ULTIMATE STRENGTH OF REINFORCED CONCRETE BEAMS IN COMBINED TORSION AND BENDING

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During the past 10 to 15 years, considerable work has been done in the investigation of torsion in reinforced concrete beams. In the past torsion has usually been treated as a secondary stress and therefore has not received the same attention as bending, transverse shear or compression. There are conditions, however, in which torsional moments could have a distinct effect on the carrying capacity of a concrete structure.

Characteristic monolithic construction is the primary cause of torsional moments in reinforced concrete frames. A good example of how torsional moments could be developed in concrete beams is the case where a concrete slab is poured monolithically with a concrete building frame. With unsymmetrical spans and loads in different bays, torisonal moments of sizable magnitude could be introduced in the supporting beams with critical beams most likely being the exterior beams.

The case of an overhang in a concrete building, such as a balcony, could produce considerable torsional moments in the side beams.

In both of these cases the torsional moments are working in combination with bending moments. The beams in the building frame have maximum bending moments acting near the center of the beam due to the vertically applied loads. The bending moments in the balcony arrangement will be due to the cantilever action of the structure with the maximum occurring at the supported end. The maximum torsional moments are also acting at these points.

Henry J. Cowan of Australia has done considerable work in the field of reinforced concrete in torsion, combined torsion and shear, and combined torsion and bending. Provisions have been made in the Australian Building Code for the design of sections subject to torsion.

His work has been performed using the concepts of elastic design. For pure torsion Cowan based his mathematical derivations for the stresses in the shear reinforcement on the hyperbolic functions of torsion for rectangular sections as derived by St Venant in 1853. Cowan states that the resistance to torsion must be supplied by the tensile resistance of the shear reinforcement and the compressive stress of the concrete.

Cowan recommended that spirals inclined at 45 degrees to the longitudinal axes would be the most efficient type of transverse reinforcement. In the case of reversal of stress this would require the presence of spiral rinforcement in both directions. He suggests that in this case of reversal of stress that the best arrangement of transverse reinforcement would be hoops perpendicular to the longitudinal reinforcement.

The Australian Code limits the maximum permissible concrete stress in diagonal tension due to torsion to $.02f'c + 20$ psi $\leqq 90$ psi. This is the same allowable stress as that for resistance due to shear. If this is exceeded, then torsional reinforcement is required. Regardless of the amount of torsional reinforcement used, the diagonal tension must not exceed $.08f'c + 80$ psi $\leqq 360$ psi.

Cowan states that "Bending moments do not reduce the capacity of reinforced concrete sections to resist torsion. In this respect reinforced concrete differs from steel and from plain concrete. In fact a moderate bending moment increases the resistance to torsion." This could partially be explained by the difference in the failures due to the two different types of actions. With bending failure there is always the presence of tension cracking in the bottom portion of the beams. Whereas torsional failure depends on the formation of diagonal tension cracking. With the compression of the upper portion of the concrete due to the bending moment the diagonal tension cracking due to torsion would be retarded.

Cowan's work has not been restricted to only rectangular sections. He has done research on T, L, and I sections under the influence of torsional moments. A fully numerical solution has not been derived but he has developed an approximate solution to determine what torsional moment can be safely carried by these sections.

Professor G. C. Ernst of the University of Nebraska carried out several experiments to determine the torsional properties of rectangular reinforced concrete sections. In his work, he used the ultimate design theory to determine the failure of the beams.

Ernst did use the elastic theory as outlined by Cowan up to the point where the first cracking occurred in the concrete. From this point he then used ultimate equations for his calculations.

Ernst pointed out that the torque at which initial cracking begins has no relationship to the amount of longitudinal or transverse reinforcement, but that initial cracking corresponds to the failure of the unreinforced concrete in torsion.

The experiments which Ernst performed pointed out that increasing the ratio of the transverse to longitudinal steel by decreasing only the longitudinal steel will decrease the torque capacity, but increasing the ratio by varying only the transverse steel will improve the torque capacity. He also found that the strains indicated by gages were larger than those calculated. This would indicate that the transverse steel was carrying more of the diagonal tension than indicated by the elastic theory.

Work was begun here at the University of Kentucky in 1959 under the direction of Dr. Hans Gesund. Financial assistance was obtained from the National Science Foundation and the Dept. of Civil Engineering by Dr. Gesund. Assistance was obtained from the Kentucky Department of Highways Scholarship students in cooperation with the Kentucky Research Lab.

The work has been performed in steps by students working for their Master of Science degrees here at the University.

The first investigation was performed by Eloy Sham. His work was restricted to square concrete beams in pure torsion.

The aim of his thesis work was to determine a crack pattern or a plane of failure of the concrete in diagonal tension. He also tried several combinations of transverse reinforcement to determine which would be the best in resistance to torsion.

Sham ran torsional test on five beams, each with a different arrangement of transverse reinforcement. Beam 1, which consisted of longitudinal steel and transverse reinforcement in the form of ties and hoops, exhibited the highest resistance against torque of all the specimens tested. The hoops were spaced at the same distance center to center as the ties. The first noticable cracking took place in the zone reinforced with the hoops. And the final failure did occur in the end of the beam reinforced with circular hoops. This would lead to the conclusion that the square ties gave more resistance to torsion than the circular hoops. The fracture cracks in this beam were very nearly 45 degrees with the longitudinal axis.

In Beam 2, Sham was trying to study the effect of having only longitudinal reinforcement. The beam was reinforced with one bar in each corner of the beam. Upon failure the beam did not exhibit any additional strength over a previously tested beam with no reinforcement.

Beam 3 was similar to Beam 2. The only difference being that a larger amount of longitudinal steel was used. The results were somewhat the same as the previous beam. The additional reinforcement did not offer any greater resistance to torsion but seemed only to hold the mechanism together.

Beam 4 was reinforced only with transverse reinforcement in the form of ties and hoops. With this type of reinforcing there was the obvious indication that the reinforcement had changed the mode of the failure. It indicated that closely spaced ties and hoops without the aid of longitudinal reinforcement change the diagonal tension crack to that of a shear fracture. Once the cracks opened, the beam did not resist any additional torsion. The beam did resist torsion above that of Beams 2 and 3.

Beam 5 was reinforced with continuous spirals at an angle of 45 degrees with the axis of the beam and a pitch of 2 inches for one half of the beam. A single spiral with an angle of almost 90 degrees with the axis of the beam and with a pitch of 1″ was used in the other half. Failure occurred in the half reinforced with the single spiral. The specimen was capable of carrying further torsional moment after the concrete had cracked. Before and at cracking load, the steel reinforcement had not been stressed very highly. This would indicate that the diagonal tension only affected the concrete and after the concrete has cracked, the steel alone is acting.

The conclusions drawn by Sham are as follows:

1. The elastic formulas as outlined by Cowan are too dangerous to apply for working loads since values obtained from these formulas are far too close to the ultimate torque values.

2. There was an indication that the parabolic shearing strain distribution at the edge as assumed in the elastic theory is in eror or at least is not applicable to ultimate torque theory. Strain gages placed on the edge of the specimens indicated the presence of shearing strains, and thus of stresses at the points where zero shearing stresses are assumed in elastic theory.

3. At ultimate torque, the concrete in compression should be considered in its plastic range rather than in its elastic range.

4. The presence of only longitudinal or only transverse reinforcement is not suitable to the resisttance of torsion. The two must be present for additionally strength in pure torsion.

5. An arrangement of transverse ties along with longitudinal bars provided the greatest resistance to torsion.

6. For shear reinforcement to be affective, the ties must be spaced at something less than the effective depth of the beam.

7. Over-reinforced beams will exhibit shear cracks between the ties rather than diagonal tension cracks characteristic of torsion. The failure will be relatively sudden.

Larry Boston took the next step in this study. His work was to introduce bending moment in combination with the torsion and to study their interaction. In his test he was working only with longitudinal reinforcement.

The first problem which he encountered was the determination of a failure surface of the concrete. Due to torsion, alone, a cracking pattern had already been determined in the work by Sham. The question was, how will the introduction of bending moment alter this crack pattern. Due to the testing that followed it was found that the only alteration was in the angle which cracking across the bottom made with the longitudinal axis. This angle may be calculated by the combination of the bending stress and shearing stress due to torsion on the bottom of the beam.

The test procedure used by Boston was as follows. With the use of a new apparatus which is capable of testing beams with a large number of combinations of bending and torsional moments, his first two beams were tested in pure torsion. The beams were square (8″x8″) with a six foot test length area. The bar placement was similar in both beams with two bars in the top and three bars in the bottom. The bar size was varied for different beams. The failure surface was the same as that found by Sham and the resulting carrying capacity was as expected.

After the first two beams, test specimens were poured in sets of two with the same reinforcement as the first two. The only variation was that bending moment was now introduced. The sets were 1:1 (Bending moment equal to torsional moment), 2:1 (Twice the bending moment as there is torsional moment), 3:1, and 4:1. Strain gages were placed on the bars to measure the strains for each load increment.

Boston approached the problem from the ultimate strength method entirely. When the concrete cracks, these must cross the longitudinal bars. With the beam cracked there is a displacement of one segment of the beam with respect to the other. This displacement is resisted by the longitudinal reinforcement in the beam. Because of the displacement of the two segments, bending moments are placed in the longitudinal bars. These bending moments must resist the torsion.

It was decided by Boston that the beams weakness to torsion was the weak link in the system. The failure would occur when the twisting of the beam would force out a segment of the resisting concrete. It is assumed that the stresses on the concrete due to the bar would be in a straight line between the cracks. When the force acting against the concrete exceeds that which the concrete is capable of carrying, a wedge shape segment is forced out. The cracking away of this segment leads to the failure of the section in torsion. Therefore by being able to determine the size of the wedge, the force acting on the segment can be calculated. At failure the concrete will exhibit plasticity which will make the forces equal in the bars. It is then a matter of multipling the forces times their lever arms about the hinge to determine the carrying capacity in torsion.

Boston found that the action of torsion and bending seem to be quite independent of each other.

Further conclusions on Boston's work are not available at this time since he has not completed his thesis.

The next phase in this research work was the placing of transverse reinforcement in beams similar to those tested by Boston. This is the section which I have been working on.

The square 8″x8″x6′-0″ test beam was used in this phase of study. The beams were poured in sets of two. The variables were the transverse steel and the ratios of bending moment to torsional moment. The ratios of bending moments to torsional moment were the same as previously used. (1:1, 2:1, 3:1, and 4:1).

The only variation in reinforcement was in the spacing of the ties. I used spacings of 5 inches and 2 inches center to center. The minimum spacing was determined on the basis of the aggregate size. The maximum spacing was based on the fact that this would just prevent the 45 degree crack angle.

Strain gages were placed on the three bottom longitudinal bars and also on six of the transverse bars.

For each case of bending-torsion ratio, two beams were poured. One beam had the ties spaced at 5 inches and the other beam had the ties at 2 inches. An attempt was made to force the failure to occur in a uniformly stressed area. That is, provisions were made to keep the cracking from occurring near the end of the beam.

The beams were under-reinforced in bending to make sure that the failure would not be sudden but would be accompanied by yielding of the longitudinal steel.

We are assuming that the failure of a reinforced concrete beam in combined bending and torsion will fail by cracking of the concrete in diagonal tension along three faces, and by compression on the fourth face with rotation about a longitudinal hinge. We are further assuming that after the first cracking of the concrete the total torsional resistance must be supplied by the reinforcing steel. This resistance is supplied by four factors.

1. The direct tensile strength of the transverse ties
2. The dowel action of the ties
3. The S-shape bending or dowel action of the longitudinal bars
4. The bending of the ties about the hinge

These forces must produce torsion about the hinge that will resist the applied torsion.

The bending moment is calculated by using the conventional ultimate design methods.

Of the testing that I have done, the results seem to indicate that the actions of bending and torsion are independent of each other. This has been pointed out previously by Boston. Strain gages on the longitudinal bars indicated that there was definite yielding of the bars. Gage readings on the ties did not indicate yielding of these bars.

A shear failure was obtained in a beam with transverse ties at 2 inches and a bending moment to torsional moment ratio of 1:1. The failure was quick and rather explosive. This did not happen in any of the other beams. All other beams failed in what appeared to be torsional failure under the influence of bending. As the ratio of the bending moment became larger, the failures were more due to the bending.

A failure mechanism such as that obtained by Boston has not been solved for at this time. Also no definite conclusions have been drawn on this phase.

Future work to be carried out at the University of Kentucky is to expand the present work to the case of true rectangular section where the width is some ratio of the depth.

Also work is needed to be done for the case of shear on the section. All work done at the University so far has been with only torsion or combined bending and torsion. Under normal circumstances shear will be a definite factor to consider.