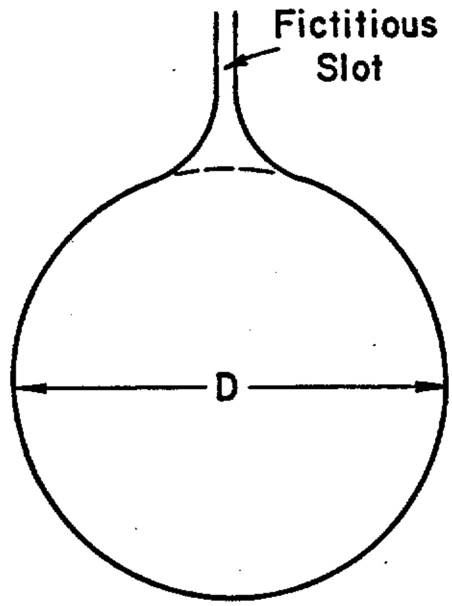


Fig. 2-1. Priessmann Slot Technique.

University in Sweden. With these models, pressurized flow is transformed into artificial open channel flow by the introduction of a ficticious slot at the sewer crest which runs the entire sewer length (Fig. 2-1). Consequently, both open channel and surcharge flow are handled using the full Saint Venant Equations.

A two phase flow hydraulic model presented by Song (29,30) handles both open channel and pressurized flow using the method of characteristics. The flow is characterized by the existence of moving interfaces which divide the system into open channel and pressurized flow. Presently the model does not account for manhole storage, junction losses or surface flooding.

The most in depth treatment of surcharge in storm sewer systems is given by Yen (24,52) in a kinematic wave surcharge model called SURKNET. The hydraulics of surcharge sewer flow along with open channel flow are developed using the kinematic wave equations together with Manning's formula to estimate the friction slope. Manhole storage and surface flooding are accounted for through use of the unsteady junction continuity equation. The present SURKNET model solves for flow in the pipes independently in a cascading manner from upstream towards downstream. A more advanced dynamic wave model which solves the system of pipes simultaneously is being developed and has not yet been published.

Wood (45,46) suggested that steady state pressurized flow theory be applied to the analysis of surcharge in storm sewer systems. A sewer network analysis is carried out by computing steady state pressure and flow conditions at a specific point in time. These steady flow conditions coupled with hydrograph inflows are used to predict the change in manhole surface water levels over the next time interval. The steady state solution is then obtained using the new manhole water levels. The time step used for the simulation must be small for accurate flow and pressure predictions.

Several other related surcharge flow models have been presented by Bettess et al. (3), Martin and King (21) and Toyokuni (39).

2.3 Other Related Surcharge Work

Very little experimental data is available for storm sewer systems operating under surcharge conditions. Land and Jobson (19) developed an unsteady flow model for a single pipe subject to surcharge conditions. The flow model was used with experimental pressure (water level) data to predict the discharge for a fully submerged section of storm sewer. It is suggested that accurate simultaneous water level data is required for reasonable model predictions.

Nearly all the published surcharge prediction models utilize a quasi-steady flow storage equation at the manhole junctions. Presenty this is an acceptable method for analyzing the hydraulics of storm sewer junctions. An unsteady pressurized junction continuity relation has yet to be developed. Joliffe (18) has developed a momentum balance steady flow continuity relation for open channel sewer flow and applied it to unsteady flow behavior ay pipe junctions.

The energy and friction losses in storm sewer analysis are handled using steady flow relations. Unsteady energy loss expressions are nonexistant. Sangster et al. (27) performed experimental studies on pressure losses at surcharged sewer junctions. Yevjevich and Barnes (53) have studied both experimental and theoretical applications of open channel flood routing through storm drains. Particular attention is given to developing expressions for unsteady junction box energy losses and these expressions need only be applied to surcharge flow analysis.

THEORY OF ONE DIMENSIONAL UNSTEADY FLOW

Two basic mechanics equations are applied to a free body of fluid to obtain two partial differential equations which describe unsteady (transient) flow in closed conduits. These include: conservation of mass (continuity) and Newton's second law of motion (momentum). In this derivation the dependent variables are centerline pressure P(x,t) and the average velocity V(x,t) at a cross section. The independent variables are position, x, measured along the axis of the pipe and time, t. For convenience the pressure, P, and velocity, V, are converted to the piezometric head, H, and flowrate, Q, respectively. These continuity and momentum equations are derived using the simplified free body approach similar to that used by Wylie and Streeter (48), Thorley et al. (38), and Bergeron (2).

3.1 Equation of Continuity (Mass Conservation)

The continuity equation is developed from the law of conservation of mass which states that the mass within a system remains constant with time. Therefore,

$$dm/dt = 0 \qquad (3\text{-}1)$$

where m is the total mass of the system.

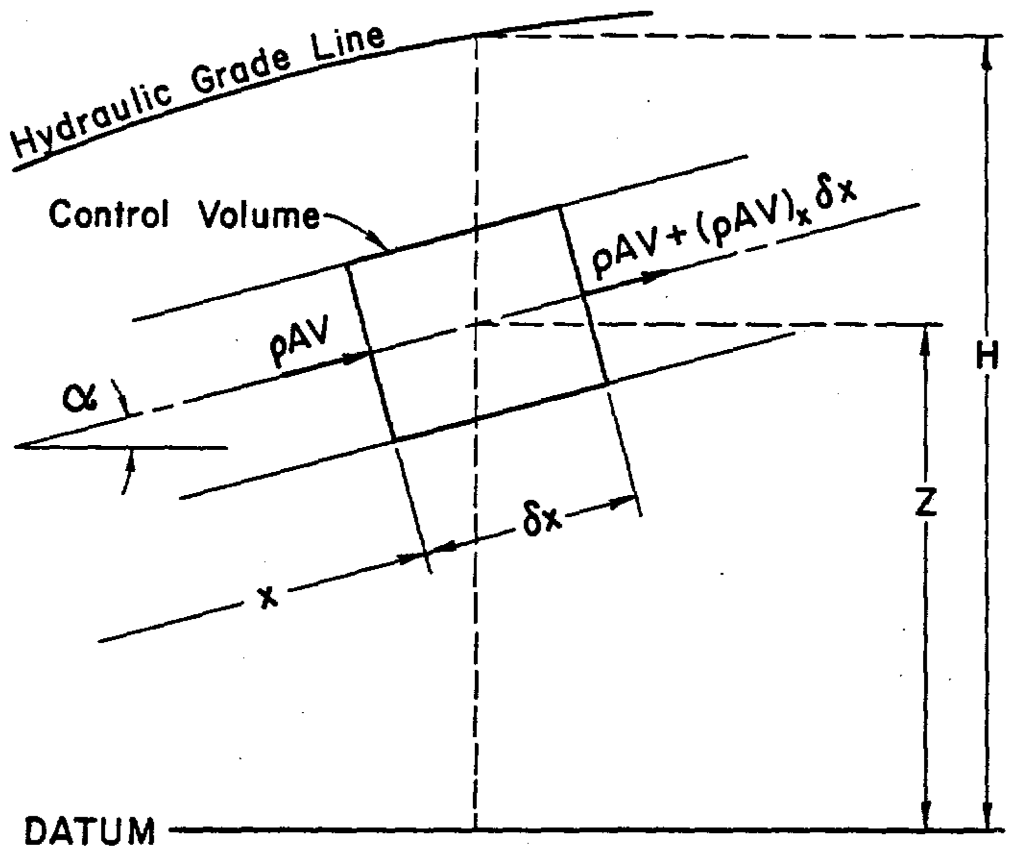


Figure 3-1. Application of Control Volume for Continuity Equation.

By applying the law of mass conservation (Eq. 3-1) to the control volume in Fig. 3-1 the continuity equation for unsteady flow is obtained:

$$\rho AV - \left[\rho AV + \frac{\partial}{\partial x}(\rho AV)\,\delta x\right] = \frac{\partial}{\partial t}(\rho A)\,\delta x \qquad (3\text{-}2)$$

The law of conservation of mass may be stated as the net rate of mass inflow into a control volume is equal to the time rate of increase of mass within the control volume.

Expanding Eq. 3-2 and rearranging yields

$$\frac{V}{A}\frac{\partial A}{\partial x} + \frac{1}{A}\frac{\partial A}{\partial t} + \frac{V}{\rho}\frac{\partial \rho}{\partial x} + \frac{1}{\rho}\frac{\partial \rho}{\partial t} + \frac{\partial V}{\partial x} = 0 \qquad (3\text{-}3)$$

The first four terms are the total derivatives of area A, and density ρ, respectively. Therefore

$$\frac{1}{A}\frac{dA}{dt} + \frac{1}{\rho}\frac{d\rho}{dt} + \frac{\partial V}{\partial x} = 0 \qquad (3\text{-}4)$$

where $\frac{d}{dt} = \left(V \frac{\partial}{\partial x} + \frac{\partial}{\partial t} \right)$

The first term of Eq. 3-4 describes the elasticity of the pipe material and its rate of deformation with varying pressures. The second term describes the compressibility of the liquid. The last term accounts for the change in flow velocity at any instant. Equation 3-4 is valid for converging or diverging pipes, liquid or gas flow since no simplifying assumptions have been made. However, this work deals with prismatic conduits and utilizes the appropriate assumptions. The reader is referred to Fluid Transients (48) for proper handling of non-prismatic conduits.

To simplify the circumferential pipe expansion term

$$\frac{1}{A}\frac{dA}{dt} = \frac{1}{A}\left(V \frac{\partial A}{\partial x} + \frac{\partial A}{\partial t} \right)$$

For prismatic conduits $\frac{\partial A}{\partial x} = 0$ and

$$\frac{1}{A}\frac{dA}{dt} = \frac{1}{\pi r^2}\frac{\partial(\pi r^2)}{\partial t} = \frac{2}{r}\frac{\partial r}{\partial t} \qquad (3\text{-}5)$$

with $$\frac{\partial r}{\partial t} = r \frac{\partial \varepsilon_2}{\partial t} \tag{3-6}$$

where r is the pipe radius and ε_2 is the circumferential or hoop strain.

By combining Eqs. 3-5 and 3-6 and utilizing $\frac{\partial \varepsilon_2}{\partial t} = \frac{\partial \varepsilon_2}{\partial P} \frac{dP}{dt}$

$$\frac{1}{A} \frac{dA}{dt} = \frac{2\partial \varepsilon_2}{\partial P} \frac{dP}{dt} \tag{3-7}$$

assuming that $\varepsilon_2 = f(P)$

From the definition of bulk modulus of elasticity of fluid (32)

$$K = \frac{dP}{(d\rho/\rho)}$$

which gives

$$\frac{1}{\rho} \frac{d\rho}{dt} = \frac{1}{K} \frac{dP}{dt} \tag{3-8}$$

Substituting Eqs. 3-7 and 3-8 into Eq. 3-4 results in the following form of the equation of continuity.

$$\left[2 \frac{\partial \varepsilon_2}{\partial P} + \frac{1}{K}\right] \frac{dP}{dt} + \rho \frac{\partial V}{\partial x} = 0 \tag{3-9}$$

In order to further expand the term $\partial \varepsilon_2/\partial P$ it is necessary to consider the manner in which the conduit deforms and various constraint conditions.

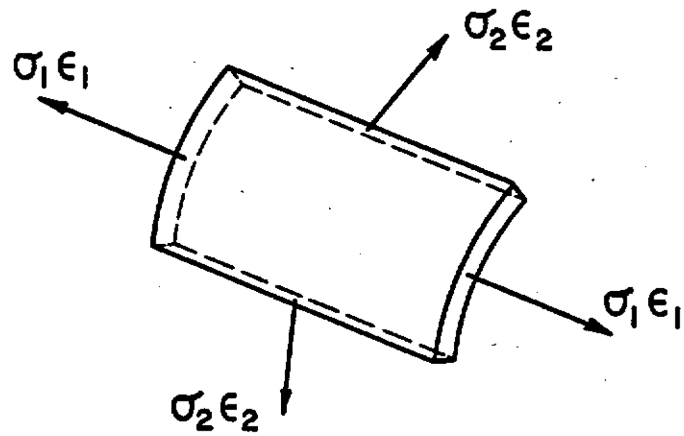


Figure 3-2. Stresses (σ) and Strains (ε) Shown Acting on a Pipe Wall.

In most water transporting systems the ratio of pipe diameter D to pipe wall thickness e is greater than 25 allowing for the application of 'thin walled' steady state stress theory.

The following conditions hold for a 'thin-walled' closed conduit subjected to changing pressures:

axial stress $$\sigma_1 = \frac{PD}{4e} \qquad (3\text{-}10)$$

hoop or circumferential stress $$\sigma_2 = \frac{PD}{2e} \qquad (3\text{-}11)$$

axial strain $$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \mu\sigma_2) \qquad (3\text{-}12)$$

hoop or circumferential strain $$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \mu\sigma_1) \qquad (3\text{-}13)$$

where E is the modulus of elasticity of the pipe material and μ is Poisson's ratio.

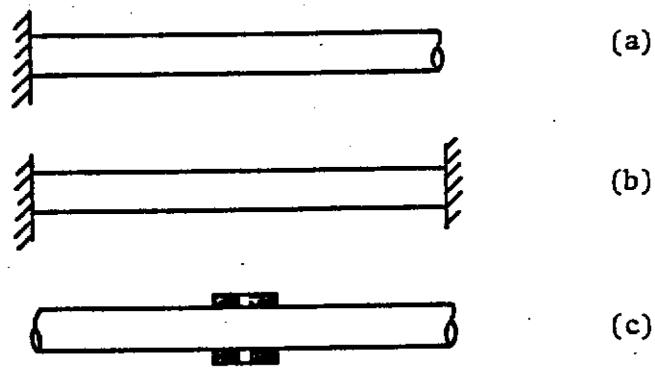


Figure 3-3. Pipeline Constraint Conditions.

Three possible constraint conditions exist as shown in Fig. 3-3:

Case a — The pipeline is anchored at the upstream end only.

From Eqs. 3-10, 3-11, 3-13 $\quad \varepsilon_2 = \frac{PD}{2eE}\left(1 - \frac{\mu}{2}\right)$

therefore

$$\frac{\partial \varepsilon_2}{\partial P} = \frac{D}{2eE}\left(1 - \frac{\mu}{2}\right) \tag{3-14}$$

Case b — The pipeline is prevented from any axial movement ($\varepsilon_1 = 0$).

From Eq. 3-12 $\quad \sigma_1 = \mu\, \sigma_2$

From Eq. 3-13 $\quad \varepsilon_2 = \frac{PD}{2eE}\left(1 - \mu^2\right)$

therefore

$$\frac{\partial \varepsilon_2}{\partial P} = \frac{D}{2eE}(1 - \mu^2) \quad (3\text{-}15)$$

Case c The pipeline has expansion joints throughout the length of the pipe ($\sigma_1 = 0$).

From Eq. 3-13 $\varepsilon_2 = \frac{PD}{4eE}$

therefore

$$\frac{\partial \varepsilon_2}{\partial P} = \frac{D}{2eE} \quad (3\text{-}16)$$

Equation 3-9 through substitution of $1/c^2$ for the coefficient of dP/dt takes the general form

$$\rho \frac{\partial V}{\partial x} + \frac{1}{c^2}\frac{dP}{dt} = 0 \quad (3\text{-}17)$$

in which

$$c^2 = \frac{K/\rho}{1 + [(K/E)(D/e)c_1]} \quad (3\text{-}18)$$

From Eqs. 3-14, 3-15 and 3-16, c_1 is defined for each case:

(a) $c_1 = 1 - \mu/2$

(b) $c_1 = 1 - \mu^2$ (3-19)

(c) $c_1 = 1$

In Eq. 3-17, c is the wave speed at which the pressure transient propagates through the fluid medium.

As stated previously most water transporting systems contain piping materials which can be classified as 'thin-walled'. In pressurized storm sewer applications corrugated steel pipe (CSP) is the most commonly found thin walled material. However, many storm sewers are constructed using vitrified clay pipe, nonreinforced concrete pipe, reinforced concrete pipe, and others. Materials such as these are classified as 'thick-walled' elastic pipe materials in which the walls are relatively thick in comparison to the diameter ($D/e <= 25$). In such cases the following c_1 coefficients should be used for the appropriate constraint condition in Eq. 3-18.

Case a — The pipeline is anchored at the upstream end only

$$c_1 = \frac{2e}{D}(1 + \mu) + \frac{D}{D+e}\left(1 - \frac{\mu}{2}\right)$$

Case b — The pipeline is prevented from any axial movement.

$$c_1 = \frac{2e}{D}(1 + \mu) + \frac{D}{D+e}(1 - \mu^2)$$

Case c — The pipeline has expansion joints throughout the length of the pipe.

$$c_1 = \frac{2e}{D}(1 + \mu) + \frac{D}{D+e}$$

It should be noted that in thick-walled pipe materials the type of constraint condition has little effect on the wave speed.

For composite materials such as reinforced concrete pipe, the dimensionless coefficient, c_1, may be estimated by replacing the actual pipe with an equivalent steel pipe based on the amount of steel reinforcing and the thickness of the pipe. An equivalent steel pipe thickness is obtained from the ratio of elastic modulus of concrete to that of steel multiplied by the concrete thickness.

Other special considerations for materials such as plastic pipes, lined concrete pipes, circular tunnels, etc. can be found in Fluid Transients (48) from which the thick walled information was obtained.

For ease of application the piezometric head, H, defined as the elevation of the hydraulic grade line above a given arbitrary datum, replaces P the fluid pressure. From Fig. 3-1

$$P = \rho g(H-z)$$

Where

$$\frac{dP}{dt} = \rho g \left(\frac{dH}{dt} - \frac{dz}{dt}\right) = \rho g\left(V\frac{\partial H}{\partial x} + \frac{\partial H}{\partial t} - V\frac{\partial z}{\partial x} - \frac{\partial z}{\partial t}\right) \qquad (3\text{-}20)$$

assuming the pipe has no motion in time $\frac{\partial z}{\partial t} = 0$ and $\frac{\partial z}{\partial x} = \sin\alpha$ Eq. 3-17 becomes

$$V\frac{\partial H}{\partial x} + \frac{\partial H}{\partial t} - V\sin\alpha + \frac{c^2}{g}\frac{\partial V}{\partial x} = 0 \qquad (3\text{-}21)$$

Eq. 3-21 is the complete governing equation of continuity (mass conservation) for one dimensional unsteady (transient) liquid flow in prismatic conduits.

3.2 Equation of Motion (Momentum)

Figure 3-4 shows a free body of fluid with cross sectional area A, and differential length, dx. The area is described as a function of x which is the centerline position of the free body measured from an arbitrary origin. The centerline x-axis of the free body is inclined at an angle α with the horizontal. This angle α is positive when the elevation increases in the positive x-direction.

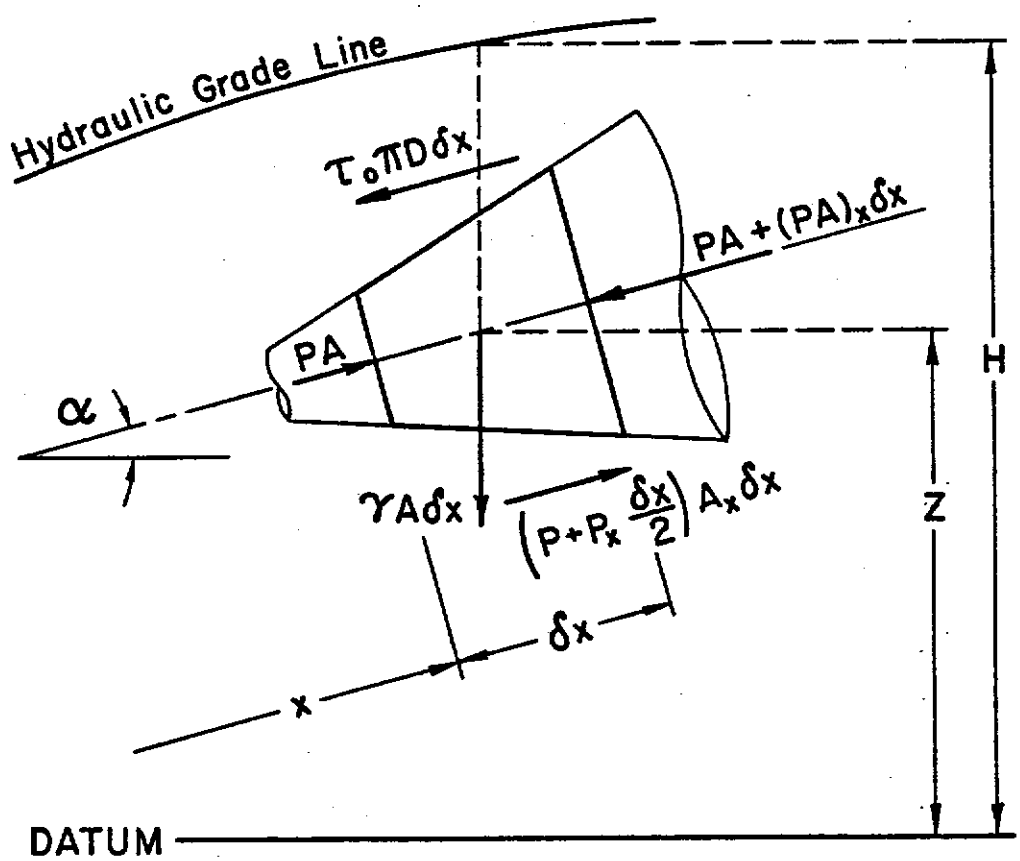


Figure 3-4. Free Body Diagram for the Momentum Equation.

Newton's second law of motion for a fluid element is defined as

$$\Sigma F = \frac{d(mv)}{dt} \tag{3-22}$$

where m is the constant mass of the element and v is the velocity of the mass center. ΣF refers to the resultant of all external forces acting on the element including body forces.

Application of Newton's second law of motion to the free body shown in Fig. 3-4 yields the following equation:

$$PA - [PA + \frac{\partial(PA)}{\partial x}\delta x] + [P + \frac{\partial P}{\partial x}\frac{\delta x}{2}]\frac{\partial A}{\partial x}\delta x$$

$$- \tau_0 \pi D \delta x - \gamma A \delta x \sin\alpha = \rho A \delta x [V\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t}]$$

The left-hand side of the equation represents the forces acting on the free body in the x-direction. They include the surface contact normal pressures, the peripheral pressure components, the frictional shear component, and the body force or gravity component. Since the shear force, τ_o, is considered a resistance to flow term it is assumed to act in the - x direction. The right hand side of the equation is simply the mass acceleration of the fluid body.

Neglecting the small quantity $(\delta x)^2$ and simplifying gives:

$$A\frac{\partial P}{\partial x} + \tau_0 \pi D + \gamma A \sin\alpha + \rho A V\frac{\partial V}{\partial x} + \rho A\frac{\partial V}{\partial t} = 0 \tag{3-23}$$

It is necessary to make some assumptions concerning the frictional shear resistance term, $\tau_o \pi D$. If it is reasonable to neglect frictional effects, the second term in Eq. 3-23 is zero. Throughout this report, however, the frictional shear resistance term is considered to be significant and is treated as if the flow is steady. In terms of the Darcy-Weisbach friction factor, f (32):

$$\tau_0 = \frac{\rho f V|V|}{8} \tag{3-24}$$

This equation is developed from the Darcy-Weisbach equation of the form

$$\Delta P = \gamma\, h_f = \frac{\rho f L V|V|}{2D} \tag{3-25}$$

and a force balance (Fig. 3-5) on a pipe under steady state flow conditions

$$\Delta P\,\frac{\pi D^2}{4} = \tau_0 \pi D \Delta L \tag{3-26}$$

The absolute value sign is applied to the velocity term in Eq. 3-24 to insure that the shear stress always opposes the direction of flow.

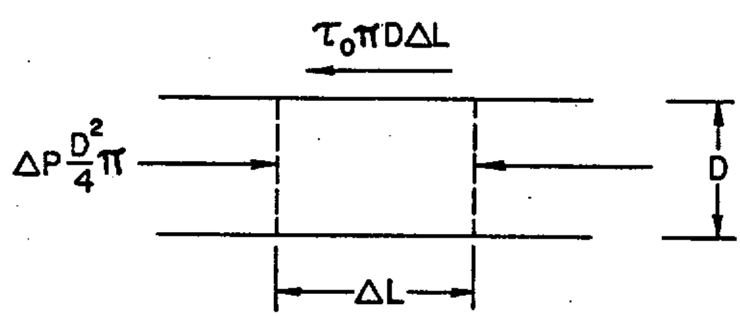


Figure 3-5. Force Balance on a Pipe.

Until recently the shear stress or friction term in an unsteady flow analysis was often neglected or the friction factor, f, was assumed constant for a given simulation. This was due primarily to the mathematical difficulty of modeling the friction term for a wide range of continuously changing flowrates. Traditional methods of determining f were based on the Moody diagram (23), a graphical procedure or empirical implicit formulas such as those developed by Colebrook (10). In 1966 Wood (43) developed the first empirical explicit friction factor relationship of the Colebrook equation. And most recently, in 1976, Swamee and Jain (33) developed the following explicit formula for f with several restrictions placed on it.

$$f = \frac{0.25}{[\log(\varepsilon/(3.7D) + 5.74/(R^{0.9}))]^2} \qquad (3\text{-}27)$$

f = friction factor

R = Reynolds Number $\quad 5000 <= R <= 10^8$

ε = roughness

D = pipe diameter $\quad 10^{-6} <= \varepsilon/D <= 10^{-2}$

The Jain equation is used to approximate the friction factor f for all flow conditions unless f is assumed to be constant. This is a valid approximation of f since nearly all storm sewers flowing under surcharged conditions have high Reynolds numbers (> 5000) and ε/D ratios within the limits $10^{-6} <= \varepsilon/D <= 10^{-2}$.

(3-28)

By combining Eq. 3-24 with Eq. 3-23 and utilizing Eq. 3-27

$$\frac{1}{\rho}\frac{\partial P}{\partial x} + V\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} + g \sin \alpha + \frac{fV|V|}{2D} = 0 \qquad (3\text{-}28)$$

which is the equation of motion (momentum) for converging or diverging fluid pipe flow.

As previously introduced the piezometric head, H, replaces the pressure, P, using

$$P = \rho g(h-z)$$

where z is the elevation of the center line of the pipe at position x.

Then

$$\frac{\partial P}{\partial x} = \rho g \left(\frac{\partial H}{\partial x} - \frac{\partial H}{\partial z} \right) = \rho g \left(\frac{\partial H}{\partial x} - \sin \alpha \right) \qquad (3\text{-}29)$$

in which ρ is assumed to be constant when compared to the fluid depth, H. Substituting Eq. 3-29 into Eq. 3-28 yields

$$g\frac{\partial H}{\partial x} + V\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} + \frac{f\,V|V|}{2D} = 0 \qquad (3\text{-}30)$$

While Eq. 3-28 is valid for any fluid (liquid or gas) Eq. 3-30 is valid only for liquids because of the assumptions considered in Eq. 3-29. This restriction does not in any way limit the unsteady flow simulation with respect to the analysis of pressurized storm sewer systems since this study is limited to surcharged flow which neglects any formation of air pockets or cavities within the system. Therefore Eq. 3-30 is the governing equation of motion (momentum) for one dimensional unsteady (transient) liquid flow as applied to pressurized liquid systems.

3.3 Governing Equations for Unsteady Surcharged Sewer Flow

Summarizing the theoretical development, the two governing differential equations for one dimensional unsteady flow in slightly deformable conduits are:

Continuity: $$V \frac{\partial H}{\partial x} + \frac{\partial H}{\partial t} - V \sin \alpha + \frac{c^2}{g} \frac{\partial V}{\partial x} = 0 \qquad (3\text{-}21)$$

Momentum: $$g \frac{\partial H}{\partial x} + V \frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} + \frac{fV|V|}{2D} = 0 \qquad (3\text{-}30)$$

These are quasi-linear hyperbolic partial differential equations containing two dependent variables (P,V) and two independent variables (x,t). The pressure and velocity of the liquid are a function of both the position and the time from which the steady state conditions are disturbed.

In general, a hydraulic analysis of a storm sewer system is considered a slowly varying transient phenomena. Therefore, several terms in Eqs. 3-21 and 3-30 can be justifiably neglected when application is restricted to this type of slowly varying flow problem.

In both Eqs. 3-21 and 3-30 the convective acceleration terms $V(\partial V/\partial x)$ and $V(\partial H/\partial x)$ are always small when compared to the local acceleration terms $\partial V/\partial t$ and $\partial H/\partial t$ respectively. They are usually of the order of V/c which is typically less than 1/100 (25, 40).

By neglecting the convective terms and substituting $V = Q/A$ the continuity and momentum equations take the familiar form

Contnuity: $$\frac{\partial H}{\partial t} - \frac{Q}{A}\sin\alpha + \frac{c^2}{gA}\frac{\partial Q}{\partial x} = 0 \qquad (3\text{-}31)$$

Momentum: $$gA\frac{\partial H}{\partial x} + \frac{\partial Q}{\partial t} + \frac{f\,Q|Q|}{2AD} = 0 \qquad (3\text{-}32)$$

For the remainder of this report the above simplified forms of Eqs. 3-21 and 3-30 will be consistently referred to as the governing equations of continuity and momentum as they apply to pressurized storm sewer systems.

3.4 Classification of Pressurized Storm Sewer Flow

Depending on the flow conditions, pressurized storm sewer flow may be classified as

a) transient (unsteady)

b) dynamic

c) kinematic

With transient or unsteady flow, the fluid is considered compressible and the transient takes the form of a moving pressure wave. The pressure wave travels through the fluid with a velocity, c, as discussed in Section 3.1. Fluid systems with severe transients are handled using a distributed parameter analysis which takes into account the fluid and pipe elasticity (capacitance), the fluid inertia and the frictional losses. The distributed parameter model is discussed in Section 4.1.

In the analysis of dynamic flow, the change in flow conditions at all points in a pipe section are assumed to occur instantaneously. The fluid is assumed to act as a rigid column in which the inertial effects are lumped over the pipe length. The dynamic flow analysis is handled using lumped parameter theory as discussed in Section 4.2.

Under kinematic flow conditions the pressure, velocity, and flowrate are determined at any instant using steady state approximations. The kinematic or steady model neglects any capacitance or inertial effects on the flow in the system.

Pressurized storm sewer flow is correctly represented using a distributed parameter analysis. However, there are situations in which a lumped (dynamic) parameter or kinematic (steady) analysis will yield satisfactory results. In general a distributed parameter analysis is necessary if $\omega L/c$ is greater than 1.0 (48,6). In this relation ω is the circular frequency, L is the pipe length, and c is the wave speed. In many transient flow situations, however, it is difficult to obtain the circular frequency, ω.

Another method is presented in order to evaluate the effects of capacitance, inertia and friction for a pressurized fluid piping system. From the governing equations of continuity and momentum (Eqs. 3-31, 3-32) the forces due to elasticity (capacitance), inertia, and friction are

$$F_e = \Delta t \frac{c^2}{gA} \frac{\Delta Q}{\Delta x}$$

$$F_i = \frac{\Delta x}{gA} \frac{\Delta Q}{\Delta t}$$

$$F_f = \frac{\Delta x f Q^2}{2A^2 D g}$$

In order to determine the relative effect of the elasticity and inertia forces to the friction force, dimensionless λ coefficients are defined as

$$\lambda_1 = \frac{F_e}{F_f} = \frac{\Delta Q}{\Delta x} \frac{c \pi D^3}{2fQ^2}$$

where $x = c\ \Delta t$

$$\lambda_2 = \frac{F_i}{F_f} = \frac{\Delta Q}{\Delta t} \frac{\pi D^3}{2fQ^2}$$

In general if λ_1 is greater than 1.0 for a system, a transient distributed parameter analysis should be used to evaluate the flow in that system. Likewise if λ_2 is greater than 1.0 the inertial effects are significant and a dynamic lumped parameter analysis is recommended. If λ_1 and λ_2 are much less than 1.0 a kinematic solution is acceptable.

The pressurized storm sewer flow models developed in Chapter 4, calculate the maximum values of λ_1 and λ_2 for each system analyzed and are presented for each example in Chapter 5.

PRESSURIZED STORM SEWER SYSTEM MODELS

Three numerical hydraulic flow models are developed for the analysis of storm sewer systems at peak flows. These include a finite element unsteady distributed parameter model, a dynamic lumped parameter model and a kinematic (steady state with storage) model. In this chapter, each model is formulated and presented with the appropriate assumptions. In addition, Sections 4.4 and 4.5 describe the boundary conditions which are incorporated into the system equations for each flow model. For each numerical method, a computer flow model program is written and presented in Chapter 7.

4.1 Finite Element Model

In this section a numerical solution of the complete governing flow equations of momentum and continuity for pressurized storm sewer systems is presented using the finite element method (FEM). The finite element method described here is the basis for an unsteady distributed parameter flow model. The solution is obtained by solving Eqs. 3.31 and 3.32 simultaneously with appropriate simplifying assumptions and boundary conditions. A brief description of the FEM follows.

The FEM is a numerical procedure for solving differential equations of physics and engineering. The fundamental concept of the FEM is that any continuous quantity such as temperature, pressure, flow or displacements can be approximated by a discrete model composed of a set of piecewise continuous functions defined over a finite number of subdomains.

The discrete model is constructed as follows:

(a) A finite number of points in the domain is identified. These points are called nodes.

(b) The value of the continuous quantity is denoted as a variable which is to be determined.

(c) The domain is divided into a finite number of subdomains called elements. These elements are connected at common nodal points and collectively approximate the shape of the domain.

(d) The continuous quantity is approximated over each element by a polynomial that is defined using the nodal values of the continuous quantity. A different polynomial is defined for each element but the element polynomials can be selected in such a way that continuity is maintained along the element boundaries.

For the FEM application to unsteady flow in storm sewer systems the governing momentum and continuity equations are solved using the Galerkin method of weighted residuals. The procedure is presented in texts by Zienkiewicz (54) and Huebner (16). In general, the Galerkin finite element technique involves:

(a) identification of the approximating polynomials $Q = Q(x)$, $H = H(x)$, etc. which contain the unknowns to be determined

(b) multiplication of Eqs. 3.31 and 3.32 by weighting functions derived from the approximating functions $Q(x)$, $H(x)$, etc.

(c) substitution of the approximating polynomials $Q(x)$, $H(x)$, etc. into Eqs. 3.31 and 3.32.

(d) integration of these modified equations over the element to form a set of ordinary differential equations in time, and

(e) integration of the ordinary differential equations over time.

The Galerkin finite element method described herein is based on representing the unknown variables, Q and H on a local element basis. The entire global solution domain is discussed following this formulation.

One of the distinct advantages of using the finite element method is the ability to choose the approximating polynomials for the dependent variables. Several possibilities exist. However, there is a direct relationship between the computational efficiency and the order of the approximating polynomials.

For the initial investigation the unknown quantities Q and H are assumed to vary linearly with x along the element, as shown in Fig. 4-1.

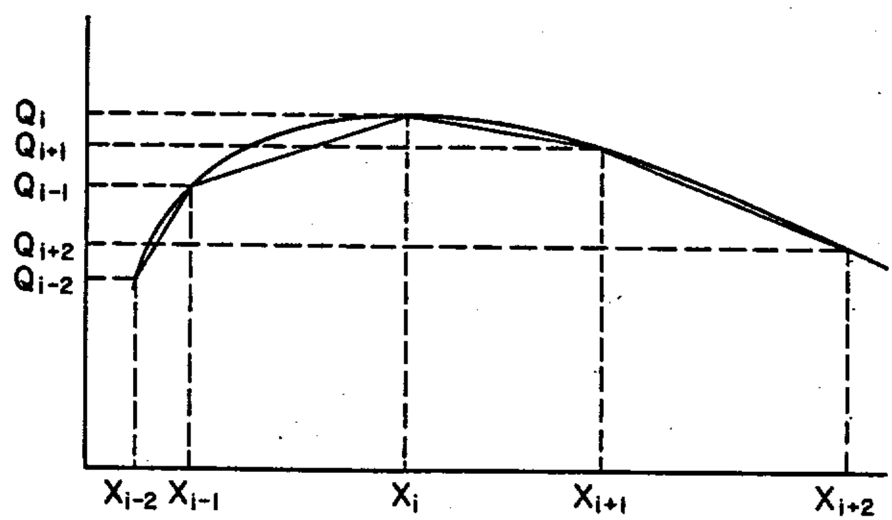


Figure 4-1. Smooth Curve Approximation by Linear Elements.

Therefore

$$Q = N_i Q_i + N_{i+1} Q_{i+1} \qquad (4\text{-}1)$$

in which N_i and N_{i+i} are shape functions for the element. From one dimensional Lagrangian interpolation over a single element

$$N_{i+1} = (x - x_{i+1})/(x_i - x_{i+1}) \qquad (4\text{-}2)$$

$$N_{i+1} = (x - x_i)/(x_{i+1} - x_i) \qquad (4\text{-}3)$$

where i and i+1 are the node numbers bounding the element, j.

From Eqs. 4-2 and 4-3, the shape functions and their first spacial derivatives for a single element of length, ℓ in Fig. 4-2 are

$$N_1 = (x - x_2)/(x_1 - x_2) = (x - \ell)/(-\ell) = 1 - (x/\ell)$$

$$(4\text{-}4)$$

$$N_2 = (x - x_1) / (x_2 - x_1) = x/\ell$$

$$d(N_1)/dx = -1/\ell \qquad d(N_2)/dx = 1/\ell \qquad (4\text{-}5)$$

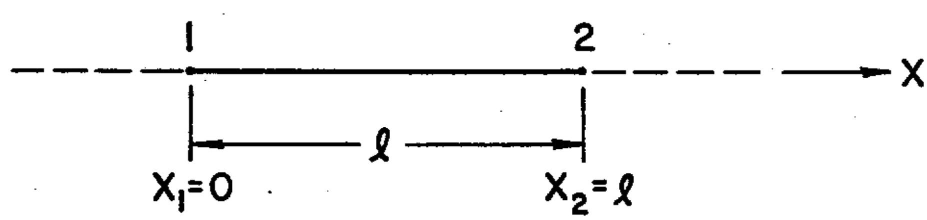


Figure 4-2. A Single, Linear Element Approximation.

Substitution of Eq. 4-4 into Eq. 4-1 and expressing in matrix form yields

$$Q = \lfloor N_1 \; N_2 \rfloor \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \lfloor N \rfloor \{Q\} \qquad (4\text{-}6)$$

A similar derivation is performed on the variable, H. Quantities such as g, A, D and c are considered constant over the element length.

The Galerkin method, in general, requires that

$$\int_{\Omega} \{N\} \mathcal{L} (\Delta) d\Omega = \{0\}$$

where $\mathcal{L}$ is the differential operator acting on the unknown field variable (Q,H) over the system domain Ω, and N are the approximating polynomials or shape functions.

Applying the Galerkin method to the governing differential equations (Eqs. 3-31 and 3-32) yields

$$\sum_{e} \int_0^{\ell} \{N\} \left(\frac{\partial H}{\partial t} - \frac{Q}{A} \sin \alpha + \frac{c^2}{gA} \frac{\partial Q}{\partial x} \right) dx = \{0\} \qquad (4\text{-}7)$$

and

$$\sum_{e} \int_0^{\ell} \{N\} \left(\frac{\partial Q}{\partial t} + gA \frac{\partial H}{\partial x} + \frac{f\, Q|Q|}{2AD} \right) dx = \{0\} \qquad (4\text{-}8)$$

Through appropriate substitution of the element shape functions (Eq. 4-6)

$$\sum_e \int_0^{\ell} \{N\}[\ \lfloor N\rfloor\{\dot{H}\} - \frac{\sin\alpha}{A}\lfloor N\rfloor\{Q\} + \frac{c^2}{gA}\frac{d\lfloor N\rfloor}{dx}\{Q\}\ ]\ dx = \{0\} \tag{4-9}$$

$$\sum_e \int_0^{\ell} \{N\}[\ \lfloor N\rfloor\{\dot{Q}\} + gA\frac{d\lfloor N\rfloor}{dx}\{H\} + \frac{L}{2AD}\lfloor N\rfloor\{f\}\lfloor N\rfloor\{|Q|\}\lfloor N\rfloor\{Q\}\ ]\ dx = \{0\} \tag{4-10}$$

where the dot over the dependent variable represents differentiation with respect to time.

Writing Eqs. 4-9 and 4-10 in complete matrix form

$$[E]\{\dot{H}\} + [F]\{Q\} + [G]\{Q\} = \{0\} \tag{4-11}$$

$$[A]\{\dot{Q}\} + [B]\{H\} + [C(Q)]\{Q\} = \{0\} \tag{4-12}$$

where

$$[A] = \sum_e \int_0^{\ell}\{N\}\lfloor N\rfloor\ dx$$

$$[B] = gA\ \sum_e \int_0^{\ell}\{N\}\ \frac{d\lfloor N\rfloor}{dx}\ dx$$

$$[C(Q)] = \frac{1}{2AD}\ \sum_e \int_0^{\ell}\{N\}\lfloor N\rfloor\{f\}\lfloor N\rfloor\{|Q|\}\lfloor N\rfloor\ dx$$

$$[E] = [A] = \sum_e \int_0^{\ell}\{N\}\lfloor N\rfloor\ dx$$

$$[F] = \frac{-\sin\alpha}{A}\ \sum_e \int_0^{\ell}\{N\}\lfloor N\rfloor\ dx$$

$$[G] = \frac{c^2}{gA} \sum_e \int_0^{\ell} \{N\} \frac{d[N]}{dx} \; dx$$

Here the nonlinear friction term, $[C(Q)]$ is linearized by assuming that the unknown variable, $|Q|$, in the friction term is known from the previous time step. This assumption is valid since the friction term does not vary substantially over the time interval.

Evaluation of [A], [B], [C(Q)], [E], [F], and [G] yields

$$[A] = L \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix}$$

$$[B] = gA \begin{bmatrix} -1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

$$[C(Q)] = \frac{L}{2AD} \begin{bmatrix} C(1,1) & C(1,2) \\ C(2,1) & C(2,2) \end{bmatrix}$$

where

$$C(1,1) = (1/5)f_1|Q_1| + (1/20)f_2|Q_1| + (1/20)f_1|Q_2| + (1/30)f_2|Q_2|$$

$$C(1,2) = (1/20)f_1|Q_1| + (1/30)f_2|Q_1| + (1/30)f_1|Q_2| + (1/20)f_2|Q_2|$$

$$C(2,1) = (1/20)f_1|Q_1| + (1/30)f_2|Q_1| + (1/30)f_1|Q_2| + (1/20)f_2|Q_2|$$

$$C(2,2) = (1/30)f_1|Q_1| + (1/20)f_2|Q_1| + (1/20)f_1|Q_2| + (1/5)f_2|Q_2|$$

$$[E] = L \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix}$$

$$[F] = \frac{-L \sin \alpha}{A} \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix}$$

$$[G] = \frac{c^2}{gA}\begin{bmatrix} -1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

This concludes the finite element formulation with respect to the spacial domain. In order to evaluate the time dimension or transient effect in the governing equations the Galerkin method is again used.

The linear polynomial is used to approximate the unknown field variables Q and H over the time domain. For a linear element in time (Fig. 4-3) the interpolation, shape functions and their first time derivatives are written in terms of local variables as

$$\begin{array}{ll} 0 \leq \xi \leq 1 & \xi = t/\Delta t \\ N_n = 1 - \xi & N_n = -1/\Delta t \\ N_{n+1} = \xi & N_{n+1} = 1/\Delta t \end{array} \tag{4-13}$$

Also by definition

$$\begin{aligned} \{\dot{Q}\} &= \frac{d\lfloor N \rfloor}{dt}\{Q\} = \lfloor \dot{N} \rfloor \{Q\} \\ \{\dot{H}\} &= \frac{d\lfloor N \rfloor}{dt}\{H\} = \lfloor \dot{N} \rfloor \{H\} \end{aligned} \tag{4-14}$$

The method of weighted residuals (MWR) requires that

$$\int_\Omega W_j \mathcal{L}(\Delta)d\Omega = 0$$

where W_j is the weighting function for element j. Application of the MWR to Eqs. 4-11 and 4-12 over a single element (j = 1) yields

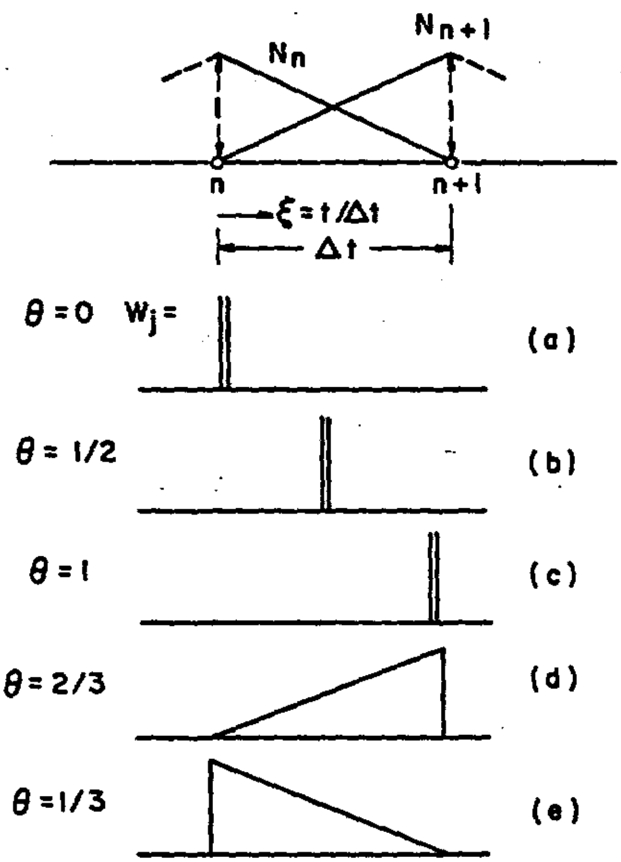


Figure 4-3. Time Shape and Weighting Functions (54).

$$\int_0^1 W_j \, [\, [E]\{\dot{H}\} + [F]\{Q\} + [G]\{Q\} \,] \, d\xi = \{0\} \qquad (4\text{-}15)$$

$$\int_0^1 W_j \, [\, [A]\{\dot{Q}\} + [B]\{H\} + [C(Q)]\{Q\} \,] \, d\xi = \{0\} \qquad (4\text{-}16)$$

Inserting Eqs. 4-13 and 4-14 into Eqs. 4-15 and 4-16 yields

$$[E] \int_0^1 W_j \, [(-\frac{1}{\Delta t})\{H\}_n + (\frac{1}{\Delta t})\{H\}_{n+1}] \, d\xi + [F] \int_0^1 W_j [(1-\xi)\{Q\}_n$$

$$+ (\xi)\{Q\}_{n+1}] \, d\xi + [G] \int_0^1 W_j \, [(1-\xi)\{Q\}_n + (\xi)\{Q\}_{n+1}] \, d\xi = \{0\} \qquad (4\text{-}17)$$

$$[A] \int_0^1 W_j \left[\left(-\frac{1}{\Delta t}\right)\{Q\}_n + \left(\frac{1}{\Delta t}\right)\{Q\}_{n+1}\right] d\xi + [B] \int_0^1 W_j[(1-\xi)\{H\}_n$$

$$+ (\xi)\{H\}_{n+1}] d\xi + [C(Q)] \int_0^1 W_j [(1-\xi)\{Q\}_n + (\xi)\{Q\}_{n+1}] d\xi = \{0\} \quad (4\text{-}18)$$

By letting $\theta = \dfrac{\int_0^1 W_j \xi \, d\xi}{\int_0^1 W_j \, d\xi}$

Equations 4-17 and 4-18 are simplified to

$$-[E]\{H\}_n + [E]\{H\}_{n+1} + (1-\theta)(\Delta t)[F]\{Q\}_n + (\theta)(\Delta t)[F]\{Q\}_{n+1}$$
$$+ (1-\theta)(\Delta t)[G]\{Q\}_n + (\theta)(\Delta t)[G]\{Q\}_{n+1} = \{0\} \quad (4\text{-}19)$$

$$-[A]\{Q\}_n + [A]\{Q\}_{n+1} + (1-\theta)(\Delta t)[B]\{H\}_n + (\theta)(\Delta t)[B]\{H\}_{n+1}$$
$$+ (1-\theta)(\Delta t)[C(Q)]\{Q\}_n + (\theta)(\Delta t)[C(Q)]\{Q\}_{n+1} = \{0\} \quad (4\text{-}20)$$

Figure 4-3 shows a series of weighting functions and the corresponding values of θ. The popular forward (Euler) difference, central difference (Crank-Nicholson) and backward difference numerical schemes are shown in Fig. 4-3 a-c. The Galerkin type schemes (54) are shown in Fig. 4-3 d and e.

Equations 4-19 and 4-20 are the complete finite element equations in matrix form. Solving Eqs. 4-19 and 4-20 for the dependent variables, Q and H, in matrix form results in

$$\begin{bmatrix} K_1^H & K_2^Q \\ K_3^H & K_4^Q \end{bmatrix} \begin{Bmatrix} H \\ Q \end{Bmatrix}_{n+1} = \begin{Bmatrix} F_1^H \\ F_2^Q \end{Bmatrix}_n \quad (4\text{-}21)$$

where the stiffness coefficient matrix, [K], components are

$$K_1^H = [E]$$

$$K_2^Q = (\theta)(\Delta t)[F] + (\theta)(\Delta t)[G]$$

$$K_3^H = (\theta)(\Delta t)[B]$$

$$K_4^Q = [A] + (\theta)(\Delta t)[C(Q)]$$

and the force vector, {F}, components are

$$\{F_1^H\} = [E]\{H\}_n - (\Delta t)(1-\theta)[F]\{Q\}_n - (\Delta t)(1-\theta)[G]\{Q\}_n$$

$$\{F_2^Q\} = [A]\{Q\}_n - (\Delta t)(1-\theta)[B]\{H\}_n - (1-\theta)(\Delta t)[C(Q)]\{Q\}_n$$

Equation 4-21 is the system solution in matrix form of the governing equations on a local coordinate (element) basis. The solution of the entire system domain is of the form

$$\sum_{j=1}^{e} [K]_j \begin{Bmatrix} H \\ Q \end{Bmatrix}_j = \{F\}_j \qquad (4\text{-}22)$$

where e is the total number of elements in the system. Equation 4-22 represents a system of e element equations in which the element nodal values of Q and H are solved for simultaneously using Gauss-Crout numerical techniques (34).

Throughout the system various initial and boundary conditions are incorporated into the system equations. These conditions are discussed in Section 4.4 and 4.5.

4.2 Dynamic Lumped Parameter Model

Since fluid piping systems are continious the complete governing equations of unsteady flow are correctly solved using a distributed parameter model, in which the elastic behavior of the fluid and pipe material, the fluid inertia and the frictional resistance are distributed along the pipeline. The finite element model described in Section 4.1 is a distributed parameter model. There is, however, a large class of fluid transient problems in which it is permissible to use a lumped parameter or rigid column theory analysis with certain simplifying assumptions. These include slowly varying transient flow situations such as that in surcharged storm sewer systems.

From Chapter 2 the governing equations for pressurized storm sewer analysis are:

Continuity:
$$\frac{\partial H}{\partial t} - \frac{Q}{A}\sin\alpha + \frac{c^2}{gA}\frac{\partial Q}{\partial x} = 0 \qquad (3\text{-}31)$$

Momentum:
$$gA\frac{\partial H}{\partial x} + \frac{\partial Q}{\partial t} + \frac{f\,Q|Q|}{2AD} = 0 \qquad (3\text{-}32)$$

The dynamic-lumped parameter model developed in this thesis makes three distinct assumptions concerning the capacitance, pipe slope and inertia of the system:

Capacitance: In this model the elastic behavior of the fluid is considered negligible as compared to friction and inertial effects. Therefore Eq. 3-31 takes the form

$$\frac{dH}{dt} - \frac{Q}{A}\sin\alpha = 0 \qquad (4\text{-}23)$$

Pipe Slope: Most storm sewer systems are designed as gravity flow (open channel) systems in which the potential head to create flow in the system is provided by a change in elevation or slope in the direction of the flow path. Typical storm sewer systems have pipes with longitudinal slopes which may vary from 0.000 ft./ft. to 0.010 ft./ft. However, in this model the effect of pipe slope on the continuity equation is considered negligible since $\sin\alpha \cong 0$ for small angles. Hence, Eq. 4-23 reduces to $dH/dt = 0$ which describes steady state continuity conditions for a pipe element. A useful form of the incompressible, steady flow continuity equations is

$$Q = AV \qquad (4\text{-}24)$$

where: Q is the discharge, A is the cross sectional flow area and V is the average velocity over area A.

Lumped Inertia: As with the continuity equation, the momentum equation is simplified for incompressible flow by assuming the pipe elements to be inelastic. Therefore, the liquid mass is treated as a rigid column (Fig. 4-4) in which the inertial forces are lumped together over the pipe length L. The modified lumped parameter momentum equation is an ordinary differential equation of the form

$$\frac{L}{gA}\frac{dQ}{dt} = H_1 - H_2 - h_L \tag{4-25}$$

The term H represents the hydraulic grade or head at a given point in the system and is measured from an arbitrary datum. The hydraulic grade is equivalent to the pressure head (P/γ) plus the elevation (z).

The head loss or friction term in Eq. 4-25 can be conveniently expressed as

$$h_L = \frac{fLQ|Q|}{A^2D2g} = KQ|Q| \tag{4-26}$$

where f is the Darcy-Weisbach friction factor, L is the pipe length, A is the pipe area and D is the pipe diameter. The pipe constant, K, is defined as

$$K = \frac{fL}{A^2D2g} = \frac{8fL}{\pi^2gD^5} \tag{4-27}$$

The pipe constant, K, can be modified to account for additional flow resistance due to entrance effects, exit losses, or any other factors which tend to dissipate energy. These losses are traditionally computed by using a minor loss coefficient (M) which is multiplied by the velocity head. Hence, the minor loss is given by

$$h_{LM} = \frac{MV^2}{2g} \tag{4-28}$$

and the pipe constant, K, is modified to

$$K = \frac{8fL}{\pi^2 gD^5} + \frac{8\Sigma M}{\pi^2 gD^4} \qquad (4\text{-}29)$$

where ΣM is the algebraic sum of the minor loss coefficients in the line segment. Equation 4-26 includes an absolute value sign on the flow terms to insure that the shear force always opposes the flow direction.

Equation 4-25 with Eq. 4-27 written in finite difference form is

$$Q_{t + \Delta t} = Q_t + \frac{gA\Delta t}{L} (H_1 - H_2 - K_t Q_t |Q_t|) \qquad (4\text{-}30)$$

where H_1, H_2, K_t and Q_t are values of hydraulic grade, pipe constant and flowrate at the beginning of the time interval.

Equation 4-30 forms the basis for an explicit forward difference numerical scheme where the unknown conditions ($Q_{t + \Delta t}$) at a later time are determined directly from conditions at the preceding time, t. The unknown values of head H_1 and H_2 are determined from junction boundary conditions and are formulated in Section 4.4.

Many numerical schemes, including both implicit and explicit schemes, have been developed to solve the governing unsteady flow equations of continuity and momentum and are discussed in Chapter 2. The explicit scheme developed herein is a simplified solution to the governing flow equations in which the fluid inertia is lumped. This numerical scheme has advantages over other methods in that it is easily programmed and requires a minimum of computer storage.

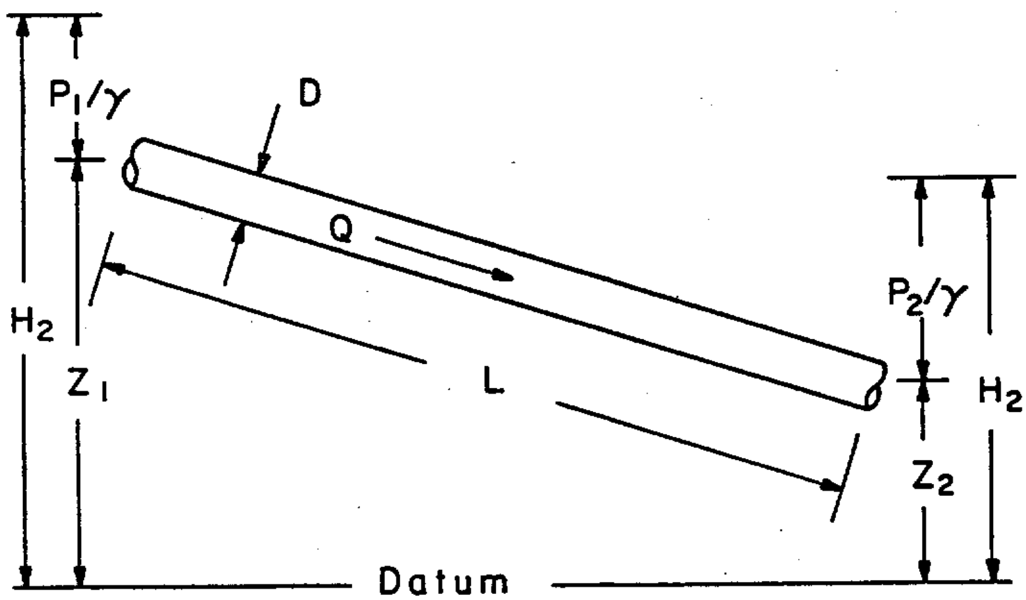


Figure 4-4. Rigid Fluid Column.

4.3 Kinematic Model (Steady State With Storage)

The third numerical flow model to be developed for surcharge storm sewer analysis is the steady state with storage model. In this model, pressure and flow conditions at any time during the simulation are calculated using steady state flow conditions. Incompressible steady flow by definition occurs when the conditions of flow, Q, pressure, P, density, ρ, and temperature, T, at any point in the fluid system, do not change with time, t; thus

$$\frac{\partial Q}{\partial t} = 0 \qquad \frac{\partial P}{\partial t} = 0 \qquad \frac{\partial \rho}{\partial t} = 0 \qquad \frac{\partial T}{\partial t} = 0 \tag{4-31}$$

Application of incompressible steady state theory to the governing unsteady flow equations of continuity and momentum (Eqs. 3.31 and

3.32) yields:

$$\frac{dQ}{dx} = 0 \tag{4-32}$$

and

$$\frac{dH}{dx} = \frac{fQ|Q|}{A^2 D2g} \tag{4-33}$$

A common useful form of Eq. 4-32 , as discussed in Section 4.2, is

$$Q = AV \tag{4-34}$$

Equation 4-33 modified for application between two distinct points in a fluid system (Fig. 4-4) yields

$$\Delta H = H_1 - H_2 = h_L \tag{4-35}$$

which is the Darcy Weisbach equation as discussed in Section 3.2. By introducing the pipe constant K, Eq. 4-35 becomes

$$\Delta H = H_1 - H_2 = KQ|Q| \tag{4-36}$$

where

$$K = \frac{8fL}{\pi^2 gD^5} + \frac{8\Sigma M}{\pi^2 gD^4} \tag{4-29}$$

Since pressure heads H_1 and H_2 are known from boundary conditions Eq.

4-36 is a quadratic equation with unknown flow Q, of the form

$$f(Q) = H_1 - H_2 - KQ|Q| \qquad (4\text{-}37)$$

Due to the complex nature of storm sewer flow it is not uncommon to experience sudden flow reversal in pipe lines, which may cause numerical stability problems when solving Eq. 4-37 using the quadratic formula. Thus, the nonlinear terms in Eq. 4-37 are linearized in terms of an approximate flowrate, Q_i. This is performed by taking the derivative of f(Q) with respect to the flowrate and evaluating f(Q) at $Q = Q_i$ using the following approximation:

$$f(Q) = f(Q_i) + \frac{\partial f}{\partial Q}\bigg|_{Q=Q_i} [\bar{Q} - Q_i] = 0$$

or

$$f(Q) = H_1 - H_2 - KQ_i|Q_i| + 2K|Q_i|[Q - Q_i] = 0$$

Solving for the unknown flowrate:

$$Q = \frac{H_1 - H_2 + 2KQ_i|Q_i|}{2KQ_i} \qquad (4\text{-}38)$$

The initial flow Q_i is always a known quantity from the previous solution. This procedure is repeated with Q replacing Q_i for each iteration until a satisfactory convergence criteria is obtained. Usually, only 3 to 5 trials are necessary for an extremely accurate solution.

4.4 System Boundary Conditions

Nearly all solutions to the unsteady flow equations of momentum and continuity involve some type of numerical time marching procedure. Three models are developed in this thesis: the finite element model, which is an implicit procedure, and the lumped and steady models, which are explicit in form. All three models, however, utilize the same method for calculation of known boundary conditions, which are consistently maintained throughout the duration of a single calculation. These boundary conditions involve pressure heads at specific points throughout the system. They are: a) fixed head manhole conditions and b) variable head manhole conditions.

In order to solve the governing equations, each system must have at least one point in which the head at that point is constant or fixed throughout the time simulation. This point, called a fixed grade, is simulated as a constant head at a manhole and is usually located at the exit of the sewer system. When solving the system equations this fixed grade is treated as a known head boundary condition.

The second boundary condition allows for variable input into each manhole with respect to time. This condition is modeled through the use of a storm hydrograph and the appropriate junction continuity equations.

The triangular hydrograph as shown in Fig. 4-5 is a simple and practical representation of the manhole inflow with only one rise, one peak and one recession. Because of its geometry, it can be easily described mathematically which makes it a useful tool for estimating manhole inflows.

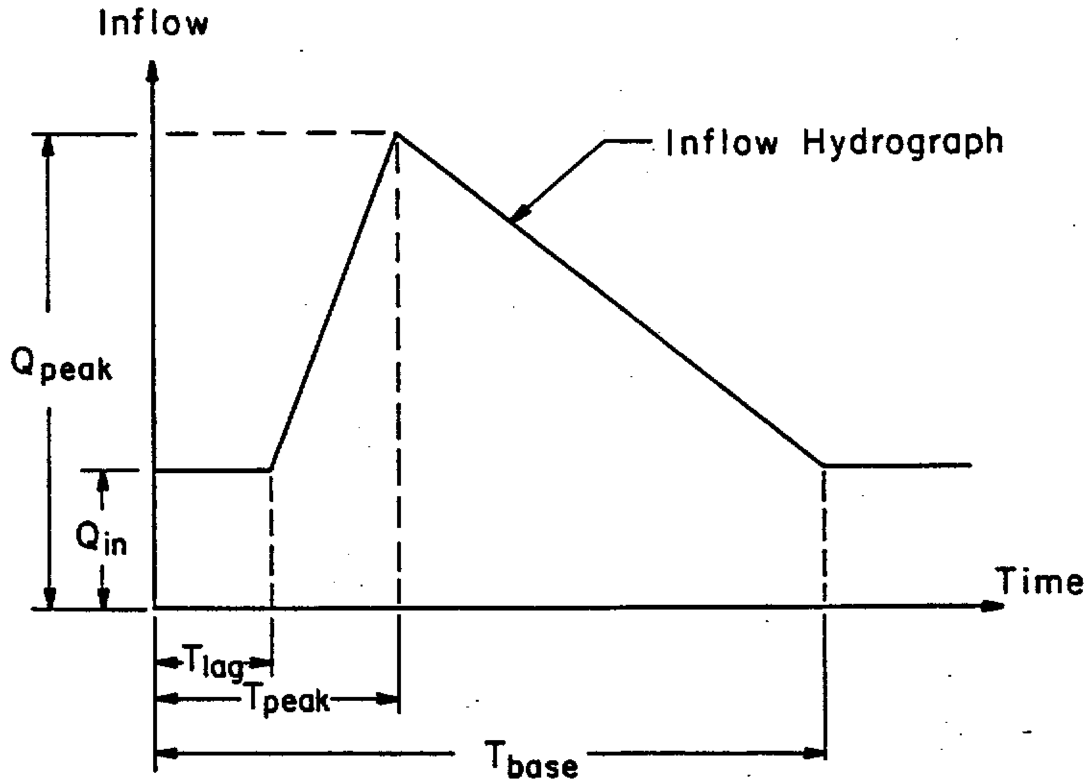


Figure 4-5. Triangular Inflow Hydrograph.

The triangular hydrograph is described by the following parameters:

Q_{in} = initial hydrograph inflow

Q_{peak} = peak hydrograph inflow

T_{lag} = time lag of hydrograph

T_{peak} = time peak of hydrograph

T_{base} = time base of hydrograph

From this inflow hydrograph the manhole inflows are known at all times throughout the simulation. The boundary conditions of pressure head at the manholes are obtained from the known inflows using the junction continuity relationship

$$\Sigma Q = \frac{dS}{dt} \qquad (4\text{-}39)$$

where ΣQ is the summation of flows into or out of the junction and dS/dt is the differential storage with time in the manhole. Writing Eq. 4-39 in numerical explicit finite difference form

$$Q_1 + Q_2 - Q_3 + I = \Delta S/\Delta t$$

or

$$(Q_1 + Q_2 - Q_3 + I)_t = \frac{A_m}{\Delta t}[H_{t+\Delta t} - H_t] \qquad (4\text{-}40)$$

where the terms in this expression (and Fig. 4-6) are:

Q_{1t} = pipe #1 lateral flow at time t

Q_{2t} = pipe #2 lateral flow at time t

Q_{3t} = pipe #3 lateral flow at time t

I = hydrograph inflow

A_m = manhole area

H_m = manhole height

H_t = junction head at time t

$H_{t+\Delta t}$ = junction head at time $t + \Delta t$

Δt = time increment

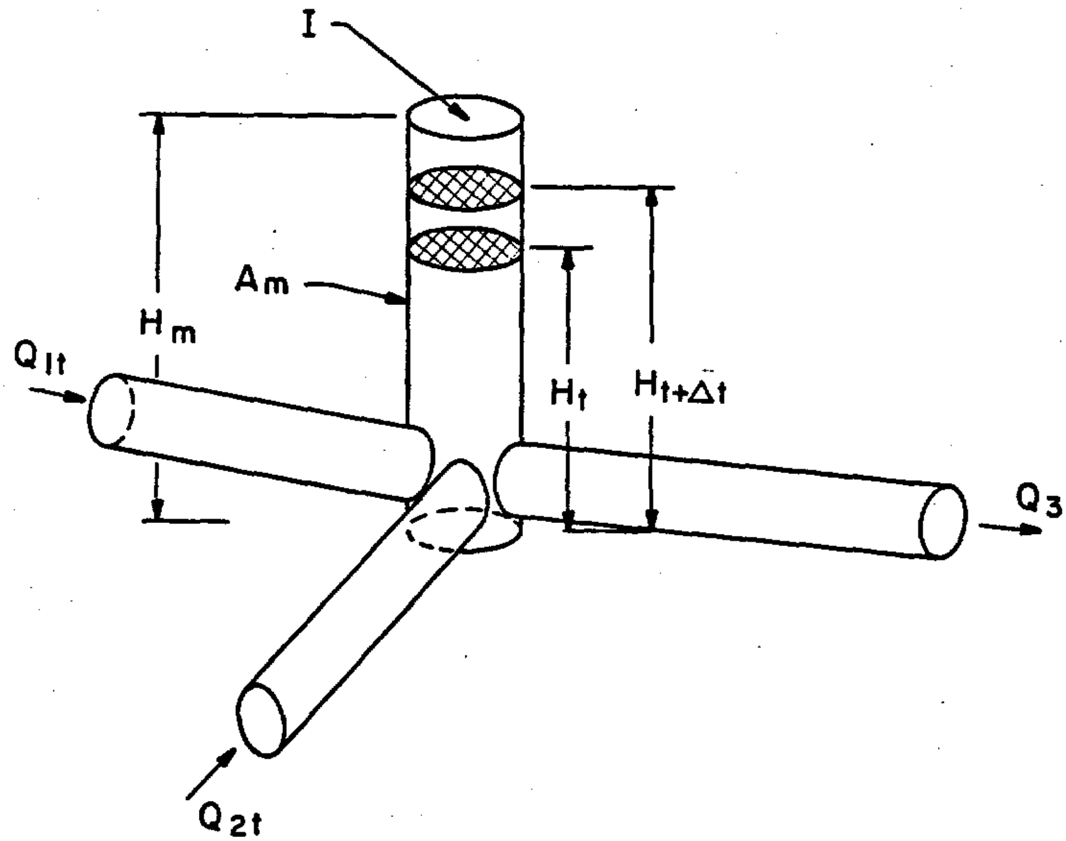


Figure 4-6. Manhole (Junction) Boundary Conditions.

Rearranging Eq. 4-40 gives

$$H_{t+\Delta t} = (Q_1 + Q_2 - Q_3 + I)_t \, [\Delta t/A_m] + H_t \qquad (4\text{-}41)$$

in which $H_{t+\Delta t}$ is computed using known values of head and flow at time, t. Thus, each manhole with a variable inflow has a known head or boundary condition which is incorporated into the system equations.

When the pressure head in the manhole exceeds the manhole height, surface flooding occurs. During flooded conditions the surface water is assumed to be temporarily stored in a detention area

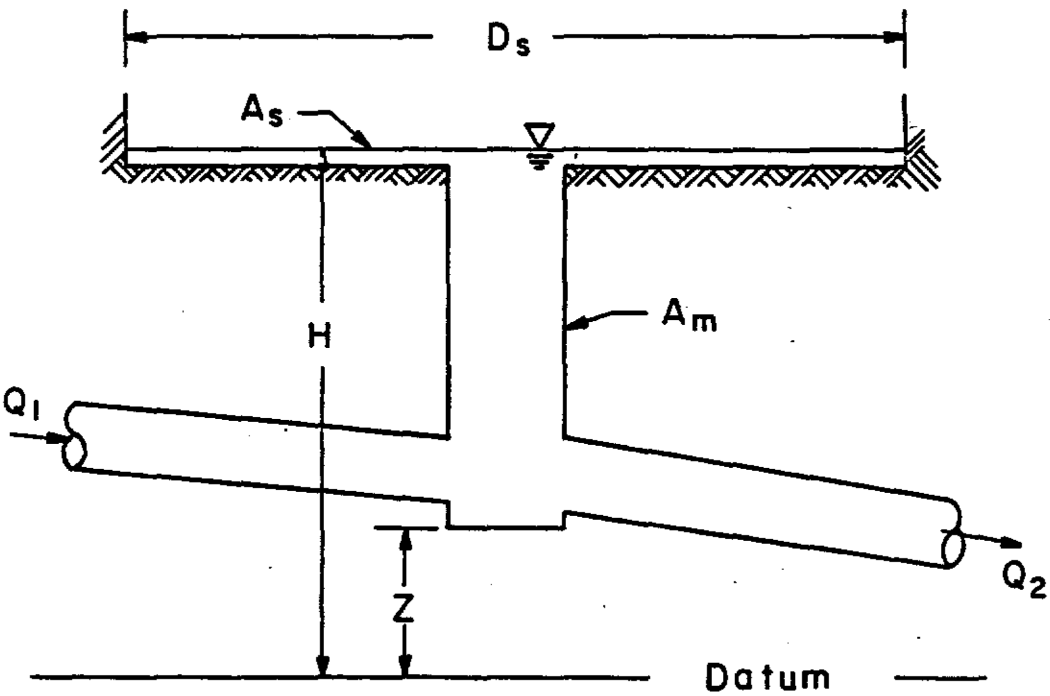


Figure 4-7. Flooded Manhole Conditions.

connected to the manhole and will return to the sewer system at a later time without any volume loss. Surface flow routing is not incorporated into the flow models.

Figure 4-7 shows a manhole with surface flooding conditons, where:

D_s = manhole surface storage diameter

A_s = manhole surface storage area

Eq. 4-42 is modified to account for surface flooding by replacing the manhole area A_m, with the surface storage area, A_s.

4.5 Initial Conditions

Numerical time schemes require that initial conditions concerning the dependent variable in the system equations be known. The three numerical models developed in this thesis require that the initial steady state values of head, H, and flow, Q, be known throughout the entire system. These initial heads and flows are obtained from a complete steady state flow analysis using a steady state pipe network model (46).

The three numerical models are developed assuming that the entire system or sub-system to be analyzed is under surcharge conditions with each pipe flowing full. Therefore, each manhole in the system will initially have a surcharged pressure head which depends on the steady state flow in the pipes connecting the manhole.

4.6 System Equation Assembly and Assumptions

With the hydraulics of sewers developed mathematically in Chapter 3 and the hydraulics of sewer junctions or manholes described in Section 4.4 a storm sewer network can be defined and analyzed. The numerical models developed in Chapter 4 provide a method to solve the system equations based on a sewer link - junction node format. Thus, at time t, the sewer network is simply disconnected at the manhole junctions and the unknown values of pressure head and flow in each link are solved independent of all other sewer links in the network. This forms an independent set of p momentum equations, p line continuity equations and j manhole or boundary equations where p is the number of pipes in the system and j is the number of junctions or manholes in the system. Upon solution of the link equations the system is reassembled to allow for modifications of the manhole boundary conditions for the next time step, $t + \Delta t$. This procedure is repeated over a finite period of time producing computed values of pressure and head throughout the time simulation.

Nearly all numerical models which solve the governing unsteady flow equations make use of a variety of assumptions which aid in the solution of the system equations and the models developed herein are no exception. The first assumption concerns the nature of the flow during the time simulation. The analysis of surcharge storm sewer flow is under investigation and therefore the models do not route sewer flow under gravity or open channel conditions. The models do, however, predict small pressure heads which would indicate that conditions are not suitable for pressurized flow.

Mathematically, it is difficult to make an exact energy analysis of flow through a junction. Instead, approximate energy expressions are assumed. In surcharged storm sewer systems, the manhole junctions are considered submerged and losses are similar to those of orifice flow with a head loss computed as $MV^2/2g$. M is a dimensionless head loss coefficient and V is the instantaneous mean velocity at the junction entrance or exit. For a sharp-edged entrance, M has an approximate value of 0.5 and for an exit, M is taken as 1.0. The lumped parameter and steady state flow models allow for junction energy losses through the use of minor loss coefficients as described in Section 4.2 and 4.3.

The wall shear or frictional head losses in the manhole itself are considered negligible since they are typically very small when compared to the sewer line losses. Evaluation of other energy losses in the manhole are only possible by relating them to line losses using the minor loss equation.

The pipe line flow resistance equation in the Darcy-Weisbach form is valid only for steady uniform flow. To this date, an unsteady, nonuniform headloss equation is unknown. Thus, the finite element and lumped parameter flow models incorporate steady state head loss theory with the unsteady governing momentum and continuity equations. Presently, this is an acceptable method for calculating such losses since the flow variations occur slowly.

The final assumption concerns the treatment of surface flooding near the manhole entrance. Detailed surface geometry of the land above the sewer network is often unknown or unavailable and its mathematical description is often difficult. The conditions imposed by the models are described in detail in Section 4.4. However, it should be emphasized that surface flow routing between manholes is not incorporated in the system equations. The surface water is assumed to be temporarily stored in a basin area directly above the manhole and will eventually return to the sewer system without volume loss. This allows for accurate flood volume predictions in the area of the manhole surface.

EXAMPLE PROBLEMS AND RESULTS

This chapter contains a comparison of the three hydraulic flow models developed in Chapter 4 which analyze storm sewer systems under peak flow conditions. Five examples are illustrated to show the ability of each model to predict pressure and flow conditions in systems under surcharge. The system geometry and properties are presented for each example.

As discussed in section 4.5, the initial conditions for each example are obtained from a steady state analysis of the storm sewer system. These conditions are based on the assumption that the entire system is flowing full under pressure.

Computer results are presented for each of the three models developed. The values of hydraulic grade and flow are plotted with time for each manhole and sewer line in the system. The dimensionless coefficients, as discussed in Section 3.4, are also calculated for each example.

The system coding instructions and typical computer output are presented in Chapter 7.

5.1 Example Problem # 1

The system shown in Fig. 5-1 illustrates complete transient flow in a single pipe sewer system. The 2 foot diameter, 900 foot long sewer line transports storm water from a single 48 inch sewer manhole to an outlet of fixed grade. The manhole input or source is in the form of an input pressure-time variation shown in Fig. 5-2. This pressure-time variation acts as a pressure forcing function at manhole number 1, raising the manhole head from 15 feet to 60 feet in 0.5 seconds. This type of forcing function is required to cause severe transients or pressure surges in a storm sewer system. For simplicity the friction factor is assumed constant for the simulation ($f = 0.05645$) and the pipe celerity (c) is 3000 feet per second.

The computer simulation results of example # 1 are shown in Figs. 5-3 and 5-4. Four methods are presented. These include: a) the wave plan model (47) b) the finite element distributed parameter model c) the dynamic lumped parameter model and d) the steady state with storage (kinematic) model. The hydraulic grade and flow values are plotted with time for the pipe midpoint (450 feet from junction 1).

As shown in Fig. 5-3, the oscillating pressure head is predicted by the wave plan and finite element methods, which are complete unsteady distributed flow methods. Both the lumped parameter model and the steady state model predict average hydraulic grades over the time simulation. The pressure surges, created by the forcing function, eventually dampen out with friction and tend to approach those of steady state conditions.

Fig. 5-4 illustrates the variable sewer flow over the time simulation. As shown here, the dynamic lumped parameter model predicts flow conditions which are similiar to the wave plan and finite element models. This gives a preliminary indication that a dynamic model, which includes inertial effects and neglects elastic considerations, may be appropriate for analyzing a certain class of slowly varying transient problems such as pressurized storm sewer systems.

For this example problem the dimensionless λ coefficient (Section 3.4) are $\lambda_1 = 6.14$ and $\lambda_2 = 13.36$. λ_1 and λ_2 are much greater than 1.0 indicating that a complete unsteady analysis including elastic, inertial and frictional effects is necessary.

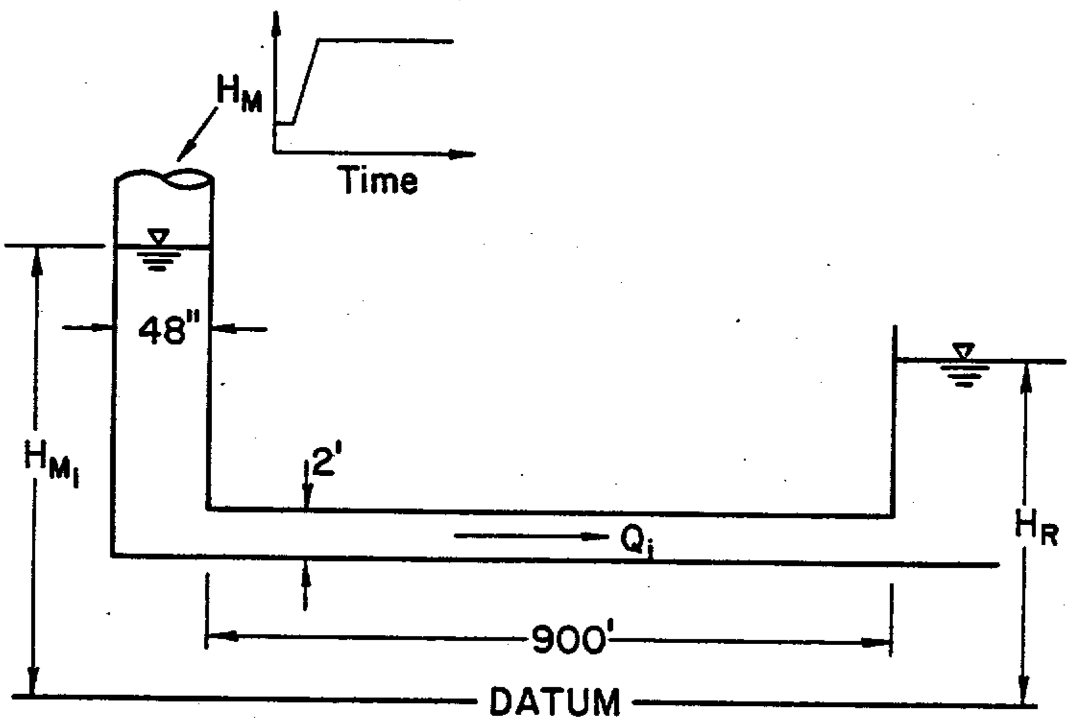


Figure 5-1. One Pipe Sewer System, Example # 1.

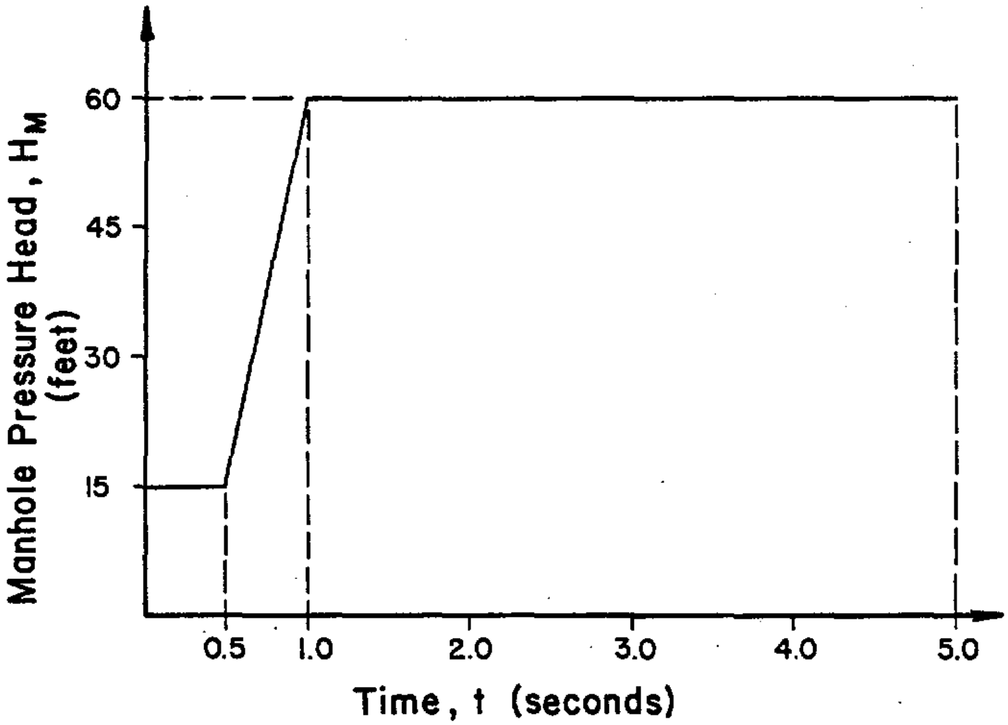


Figure 5-2. Input Pressure-Time Variation for Example # 1.

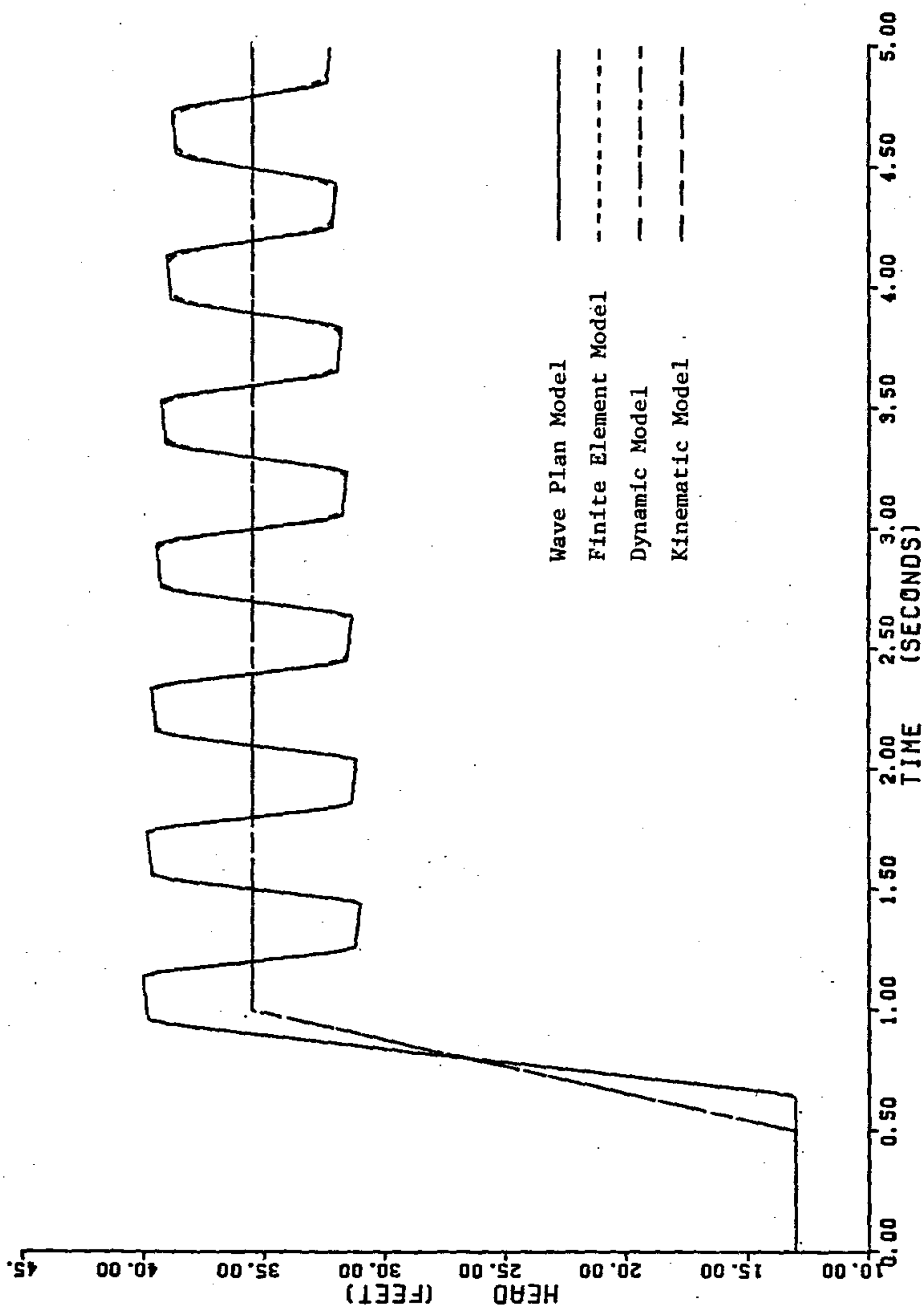


Figure 5-3. Total Head Graph for Junction 1.

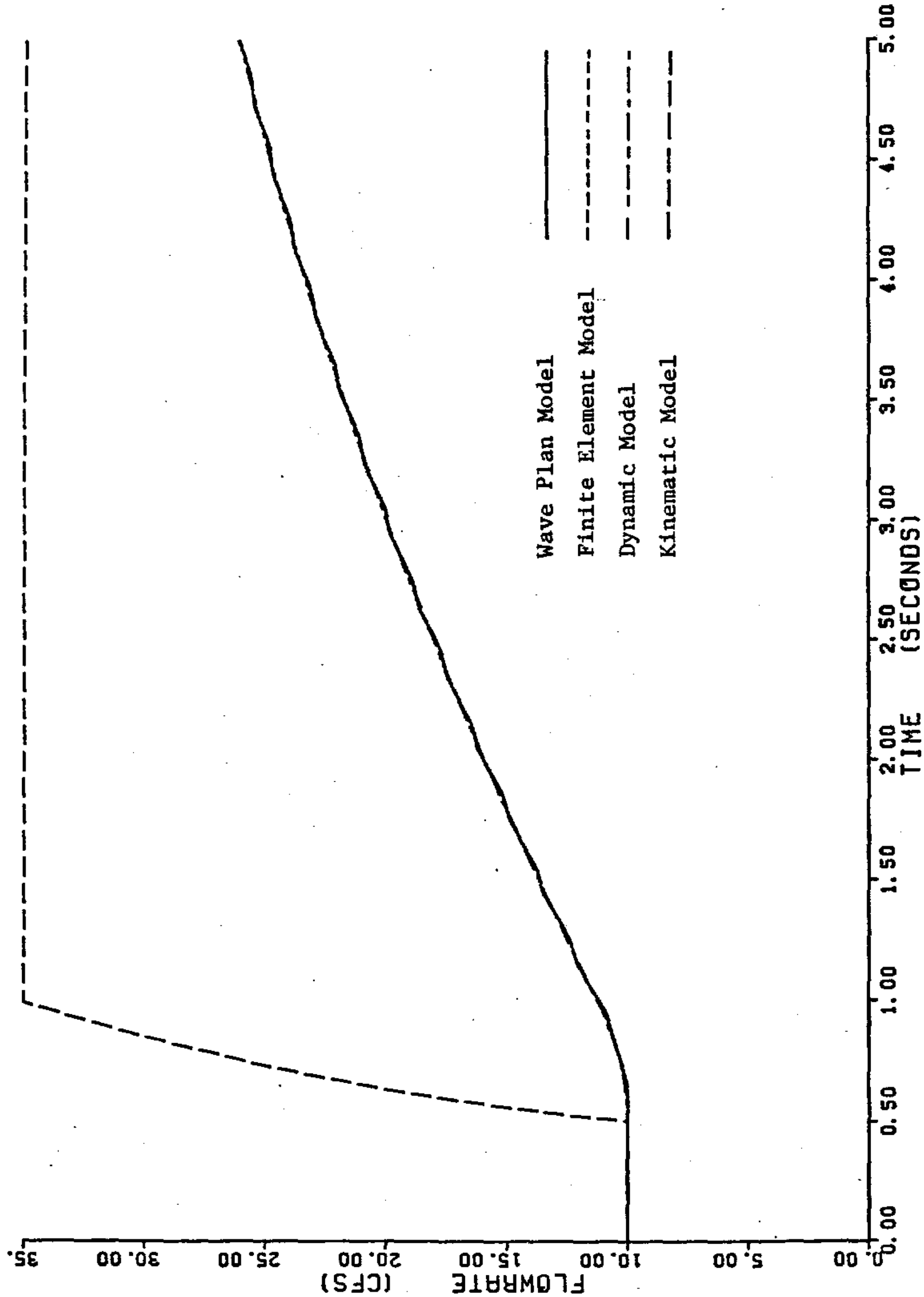


Figure 5-4. Flow Graph for Pipe 1.

5.2 Example Problem # 2

The system shown in Fig. 5-5 illustrates the effects of variable inflow into a manhole of infinite height connected to a one pipe storm sewer system. The system properties are shown in Table 5-1. In order to determine the ability of each model to accurately predict hydraulic grade and flow, the one pipe system is analyzed with three different inflow hydrographs of equal volume. The hydrograph characteristics shown in Table 5-2, corrseponding to Fig. 5-6. With this type of analysis, the system behavior is analyzed under various peak flow conditions.

As shown in Figs. 5-7 through 5-9, the dynamic lumped parameter solution yields results which are nearly identical to those of the finite element distributed parameter model. The steady state solution does not predict the pressure and flow oscillations, however, it does predict accurate peak head and flow values at the correct point in time. As the H-t and Q-t plots indicate, the true transient solution tends to approach a stable steady state condition.

From this analysis and numerous other data runs, it is concluded that severe pressure surges or transients are not a significant problem in pressurized storm sewer systems under peak flow conditions. Peak storm events, such as that in example 2c, may create extended surcharge and flooding, however, pressure surges, as shown in example 1, are not commonly found in completely pressurized sewer systems under normal conditions. Only events such as two-phase flow transitions and in-line pump failures cause significant pressure surges. In the event that pressure surges do exist, it is likely that they will be contained in the sewer link in which they formed. This is due primarily to the connecting manholes which act as surge tanks, absorbing the system pressures. Manhole water levels, however, will rise rapidly causing the most severe surcharge condition which can exist.

For this analysis the maximum dimensionless values were obtained for example 2c in which $\lambda_1 = 0.03$ and $\lambda_2 = 1.59$. These values indicate that the steady state with storage model yields sufficient results for this one pipe - one manhole problem.

PIPE	LENGTH (FT)	DIAMETER (IN)	SLOPE (FT/FT)	ROUGHNESS (FT)	CELERITY (FT/SEC)	INITAL Q (CFS)
1	300	24	0.005	0.001	3000.0	20.0

MANHOLE	ELEVATION (FT)	HEIGHT (FT)	DIAMETER (IN)	INITIAL HEAD (FT)
1	8.0		48.0	11.63
2	6.5	--		10.00 (FIXED)

Table 5-1. Sewer Systems Properties for Example # 2.

EXAMPLE	Qin (CFS)	Qpeak (CFS)	Tpeak (MIN)	Tbase (MIN)	Vhydrograph (CU FT)
2a	20.0	70.0	8.0	24.0	36000
2b	20.0	120.0	4.0	12.0	36000
2c	20.0	220.0	2.0	6.0	36000

Table 5-2. Hydrograph Properties for Example # 2.

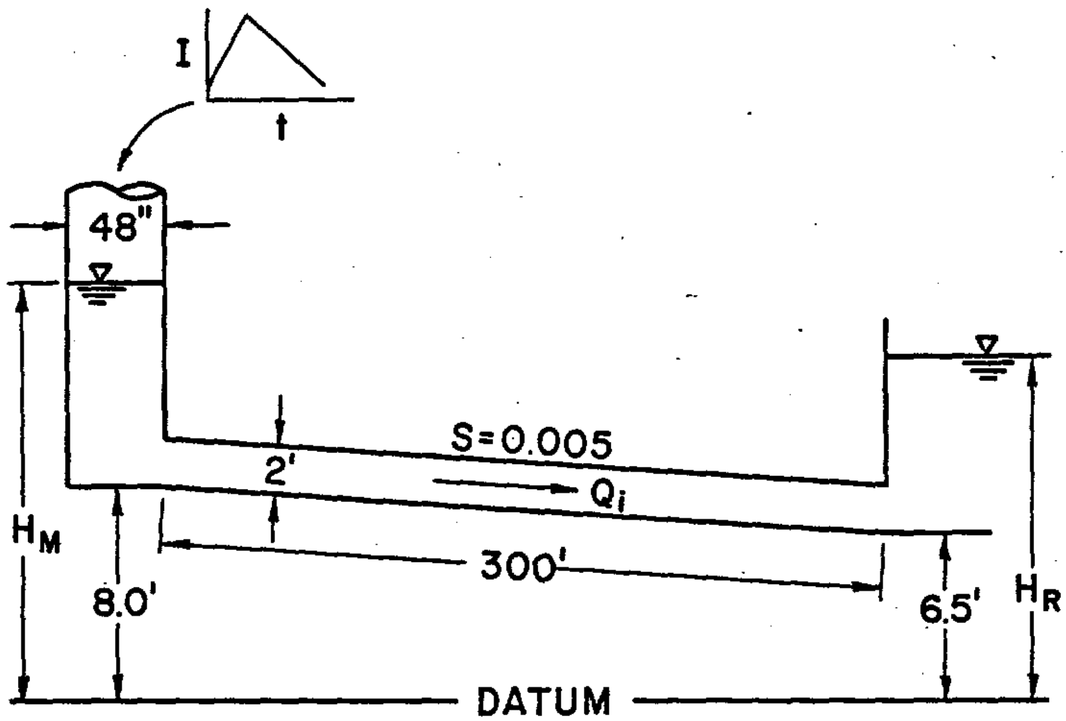


Figure 5-5. One Pipe Sewer System, Example # 2.

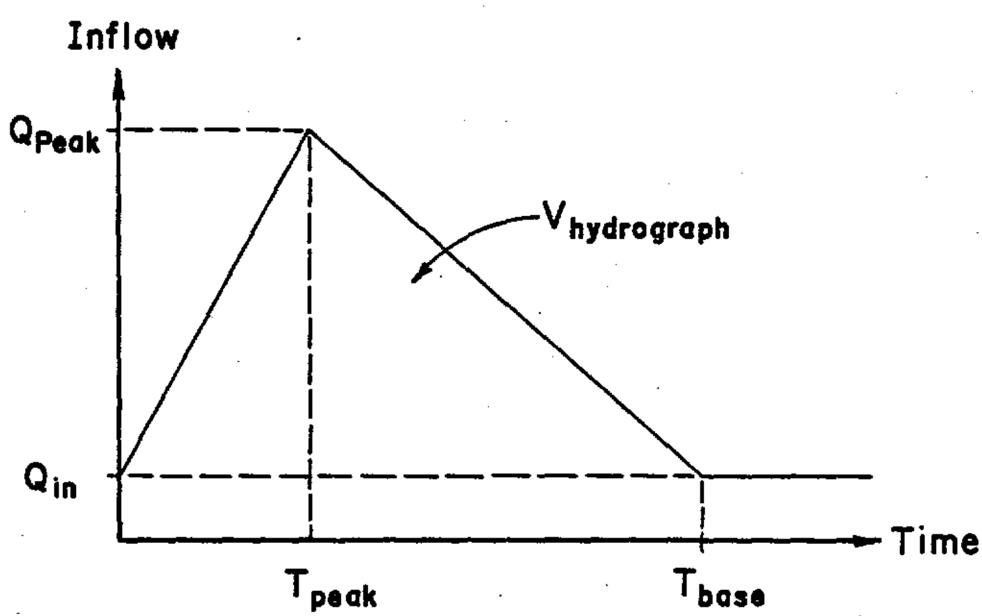


Figure 5-6. Inflow Hydrograph for Example # 2 (Table 5-2).

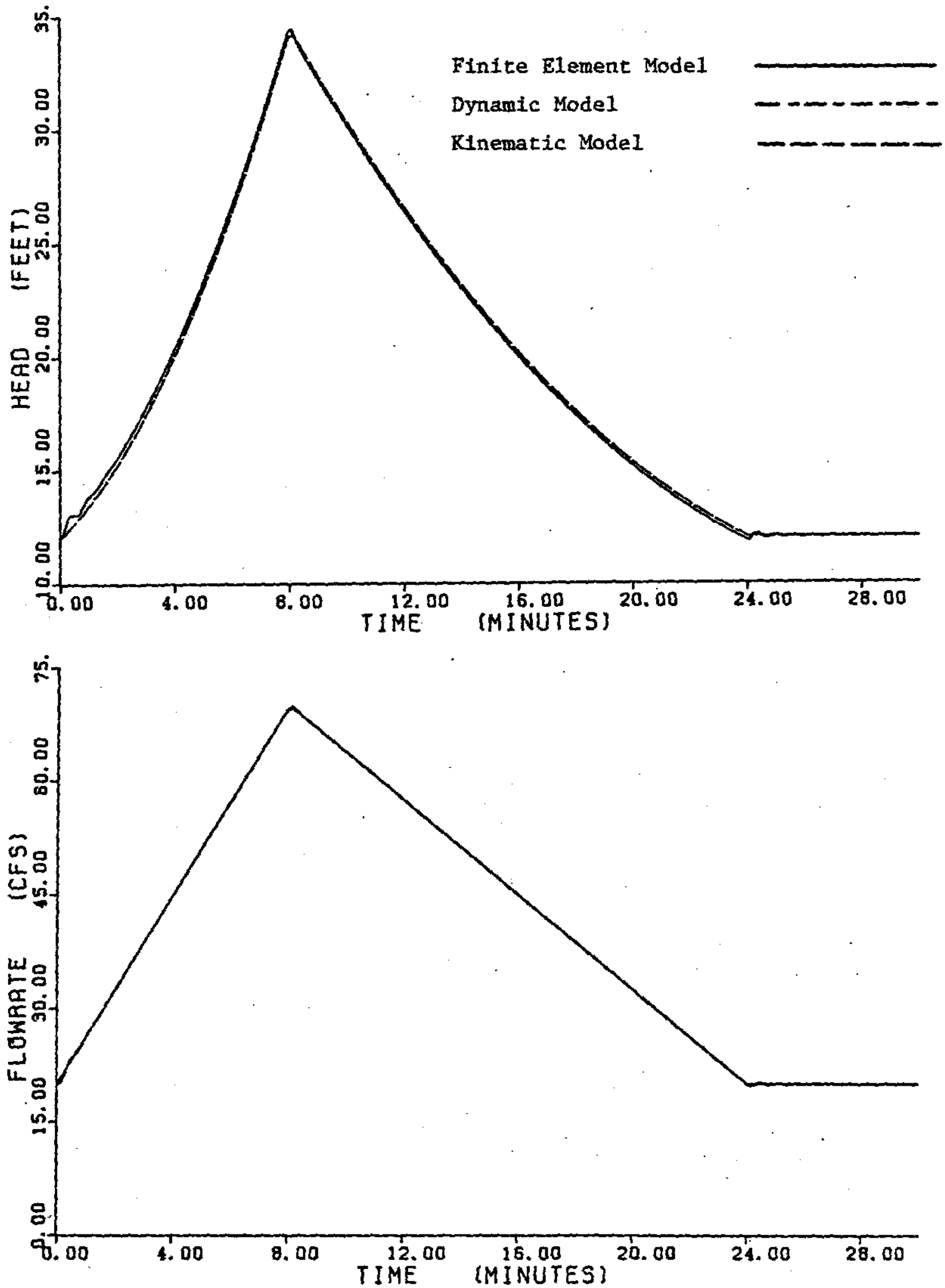


Figure 5-7. Head and Flow Graphs for Junction 1 and Pipe 1, Ex. # 2a.

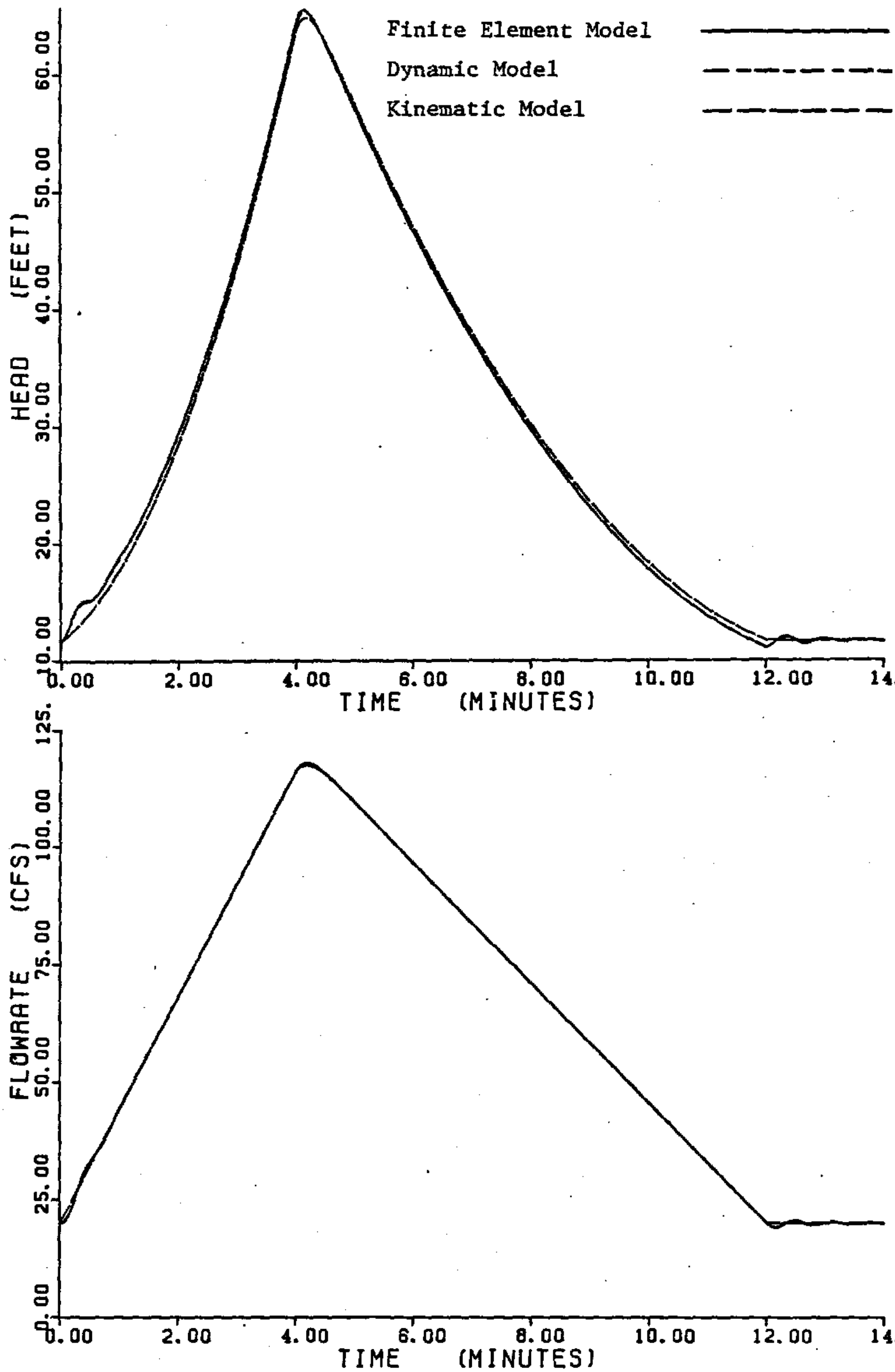


Figure 5-8. Head and Flow Graphs for Junction 1 and Pipe 1, Ex. # 2b.

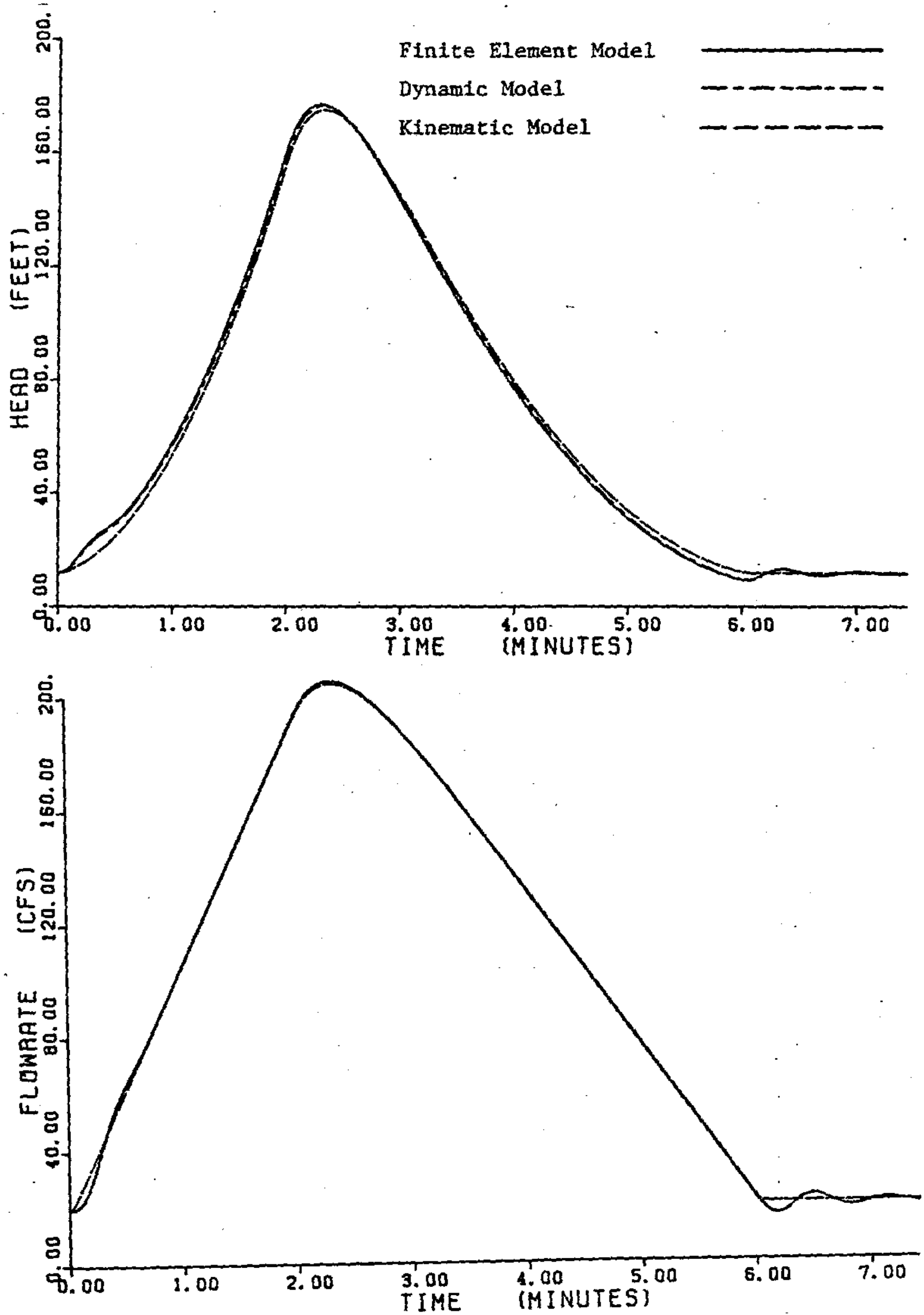


Figure 5-9. Head and Flow Graphs for Junction 1 and Pipe 1, Ex. # 2c.

5.3 Example Problem # 3

This five pipe storm sewer system illustrates the effect of severe surcharge throughout the system including surface flooding at several manholes. The system includes sewer pipes of different lengths, diameters, and slopes as listed in Table 5-3 and shown in Fig. 5-10. An additional component in this analysis is the introduction of minor loss coefficients, which account for the manhole junction losses. For this example the entrance loss is taken as 0.5 while 1.0 is used for the exit loss. Concrete pipe is used with a roughness of 0.001 feet. The system equations are solved using a 1.0 second time step.

Hydraulic grade (head) and flow graphs with time are presented in Figs. 5-11 through 5-15. From this analysis, severe surface flooding is seen to occur at manholes 1, 2 and 3 with minor flooding at manhole 4. Flooding is a stabilizing factor in the analysis and allows for extremely accurate predictions with the kinematic flow model. However, as surface flooding subsides, and the water surface recedes back into the manhole, the flow calculations are unstable using the kinematic model. At this point only the dynamic model accurately predicts the system pressures and flows. The kinematic model consistently predicts average pressure and flow values throughout the instability.

In this example several situations occur in which the water surface in the manholes drops below the pipe crown (eg. manhole 1 at time = 21.0 minutes). This indicates a highly unstable two-phase flow condition in which both pressurized and open channel flow co-exist. The flow models developed herein, do not incorporate two-phase flow and assume pressurized flow throughout the simulation.

A storm sewer network behaves as a system and the feedback between parts of the network is directly shown in the results. In this example, junctions 3 and 5 play key roles in determining the system pressure and flow patterns throughout the simulation. A major disturbance at time = 12.5 minutes at junction 3 (residing water surface in manhole 3) causes severe flow disturbances at that same time in all sewer lines connecting that junction. This cause and effect relationship occurs whenever a substantial flow disturbance is encountered during the simulation.

The dimensionless coefficient λ_2 has a value of 5.84. This indicates that a dynamic flow analysis is desirable for this example problem.

***** ORIGINAL DATA SUMMARY *****

THE DARCY-WEISBACH HEAD LOSS EQUATION IS USED, THE KINEMATIC VISCOSITY = 0.00001059 SQ.FT./SEC.

PIPE NO.	NODE NUMBERS		LENGTH (FEET)	DIAMETER (INCHES)	ROUGHNESS (FEET)	M-LOSS	INITIAL FLOWRATE (CFS)
1	1	3	300.00	24.00	0.00100	1.0	20.00
2	2	3	100.00	24.00	0.00100	1.0	20.00
3	3	5	400.00	36.00	0.00100	2.0	60.00
4	4	5	200.00	30.00	0.00100	1.0	20.00
5	5	6	500.00	48.00	0.00100	2.0	100.00

MANHOLE DATA

JUNCTION NO.	ELEVATION (FEET)	HEIGHT (FEET)	DIAMETER (INCHES)	STORAGE DIAMETER (FEET)	INITIAL HEAD (FEET)
1	63.40	14.00	36.0	200.00	65.600
2	62.10	14.00	36.0	150.00	64.520
3	60.10	14.00	48.0	200.00	63.350
4	56.70	13.00	36.0	150.00	59.380
5	54.50	13.00	60.0	250.00	58.780
6	50.00	THIS JUNCTION HAS A FIXED HEAD OF 55.00 FEET			

HYDROGRAPH INFORMATION

JUNCTION NO.	INITIAL FLOW (CFS)	PEAK FLOW (CFS)	TIME LAG (MINUTES)	TIME TO PEAK (MINUTES)	TIME BASE (MINUTES)
1	20.00	50.00	0.00	4.00	12.00
2	20.00	50.00	0.00	4.00	12.00
3	20.00	50.00	0.00	4.00	12.00
4	20.00	50.00	0.00	4.00	12.00
5	20.00	50.00	0.00	4.00	12.00

Table 5-3. Sewer System Properties for Example # 3.

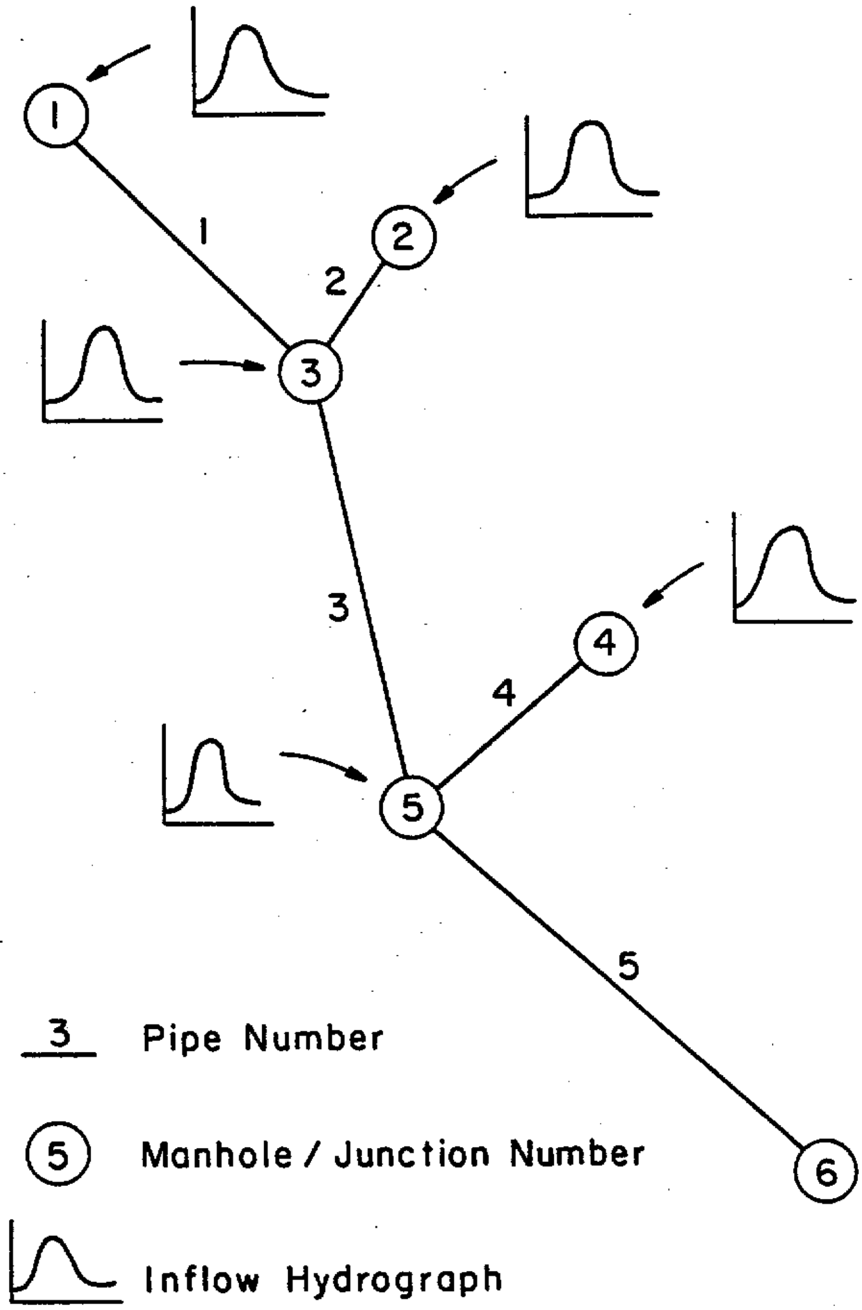


Figure 5-10. Five Pipe Sewer System, Example # 3.

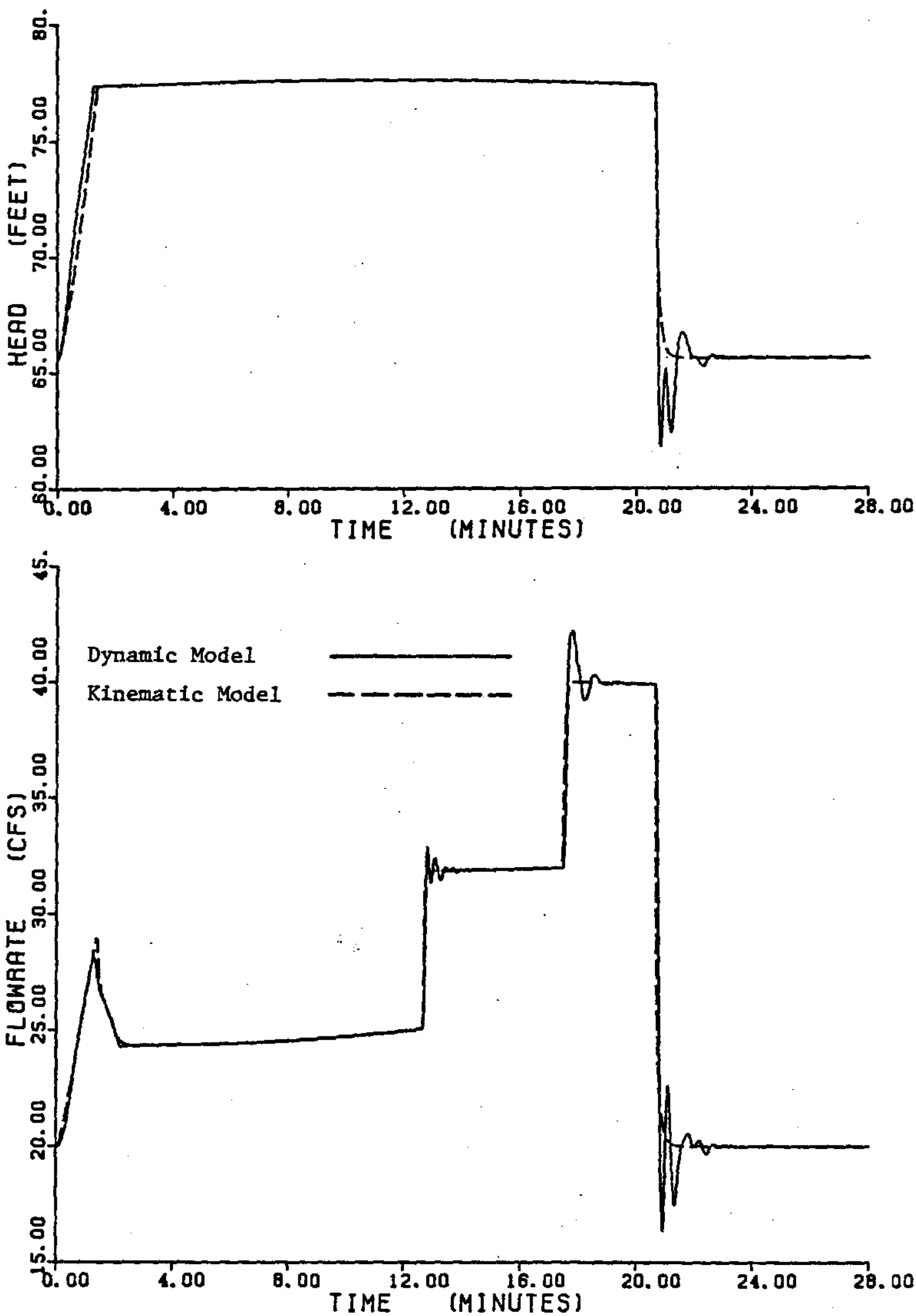


Figure 5-11. Head and Flow Graphs for Junction 1 and Pipe 1, Ex. # 3.

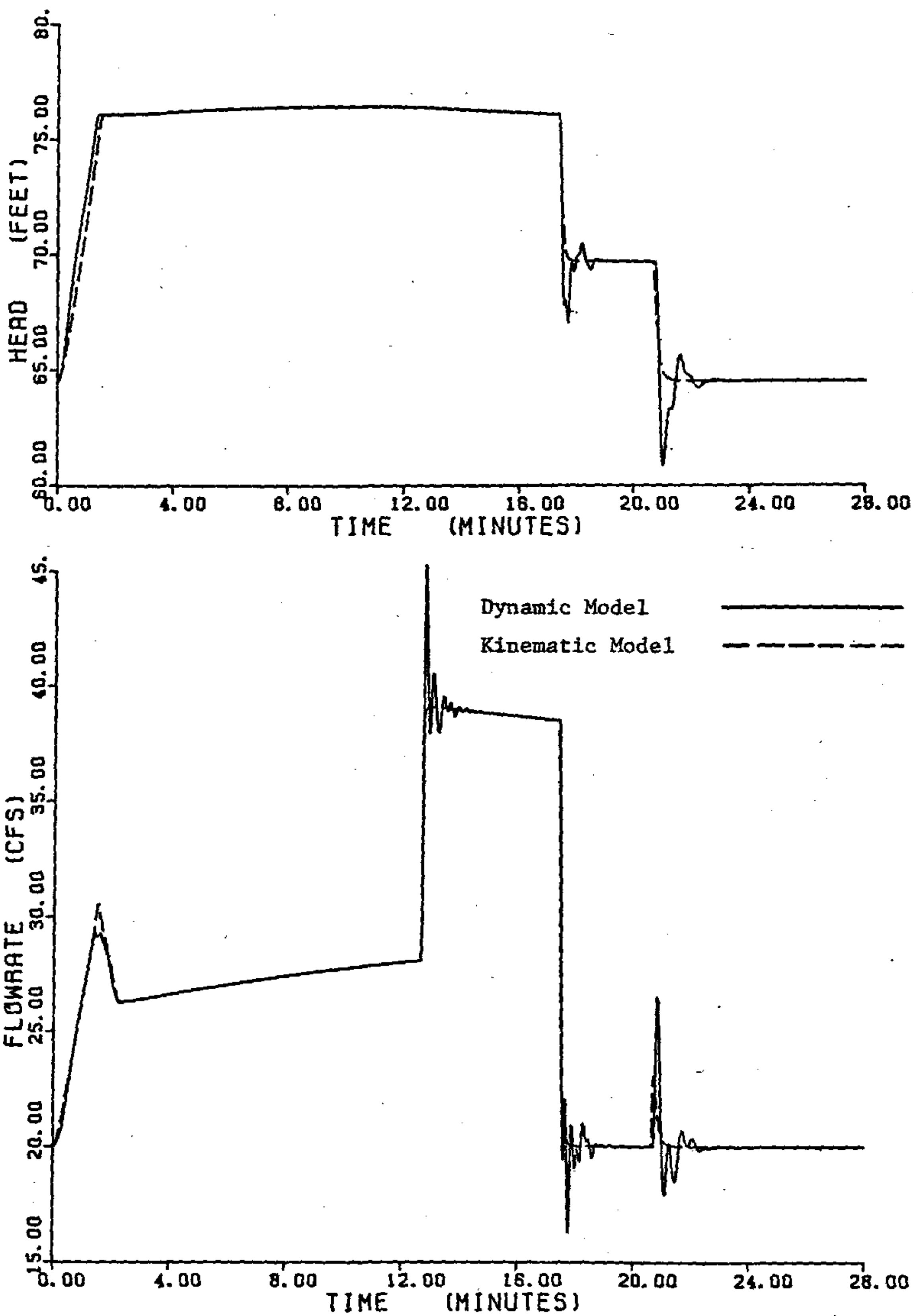


Figure 5-12. Head and Flow Graphs for Junction 2 and Pipe 2, Ex. # 3.

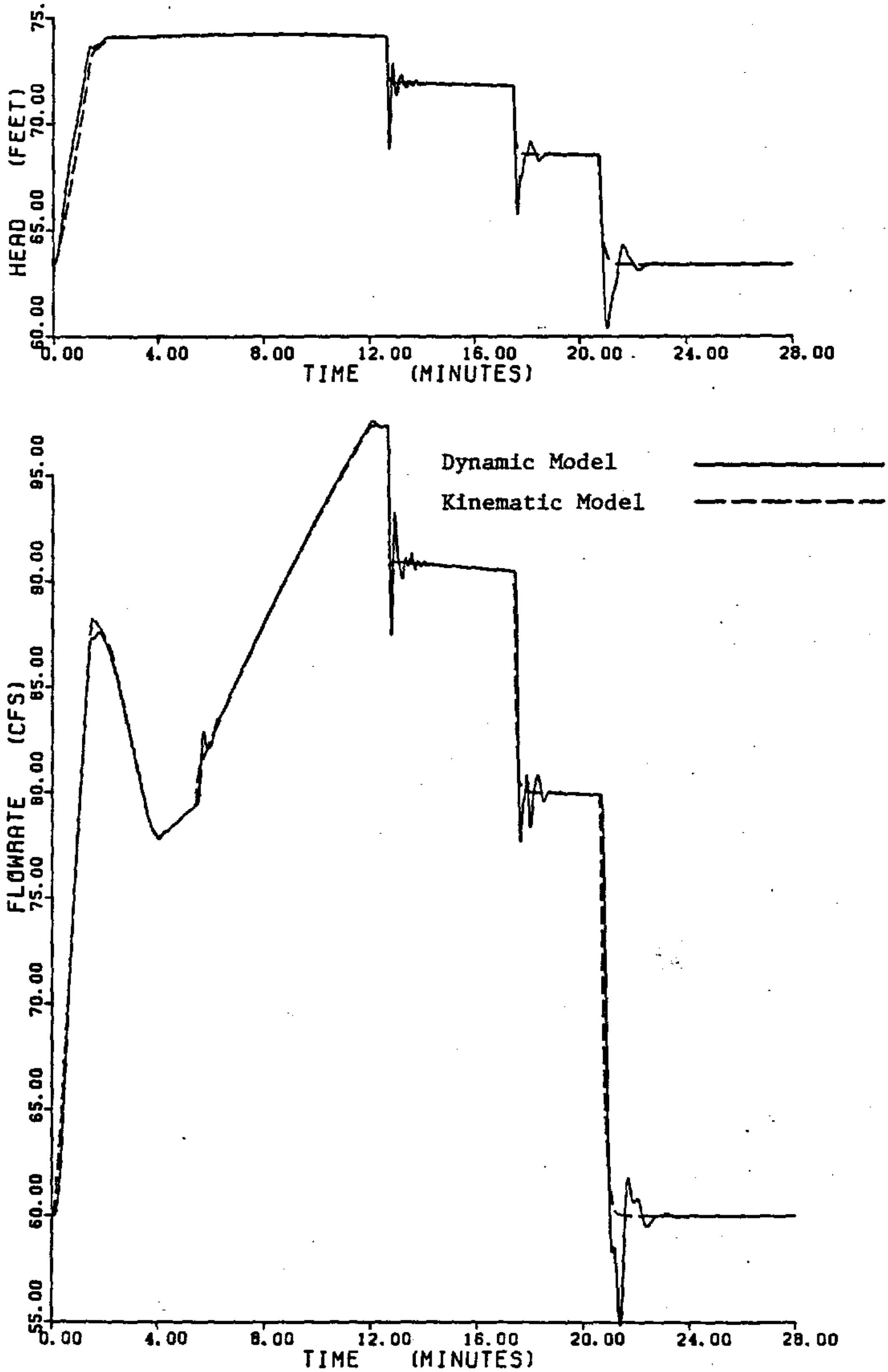


Figure 5-13. Head and Flow Graphs for Junction 3 and Pipe 3, Ex. # 3.

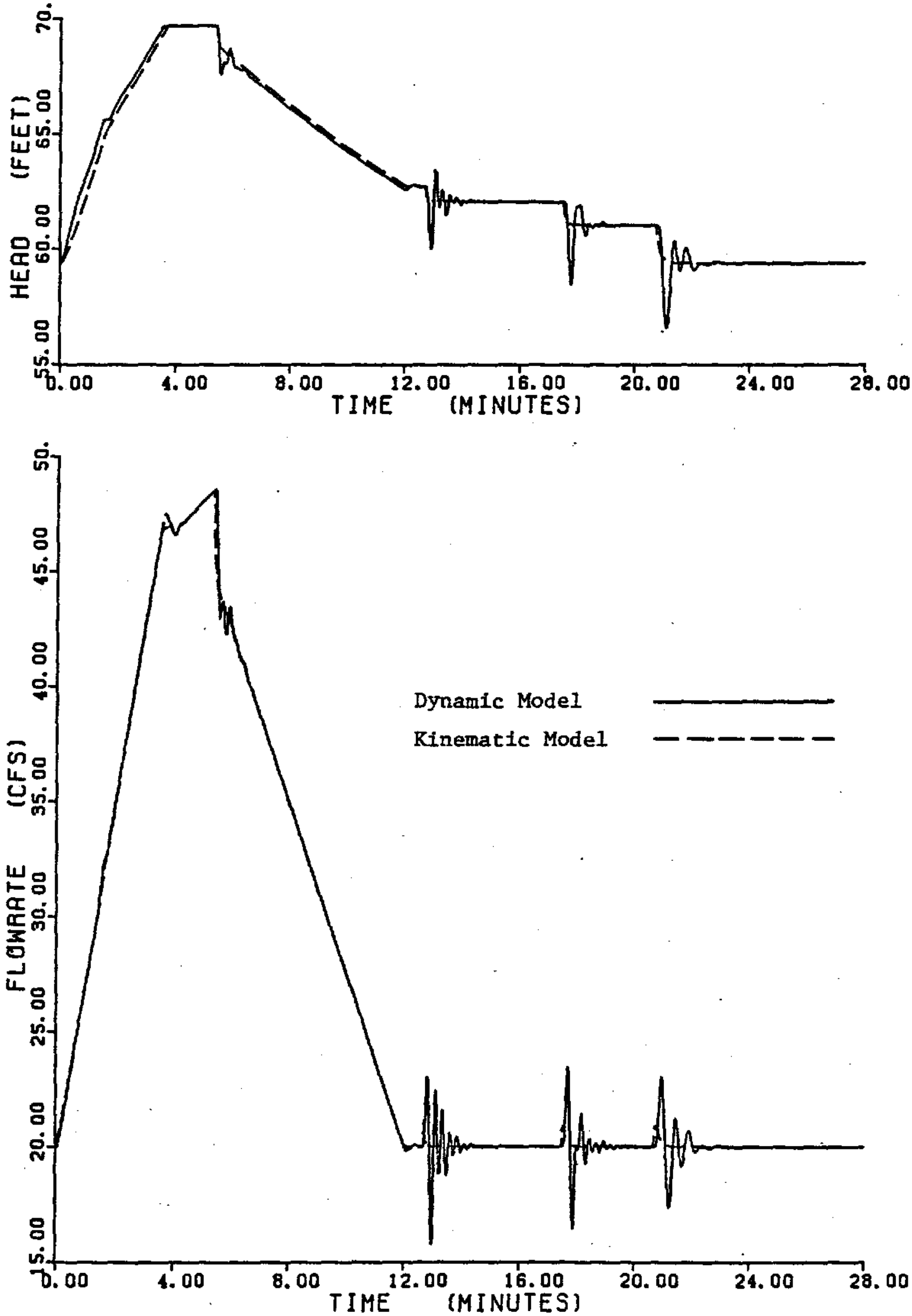


Figure 5-14. Head and Flow Graphs for Junction 4 and Pipe 4, Ex. # 3.

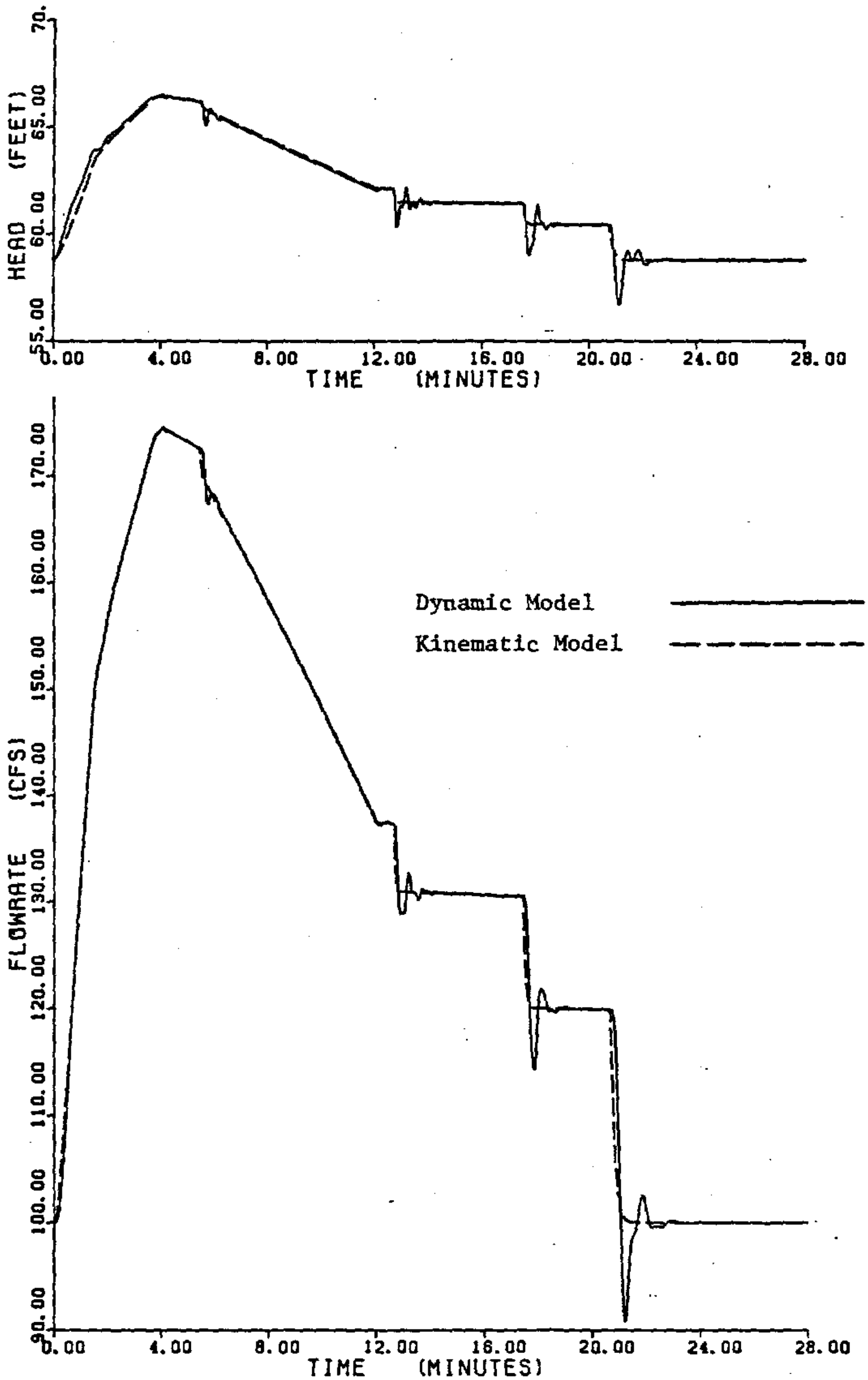


Figure 5-15. Head and Flow Graphs for Junction 5 and Pipe 5, Ex. # 3.

5.4 Example Problem # 4

This seven pipe system shown in Fig. 5-16 is the largest system analyzed and illustrates the time lag effect of storm events. The system properties are shown in Table 5-4. For this system, different inflow hydrographs with appropriate hydrograph time lags are applied to each manhole. Manhole 5 represents a junction box connecting the lateral sewer lines with a vertical drop inlet which restricts surface inflow. Surcharge and surface flooding, however, are allowed at this manhole. The total time of the simulation is 30 minutes with a 0.5 second time step.

The hydraulic grade and flow values are plotted in Figs. 5-17 through 5-23 for each manhole and sewer line respectively. Manholes 1,2,3,4, and 6 experience severe flooding while manholes 5 and 7 completely contain the surcharge within the system. The hydrograph time lag of 1 (manhole 3,4) and 2 minutes (manhole 5,6) directly effects the flow patterns in the corresponding downstream pipes as shown in Figs. 5-22 through 5-25.

As compared to example 3, this system has relatively stable pressure and flow conditions throughout the simulation. The kinematic solution yields very accurate results when compared to the dynamic solution. This indicates that a steady state model may be used for reliable predictions of maximum pressure and peak flows if small time steps are used.

The dimensionless coefficient λ_2 has a maximum value of 0.23 for this problem. This indicates the importance of the frictional forces as compared to the body and inertial forces. For this reason the kinematic solution provides reasonable results.

***** ORIGINAL DATA SUMMARY *****

THE DARCY-WEISBACH HEAD LOSS EQUATION IS USED, THE KINEMATIC VISCOSITY = 0.00001059 SQ.FT./SEC.

PIPE NO.	NODE NUMBERS		LENGTH (FEET)	DIAMETER (INCHES)	ROUGHNESS (FEET)	M-LOSS	INITIAL FLOWRATE (CFS)
1	1	2	600.00	15.00	0.00150	1.0	6.00
2	2	5	1200.00	18.00	0.00150	2.0	12.00
3	3	5	1000.00	15.00	0.00150	1.0	8.00
4	4	5	800.00	15.00	0.00150	1.0	8.00
5	5	7	3000.00	30.00	0.00150	4.0	28.00
6	6	7	600.00	18.00	0.00150	1.0	10.00
7	7	8	1000.00	36.00	0.00150	3.0	48.00

MANHOLE DATA

JUNCTION NO.	ELEVATION (FEET)	HEIGHT (FEET)	DIAMETER (INCHES)	STORAGE DIAMETER (FEET)	INITIAL HEAD (FEET)
1	89.50	16.00	36.0	150.00	91.080
2	84.10	16.00	36.0	150.00	86.950
3	83.90	16.00	48.0	200.00	85.770
4	82.10	16.00	48.0	200.00	83.560
5	70.90	15.00	60.0	200.00	74.040
6	64.00	14.00	48.0	250.00	65.710
7	58.00	12.00	60.0	250.00	61.210
8	50.00	THIS JUNCTION HAS A FIXED HEAD OF 55.00 FEET			

HYDROGRAPH INFORMATION

JUNCTION NO.	INITIAL FLOW (CFS)	PEAK FLOW (CFS)	TIME LAG (MINUTES)	TIME TO PEAK (MINUTES)	TIME BASE (MINUTES)
1	6.00	12.00	0.00	2.00	6.00
2	6.00	12.00	0.00	2.00	6.00
3	8.00	16.00	1.00	4.00	10.00
4	8.00	16.00	1.00	4.00	10.00
5	0.00	0.00	0.00	0.00	0.00
6	10.00	20.00	2.00	6.00	14.00
7	10.00	20.00	2.00	6.00	14.00

Table 5-4. Sewer System Properties for Example # 4.

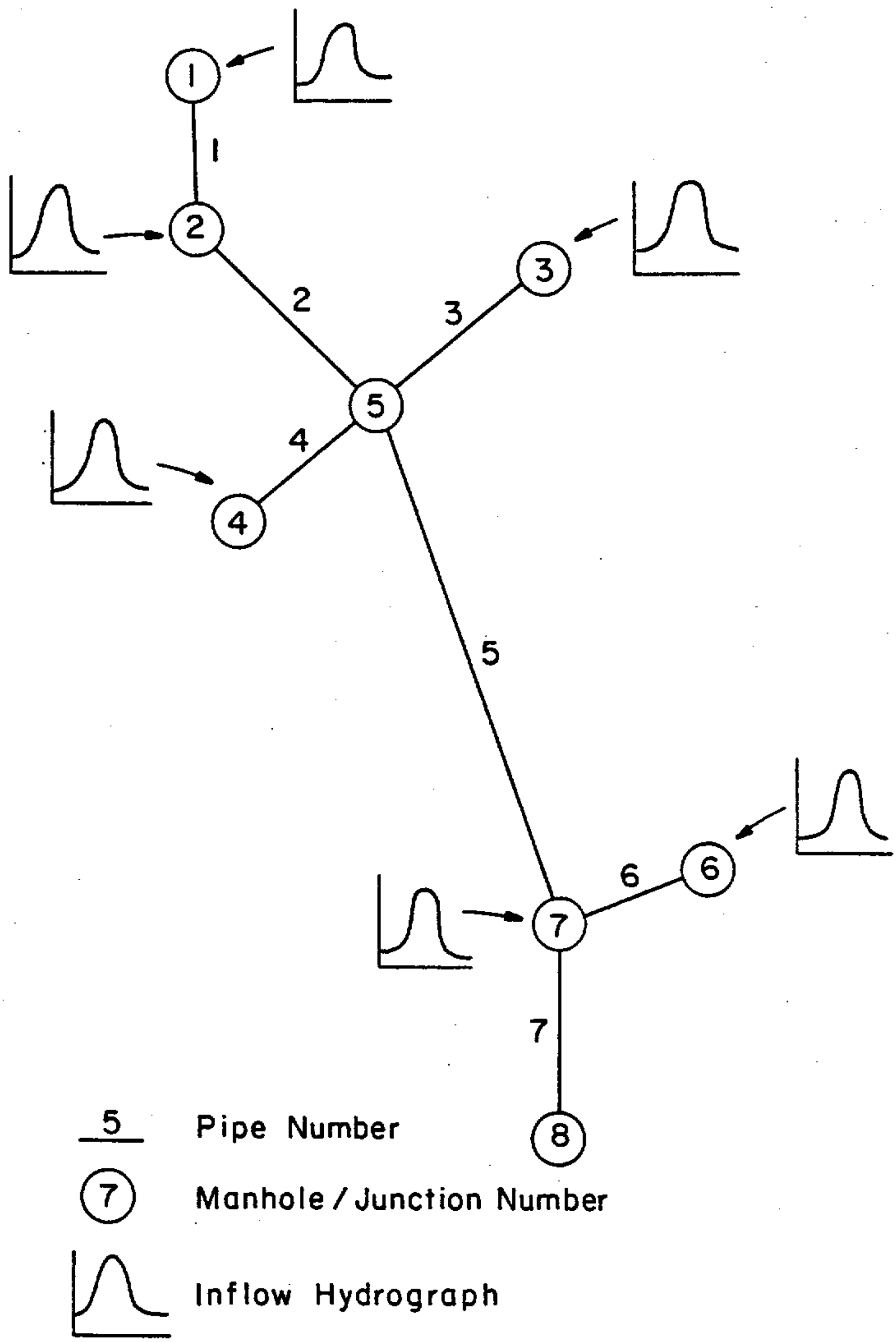


Figure 5-16. Seven Pipe Sewer System, Example # 4.

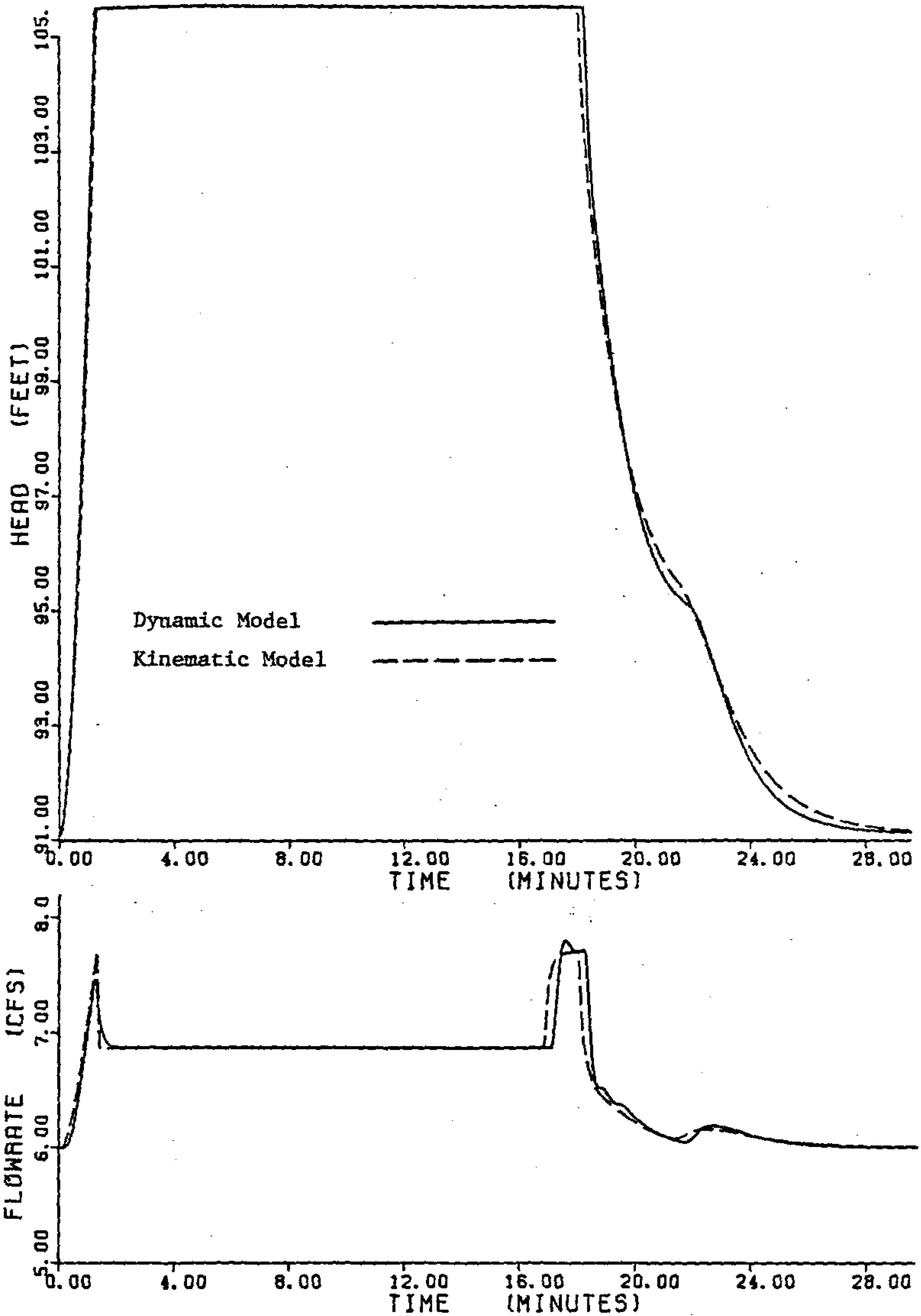
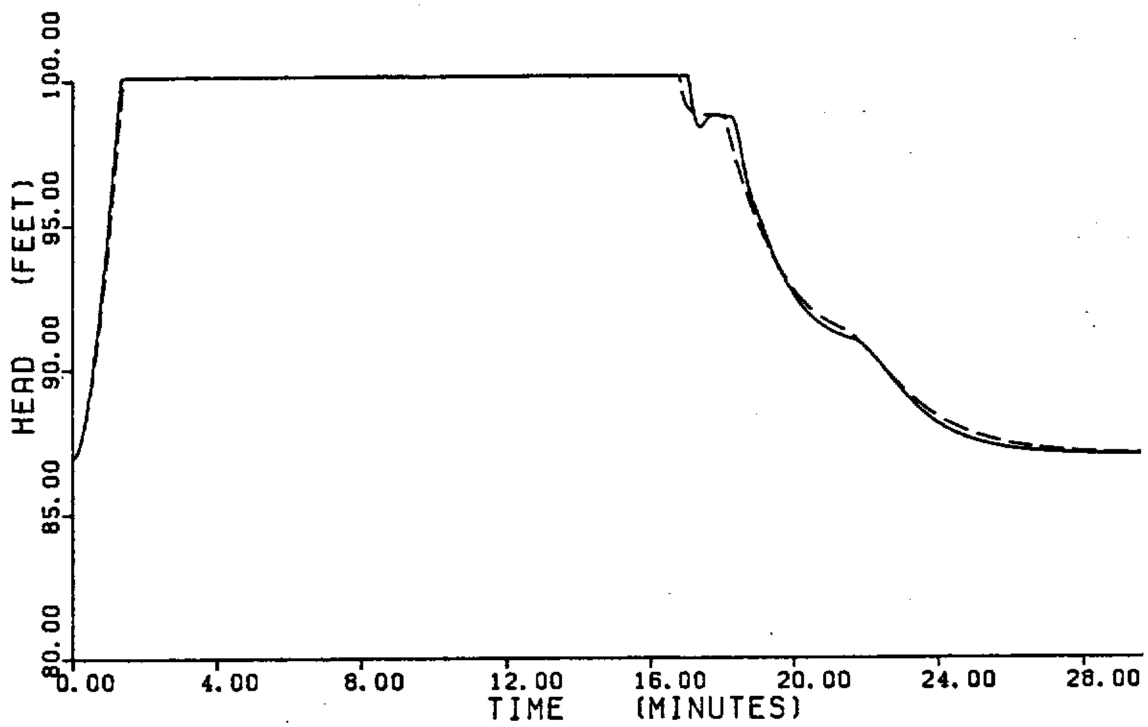


Figure 5-17. Head and Flow Graphs for Junction 1 and Pipe 1, Ex. # 4.



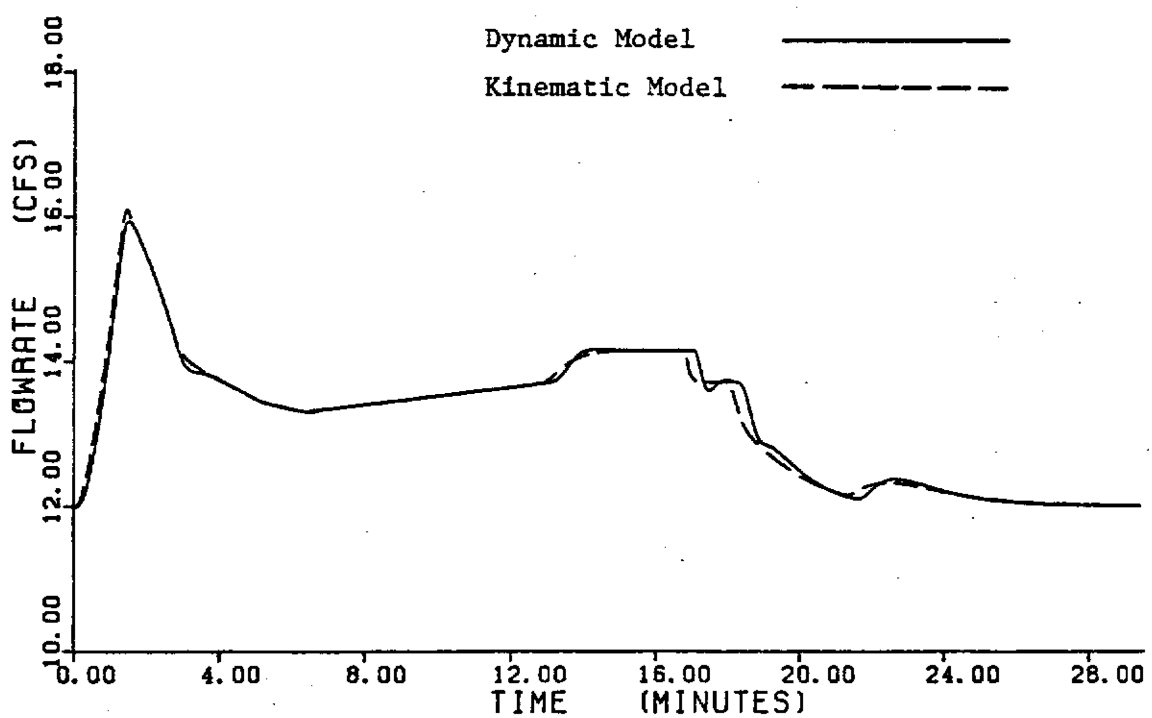


Figure 5-18. Head and Flow Graphs for Junction 2 and Pipe 2, Ex. # 4.

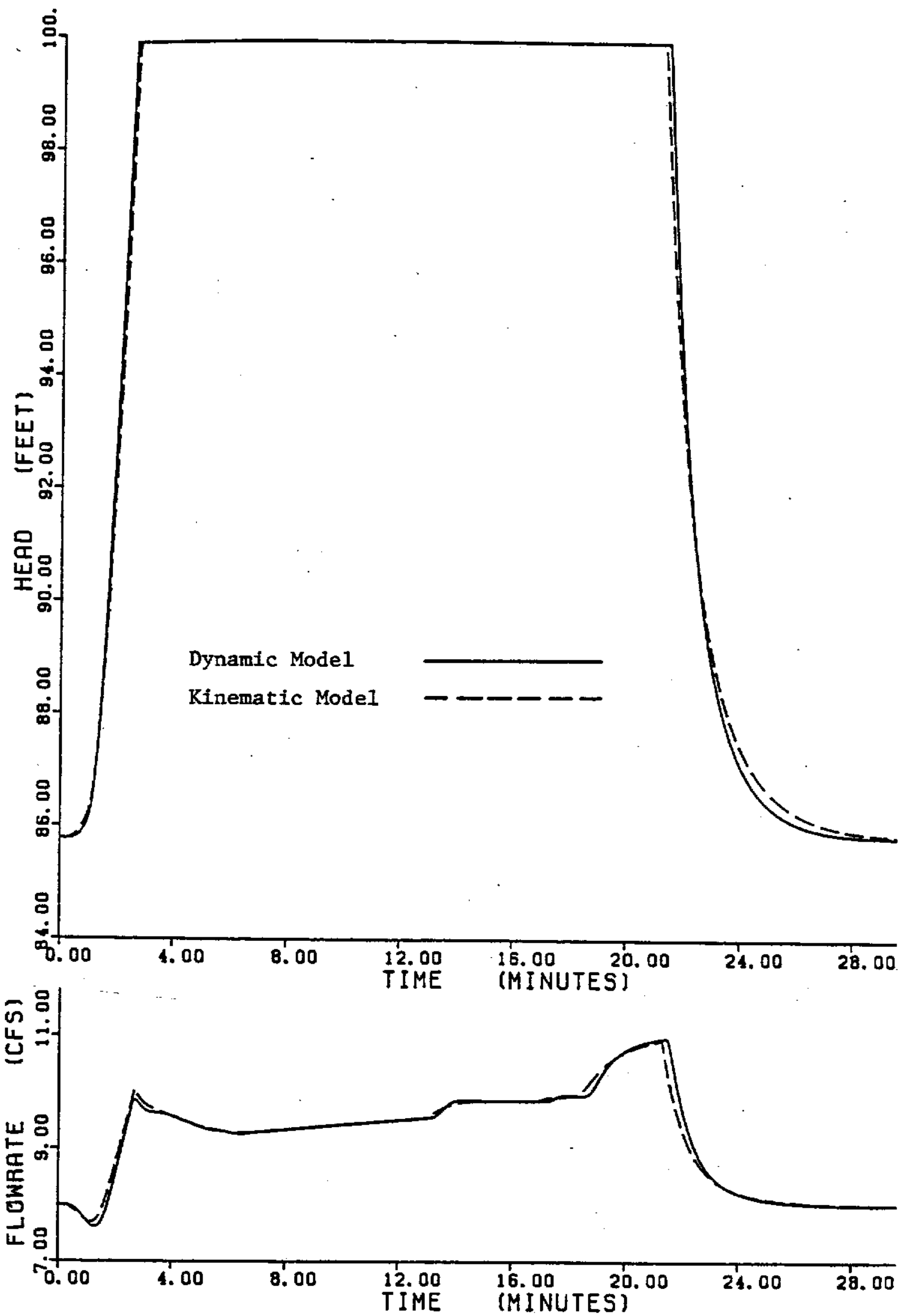


Figure 5-19. Head and Flow Graphs for Junction 3 and Pipe 3, Ex. # 4.

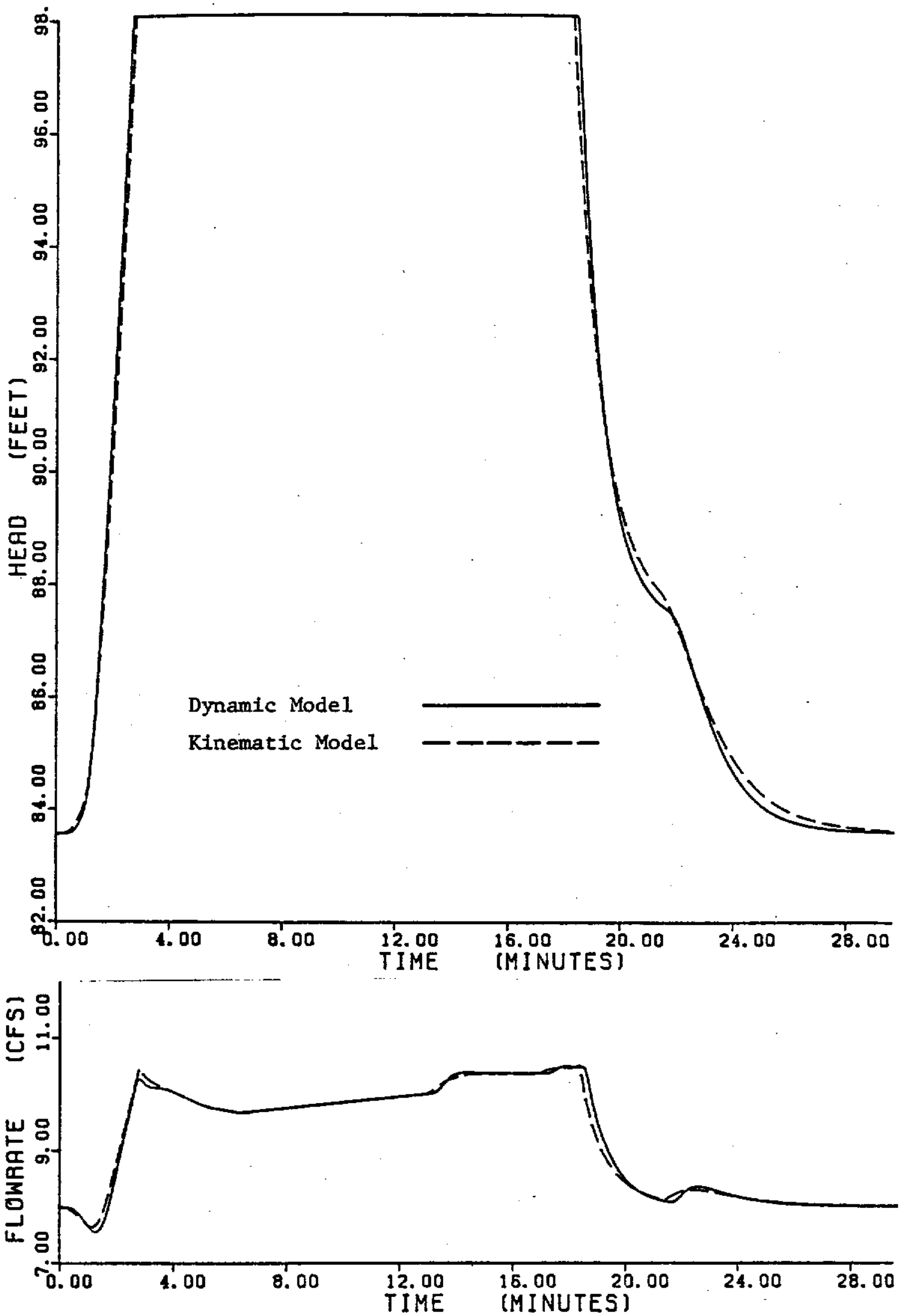
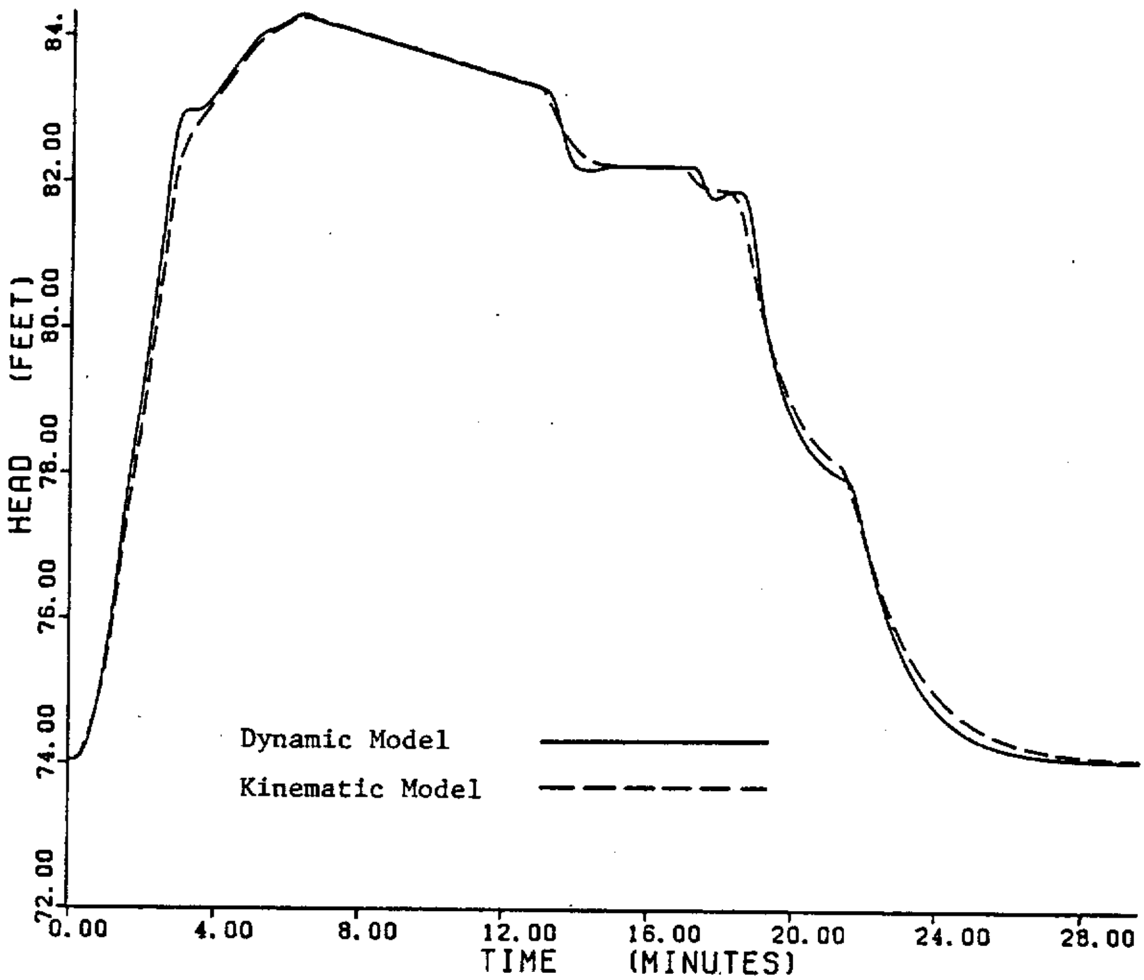


Figure 5-20. Head and Flow Graphs for Junction 4 and Pipe 4, Ex. # 4.



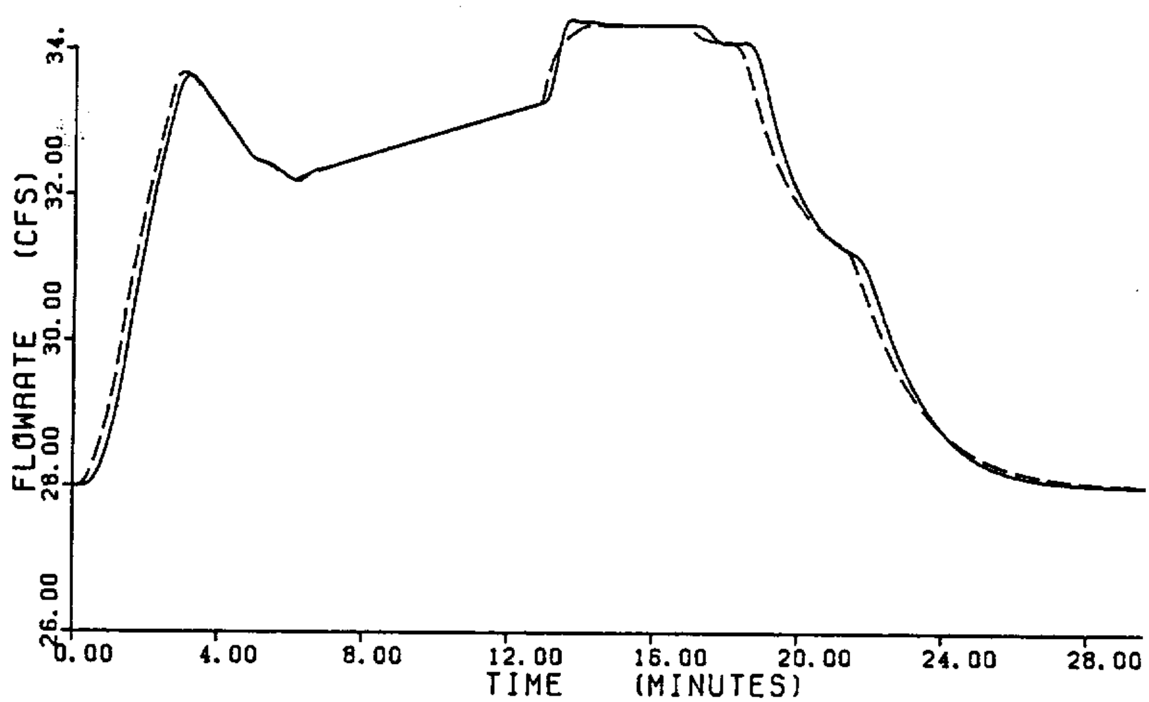


Figure 5-21. Head and Flow Graphs for Junction 5 and Pipe 5, Ex. # 4.

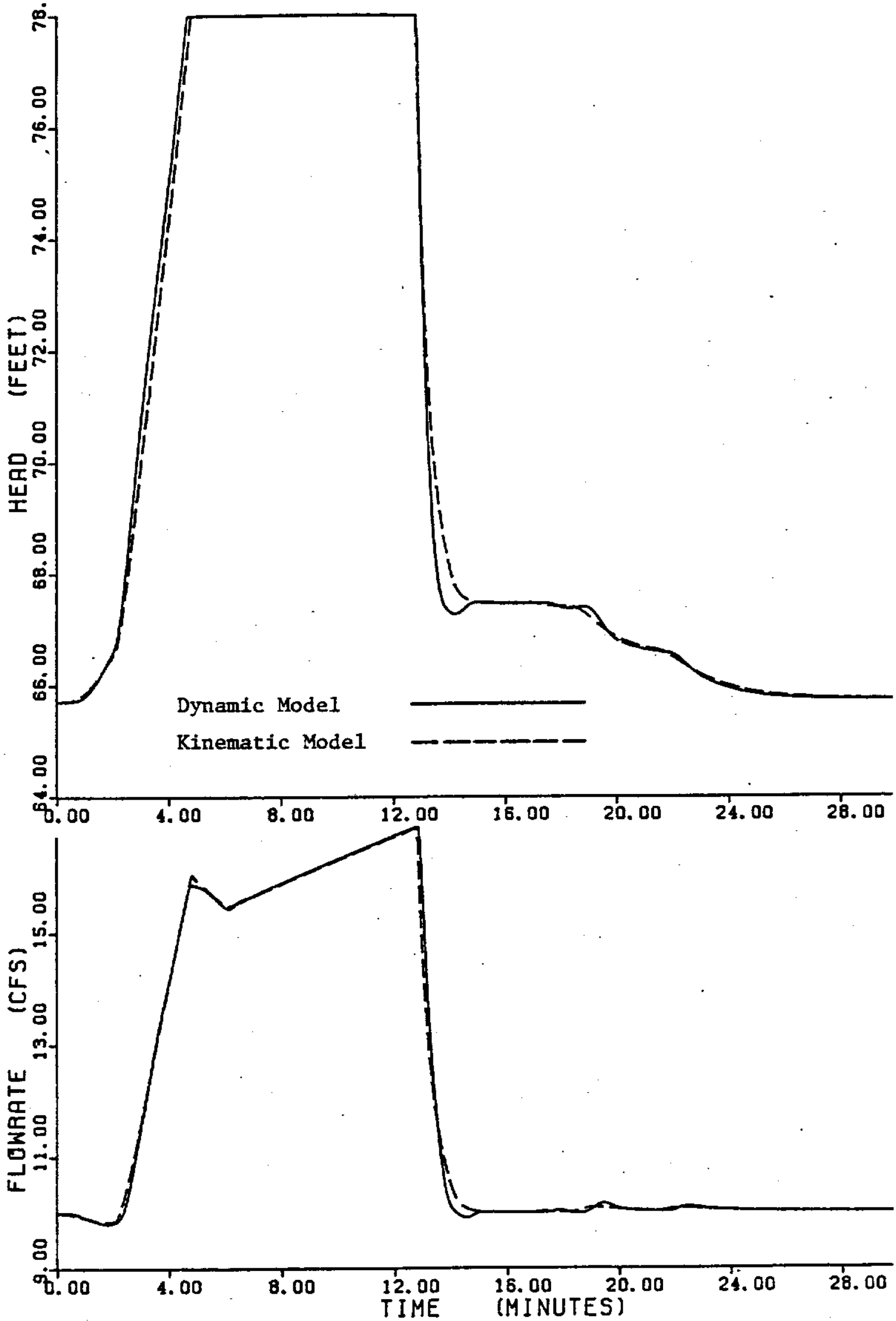
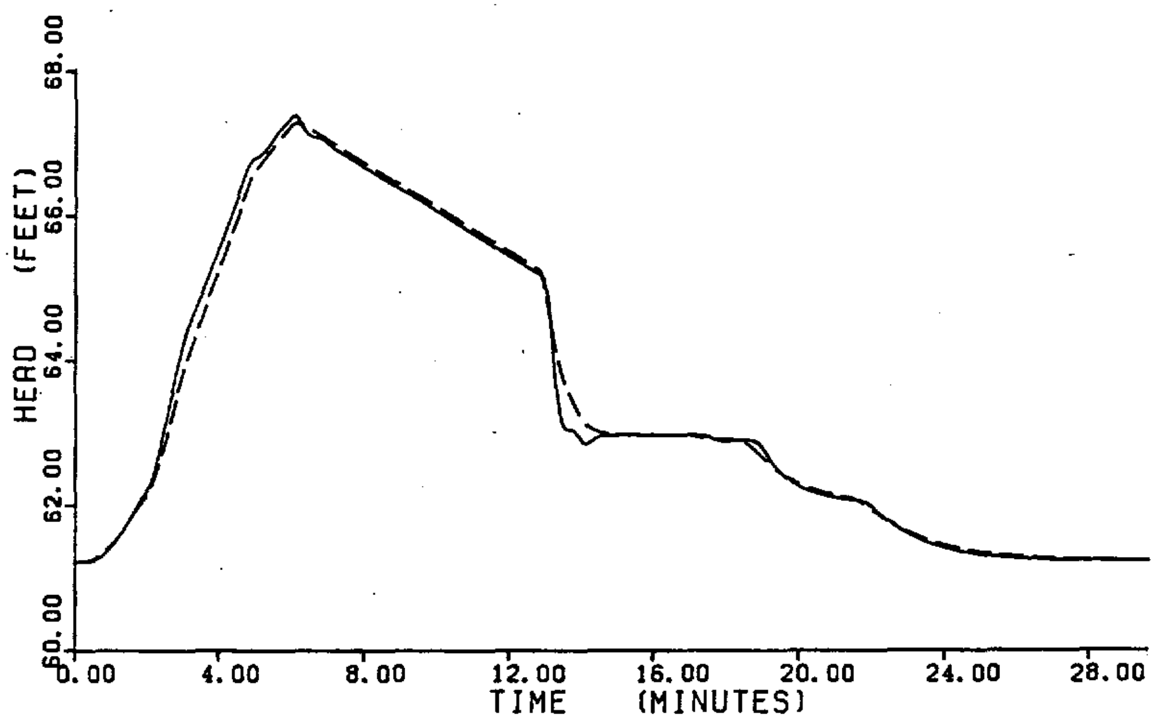


Figure 5-22. Head and Flow Graphs for Junction 6 and Pipe 6, Ex. # 4.



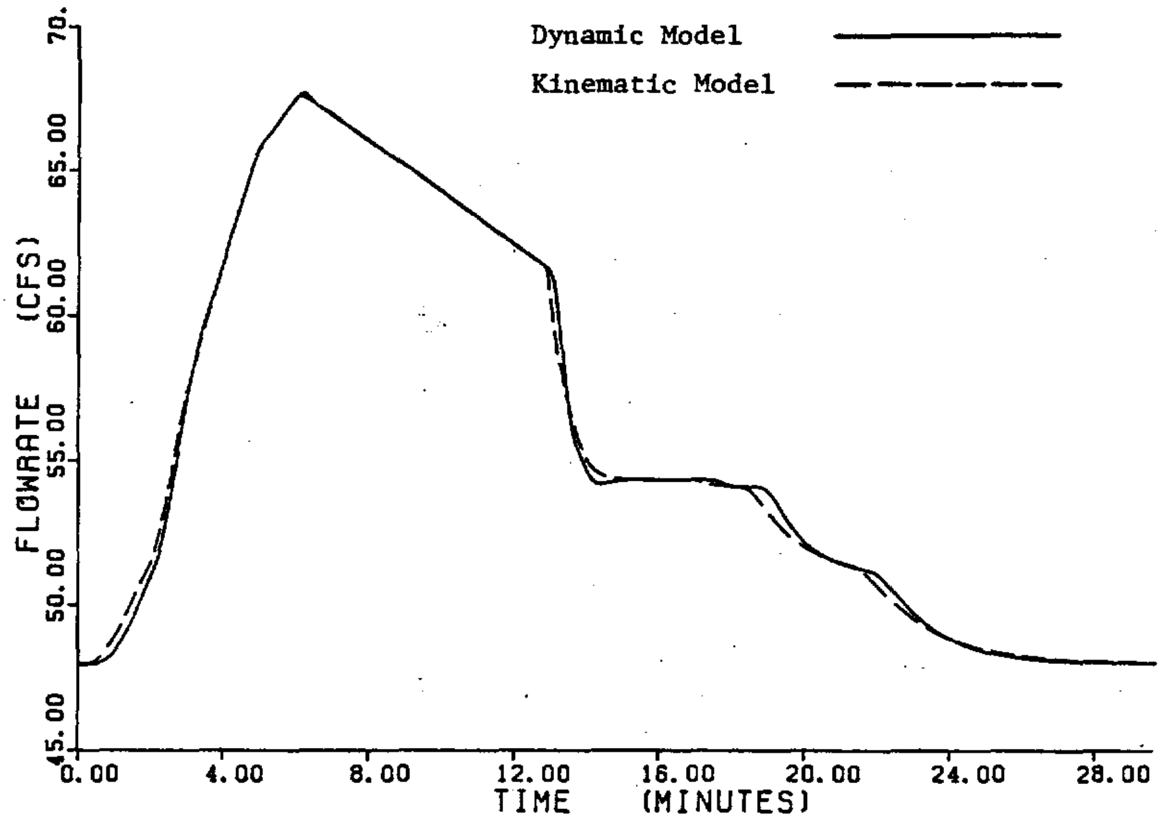


Figure 5-23. Head and flow Graphs for Junction 7 and Pipe 7, Ex. # 4.

5.5 Example Problem # 5

Example # 5 is a 3 sewer line system shown in Fig. 5-24. The system includes sewer pipes of various lengths, diameters and slopes as listed in Table 5-5. Concrete sewer lines are illustrated with a roughness of 0.001 feet. The manhole and inflow hydrograph properties are also shown in Table 5-5.

This 3 pipe storm sewer system is relatively very flat with pipe slopes ranging from 0.001 ft/ft to 0.002 ft/ft. Systems, such as this, generally have substantial surcharge and flooding problems and are often physically and numerically unstable. This is due primarily to the small difference in head between adjacent manholes resulting in unstable flowrates. In addition, this small potential head tends to minimize the system flows resulting in larger storm detention and increased chance of surface flooding. These factors make this 3 pipe system ideal for testing the stability of the three flow models with respect to the time step (Δt) used.

The 3 pipe system is analyzed using the unsteady (transient), dynamic, and kinematic (steady) models in order to determine the relative numerical stability of each model. The finite element distributed parameter model is assumed to provide the most reliable and accurate solution. The maximum time step allowed is 0.0667 seconds (200ft/3000 ft/s) therefore Δt = 0.05 seconds and Δx = 10.0 feet are used for the finite element distributed parameter solution. The dynamic and kinematic models are each tested for time steps (Δt) of 0.1, 1.0 and 5.0 seconds.

The results of hydraulic grade (head) and flow for each run are plotted for comparison in Figs. 5-25 through 5-31 . For this problem the kinematic model using time steps of 1.0 and 5.0 seconds becomes unstable at time t ≅ 11 minutes, producing oscillating positive-negative values of flow for each time step. Kinematic solutions for $\Delta t = 1.0$ and 5.0 seconds are not shown in Figs. 5-25 to 5-31 because of the severe instabilities.

An analysis of the solution plots indicates the highly unstable nature of flow for this system. Both the unsteady and dynamic models yield similar results for small steps ($\Delta t \leq 1.0$). The dynamic solution for $\Delta t = 5.0$ seconds provides good results for peak head and flow predictions, however minor instabilities are encountered when manhole surcharge subsides ($11 \leq t \leq 15$ minutes). As expected the kinematic solution, with $\Delta t = 0.1$ seconds, provides reliable predictions of head and flow values throughout the time simulation. For periods of unstable flow ($t \geq 11$ minutes), the kinematic model predicts average values of head and flow in each manhole and sewer line.

***** ORIGINAL DATA SUMMARY *****

THE DARCY-WEISBACH HEAD LOSS EQUATION IS USED, THE KINEMATIC VISCOSITY = 0.00001059 SQ.FT./SEC.

PIPE NO.	NODE NUMBERS		LENGTH (FEET)	DIAMETER (INCHES)	ROUGHNESS (FEET)	M-LOSS	INITIAL FLOWRATE (CFS)
1	1	3	200.00	18.00	0.00100	0.0	5.00
2	2	3	300.00	24.00	0.00100	0.0	5.00
3	3	4	500.00	30.00	0.00100	0.0	15.00

MANHOLE DATA

JUNCTION NO.	ELEVATION (FEET)	HEIGHT (FEET)	DIAMETER (INCHES)	STORAGE DIAMETER (FEET)	INITIAL HEAD (FEET)
1	52.90	15.00	36.0	150.00	55.800
2	53.10	15.00	36.0	150.00	55.600
3	52.50	15.00	48.0	150.00	55.490
4	52.00	THIS JUNCTION HAS A FIXED HEAD OF 55.00 FEET			

HYDROGRAPH INFORMATION

JUNCTION NO.	INITIAL FLOW (CFS)	PEAK FLOW (CFS)	TIME LAG (MINUTES)	TIME TO PEAK (MINUTES)	TIME BASE (MINUTES)
1	5.00	30.00	0.00	4.00	12.00
2	5.00	30.00	0.00	4.00	12.00
3	5.00	30.00	0.00	4.00	12.00

Table 5-5. Sewer System Properties for Example # 5.

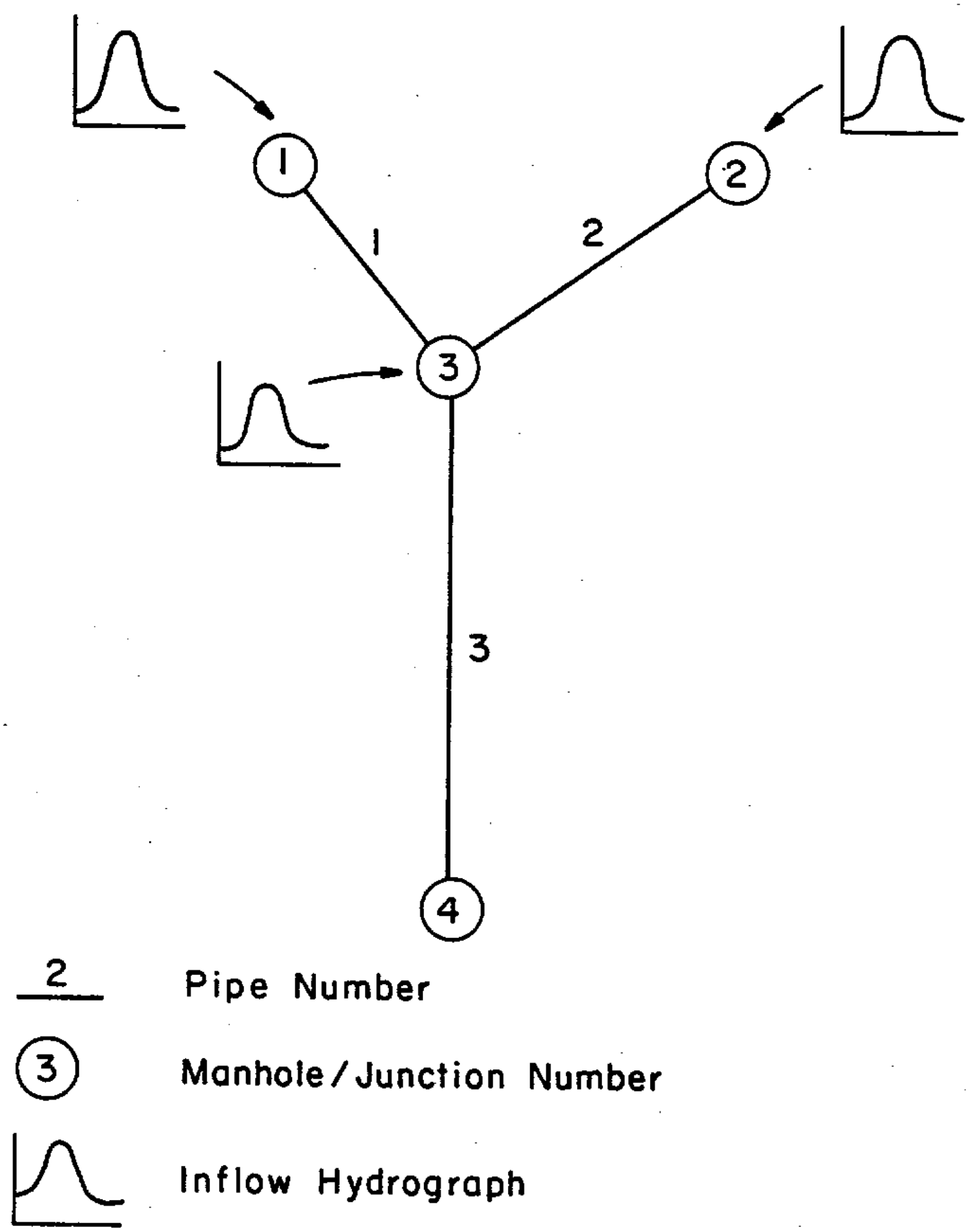


Figure 5-24. Three Pipe Sewer System, Example # 5.

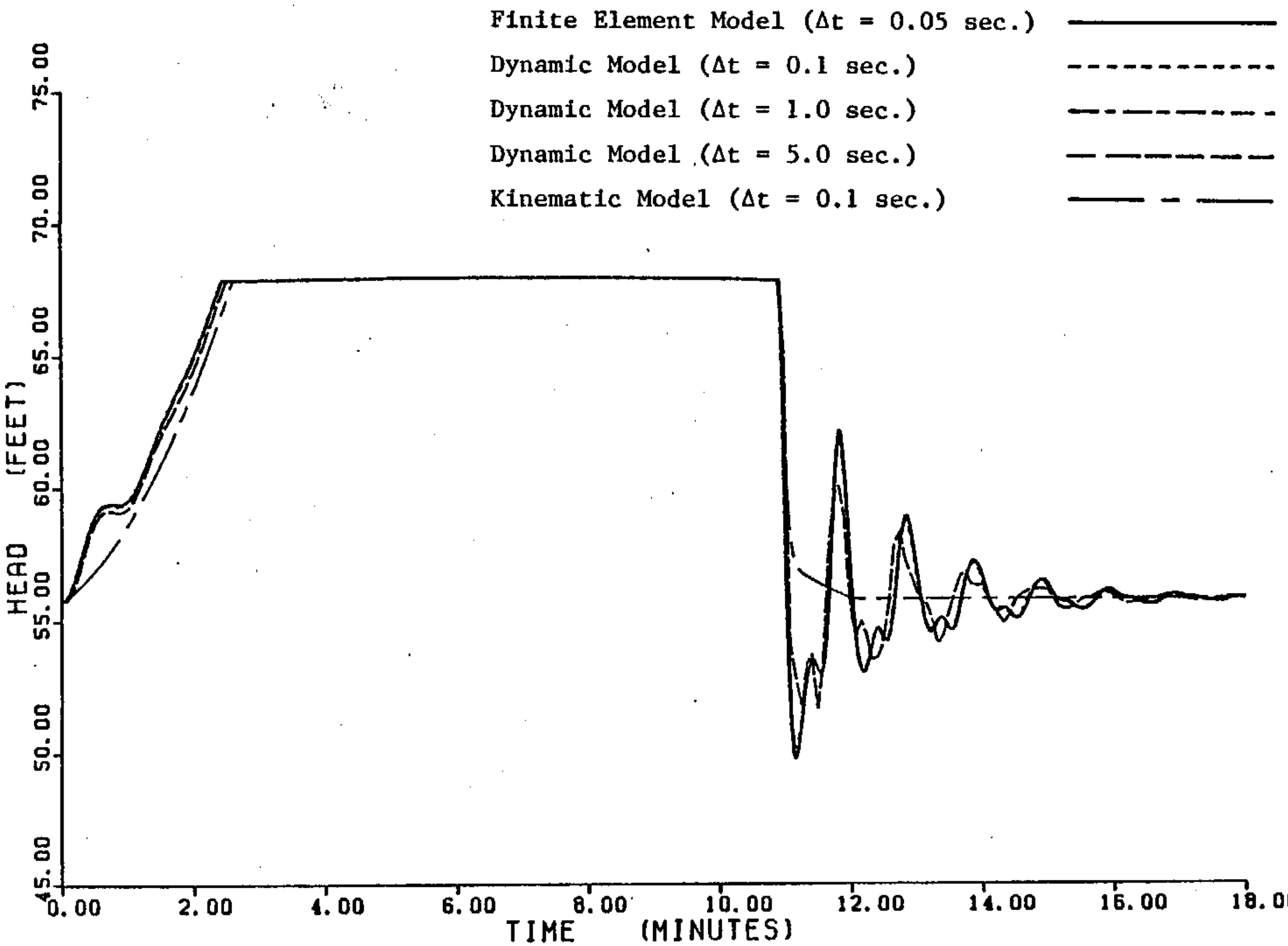


Figure 5-25. Total Head Graph for Junction 1, Example # 5.

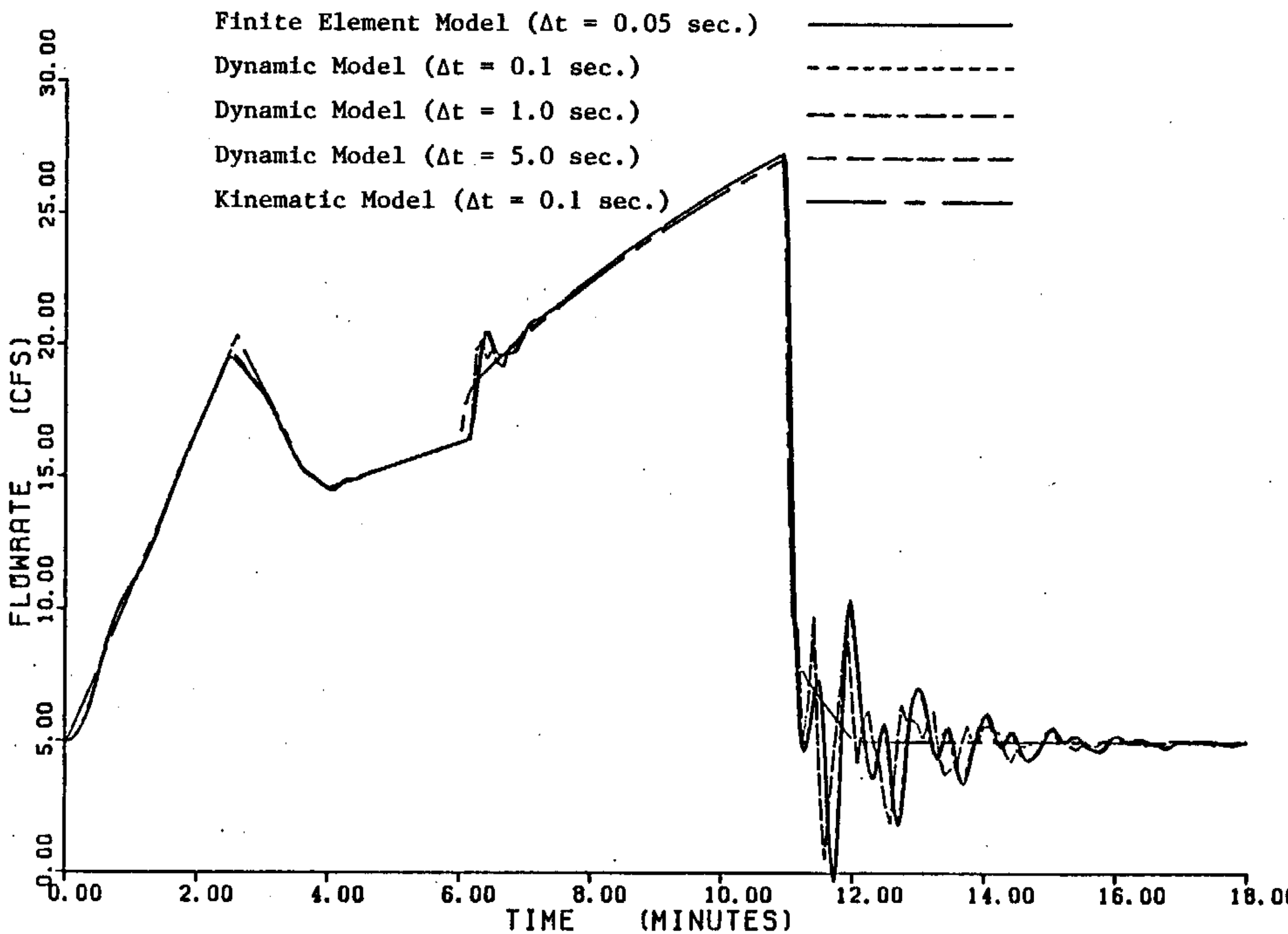


Figure 5-26. Flow Graph for Pipe 1, Example # 5.

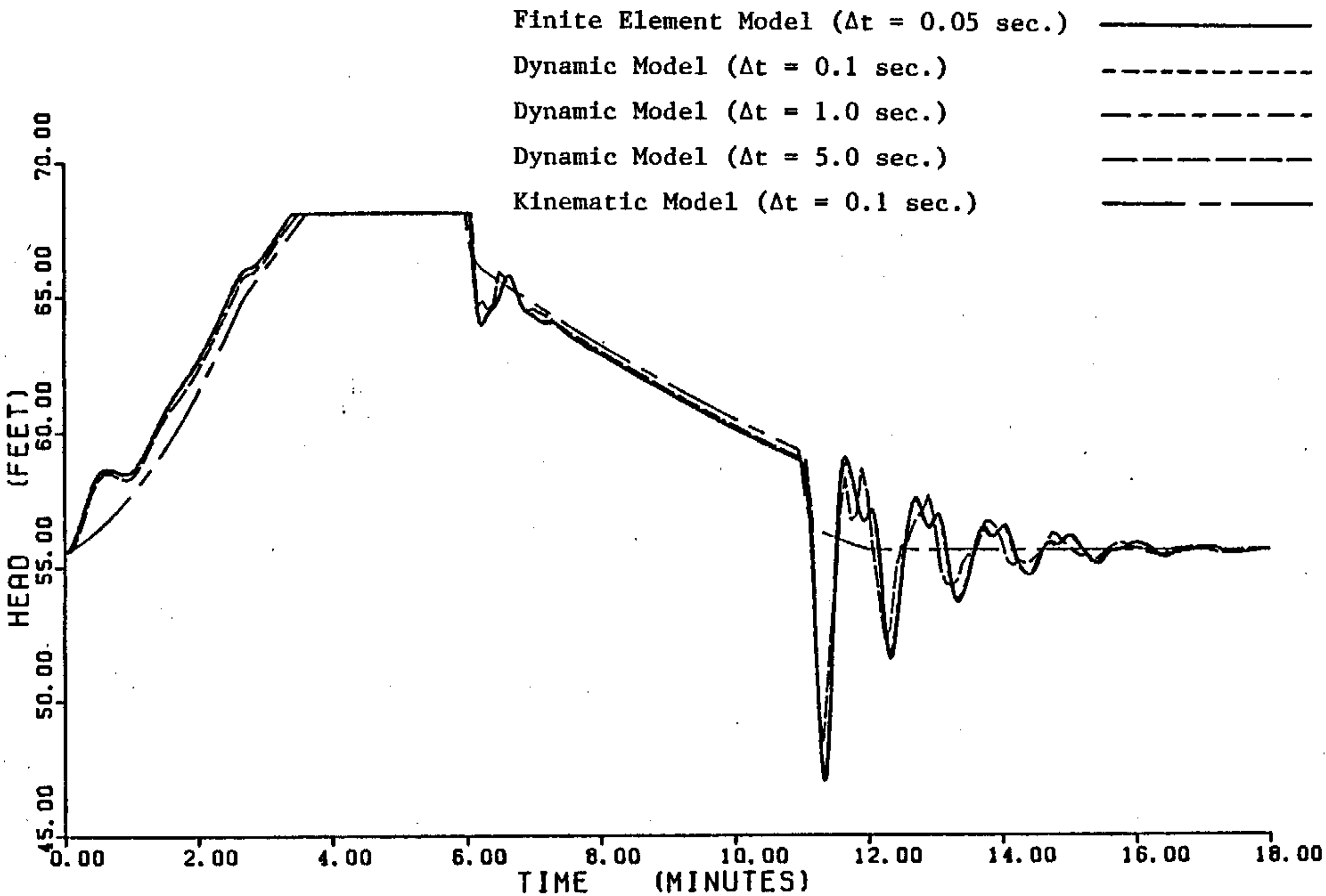


Figure 5-27. Total Head Graph for Junction 2, Example # 5.

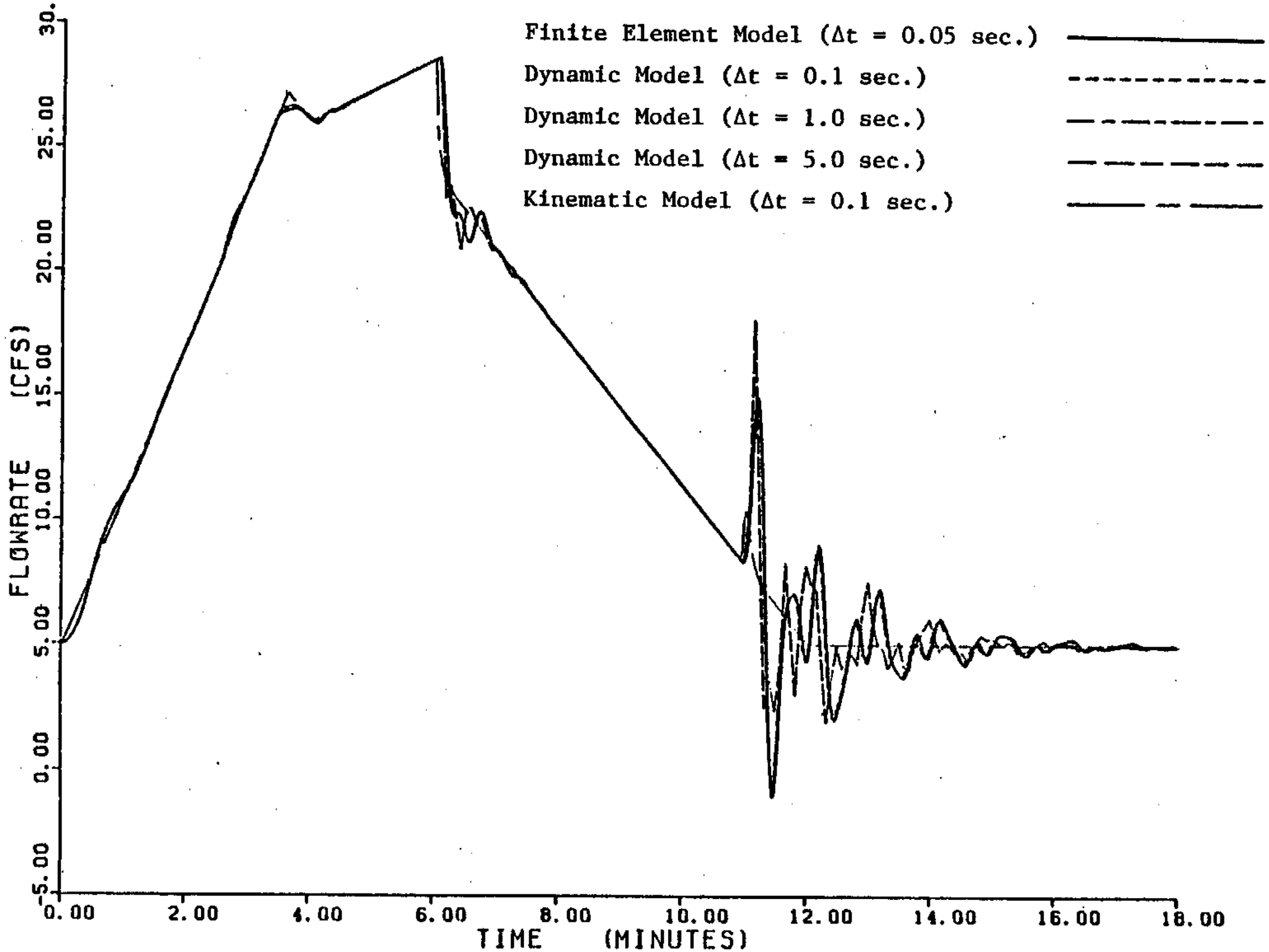


Figure 5-28. Flow Graph for Pipe 2, Example # 5.

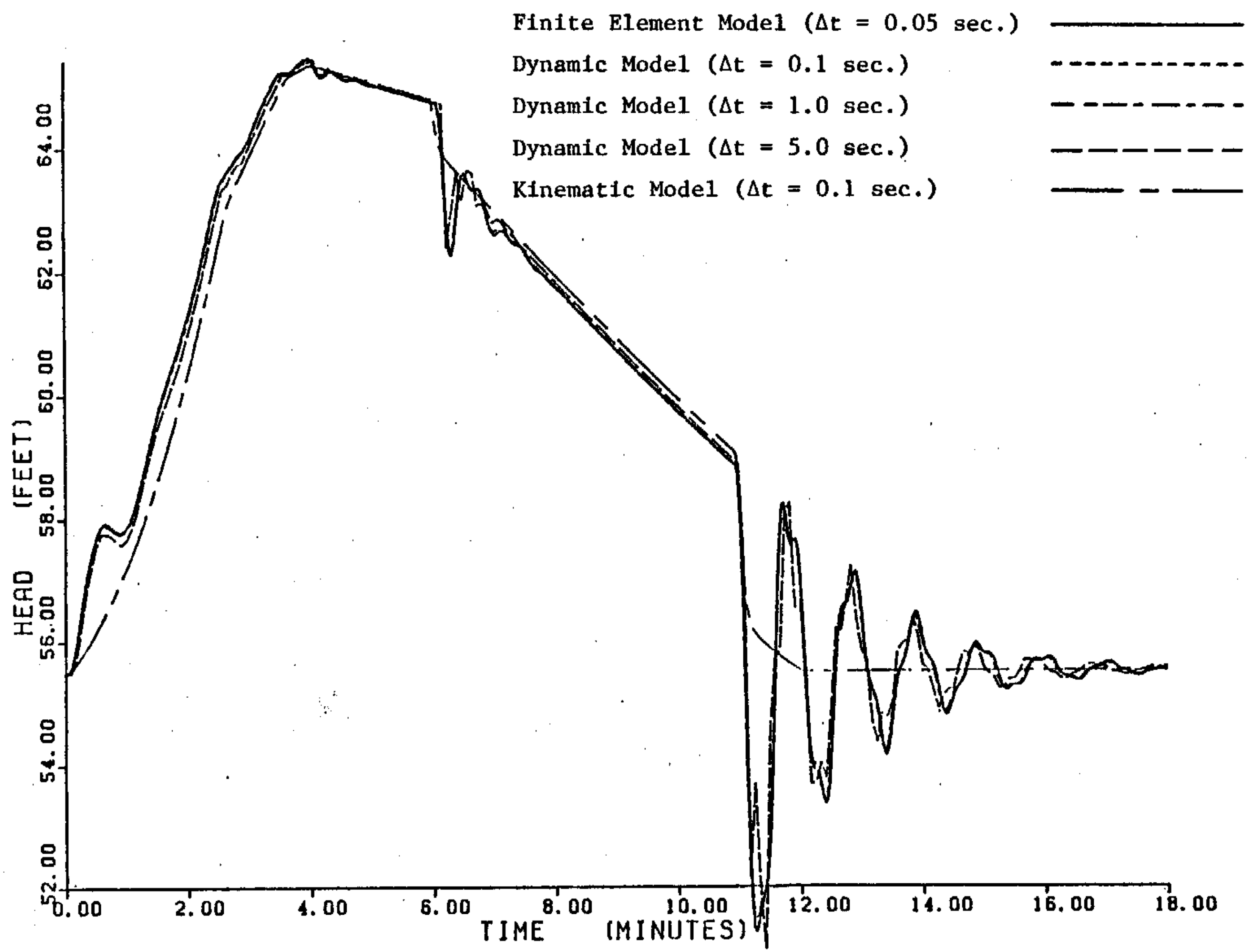


Figure 5-29. Total Head Graph for Junction 3, Example # 5.

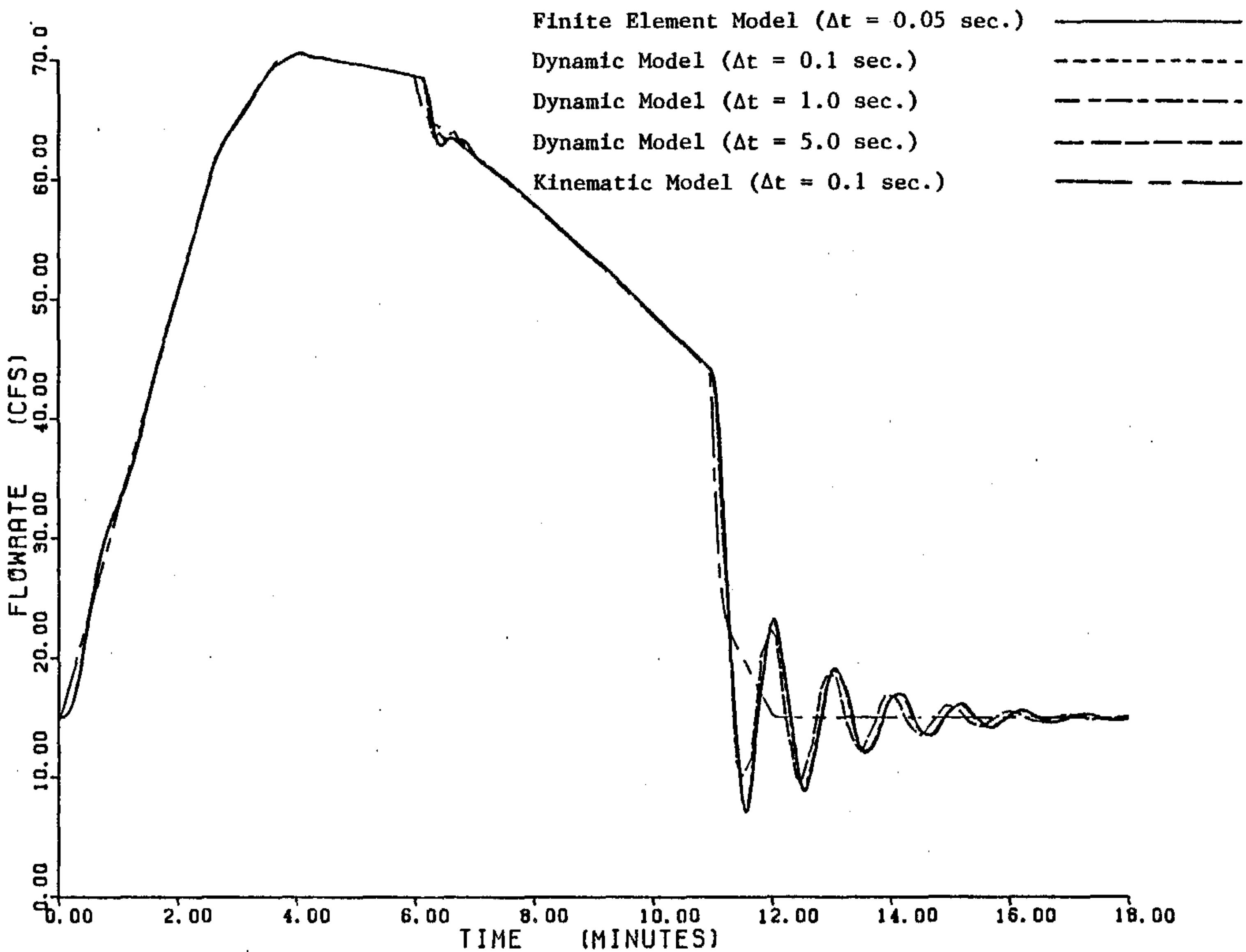


Figure 5-30. Flow Graph for Pipe 3, Example # 5.

CONCLUSIONS AND RECOMMENDATIONS

The primary objective of this thesis was to carry out a preliminary investigation which would provide the basis for the eventual development of a hydraulic flow model for the analysis of storm sewer systems at peak flows. This investigation includes the initial development of three hydraulic flow models. They are a) a finite element distributed parameter unsteady flow model b) a dynamic lumped parameter flow model and c) a kinematic steady flow model. Comments on each of the flow models and future research recommendations are presented here.

6.1 Finite Element Model

The unsteady distributed parameter finite element model provided the most accurate and reliable solutions of the system continuity and momentum equations. This method, however, by its distributed parameter nature, requires a large amount of computer storage and small time steps for the even the simplest of system simulations. It is not uncommon to find storm sewer systems in excess of 15 sewer links of various lengths which may require the use of several hundred elements. Each element in turn, has properties such as pipe length, diameter, roughness, slope, celerity, etc. which must be stored throughout the time simulation. Further complications arise with the small time steps required for reliable solutions. Transient pressure waves in pressurized storm sewer systems often travel in excess of the speed of sound (3000 ft/s). In order to correctly model the transient pressure waves the computational time step must not exceed L_s/c, where L_s is the minimum sewer element length in the sewer system and c is the celerity or pressure wave velocity. For most storm sewer systems, L_s/c is less than 0.1 seconds. Many design storms, however, exceed 20 minutes and would require in excess of 12000 time step calculations. Therefore, generally speaking, the finite element solution or any distributed parameter solution applied to pressurized storm sewer analysis (method of characteristics, wave plan, etc.) may be computationally inefficient from a numerical solution viewpoint.

The finite element model is also an implicit numerical model which allows for a variety of time marching schemes. After thorough investigation of the solutions and numerous unstable data runs it was decided that the FEM yields inaccurate solutions for values of the time weighting constant (θ) less than 0.5. These accuracy problems were anticipated considering the nature of the problem. According to Zienkiewicz (54), the Euler explicit ($\theta = 0$) time marching scheme often yields oscillatory or divergent results. From the author's experience $.55 \leq \theta \leq 0.8$ yields satisfactory stable, nonoscillating solutions. For this reason Galerkin weighting ($\theta = 0.67$) is used for all FEM solutions in this thesis.

Although not computationally appropriate for the analysis of pressurized storm sewer systems, the FEM may well be applied to general flow problems of fluid transients. These include both open channel and closed conduit applications. Cooley and Moin (11) have developed a FEM solution for open channel transients. However, after a thorough literature search very little material has been published concerning the application of the FEM to pressurized fluid transients.

For future research it is recommended that a working finite element model would provide a useful tool for the analysis of transients in closed conduits. With minor modifications, the existing finite element flow model developed herein, could be modified to handle routine transient flow problems, such as pump failures, value closures, cavitation, etc. Particular attention would be given to the spatial approximations and the non-linear flow resistant term. Presently the FEM uses linear shape functions to describe spatial variations of the unknown variables, Q and H. A more accurate quadratic, cubic or cubic hermitian approximation needs investigation.

The friction loss term is linearized in the present finite element model and an alternate method of treating the friction term is recommended. Possibilities include a modified Newton-Raphson iteration procedure or a direct nonlinear solution of the governing unsteady flow equations. One distinct advantage of the FEM would be the quasi-steady modeling of the non-linear head loss term. The friction loss for a given line length would vary over each element allowing it to be an independent function of the nodal flow values. Presently, several available transient flow models approximate the friction loss based on the initial line flow. Consequently the head loss is constant for a given line length throughout the entire simulation.

As with pressurized transients, the FEM could be easily applied to the governing unsteady open channel flow equations for application to sewer flow analysis. Such a model could then be coupled with the present surcharged sewer model developed here to form a complete finite element storm sewer analysis package. The FEM would provide a useful alternative to the presently available numerical schemes.

6.2 Dynamic Model

The dynamic lumped parameter model provides a simple and reliable method for the analysis of storm sewer systems under surcharge. As seen from the examples, the dynamic model yields accurate solutions for each problem investigated. From a computational viewpoint, the dynamic model is comparatively efficient due to its lumped parameter nature. In comparison to the kinematic model, the dynamic model is much more stable and allows for the use of larger time steps (Δt). In addition, the dynamic model, which includes inertial effects, models unstable flow conditions quite well, whereas, the kinematic model predicts mean values of head and flow throughout the instability.

The dynamic model is an explicit Euler ($\theta = 0$) forward differencing time marching scheme. As previously indicated, the Euler scheme often yields numerical instability problems for larger time steps. For this reason, small time steps ($\Delta t < 5.0$ seconds) are used for all simulations.

From this initial investigation, the dynamic lumped parameter model provides stable and accurate results for the analysis of pressurized storm sewer systems. Additional research concerning the numerical stability of the dynamic model needs to be conducted. With minor modifications an implicit ($\theta \neq 0$) time marching scheme can be developed for the dynamic model. This would allow for the use of much larger time steps, thereby reducing the total number of calculations and cost for each simulation.

With an implicit time marching scheme the boundary conditions should also be modified accordingly. Because of the implicit nature of these boundary conditions a modified Newton-Raphson iteration procedure is recommended when solving for the unknown junction head.

6.3 Kinematic Model

The kinematic model is simply a time modified steady flow model which predicts total head and flow values from previously known steady state conditions. Of the three models presented, the kinematic model is the simplest and most easily programmed. The solution stability, however, is a significant problem. For very small time steps ($\Delta t \leq 1.0$ seconds) the kinematic solution provides reasonable results for each example investigated. For larger time steps, however, this method gives extremely unstable oscillatory results. For this reason the dynamic model is preferred over the kinematic steady flow model. Some improvements in stability may be possible if the method for updating boundary conditions is modified. An implicit procedure using a modified Newton-Raphson iteration technique is suggested.

There are several advantages to the kinematic steady flow model. Presently, steady state pressurized flow theory is well understood and several well documented programs (46,31) are available which analyze pressurized water systems. Although requiring small time steps, the kinematic theory for pressurized storm sewer analysis could be incorporated into these existing steady flow models. The dynamic theory, which can not be adapted to the steady state models, is not well documented or readily available at this time.

Presently Wood (46) has developed a extended steady flow model which performs time simulations similiar to that of the kinematic model presented here. The model is stable and accurate for pressurized storm sewer analysis, provided that small time steps are used.

COMPUTER PROGRAMS

Two programs, called FESSA and DYN/KIN, are written for the unsteady and dynamic/kinematic flow methods, respectively, developed in Chapter 4. A description of the programs and data coding instructions for their use are presented in this chapter. The programs are listed in Appendix I.

7.1 Fortran Programs

FESSA, the finite element unsteady program was written and debugged in Fortran IV, WATFIV and compiled and executed in Fortran IV, G level. FESSA was initially programmed and run on the IBM-370 computer at the University of Kentucky. For additional versatility, the program was transferred for execution to the DEC-10 computer at the University of Louisville.

The dynamic and kinematic models are combined into one program, called DYN/KIN, for execution and plotting convenience. For a program check the DYN/KIN program was written in both Fortran and Basic computer languages. These programs are also executed on both the IBM-370 and DEC-10 computer systems.

In each program the solutions of hydraulic head and flow are stored in plotting arrays on either tapes or cards. These time solutions are then plotted using a Nicolet Zeta plotter by executing a plotting program called PLTFLO written by the author. In producing the plot, PLTFLO calls several plotting subroutines which are described in the University of Kentucky Computing Center Plotting Manual (36). PLTFLO is written and executed in Fortran IV, WATFIV on the IBM-370 computer. The resulting graphical solutions of head and flow are presented in Chapter 5.

7.2 Data Coding Instructions

The data coding for the unsteady, dynamic and kinematic flow models are very similiar. The coding consists of the original data which describes the system geometry, an initial set of pressure and flow conditions and optional plotting information. The data requirements are summarized in Table 7-1 and 7-2 for the FESSA and DYN/KIN programs respectively. In the data coding instructions, integer numbers are represented by an 'I' followed by the ending column field number and real numbers are represented by a 'R' followed by the column field. All data fields are either 1 card column or a multiple of 5 card columns. For example, (I:5) represents an integer variable number which ends in card column 5 while (R:11-20) represents a real number placed within card columns 11 through 20. Real variable numbers should contain a decimal for user convenience. Example data and solution results are presented in Tables 7-3 through 7-6.

1. SYSTEM DATA (one card)

a) type of simulation, (1-unsteady, 2-dynamic); (I:1)
b) number of sewer lines; (I:5)
c) number of junctions/manholes; (I:10)
d) time weighting constant; (R:11-20)
f) time step, (seconds); (R:21-30)
e) total simulation time, (minutes); (R:31-40)
g) print time step, (minutes); (R:41-50)
h) system kinematic viscosity, (sq-ft/sec); (R:51-60)

2. SEWER LINE DATA (one card for each pipe)

a) sewer pipe number; (I:5)
b) connecting node # 1; (I:10)
c) connecting node # 2; (I:15)
d) sewer pipe length, (ft); (R:16-25)
e) sewer diameter, (ft); (R:26-35)
f) sewer roughness, (ft); (R:36-45)
g) initial sewer flow, (cfs); (R:46-55)
h) sewer pipe celerity, (ft/sec); (R:56-65)
i) element length, (ft); (R:66-75)

3. JUNCTION/MANHOLE/HYDROGRAPH DATA (one card for each)

a) junction number; (I:5)
b) junction elevation, (ft); (R:6-15)
c) number of connecting sewer pipes; (I:20)
d) connecting sewer pipe numbers; (I:25,30,35,40,45)

aa) manhole type, (1-fixed head, 2-variable head); (I:1)
bb) manhole height, (ft); (R:2-10)
cc) manhole diameter, (inches); (R:11-20)
dd) manhole surface area diameter, (ft); (R:21-30)
ee) initial manhole head, (ft); (R:31-40)

aaa) initial hydrograph inflow, (cfs); (R:1-10)
bbb) hydrograph peak flow, (cfs); (R:11-20)
ccc) hydrograph time peak, (minutes); (R:21-30)
ddd) hydrograph time base, (minutes); (R:31-40)

4. PLOTTING DATA (one card)

a) number of plots; (I:5)
b) plot time step, (seconds); (R:6-10)
c) time-axis increment, (minutes); (R:11-15)
d) initial head-axis value at time = 0, (ft); (R:16-20)
e) head-axis increment, (minutes); (R:21-25)
f) flow-axis increment, (cfs); (R:26-30)
g) sewer pipe/junctions numbers for flow/head plots; (I:35,40, 45,50,55)

TABLE 7-1. Data Coding Instructions for FESSA Program.

1. SYSTEM DATA (one card)

   a) number of sewer lines; (I:5)
   b) number of junctions/manholes; (I:10)
   c) time step, (seconds); (R:11-20)
   d) print time step, (minutes); (R:21:30)
   e) total simulation time, (minutes); (R:31-40)
   f) system kinematic viscosity, (sq-ft/sec); (R:41-50)

2. SEWER LINE DATA (one card for each pipe)

   a) sewer pipe number; (I:5)
   b) connecting node # 1; (I:10)
   c) connecting node # 2; (I:15)
   d) sewer pipe length, (ft); (R:16-25)
   e) sewer diameter, (ft); (R:26-35)
   f) sewer roughness, (ft); (R:36-45)
   g) sewer minor loss; (R:46-55)
   h) initial sewer flow, (cfs); (R:56-65)

3. JUNCTION/MANHOLE/HYDROGRAPH DATA (one card for each)

   a) junction number; (I:5)
   b) junction elevation, (ft); (R:6-15)
   c) number of connecting sewer pipes; (I:20)
   d) connecting sewer pipe numbers; (I:25,30,35,40,45)

   aa) manhole type, (1-fixed head, 2-variable head); (I:1)
   bb) manhole height, (ft); (R:2-10)
   cc) manhole diameter, (inches); (R:11-20)
   dd) manhole surface area diameter, (ft); (R:21-30)
   ee) initial manhole head, (ft); (R:31-40)

   aaa) initial hydrograph inflow, (cfs); (R:1-10)
   bbb) hydrograph peak flow, (cfs); (R:11-20)
   ccc) hydrograph time lag, (minutes); (R:21-30)
   ddd) hydrograph time peak, (minutes); (R:31-40)
   eee) hydrograph time base, (minutes); (R:41-50)

4. PLOTTING DATA (one card)

   a) number of plots; (I:5)
   b) plot time step, (seconds); (R:6-10)
   c) time-axis increment, (minutes); (R:11-15)
   d) initial head-axis value at time = 0, (ft); (R:16-20)
   e) head-axis increment, (minutes); (R:21-25)
   f) flow-axis increment, (cfs); (R:26-30)
   g) sewer pipe/junctions numbers for flow/head plots; (I:35,40,45,50,55)

TABLE 7-2. Data Coding Instructions for DYN/KIN Program.

DATA CODING SHEET - PIPE SYSTEMS ANALYSIS

NAME Heitzman

PROBLEM FESSA Example # 5

PAGE ___ OF ___

COLUMN NUMBERS

```
         1         2         3         4         5         6         7         8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
0   3    4 0.67     1.0       2.0       1.0       .00001059
    1    1    3 200.0    18.0      0.001     5.0       3000.0    50.0
    2    2    3 300.0    24.0      0.001     5.0       3000.0    50.0
    3    3    4 500.0    30.0      0.001     15.0      3000.0    50.0
    1 52.90        1    1
  15.0    36.0      150.0     55.80
5.0       30.0      4.0       12.0
    2 53.10        1    2
  15.0    36.0      150.0     55.60
5.0       30.0      4.0       12.0
    3 52.50        3    1    2    3
  15.0    48.0      150.0     55.49
5.0       30.0      4.0       12.0
    4 52.00        1    3
1 15.0    48.0      150.0     55.00
    0 5.0 2.0  0.0  1.0  1.0      1    2    3
```

Table 7-3. Example FESSA Data Coding for Example # 3.

DATA CODING SHEET - PIPE SYSTEMS ANALYSIS

NAME Heitzman

PROBLEM DYN/KIN Example # 5

PAGE ___ OF ___

COLUMN NUMBERS

```
         1         2         3         4         5         6         7         8
12345678901234567890123456789012345678901234567890123456789012345678901234567890
    3    4 1.0      1.0       2.0       .00001059
    1    1    3 200.0    18.0      0.001     0.0       5.0
    2    2    3 300.0    24.0      0.001     0.0       5.0
    3    3    4 500.0    30.0      0.001     0.0       15.0
    1 52.90        1    1
  15.0    36.0      150.0     55.80
5.0       30.0      0.0       4.0       12.0
    2 53.1         1    2
  15.0    36.0      150.0     55.60
5.0       30.0      0.0       4.0       12.0
    3 52.5         3    1    2    3
  15.0    48.0      150.0     55.49
5.0       30.0      0.0       4.0       12.0
    4 52.0         1    3
1 15.0    48.0      150.0     55.00
    0 5.0 2.0  0.0  1.0  1.0      1    2    3
```

Table 7-4. Example DYN/KIN Data Coding for Example # 3.

***** ORIGINAL DATA SUMMARY *****

SOLUTION TYPE: DISTRIBUTED PARAMETER (TRANSIENT)

THE TIME WEIGHTING CONSTANT THETA EQUALS 0.67

THE DARCY-WEISBACH HEAD LOSS EQUATION IS USED, THE KINEMATIC VISCOSITY = 0.00001059 SQ.FT./SEC.

PIPE NO.	NODE NUMBERS		LENGTH (FEET)	DIAMETER (INCHES)	ROUGHNESS (FEET)	CELERITY (FT/S)	INITIAL FLOWRATE (CFS)
1	1	3	200.00	18.00	0.00100	3000.00	5.00
2	2	3	300.00	24.00	0.00100	3000.00	5.00
3	3	4	500.00	30.00	0.00100	3000.00	15.00

MANHOLE DATA

JUNCTION NO.	ELEVATION (FEET)	HEIGHT (FEET)	DIAMETER (INCHES)	STORAGE DIAMETER (FEET)	INITIAL HEAD (FEET)
1	52.90	15.0	36.0	150.0	55.800
2	53.10	15.0	36.0	150.0	55.600
3	52.50	15.0	48.0	150.0	55.490
4	52.00	THIS JUNCTION HAS A FIXED HEAD OF 55.00 FEET			

HYDROGRAPH INFORMATION

JUNCTION NO.	INITIAL FLOW (CFS)	PEAK FLOW (CFS)	TIME TO PEAK (MINUTES	TIME BASE (MINUTES)
1	5.00	30.00	4.00	12.00
2	5.00	30.00	4.00	12.00
3	5.00	30.00	4.00	12.00

Table 7-5. Example FESSA Solution Results for Example # 3.

GLOBAL ELEMENT-NODE CONNECTIVITY

ELEMENT	CONNECTING NODES		ELEMENT LENGTH (FEET)
1	1	2	50.00
2	2	3	50.00
3	3	4	50.00
4	4	5	50.00
5	6	7	50.00
6	7	8	50.00
7	8	9	50.00
8	9	10	50.00
9	10	11	50.00
10	11	12	50.00
11	13	14	50.00
12	14	15	50.00
13	15	16	50.00
14	16	17	50.00
15	17	18	50.00
16	18	19	50.00
17	19	20	50.00
18	20	21	50.00
19	21	22	50.00
20	22	23	50.00

SYSTEM EQUATIONS ARE SOLVED USING A 1.00 SEC. TIME INCREMENT

RESULTS ARE OUTPUT EVERY 1.0000 MINUTES

TOTAL TIME OF SIMULATION = 2.0000 MINUTES

TIME FROM START OF SIMULATION = 1.0000 MINUTES

PIPE NUMBER	FLOWRATE (CFS)
1 (3)	10.951
2 (10)	11.010
3 (20)	32.936

JUNCTION NUMBER	INFLOW (CFS)	GRADE LINE (FEET)	HEAD ABOVE PIPE (FEET)
1 (1)	11.198	59.532	6.632
2 (6)	11.198	58.537	5.437
3 (5)	11.198	57.857	5.357
4 (23)	0.0	55.000	3.000

Table 7-5. Example FESSA solution Results (continued).

***** ORIGINAL DATA SUMMARY *****

THE DARCY-WEISBACH HEAD LOSS EQUATION IS USED, THE KINEMATIC VISCOSITY = 0.00001059 SQ.FT./SEC.

PIPE NO.	NODE NUMBERS		LENGTH (FEET)	DIAMETER (INCHES)	ROUGHNESS (FEET)	M-LOSS	INITIAL FLOWRATE (CFS)
1	1	3	200.00	18.00	0.00100	0.0	5.00
2	2	3	300.00	24.00	0.00100	0.0	5.00
3	3	4	500.00	30.00	0.00100	0.0	15.00

MANHOLE DATA

JUNCTION NO.	ELEVATION (FEET)	HEIGHT (FEET)	DIAMETER (INCHES)	STORAGE DIAMETER (FEET)	INITIAL HEAD (FEET)
1	52.90	15.00	36.0	150.00	55.800
2	53.10	15.00	36.0	150.00	55.600
3	52.50	15.00	48.0	150.00	55.490
4	52.00	THIS JUNCTION HAS A FIXED HEAD OF 55.00 FEET			

HYDROGRAPH INFORMATION

JUNCTION NO.	INITIAL FLOW (CFS)	PEAK FLOW (CFS)	TIME LAG (MINUTES)	TIME TO PEAK (MINUTES)	TIME BASE (MINUTES)
1	5.00	30.00	0.0	4.00	12.00
2	5.00	30.00	0.0	4.00	12.00
3	5.00	30.00	0.0	4.00	12.00

SYSTEM EQUATIONS ARE SOLVED USING A 1.00 SECOND TIME INCREMENT

RESULTS ARE OUTPUT EVERY 1.0000 MINUTES

TOTAL TIME OF SIMULATION = 2.0000 MINUTES

Table 7-6. Example DYN/KIN Solution Results for Example # 3.

Table 7-6. Example DYN/KIN Solution Results (continued).

TIME FROM START OF SIMULATION = 1.0000 MINUTES

***** SOLUTION TYPE *****

PIPE NUMBER	STEADY STATE W/STORAGE VELOCITY (FT/SEC)	STEADY STATE W/STORAGE FLOWRATE (CFS)	LUMPED PARAMETER VELOCITY (FT/SEC)	LUMPED PARAMETER FLOWRATE (CFS)
1	15.069	26.630	6.188	10.935
2	15.109	47.467	3.501	10.998
3	0.638	3.130	6.696	32.870

JUNCTION NUMBER	INFLOW (CFS)	STEADY STATE W/STORAGE GRADE LINE (FEET)	STEADY STATE W/STORAGE HEAD ABOVE PIPE (FEET)	LUMPED PARAMETER GRADE LINE (FEET)	LUMPED PARAMETER HEAD ABOVE PIPE (FEET)
1	11.15	63.537	10.637	59.506	6.606
2	11.15	64.039	10.939	58.524	5.424
3	11.15	55.024	2.524	57.839	5.339
4	0.0	55.000	3.000	55.000	3.000

APPENDIX A FESSA FORTRAN SOURCE PROGRAM

```
C
C
C
C
C   ******************************************************************
C   ******************************************************************
C   *                                                                *
C   *                                                                *
C   *                                                                *
C   *                           F E S S A                            *
C   *                                                                *
C   *        ( FINITE ELEMENT SURCHARGED STORMSEWER ANALYSIS )       *
C   *                                                                *
C   *                                                                *
C   *  THIS PROGRAM SOLVES THE GOVERNING DIFFERENTIAL EQUATIONS FOR ONE *
C   *  DIMENSIONAL UNSTEADY (TRANSIENT) FLOW IN PRESSURIZED STORM SEWER *
C   *  SYSTEMS USING THE FINITE ELEMENT METHOD (FEM). THE GALERKIN      *
C   *  METHOD USING LINEAR ELEMENTS IS APPLIED IN ONE DIMENSIONAL SPACE.*
C   *                                                                *
C   *                                                                *
C   *                                                                *
C   *                                                                *
C   *                            THESIS                              *
C   *                             1982                               *
C   *                                                                *
C   *                       GREG C. HEITZMAN                         *
C   *                 GRADUATE RESEARCH ASSISTANT                    *
C   *               DEPARTMENT OF CIVIL ENGINEERING                  *
C   *                    UNIVERSITY OF KENTUCKY                      *
C   *                                                                *
C   *                                                                *
C   *                                                                *
C   ******************************************************************
C   ******************************************************************
C
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL TPLOTM,HPLOT1,QPLOT1,HPLOT2,QPLOT2,HPLOT3,QPLOT3
      REAL HPLOT4,QPLOT4,HPLOT5,QPLOT5
      DIMENSION DATA(1),TPLOTM(1500),HPLOT1(1500),QPLOT1(1500)
      DIMENSION HPLOT2(1500),QPLOT2(1500),HPLOT3(1500),QPLOT3(1500)
      DIMENSION HPLOT4(1500),QPLOT4(1500),HPLOT5(1500),QPLOT5(1500)
      COMMON/B1/NOPIPE,NOJUNC,TT,TT1,TINCR,VISC
      COMMON/B2/PNUM(10),JUNC1(10),JUNC2(10)
      COMMON/B3/PLEN(10),PDIA(10),PRUF(10),PFLO(10),PCEL(10),PELEN(10)
      COMMON/B4/JNUM(10),JELV(10),NOCP(10),CPNUM(10,5)
      COMMON/B5/KEY(10),MHT(10),MDIA1(10),MDIA2(10),MHED(10)
      COMMON/B6/QI(10),QIN(10),QPK(10),TPK(10),TBAS(10),TIME
      COMMON/B7/JNOD(10,5),NPIPE(95),JUNC(95)
      COMMON/B8/Q(95),H(95),AMAT(2,2),BMAT(2,2)
      COMMON/B9/ELEN(95),EDIA(95),ERUF(95),EFLO(95),ECEL(95),EALPHA(95)
      COMMON/B11/ID(2,95),HPBV(95)
      COMMON/B12/IEL,NEQ,NEDF,NEND,NNDF,NOEL,NOND,LD(4,95),NELCON(2,95)
      COMMON/B13/SA,SF,SG
```

```
      COMMON/RDATA/DT,ONE,RCT,RTH,RTT,TWO,ZERO
      COMMON/IEQN/JDIAG(2ØØ)
      COMMON/RLOG/AFAC,AFL,BACK,BFL
      COMMON/REQN/A(15ØØ),B(2ØØ),C(15ØØ)
      COMMON/WORK/S(4,4),P(4)
      INTEGER PNUM, CPNUM
      REAL*8 JELV,MHT,MDIA1,MDIA2,MHED
      LOGICAL AFAC,AFL,BACK,BFL
  5ØØ FORMAT(I1,I4,I5,F1Ø.6,3F1Ø.4,F1Ø.8,I1)
  51Ø FORMAT(3I5,2F1Ø.2,F1Ø.6,3F1Ø.2)
  52Ø FORMAT(I5,F1Ø.2,6I5)
  53Ø FORMAT(I1,F9.2,3F1Ø.2)
  54Ø FORMAT(4F1Ø.2)
  55Ø FORMAT(I5,5F5.1,5I5)
  6ØØ FORMAT(//////////'                                          ***** ORIGINAL DAT
     QA SUMMARY *****')
  61Ø FORMAT(/////' SOLUTION TYPE:   DISTRIBUTED PARAMETER (TRANSIENT)')
  62Ø FORMAT(/////' SOLUTION TYPE:   LUMPED PARAMETER')
  63Ø FORMAT(//' THE TIME WEIGHTING CONSTANT THETA EQUALS ',F8.2)
  64Ø FORMAT(//' THE DARCY-WEISBACH HEAD LOSS EQUATION IS USED, THE KINE
     QMATIC VISCOSITY = ',F1Ø.8,' SQ.FT./SEC.')
  65Ø FORMAT(/////'        PIPE NO.   NODE NUMBERS      LENGTH   DIAMETER
     Q   ROUGHNESS   CELERITY   INITIAL FLOWRATE ')
  66Ø FORMAT('                                         (FEET)   (INCHES)    (
     QFEET)      (FT/S)        (CFS) ')
  67Ø FORMAT(/I1Ø,I11,I5,9X,F7.2,4X,F5.2,6X,F7.5,5X,F7.2,6X,F7.2)
  68Ø FORMAT(/////)
  69Ø FORMAT(///'
     Q          MANHOLE  DATA ')
  7ØØ FORMAT(//'   JUNCTION NO.    ELEVATION ')
  71Ø FORMAT('                                                  HEIGHT      D
     QIAMETER  STORAGE DIAMETER  INITIAL HEAD')
  72Ø FORMAT('                         (FEET)                  (FEET)      (
     QINCHES)       (FEET)           (FEET)')
  73Ø FORMAT(/I1Ø,1ØX,F6.2,19X,F6.1,7X,F6.1,8X,F6.1,9X,F7.3)
  74Ø FORMAT(/I1Ø,1ØX,F6.2,19X,'THIS JUNCTION HAS A FIXED HEAD OF '
     Q      F6.2,' FEET ')
  75Ø FORMAT(//////////,'                                              HYDROGRAP
     QH  INFORMATION ')
  76Ø FORMAT(//' JUNCTION NO.          INITIAL FLOW    PEAK FLOW    TIME
     QTO PEAK    TIME BASE ')
  77Ø FORMAT('                           (CFS)          (CFS)        (MINUT
     QES       (MINUTES) ')
  78Ø FORMAT(/I1Ø,15X,F6.2,8X,F6.2,8X,F6.2,7X,F6.2,1ØX,F6.2)
  79Ø FORMAT(//////////,'                         GLOBAL ELEMENT-NODE CONNECTIVIT
     QY ')
  8ØØ FORMAT(//'     ELEMENT     CONNECTING NODES     ELEMENT LENGTH (FE
     QET)')
  81Ø FORMAT(/I9,I12,I9,13X,F7.2)
  82Ø FORMAT(/////' SYSTEM EQUATIONS ARE SOLVED USING A',F5.2,' SEC. TIM
     QE INCREMENT')
  83Ø FORMAT(/' RESULTS ARE OUTPUT EVERY ',F7.4,' MINUTES')
  84Ø FORMAT(/' TOTAL TIME OF SIMULATION = ',F7.4,' MINUTES'//////////)
  85Ø FORMAT(///////////' TIME FROM START OF SIMULATION = ',F1Ø.4,' MINU
     QTES')
  86Ø FORMAT(/////'         PIPE NUMBER                     FLOWRATE')
  87Ø FORMAT('                                             (CFS)')
  88Ø FORMAT(/I1Ø,'   (',I2,')',18X,F8.3)
  89Ø FORMAT(/////'      JUNCTION NUMBER                INFLOW
     Q GRADE LINE       HEAD ABOVE PIPE')
  9ØØ FORMAT('                                          (CFS)             (F
     QEET)           (FEET)')
  91Ø FORMAT(/I1Ø,'   (',I2,')',18X,F7.3,14X,F7.3,13X,F7.3)
  92Ø FORMAT(///)
```

```
  930 FORMAT(2I10,4F10.1)
  940 FORMAT(10F8.4)
C
C      INITIALIZE THE ASSEMBLY AND SOLUTION LOGICAL VARIABLES
C
       AFL = .TRUE.
       BFL = .TRUE.
       AFAC = .TRUE.
       BACK = .TRUE.
C
C      INITIALIZE THE SUBROUTINE CONSTANTS
C
       ZERO = 0.0D0
       NNDF = 2
       NEDF = 4
       NEND = 2
       READ(5,500)KSOL,NOPIPE,NOJUNC,THETA,TINCR,TTOTL,TPRINT,VISC
       WRITE(6,600)
C
C      INITIALIZE THE SOLUTION COEFFICIENTS TO:
C       DYNAMIC SOLUTION (KSOL = 0)
C       LUMPED PARAMETER SOLUTION (KSOL = 1)
C
       IF (KSOL .EQ. 1) GO TO 10
       SA = 1.0
       SF = 1.0
       SG = 1.0
       WRITE(6,610)
       GO TO 20
   10 SA = 1.0
       SF = 0.0
       SG = 0.0
       WRITE(6,620)
   20 DO 30 I = 1,NOPIPE
   30 READ(5,510)PNUM(I),JUNC1(I),JUNC2(I),PLEN(I),PDIA(I),PRUF(I),PFLO(
     QI),PCEL(I),PELEN(I)
       DO 40 I = 1,NOJUNC
       READ(5,520) JNUM(I), JELV(I), NOCP(I),(CPNUM(I,J),J=1,5)
       READ(5,530) KEY(I), MHT(I), MDIA1(I),MDIA2(I),MHED(I)
       IF (KEY(I) .EQ. 1) GO TO 40
       READ(5,540) QIN(I),QPK(I),TPK(I),TBAS(I)
   40 CONTINUE
       READ(5,550) NOPLOT,TPLT,DELTAX,PLTHD,DELTHY,DELTQY,IP1,IP2,IP3,IP5
     QIP5
       WRITE(6,630) THETA
       WRITE(6,640) VISC
       WRITE(6,650)
       WRITE(6,660)
       WRITE(6,670)(PNUM(I),JUNC1(I),JUNC2(I),PLEN(I),PDIA(I),PRUF(I),PCE
     QL(I),PFLO(I),I=1,NOPIPE)
       WRITE(6,680)
       WRITE(6,690)
       WRITE(6,700)
       WRITE(6,710)
       WRITE(6,720)
       DO 60 J = 1,NOJUNC
       IF (KEY(J) .EQ. 1) GO TO 50
       WRITE(6,730)JNUM(J),JELV(J),MHT(J),MDIA1(J),MDIA2(J),MHED(J)
       GO TO 60
   50 WRITE(6,740)JNUM(J),JELV(J),MHED(J)
   60 CONTINUE
       WRITE(6,750)
       WRITE(6,760)
       WRITE(6,770)
```

```
          DO 70 J = 1,NOJUNC
          IF (KEY(J) .EQ. 1) GO TO 70
          WRITE(6,780)JNUM(J),QIN(J),QPK(J),TPK(J),TBAS(J)
          TPK(J) = TPK(J) * 60.0
          TBAS(J) = TBAS(J) * 60.0
       70 CONTINUE
          CALL ELEMNT
          ISIZE = 2 * NOND
          TP = 1.0
          ITPT = 0
          CALL JUNCTN
          WRITE(6,790)
          WRITE(6,800)
          WRITE(6,810)(L,NELCON(1,L),NELCON(2,L),ELEN(L),L = 1,NOEL)
          WRITE(6,820) TINCR
          WRITE(6,830) TPRINT
          WRITE(6,840) TTOTL
          TT = TINCR * THETA
          TT1 = TINCR * (1.0 - THETA)
          CALL NODE
          CALL PROFIL
          CALL ELEQN
C
C         INITIALIZE THE PLOTTER AND PLOTTING ARRAYS
C
          IF (NOPLOT .EQ. 0 ) GO TO 90
          DO 80 N = 1,1500
          TPLOTM(N) = 0.0
          HPLOT1(N) = 0.0
          HPLOT2(N) = 0.0
          HPLOT3(N) = 0.0
          HPLOT4(N) = 0.0
          HPLOT5(N) = 0.0
          QPLOT1(N) = 0.0
          QPLOT2(N) = 0.0
          QPLOT3(N) = 0.0
          QPLOT4(N) = 0.0
          QPLOT5(N) = 0.0
       80 CONTINUE
       90 TIME = 0.0
          TTOTL = TTOTL * 60.
          TPRINT = TPRINT * 60.
C
C         ASSIGNMENT OF PLOTTING ARRAYS
C
      100 IF (NOPLOT .EQ. 0) GO TO 120
          TPLO = TPLT * ITPT
          IF ((TPLO - TIME) .LT. .01) GO TO 110
          GO TO 120
      110 ITPT = ITPT + 1
          TPLOTM(ITPT) = TIME / (60.0 * DELTAX)
          HPLOT1(ITPT) = (H(IP1) - PLTHD) / DELTHY
          QPLOT1(ITPT) = Q(IP1) / DELTQY
          IF (NOPLOT .LE. 1) GO TO 120
          HPLOT2(ITPT) = (H(IP2) - PLTHD) / DELTHY
          QPLOT2(ITPT) = Q(IP2) / DELTQY
          IF (NOPLOT .LE. 2) GO TO 120
          HPLOT3(ITPT) = (H(IP3) - PLTHD) / DELTHY
          QPLOT3(ITPT) = Q(IP3) / DELTQY
          IF (NOPLOT .LE. 3) GO TO 120
          HPLOT4(ITPT) = (H(IP4) - PLTHD) / DELTHY
          QPLOT4(ITPT) = Q(IP4) / DELTQY
          IF (NOPLOT .LE. 4) GO TO 120
          HPLOT5(ITPT) = (H(IP5) - PLTHD) / DELTHY
```

```
          QPLOT5(ITPT) = Q(IP5) / DELTQY
C
      120 TIME = TIME + TINCR
          IF (TIME .GT. TTOTL) GO TO 200
C
C         SOLVING FOR THE UNKNOWN NODAL HEAD AND FLOW VALUES
C
          CALL HEAD
          CALL SFORCE
          CALL UACTCL
          DO 150 NOD = 1,NOND
          IF (ID(1,NOD) .EQ. 0 ) GO TO 130
          H(NOD) = B(ID(1,NOD))
          GO TO 140
      130 H(NOD) = HPBV(NOD)
      140 IF (ID(2,NOD) .EQ. 0 ) GO TO 150
          Q(NOD) = B(ID(2,NOD))
      150 CONTINUE
          TPRIN = TPRINT * TP
          IF ((TPRIN - TIME) .LT. 0.01) GO TO 160
          GO TO 100
      160 TIME = TIME / 60.0
          WRITE(6,850) TIME
          TIME = TIME * 60.0
          WRITE(6,860)
          WRITE(6,870)
          PNOEL = 0.0
          DO 170 IP = 1,NOPIPE
          NOD = IDINT(PNOEL)  + IDINT((PLEN(IP) / PELEN(IP)) / 2 + .49 ) + 1
          PNOEL = PNOEL + PLEN(IP) / PELEN(IP) + 2.0
          WRITE(6,880) IP,NOD,Q(NOD)
      170 CONTINUE
          WRITE(6,890)
          WRITE(6,900)
          DO 190 IJ = 1,NOJUNC
          DO 180 NOD = 1,NOND
          IF(JUNC(NOD) .NE. IJ) GO TO 180
          HP = H(NOD) - JELV(JUNC(NOD))
          WRITE(6,910)JUNC(NOD),NOD,QI(IJ),H(NOD),HP
          GO TO 190
      180 CONTINUE
      190 CONTINUE
          WRITE(6,920)
          TP = TP + 1.0
          GO TO 100
      200 IF (NOPLOT .EQ. 0) GO TO 260
C
C         ASSIGNMENT OF PLOTTING ARRAY SCALE FACTORS
C
          TPLOTM(ITPT+2) = 1.0
          HPLOT1(ITPT+2) = 1.0
          QPLOT1(ITPT+2) = 1.0
          HPLOT2(ITPT+2) = 1.0
          QPLOT2(ITPT+2) = 1.0
          HPLOT3(ITPT+2) = 1.0
          QPLOT3(ITPT+2) = 1.0
          HPLOT4(ITPT+2) = 1.0
          QPLOT4(ITPT+2) = 1.0
          HPLOT5(ITPT+2) = 1.0
          QPLOT5(ITPT+2) = 1.0
C
C         PUNCHING PLOTTING ARRAY VALUES ON CARDS
C
          ITPT = ITPT+2
```

```
      WRITE(7,930)NOPLOT,ITPT,PLTHD,DELTAX,DELTHY,DELTQY
      DO 210 IT = 1,ITPT,3
      WRITE(7,940) TPLOTM(IT),HPLOT1(IT),QPLOT1(IT),TPLOTM(IT+1),HPLOT1(
     QIT+1),QPLOT1(IT+1),TPLOTM(IT+2),HPLOT1(IT+2),QPLOT1(IT+2)
  210 CONTINUE
      IF (NOPLOT .LE. 1) GO TO 260
      DO 220 IT = 1,ITPT,3
      WRITE(7,940) TPLOTM(IT),HPLOT2(IT),QPLOT2(IT),TPLOTM(IT+1),HPLOT2(
     QIT+1),QPLOT2(IT+1),TPLOTM(IT+2),HPLOT2(IT+2),QPLOT2(IT+2)
  220 CONTINUE
      IF (NOPLOT .LE. 2) GO TO 260
      DO 230 IT = 1,ITPT,3
      WRITE(7,940) TPLOTM(IT),HPLOT3(IT),QPLOT3(IT),TPLOTM(IT+1),HPLOT3(
     QIT+1),QPLOT3(IT+1),TPLOTM(IT+2),HPLOT3(IT+2),QPLOT3(IT+2)
  230 CONTINUE
      IF (NOPLOT .LE. 3) GO TO 260
      DO 240 IT = 1,ITPT,3
      WRITE(7,940) TPLOTM(IT),HPLOT4(IT),QPLOT4(IT),TPLOTM(IT+1),HPLOT4(
     QIT+1),QPLOT4(IT+1),TPLOTM(IT+2),HPLOT4(IT+2),QPLOT4(IT+2)
  240 CONTINUE
      IF (NOPLOT .LE. 4) GO TO 260
      DO 250 IT = 1,ITPT,3
      WRITE(7,940) TPLOTM(IT),HPLOT5(IT),QPLOT5(IT),TPLOTM(IT+1),HPLOT5(
     QIT+1),QPLOT5(IT+1),TPLOTM(IT+2),HPLOT5(IT+2),QPLOT5(IT+2)
  250 CONTINUE
  260 CONTINUE
      STOP
      END
C
      BLOCK DATA
C
C ********************************************************************
C *                                                                  *
C *  THE PURPOSE OF THIS BLOCK DATA SUBPROGRAM IS TO INITIALIZE THE  *
C *  PROGRAM MATRIX VALUES                                           *
C *                                                                  *
C ********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/B8/Q(95),H(95),AMAT(2,2),BMAT(2,2)
      COMMON/B11/ID(2,95),HPBV(95)
      COMMON/B12/IEL,NEQ,NEDF,NEND,NNDF,NOEL,NOND,LD(4,95),NELCON(2,95)
      COMMON/IEQN/JDIAG(200)
      COMMON/REQN/A(1500),B(200),C(1500)
      COMMON/MATR1/AM(2,2),BM(2,2),CM(2,2),EM(2,2),FM(2,2),GM(2,2)
      DATA AMAT/0.333333333333333,0.166666666666667,0.16666666666667,
     Q          0.333333333333333/
      DATA BMAT/-0.50D0,-0.50D0,0.50D0,0.50D0/
      DATA ID/190*0/,LD/380*0/,NELCON/190*0/,JDIAG/200*0/
      DATA A/1500*0.0D0/,B/200*0.0D0/,C/1500*0.0D0/
      END
C
      FUNCTION DOT(A,B,N)
C
C ********************************************************************
C *                                                                  *
C *  THE PURPOSE OF THIS FUNCTION SUBPROGRAM IS TO EVALUATE DOT      *
C *  PRODUCTS.                                                       *
C *                                                                  *
C ********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/RDATA/DT,ONE,RCT,RTH,RTT,TWO,ZERO
      DIMENSION A(1),B(1)
```

```
      DOT = ZERO
      DO 10 I=1,N
      DOT = DOT + A(I)*B(I)
10    CONTINUE
      RETURN
      END
C
      SUBROUTINE ELEMNT
C
C  ***********************************************************************
C  *                                                                     *
C  *  THIS SUBROUTINE DIVIDES EACH PIPE INTO ELEMENTS AND ASSIGNS       *
C  *  GLOBAL ELEMENT AND NODE NUMBERS. IT ALSO ASSIGNS THE NECESSARY    *
C  *  PIPE PARAMETERS TO EACH ELEMENT.                                  *
C  *                                                                     *
C  *  IEL    = ELEMENT NUMBER                                            *
C  *  NOD    = NODE NUMBER                                               *
C  *  IP     = PIPE NUMBER                                               *
C  *  PL     = CUMLATIVE ELEMENT LENGTH ALONG EACH PIPE                  *
C  *  PLEFT  = REMAINDER OF PIPE LENGTH NOT YET OCCUPIED BY ELEMENTS     *
C  *  JUNC1  = LOWEST NUMBERED JUNCTION CONNECTED TO EACH PIPE           *
C  *  JUNC2  = HIGHEST NUMBERED JUNCTION CONNECTED TO EACH PIPE          *
C  *  NOPIPE = TOTAL NUMBER OF PIPES IN THE SYSTEM                       *
C  *  EDIA   = ELEMENT DIAMETER                                          *
C  *  ERUF   = ELEMENT ROUGHNESS                                         *
C  *  ECEL   = ELEMENT CELERITY                                          *
C  *  EFLO   = ELEMENT INITIAL FLOW                                      *
C  *  EALPHA = ELEMENT ANGLE ALPHA                                       *
C  *  ELEN   = ELEMENT LENGTH                                            *
C  *  PDIA   = PIPE DIAMETER                                             *
C  *  PRUF   = PIPE ROUGHNESS                                            *
C  *  PCEL   = PIPE CELERITY                                             *
C  *  PFLO   = PIPE INITIAL FLOW                                         *
C  *  PLEN   = PIPE LENGTH                                               *
C  *  PELEN  = PIPE ELEMENT LENGTH                                       *
C  *  NELCON = ELEMENT CONNECTIVITY MATRIX                               *
C  *  NOEL   = TOTAL NUMBER OF ELEMENTS                                  *
C  *  NOND   = TOTAL NUMBER OF NODES                                     *
C  *                                                                     *
C  ***********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/B1/NOPIPE,NOJUNC,TT,TT1,TINCR,VISC
      COMMON/B2/PNUM(10),JUNC1(10),JUNC2(10)
      COMMON/B3/PLEN(10),PDIA(10),PRUF(10),PFLO(10),PCEL(10),PELEN(10)
      COMMON/B3A/PALPHA(10)
      COMMON/B4/JNUM(10),JELV(10),NOCP(10),CPNUM(10,5)
      COMMON/B9/ELEN(95),EDIA(95),ERUF(95),EFLO(95),ECEL(95),EALPHA(95)
      COMMON/B12/IEL,NEQ,NEDF,NEND,NNDF,NOEL,NOND,LD(4,95),NELCON(2,95)
      INTEGER PNUM, CPNUM
      REAL*8 JELV,MHT,MDIA1,MDIA2,MHED
      IEL = 0
      NOD = 1
      IP = 1
   10 PL = 0.0
   20 PLEFT = PLEN(IP) - PL
      IEL = IEL + 1
      EALPHA(IEL) = ((JELV(JUNC2(IP)) - JELV(JUNC1(IP)))/PLEN(IP))
      EDIA(IEL) = PDIA(IP)
      ERUF(IEL) = PRUF(IP)
      ECEL(IEL) = PCEL(IP)
      EFLO(IEL) = PFLO(IP)
      NELCON(1,IEL) = NOD
      NOD = NOD + 1
```

```
      NELCON(2,IEL) = NOD
      IF (IDINT(PLEFT).LE.PELEN(IP)) GO TO 30
      ELEN(IEL) = PELEN(IP)
      PL = PL + PELEN(IP)
      GO TO 20
   30 ELEN(IEL) = PLEFT
      PALPHA(IP) = EALPHA(IEL)
      IF (IP .EQ. NOPIPE) GO TO 40
      IP = IP + 1
      NOD = NOD + 1
      GO TO 10
   40 CONTINUE
      NOND = NOD
      NOEL = IEL
      RETURN
      END
C
      SUBROUTINE JUNCTN
C
C   ***********************************************************************
C   *                                                                     *
C   *  THIS SUBROUTINE IDENTIFIES THE CONNECTING ELEMENT NODES AT EACH    *
C   *  PIPE JUNCTION AND IDENTIFIES THE NODE NUMBERS IN WHICH HEAD        *
C   *  VALUES ARE PRESCRIBED.                                             *
C   *                                                                     *
C   *  IEL     = ELEMENT NUMBER                                           *
C   *  IJ      = JUNCTION NUMBER                                          *
C   *  IP      = PIPE NUMBER                                              *
C   *  NOD     = NODE NUMBER                                              *
C   *  PL      = CUMLATIVE NODE LENGTH ALONG EACH PIPE                    *
C   *  JUNC    = JUNCTION NUMBER CONNECTED TO EACH NODE (OR ZERO)         *
C   *  NPIPE   = PIPE NUMBER CORRESPONDING TO EACH NODE                   *
C   *  PLEFT   = REMAINDER OF PIPE LENGTH NOT YET OCCUPIED BY NODES       *
C   *  JNOD    = NODE NUMBERS CONNECTED TO EACH JUNCTION                  *
C   *  ID      = ARRAY TO IDENTIFY PRESCRIBED NODAL VALUES OF HEAD        *
C   *            ( 1 = PRESCRIBED, 0 = FREE )                             *
C   *  NOND    = NUMBER OF NODES                                          *
C   *  NOCP    = NUMBER OF CONNECTING PIPES TO EACH JUNCTION              *
C   *  NOPIPE  = NUMBER OF PIPES IN THE SYSTEM                            *
C   *  PLEN    = PIPE LENGTH                                              *
C   *  PELEN   = PIPE ELEMENT LENGTH                                      *
C   *                                                                     *
C   ***********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/B1/NOPIPE,NOJUNC,TT,TT1,TINCR,VISC
      COMMON/B2/PNUM(10),JUNC1(10),JUNC2(10)
      COMMON/B3/PLEN(10),PDIA(10),PRUF(10),PFLO(10),PCEL(10),PELEN(10)
      COMMON/B3A/PALPHA(10)
      COMMON/B4/JNUM(10),JELV(10),NOCP(10),CPNUM(10,5)
      COMMON/B7/JNOD(10,5),NPIPE(95),JUNC(95)
      COMMON/B11/ID(2,95),HPBV(95)
      COMMON/B12/IEL,NEQ,NEDF,NEND,NNDF,NOEL,NOND,LD(4,95),NELCON(2,95)
      INTEGER PNUM,CPNUM
      REAL*8 JELV,MHT,MDIA1,MDIA2,MHED
      IP = 1
      IEL = 1
      NOD = 1
   10 PL = PELEN(IP)
      JUNC(NOD) = JUNC1(IP)
      NPIPE(NOD) = IP
      IF (PELEN(IP) .GE. PLEN(IP)) GO TO 30
   20 NOD = NOD + 1
      JUNC(NOD) = 0
```

```
      PLEFT = PLEN(IP) - PL
      NPIPE(NOD) = IP
      PL = PL + PELEN(IP)
      IF (IDINT(PLEFT) .LE. PELEN(IP)) GO TO 30
      GO TO 20
   30 NOD = NOD + 1
      JUNC(NOD) = JUNC2(IP)
      NPIPE(NOD) = IP
      IF (IP .EQ. NOPIPE) GO TO 40
      IP = IP + 1
      NOD = NOD + 1
      IEL = IEL + 1
      GO TO 10
   40 DO 60 IJ = 1,NOJUNC
      IP = 1
      NOD = 1
   50 IF (JUNC(NOD) .EQ. IJ) GO TO 55
      NOD = NOD + 1
      GO TO 50
   55 JNOD(IJ,IP) = NOD
      IF (IP .EQ. NOCP(IJ)) GO TO 60
      IP = IP + 1
      NOD = NOD + 1
      GO TO 50
   60 CONTINUE
      DO 100 N = 1,NOND
      IF (JUNC(N) .EQ. 0) GO TO 100
      ID(1,N) = 1
  100 CONTINUE
      RETURN
      END
C
      SUBROUTINE NODE
C
C  *********************************************************************
C  *                                                                   *
C  *  THIS SUBROUTINE ASSIGNS INITIAL FLOWS AND HEADS TO EACH NODE.    *
C  *                                                                   *
C  *  IEL     = ELEMENT NUMBER                                         *
C  *  IP      = PIPE NUMBER                                            *
C  *  NOD     = NODE NUMBER                                            *
C  *  PL      = CUMULATIVE NODE LENGTH ALONG EACH PIPE                 *
C  *  PLEFT   = REMAINDER OF PIPE LENGTH NOT YET OCCUPIED BY NODES     *
C  *  JUNC1S  = LOWEST NUMBERED JUNCTION CONNECTED TO EACH PIPE        *
C  *  JUNC2S  = HIGHEST NUMBERED JUNCTION CONNECTED TO EACH PIPE       *
C  *  NOPIPE  = NUMBER OF PIPES IN THE SYSTEM                          *
C  *  MHED    = MANHOLE INITIAL HEAD                                   *
C  *  PLEN    = PIPE LENGTH                                            *
C  *  PELEN   = PIPE ELEMENTLENGTH                                     *
C  *  PFLO    = PIPE INITIAL FLOW                                      *
C  *  H       = KNOWN NODAL HEAD VALUE                                 *
C  *  Q       = KNOWN NODAL FLOW VALUE                                 *
C  *                                                                   *
C  *********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/B1/NOPIPE,NOJUNC,TT,TT1,TINCR,VISC
      COMMON/B2/PNUM(10),JUNC1(10),JUNC2(10)
      COMMON/B3/PLEN(10),PDIA(10),PRUF(10),PFLO(10),PCEL(10),PELEN(10)
      COMMON/B3A/PALPHA(10)
      COMMON/B4/JNUM(10),JELV(10),NOCP(10),CPNUM(10,5)
      COMMON/B5/KEY(10),MHT(10),MDIA1(10),MDIA2(10),MHED(10)
      COMMON/B8/Q(95),H(95),AMAT(2,2),BMAT(2,2)
      COMMON/B9/ELEN(95),EDIA(95),ERUF(95),EFLO(95),ECEL(95),EALPHA(95)
```

```
      INTEGER PNUM, CPNUM
      REAL*8 JELV,MHT,MDIA1,MDIA2,MHED
      IP = 1
      IEL = 1
      NOD = 1
   10 PL = 0.0
      JUNC1S = JUNC1(IP)
      JUNC2S = JUNC2(IP)
      H(NOD) = MHED(JUNC1S)
      Q(NOD) = PFLO(IP)
   20 PLEFT = PLEN(IP) - PL
      PL = PL + PELEN(IP)
      IF (IDINT(PLEFT).LE.PELEN(IP)) GO TO 30
      IEL = IEL + 1
      NOD = NOD + 1
      H(NOD) = MHED(JUNC1S) + PL * (MHED(JUNC2S) - MHED(JUNC1S)) /
     C         PLEN(IP)
      Q(NOD) = PFLO(IP)
      GO TO 20
   30 NOD = NOD + 1
      H(NOD) = MHED(JUNC2S)
      Q(NOD) = PFLO(IP)
      IF (IP .EQ. NOPIPE) GO TO 40
      IP = IP + 1
      IEL = IEL + 1
      NOD = NOD + 1
      GO TO 10
   40 CONTINUE
      RETURN
      END
C
      SUBROUTINE PROFIL
C
C ********************************************************************
C *                                                                  *
C *  THE PURPOSE OF THIS SUBROUTINE IS TO CALCULATE THE NUMBER OF    *
C *  EQUATIONS AND ESTABLISH THE DIAGONAL ENTRY ADDRESSES.           *
C *                                                                  *
C *  NEQ    = NUMBER OF EQUATIONS                                    *
C *  JDIAG  = DIAGONAL ARGUMENT NUMBERS OF COEFFICIENT MATRIX        *
C *  NOEL   = NUMBER OF ELEMENTS                                     *
C *  NOND   = NUMBER OF NODES                                        *
C *                                                                  *
C ********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/B11/ID(2,95),HPBV(95)
      COMMON/B12/IEL,NEQ,NEDF,NEND,NNDF,NOEL,NOND,LD(4,95),NELCON(2,95)
      COMMON/IEQN/JDIAG(200)
C
C     SET UP THE EQUATION NUMBERS.
C
      NEQ = 0
      DO 40 N = 1,NOND
      DO 30 I=1,NNDF
      J = ID(I,N)
      IF (J)  30,10,20
   10 NEQ = NEQ + 1
      ID(I,N) = NEQ
      GO TO 30
   20 ID(I,N) = 0
   30 CONTINUE
   40 CONTINUE
C
```

```
C       COMPUTE THE COLUMN HEIGHTS.
C
        DO 9Ø N=1,NOEL
        DO 8Ø I=1,NEND
        II = NELCON(I,N)
        DO 7Ø K=1,NNDF
        KK = ID(K,II)
        IF (KK .LE. Ø) GO TO 7Ø
        DO 6Ø J=I,NEND
        JJ = NELCON(J,N)
        DO 5Ø L=1,NNDF
        LL = ID(L,JJ)
        IF (LL .LE. Ø) GO TO 5Ø
        M = MAXØ(KK,LL)
        JDIAG(M) = MAXØ(JDIAG(M),IABS(KK-LL))
  5Ø    CONTINUE
  6Ø    CONTINUE
  7Ø    CONTINUE
  8Ø    CONTINUE
  9Ø    CONTINUE
C
C       COMPUTE THE DIAGONAL ADDRESSES WITHIN THE PROFILE.
C
        JDIAG(1) = 1
        IF (NEQ .EQ. 1) RETURN
        DO 1ØØ N=2,NEQ
        JDIAG(N) = JDIAG(N) + JDIAG(N-1) + 1
  1ØØ   CONTINUE
        RETURN
        END
C
        SUBROUTINE ELEQN
C
C   *********************************************************************
C   *                                                                   *
C   *   THE PURPOSE OF THIS SUBROUTINE IS TO GENERATE THE ELEMENT       *
C   *   EQUATION NUMBER ARRAY.                                          *
C   *                                                                   *
C   *   IEL    = ELEMENT NUMBER                                         *
C   *   NOD    = NODE NUMBER                                            *
C   *   NOEL   = NUMBER OF ELEMENTS                                     *
C   *   NOND   = NUMBER OF NODES                                        *
C   *   NELCON = ELEMENT CONNECTIVITY MATRIX                            *
C   *                                                                   *
C   *********************************************************************
C
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON/B11/ID(2,95),HPBV(95)
        COMMON/B12/IEL,NEQ,NEDF,NEND,NNDF,NOEL,NOND,LD(4,95),NELCON(2,95)
C
C       LOOPING OVER THE ELEMENTS.
C
        DO 3Ø IEL=1,NOEL
C
C       DETERMINING THE ELEMENT EQUATION NUMBER ARRAY.
C
        DO 2Ø NOD=1,NEND
        IC = (NOD - 1)*2
        NODE = NELCON(NOD,IEL)
        DO 1Ø J=1,2
        IDC = ID(J,NODE)
        IF (IDC .GT. Ø) LD(J+IC,IEL) = IDC
  1Ø    CONTINUE
  2Ø    CONTINUE
```

```
 3Ø   CONTINUE
      RETURN
      END
C
      SUBROUTINE ESTIFF
C
C ********************************************************************
C *                                                                  *
C *   THIS SUBROUTINE GENERATES THE ELEMENT STIFFNESS MATRIX        *
C *                                                                  *
C *   STIFF  = STIFFNESS COEFICIENT MATRIX                          *
C *   TT     = TINCR * THETA                                        *
C *   TT1    = TINCR * (1.Ø - THETA)                                *
C *   AM     = "A" MATRIX (SEE GOVERNING EQUATIONS)                 *
C *   BM     = "B" MATRIX (SEE GOVERNING EQUATIONS)                 *
C *   CM     = "C(Q)" MATRIX (SEE GOVERNING EQUATIONS)              *
C *   EM     = "E" MATRIX (SEE GOVERNING EQUATIONS)                 *
C *   FM     = "F" MATRIX (SEE GOVERNING EQUATIONS)                 *
C *   GM     = "G" MATRIX (SEE GOVERNING EQUATIONS)                 *
C *                                                                  *
C ********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/B1/NOPIPE,NOJUNC,TT,TT1,TINCR,VISC
      COMMON/WORK/STIFF(4,4),FORCE(4)
      COMMON/MATR1/AM(2,2),BM(2,2),CM(2,2),EM(2,2),FM(2,2),GM(2,2)
C
C     ASSIGNMENT OF ELEMENT STIFFNESS MATRIX VALUES
C
      STIFF(1,1) = EM(1,1)
      STIFF(1,2) = TT * (FM(1,1) + GM(1,1))
      STIFF(1,3) = EM(1,2)
      STIFF(1,4) = TT * (FM(1,2) + GM(1,2))
      STIFF(2,1) = TT * BM(1,1)
      STIFF(2,2) = AM(1,1) + TT * CM(1,1)
      STIFF(2,3) = TT * BM(1,2)
      STIFF(2,4) = AM(1,2) + TT * CM(1,2)
      STIFF(3,1) = EM(2,1)
      STIFF(3,2) = TT * (FM(2,1) + GM(2,1))
      STIFF(3,3) = EM(2,2)
      STIFF(3,4) = TT * (FM(2,2) + GM(2,2))
      STIFF(4,1) = TT * BM(2,1)
      STIFF(4,2) = AM(2,1) + TT * CM(2,1)
      STIFF(4,3) = TT * BM(2,2)
      STIFF(4,4) = AM(2,2) + TT * CM(2,2)
      RETURN
      END
C
      SUBROUTINE HEAD
C
C
C ********************************************************************
C *                                                                  *
C *   THIS SUBROUTINE CALCULATES THE NODAL HEAD (H(N+1)) VALUES AT  *
C *   EACH MANHOLE TO BE USED AS PRESCRIBED BOUNDARY VALUES (HPBV)  *
C *   BEFORE EACH GENERAL SYSTEM SOLUTION.                          *
C *                                                                  *
C *   NOCP   = NUMBER OF CONNECTING NODES FOR EACH JUNCTION         *
C *   JNOD   = NODE NUMBERS CONNECTED TO EACH JUNCTION              *
C *   NPIPE  = PIPE NUMBER CORRESPONDING TO EACH HEAD               *
C *   QJ     = FLOW INTO JUNCTION FROM A NODE                       *
C *   Q      = KNOWN NODAL FLOW VALUE                               *
C *   TIME   = TIME SINCE SIMULATION STARTED                        *
C *   TINCR  = TIME INCREMENT                                       *
```

```
C  *  QIN     = HYDROGRAPH INITIAL FLOW                                      *
C  *  QPK     = HYDROGRAPH PEAK FLOW                                         *
C  *  TPK     = HYDROGRAPH TIME TO PEAK                                      *
C  *  TBAS    = HYDROGRAPH TIME BASE                                         *
C  *  QI      = MANHOLE INFLOW                                               *
C  *  QSUM    = SUM OF FLOWS INTO JUNCTION                                   *
C  *  TQSUM   = VOLUME OF WATER INTO JUNCTION                                *
C  *  AM1     = MANHOLE AREA                                                 *
C  *  AM2     = OVER FLOW STORAGE AREA                                       *
C  *  VM      = MANHOLE VOLUME CAPACITY                                      *
C  *  WHT1    = WATER HEIGHT AT JUNCTION FROM PREVIOUS TIME STEP             *
C  *  WHT     = WATER HEIGHT ABOVE MANHOLE FROM PREVIOUS TIME STEP           *
C  *  WHT2    = WATER HEIGHT AT JUNCTION FOR PRESENT TIME STEP               *
C  *  HPBV    = HEAD PRESCRIBED BOUNDARY VALUE (JUNCTION HEAD FOR            *
C  *            PRESENT TIME STEP)                                           *
C  *                                                                         *
C  ***************************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION QJ(95)
      COMMON/B1/NOPIPE,NOJUNC,TT,TT1,TINCR,VISC
      COMMON/B2/PNUM(1Ø),JUNC1(1Ø),JUNC2(1Ø)
      COMMON/B4/JNUM(1Ø),JELV(1Ø),NOCP(1Ø),CPNUM(1Ø,5)
      COMMON/B5/KEY(1Ø),MHT(1Ø),MDIA1(1Ø),MDIA2(1Ø),MHED(1Ø)
      COMMON/B6/QI(1Ø),QIN(1Ø),QPK(1Ø),TPK(1Ø),TBAS(1Ø),TIME
      COMMON/B7/JNOD(1Ø,5),NPIPE(95),JUNC(95)
      COMMON/B8/Q(95),H(95),AMAT(2,2),BMAT(2,2)
      COMMON/B9/ELEN(95),EDIA(95),ERUF(95),EFLO(95),ECEL(95),EALPHA(95)
      COMMON/B11/ID(2,95),HPBV(95)
      COMMON/B12/IEL,NEQ,NEDF,NEND,NNDF,NOEL,NOND,LD(4,95),NELCON(2,95)
      INTEGER PNUM, CPNUM
      REAL*8 JELV,MHT,MDIA1,MDIA2,MHED
      PI = 3.141592653589793DØ
      DO 1Ø NOD = 1,NOND
   1Ø HPBV(NOD) = Ø.Ø
C
C     GEOMETRY CALCULATION FOR THE PIPES CONNECTING EACH JUNCTION
C
      DO 15Ø IJ = 1,NOJUNC
      NOCPS = NOCP(IJ)
      IF (KEY(IJ) .EQ. 1) GO TO 13Ø
      DO 3Ø N = 1,NOCPS
      JNODS = JNOD(IJ,N)
      NPIPES = NPIPE(JNODS)
      IF (JUNC1(NPIPES) .EQ. IJ) GO TO 2Ø
      QJ(JNODS) = Q(JNODS)
      GO TO 3Ø
   2Ø QJ(JNODS) = Ø.Ø - Q(JNODS)
   3Ø CONTINUE
C
C     CALCULATION OF HYDROGRAPH INFLOW
C
      IF (TIME .GT. TBAS(IJ)) GO TO 5Ø
      IF (TIME .GT. TPK(IJ)) GO TO 4Ø
      QI(IJ) = QIN(IJ) + (TIME - TINCR) * (QPK(IJ) - QIN(IJ)) / TPK(IJ)
      GO TO 6Ø
   4Ø QI(IJ) = QPK(IJ) + ((QIN(IJ) - QPK(IJ)) * (TIME - TINCR - TPK(IJ))
     Q          / (TBAS(IJ) - TPK(IJ)))
      GO TO 6Ø
   5Ø QI(IJ) = QIN(IJ)
   6Ø QSUM = QI(IJ)
C
C     CALCULATION OF HPBV AT EACH MANHOLE (JUNCTION CONTINUITY EQUATION)
C
```

```
      DO 70 N = 1,NOCPS
      JNODS = JNOD(IJ,N)
   70 QSUM = QSUM + QJ(JNODS)
      TQSUM = QSUM * TINCR
      AM1 = PI * ((MDIA1(IJ) / 12.0) ** 2.0) / 4.0
      VM = AM1 * MHT(IJ)
      AM2 = PI * ((MDIA2(IJ)) ** 2.0) / 4.0
      JNODS = JNOD(IJ,1)
      WHT1 = H(JNODS) - JELV(IJ)
      IF (WHT1 .GT. MHT(IJ)) GO TO 90
      TQSUM = TQSUM + WHT1 * AM1
      IF (TQSUM .GT. VM) GO TO 100
   80 WHT2 = TQSUM / AM1
      GO TO 110
   90 WHT = WHT1 - MHT(IJ)
      TQSUM = TQSUM + MHT(IJ) * AM1 + WHT * AM2
      IF (TQSUM .LE. VM) GO TO 80
  100 WHT2 = MHT(IJ) + (TQSUM - VM) / AM2
  110 DO 120 N = 1,NOCPS
      JNODS = JNOD(IJ,N)
  120 HPBV(JNODS) = WHT2 + JELV(IJ)
      GO TO 150
  130 QI(IJ) = 0.0
      DO 140 N = 1,NOCPS
      JNODS = JNOD(IJ,N)
  140 HPBV(JNODS) = H(JNODS)
  150 CONTINUE
      RETURN
      END
C
      SUBROUTINE SFORCE
C
C  ********************************************************************
C  *                                                                  *
C  *  THIS SUBROUTINE GENERATES THE ELEMENT FORCE VECTOR              *
C  *                                                                  *
C  *  FORCE  = RIGHT HAND SIDE FORCE VECTOR                           *
C  *  AM     = "A" MATRIX (SEE GOVERNING EQUATIONS)                   *
C  *  BM     = "B" MATRIX (SEE GOVERNING EQUATIONS)                   *
C  *  CM     = "C(Q)" MATRIX (SEE GOVERNING EQUATIONS)                *
C  *  EM     = "E" MATRIX (SEE GOVERNING EQUATIONS)                   *
C  *  FM     = "F" MATRIX (SEE GOVERNING EQUATIONS)                   *
C  *  GM     = "G" MATRIX (SEE GOVERNING EQUATIONS)                   *
C  *  Q1     = KNOWN FLOW VALUE AT NODE 1                             *
C  *  Q2     = KNOWN FLOW VALUE AT NODE 2                             *
C  *  FF1    = KNOWN DARCY FRICTION FACTOR AT NODE 1                  *
C  *  FF2    = KNOWN DARCY FRICTION FACTOR AT NODE 2                  *
C  *  RE1    = KNOWN REYNOLDS NUMBER AT NODE 1                        *
C  *  RE2    = KNOWN REYNOLDS NUMBER AT NODE 2                        *
C  *  VISC   = KINEMATIC VISCOSITY OF FLUID                           *
C  *  TT1    = TINCR * (1.0 - THETA)                                  *
C  *  G      = ACCELERATION DUE TO GRAVITY                            *
C  *  ELEN   = ELEMENT LENGTH                                         *
C  *  EDIA   = ELEMENT DIAMETER                                       *
C  *  EAREA  = ELEMENT CROSS SECTIONAL AREA                           *
C  *  ECEL   = ELEMENT CELERITY                                       *
C  *  EALPHA = ELEMENT ANGLE ALPHA                                    *
C  *  ERUF   = ELEMENT ROUGHNESS                                      *
C  *  HPBV   = HEAD PRESCRIBED BOUNDARY VALUE                         *
C  *  NELCON = ELEMENT CONNECTIVTY MATRIX                             *
C  *  A      = GLOBAL COEFFICIENT ARRAY FOR GAUS-CROUT SOLUTION ROUTINE*
C  *  B      = GLOBAL RHS VECTOR FOR GAUS-CROUT SOLUTION ROUTINE      *
C  *  C      = GLOBAL COEFFICIENT ARRAY FOR GAUS-CROUT SOLUTION ROUTINE*
C  *                                                                  *
```

```
C     *********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION EAREA(95)
      COMMON/B1/NOPIPE,NOJUNC,TT,TT1,TINCR,VISC
      COMMON/B8/Q(95),H(95),AMAT(2,2),BMAT(2,2)
      COMMON/B9/ELEN(95),EDIA(95),ERUF(95),EFLO(95),ECEL(95),EALPHA(95)
      COMMON/B11/ID(2,95),HPBV(95)
      COMMON/B12/IEL,NEQ,NEDF,NEND,NNDF,NOEL,NOND,LD(4,95),NELCON(2,95)
      COMMON/B13/SA,SF,SG
      COMMON/RLOG/AFAC,AFL,BACK,BFL
      COMMON/IEQN/JDIAG(200)
      COMMON/REQN/A(1500),B(200),C(1500)
      COMMON/WORK/STIFF(4,4),FORCE(4)
      COMMON/MATR1/AM(2,2),BM(2,2),CM(2,2),EM(2,2),FM(2,2),GM(2,2)
      LOGICAL AFAC,AFL,BACK,BFL
      ZERO = 0.0D0
C
C     INITIALIZING THE COEFFICIENT ARRAYS
C
      NAD = JDIAG(NEQ)
      DO 40 IAD = 1,NAD
      A(IAD) = ZERO
      C(IAD) = ZERO
   40 CONTINUE
C
C     INITIALIZING THE GLOBAL RIGHT HAND SIDE VECTOR
C
      DO 50 NN = 1,NEQ
      B(NN) = ZERO
   50 CONTINUE
      G = 32.174D0
      DO 1000 IEL = 1,NOEL
      EAREA(IEL) = 3.141592653589793D0 * ((EDIA(IEL)/12.0) **2.0) / 4.0
C
C     CALCULATION OF THE STIFFNESS ARRAY COMPONENTS
C
      DO 60 I = 1,2
      DO 60 J = 1,2
      AM(I,J) = SA * ELEN(IEL) * AMAT(I,J)
      BM(I,J) = G * EAREA(IEL) * BMAT(I,J)
      EM(I,J) = ELEN(IEL) * AMAT(I,J)
      FM(I,J) = SF*ELEN(IEL) * EALPHA(IEL) * AMAT(I,J)/EAREA(IEL)
      GM(I,J) = SG*ECEL(IEL) * ECEL(IEL) * BMAT(I,J) / (G * EAREA(IEL))
   60 CONTINUE
C
C     UPDATING THE ELEMENT FRICTION LOSS TERM
C
      Q1 = Q(NELCON(1,IEL))
      Q2 = Q(NELCON(2,IEL))
      RE1 = DABS(Q1 * (EDIA(IEL) / 12.0) / (EAREA(IEL) * VISC))
      RE2 = DABS(Q2 * (EDIA(IEL) / 12.0) / (EAREA(IEL) * VISC))
      FF1 = 0.25D0 / ((DLOG10((ERUF(IEL) / (3.7D0 * EDIA(IEL) / 12.0) +
     Q      5.74D0 / RE1 ** 0.9D0))) ** 2)
      FF2 = 0.25D0 / ((DLOG10((ERUF(IEL) / (3.7D0 * EDIA(IEL) / 12.0) +
     Q      5.74D0 / RE2 ** 0.9D0))) ** 2)
      CM(1,1) = ELEN(IEL) * (0.20D0 * FF1 * DABS(Q1) + 0.05D0
     Q          * FF2 * DABS(Q1) + 0.05D0 * FF1 * DABS(Q2) + FF2 *
     Q          DABS(Q2) / 30.0) / (2.0 * EAREA(IEL) * EDIA(IEL) / 12.0)
      CM(1,2) = ELEN(IEL) * (0.05D0 * FF1 * DABS(Q1) +
     Q          FF2 * DABS(Q1) / 30.0 + FF1 * DABS(Q2) / 30.0 + 0.05D0 *
     Q          FF2 * DABS(Q2)) / (2.0 * EAREA(IEL) * EDIA(IEL) / 12.0)
      CM(2,1) = ELEN(IEL) * (0.05D0 * FF1 * DABS(Q1) +
     Q          FF2 * DABS(Q1) / 30.0 + FF1 * DABS(Q2) / 30.0 + 0.05D0 *
```

```
     Q           FF2 * DABS(Q2)) / (2.0 * EAREA(IEL) * EDIA(IEL) / 12.0)
      CM(2,2) = ELEN(IEL) * (FF1 * DABS(Q1) / 30.0 + 0.05D0
     Q           * FF2 * DABS(Q1) + 0.05D0 * FF1 * DABS(Q2) + 0.2D0 *
     Q           FF2 * DABS(Q2)) / (2.0 * EAREA(IEL) * EDIA(IEL) / 12.0)
      DO 110 NOD = 1,2
C
C     INITIALIZING THE FORCE VECTOR
C
      FORCE(2 * NOD - 1) = ZERO
      FORCE(2 * NOD) = ZERO
C
C     CALCULATION OF ELEMENT RIGHT HAND SIDE VECTOR (FORCE VECTOR)
C
      DO 100 IDF = 1,NNDF
      FORCE(2*NOD-1) = FORCE(2*NOD-1) + EM(NOD,IDF) * H(NELCON(IDF,IEL))
     Q                 - TT1 * (FM(NOD,IDF) + GM(NOD,IDF)) *
     Q                 Q(NELCON(IDF,IEL))
      FORCE(2 * NOD) = FORCE(2 * NOD) - TT1 * BM(NOD,IDF) *
     Q                 H(NELCON(IDF,IEL)) + (AM(NOD,IDF) -
     Q                 TT1 * CM(NOD,IDF)) * Q(NELCON(IDF,IEL))
  100 CONTINUE
  110 CONTINUE
C
C     MODIFYING THE ELEMENT RIGHT HAND SIDE VECTOR TO INCLUDE NONZERO
C     PRESCRIBED BOUNDARY VALUES.
C
      DO 500 NOD = 1,NEND
      DO 500 IDF = 1,NNDF
      IF (ID(IDF,NELCON(NOD,IEL)) .GT. 0)GO TO 500
      CALL ESTIFF
      IC = (NOD - 1) * NNDF + IDF
      DO 450 I = 1,NEDF
      FORCE(I) = FORCE(I) - STIFF(I,IC) * HPBV(NELCON(NOD,IEL))
  450 CONTINUE
  500 CONTINUE
      CALL ADDSTF
 1000 CONTINUE
      RETURN
      END
C
      SUBROUTINE ADDSTF
C
C  *********************************************************************
C  *                                                                   *
C  *  THE PURPOSE OF THIS SUBROUTINE IS TO ASSEMBLE THE ELEMENT        *
C  *  FORCE VECTOR AND COEFFICIENT MATRIX INTO THE GLOBAL FORCE        *
C  *  VECTOR AND COEFFICIENT MATRIX CONSISTENT WITH THE PROFILE        *
C  *  SOLVER.                                                          *
C  *                                                                   *
C  *  NEDF    = NUMBER OF ELEMENT DEGREES OF FREEDOM                   *
C  *  IEL     = ELEMENT NUMBER                                         *
C  *  A       = UPPER PROFILE COEFFICIENT MATRIX ARGUMENTS             *
C  *  B       = RIGHT HAND SIDE VECTOR ARGUMENTS                       *
C  *  C       = LOWER PROFILE COEFFICIENT MATRIX ARGUMENTS             *
C  *                                                                   *
C  *********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/B12/IEL,NEQ,NEDF,NEND,NNDF,NOEL,NOND,LD(4,95),NELCON(2,95)
      COMMON/RLOG/AFAC,AFL,BACK,BFL
      COMMON/IEQN/JDIAG(200)
      COMMON/REQN/A(1500),B(200),C(1500)
      COMMON/WORK/S(4,4),P(4)
      LOGICAL AFAC,AFL,BACK,BFL
```

```
C
      DO 200 J=1,NEDF
C
C     ASSEMBLING THE RIGHT HAND SIDE VECTOR.
C
      K = LD(J,IEL)
      IF (K .EQ. 0) GO TO 200
      IF (BFL)  B(K) = B(K) + P(J)
      IF (.NOT. AFL)  GO TO 200
      L = JDIAG(K) - K
      DO 100 I=1,NEDF
      M = LD(I,IEL)
C
C     ASSEMBLING THE UPPER AND LOWER PROFILE COEFFICIENT MATRICES.
C
      IF (M .GT. K  .OR.  M .EQ. 0) GO TO 100
      M = L + M
      A(M) = A(M) + S(I,J)
      C(M) = C(M) + S(J,I)
100   CONTINUE
200   CONTINUE
C
      RETURN
      END
C
      SUBROUTINE UACTCL
C
C  ***********************************************************************
C  *                                                                     *
C  *  THE PURPOSES OF THIS SUBROUTINE ARE TO PERFORM FORWARD ELIMI-      *
C  *  NATION AND BACKSUBSTITUTION OPERATIONS ON AN UNSYMMETRIC           *
C  *  COEFFICIENT MATRIX WITH A SYMMETRIC PROFILE USING GAUSS-CROUT      *
C  *  ELIMINATION.                                                       *
C  *                                                                     *
C  *  NEQ     = NUMBER OF EQUATIONS                                      *
C  *  JDIAG   = DIAGONAL ARGUMENT NUMBERS OF COEFFICIENT MATRIX          *
C  *  A       = UPPER PROFILE COEFFICIENT MATRIX ARGUMENTS               *
C  *  B       = RIGHT HAND SIDE VECTOR ARGUMENTS                         *
C  *  C       = LOWER PROFILE COEFFICIENT MATRIX ARGUMENTS               *
C  *                                                                     *
C  ***********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/B12/IEL,NEQ,NEDF,NEND,NNDF,NOEL,NOND,LD(4,95),NELCON(2,95)
      COMMON/RDATA/DT,ONE,RCT,RTH,RTT,TWO,ZERO
      COMMON/RLOG/AFAC,AFL,BACK,BFL
      COMMON/IEQN/JDIAG(200)
      COMMON/REQN/A(1500),B(200),C(1500)
      LOGICAL AFAC,AFL,BACK,BFL
C
C     FACTOR THE COEFFICIENT MATRIX A INTO UT*D*U AND REDUCE THE
C     RIGHT HAND SIDE VECTOR B.
C
      JR = 0
      DO 300 J=1,NEQ
      JD = JDIAG(J)
      JH = JD - JR
      IF (JH .LE. 1) GO TO 300
      IS = J + 1 - JH
      IE = J - 1
      IF (.NOT. AFAC) GO TO 250
      K = JR + 1
      ID = 0
C
```

```
C      REDUCE ALL EQUATIONS EXECPT THE DIAGONAL.
C
       DO 200 I=IS,IE
       IR = ID
       ID = JDIAG(I)
       IH = MIN0(ID-IR-1,I-IS)
       IF (IH .EQ. 0) GO TO 150
       A(K) = A(K) - DOT(A(K-IH),C(ID-IH),IH)
       C(K) = C(K) - DOT(C(K-IH),A(ID-IH),IH)
 150   IF (A(ID) .NE. ZERO) C(K) = C(K)/A(ID)
       K = K + 1
 200   CONTINUE
C
C      REDUCE THE DIAGONAL TERM.
C
       A(JD) = A(JD) - DOT(A(JR+1),C(JR+1),JH-1)
C
C      FORWARD ELIMINATION OF THE RIGHT HAND SIDE VECTOR.
C
 250   IF (BACK) B(J) = B(J) - DOT(C(JR+1),B(IS),JH-1)
 300   JR = JD
       IF (.NOT. BACK) RETURN
C
C      BACKSUBSTITUTION.
C
       J = NEQ
       JD = JDIAG(J)
 500   IF (A(JD) .NE. ZERO) B(J) = B(J)/A(JD)
       D = B(J)
       J = J - 1
       IF (J .LE. 0) RETURN
       JR = JDIAG(J)
       IF (JD-JR .LE. 1) GO TO 700
       IS = J - JD + JR + 2
       K = JR - IS + 1
       DO 600 I=IS,J
       B(I) = B(I) - D*A(I+K)
 600   CONTINUE
 700   JD = JR
       GO TO 500
       END
```

<u>APPENDIX B</u>   <u>DYN/KIN FORTRAN SOURCE PROGRAM</u>

```
C
C
C
C
C     *********************************************************************
C     *********************************************************************
C     *                                                                   *
C     *                                                                   *
C     *                                                                   *
C     *                              DYN/KIN                              *
C     *                                                                   *
C     *                  ( DYNAMIC AND KINEMATIC MODEL )                  *
C     *                                                                   *
C     *               ( STORMSEWER ANALYSIS AT PEAK FLOWS )               *
C     *                                                                   *
C     *                                                                   *
C     *  THIS PROGRAM SOLVES THE GOVERNING EQUATIONS FOR FLOW IN          *
C     *  PRESSURIZED STORM SEWER SYSTEMS  USING NUMERICAL FORWARD (EULER) *
C     *  DIFFERENCING TECHNIQUES. TWO TYPES OF SOLUTIONS ARE OBTAINED:    *
C     *                                                                   *
C     *                  A)  KINEMATIC (STEADY STATE W/STORAGE)           *
C     *                  B)  DYNAMIC LUMPED PARAMETER                     *
C     *                                                                   *
C     *                                                                   *
C     *                                                                   *
C     *                                                                   *
C     *                              THESIS                               *
C     *                               1982                                *
C     *                                                                   *
C     *                          GREG C. HEITZMAN                         *
C     *                    GRADUATE RESEARCH ASSISTANT                    *
C     *                  DEPARTMENT OF CIVIL ENGINEERING                  *
C     *                      UNIVERSITY OF KENTUCKY                       *
C     *                                                                   *
C     *.                                                                  *
C     *                                                                   *
C     *********************************************************************
C     *********************************************************************
C
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/M1/TPLOTM(1500),HPLO1A(1500),HPLO1B(1500)
      COMMON/M2/HPLO2A(1500),HPLO2B(1500),HPLO3A(1500),HPLO3B(1500)
      COMMON/M3/HPLO4A(1500),HPLO4B(1500),HPLO5A(1500),HPLO5B(1500)
      COMMON/M4/QPLO1A(1500),QPLO1B(1500),QPLO2A(1500),QPLO2B(1500)
      COMMON/M5/QPLO3A(1500),QPLO3B(1500),QPLO4A(1500),QPLO4B(1500)
      COMMON/M6/QPLO5A(1500),QPLO5B(1500)
      COMMON/B1/NOPIPE,NOJUNC,TINCR,VISC
      COMMON/B2/PNUM(10),JUNC1(10),JUNC2(10)
      COMMON/B3/PLEN(10),PDIA(10),PRUF(10),MLOSS(10),PFLO(10)
      COMMON/B4/JNUM(10),JELV(10),NOCP(10),CPNUM(10,5)
      COMMON/B5/KEY(10),MHT(10),MDIA1(10),MDIA2(10),MHED(10)
      COMMON/B6/QI(10),QIN(10),QPK(10),TLAG(10),TPK(10),TBAS(10),TIME
      COMMON/B8/Q(10),QA(10),QB(10),HA(10),HB(10)
```

```
      REAL*8 JELV,MHT,MDIA1,MDIA2,MHED,MLOSS
      REAL TPLOTM,HPLO1A,HPLO1B,HPLO2A,HPLO2B,HPLO3A,HPLO3B,HPLO4A
      REAL HPLO4B,HPLO5A,HPLO5B,QPLO1A,QPLO1B,QPLO2A,QPLO2B,QPLO3A
      REAL QPLO3B,QPLO4A,QPLO4B,QPLO5A,QPLO5B
      INTEGER PNUM,CPNUM
  500 FORMAT(I5,I5,3F10.5,F10.8)
  510 FORMAT(3I5,2F10.2,F10.6,2F10.2)
  520 FORMAT(I5,F10.2,6I5)
  530 FORMAT(I1,F9.2,3F10.2)
  540 FORMAT(5F10.2)
  550 FORMAT(I5,5F5.1,5I5)
  600 FORMAT(//////////'                                    ***** ORIGINAL
     QDATA SUMMARY *****')
  610 FORMAT(//'   THE DARCY-WEISBACH HEAD LOSS EQUATION IS USED, THE KI
     QNEMATIC VISCOSITY = ',F10.8,' SQ.FT./SEC.')
  620 FORMAT(  ///'       PIPE NO.   NODE NUMBERS         LENGTH    DIAMETER
     Q   ROUGHNESS    M-LOSS     INITIAL FLOWRATE ')
  630 FORMAT('                                          (FEET)    (INCHES)     (
     QFEET)                      (CFS) '/)
  640 FORMAT( I10,I11,I5,9X,F7.2,4X,F5.2,6X,F7.5,5X,F4.1,9X,F7.2)
  650 FORMAT(///  )
  660 FORMAT( //'
     Q            MANHOLE  DATA ')
  670 FORMAT(//'   JUNCTION NO.    ELEVATION ')
  680 FORMAT('                                                HEIGHT      D
     QIAMETER  STORAGE DIAMETER  INITIAL HEAD')
  690 FORMAT('                          (FEET)                 (FEET)      (
     QINCHES)        (FEET)          (FEET)'/)
  700 FORMAT( I10,10X,F6.2,19X,F6.2,7X,F6.1,8X,F6.2,8X,F7.3)
  710 FORMAT( I10,10X,F6.2,19X,'THIS JUNCTION HAS A FIXED HEAD OF ',
     Q       F7.2,'  FEET ')
  720 FORMAT(          ///,'                                             HY
     QDROGRAPH  INFORMATION ')
  730 FORMAT(//'   JUNCTION NO.        INITIAL FLOW    PEAK FLOW      TIME
     Q LAG     TIME TO PEAK     TIME BASE ')
  740 FORMAT('                             (CFS)          (CFS)        (MINUT
     QES)     (MINUTES)     (MINUTES) '/)
  750 FORMAT( I10,15X,F6.2,8X,F6.2,8X,F6.2,8X,F6.2,8X,F6.2,8X,F6.2)
  760 FORMAT(/////' SYSTEM EQUATIONS ARE SOLVED USING A',F5.2,' SECOND T
     QIME INCREMENT')
  770 FORMAT(/' RESULTS ARE OUTPUT EVERY ',F7.4,' MINUTES')
  780 FORMAT(/' TOTAL TIME OF SIMULATION = ',F7.4,' MINUTES'//////////)
  790 FORMAT(//////////' TIME FROM START OF SIMULATION = ',F8.4,'   MIN
     QUTES')
  800 FORMAT(////'
     Q                         *****   SOLUTION TYPE    *****')
  810 FORMAT(///'
     Q     STEADY STATE W/STORAGE                        LUMPED PARAMETER')
  820 FORMAT(//'          PIPE NUMBER
     QVELOCITY            FLOWRATE                VELOCITY            FLOWRAT
     QE')
  830 FORMAT('                                                             (F
     QT/SEC)              (CFS)                (FT/SEC)              (CFS)')
  840 FORMAT(/I13,39X,F10.3,10X,F10.3,10X,F10.3,10X,F10.3)
  850 FORMAT(/////'      JUNCTION NUMBER                 INFLOW
     Q GRADE LINE           HEAD ABOVE PIPE          GRADE LINE           HEAD AB
     QOVE PIPE')
  860 FORMAT('                                      (CFS)                  (F
     QEET)               (FEET)                (FEET)              (FEET)')
  870 FORMAT(/I13,13X,F10.2,16X,F10.3,10X,F10.3,10X,F10.3,10X,F10.3)
  880 FORMAT(///)
  900 FORMAT(2I10,4F10.1)
  910 FORMAT(10F8.4)
      READ(5,500)NOPIPE,NOJUNC,TINCR,TPRINT,TTOTL,VISC
```

```
      WRITE(6,600)
   20 DO 30 I = 1,NOPIPE
      READ(5,510)PNUM(I),JUNC1(I),JUNC2(I),PLEN(I),PDIA(I),PRUF(I),MLOSS
     Q(I),PFLO(I)
      QA(I) = PFLO(I)
   30 QB(I) = PFLO(I)
      DO 40 I = 1,NOJUNC
      READ(5,520) JNUM(I), JELV(I), NOCP(I),(CPNUM(I,J),J=1,5)
      READ(5,530) KEY(I), MHT(I), MDIA1(I),MDIA2(I),MHED(I)
      HA(I) = MHED(I)
      HB(I) = MHED(I)
      IF (KEY(I) .EQ. 1) GO TO 40
      READ(5,540) QIN(I),QPK(I),TLAG(I),TPK(I),TBAS(I)
   40 CONTINUE
      READ(5,550) NOPLOT,TPLT,DELTAX,PLTHD,DELTHY,DELTQY,IP1,IP2,IP3,IP4
     Q,IP5
      WRITE(6,610) VISC
      WRITE(6,620)
      WRITE(6,630)
      WRITE(6,640)(PNUM(I),JUNC1(I),JUNC2(I),PLEN(I),PDIA(I),PRUF(I),MLO
     QSS(I),PFLO(I),I=1,NOPIPE)
      WRITE(6,650)
      WRITE(6,660)
      WRITE(6,670)
      WRITE(6,680)
      WRITE(6,690)
      DO 60 J = 1,NOJUNC
      IF (KEY(J) .EQ. 1) GO TO 50
      WRITE(6,700)JNUM(J),JELV(J),MHT(J),MDIA1(J),MDIA2(J),MHED(J)
      GO TO 60
   50 WRITE(6,710)JNUM(J),JELV(J),MHED(J)
   60 CONTINUE
      WRITE(6,720)
      WRITE(6,730)
      WRITE(6,740)
      DO 70 J = 1,NOJUNC
      IF (KEY(J) .EQ. 1) GO TO 70
      WRITE(6,750)JNUM(J),QIN(J),QPK(J),TLAG(J),TPK(J),TBAS(J)
      TLAG(J) = TLAG(J) * 60.0
      TPK(J) = TPK(J) * 60.0
      TBAS(J) = TBAS(J) * 60.0
   70 CONTINUE
      TP = 1.0
      ITPT = 0
      WRITE(6,760) TINCR
      WRITE(6,770) TPRINT
      WRITE(6,780) TTOTL
      TIME = 0.0
      TTOTL = TTOTL * 60.0
      TPRINT = TPRINT * 60.0
C
C     ASSIGNMENT OF PLOTTING ARRAYS WITH SOLUTION VALUES
C
   80 IF(NOPLOT .EQ. 0) GO TO 100
      TPLO = TPLT * ITPT
      IF ((TPLO - TIME) .LT. .01) GO TO 90
      GO TO 100
   90 ITPT = ITPT + 1
      TPLOTM(ITPT) = TIME / (60.0 * DELTAX)
      HPLO1A(ITPT) = (HA(IP1) - PLTHD) / DELTHY
      HPLO1B(ITPT) = (HB(IP1) - PLTHD) / DELTHY
      QPLO1A(ITPT) = QA(IP1) / DELTQY
      QPLO1B(ITPT) = QB(IP1) / DELTQY
      IF (NOPLOT .LE. 1) GO TO 100
```

```
      HPLO2A(ITPT) = (HA(IP2) - PLTHD) / DELTHY
      HPLO2B(ITPT) = (HB(IP2) - PLTHD) / DELTHY
      QPLO2A(ITPT) = QA(IP2) / DELTQY
      QPLO2B(ITPT) = QB(IP2) / DELTQY
      IF (NOPLOT .LE. 2) GO TO 100
      HPLO3A(ITPT) = (HA(IP3) - PLTHD) / DELTHY
      HPLO3B(ITPT) = (HB(IP3) - PLTHD) / DELTHY
      QPLO3A(ITPT) = QA(IP3) / DELTQY
      QPLO3B(ITPT) = QB(IP3) / DELTQY
      IF (NOPLOT .LE. 3) GO TO 100
      HPLO4A(ITPT) = (HA(IP4) - PLTHD) / DELTHY
      HPLO4B(ITPT) = (HB(IP4) - PLTHD) / DELTHY
      QPLO4A(ITPT) = QA(IP4) / DELTQY
      QPLO4B(ITPT) = QB(IP4) / DELTQY
      IF (NOPLOT .LE. 4) GO TO 100
      HPLO5A(ITPT) = (HA(IP5) - PLTHD) / DELTHY
      HPLO5B(ITPT) = (HB(IP5) - PLTHD) / DELTHY
      QPLO5A(ITPT) = QA(IP5) / DELTQY
      QPLO5B(ITPT) = QB(IP5) / DELTQY
C
  100 TIME = TIME + TINCR
      IF (TIME .GT. TTOTL) GO TO 140
      CALL HEADA
      CALL FLOWA
      CALL HEADB
      CALL FLOWB
      TPRIN = TPRINT * TP
      IF ((TPRIN - TIME) .LT. 0.01) GO TO 110
      GO TO 80
  110 TIME = TIME / 60.0
      WRITE(6,790) TIME
      TIME = TIME * 60.0
      WRITE(6,800)
      WRITE(6,810)
      WRITE(6,820)
      WRITE(6,830)
      PI = 3.141592653589793D0
      DO 120 IP = 1,NOPIPE
      VELA = 4.0 * QA(IP) / (PI * (PDIA(IP) / 12.0) ** 2)
      VELB = 4.0 * QB(IP) / (PI * (PDIA(IP) / 12.0) ** 2)
      WRITE(6,840)IP,VELA,QA(IP),VELB,QB(IP)
  120 CONTINUE
      WRITE(6,850)
      WRITE(6,860)
      DO 130 IJ = 1,NOJUNC
      HPA = HA(IJ) - JELV(IJ)
      HPB = HB(IJ) - JELV(IJ)
      WRITE(6,870)IJ,QI(IJ),HA(IJ),HPA,HB(IJ),HPB
  130 CONTINUE
      WRITE(6,880)
      TP = TP + 1.0
      GO TO 80
C
C     INITIALIZE THE SCALE FACTORS FOR PLOTTING
C
  140 IF (NOPLOT .EQ. 0) GO TO 200
      ITPT = ITPT + 2
      TPLOTM(ITPT) = 1.0
      HPLO1A(ITPT) = 1.0
      HPLO1B(ITPT) = 1.0
      HPLO2A(ITPT) = 1.0
      HPLO2B(ITPT) = 1.0
      HPLO3A(ITPT) = 1.0
      HPLO3B(ITPT) = 1.0
```

```
      QPLO1A(ITPT) = 1.0
      QPLO1B(ITPT) = 1.0
      QPLO2A(ITPT) = 1.0
      QPLO2B(ITPT) = 1.0
      QPLO3A(ITPT) = 1.0
      QPLO3B(ITPT) = 1.0
C
C     PUNCHING THE PLOTTING ARRAY VALUES ON CARDS
C
      WRITE(7,900)NOPLOT,ITPT,PLTHD,DELTAX,DELTHY,DELTQY
      DO 150 IT = 1,ITPT,2
      WRITE(7,910)TPLOTM(IT),HPLO1A(IT),HPLO1B(IT),QPLO1A(IT),QPLO1B(IT)
     Q,TPLOTM(IT+1),HPLO1A(IT+1),HPLO1B(IT+1),QPLO1A(IT+1),QPLO1B(IT+1)
  150 CONTINUE
      IF (NOPLOT .LE. 1) GO TO 200
      DO 160 IT = 1,ITPT,2
      WRITE(7,910)TPLOTM(IT),HPLO2A(IT),HPLO2B(IT),QPLO2A(IT),QPLO2B(IT)
     Q,TPLOTM(IT+1),HPLO2A(IT+1),HPLO2B(IT+1),QPLO2A(IT+1),QPLO2B(IT+1)
  160 CONTINUE
      IF (NOPLOT .LE. 2) GO TO 200
      DO 170 IT = 1,ITPT,2
      WRITE(7,910)TPLOTM(IT),HPLO3A(IT),HPLO3B(IT),QPLO3A(IT),QPLO3B(IT)
     Q,TPLOTM(IT+1),HPLO3A(IT+1),HPLO3B(IT+1),QPLO3A(IT+1),QPLO3B(IT+1)
  170 CONTINUE
      IF (NOPLOT .LE. 3) GO TO 200
      DO 180 IT = 1,ITPT,2
      WRITE(7,910)TPLOTM(IT),HPLO4A(IT),HPLO4B(IT),QPLO4A(IT),QPLO4B(IT)
     Q,TPLOTM(IT+1),HPLO4A(IT+1),HPLO4B(IT+1),QPLO4A(IT+1),QPLO4B(IT+1)
  180 CONTINUE
      IF (NOPLOT .LE. 4) GO TO 200
      DO 190 IT = 1,ITPT,2
      WRITE(7,910)TPLOTM(IT),HPLO5A(IT),HPLO5B(IT),QPLO5A(IT),QPLO5B(IT)
     Q,TPLOTM(IT+1),HPLO5A(IT+1),HPLO5B(IT+1),QPLO5A(IT+1),QPLO5B(IT+1)
  190 CONTINUE
  200 CONTINUE
C
      STOP
      END
C
      BLOCK DATA
C
C  ********************************************************************
C  *                                                                  *
C  *  THE PURPOSE OF THIS BLOCK DATA IS TO INITIALIZE THE PLOTTING    *
C  *  MATIX VALUES TO ZERO.                                           *
C  *                                                                  *
C  ********************************************************************
      COMMON/M1/TPLOTM(1500),HPLO1A(1500),HPLO1B(1500)
      COMMON/M2/HPLO2A(1500),HPLO2B(1500),HPLO3A(1500),HPLO3B(1500)
      COMMON/M3/HPLO4A(1500),HPLO4B(1500),HPLO5A(1500),HPLO5B(1500)
      COMMON/M4/QPLO1A(1500),QPLO1B(1500),QPLO2A(1500),QPLO2B(1500)
      COMMON/M5/QPLO3A(1500),QPLO3B(1500),QPLO4A(1500),QPLO4B(1500)
      COMMON/M6/QPLO5A(1500),QPLO5B(1500)
      REAL TPLOTM,HPLO1A,HPLO1B,HPLO2A,HPLO2B,HPLO3A,HPLO3B,HPLO4A
      REAL HPLO4B,HPLO5A,HPLO5B,QPLO1A,QPLO1B,QPLO2A,QPLO2B,QPLO3A
      REAL QPLO3B,QPLO4A,QPLO4B,QPLO5A,QPLO5B
      DATA TPLOTM/1500*0.0/,HPLO1A/1500*0.0/,HPLO1B/1500*0.0/
      DATA HPLO2A/1500*0.0/,HPLO2B/1500*0.0/,HPLO3A/1500*0.0/
      DATA HPLO3B/1500*0.0/,HPLO4A/1500*0.0/,HPLO4B/1500*0.0/
      DATA HPLO5A/1500*0.0/,HPLO5B/1500*0.0/,QPLO1A/1500*0.0/
      DATA QPLO1B/1500*0.0/,QPLO2A/1500*0.0/,QPLO2B/1500*0.0/
      DATA QPLO3A/1500*0.0/,QPLO3B/1500*0.0/,QPLO4A/1500*0.0/
      DATA QPLO4B/1500*0.0/,QPLO5A/1500*0.0/,QPLO5B/1500*0.0/
      END
```

```
C
      SUBROUTINE HEADA
C
C
C ********************************************************************
C *                                                                  *
C *   THIS SUBROUTINE CALCULATES THE STEADY STATE HEAD (HA(N+1))     *
C *   VALUES AT EACH JUNCTION (MANHOLE) OF THE SYSTEM                *
C *                                                                  *
C *                                                                  *
C *   NOPIPE = NUMBER OF PIPES IN THE SYSTEM                         *
C *   PNUM   = PIPE NUMBER                                           *
C *   NOJUNC = NUMBER OF JUNCTIONS IN THE SYSTEM                     *
C *   JNUM   = JUNCTION NODE NUMBER                                  *
C *   JUNC1  = JUNCTION NODE 1 FOR PIPE                              *
C *   JUNC2  = JUNCTION NODE 2 FOR PIPE                              *
C *   JELV   = JUNCTION ELEVATION                                    *
C *   CPNUM  = CONNECTING PIPE NUMBER TO EACH JUNCTION               *
C *   NOCP   = NUMBER OF CONNECTING PIPES AT EACH JUNCTION           *
C *   TIME   = TIME SINCE SIMULATION STARTED                         *
C *   TINCR  = TIME INCREMENT (TIME STEP)                            *
C *   QIN    = HYDROGRAPH INITIAL FLOW                               *
C *   QPK    = HYDROGRAPH PEAK FLOW                                  *
C *   TLAG   = HYDROGRAPH TIME LAG                                   *
C *   TPK    = HYDROGRAPH TIME TO PEAK                               *
C *   TBAS   = HYDROGRAPH TIME BASE                                  *
C *   QI     = MANHOLE INFLOW                                        *
C *   QSUM   = SUM OF FLOWS INTO JUNCTION                            *
C *   TQSUM  = VOLUME OF WATER INTO JUNCTION                         *
C *   MHT    = MANHOLE HEIGHT                                        *
C *   MHED   = INITIAL MANHOLE HEAD                                  *
C *   MDIA1  = MANHOLE DIAMETER                                      *
C *   MDIA2  = MANHOLE SURFACE OVERFLOW DIAMETER                     *
C *   AM1    = MANHOLE AREA                                          *
C *   AM2    = OVER FLOW STORAGE AREA                                *
C *   VM     = MANHOLE VOLUME CAPACITY                               *
C *   WHT1   = WATER HEIGHT AT JUNCTION FROM PREVIOUS TIME STEP      *
C *   WHT    = WATER HEIGHT ABOVE MANHOLE FROM PREVIOUS TIME STEP    *
C *   WHT2   = WATER HEIGHT AT JUNCTION FOR PRESENT TIME STEP        *
C *   HA     = JUNCTION HEAD FOR STEADY STATE FLOW CONDITIONS        *
C *   QA     = STEADY STATE PIPE FLOW                                *
C *                                                                  *
C ********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION QJ(5)
      COMMON/B1/NOPIPE,NOJUNC,TINCR,VISC
      COMMON/B2/PNUM(10),JUNC1(10),JUNC2(10)
      COMMON/B4/JNUM(10),JELV(10),NOCP(10),CPNUM(10,5)
      COMMON/B5/KEY(10),MHT(10),MDIA1(10),MDIA2(10),MHED(10)
      COMMON/B6/QI(10),QIN(10),QPK(10),TLAG(10),TPK(10),TBAS(10),TIME
      COMMON/B8/Q(10),QA(10),QB(10),HA(10),HB(10)
      REAL*8 JELV,MHT,MDIA1,MDIA2,MHED,MLOSS
      INTEGER PNUM,CPNUM
      PI = 3.141592653589793D0
      DO 150 IJ = 1,NOJUNC
      IF (KEY(IJ) .EQ. 1) GO TO 140
      NOCPS = NOCP(IJ)
      DO 30 N = 1,NOCPS
      IF (JUNC1(CPNUM(IJ,N)) .EQ. IJ) GO TO 20
      QJ(N) = QA(CPNUM(IJ,N))
      GO TO 30
   20 QJ(N) = 0.0 - QA(CPNUM(IJ,N))
   30 CONTINUE
```

```
      IF (TIME .GT. TBAS(IJ)) GO TO 50
      IF (TIME .GT. TPK(IJ)) GO TO 40
      IF (TIME .LE. TLAG(IJ)) GO TO 50
      QI(IJ) = QIN(IJ) + ((TIME-TINCR)- TLAG(IJ)) * (QPK(IJ) - QIN(IJ))
     Q         / (TPK(IJ) - TLAG(IJ))
      GO TO 60
   40 QI(IJ) = QPK(IJ) + ((QIN(IJ) - QPK(IJ)) * ((TIME-TINCR)- TPK(IJ))
     Q         / (TBAS(IJ) - TPK(IJ)))
      GO TO 60
   50 QI(IJ) = QIN(IJ)
   60 QSUM = QI(IJ)
      NOCPS = NOCP(IJ)
      DO 70 N = 1,NOCPS
   70 QSUM = QSUM + QJ(N)
      TQSUM = QSUM * TINCR
      AM1 = PI * ((MDIA1(IJ) / 12.0) ** 2.0) / 4.0
      VM = AM1 * MHT(IJ)
      AM2 = PI * ((MDIA2(IJ)) ** 2.0) / 4.0
      WHT1 = HA(IJ) - JELV(IJ)
      IF (WHT1 .GT. MHT(IJ)) GO TO 90
      TQSUM = TQSUM + WHT1 * AM1
      IF (TQSUM .GT. VM) GO TO 100
   80 WHT2 = TQSUM / AM1
      GO TO 120
   90 WHT = WHT1 - MHT(IJ)
      TQSUM = TQSUM + MHT(IJ) * AM1 + WHT * AM2
      IF (TQSUM .LE. VM) GO TO 80
  100 WHT2 = MHT(IJ) + (TQSUM - VM) / AM2
  120 HA(IJ) = WHT2 + JELV(IJ)
      GO TO 150
  140 QI(IJ) = 0.0
  150 CONTINUE
      RETURN
      END
C
      SUBROUTINE FLOWA
C
C
C ********************************************************************
C *                                                                  *
C *  THIS SUBROUTINE CALCULATES THE STEADY STATE FLOWRATE (QA(N+1))  *
C *  IN EACH PIPE                                                    *
C *                                                                  *
C *                                                                  *
C *  NOPIPE = NUMBER OF PIPES IN THE SYSTEM                          *
C *  JUNC1  = JUNCTION NODE 1 FOR PIPE                               *
C *  JUNC2  = JUNCTION NODE 2 FOR PIPE                               *
C *  HA     = JUNCTION HEAD FOR STEADY STATE FLOW CONDTIONS          *
C *  QA     = STEADY STATE PIPE FLOW                                 *
C *  PLEN   = PIPE LENGTH                                            *
C *  PDIA   = PIPE DIAMETER                                          *
C *  PAREA  = PIPE AREA                                              *
C *  PRUF   = PIPE ROUGHNESS (EPSILON)                               *
C *  MLOSS  = SUM OF MINOR LOSSES                                    *
C *  PFLO   = INITIAL STEADY STATE PIPE FLOW                         *
C *  VISC   = SYSTEM KINEMATIC VISCOSITY                             *
C *  G      = ACCELERATION DUE TO GRAVITY                            *
C *  RE     = REYNOLDS NUMBER                                        *
C *  FF     = DARCY FRICTION FACTOR (JAIN EQUATION)                  *
C *  CCOEF  = HEAD LOSS COEFFICIENT                                  *
C *  DELTQA = CHANGE IN STEADY STATE FLOWRATE                        *
C *                                                                  *
C ********************************************************************
C
```

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION PAREA(10)
      COMMON/B1/NOPIPE,NOJUNC,TINCR,VISC
      COMMON/B2/PNUM(10),JUNC1(10),JUNC2(10)
      COMMON/B3/PLEN(10),PDIA(10),PRUF(10),MLOSS(10),PFLO(10)
      COMMON/B4/JNUM(10),JELV(10),NOCP(10),CPNUM(10,5)
      COMMON/B8/Q(10),QA(10),QB(10),HA(10),HB(10)
      REAL*8 JELV,MHT,MDIA1,MDIA2,MHED,MLOSS
      INTEGER PNUM,CPNUM
      G = 32.174D0
      PI = 3.141592653589793D0
      DO 1000 IP = 1,NOPIPE
      PAREA(IP) = PI * ((PDIA(IP)/12.0) **2.0) / 4.0
      DO 100 N = 1,20
      RE = DABS(QA(IP) * (PDIA(IP) / 12.0) / (PAREA(IP) * VISC))
      FF = 0.25D0 / ((DLOG10((PRUF(IP) / (3.7D0 * PDIA(IP) / 12.0) +
     Q     5.74D0 / RE ** 0.9D0))) ** 2)
      CCOEF = 8 * (FF * PLEN(IP) * 12.0 / PDIA(IP) + MLOSS(IP)) / (PI **
     Q       2 * (PDIA(IP) / 12.0) ** 4 * G)
      DELTQA = (HA(JUNC1(IP)) - HA(JUNC2(IP)) - CCOEF * QA(IP) * DABS(
     Q         QA(IP))) / (-2.0  * CCOEF * DABS(QA(IP)))
      QA(IP) = QA(IP) - DELTQA
  100 CONTINUE
      IF (DELTQA .LE. .00001) GO TO 1000
      STOP
 1000 CONTINUE
      RETURN
      END
C
      SUBROUTINE HEADB
C
C
C  *******************************************************************
C  *                                                                 *
C  *   THIS SUBROUTINE CALCULATES THE STEADY STATE HEAD (HA(N+1))    *
C  *   VALUES AT EACH JUNCTION (MANHOLE) OF THE SYSTEM               *
C  *                                                                 *
C  *                                                                 *
C  *   NOPIPE = NUMBER OF PIPES IN THE SYSTEM                        *
C  *   PNUM   = PIPE NUMBER                                          *
C  *   NOJUNC = NUMBER OF JUNCTIONS IN THE SYSTEM                    *
C  *   JNUM   = JUNCTION NODE NUMBER                                 *
C  *   JUNC1  = JUNCTION NODE 1 FOR PIPE                             *
C  *   JUNC2  = JUNCTION NODE 2 FOR PIPE                             *
C  *   JELV   = JUNCTION ELEVATION                                   *
C  *   CPNUM  = CONNECTING PIPE NUMBER TO EACH JUNCTION              *
C  *   NOCP   = NUMBER OF CONNECTING PIPES AT EACH JUNCTION          *
C  *   TIME   = TIME SINCE SIMULATION STARTED                        *
C  *   TINCR  = TIME INCREMENT (TIME STEP)                           *
C  *   QIN    = HYDROGRAPH INITIAL FLOW                              *
C  *   QPK    = HYDROGRAPH PEAK FLOW                                 *
C  *   TLAG   = HYDROGRAPH TIME LAG                                  *
C  *   TPK    = HYDROGRAPH TIME TO PEAK                              *
C  *   TBAS   = HYDROGRAPH TIME BASE                                 *
C  *   QI     = MANHOLE INFLOW                                       *
C  *   QSUM   = SUM OF FLOWS INTO JUNCTION                           *
C  *   TQSUM  = VOLUME OF WATER INTO JUNCTION                        *
C  *   MHT    = MANHOLE HEIGHT                                       *
C  *   MHED   = INITIAL MANHOLE HEAD                                 *
C  *   MDIA1  = MANHOLE DIAMETER                                     *
C  *   MDIA2  = MANHOLE SURFACE OVERFLOW DIAMETER                    *
C  *   AM1    = MANHOLE AREA                                         *
C  *   AM2    = OVER FLOW STORAGE AREA                               *
C  *   VM     = MANHOLE VOLUME CAPACITY                              *
```

```
C  *  WHT1    = WATER HEIGHT AT JUNCTION FROM PREVIOUS TIME STEP           *
C  *  WHT     = WATER HEIGHT ABOVE MANHOLE FROM PREVIOUS TIME STEP         *
C  *  WHT2    = WATER HEIGHT AT JUNCTION FOR PRESENT TIME STEP             *
C  *  HA      = JUNCTION HEAD FOR STEADY STATE FLOW CONDITIONS             *
C  *  QA      = STEADY STATE PIPE FLOW                                     *
C  *                                                                       *
C  *************************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION QJ(5)
      COMMON/B1/NOPIPE,NOJUNC,TINCR,VISC
      COMMON/B2/PNUM(10),JUNC1(10),JUNC2(10)
      COMMON/B4/JNUM(10),JELV(10),NOCP(10),CPNUM(10,5)
      COMMON/B5/KEY(10),MHT(10),MDIA1(10),MDIA2(10),MHED(10)
      COMMON/B6/QI(10),QIN(10),QPK(10),TLAG(10),TPK(10),TBAS(10),TIME
      COMMON/B8/Q(10),QA(10),QB(10),HA(10),HB(10)
      REAL*8 JELV,MHT,MDIA1,MDIA2,MHED,MLOSS
      INTEGER PNUM,CPNUM
      PI = 3.141592653589793D0
      DO 150 IJ = 1,NOJUNC
      IF (KEY(IJ) .EQ. 1) GO TO 140
      NOCPS = NOCP(IJ)
      DO 30 N = 1,NOCPS
      IF (JUNC1(CPNUM(IJ,N)) .EQ. IJ) GO TO 20
      QJ(N) = QB(CPNUM(IJ,N))
      GO TO 30
   20 QJ(N) = 0.0 - QB(CPNUM(IJ,N))
   30 CONTINUE
      IF (TIME .GT. TBAS(IJ)) GO TO 50
      IF (TIME .GT. TPK(IJ)) GO TO 40
      IF (TIME .LE. TLAG(IJ)) GO TO 50
      QI(IJ) = QIN(IJ) + ((TIME-TINCR)- TLAG(IJ)) * (QPK(IJ) - QIN(IJ))
     Q          / (TPK(IJ) - TLAG(IJ))
      GO TO 60
   40 QI(IJ) = QPK(IJ) + ((QIN(IJ) - QPK(IJ)) * ((TIME-TINCR) - TPK(IJ))
     Q          /(TBAS(IJ) - TPK(IJ)))
      GO TO 60
   50 QI(IJ) = QIN(IJ)
   60 QSUM = QI(IJ)
      NOCPS = NOCP(IJ)
      DO 70 N = 1,NOCPS
   70 QSUM = QSUM + QJ(N)
      TQSUM = QSUM * TINCR
      AM1 = PI * ((MDIA1(IJ) / 12.0) ** 2.0) / 4.0
      VM = AM1 * MHT(IJ)
      AM2 = PI * ((MDIA2(IJ)) ** 2.0) / 4.0
      WHT1 = HB(IJ) - JELV(IJ)
      IF (WHT1 .GT. MHT(IJ)) GO TO 90
      TQSUM = TQSUM + WHT1 * AM1
      IF (TQSUM .GT. VM) GO TO 100
   80 WHT2 = TQSUM / AM1
      GO TO 120
   90 WHT = WHT1 - MHT(IJ)
      TQSUM = TQSUM + MHT(IJ) * AM1 + WHT * AM2
      IF (TQSUM .LE. VM) GO TO 80
  100 WHT2 = MHT(IJ) + (TQSUM - VM) / AM2
  120 HB(IJ) = WHT2 + JELV(IJ)
      GO TO 150
  140 QI(IJ) = 0.0
  150 CONTINUE
      RETURN
      END
C
      SUBROUTINE FLOWB
```

```
C
C
C   ********************************************************************
C   *                                                                  *
C   *  THIS SUBROUTINE CALCULATES THE STEADY STATE FLOWRATE (QA(N+1))  *
C   *  IN EACH PIPE                                                    *
C   *                                                                  *
C   *                                                                  *
C   *  NOPIPE = NUMBER OF PIPES IN THE SYSTEM                          *
C   *  JUNC1  = JUNCTION NODE 1 FOR PIPE                               *
C   *  JUNC2  = JUNCTION NODE 2 FOR PIPE                               *
C   *  HA     = JUNCTION HEAD FOR STEADY STATE FLOW CONDTIONS          *
C   *  QA     = STEADY STATE PIPE FLOW     -                           *
C   *  PLEN   = PIPE LENGTH                                            *
C   *  PDIA   = PIPE DIAMETER                                          *
C   *  PAREA  = PIPE AREA                                              *
C   *  PRUF   = PIPE ROUGHNESS (EPSILON)                               *
C   *  MLOSS  = SUM OF MINOR LOSSES                                    *
C   *  PFLO   = INITIAL STEADY STATE PIPE FLOW                         *
C   *  VISC   = SYSTEM KINEMATIC VISCOSITY                             *
C   *  G      = ACCELERATION DUE TO GRAVITY                            *
C   *  RE     = REYNOLDS NUMBER                                        *
C   *  FF     = DARCY FRICTION FACTOR (JAIN EQUATION)                  *
C   *  CCOEF  = HEAD LOSS COEFFICIENT                                  *
C   *  DELTQA = CHANGE IN STEADY STATE FLOWRATE                        *
C   *                                                                  *
C   ********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION PAREA(95)
      COMMON/B1/NOPIPE,NOJUNC,TINCR,VISC
      COMMON/B2/PNUM(10),JUNC1(10),JUNC2(10)
      COMMON/B3/PLEN(10),PDIA(10),PRUF(10),MLOSS(10),PFLO(10)
      COMMON/B4/JNUM(10),JELV(10),NOCP(10),CPNUM(10,5)
      COMMON/B8/Q(10),QA(10),QB(10),HA(10),HB(10)
      REAL*8 JELV,MHT,MDIA1,MDIA2,MHED,MLOSS
      INTEGER PNUM,CPNUM
      G = 32.174D0
      PI = 3.141592653589793D0
      DO 1000 IP = 1,NOPIPE
      PAREA(IP) = PI * ((PDIA(IP)/12.0) **2.0) / 4.0
      RE = DABS(QB(IP) * (PDIA(IP) / 12.0) / (PAREA(IP) * VISC))
      FF = 0.25D0 / ((DLOG10((PRUF(IP) / (3.7D0 * PDIA(IP) / 12.0) +
     Q     5.74D0 / RE ** 0.9D0))) ** 2)
      CCOEF = 8 * (FF * PLEN(IP) * 12.0 / PDIA(IP) + MLOSS(IP)) / (PI **
     Q        2 * (PDIA(IP) / 12.0) ** 4 * G)
      DELTQB = TINCR * G * PI * (PDIA(IP) / 12.0) ** 2 * (HB(JUNC1(IP))
     Q         - HB(JUNC2(IP)) - CCOEF * QB(IP) * DABS(QB(IP))) / ( 4.0
     Q         * PLEN(IP))
      QB(IP) = QB(IP) + DELTQB
 1000 CONTINUE
      RETURN
      END
```

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