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ABSTRACT

MODELING OF OVERLAND FLOW BY THE DIFFUSION WAVE APPROACH

One of the major issues of present times, i.e. water quality degradation and a need for precise answers to transport of pollutants by overland flow, is addressed with special reference to the evaporator pits located adjacent to streams in the oil-producing regions of Eastern Kentucky. The practical shortcomings of the state-of-the-art kinematic wave are discussed and a new mathematical modeling approach for overland flows using the more comprehensive diffusion wave is attempted as the first step in solving this problem. A Fourier series representation of the solution to the diffusion wave is adopted and found to perform well. The physically justified boundary conditions for steep slopes is considered and both numerical and analytical schemes are developed. The zero-depth-gradient lower condition is used and found to be adequate. The steady state analysis for mild slopes is found to be informative and both analytical and numerical solutions are found. The effect of imposing transients on the steady state solution are considered. Finally the cases for which these techniques can be used are presented.

Descriptors: Model Studies, Overland Flow, Oil Fields, Oily Pollution, Flow Characteristics, Flow Pattern

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CHAPTER 1. INTRODUCTION

1.1 MOTIVATION FOR THE PROJECT:-

The mathematical modeling of overland flow is an important problem when considering issues such as water quality degradation and transport of pollutants from land surface to streams. Most of the evaporator pits in the oil producing regions of Eastern Kentucky are located on land adjacent to streams. The oil obtained from secondary oil recovery operations contains significant quantities of brines (or saltwater). These brines are separated from the oil and dumped into evaporator pits and are then transported by overland flow into the nearby streams. This phenomenon causes serious degradation of surface water quality in the streams in these regions.

In order to be able to develop surface water pollution abatement strategies, the time-space evolution of the pollutants needs to be determined. The effects of pollution sources can be precisely qualified only after such a determination. The transport of pollutants in surface waters is a complicated problem and a superficial empirical analysis will leave the solution with many uncertainties. This will lead to undermining of the reliability of any abatement strategies based upon such an analysis. A precise mathematical modeling approach is needed to provide accurate numerical answers to the time-space evolution of pollutants in surface waters.

The transport process of pollutants by surface waters can be separated into the overland flow and the channel flow phases. The first phase of transfer through overland flow carries the pollutant from land surface to neighbouring streams as in the evaporator pits of Eastern Kentucky's oil producing regions. In order to solve this problem one needs to know the flow depth h(x,t) and the discharge per unit width q(x,t) over the flow domain. This study will provide approximate analytical solutions to the hydraulic problem of flow using the diffusion wave approximation. The state-of-the-art approach to the overland flow modeling is the kinematic wave. The practical shortcomings of this method are discussed and the more comprehensive diffusion wave is used for the purposes of this study. The solutions obtained will provide the requisite depth and discharge values over the overland flow domain under realistic initial and end conditions.

The problem attempted in this report is of a very fundamental nature and therefore has application in many larger problems. It is a small element when considering catchment-stream problems. The equations of channel flow are of similar nature and the concepts developed during this study may be extended to solve the equations governing such flows. Thus it may be possible to obtain analytical (or semi-analytical) solutions for flood propagation in channels and new solutions to flows in channel networks. These networks may be looked upon as a collection of overland flow reaches and converging sections interlaced by channels. The problem under consideration is a simpler version of the two dimensional overland flow and thus needs to be solved before attempting the more difficult case.

1.2 FRAMEWORK OF THE REPORT:-

This report presents a new solution to the diffusion wave equation under some acceptable initial and end conditions. It has been presumed that the wave profile is made up of many components which when properly superimposed together sum up to the true wave form. A Fourier series representation to the solution has been found to be most appropriate for this purpose and has been adopted for a major part of this study.

The present chapter considers the justification of such an effort. Pollution transport, flood waves, channel networks, two-dimensional overland flows are some of the benefits to be derived from this solution. This chapter also states the objectives and aims of the project. A brief review of past work directly connected with the problem of interest has also been included. A critical appraisal of the methods adopted by previous researchers and their relative merits and demerits have been discussed.

The second section deals with the solution to the flow equation when applied to steep slopes. The associated end conditions and numerical and analytical solutions of the resulting system have been developed. A sine series solution has been found to be very effective numerically and many cases have been demonstrated to show its performance (Fig. 2.2). The analytical solution for this case reduces to an eigenvalue problem after effecting a simple Taylor series expansion. The resulting solution is found to be good near the steady state and a similar procedure adopted for the recession region has been found to give reasonable results (see Fig. 2.3).

The solution for mild slopes is dealt with in the third chapter. The solution is split up into two components; a steady state solution and transient solutions. The complete solution is obtained when sufficient number of transients are added on the steady state, this number being dictated by the accuracy desired by the user. The steady state numerical solutions are presented for the diffusion wave (Fig. 3.4). The effect of adding one term transient on to the steady state solution is considered for the zero-depth-gradient downstream boundary condition (Fig. 3.5). Analytical solutions for the steady state for this case are considered in the form of polynomials (Fig. 3.3). The numerical solution to the steady state for critical flow downstream condition is also presented (Fig. 3.4).

The last chapter presents a global picture of the report and states the conditions under which any particular approximation may be used. Important conclusions regarding the accuracy and justification of these solutions are presented. The consequences of this solution procedure are stated. Results obtained by other investigators have also been included in the figures for comparison purposes. References for these are given in the Appendix.

1.3 SURVEY OF PREVIOUS WORK :-

It is known from literature that the flow in open channels is governed by the gradually varied, unsteady,one dimensional shallow water equations known as Saint- Venant equations (see, for example, Vieira [1982]). These are given by equations (1.1) and (1.2). The flow is one dimensional in the x- direction, t is the time, h is the depth, u is the average velocity at (x,t) and q is the lateral inflow per unit area per unit time.

The continuity equation, then, for unit width of plane is:

$$\frac{\partial h}{\partial t} + U\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} = q \qquad (1.1)$$

and the momentum equation is:

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} + g\cos\theta\frac{\partial h}{\partial x} = g(\sin\theta - S_f) - \frac{qu}{h} \qquad (1.2)$$

where g is the acceleration due to gravity; θ the angle of the slope, assumed constant; and $\rho g h S_f$ = the frictional retarding force exerted by the plane on the water. S_f is usually defined by the Chezy equation:

$$S_f = \frac{u^2}{C^2 h} \qquad (1.3)$$

C being the Chezy roughness of the plane. In the case of overland flow , q is the rainfall plus any seepage less any infiltration from the ground .

Grace and Eagleson [1966] developed the continuity and momentum equations governing vertical two dimensional flow over a plane surface. All equations are expressed in vector form with components in the horizontal (x) and the vertical (η) directions as shown in figure (1.1).

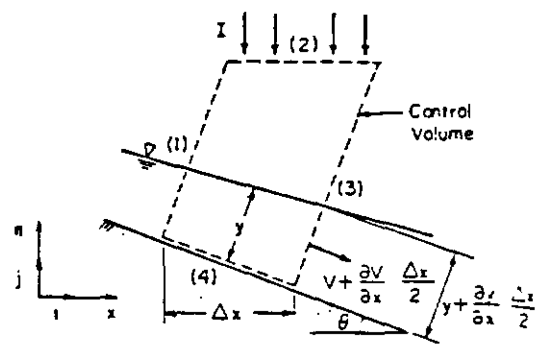

Fig. 1.1 OVERLAND FLOW

The momentum equation for flow through a control volume fixed in inertial space is:

$$\vec{F}_S + \iiint_{CV} \vec{B} \cdot \rho dv = \iint_{CS} \vec{V}(\rho \vec{V} \cdot \vec{dA}) + \frac{\partial}{\partial t} \iiint_{CV} \vec{V} \cdot (\rho dv) \qquad (1.4)$$

and the continuity equation for incompressible flows is:

$$\iint \vec{V}.\vec{dA} = -\frac{\partial}{\partial t} \iiint dv \qquad (1.5)$$

Grace and Eagleson [1966] assumed the velocity vector to be varying linearly with depth, parallel to channel bottom, the mean velocity vector being given by:

$$V = v(i\cos\theta + j\sin\theta) \qquad (1.6)$$

When the momentum equation for overland flow is expressed in its 'x' and ' η ' components , we have

$$[\gamma y\cos^2\theta + p_* \cos\theta + \beta\rho v^2\cos\theta]\frac{\partial y}{\partial x} + [2\beta\rho V y\cos\theta]\frac{\partial V}{\partial x}$$

$$+ (\rho y)\frac{\partial V}{\partial t} + y\frac{\partial p_*}{\partial x}\cos\theta = 2p_*\tan\theta + \gamma y\sin\theta - \tau \qquad (1.7a)$$

$$y\frac{\partial p_*}{\partial x}\sin\theta + p_*(2 + \frac{\partial y}{\partial x}\sin\theta) = -[\gamma y\sin\theta\cos\theta + \beta\rho v^2\sin\theta]\frac{\partial y}{\partial x}$$

$$- [2\beta\rho V y\sin\theta]\frac{\partial y}{\partial x} - [\rho y\tan\theta]\frac{\partial v}{\partial t} - (\rho V\tan\theta)\frac{\partial y}{\partial t}$$

$$- \tau\tan\theta - \gamma y\cos\theta + \gamma y \qquad (1.7b)$$

where, p_* is the average pressure intensity in excess of the hydrostatic value and is caused by momentum flux in the vertical direction, γ = specific weight of the fluid, β = momentum correction factor, θ = slope angle, ρ = fluid density, τ = shear stress and $\mathcal{I}$ = infiltration intensity. The other variables appearing in the equation are described in Fig. 1.1.

The continuity equation becomes:

$$\frac{\partial y}{\partial t}\sec\theta + \frac{\partial}{\partial x}(vy) = I - \mathcal{I} \qquad (1.8)$$

No known solutions exist for equations 1.7a,b and 1.8 in their present form. Approximate solutions to these have however been obtained after some oversimplifying assumptions and the discarding of many terms. The method of characteristics seems to have been popular with most researchers (see eg. Henderson and Wooding [1964]).

Grace and Eagleson have carried out a systematic study of the orders of magnitude of the terms appearing in equations 1.7a and 1.7b. This then provides physically justifiable reasons for discarding or retaining a term. Normalization of these dimensional equations was then carried out by defining dimensionless ratios linking all variables with appropriate reference variables. A similar analysis was carried out on the non dimensional equations. In the process of carrying out an 'Order of Magnitude Analysis' the following assumptions were made

1) Surface tension effects are negligible in both model and prototype.

2) Roll wave formation, if present is dynamically similar in model and prototype.

3) There is no infiltration in the model.

4) Depth to length ratio of the model, i.e. (Y/L) should be less than 0.003.

5) Slope of the bed surface of the model should be greater than 5 degrees.

6) For both prototype and model overland flow

$$C_f \tan\theta << 4.$$

where C_f is the non dimensional frictional coefficient.

7) The overland flow is two dimensional.

It may be noted that 1.7a and 1.7b are the most general forms of these equations. The expression for the dimensionless over-pressure term p_* may be obtained from the non dimensional form of the momentum equation in the η direction (obtained from 1.7b). From the 'Order of Magnitude Analysis' presented by Grace and Eagleson it follows that all terms regarding rainfall, infiltration and Y/L may be neglected in the expression for the over-pressure for the prototype in case of steep bed slopes. A similar analysis for the model gives an expression for the normalized over-pressure which is identical to the one obtained for the prototype. For reasonable modelling, the model slope should be greater than 5^{o} to include the frictional and gravitational effects. This expression for the over-pressure term may be substituted into the dimensionless momentum equation in the x direction obtained from 1.7a. Then for small slopes and for $\beta = 1$ (where β = momentum correction factor) the commonly used momentum equation may be obtained after some further simplification.

Finite difference solutions adopting various schemes for solving the differential equations were investigated. Woolhiser and Liggett [1967] considered the acceptable numerical methods which can be used in connection with shallow water equations primarily with overland flow applications. Their study provides suitable guidelines for choosing stable finite difference schemes. For small values of Θ, i.e. for mild slopes, (1.1) and (1.2) may be written as :

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + h\,\frac{\partial u}{\partial t} = q \qquad (1.9)$$

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} = g(S_0 - S_f) - \frac{qu}{h} \qquad (1.10)$$

where S_0 is the sine of the slope angle for small slopes. The shallow water equations can be treated more generally by adopting a dimensionless representation. Then

$$S_0 = \frac{V_0^2}{C^2 H_0} \qquad (1.11)$$

in which, H_0 = normal flow depth for flow $Q_0 = qL_0$ at the end of the reach under consideration (at $x = L_0$); V_0 is the normal velocity for $Q_0 = qL_0$ at $x = L_0$. When the flow in the reach of length L_0 comes in as lateral inflow q,

$$Q_0 = H_0 V_0 = qL_0 \qquad (1.12)$$

The quantities H_0, V_0 and L_0 are frequently used as normalizing constants and the following dimensionless variables are defined:

$$U_* = \frac{u}{V_0}\ ;\ h_* = \frac{h}{H_0}\ ;\ x_* = \frac{x}{L_0}\ ;\ t_* = t\frac{V_0}{L_0} \qquad (1.13)$$

Also we have,

$$F_0 = \frac{V_0}{\sqrt{gH_0}}\ ;\quad k = \frac{S_0 L_0}{F_0^2\ H_0} \qquad (1.14)$$

Then the dimensionless shallow water equations are:

$$\frac{\partial h_*}{\partial t_*} + U_*\frac{\partial h_*}{\partial x_*} + h_*\frac{\partial U_*}{\partial x_*} = R_* \qquad (1.15)$$

$$\frac{\partial U_*}{\partial t_*} + U_*\frac{\partial U_*}{\partial x_*} + \frac{1}{F_0^2}\frac{\partial h_*}{\partial x_*} = k\left(1 - \frac{U_*^2}{h_*}\right) - \frac{U_* R_*}{h_*} \qquad (1.16)$$

The dimensionless lateral inflow, R_*, is obtained by dividing the lateral inflow by the maximum rate, q_{max}. It is worth noting that the dimensionless time is related to the 'time of equilibrium', a t_* value of 1.0 being the time required for a fluid particle to traverse the reach under the normal flow conditions. The dimensionless flow equations have only two parameters F_0 and K (excluding R) instead of four as in the original equations. Fig. 1.2 shows the sketch defining the general one dimensional flow problem:

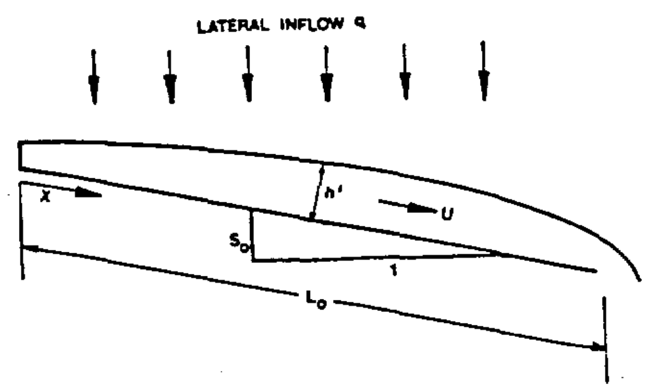

Fig. 1.2 DEFINITION SKETCH OF OVERLAND FLOW

In any finite difference scheme to solve equations (1.15) and (1.16), the partial derivatives are approximated by finite differences. Where non-linear equations are involved, these differencing techniques can be very complicated. Liggett and Woolhiser [1967] have tried to determine how well various finite difference schemes work for the overland flow problem.

As a numerical scheme, the characteristics method has some advantages as pointed by Liggett and Woolhiser. It is accurate because the characteristics trace the path of the disturbances. The characteristics have an adaptive property i.e. they tend to be closer together in areas of rapid change. The method of characteristics is also reasonably fast and for a given accuracy criteria it covers maximum ground on the x-t plane. Yet another advantage is that it does not have to face the 'starting problem'. The usual initial condition of dry surface often leads to singularities which create certain difficulties. However, it suffers from the chief disadvantage of not having a uniform mesh spacing. A special interpolation subroutine needs to be incorporated which consumes time of both man and machine, not to mention the extra large high speed memory space required to handle medium sized problems.

Explicit methods refer to those finite difference schemes where the results at any time step may be explicitly obtained using values from previous time steps. Unfortunately, in non-linear partial differential equations, precise stability criteria can rarely be found. It was commonly agreed among investigators that the Courant condition

$$\frac{\Delta t}{\Delta x}\left(|U| + C\right) \leq 1 \qquad (1.17)$$

is a necessary condition for stability of an explicit finite difference scheme. However it is by no means sufficient. In general explicit methods are unsatisfactory for even very approximate calculations.

Implicit Method is usually a safe method and involves solving a set of simultaneous equations for each row of points at every time step. Centered differencing is used to ensure stability. Newton's method is commonly used for solving the non-linear system of equations. The user is left with the onus of prescribing an initial guess. Therefore prior knowledge as to the behaviour of the solution must be known to the user. A 'double sweep method' which requires only a 2 by 2 matrix inversion and is very efficient has been suggested. This method was subsequently used by other investigators in their work (e.g. Morris [1980]). Liggett and Woolhiser [1967] have made a study into the various methods and presented them in a tabular form.

Amein [1968] considered the need for a fast (i.e. rapidly convergent) and accurate method for numerical solutions of unsteady flows. He evolved such a method consisting of a centered finite difference scheme and solving the resulting system of non-linear simultaneous equations by the generalized Newton's method.

To illustrate the validity of his method, Amein solved a problem originally considered by some other researchers. He has presented an excellent comparison with three different methods viz. storage routing equation, explicit method and method of characteristics. It was seen that the solutions of the problems obtained by the direct implicit method are in very good agreement with the best results (i.e. when smallest time steps are used) of the characteristics method and are quite close to the results obtained from the explicit method. He also observed that the linear system of simultaneous equations (arising from Newton's method) has very few non zero elements, and that these are clustered around the diagonal. This property of the matrix can be exploited for fast solutions.

It was soon realized that the complete Saint-Venant equations are too complex to be solved analytically. Hence, since the early sixties hydrologists have tried to obtain physically justifiable approximations which are easier to handle and operate. Lighthill and Whitham [1955] have considered a class of wave motions which are physically quite distinct from the classical wave motions encountered in dynamical systems. They stated that the kinematic waves possessed one wave velocity at each point because of the conservation law or the continuity equation:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \qquad (1.18)$$

where, q is the flow quantity passing a given point in unit time and k is the concentration (i.e. quantity per unit distance). Kinematic waves are non dispersive but they may change form due to non linearity (i.e. wave velocity, c, depends on q). Hence continuous wave forms may develop discontinuities due to faster waves overtaking slower ones. These are called shock waves. The properties of such shock waves have been described. A detailed treatment of flood movement in long rivers is then considered where kinematic waves play a leading role. It is the contention of the authors that the dynamic waves are rapidly attenuated and the main disturbances are then carried downstream by kinematic waves. It was found that if $(3/2)V_0$ and $v_0 + \sqrt{gh_0}$ are taken as typical wave velocities for kinematic and dynamic waves respectively, then F = 2 (where F is the Froude number $v_0/\sqrt{gh_0}$) is the value at which these velocities are approximately equal. It appears that kinematic conditions prevail and dynamic effects die out exponentially when F < 1 (subcritical flows). For F > 2, the approximate theory ceases to apply. For subcritical flow case the equations were further linearized and it was noticed that the complete solution contains both kinematic and dynamic wave fronts.

In 1964, Henderson and Wooding developed the kinematic wave approximation to the equations of overland flow on a plane. This involves significant simplification in the momentum equation where the friction slope is considered equal to the bed slope and all other terms are neglected. An analytical solution to the kinematic wave model for overland flow on a sloping plane was obtained by them. However their analytical solution was valid only for constant rainfall and constant infiltration in time and space. Further, they did not specify any lower boundary condition to the stream.

In a comprehensive treatment of overland flow on a plane surface, Woolhiser and Liggett [1967] determined an exact series solution to the dimensionless equations in Zone A (see Fig.1.3), the portion of the solution domain enclosed

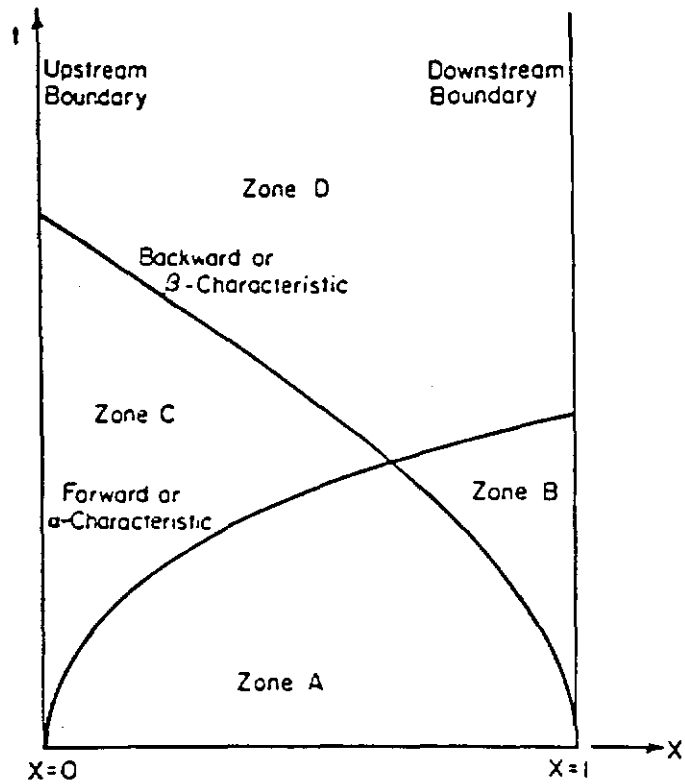

Fig. 1.3 ZONES IN THE (x,t) PLANE

by characteristics originating at x = 0, t = 0, and x = 1, t = 0 and the line t = 0. Zone A is the region of the x-t plane where the solution is dependent only upon the initial condition. Zone B is the region where the solution is dependent on the initial and the downstream boundary condition but not on the upstream boundary condition. Zone C is the region where the solution depends on the initial and upstream boundary condition. Zone D is the region where the solution is influenced by the initial condition and both the boundary conditions.

Woolhiser and Liggett [1967] considered an algebraic approximation to the series solution for the velocity of flow and the possible error involved in adopting such a technique. Normal depth normalizing was adopted for non dimensionalising the shallow water equations where the lateral inflow was represented by a step function. However for very small values of $F_0 = V_0 / \sqrt{gH_0}$ (Froude number for normal flow at $x_* = 1$), critical depth normalizing was used. They obtained most numerical solutions by the method of characteristics described earlier. For very large values of K ($= S_0 L_0 / H_0 F_0^2$) many difficulties arose with the numerical integration. The problem was reformulated and solved. It was noticed that the outflow hydrograph rises as $t_*^{3/2}$ until equilibrium is reached and then remains flat.

The difficulties in solving equations (1.15) and (1.16) for large K are in part associated with boundary conditions. The problem was clearly defined including all boundary conditions and approximations by Vieira [1983] and appears later in the text (see page 25). It was noticed that supercritical flows have numerical difficulties at upstream boundary while the numerical integration rapidly lost accuracy at the downstream end for subcritical flows. Woolhiser and Liggett have solved for the intersection point of the characteristics beginning at (0,0) and $x_* = 1$, $t_* = 0$ -the time of intersection of these characteristics can be found by setting $x_\alpha = x_\beta$ and was found to be dependant only on F_0 and R:

$$T_{\alpha\beta} = \left(3/4 \frac{F_0}{\sqrt{R}}\right)^{2/3} \qquad (1.19)$$

where α and β represent the forward and backward characteristics respectively (see Fig.1.4).

The variation of Zone A domain with changes in F_0 and S_0L_0 / H_0 were clearly demonstrated in a graph by Woolhiser and Liggett. The effect of the dimensionless parameters F_0 and K on the rising hydrographs were studied by them and the results compared with those obtained by previous investigators. They concluded that there is no single unique dimensionless rising hydrograph. Also as K becomes larger the hydrographs are independent of F_0 and approach the case for K = infinite. This case corresponds to the kinematic solution given by Henderson and Wooding [1964]. Woolhiser and Liggett concluded that the kinematic wave approximates most physical cases. No recession cases were considered till later by Morris [1978].

An exact analytical solution to the full Saint-Venant equations describing flow over a plane on a wide channel with general turbulent friction was obtained by Brutsaert [1968]. His solution is applicable only to Zone A. The lateral inflow was assumed uniform. He concluded that for very large slopes or for a large roughness of the plane, the solution reduces to the one obtained using the Kinematic Wave approximation.

Woolhiser [1974] considered the kinematic approximation and the effects of varying F_0 and K on the rising portion of the hydrographs. He stated that the kinematic model is a gross simplification of the momentum equation (1.2). In fact, the kinematic wave yields analytical solutions only for rainfall rate independent of any space or time variation and when very simple geometries are being taken into consideration. These solutions give an insight into the problem. Care must be taken while modelling flows using this approach since it is incapable of accounting for backwater effects. Woolhiser has further looked into the friction factors and hydraulic resistance offered to overland flow - especially those arising from boundary roughness. He presented a very informative table containing the resistance parameters for overland flows giving values of laminar resistance, K_0, Manning's coefficient η and Chezy's C for typical surfaces. It has been observed that the effect of impact of raindrops may have non-negligible effect on flows. Since incorporating this 'over-pressure' term directly into the equations governing flows makes them too complicated, their effects may be introduced by a judicious control over the friction factor coefficients. Hydrologic applications include modeling real life problems; the first step of which is to decide upon the model geometry. An example of geometric simplification, involving maintaining a close geometric similarity between prototype watershed and the idealized network or cascade of planes and channels that specify the model geometry has been considered by Woolhiser [1974]. He concluded that the Kinematic approximation is good enough for most urban and rural watersheds.

Morris [1978] considered a new implicit finite difference method for the solution of the equations (1.1) and (1.2). The method described uses central differencing for inside points and backward and forward differencing for upstream and downstream boundaries respectively. The double sweep algorithm adopted from Richtmyer's method was used (see also Liggett and Woolhiser [1967]). Morris introduced the zero-depth-gradient condition for subcritical flows. Using this downstream boundary condition she was able to obtain stable results for cases where the numerical schemes of Liggett and Woohiser [1967] failed. She demonstrated experimentally that the difference between the subcritical flow hydrographs obtained from critical flow boundary condition and zero-depth-gradient boundary condition decreases as F_0 and K increase. The results were compared with those obtained from the method of characteristics and variation in the hydrographs with changes in F_0 and K were also studied. It was noticed that the solutions could be improved by reducing Δx and Δt ; reduction in Δt being more effective. The recession curves for various values of F_0 and K were also provided as new material in this paper. The recession becomes more rapid (in terms of normalized time unit) as F_0 and K increase. Morris [1978] later discusses the validity of the zero-depth-gradient boundary condition and the range of the parameters for which it is applicable. It is also observed that the kinematic wave, though simpler, needs to be used with caution as its applicability is limited to rather steep slopes.

Beven [1979] described a more general kinematic channel network routing model which has a flow relationship that can accommodate both high and low flow characteristics. Velocities of flows in networks of steep and rough channels have been shown to vary non-linearly, both with increasing discharge and downstream distance in the network. He observed that while the overall velocities of the flow of water were markedly non-linear, they approached a nearly constant value at high discharges. He presented a generalized kinematic routing method with a more flexible approach to specification of velocity-discharge relationships so that they can incorporate the case of a non-linear channel system at low flows and a linear system at high flows into a single model. Routing experiments were carried out for a channel network system to compare i) a simple additive 'routing' method in which inputs to each reach at a given time are merely added to give catchment flow neglecting all channel effects, ii) a non-linear kinematic routing model based on reach measurements and usual flow relationships like the Chezy or Manning equation and iii) the generalized kinematic routing model described in the paper. He demonstrated that for low to medium discharges (ii) and (iii) lead to very similar hydrographs which were different from the additive method. He has then drawn conclusions regarding the use of a particular velocity-discharge relationship under various physical conditions. It was observed that for low discharges the non-linear relationship needs to be employed but that for high discharges there exists a nearly constant relation between the velocity and discharge.

Parlange et al. [1981] presented a more general analytical solution to the kinematic flow approximation by considering the excess rainfall (usually precipitation - infiltration) as an arbitrary function of time. This is a generalisation of the earlier solutions which were applicable when excess rainfall is constant for a finite time interval. They have shown that for the case of constant rainfall their solutions reduce to those obtained by previous investigators. However, their solution cannot incorporate spatially varying rainfall.

Hjelmfelt [1981] studied the influence of time distribution of rainfall on peak discharge using the kinematic approach. Most previous efforts had used constant rainfall intensity to estimate overland flow hydrographs. The peak discharge has been shown by him to be a function of surface length, total precipitation, storm duration and time to equilibrium for an equivalent rainfall of constant intensity. Time of concentration depends on watershed characteristics but also varies with rainfall intensity which in turn varies with time. Hjelmfelt observed that the thunderstorm time distribution of rainfall causes the watershed to respond more rapidly than the same amount of rainfall distributed uniformly over the same period. The time of concentration for thunderstorm rainfall was found to be less than that of rainfall of constant intensity. The time duration of rainfall has no influence on the peak discharge if this time is less than the time of concentration. Consider the ratio of the duration of rainfall and the time to equilibrium. If this ratio lies between 0.87 and 1.08, the uniform rainfall leads to a peak greater than the true thunderstorm rain distribution, possibly because slower time for total watershed contribution with uniform rainfall allows more water to accumulate. On the other hand storms of longer duration yield higher peaks when distributed according to thunderstorm rain pattern. This is because the intensity of the early portion of the thunderstorm distribution is greater than that used for the equivalent uniform rainfall and the peak discharge is determined by this early portion. Usual design procedures use rainfall of constant intensity for a duration equal to the time of equilibrium. Hjelmfelt's analysis indicates that the peak discharge at the design condition will be slightly greater for rainfall of constant intensity than for rainfall with thunderstorm time distribution. A correct value of time to equilibrium is essential for estimation of peak discharge which is however obtained by trial and error for time variant rainfall. The constant intensity approximation is valid for rainfall durations which are close to the time to equilibrium or less. As the relative duration increases, the approximation becomes less valid. Therefore an erroneous value of time to equilibrium can lead to significant under design.

Morris and Woolhiser [1980] examined the partial equilibrium hydrographs (i.e. hydrographs generated by lateral inflow that ceases before the steady state is reached) and recession hydrographs. Both the Diffusion and Kinematic models were compared with the Saint-Venant equations. An extra variable D_* (or normalized duration of rain) was also needed in partial equilibrium cases. In general, the kinematic hydrographs receded much faster. Unless the values of F_0 and K were very large, kinematic approximation did not do a very good job of replicating the original equations.

The diffusion approximation is especially good for small F_0 and large K. The dimensionless momentum equation becomes:

$$\frac{\partial h_*}{\partial x_*} \approx F_0^2 k \left(1 - \frac{U_*^2}{h_*}\right) \qquad (1.20)$$

and, substitution into the continuity equation gives:

$$\frac{\partial h_*}{\partial x_*} + \frac{\partial}{\partial x_*}\left(\left(h_*^{3/2}\left(1 - \frac{1}{F_0^2 k}\frac{\partial h_*}{\partial x_*}\right)\right)^{1/2}\right) = R_* \qquad (1.21)$$

The initial and boundary conditions are similar to those discussed by Vieira [1983] and appear later in the text(see page 25). Morris and Woolhiser [1980] used an implicit finite difference scheme to solve equation (1.21) and have compared the results with the Saint-Venant equations. The diffusion wave does a very good job of replicating the rising limbs of the partial equilibrium hydrographs. As F_0 tends to zero and K tends to infinite the diffusion equation approaches the full Saint-Venant equations. Morris and Woolhiser concluded that the complete Saint Venant equations or at least the diffusion equation must be used for overland flows on flat grassy slopes, even though K may be very large.

In his paper (referred to earlier in the this chapter) Vieira [1983] examines the validity of approximations for the Saint-Venant equations for overland flow. He started by considering the solution through characteristic curves and stated that, for subcritical flows, the solution domain may be divided into four zones viz. A,B,C,D, see e.g. Woolhiser and Liggett [1967]. He has then further analyzed the nature of the solution in each zone. The dimensionless equations (1.15) and (1.16) are re-written here for convenience in the notation used by Vieira:

$$\frac{\partial H}{\partial T} + U\frac{\partial H}{\partial X} + H\frac{\partial U}{\partial X} = 1 \qquad (1.22a)$$

$$\frac{\partial U}{\partial T} + U\frac{\partial U}{\partial X} + F_0^{-2}\left(\frac{\partial H}{\partial X}\right) = k\left(1 - \frac{U^2}{H}\right) - \frac{U}{H} \qquad (1.22b)$$

(where all symbols have usual significance) .

In Zone A, the solutions are dependent only on the initial condition of the dry channel. Hence equations (1.22a) and (1.22b) reduce to ordinary differential equations. The differential equations have been solved analytically using the initial conditions (1.23) by Brutsaert [1968]:

$$U(X,0) = H(X,0) = 0 \qquad (1.23)$$

For large K it was found that the solution is of the form

$$H = T \ , \quad U = T^{1/2} \qquad (1.24)$$

As T increases, the upper and lower boundary conditions begin to take effect. $T_{\alpha\beta}$ (see equation (1.19)) is the value where the Zone A solution ceases. In case of supercritical flows Zone A is bounded by line T = 0, X = 1 and the forward characteristic starting at the point X = 0, T = 0. The flow is influenced only by initial and upper boundary conditions.

In Zones B, C and D, Woolhiser and Liggett [1967] used characteristics method for solving the equations for subcritical flow using the following boundary conditions.

$$U(0,T) = 0 \qquad (1.25)$$

$$U(1,T) = H(1,T)^{1/2}/F_0 \qquad (1.26)$$

When the characteristics method was not suitable, they used an implicit finite difference scheme which is generally suitable for medium sized computers. However solutions for $F_0 < 0.4$ and for large values of K could not be obtained for critical flow downstream boundary condition (1.26). Morris [1979] used the zero-depth-gradient lower boundary condition i.e. $\partial H/\partial x = 0$. It was found that the difference in solutions using the two lower boundary conditions decreases for increasing F_0 and K.

The following thus are the end conditions for solving equations (1.22a) and (1.22b):

i) Initial Condition :- This is the dry slope or empty channel condition

$$H(X,0) = U(X,0) = 0 \qquad (1.23)$$

ii) Upper Boundary Condition :-

$$U(0,T) = 0 \qquad (1.27)$$

This condition influences both supercritical and subcritical flow outside Zone A.

iii) Lower Boundary Condition :-

a) The Critical Flow Condition

$$U(1,T) = H(1,T)^{1/2}/F_0 \qquad (1.26)$$

is usually created when slope ends at the steep bank of a river.

b) The Zero-Depth-Gradient Condition

$$\frac{\partial H(1,T)}{\partial X} = 0 \qquad (1.28)$$

For supercritical flows,

$$U(1,T) > H(1,T)^{1/2}/F_0 \qquad (1.29)$$

The kinematic wave approximation is valid for K > 50, and all the other terms in the momentum equation (1.19) are negligible when compared to the term $k(1 - U^2/H)$.

Equation (1.22b) reduces to

$$k(1-U^2H) = 0 \qquad (1.30)$$

Hence,

$$U^2 = H \qquad (1.31)$$

which, on substitution into equation (1.22b) yields the Kinematic Wave Equation:

$$\frac{\partial H}{\partial T} + \frac{\partial (H^{3/2})}{\partial X} = 1 \qquad (1.32)$$

Equation (1.32) may be solved using the initial condition (1.23) to obtain the solution given by equation (1.24). The rising portion of the hydrograph is given by

$$Q = HU = T^{3/2} \qquad (1.33)$$

There is only one hydrograph for this approximation as the equation (1.32) is independent of F_0 and K. The hydrograph rises as $T^{3/2}$ till equilibrium when T = 1, then Q = 1 for all values of T > 1.

The diffusion wave is very realistic for slow subcritical flows on mild slopes. For small F_0, (say F_0 << 0.8) and large K, the product $F_0{}^2K$ is not negligible and the momentum equation reduces to:

$$\frac{\partial H}{\partial X} = F_0^2K(1 - U^2/H) \qquad (1.34)$$

which when substituted into (1.22b) gives the diffusion wave equation :

$$\frac{\partial H}{\partial T} + \partial [H^{3/2} \{1 - (F_0^2 k)^{-1} \frac{\partial H}{\partial X} \}^{1/2}] / \partial X = 1 \qquad (1.35)$$

The upstream boundary condition (1.27) substituted into (1.34) gives:

$$\frac{\partial H(0,T)}{\partial X} = F_0^2 k \qquad (1.36)$$

Substituting the critical flow downstream boundary condition in equation (1.34) gives:

$$\frac{\partial H(1,T)}{\partial X} = k(F_0^2 - 1) \qquad (1.37)$$

Solutions of (1.35) under (1.36) and (1.37) for constant F_0 and K have been obtained by Morris and Woolhiser [1980] using an implicit finite difference scheme. The results are dependent on the parameters F_0 and K.

Substituting the zero-depth-gradient boundary condition (1.28) into (1.34) we have :

$$F_0^2 k(1-U^2/H) = 0 \qquad (1.38)$$

Vieira has used an implicit finite difference scheme to obtain solutions to (1.35) with this boundary condition. The solutions are dependent on $F_0^2 K$, and reduce to the kinematic case for $F_0^2 K$ tending to ∞ . Hence the kinematic wave is a special case of the diffusion approximation. Vieira [1983] also considers two other less popular approximations; namely the gravity wave approximation and the zero depth gradient approximation.

The solutions obtained under various approximations have then been compared for different boundary conditions. Vieira has then drawn figures to illustrate where these approximations are valid in terms of F_0 and K (see Fig. 1.4). The following conclusions may be drawn:

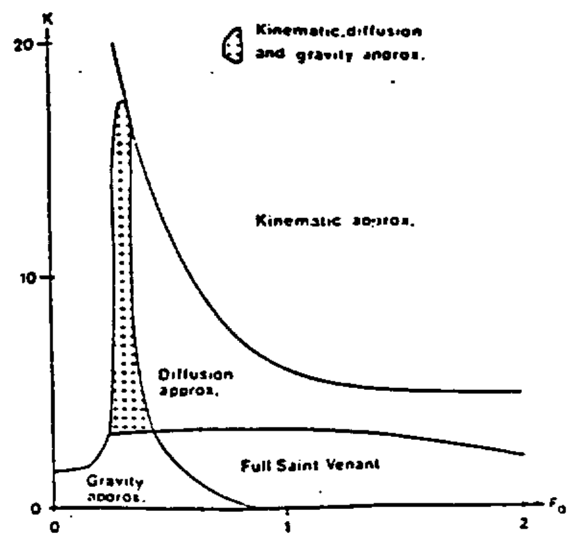

Fig. 1.4a

FOUR ZONES OF FO,K FIELD

CRITICAL-FLOW

LOWER BOUNDARY CONDITION

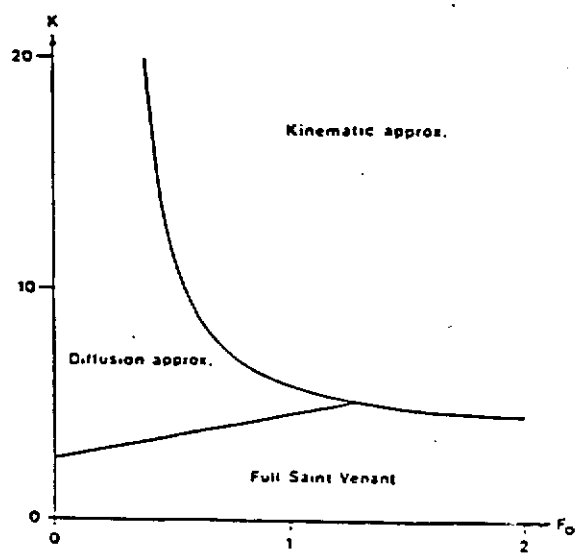

Fig. 1.4b

THREE ZONES OF FO,K FIELD

ZERO-DEPTH-GRADIENT

LOWER BOUNDARY CONDITION

i) The kinematic wave approximation is independent of F_0 and K and as such is valid only for large values of F_0 and K (say K > 20 , F_0 < 0.5). It is applicable for steep hill slopes and may be used with either of the downstream boundary conditions.

ii) On smooth urban slopes, values of K lie between 5 and 20; both kinematic and diffusion approximations are valid and either of them may be adopted. However for lower values of F_0, (say $F_0 < 0.5$) the diffusion wave is very good. It performs better than the kinematic wave for all cases.

iii) The zero-depth gradient condition should be used for higher values of F_0 and K. The Diffusion Wave results are sensitive to F_0^2K. It is interesting to note that the hydrographs are dependant on the product F_0^2K, and not on F_0 and K individually.

The study presented by Vieira has clearly stated the conditions under which any particular approximation may be valid - very useful information for the practising engineer. For reasons already mentioned, the full Saint-Venant equations are rarely used.

1.4 <u>PROJECT OBJECTIVES</u> :-

The overall objective of this report is to determine new analytical solutions to the hydraulics of the overland flow problem to provide depth and discharge values over the flow domain under physically justifiable initial and upstream-downstream boundary conditions. This is achieved through the following:

a) The hydraulic equations of gradually varied unsteady overland flow (Saint Venant Equations) are approximated by the diffusion wave which is more realistic than the kinematic wave. The diffusion wave model is developed for the overland flow plane and many cases of initial and end conditions of practical interest are considered.

b) The approximate analytical solutions for the diffusion wave model of overland flow are developed and then tested by

i) the comparison of these solutions with the corresponding numerical solutions of the diffusion wave and the full Saint Venant equations under various hydraulic, topographic, boundary and rainfall-infiltration conditions which are typical of Eastern Kentucky watersheds. The numerical kinematic wave solutions are also obtained to compare the performance of the diffusion wave against the kinematic wave.

ii) the comparison of these approximate analytical solutions of the diffusion wave to those numerical results of overland flow studies given in the literature.

CHAPTER 2 . SOLUTION FOR STEEP SLOPES

2.1 DESCRIPTION :-

As discussed in the previous chapter, analytical solutions to the full Saint-Venant equations are very difficult to obtain if not impossible. The diffusion wave approximation itself is not amenable to complete analytical solution without some further simplifications. It has been stated in the literature (e.g. Morris [1978], Vieira [1982]) that under the cases of large F_0 and K, the zero-depth-gradient downstream boundary condition is a justifiable substitute to the critical depth condition. The upstream boundary condition is one of zero influx. This implies that while there is no inflow at x = 0, it is possible to have a finite depth with time. This condition does not match with the initial condition of zero depth (i.e. the dry channel condition) and hence causes some uncertainty at the point x = 0 and t = 0. This difficulty may be avoided while using finite difference schemes by using backward differencing for the first few time steps and then switching to a more accurate Crank - Nicolson type scheme. In this way no decision needs to be made about values at the initial/boundary points at the corners.

However when steep slopes are being considered, there is not much scope for the water to accumulate at the top of the overland flow plane ($x = 0$) as it will flow away immediately. Hence the water depth here is practically negligible for all times $t > 0$. Under these circumstances it is reasonable to assume

$$h(0,t) = 0 \tag{2.1}$$

as the upstream boundary condition. This assumption is further strengthened when performing the steady state analysis for the diffusion wave. This chapter deals with a numerical series solution for the diffusion wave under the upstream boundary condition of (2.1) and zero-depth-gradient downstream boundary condition. The first two and three terms in the series are considered to demonstrate the efficacy of this method as compared to finite differencing or the method of characteristics. Semi analytical solutions are also obtained under the above mentioned conditions. These analytical solutions are compared to the complete Saint Venant equations (see Fig. 2.2,2.3).

2.2 THEORY OF THE NUMERICAL SERIES SOLUTION :-

The governing differential equation is

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \{h^{3/2} \left(1 - \varepsilon \frac{\partial h}{\partial x}\right)^{1/2}\} = q(x,t) \tag{2.2}$$

where,

$$\varepsilon = 1/(F_0^2 K) \tag{2.3}$$

subject to an initial condition of

$$h(x,0) = 0 \qquad (2.3a)$$

and the boundary conditions

$$h(0,t) = 0 \qquad (2.4a)$$

$$\frac{\partial h(1,t)}{\partial x} = 0 \qquad (2.4b)$$

Assume that the solution is of the form given by the following infinite sine series ,

$$\bar{h} = \sum_{n=1}^{\infty} h_n(t) \sin(\frac{2n-1}{2}) \pi x \qquad (2.5)$$

where $h_n(t)$, $n = 1,2,...,$, are all functions of time only and the x dependence comes from the sine terms. It may be further assumed that the first few terms of the series are the dominant ones and the contribution of terms for higher n is negligible. In fact, it was found that for most instances, two or three terms are quite adequate. Hence

$$\bar{h} = \sum_{n=1}^{N} h_n(t) \sin(\frac{2n-1}{2}) \pi x \qquad (2.6)$$

where $\bar{h}$ is an approximation to h and is equal to h for sufficiently large N.

A variant of Galerkin's method is now adopted. The interval of interest [0,1] is partitioned into N equal subintervals. The residual $\mathcal{R}$ is defined as

$$\mathcal{R} = \frac{\partial \bar{h}}{\partial t} + \frac{\partial}{\partial x} \{\bar{h}^{3/2} (1 - \varepsilon \frac{\partial \bar{h}}{\partial x})^{1/2} \} - q(x,t) \qquad (2.7)$$

From (2.6) we notice that there are N unknowns (i.e. $h_n(t)$, $n = 1,2,....,N$) and hence N independent differential equations are required. These are obtained by integrating the residual over each of the N subintervals and setting it to zero. The partition of the interval is given by

$$\Delta : 0 = \frac{0}{N} < \frac{1}{N} < \frac{2}{N} < \cdots < \frac{N-1}{N} < \frac{N}{N} = 1$$

and the N differential equations are obtained from

$$\int_{\frac{K-1}{N}}^{K/N} \mathcal{R} \, dx = 0 \quad , k = 1,2,\cdots,N \qquad (2.8)$$

$$\int_{\frac{K-1}{N}}^{K/N} \frac{\partial \bar{h}}{\partial t} dx + \int_{\frac{K-1}{N}}^{K/N} \frac{\partial}{\partial x} \{ \bar{h}^{3/2} (1 - \varepsilon \frac{\partial \bar{h}}{\partial x})^{1/2} \} dx - \int_{\frac{K-1}{N}}^{K/N} q(x,t) dx = 0$$

$$k = 1,2,\ldots,N \qquad (2.9)$$

The equation (2.9) may be stated in matrix form as

$$[R]\{\dot{h}\} + \{F\} - \{Q\} = \{0\} \qquad (2.10)$$

where the following results are obtained after simplification

$$[R] = r_{kn} = \frac{2}{(2n-1)\pi} [\cos \{ (\frac{2n-1}{2}) \pi (\frac{k-1}{N}) \} - \cos \{ (\frac{2n-1}{2}) \frac{\pi k}{N} \}]$$

$$\text{for } n,k = 1,2,\ldots\ldots\ldots,N \qquad (2.11)$$

$$\{F\} = f_k = \bar{h}^{3/2} (1 - \varepsilon \frac{\partial \bar{h}}{\partial x}) \Big|_{\frac{K-1}{N}}^{K/N}$$

$$k = 1,2,\ldots\ldots, N \qquad (2.12)$$

and , $\{Q\} = q_k = \int_{\frac{K-1}{N}}^{K/N} q(x,t) dx$

$$k = 1,2,\ldots\ldots, N \qquad (2.13)$$

and $\{\dot{h}\}$ is the vector denoting the derivatives of the components $\bar{h}_n$ ($n = 1,2,...,N$) with respect to time.

This then provides a system of ordinary differential equations. Note that the form of the solution chosen in equation (2.5) automatically satisfies the boundary conditions stated above. The initial condition is satisfied when solving the N initial value problems obtained from (2.10) under the following starting conditions

$$h_n(0) = 0 \ , \quad n = 1,2,......,N \qquad (2.14)$$

The system of ordinary differential equations given by (2.10) has been solved numerically by using the IMSL subroutine DVERK which employs a Runge-Kutta-Verner fifth and sixth order method. It has automatic error and step size control capabilities and was found to be reliable for this problem. For a large N, the matrix R has been inverted using another IMSL routine LINV2F which produces high accuracy solutions. The performance of this numerical technique has been shown in Fig. 2.2.

2.3 THEORY OF THE ANALYTICAL SOLUTION :-

The numerical solution discussed in the previous chapter provides sufficient insight to obtain analytical (or semi-analytical) solutions. These are dependent on the number of terms chosen in the series solution. The theory involved in using two terms is explained in this section and may be easily generalised to include cases for larger N.

It may be assumed that $h(x,t)$, and hence all the components $h_n(t)$, approach a constant asymptotic value for

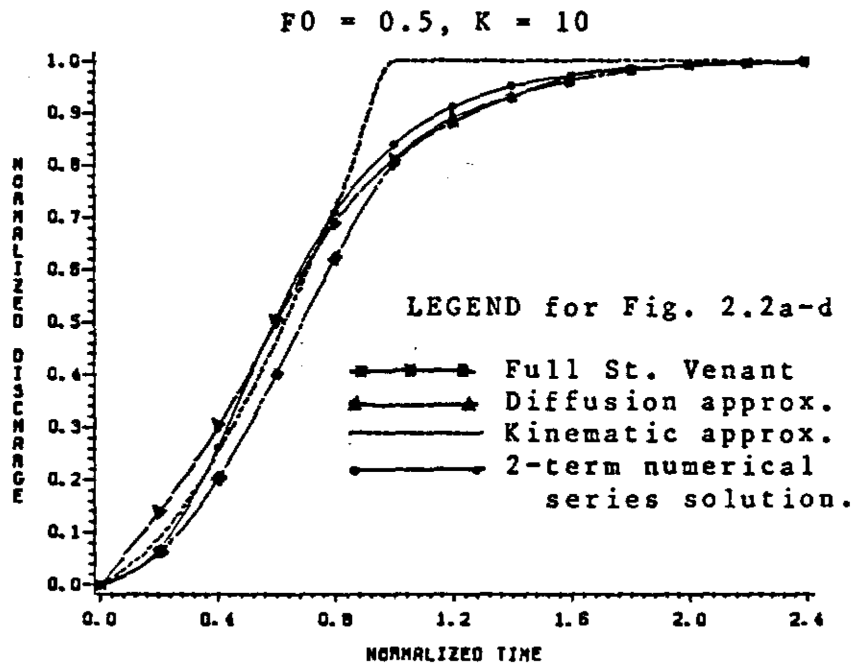

Fig. 2.2a RISING OUTFLOW HYDROGRAPH

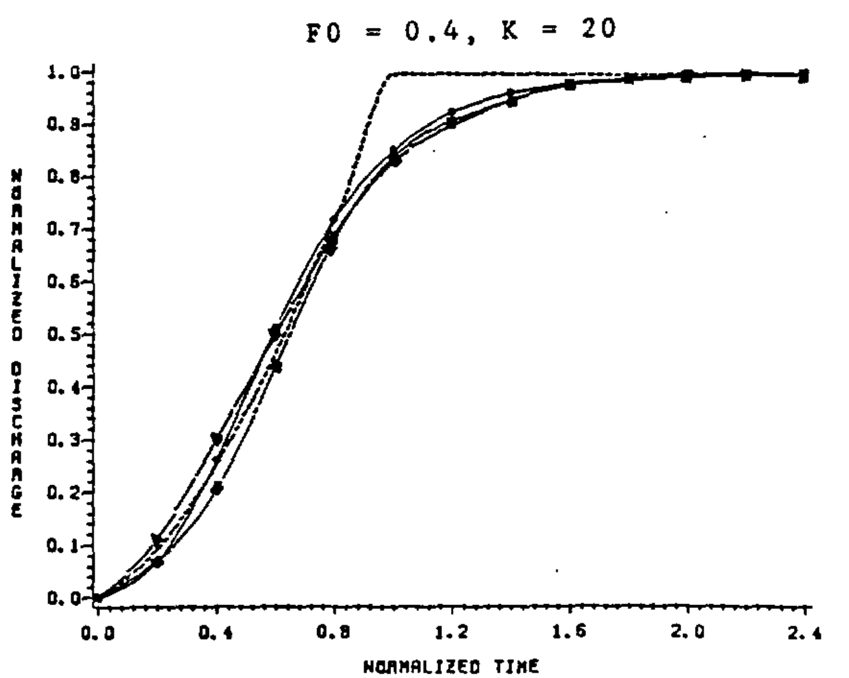

Fig. 2.2b RISING OUTFLOW HYDROGRAPH

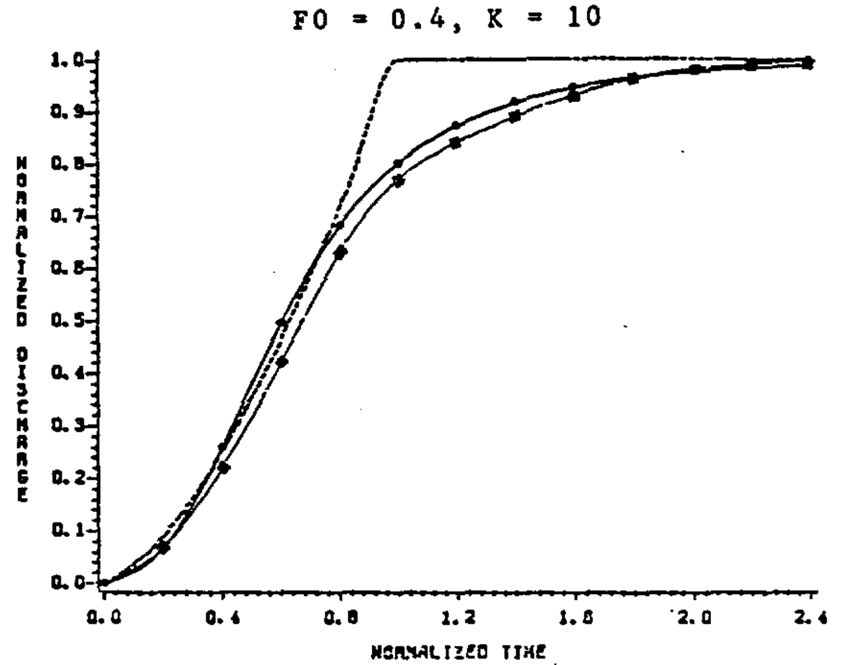

Fig. 2.2c RISING OUTFLOW HYDROGRAPH

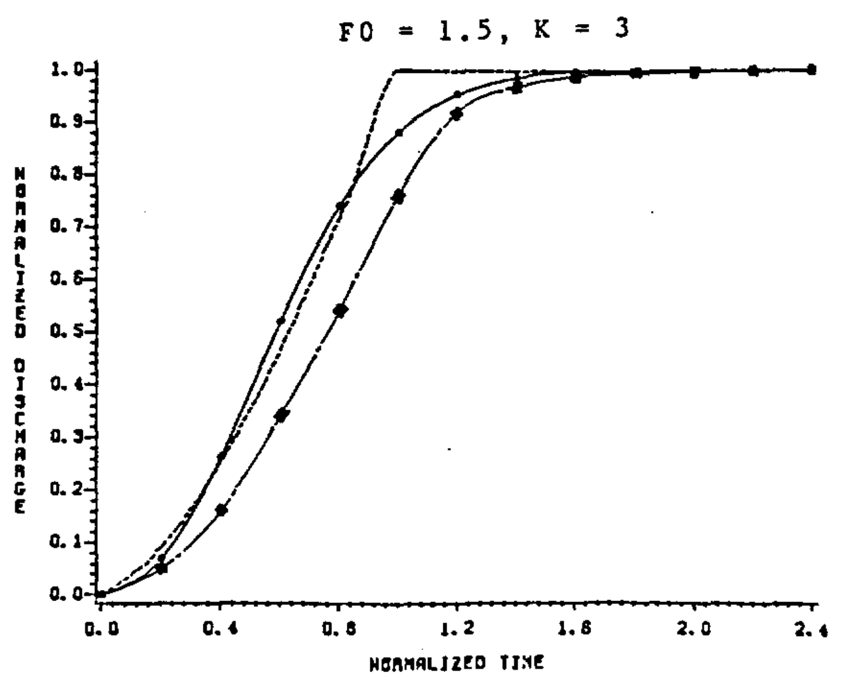

Fig. 2.2d RISING OUTFLOW HYDROGRAPH

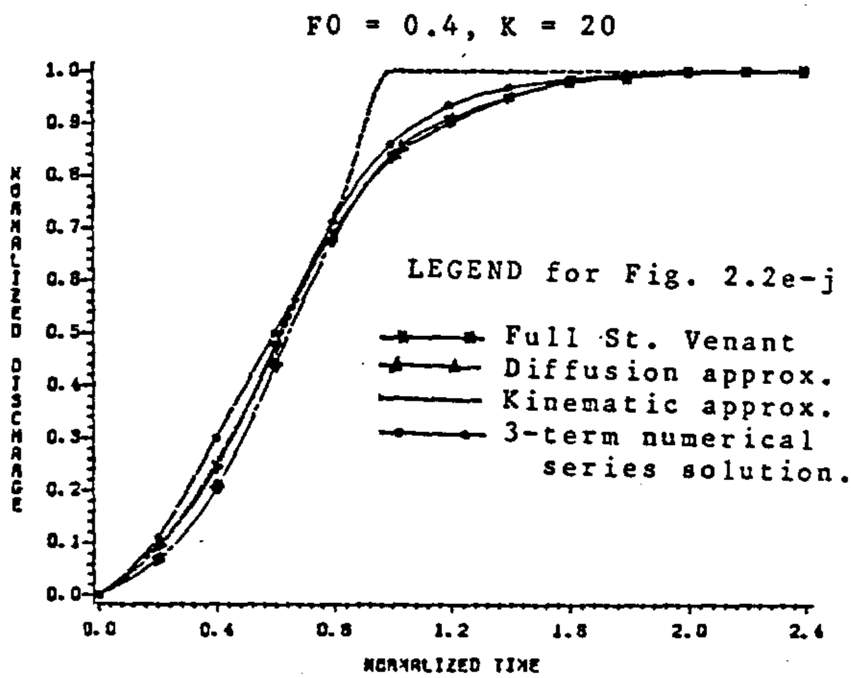

Fig. 2.2e RISING OUTFLOW HYDROGRAPH

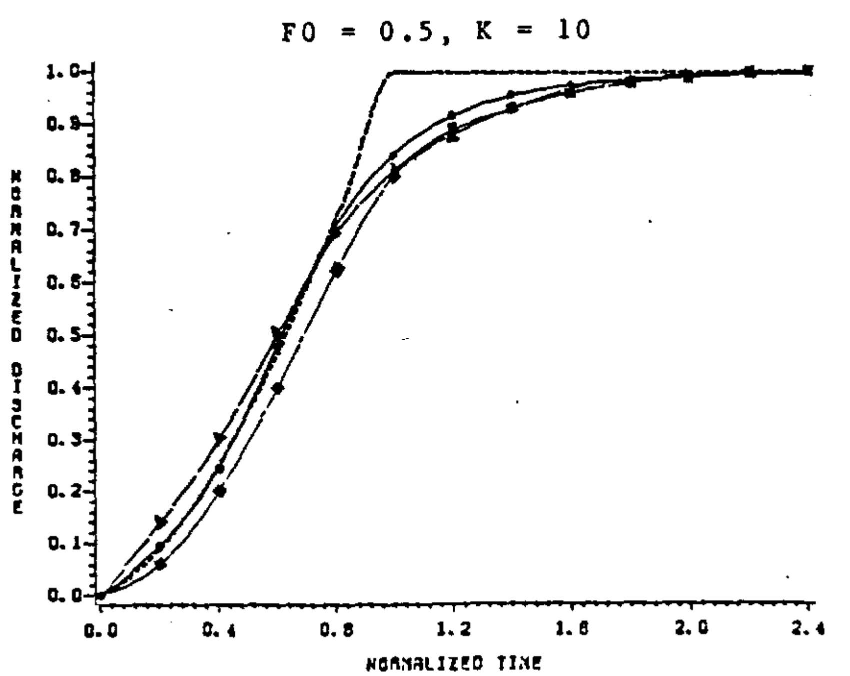

Fig. 2.2f RISING OUTFLOW HYDROGRAPH

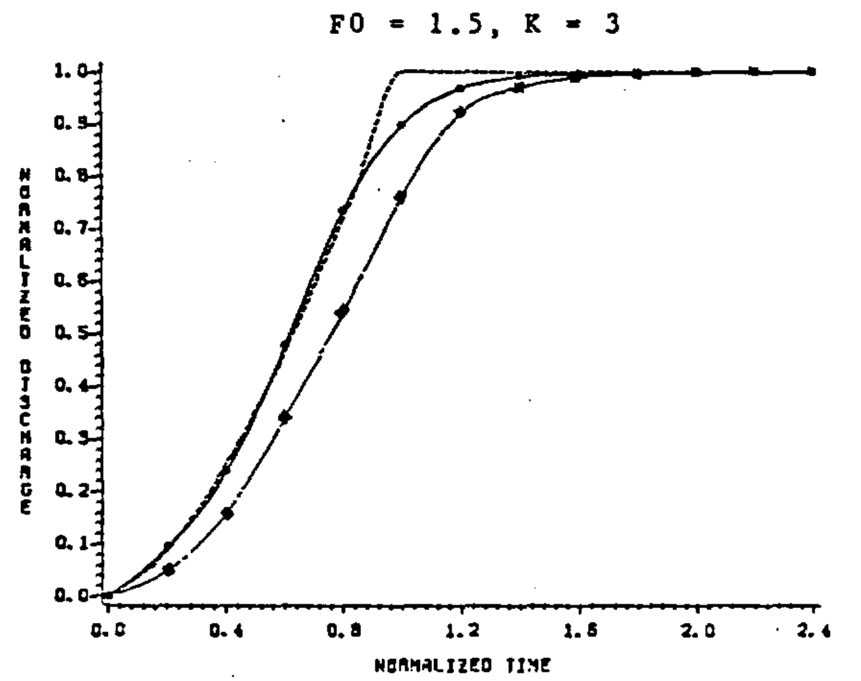

Fig. 2.2g RISING OUTFLOW HYDROGRAPH

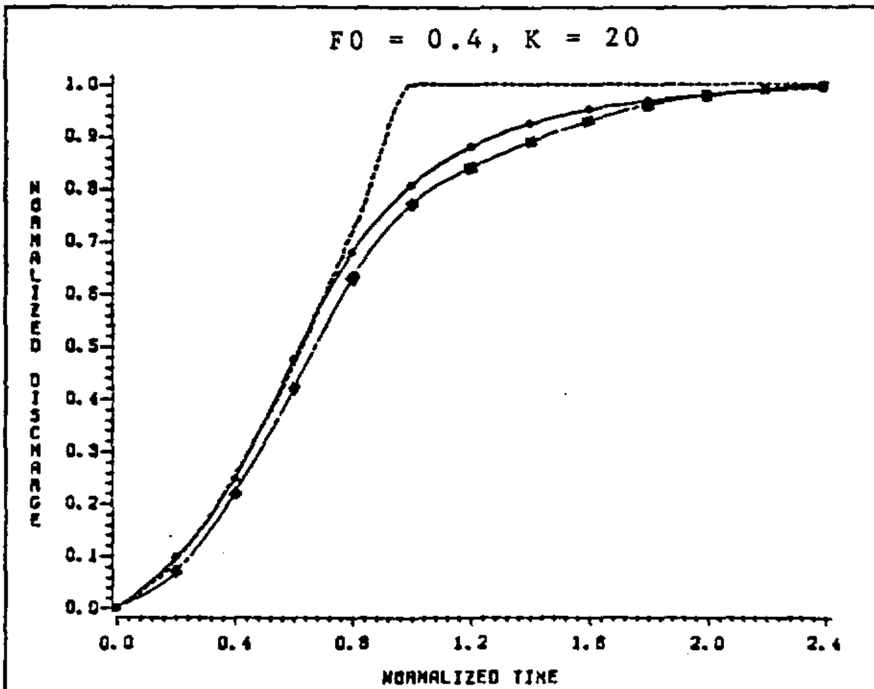

Fig. 2.2h RISING OUTFLOW HYDROGRAPH

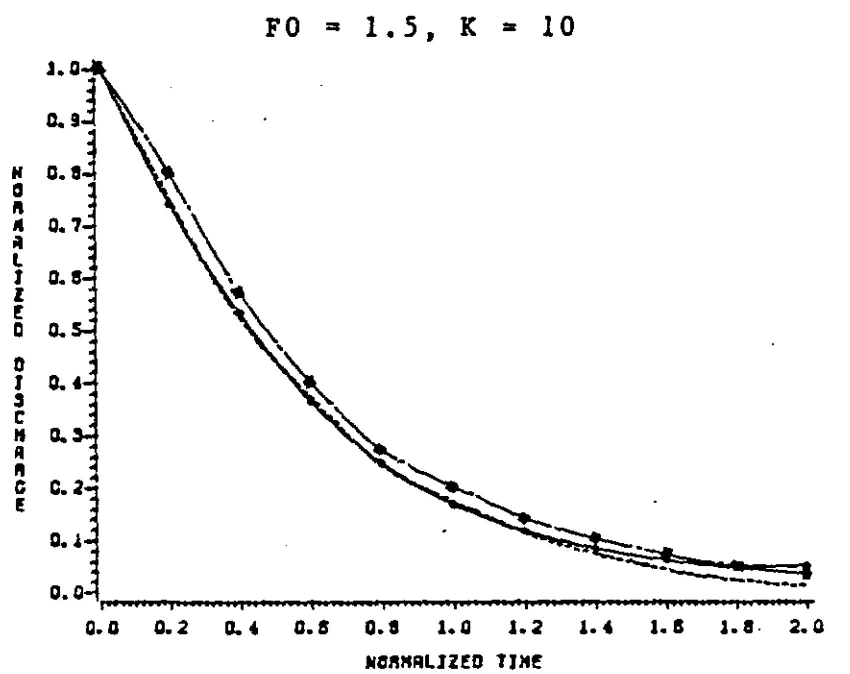

Fig. 2.2i RECESSION OUTFLOW HYDROGRAPH

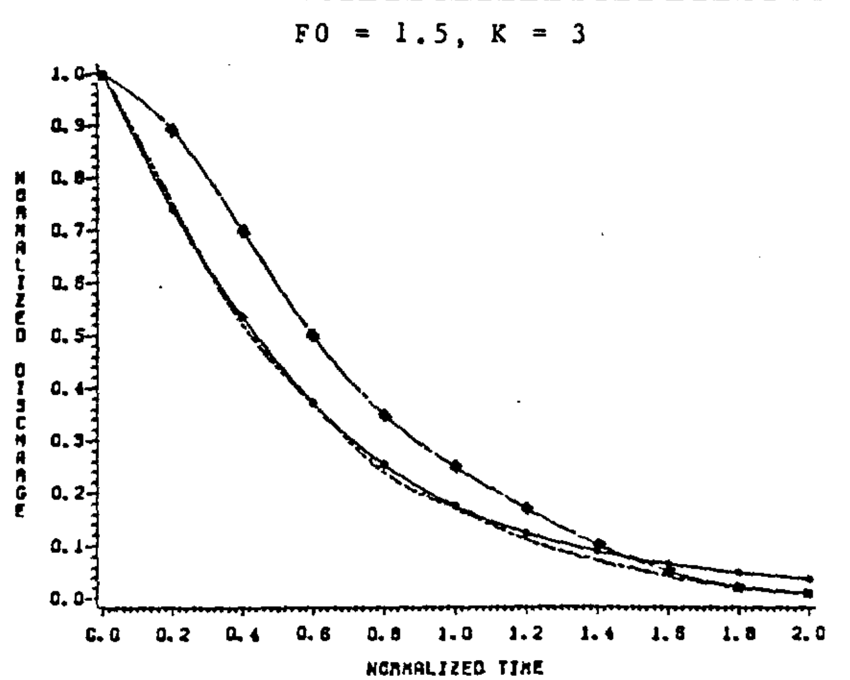

Fig. 2.2j RECESSION OUTFLOW HYDROGRAPH

time $t = \infty$ under continuous uniform rainfall. From equation (2.10) the equations for $N = 2$ emerge as

$$r_{11}\dot{h}_1 + r_{12}\dot{h}_2 = \frac{1}{2} - F = f_1(h_1,h_2) \qquad (2.15a)$$

$$r_{21}\dot{h}_1 + r_{22}\dot{h}_2 = \frac{1}{2} - (h_1 - h_2)^{3/2} + F = f_2(h_1,h_2) \qquad (2.15b)$$

where,

$$F = \left(\frac{1}{\sqrt{2}}\right)^{3/2} (h_1 + h_2)^{3/2} \{1 - \frac{\varepsilon\pi\sqrt{2}}{4} (h_1 - 3h_2)\}^{1/2} \qquad (2.16)$$

Expanding equations (2.15a) and (2.15b) we have the following equations

$$\bar{h}_1 - \bar{h}_2 = 1 \qquad (2.17)$$

and

$$(1 + 2\bar{h}_2)^3 \, [1 - \frac{\varepsilon\pi\sqrt{2}}{4} (1 - 2\bar{h}_2)] = \frac{1}{\sqrt{2}} \qquad (2.18)$$

Equation (2.18) is a non-linear equation in $\bar{h}_2$ but may be solved by using Newton's method. More terms would similarly lead to more equations, albeit non-linear. The solution to this set of equations gives the asymptotic steady states.

The right hand sides of equations (2.15a,b) may be expanded in Taylor series about the steady states as follows

$$f_1(h_1,h_2) = f_1(\bar{h}_1,\bar{h}_2) + \frac{\partial f_1}{\partial h_1}(\bar{h}_1,\bar{h}_2)\,(h_1 - \bar{h}_1) + \frac{\partial f_1}{\partial h_2}(\bar{h}_1,\bar{h}_2)\,(h_2 - \bar{h}_2) \qquad (2.19a)$$

$$f_2(h_1,h_2) = f_2(\bar{h}_1,\bar{h}_2) + \frac{\partial f_2}{\partial h_1}(\bar{h}_1,\bar{h}_2)\,(h_1 - \bar{h}_1) + \frac{\partial f_2}{\partial h_2}(\bar{h}_1,\bar{h}_2)\,(h_2 - \bar{h}_2) \qquad (2.19b)$$

From (2.15a,b), we have

$$f_1(\bar{h}_1,\bar{h}_2) = f_2(\bar{h}_1,\bar{h}_2) = 0 \qquad (2.20)$$

Equations (2.19a,b) may be written as

$$f_1(h_1,h_2) = a_{11}h_1 + a_{12}h_2 + b_1 \qquad (2.21a)$$

$$f_2(h_1,h_2) = a_{21}h_1 + a_{22}h_2 + b_2 \qquad (2.21b)$$

where,

$$[A] = [a_{ij}] = \frac{\partial f_i(\bar{h}_1,\bar{h}_2)}{\partial hj} \qquad i,j = 1,2 \qquad (2.22a)$$

$$\{K\} = b_k = -\{\frac{\partial f_k(\bar{h}_1,\bar{h}_2)}{\partial h_1}\bar{h}_1 + \frac{\partial f_k(\bar{h}_1,\bar{h}_2)}{\partial h_2}\bar{h}_2\} \quad k = 1,2$$

.... (2.22b)

Say $[B] = [R]^{-1}[A]$ and $\{D\} = [R]^{-1}\{K\}$. Substituting these relations in (2.21) we have

$$\{\dot{h}\} = [B].\{h\} + \{D\} \qquad (2.23)$$

where $\{h\} = \{h_k\}$, is the vector of solution components to be substituted in (2.6).

This is a linear system of differential equations with constant coefficients. Its solution may be expressed as follows

$$\{h(t)\} = e^{[B](t-t_0)}\{c\} + \int_{t_0}^{t} e^{[B](t-s)}\{D\}ds \qquad (2.24)$$

where the column vector $\{C\}$ is obtained from the initial condition at time $t = t_0$. Therefore

$$\{h(t_0)\} = \{C\} \qquad (2.25)$$

2.3.1 The Rising Hydrograph:-

For the rising portion of the hydrograph, $t_0 = 0$ and $\{c\} = \{0\}$. Hence the solution reduces to the form

$$\{h(t)\} = \int_0^t e^{[B](t-s)}\{D\}\,ds \qquad (2.26)$$

This expression has been evaluated by using eigenvalue theory (see Bronson [1973], chp. 29) to obtain the rising portion of the hydrographs.

2.3.2 The Recession Portion:-

Recession starts once the rainfall stops. Assuming t_r to be the duration of rain, the outflow hydrograph may developed as in section 2.3.1. At time $t = t_r$, we obtain the solution for $h(t_r)$ from equation (2.26). This serves as the initial condition for the recession portion. The procedure followed is similar to the one explained above except that the lateral inflow $q(x,t)$ is zero and the Taylor series expansion is about (h_{1*}, h_{2*}) which are the two components of the solution evaluated at the time when the lateral inflow ceases. Maintaining the same notation for convenience, the solution to the recession portion may be given as

$$\{h(t)\} = \exp([B](t-t_r))\{C\} + \exp([B]t).$$

$$\int_{t_r}^{t} \exp(-[B]s)ds.\{D\} \quad ..... \qquad (2.27)$$

where $\{C\} = \{h(t_r)\}$ obtained from equation (2.26).

This is the analytical solution to the individual components h_1 and h_2 for both rising and recession parts.

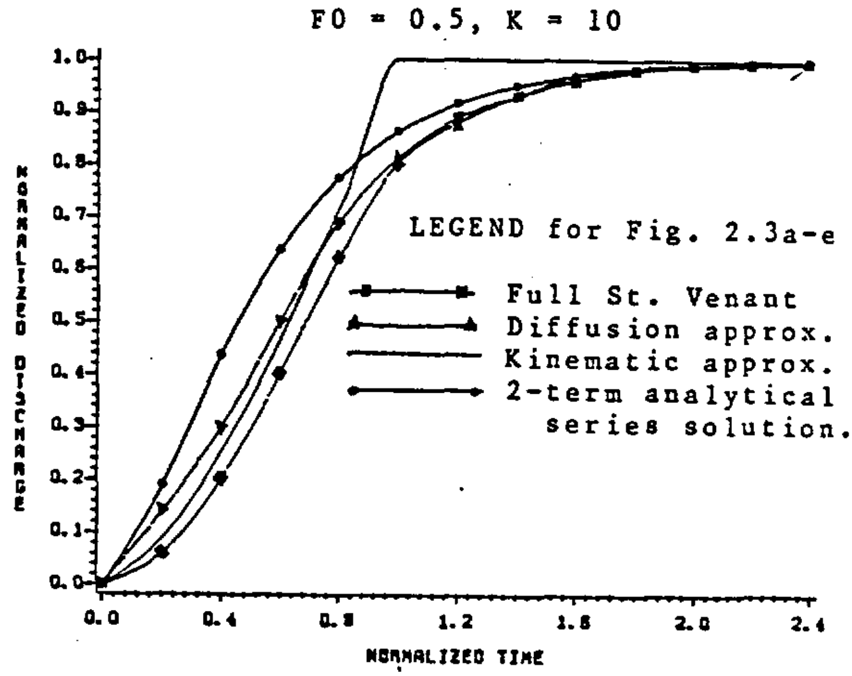

Fig. 2.3a RISING OUTFLOW HYDROGRAPH

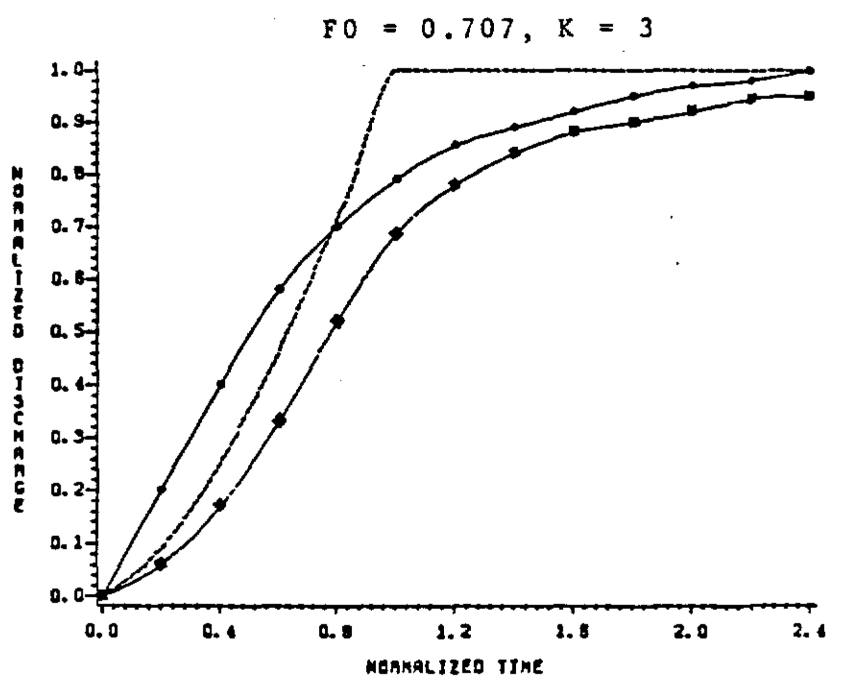

Fig. 2.3b RISING OUTFLOW HYDROGRAPH

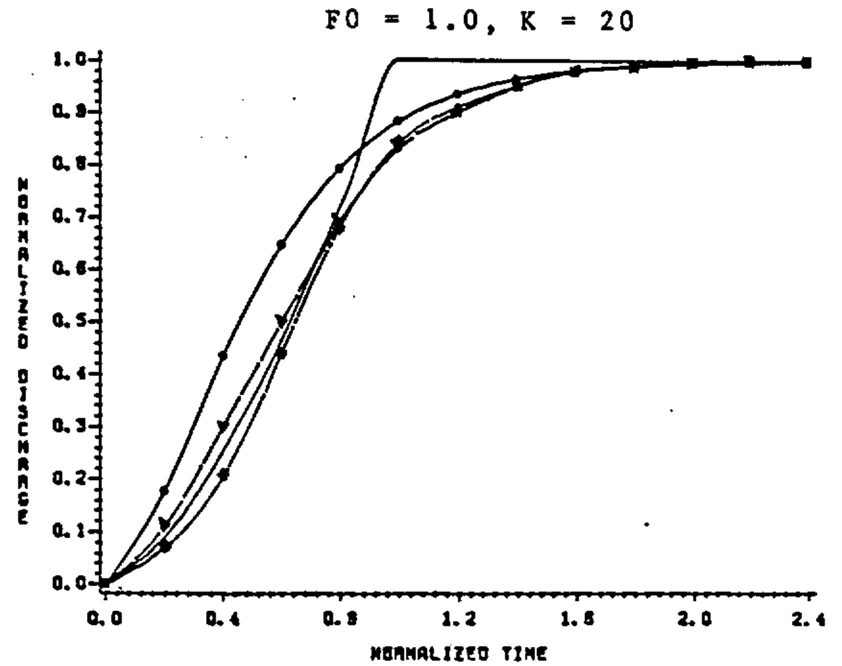

Fig. 2.3c RISING OUTFLOW HYDROGRAPH

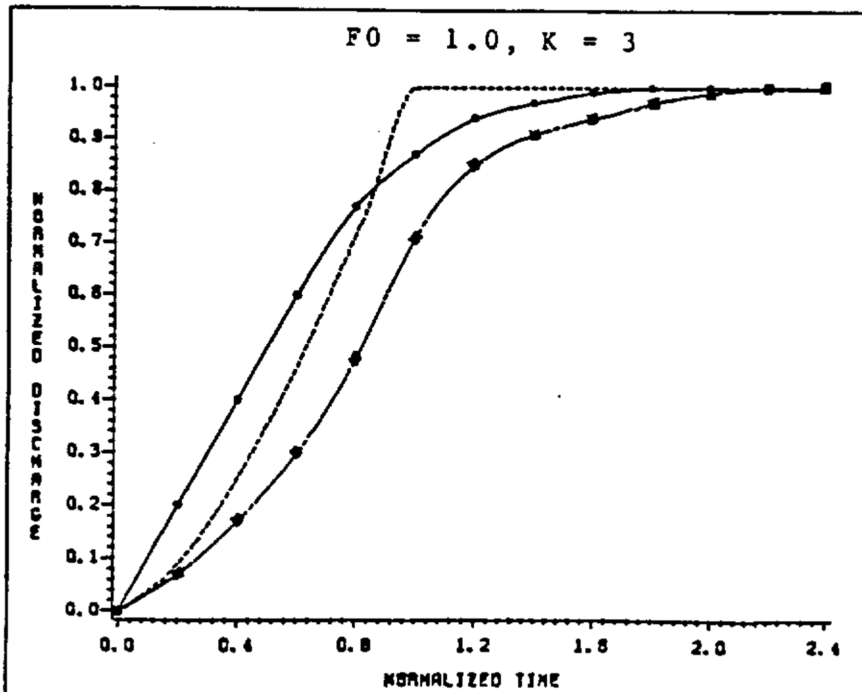

Fig. 2.3d RISING OUTFLOW HYDROGRAPH

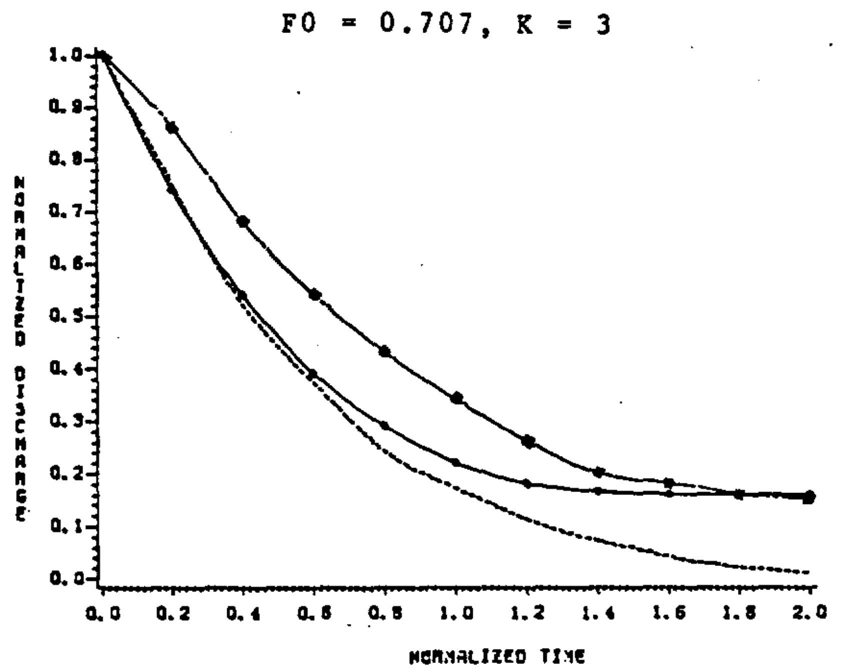

Fig. 2.3e RECESSION OUTFLOW HYDROGRAPH

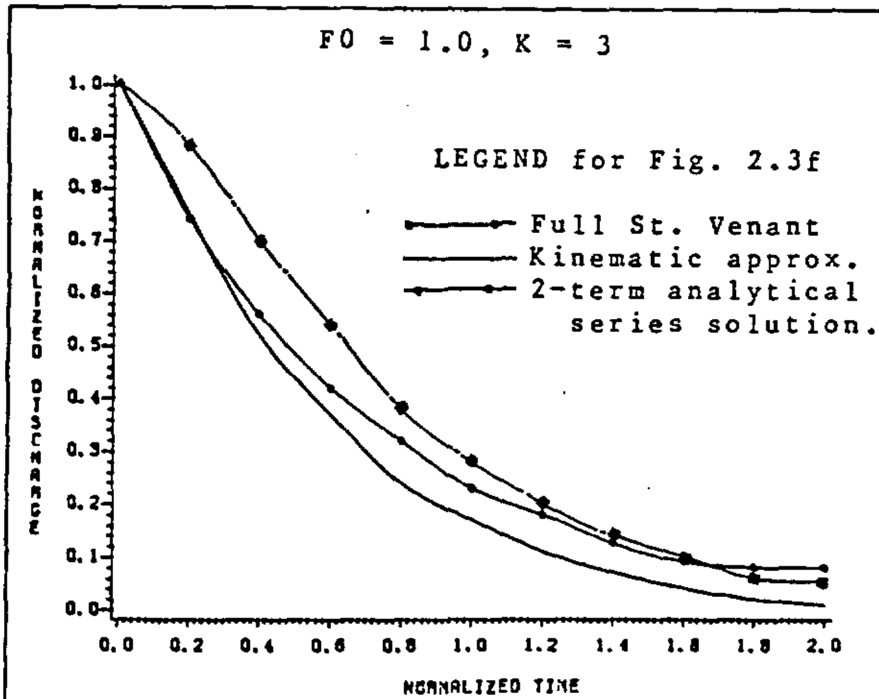

Fig. 2.3f RECESSION OUTFLOW HYDROGRAPH

The complete solution with space dependence may be obtained by substituting in equation (2.6). Fig. 2.3 shows the performance of this technique.

2.4 PERFORMANCE OF THE SERIES SOLUTION SCHEME :-

The numerical solution discussed in section 2.2 has many advantages in comparison to the method of characteristics and finite differencing. It is computationally more efficient and simpler to formulate. It does not use any variant of Newton's method for solving a set of non-linear equations as in the method of characteristics or finite differencing and hence saves a lot of time. It involves solving a system of initial value problems for which good software packages are available. The IMSL subroutine DVERK is found to be adequate for this problem. The program performs eight function evaluations per step and from these two estimates of the dependent variable are obtained based on the fifth and sixth order approximations. A comparison of these two estimates provides a basis for step size selection. The equations appearing in the analysis are simple in form and structure. The level of accuracy may be increased by increasing the number of terms considered in the sine series.

The major setback of the analytical solution seems to be the solution of a system of non linear equations to obtain the steady state values for each case of F_0, K and rainfall intensity. This can be avoided by using TABLE 2.1 which presents the steady state values for two term sine series solutions under a constant and uniform dimensionless lateral inflow of 1.0. Intermediate values may be obtained through a similar procedure (Section 2.3).

The numerical series solution for the case of N = 2 performs better than both the method of characteristics and finite differencing. The solution obtained on considering three terms in the series is better than the two term approximation and is practically coincident to the solution obtained from the numerical solution to the Saint Venant equations. This means that convergence is very fast and very few terms are required for most practical cases. This is perhaps the most desirable feature of this new method.

This technique also provides valuable insight into the problem and provides guidelines for developing an analytical solution for both rising and recession portions. The two term analytical solution overestimates the outflow hydrographs in the initial region. However as time increases and the steady state is approached, the solution becomes very good. This is expected since, for the rising portion of the hydrograph, the right hand sides of equations (2.15a,b) are expanded in Taylor series about the steady state. The truncated series is therefore a good approximation in the neighbourhood of the steady state but looses accuracy as it moves further away in time. The overall shape of the profile is similar to the numerical Saint Venant solution. The recession portion has typical exponential behaviour and is

TABLE 2.1

Table showing steady state values for two-term analytical solution

K	FO = 0.25 $\bar{h}_1$	FO = 0.25 $\bar{h}_2$	FO = 0.50 $\bar{h}_1$	FO = 0.50 $\bar{h}_2$
2	0.3350	-0.6650	1.3121	0.3121
4	0.2914	-0.7086	1.1777	0.1777
6	1.3550	0.3550	1.1004	0.1004
8	1.3121	0.3121	1.0579	0.0579
10	1.2726	0.2726	1.0327	0.0327
12	1.2369	0.2369	1.0164	0.0164
14	1.2053	0.2053	1.0051	0.0051
16	1.1777	0.1777	0.9969	-0.0031
18	1.1538	0.1538	0.9906	-0.0094
20	1.1332	0.1332	0.9857	-0.0143
22	1.1156	0.1156	0.9817	-0.0183
24	1.1004	0.1004	0.9784	-0.0216
26	1.0874	0.0874	0.9757	-0.0243
28	1.0762	0.0762	0.9734	-0.0266
30	1.0664	0.0664	0.9715	-0.0285
32	1.0579	0.0579	0.9698	-0.0302
34	1.0504	0.0504	0.9683	-0.0317
36	1.0438	0.0438	0.9669	-0.0331
38	1.0379	0.0379	0.9657	-0.0343
40	1.0327	0.0327	0.9646	-0.0354
42	1.0280	0.0280	0.9637	-0.0363
44	1.0238	0.0238	0.9629	-0.0371

TABLE 2.1 (continued)

K	F0 = 0.75		F0 = 1.00	
	$\bar{h}_1$	$\bar{h}2$	$\bar{h}1$	$\bar{h}2$
2	1.1538	0.1538	1.0579	0.0579
4	1.0438	0.0438	0.9969	-0.0031
6	1.0076	0.0076	0.9784	-0.0216
8	0.9906	-0.0094	0.9698	-0.0302
10	0.9808	-0.0192	0.9646	-0.0354
12	0.9746	-0.0254	0.9613	-0.0387
14	0.9701	-0.0299	0.9590	-0.0410
16	0.9669	-0.0331	0.9573	-0.0427
18	0.9644	-0.0356	0.9559	-0.0441
20	0.9624	-0.0376	0.9548	-0.0452
22	0.9608	-0.0392	0.9540	-0.0460
24	0.9595	-0.0405	0.9533	-0.0467
26	0.9584	-0.0416	0.9526	-0.0474
28	0.9574	-0.0426	0.9521	-0.0479
30	0.9566	-0.0434	0.9516	-0.0484
32	0.9559	-0.0441	0.9513	-0.0487
34	0.9553	-0.0447	0.9509	-0.0491
36	0.9547	-0.0453	0.9506	-0.0494
38	0.9542	-0.0458	0.9503	-0.0497
40	0.9537	-0.0463	0.9501	-0.0499
42	0.9534	-0.0466	0.9498	-0.0502
44	0.9530	-0.0470	0.9496	-0.0504
46	0.9527	-0.0473	0.9494	-0.0506

reasonably close to the corresponding numerical results.

CHAPTER 3. ANALYSIS OF STEADY STATE

3.1 DESCRIPTION :-

One of the ways of tackling highly non linear time and space dependent partial differential equations is to assume the solution to be composed of two parts. The first part consists of solving the problem assuming steady state conditions exist. This is physically justifiable for the problem under consideration since if the rain duration is infinite (practically speaking t_r greater than time of concentration of the reach) a steady state is achieved. The problem reduces to an ordinary non linear differential equation for the one dimensional overland flow. The second part of the solution process involves finding transient solutions which when superimposed on the steady state solution, yield the complete solution.

The steady state solution also provides a better understanding of the nature of the water profile. It may, for example, provide information on the cases when zero depth upstream boundary condition may be used instead of the zero influx condition. This important aspect of the solution of the process has received very little attention in the literature. An efficient method for numerically evaluating the steady state profiles and analytical approximations for some of the cases are developed in the following sections.

3.2 THE STEADY STATE THEORY :-

The steady state diffusion equation is given by

$$\frac{d}{dx}\{\bar{h}^{3/2}(1 - \varepsilon\frac{d\bar{h}}{dx})^{1/2}\} = \bar{q}(x) \qquad (3.1)$$

where, $\bar{h}(x)$ is the steady state solution,

$$\lim_{t\to\infty} q(x,t) = \bar{q}(x) \qquad (3.2a)$$

$$\lim_{t\to\infty} h(x,t) = \bar{h}(x) \qquad (3.2b)$$

The steady state solution $\bar{h}(x)$ is such that it satisfies the boundary conditions for the complete solution h(x,t). Therefore the initial condition is satisfied by the transient solution. Hence

$$\frac{d\bar{h}(0)}{dx} = a \qquad (3.3a)$$

$$\frac{d\bar{h}(1)}{dx} = b \qquad (3.3b)$$

where $a = F_0{}^2K$ and $b = K(F_0{}^2-1)$ for critical depth downstream boundary condition and b = 0 when zero-depth-gradient downstream boundary condition is used.

Equations (3.1) and (3.3) constitute a non linear two point boundary value problem which is rather difficult to solve. Integrating equation (3.1) over the interval [0,1], we obtain

$$\bar{h}(1)^{3/2}(1 - \varepsilon\frac{d\bar{h}(1)}{dx})^{1/2} - \bar{h}(0)^{3/2}(1 - \varepsilon\frac{d\bar{h}(0)}{dx})^{1/2} = \int_0^1 \bar{q}(x)dx \qquad (3.4)$$

Substituting the boundary values from (3.3), we have

$$\bar{h}(1)^{3/2}(1 - \varepsilon b)^{1/2} - \bar{h}(0)^{3/2}(1 - \varepsilon a)^{1/2} = \int_0^1 \bar{q}(x)dx \qquad (3.5)$$

which reduces after simplification to

$$\bar{h}(1) = \{(1 - \varepsilon b)^{-1/2} \int_0^1 \bar{q}(x)dx\}^{2/3} \qquad (3.6)$$

This provides an analytical expression for the steady state ordinate at x = 1. For constant and uniform rain q(x) = 1, and

$$\bar{h}(1) = (1 - \varepsilon b)^{-1/3} \qquad (3.7)$$

For critical depth lower boundary condition, we have from equation (3.7)

$$\bar{h}(1) = F_0 2/3 \qquad (3.8)$$

and the corresponding expression for the zero-depth-gradient downstream condition is

$$\bar{h}(1) = 1 \qquad (3.9)$$

It may also be noticed that for $F_0 = 1$, b is identically zero for all values of K. Under this condition the steady state profile for either lower boundary condition is the same.

3.3 THE NUMERICAL STEADY STATE SOLUTION :-

Integrating equation (3.1) over [0,x] we have

$$\bar{h}(x)^{3/2}(1 - \varepsilon\frac{d\bar{h}}{dx})^{1/2} = \int_0^x \bar{q}(x)dx \qquad (3.10)$$

Under conditions of constant rainfall, equation (3.10) is

$$\bar{h}^3(1 - \varepsilon\frac{d\bar{h}}{dx}) = x^2 \qquad (3.11)$$

Using the transformation x + z = 1, we have

$$\frac{d\bar{h}}{dz} = \{\frac{(1-z)^2}{\bar{h}^3} - 1\} /\varepsilon \qquad (3.12)$$

where the initial condition for $\bar{h}(z)$ (obtained from equation (3.7) after transforming the variables) is

$$\bar{h}(z)|_{z=0} = (1 - \varepsilon b)^{-1/3} \qquad (3.13)$$

Equations (3.12) and (3.13) form an initial value problem. The numerical solution may be obtained by using the IMSL routine DVERK. The results obtained for the steady state are very good (see Fig. 3.3, 3.4).

3.4 ANALYTICAL APPROXIMATIONS TO STEADY STATE :-

Polynomials have been adopted for most approximations in this section because of the relative ease in handling them during integration and differentiation. The case for the zero-depth-gradient boundary condition is much simpler to replicate. All the analytical expressions satisfy the boundary conditions i.e. matching slopes at both ends x = 0 and x = 1. The complete behaviour of the steady state is known at the lower boundary. The curvature and higher derivatives at this point may be determined by succesive differentiation of the governing equation.

Among the many polynomials tried, the cubic which matches the slope at x = 0 and the ordinate, slope and

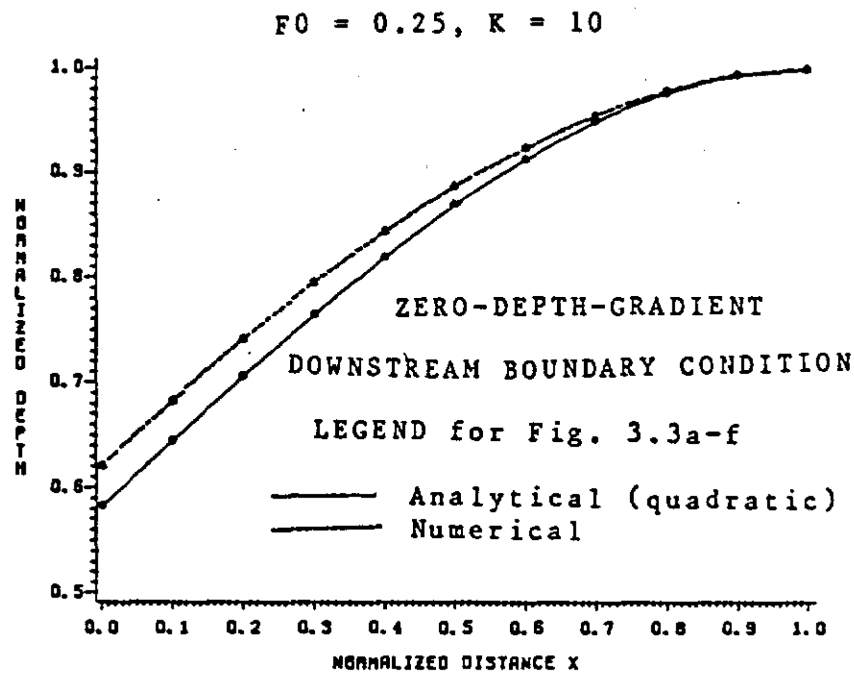

Fig. 3.3a STEADY STATE PROFILES

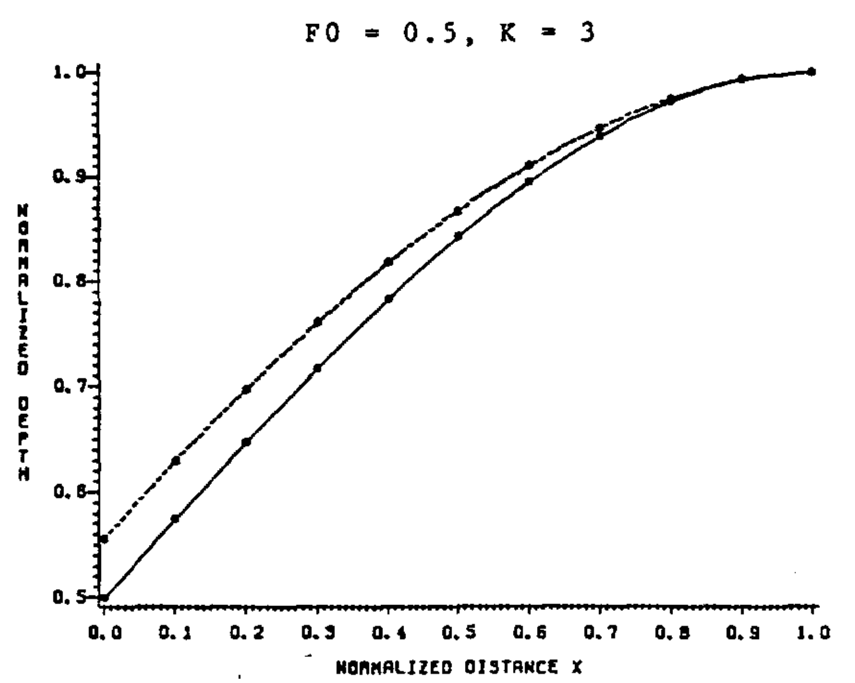

Fig. 3.3b STEADY STATE PROFILES

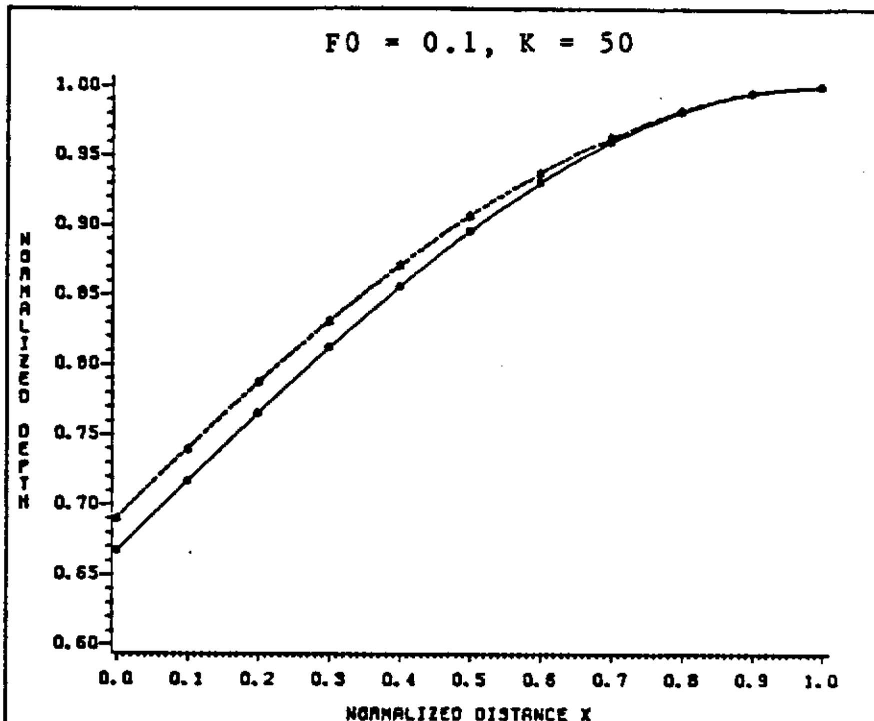

Fig. 3.3c STEADY STATE PROFILES

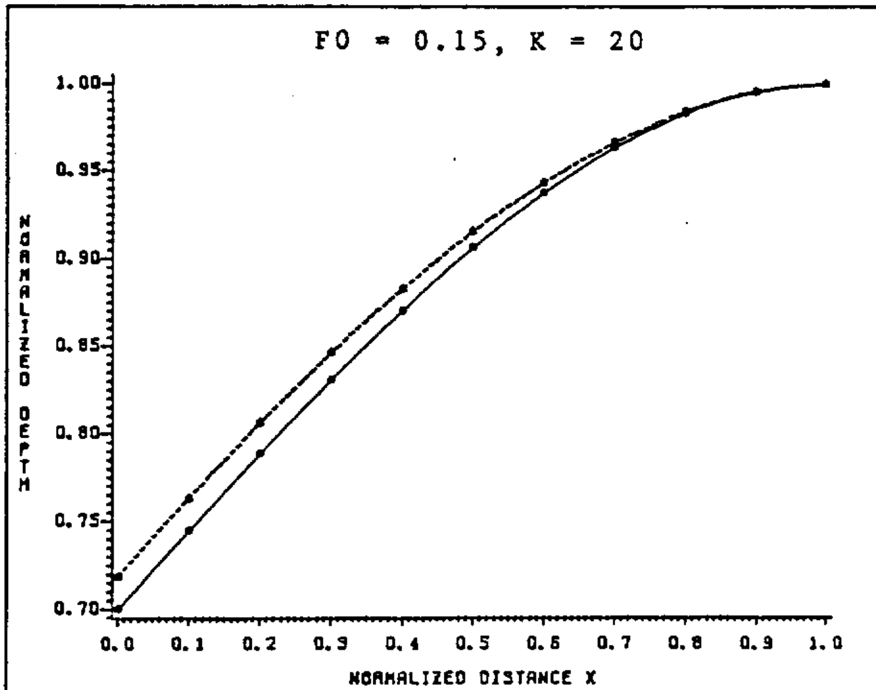

Fig. 3.3d STEADY STATE PROFILES

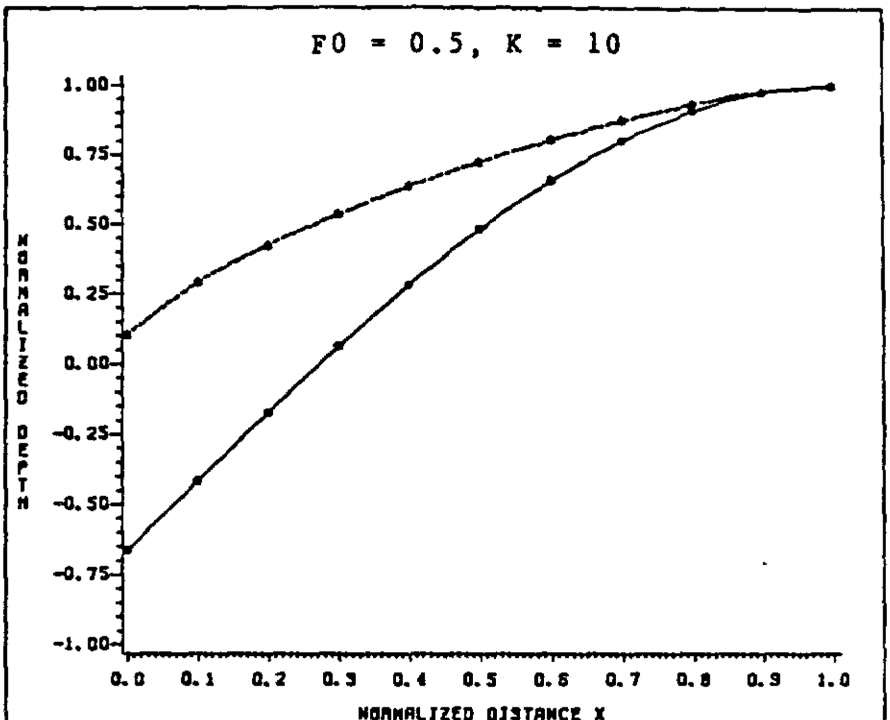

Fig. 3.3e STEADY STATE PROFILES

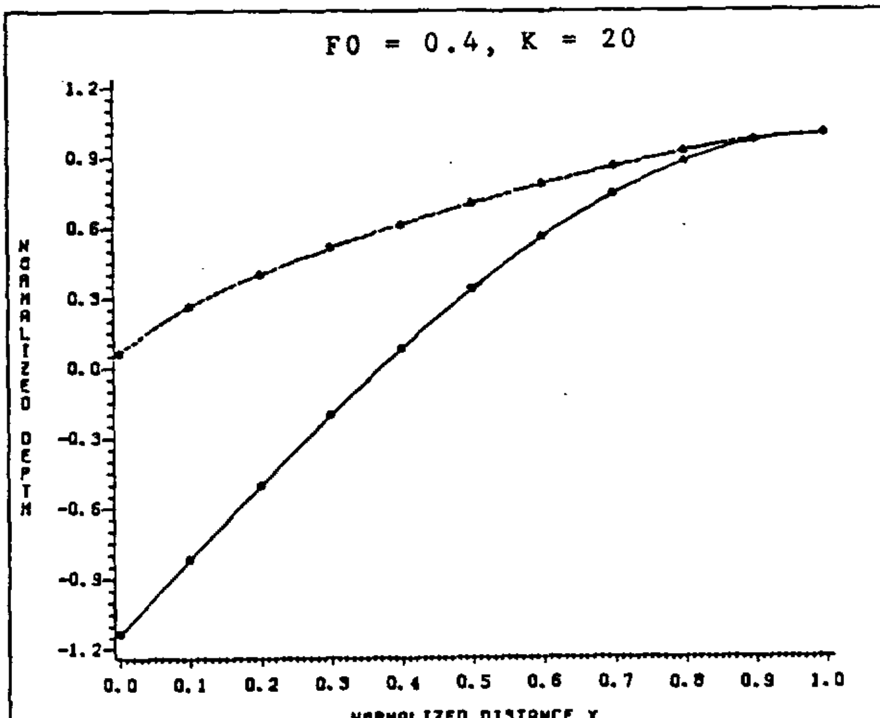

Fig. 3.3f STEADY STATE PROFILES

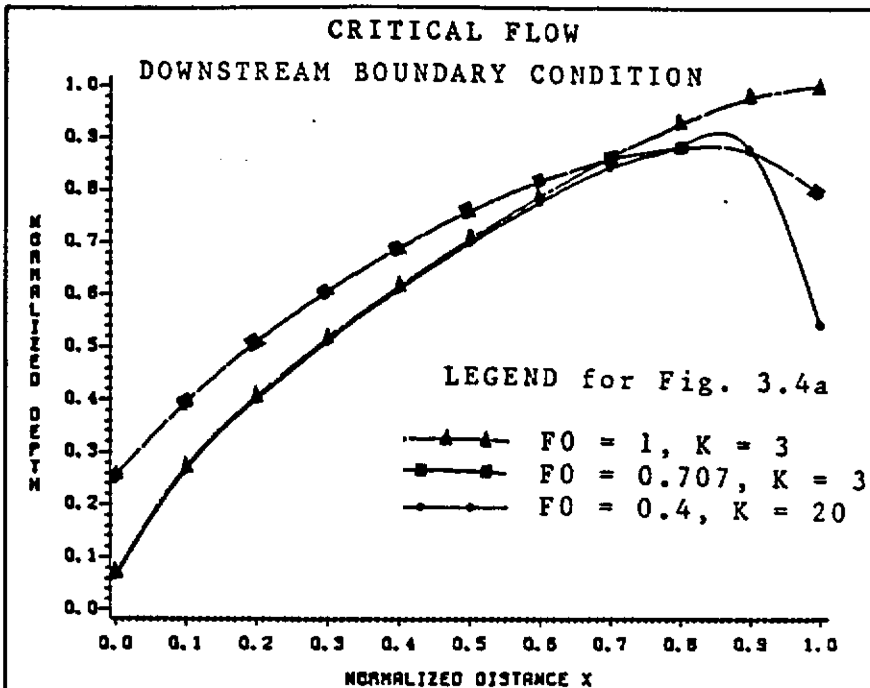

Fig. 3.4a NUMERICAL STEADY STATE PROFILES

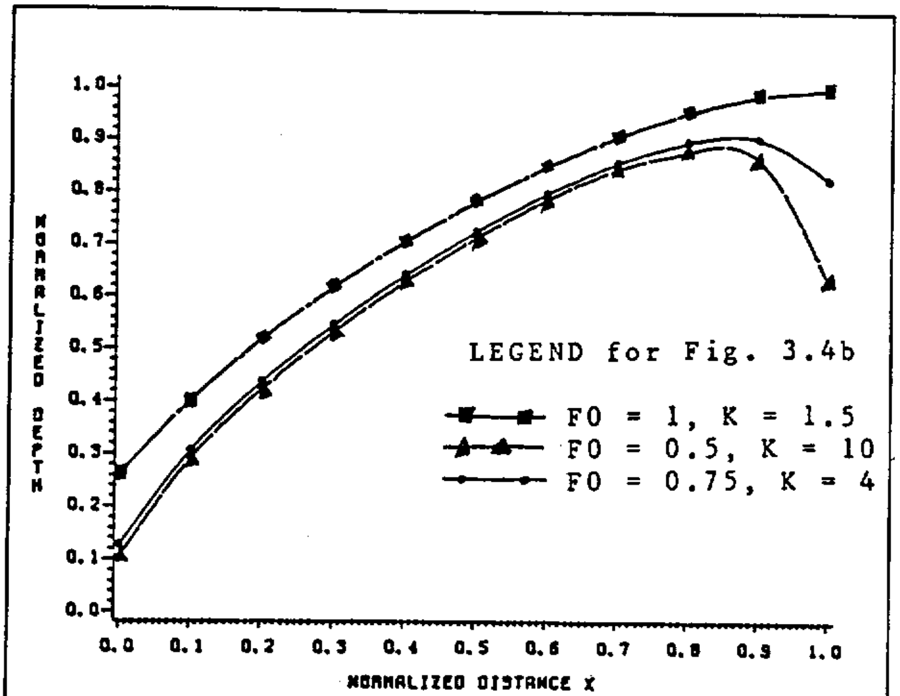

Fig. 3.4b NUMERICAL STEADY STATE PROFILES

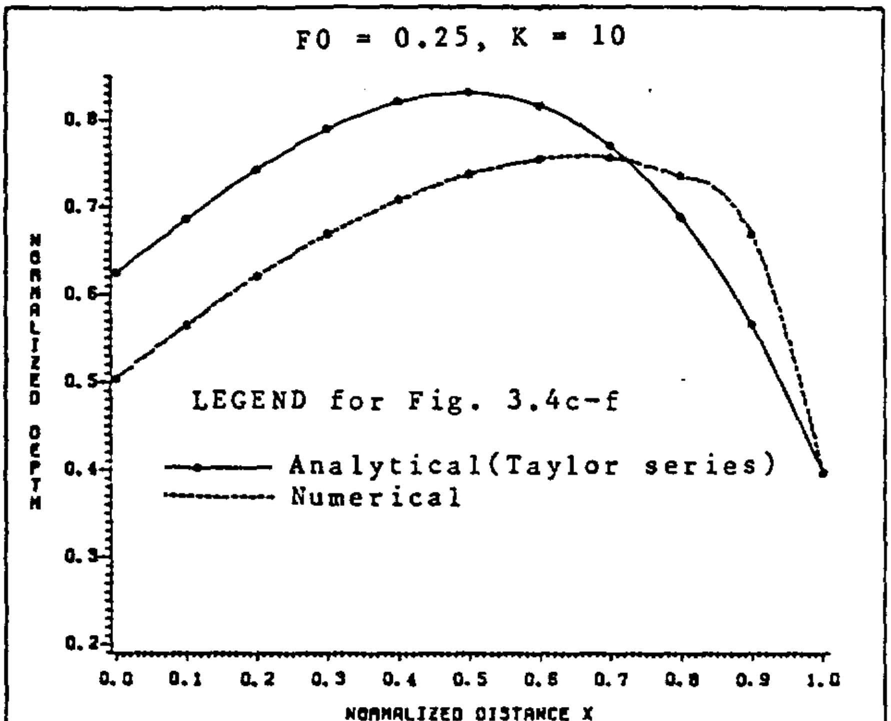

Fig. 3.4c STEADY STATE PROFILES

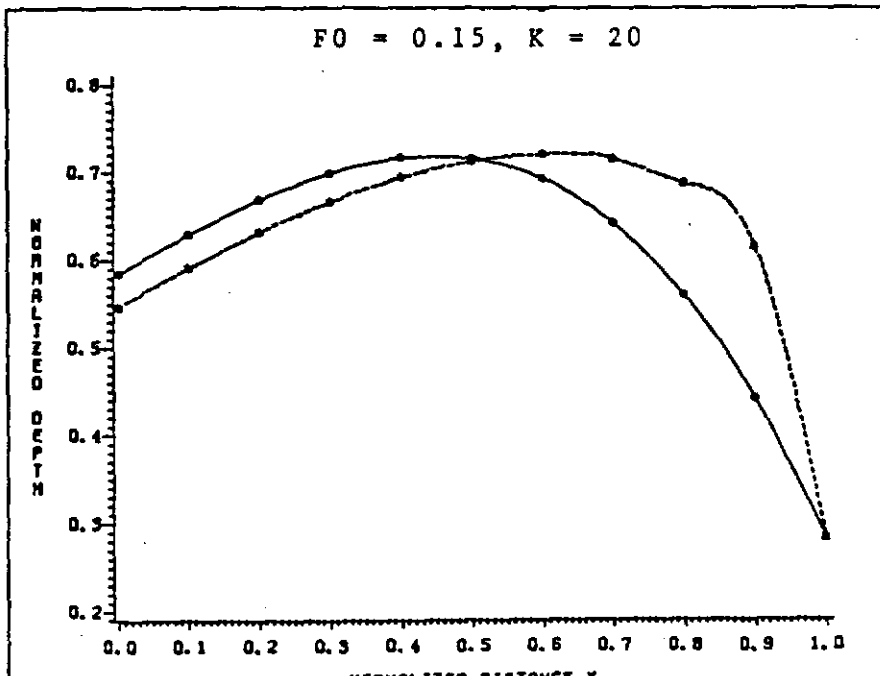

Fig. 3.4d STEADY STATE PROFILES

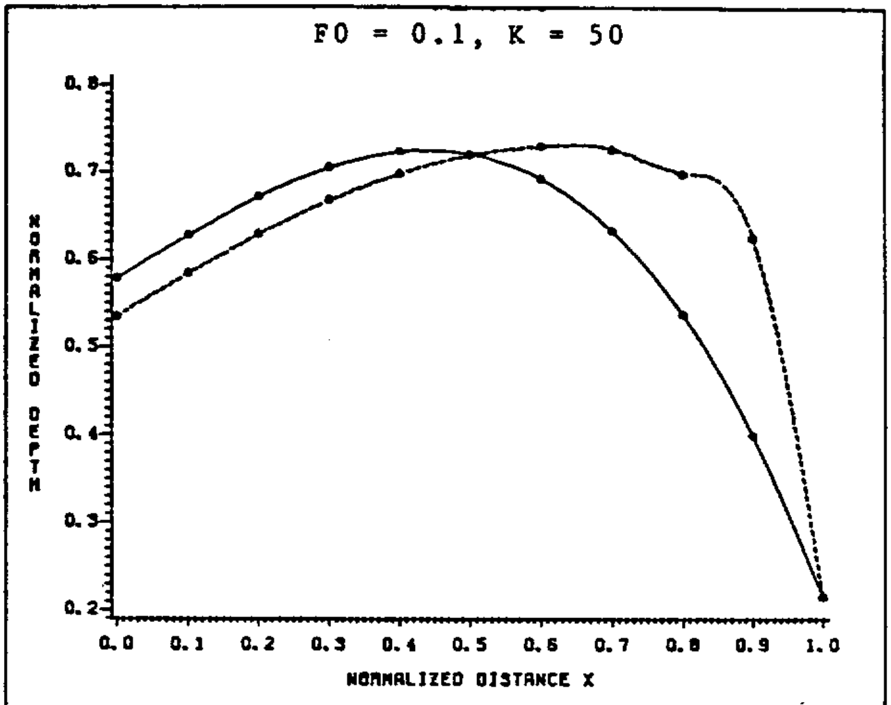

Fig. 3.4e STEADY STATE PROFILES

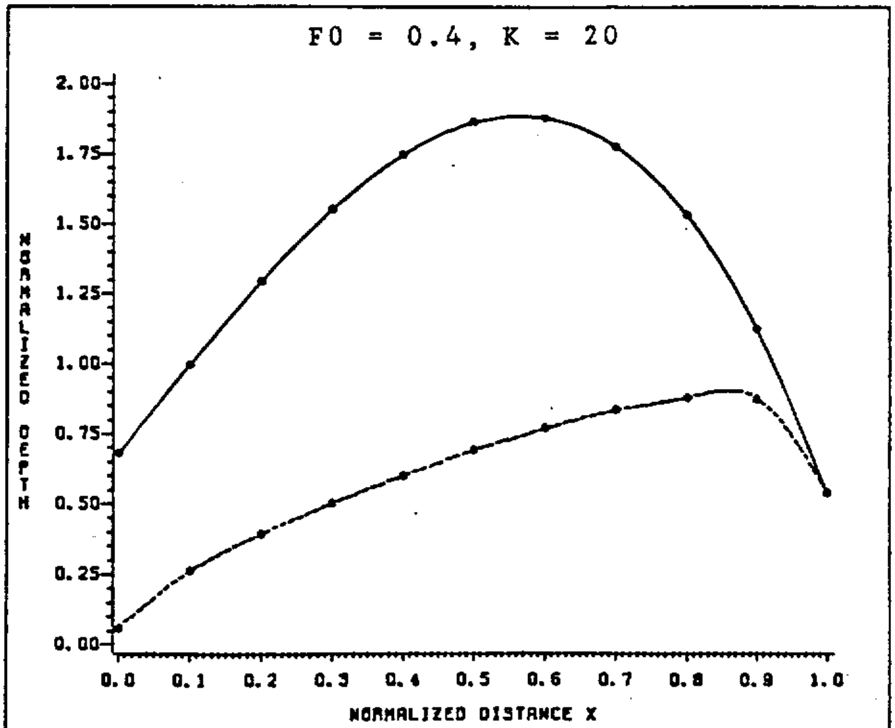

Fig. 3.4f STEADY STATE PROFILES

curvature at x = 1 was found to be closest to the numerical solution (see Fig. 3.3). This is however only valid for cases where F_0^2K is less than 1.5 (Fig. 3.3e,f). For greater values of the product the solution for steep slopes discussed in the previous chapter is applicable (see Fig 2.2, 2.3). Analytical approximations to the steady state solution for critical flow lower boundary condition are very difficult to simulate. Taylor series expansion about either end are valid only in the immediate neighbourhood of the particular point and hence cannot be used in general (see Fig. 3.4).

3.5 TRANSIENT SOLUTIONS :-

The complete solution to the diffusion equation is of the form

$$h = \bar{h}(x) + \phi(x,t) \qquad (3.14)$$

where $\phi(x,t)$ forms the transient portion and is usually dependent on both x and t. This transient solution may be, for example, a cosine series such as

$$\phi(x,t) = \phi_0(t) + \sum_{n=1}^{\infty} \phi_n(t) \cos n\pi x \qquad (3.15)$$

Since the steady state solution already satisfies the boundary conditions, the slope for the transient solution must be zero at both ends, i.e.

$$\partial\phi(0,t)/\partial x = 0 \qquad (3.16a)$$

$$\partial\phi(1,t)/\partial x = 0 \qquad (3.16b)$$

The initial condition for the transient reduces to equation (3.17) after simplification

$$\Phi(x,0) = \Phi_0(0)+\sum_{n=1}^{\infty} \Phi_n(0) \cos n\pi x = -\bar{h}(x) \qquad (3.17)$$

Considering a one term approximation for the transient, we have

$$\Phi_0(0) = -\int_0^1 \bar{h}(x)\,dx \qquad (3.18)$$

which means that the initial condition is satisfied for at least one point in [0,1] from the mean value theorem. There is an error in such an approximation, but this can be improved upon by considering more terms in the transient and satisfying the initial condition at more points. Following the procedure outlined in section 2.2, i.e. integrating the residual over the interval [0,1] converts the problem to an initial value problem. This solution mode has been investigated for the zero-depth-gradient condition and the results for the outflow hydrograph are in close agreement with the corresponding numerical solutions (see Fig.3.5). There is an error caused at x = 0 due to considering only one term. The scope and applicability of this solution procedure are discussed in the next chapter. The scheme developed in this section is not applicable to flows with critical depth at the downstream end.

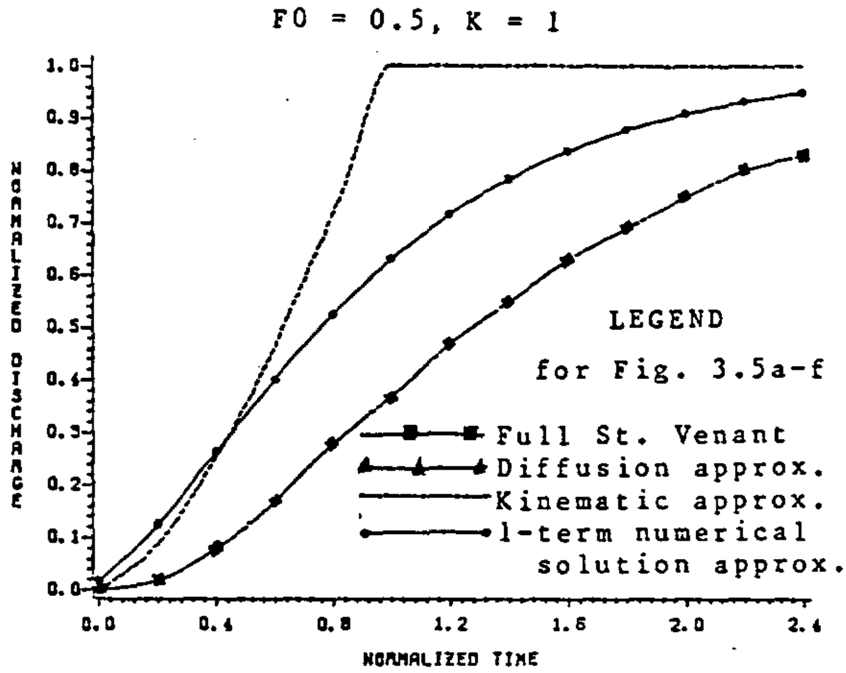

Fig. 3.5a RISING OUTFLOW HYDROGRAPH

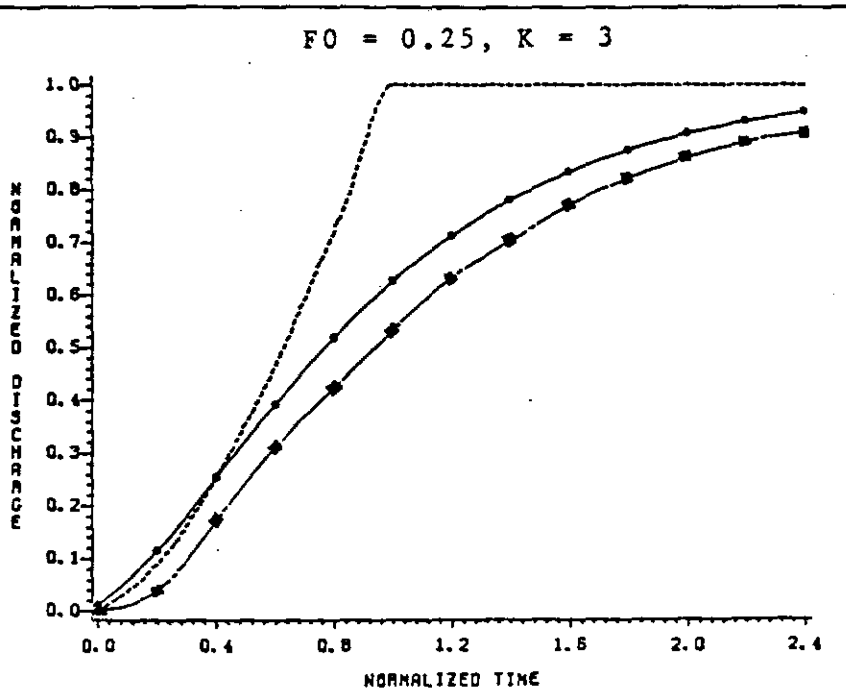

Fig. 3.5b RISING OUTFLOW HYDROGRAPH

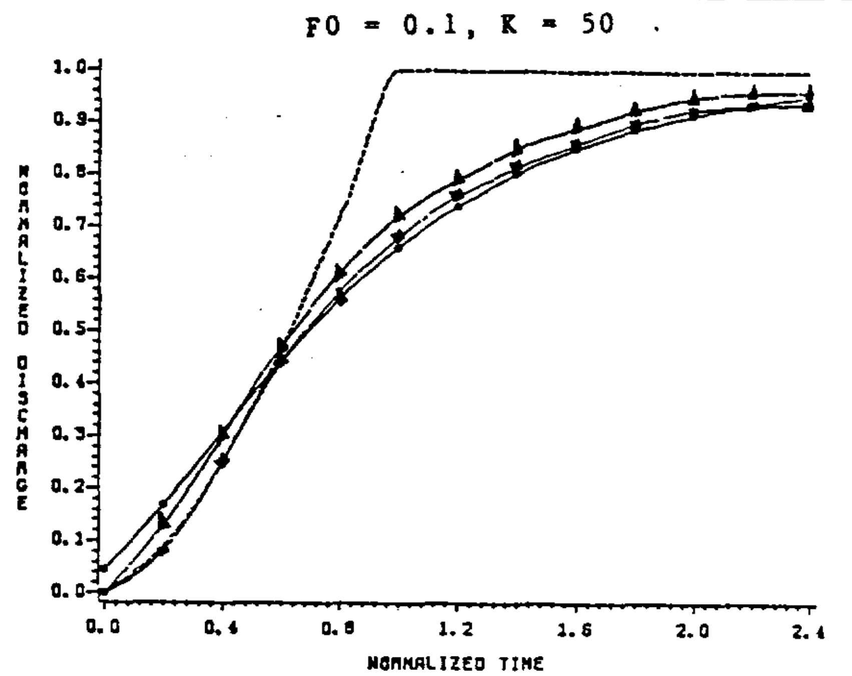

Fig. 3.5c RISING OUTFLOW HYDROGRAPH

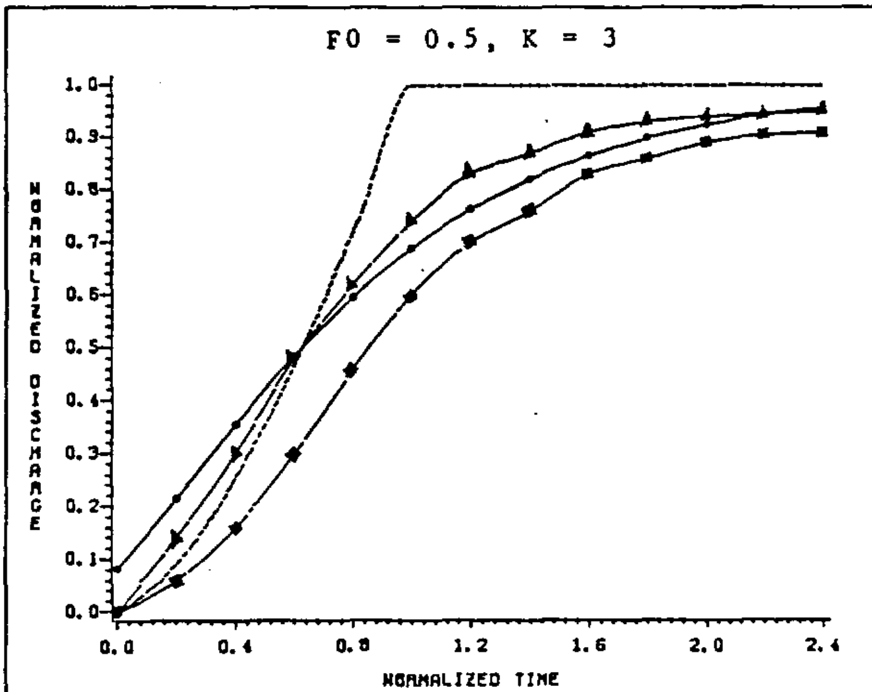

Fig. 3.5d RISING OUTFLOW HYDROGRAPH

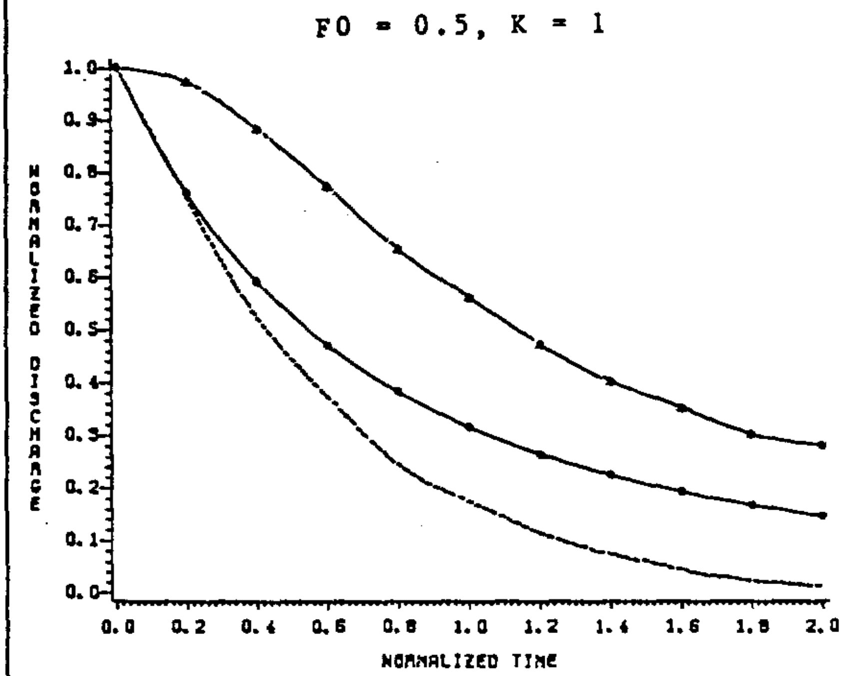

Fig. 3.5e RECESSION OUTFLOW HYDROGRAPH

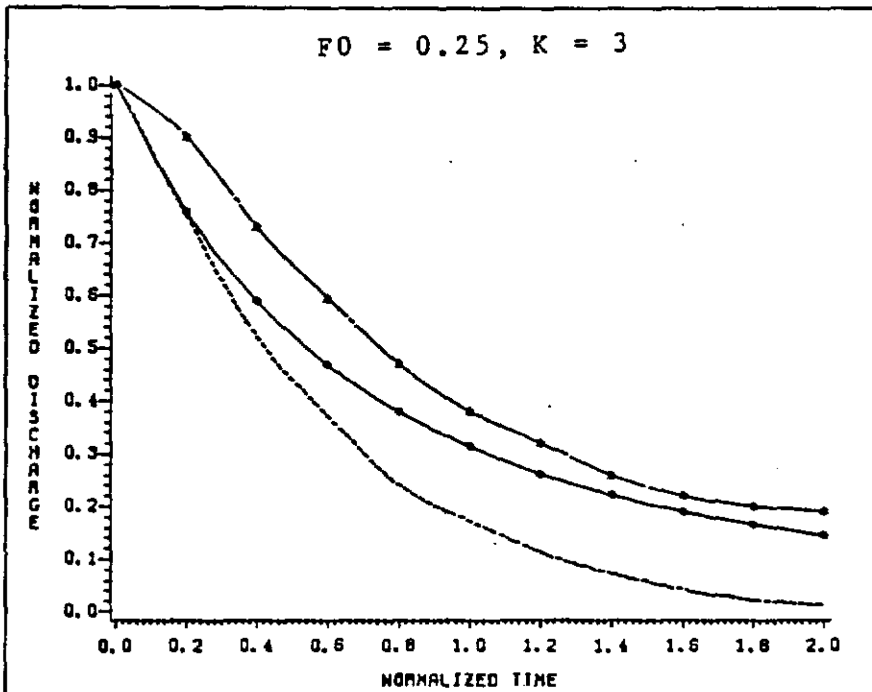

Fig. 3.5f RECESSION OUTFLOW HYDROGRAPH

CHAPTER 4 . DISCUSSION AND CONCLUSIONS

4.1 DISCUSSION :-

A good numerical scheme is one which is stable and converges rapidly. Non-linear systems of differential equations frequently involve the solution of non linear simultaneous equations. Most schemes for solving such a system use a variant of Newton's method and are economical if convergence is achieved within a couple of iterations. Some researchers have found it preferable to use Richtmyer's algorithm or the 'double sweep method' which involves the inversion of a 2 * 2 matrix for which expressions can be written directly.

Both the method of characteristics and finite difference schemes are acceptable techniques. They suffer from the disadvantage of involving long and formidable equations. Their stability can be ensured by using appropriate time steps and space discretization. The series solutions discussed in the text are very simple to formulate and convergence is very rapid. For most cases two terms are adequate. The many instances considered show that this scheme performs very well (see Fig. 2.2, 2.3). It is an entirely new solution procedure adopting a different approach to the problem and is therefore important in its own right. It has all the properties of a good numerical solution.

Analytical solutions developed by previous researchers involve linearization of the differential equation (usually through regular perturbations). The analytical procedure described in the text preserves the non-linearity in the problem to a greater extent. The system of non-linear equations solved for preparing TABLE 2.1 are solved only once for each combination of F_0 and K and may be used for later work.

It is clear from this study that the zero-depth-gradient at the downstream is a much simpler condition to handle than the critical flow requirement at the lower end. The concept of breaking up the solution procedure into two parts is reasonable when the physical system represented by the differential equation reaches a steady state. A difficult problem may therefore be solved by superimposing the solutions of two simpler ones. Polynomials cannot capture the steep behaviour of the steady state profiles in cases of critical flow downstream boundary condition (see Fig 3.4). Hence superimposing the transients also becomes difficult because the initial condition for the transient components are determined by the steady state.

The diffusion wave theory is superior to the kinematic wave though it is more difficult to solve. The insensitivity of the kinematic wave to the field conditions and its inability to account for backwater effects are among its severe drawbacks. Therefore it cannot be used with complete confidence in problems of channel networks. Hence a solution which is simple in structure and can incorporate the effect of F_0 and K is a signicant contribution to this field.

4.2 CONCLUSIONS :-

The diffusion wave is too difficult a problem to attempt a blanket solution for all cases using the Fourier series approach. Hence different solution procedures were adopted for steep slopes and mild slopes. Since the kinematic wave is just a single profile a simple solution is feasible in this instance.

The diffusion wave is known to perform well for small F_0 and large K. A large value of K usually indicates steep bed slopes and the solution discussed in section 2.2 is applicable in such cases. The performance of the solution indicates that the boundary condition of zero depth at the upstream produces similar outflow profile to the one produced by the zero influx upper end condition (see Fig. 2.2, 2.3). Physically this implies that not much water accumulates at the beginning of the overland flow plane for such cases. An inspection of the steady state profiles shows that the depth at x = 0 is practically zero for cases of F_0^2K greater than 1.5 (Fig. 3.3e,f, 3.4e,f). Hence the solution procedure of section 2.2 may be used under the condition stated above. It may be noted here that the zero-depth-gradient condition is valid for large vaues of $F_0{}^2K$. However the Diffusion wave is a good approximation for small F_0 (< 0.5) and large K (> 10 ,say).

It may also be observed that the upstream boundary condition has little or no effect on the outflow hydrographs. For important problems like channel networks and catchment-stream problems it is the profile at the outflow section of the overland flow plane that is of primary significance. The efforts of most previous researchers have been concentrated in this phase of the solution. Therefore, when the profile at $x = 1$ is of interest, the solution procedure outlined in section 2.2 may be used for all those cases where the zero-depth-gradient downstream is a valid approximation.

The analytical expression (obtained as an approximate solution for steep slopes) is found to perform very well in the neighbourhood of the steady state (Fig.2.3). The recession profiles beginning close to the steady state are more accurate as a result. For design purposes, the most critical profiles are of major interest. This implies that the full profile needs to be developed before the recession starts. The steady state would therefore seem to be a good indicator of peak flood discharge and the analytical expression (discussed in section 2.3) may be employed in such cases.

Section 3.4 deals with the steady state solution procedure for cases where F_0^2K is less than 1.5 and when zero-depth-gradient downstream end condition and zero-inflow upstream boundary condition are used (see Fig 3.3). This therefore covers all cases for this end condition. The case for critical flow at x = 1 is not amenable to a solution of this form because the steady state has very steep slope requirements and cannot be matched by using polynomials (Fig. 3.4).

The steady state analysis reveals several interesting features. Complete knowledge of the behaviour of the steady state solution at the outflow section is obtained. For the zero-depth-gradient condition, the steady state reaches its maximum normalized depth of 1.0 here. However the critical flow downstream boundary requirement yields a different profile. The maximum does not occur at x = 1 because of the severe negative slope at that end. The case for $F_0 = 1$ reduces identically to the zero-depth-gradient condition for all values of K as far as the steady state profile is concerned.

The key variable in both kinematic and diffusion approximations is K. The smplification of the momentum equation is valid only for large K. For smaller values of K (say $K < 5$), the terms in the momentum equation are not negligible. Hence, results for greater values of K are found to be better than those for smaller K (see Fig. 2.2). It may be noticed that the kinematic wave approximation naturally satisfies the zero-depth-gradient downstream boundary condition and the zero depth condition at $x = 0$. Therefore it is analogous to the case of $F_0 = 1$. This is physically reasonable since the kinematic wave works well for steep bed slopes on which the flows are close to being critical.

NOMENCLATURE

Symbol	Definition
C	Chezy's coefficient
F0	Froude number for normal flow conditions
g	Acceleration due to gravity
h	Depth of flow
H_0	Normal flow depth
h*	Non-dimensional depth
$\bar{h}$	Truncated series solution ,or
	Steady state values
K	$S0L0/H0F0^2$
L_0	Length of overland flow reach
q	Lateral inflow per unit area per unit time
R_*	Non-dimensional lateral inflow
S_f	Dimensionless 'friction slope'
S_0	Non-dimensional slope of the plane
t	Time
t_r	Duration of rain
t_*,T	Non-dimensional time
u	Average velocity of flow
u_*,U	Non-dimensional velocity
V_0	Velocity for normal flow at $X = L_0$
x	Co-ordinate in direction of flow

NOMENCLATURE(Contnd.)

Symbol	Definition
x_*, X	Non-dimensional x
β	Momentum correction factor
γ	Specific weight of water
ϵ	$1/F_0^2K$
η	Manning's coefficient
ρ	Density of water
θ	Angle of slope

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APPENDIX

Reference for Outflow Profiles

This list contains references to all those profiles used for comparison purposes. Profiles developed by the author are not listed.

Figure No.	Description	Reference
2.2a,b	Full St. Venant	Vieira [1983]
2.2a,b	Diffusion approx.	Vieira [1983]
2.2c,d	Full St. Venant	Morris [1978]
2.2e,f	Full St. Venant	Vieira [1983]
2.2e,f	Diffusion approx.	Vieira [1983]
2.2g,h	Full St. Venant	Morris [1978]
2.2i,j	Full St. Venant	Morris [1978]
2.3a,c	Full St. Venant	Vieira [1983]
2.3a,c	Diffusion approx.	Vieira [1983]
2.3b,d	Full St. Venant	Morris [1978]
2.3e,f	Full St. Venant	Morris [1978]
3.5a,b	Full St. Venant	Morris [1978]
3.5c,d	Full St. Venant	Vieira [1983]
3.5c,d	Diffusion approx.	Vieira [1983]
3.5e,f	Full St.Venant	Morris [1978]