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Upward Water Movement in Field Cores

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THE present world population growth rate presents the agricultural industry with a significant challenge to provide adequate supplies of food and fiber. In addition, demand for water and land by other elements of society continues to increase, restricting the amount of these vital resources which are available to agriculture. Thus efficient use of our land and water resources becomes increasingly important.

Many agricultural lands in the United States exhibit shallow natural water tables which require artificial drainage systems to insure suitable conditions for growing crops. In some cases, these systems can also be used to supply water to crops via subirrigation (Fox et al. 1956). To properly design such systems, it is necessary to accurately describe water table rise under field conditions.

This paper presents the results of a study to evaluate exact and approximate theoretical methods of predicting upward water movement. The specific objectives of the study were:

1 To formulate an approximate method of predicting transient upward water movement during subirrigation.

2 To determine the hydraulic properties of two field soils and apply the approximate method as well as the so-called exact theory of water movement for subirrigation.

3 To test the validity of both methods experimentally on large undisturbed cores of the two field soils.

4 To evaluate the relative utility of the two methods from the standpoint of engineering design.

BACKGROUND

In this study, subirrigation denotes the transient upward movement of a water table and the water in the overlying unsaturated zone resulting from artificially imposed boundary conditions. This mechanism has been recognized for some time as a potential means of supplying water to the root zone when the need exists. Water table movement during subirrigation has been measured for various spacings of parallel water conduits in a field soil by Skaggs et al. (1972). It was shown that the time required to artificially raise the water table is dependent on such spacing. Fox et al. (1956) discussed various factors which are important in the design of subirrigation systems. They concluded that a high natural water table or a relatively shallow restrictive sublayer is needed if subirrigation is to be practiced. Perhaps because of these restrictions the process has not received as much attention among investigators as have infiltration and drainage.

Because subirrigation artificially raises the water table to supply water to crop roots, upward unsaturated water movement via capillary rise is important. Transient capillary rise has been investigated by Philip (1966) and Parlange and Aylor (1972). Steady capillary rise above a fixed water table position is probably of more relevance in supplying water to the root zone. This process is discussed by Gardner (1957), Anat et al. (1965) and Whisler et al. (1968) in relation to soil properties and evaporation potential at the soil surface.

While steady capillary rise is an important phenomenon in subirrigation, the transient process, i.e. the manner in which the water table rises, is of equal importance from the standpoint of system design. Such design should specify the response time required to raise the water table by a desired amount. Therefore methods are needed to predict transient water table rise in an unsaturated soil.

Theoretical studies of steady two-dimensional water movement during subirrigation were presented by Bouwer (1959) and Sewell and van Schilfgaarde (1963). Skaggs (1973) neglected lateral movement in the unsaturated zone and presented solutions for water table rise between two parallel conduits. Solutions for transient water movement under subirrigation conditions which consider both saturated and unsaturated flow are not available in the literature.

This study considers upward, one-dimensional water movement under subirrigation conditions. The results provide a basis for determining the effects of variation in soil properties on subirrigation and for evaluating the relative utility of various theoretical methods of describing the process.

THEORY

Vertical water movement in soil can be characterized by a relationship proposed by Richards (1931) and given as

$$C(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left[K\frac{\partial h}{\partial z}\right] + \frac{\partial K}{\partial z} \quad ..... \quad [1]$$

where h is pressure head, z is vertical displacement (measured positively upward from the base of the column), t is time, K is the hydraulic conductivity which is a function of pressure head, $K = K(h)$, and $C(h)$ is the soil water capacity function. $C(h) \equiv d\Theta/dh$, where Θ is the volumetric water content. For saturated conditions, $C(h) = 0$ and equation [1] reduces to Laplace's equation; thus it can be solved numerically to describe combined saturated-unsaturated vertical water movement in the soil profile. The initial and boundary

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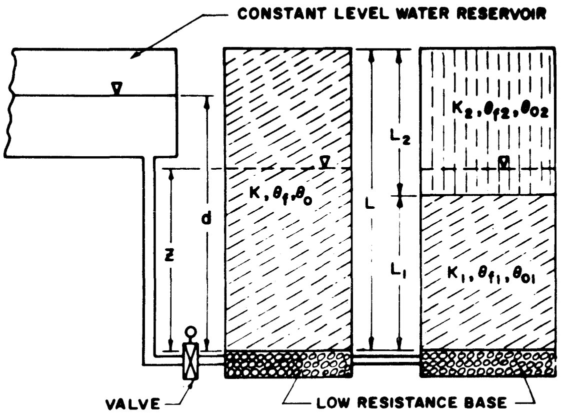

A. UNIFORM SOIL COLUMN　B. LAYERED SOIL COLUMN

FIG. 1 Illustration of soil columns with rising water table.

conditions considered here may be written as:

$$h = h_o(z) \qquad t = 0,\ 0 \leqslant z \leqslant L$$
$$h = d \qquad t > 0,\ z = 0$$
$$q = -K\left(\frac{\partial h}{\partial z} + 1\right) = 0 \qquad t > 0,\ z = L \quad \ldots \ [2]$$

where h_o (z) is a known initial pressure head distribution in the soil column, L is the length of the column, d is the constant head imposed at the base of the column (z=0), and q is flux, which is assumed to be zero at the soil surface. Equation [1] subject to conditions of equation [2] was solved according to an implicit numerical scheme outlined by Skaggs et al. (1970).

Equation [1], Richards equaiton, has been used to describe water movement in soils during infiltration (e.g., Rubin and Steinhardt (1963), Whisler and Klute (1965)) and drainage (e.g., Day and Luthin (1956), Remson et al. (1965)). However, approximate theories which require simpler inputs in terms of soil properties and boundary conditions are also frequently used. Approximate models describing one-dimensional infiltration (Green and Ampt (1911), Horton (1940)) and drainage (Youngs (1960), Jackson and Whisler (1970)) are some examples. An approximate method of describing water table rise during subirrigation is presented here. The derivation is similar in many respects to that of the Green-Ampt equation presented by Swartzendruber et al. (1968).

Fig. 1(A) illustrates a homogeneous soil column with water application at the base via an elevated, constant head reservoir. The initial water content is uniform, $\Theta = \Theta_o$ at $t = 0$, $0 \leqslant z \leqslant L$. The base offers negligible resistance to flow and is initially filled with water. When the valve is opened, at $t = 0$, water begins to enter the soil profile at $z = 0$. Thus for $t > 0$, the water table rises into the profile and at some time t, the water table is located at z. The flux at the base, $z = 0$, is defined as $q = f_s$ and is given by

$$f_s = \left[-K_1 \frac{\partial H}{\partial z}\right]_{0,t} \ \ldots\ldots\ldots\ldots\ [3]$$

where K_1 is the saturated hydraulic conductivity and H is the total hydraulic head, $H = h + z$. By definition the pressure head at the water table is zero, so by assuming the hydraulic gradient beneath the water table is constant, the gradient may be expressed as $\left.\frac{\partial H}{\partial z}\right|_{z=0} = -\frac{d - Z}{Z}$

Where Z is the distance of the water table above the base.
Therefore,

$$\frac{dF_s}{dt} = f_s = K_1\left(\frac{d-Z}{Z}\right) \ \ldots\ldots\ldots\ldots\ [4]$$

where F_s is the cumulative water volume entering the profile.

If it is assumed that all the water entering the profile is contained beneath the water table, F_s can be expressed as

$$F_s = (\Theta_f - \Theta_o)Z \ \ldots\ldots\ldots\ldots\ [5]$$

where Θ_f is the volumetric water content below the water table, and Θ_o is the initial water content. Solving for Z in equation [5], and substituting into equation [4] yields

$$f_s = \frac{dF_s}{dt} = K_1\left(\frac{1 - F_s/a_1}{F_s/a_1}\right) \ \ldots\ldots\ [6]$$

where $a_1 = d(\Theta_f - \Theta_o)$. Separating variables, integrating, and requiring $F_s = 0$ when $t = 0$ results in

$$\frac{F_s}{a_1} + \ln\left(1 - \frac{F_s}{a_1}\right) = -\frac{K_1}{a_1} t \ \ldots\ldots\ [7]$$

which is an implicit relationship between F_s and t, where a_1 and K_1 are parameters which depend on the soil properties and boundary conditions.

The cumulative volume entering the profile while the water table is rising is approximated by equation [7]. It is assumed that water moves upward in the profile via saturated flow below the water table. Water movement in advance of the water table is neglected and thus the model should underestimate the actual volume entering the profile at any time. For cases where the initial water content is relatively low, this error should be small because of the low hydraulic conductivity in advance of the water table.

Another aspect of unsaturated flow neglected by the model is subsequent movement of water into the profile above a final water table position. The parameter a_1 in equation [7] represents the storage volume in the profile below the water table under previous assumptions. However, if a_1 is modified to represent the total volume of water added at equilibrium, equation [7] can be used to approximate water inflow during the entire subirrigation event. Evaluating the parameter a_1 requires a knowledge of $\Theta(h)$ for the profile in question as well as the appropriate initial and boundary conditions.

Uniform soil profiles in the field are rare, thus it is desirable to extend the foregoing model to include cases of layered soil profiles. Referring to Fig. 1 (B), the soil profile is assumed to be composed of two uniform layers. As long as $Z \leqslant L_1$, water movement in the bottom layer is described by equation [7]. However, when $Z > L_1$ the parameters in this equation are no longer valid. To deal with this situation, supose that at $t = t^*$, the water table is located at $Z = L_1$. For $Z > L_1$ the effective hydraulic con-

TABLE 1. SUMMARY OF SUBIRRIGATION TESTS CONDUCTED.

Soil	Initial water table depth	Final water table depth	Number of cores tested
Wagram	76.2	0	4
Wagram	76.2	25.4	4
Lumbee	61.0	0	4
Lumbee	61.0	25.4	4

ductivity, K_e, beneath the water table can be expressed as

$$K_e = \frac{K_1K_2Z}{L_1K_2 + (Z-L_1)K_1} \quad \ldots\ldots\ldots \quad [8]$$

where K_1 is the saturated hydraulic conductivity of the bottom layer and K_2 is that of the top layer. Letting F_s' denote the cumulative water volume flowing into the profile above $z = L_1$ and denoting the corresponding time as t', the analogue of equation [5] is

$$F_s' = (\Theta_{f2} - \Theta_{02})(Z - L_1) \quad \ldots\ldots\ldots \quad [9]$$

where Θ_{f2} is the volumetric water content below the water table in the top layer and Θ_{02} is the corresponding initial water content. Thus

$$\frac{DF_s'}{dt'} = K_e \left(\frac{d-Z}{Z}\right) \quad \ldots\ldots\ldots \quad [10]$$

where k_e is defined in equation [8]. Combining equations [8], [9], and [10] yields

$$f_s' = \frac{dF_s'}{dt'} = \frac{K_2(1 - F_s'/a_2)}{(\beta_2 + F_s'/a_2)} \quad \ldots\ldots \quad [11]$$

where $a_2 = (d - L_1)(\Theta_{02})$ and $\beta_2 = L_1K_2/K_1(d-L_1)$. Separating variables, integrating and requiring $F_s' = 0$ when $t' = 0$ results in

$$\frac{F_s'}{a_2} + (1 + \beta_2)\ln\left(1 - \frac{F_s'}{a_2}\right) = -\frac{K_2}{a_2}t' \quad \ldots\ldots\ldots \quad [12]$$

which approximates water table rise in the top layer.

The model is now capable of estimating water movement during subirrigation for one or two uniform soil layers. For one layer equation [7] is used. When the water table reaches the interface in a layered soil, the cumulative water added to the profile can be denoted as F_s^*. For this case equation [7] is used when $F_s < F_s^*$. When $F_s = F_s^*$, equation [12] is then employed to describe water movement into the top layer. While the details are not presented here, the model can be extended in like manner for as many layers as desired.

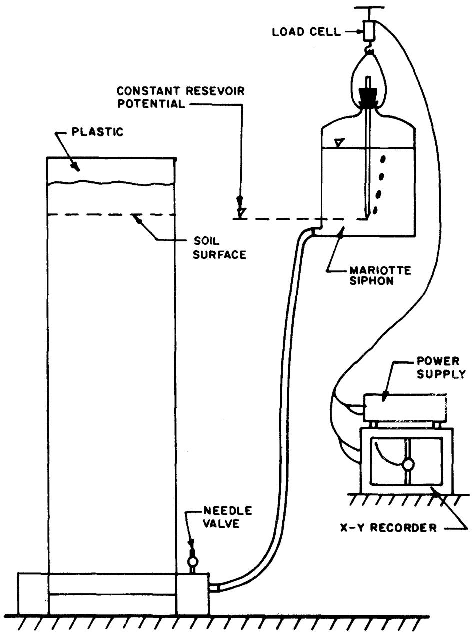

FIG. 2 Illustration of apparatus used for subirrigation volume measurement.

EXPERIMENTAL METHODS

Large undisturbed soil cores, 51 cm in diameter, were collected from two field soils, a Wagram loamy sand with a core depth of 86 cm, and a Lumbee sandy loam with a core depth of 61 cm. The cores were obtained by driving empty 16 gauge galvanized cylinders into the soil with an anchored hydraulic ram device. Upon removal, the cores were brought to the laboratory and placed atop metal bases filled with coarse gravel. The bases were constructed with ports for water addition and removal and evacuation of air. Five cores were collected for each soil type in a field proximity of less than 9 m.

Subirrigation tests were conducted on each core. The initial condition was a profile drained to equilibrium above a water table near the base of the core. Tests conducted for each soil are summarized in Table 1. Additional tests were conducted for initially dry cores but were not replicated. The results of these tests along with details of the experimental procedures were described by Wells (1975). Water was introduced through the base of the core from a constant head reservoir which was suspended on a load cell at a specified level (Fig. 2). The load cell output was recorded on an x-y plotter; thus a continuous record of subirrigation volume versus time was obtained. Errors in these plots did not exceed ±0.5 percent for the inflow volume.

The tops of the cores were covered to prevent evaporation. Tensiometers were installed in the Wagram cores at depths of 5.1, 12.7, 22.9, 33.0, 43.2, 53.3, 63.5, and 73.7 cm with an additional tensiometer placed at 3.8 cm from the base of two cores. The placements in the Lumbee soil were at depths of 2.5, 5.1, 12.7, 22.9, 33.0, 43.2, 53.3, and 57.2 cm. The tensiometers were connected to a rotary valve with the common port connected to a pressure transducer. Pressure heads were automatically recorded on teletype punched tape at 15 sec intervals during the experiments and the data were subsequently analyzed on a digital computer. The pressure transducer was calibrated prior to each experiment and the ambient temperature was monitored. Static checks indicated that the combined error associated with leakage and temperature variation did not exceed ±0.5 cm of water in the measurement of pressure head.

Three soil water characteristic determinations were made for each soil. The desorption or drainage branch of the characteristic was determined by a method similar to the

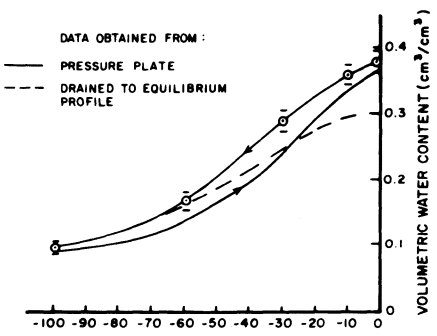

FIG. 3 Soil water characteristic for Wagram Loamy sand [bars indicate ± one standard deviation].

one described by Richards (1965). Small undisturbed soil samples were collected at two depths from three proximate locations at the field sites when the cores were removed. The samples were saturated and placed in a pressure plate apparatus and pressure steps of 2, 10, 30, 60, 100, 200, 400, 600, 800, and 1000 cm water were applied. These data were supplemented by determinations on samples collected from one of the large cores. After experiments were completed, replicate samples 10 cm in diameter and 2 cm deep were taken from the large cores and the imbibition branch of the soil water characteristic measured using the pressure plate apparatus described by Tanner and Elrick (1958). Because of evidence of air entrapment in the large cores which was not reflected in the above determinations on small samples, an effective soil water characteristic was determined directly from a large core for each soil. The core was saturated by raising the water table to the surface from the base; then drained to equilibrium to water table depths of 76.2 cm for Wagram and 61 cm for Lumbee. Triplicate soil samples were taken at the tensiometer depths and the volumetric water content determined. Since the pressure at each depth was known from the equilibrium relationship (and confirmed by tensiometer measurement prior to sampling) the soil water characteristic could be plotted directly for a range in h of 0 to -76.2 cm.

The apparent saturated hydraulic conductivity was determined for each core. Steady pressure head profiles were measured using tensiometers to determine possible variation of conductivity along the length of the cores.

The hydraulic conductivity-pressure head relationship, K(h), was determined using a method similar to that described by Nielsen et al. (1973). One core of each soil type was saturated and allowed to drain to a final water table position near the base. Pressure head values at each tensiometer position were continuously measured during the tests. Using this data and the effective Θ(h) relationships for each soil, the flux at each tensiometer position during an arbitrary increment of time was computed and the conductivity was determined from the corresponding measured hydraulic gradients. Data collection between 1 and 4 hr during these tests was used to characterize the K(h) relationship for each soil.

RESULTS AND ANALYSIS

Soil Properties

Soil Water Characteristics

The soil water characteristics are plotted in Figs. 3 and 4 for the Wagram and Lumbee soils respectively. As previously indicated the desorption curves were measured on field samples taken at different depths and locations in the proximity of the cores. Thus these data reflected the field variabilty of the Θ(h) relationship. However the variability associated with different sampling depths was of the same magnitude as that resulting from different proximate locations, so the desorption branches for both soils were obtained by grouping the Θ(h) measurements for all depths and locations. The mean value of Θ and the standard deviation are plotted in Figs. 3 and 4 for each pressure increment. The average standard deviation for the Wagram soil was 0.0281 cm³/cm³ and that for the Lumbee soil was 0.0341 cm³/cm³. These values are within the variability range reported by Nielsen et al. (1973) for a Panoche soil.

The imbibition branches of the soil water characteristics are also plotted in Figs. 3 and 4. As expected (see, for example, Topp and Miller 1966) both soils exhibit hysteresis and the water content corresponding to h = 0 on the imbibition curve was less than the saturated value. This difference is due to a small amount of entrapped air which is not present when samples are slowly saturated or when saturated under suction. However, early subirrigation experiments on the Wagram soil indicated that air entrapment is of much greater significance than would be expected from these data. Specifically, soil core 1, initially drained to equilibrium with the water table 76.2 cm deep, was wetted from the base by positioning a constant potential reservoir at the soil surface. The volume of water required to raise the water table to the surface was equivalent to a depth of 5.2 cm. This was compared to predicted volumes of 10.4 cm when the drainage branch of Θ(h) was assumed and 10.3 cm when the imbibition branch was used. The difference was attributed to air entrapment and an effective Θ(h) relationship determined from a drained to equilibrium profile as discussed previously. The effective Θ(h) relationships are shown as the broken curves in Figs. 3 and 4. Since these curves were determined from a drained profile, they represent drainage branches of Θ(h) relationships which may also exhibit hysteresis. However a more important implication is that under the conditions tested here, the Θ(h) relationship is not unique. That is, because of air entrapment, the Θ(h) relationship measured in small samples is apparently different than that which exists in large cores under subirrigation conditions. Such observations are not new. Philip (1957) concluded that the transition zone observed in the infiltration studies of Bodman and Colman (1943) was due to the non-uniqueness of the soil water characteristic. He hypothesized that the non-uniqueness was due to air entrapment and suggsted that Θ(h) measurements be conducted on samples large enough to incorporate these effects.

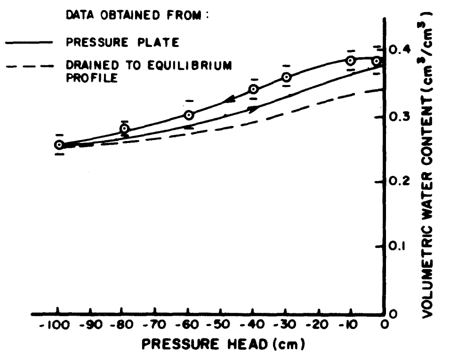

FIG. 4 Soil Water characteristic for Lumbee sandy loam [bars indicate ± one standard deviation].

The broken curves in Figs. 3 and 4 were used as the effective relationships for all subsequent soil property determinations and calculations requiring Θ(h). Based on this relation-ship for Wagram the volume of water required to raise the water table from a 76.2 cm depth to the surface was 5.7 cm compared to the observed 5.2 cm. This represents an overprediction of

TABLE 2. APPARENT SATURATED HYDRAULIC CONDUCTIVITIES.

Wagram loamy sand		Lumbee sandy loam	
core	K_e (cm/hr)	core	K_e (cm/hr)
1	5.92	1	21.3
2	7.66	2	11.4
3	8.49	3	13.2
4	4.21	4	1.18

the observed inflow by about 10 percent as compared to 100 percent when air entrapment is neglected.

To characterize the amount of entrapped air escaping via diffusion over long periods of time, a Wagram core, initially drained to equilibrium above a water table 76.2 cm deep, was saturated from the base in the same manner as described above. After the water table had risen to the surface and inflow ceased, the total influx volume was noted and recorded. The supply reservoir was maintained in position such that water could enter the core and replace air that slowly diffused out. After 22 days only 0.8 cm of water had entered the profile, indicating that entrapped air is released very slowly via diffusion.

Hydraulic Conductivity

Values of the apparent saturated hydraulic conductivities measured for each core used in this study are compiled in Table 2. The cores were wetted under the normal subirrigation process so the apparent values given in Table 2 include the effects of entrapped air as previously discussed. These values are somewhat lower than the actual saturated K values which would have been obtained if the cores had been saturated under suction to remove all of the air. The results indicate significant variability within relatively close field proximity for both soils. Steady pressure head profiles were measured and used to characterize stratification in the cores with respect to apparent saturated hydraulic conductivity. Specifically, core 1 of the Wagram soil had a conductivity, $K_2 = 6.49$ cm/hr in the top 73.6 cm and $K_1 = 3.98$ cm/hr in the remaining portion of the profile. Likewise core 2 of the Lumbee soil was characterized in the top 43.2 cm by $K_2 = 9.34$ cm/hr and in the remaining portion of the profile by $K_1 = 25.1$ cm/hr. These descriptions are used to approximate stratification effects in subsequent analysis.

Hydraulic conductivity-pressure head relationships are plotted in Figs. 5 and 6. Core 1 of the Wagram and core 2 of the Lumbee were used in determining these K(h) relationships. The value of K corresponding to $h = 0$ for each soil is the apparent saturated hydraulic conductivity listed in Table 2. The shaded portions of Figs. 5 and 6 represent ± one standard deviation of the measured K values at various h levels.

Subirrigation Experiments

Subirrigation experiments were conducted on all soil cores with the profiles initially drained to equilibrium above a water table 76.2 cm deep for the Wagram soil and 61 cm deep for the Lumbee. The cumulative subirrigation volume-time relationship, $F_S(t)$, was measured for each core and is plotted in Fig. 7 for the Wagram soil. The tests were conducted until inflow ceased and the total hydraulic head, as indicated by the tensiometers, was constant throughout the profile. The final total influx is indicated by the slashed symbols at the right margin of Fig. 7 and ranged from 4.27 cm for core 3 to 5.27 cm for core 2. These values may be compared to the theoretical storage volume of 5.7 cm which was computed from the soil water characteristic (broken curve in Fig. 3). Tests were repeated on some of the cores and the results were found to be reproducible within 4 percent of the total influx volume at any time. The reduced subirrigation volume was obtained by dividing the volume by the total for each core. The results are plotted in Fig. 8 and show, at $t = 1.5$ hr, a difference in reduced volume between cores of 13 percent as compared to 22 percent when the effect in total inflow volumes was not removed. This demonstrates that differences in the volume-time relationships between cores were not solely due to variation in the total available storage volume but were also dependent on the variation of other factors such as the hydraulic conductivity.

Predicted $F_S(t)$ relationships were obtained by solving equation [1] numerically and by using the approximate subirrigation model. Equation [7] was employed under the assumption that the profile was uniform. The saturated hydraulic conductivity was taken as the apparent value plotted for $h = 0$ in Figs. 5 and 6 for the two soils. Analysis of water content-pressure head data indicated little stratification insofar as the Θ(h) relationship is concerned. However, there was evidence of stratification with respect to apparent saturated hydraulic conductivity, as previously discussed. Using equations [7] and [12] it was possible to consider this effect by employing a two-layer model. For both the one-layer and two-layer models, the parameters were either measured directly or computed using the Θ(h) relationship and are compiled in Table 3. For the two-layer case, the subscripts 1 and 2 refer to the lower and upper layers, respectively.

Predicted relationships for cumulative subirrigation volume versus time, $F_S(t)$, are presented for the Wagram soil in Fig. 7. The numerical solution to equation [1] agrees well with observations for small times, but tends to overpredict as time increases. Conversely, the approximate models overpredict in the initial stages and become more accurate for the in-

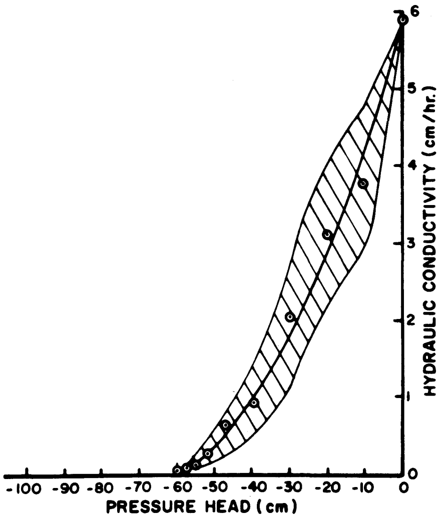

FIG. 5 Hydraulic conductivity-pressure head relationship for Wagram loamy sand [Values of K only approach zero].

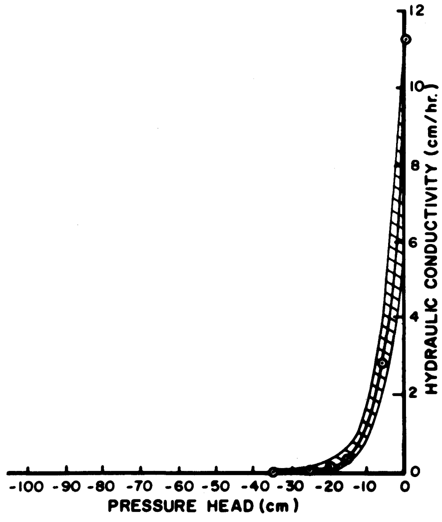

FIG. 6 Hydraulic conductivity-pressure head relationship for Lumbee sandy loam Values of K only approach zero].

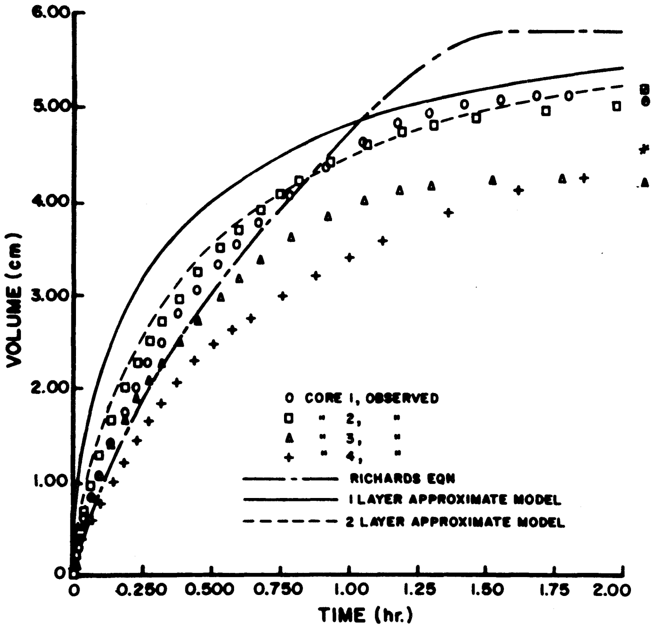

FIG. 7 Cumulative subirrigation volume versus time, Wagram loamy sand, initial water table depth equal 76.2 cm, final water table level at soil surface. [Slashed symbols at right margin represent equilibrium values.]

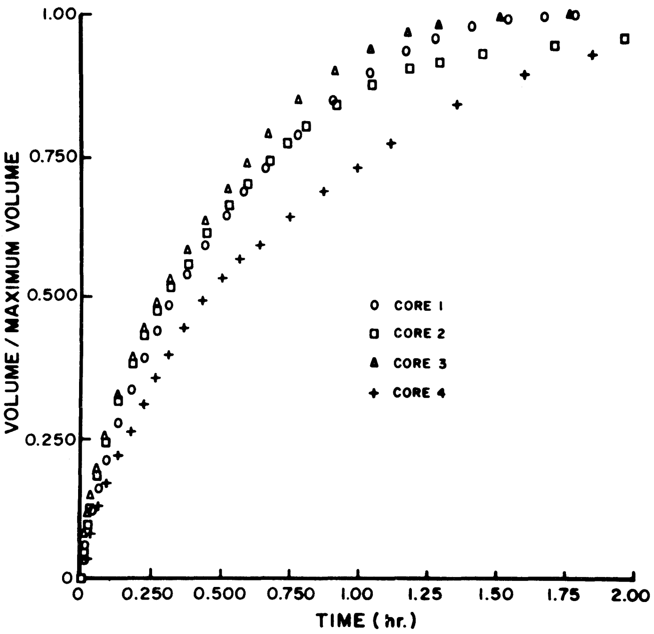

FIG. 8 Reduced subirrigation volume versus time, Wagram loamy sand.

creasing times. This is probably due to the fact that the initial water content was non-uniform, corresponding to a drained to equilibrium profile, whereas a uniform initial water content was assumed in the derivation of the equation. The agreement between measured results and predictions of the various models was quantified by computing an estimate or error for each combination of observed and predicted $F_s(t)$ relationships presented in Fig. 7. The estimate of error, Φ, was defined in the same manner as the standard error of estimate for regression equations and was computed using the formula

$$\Phi = [\sum_{i=1}^{N} (F_{si} - \hat{F}_{si})^2/(N-1)]^{1/2} \quad \ldots [13]$$

where N is the total number of observations for all cores and F_{si}, $\hat{F}_{si}$ are observed and predicted values of total influx, respectively. The estimates of error for each model are as follows for the Wagram soil: 0.64 cm for equation [1], 0.44 cm for the two-layer model, and 0.87 for the one-layer model. The estimate of error is biased toward agreement during initial stages because of more frequent observations for small times.

Measured cumulative subirrigation volume-time relationships for the Lumbee soil are presented in Fig. 9. These results show considerable variation in the $F_s(t)$ relationships. For example, at t = 1.5 hr, there is a difference of 53 percent between cores 2 and 4. When the variation in the $F_s(t)$ relationships due to total water influx (as shown in Fig. 9 for Wagram) was removed, the results showed a maximum difference of 31 percent between cores for t = 1.5 hr.

Theoretical $F_s(t)$ relationships were obtained in the same manner as discussed for the Wagram soil. The values of parameters used in the approximate subirrigation models are listed in Table 3. The predicted subirrigation volume-time relationships are plotted in Fig. 9. The total predicted subirrigation volume for each theoretical model is 2.13 cm. As was the case for Wagram, the numerical solution to equation [1] agrees well with observations for small times but tends to predict complete saturation of the profiles much sooner than was observed. The approximate models seem to conform more closely to the observed $F_s(t)$ relationships for intermediate times with the one layer version appearing to show the best agreement. The estimates of error were as follows: 0.59 cm for equation [1], 0.46 for the one-layer model, and 0.90 for the two-layer model.

An analysis of the results presented in Figs. 7 and 9 indicate that subirrigation relationships predicted by the approximate equations are in somewhat better agreement with experimental results than are the solutions to the Richard's equation. This is probably due to the failure to consider hysteresis in the soil properties used in the Richard's equation. Although subirrigation is an imbibition process, both the effective Θ(h) and K(h) were obtained from drainage events. Because the values of both Θ and K corresponding to a given h are higher for the drainage than for the

TABLE 3. SOIL PARAMETERS FOR THE APPROXIMATE SUBIRRIGATION MODEL.

Soil	Final water table depth	Model	K_1(cm/hr)	K_2(cm/hr)	a_1(cm)	d(cm)	L_1(cm)	F_s*(cm)
Wagram	0.0	1 layer	5.92		5.70	86.9		
”	0.0	2 layers	3.98	6.49	”	”	13.3	0.05
”	25.4	1 layer	5.92		5.46	61.5		
”	25.4	2 layers	3.98	6.49	”	”	13.3	0.05
”	25.4	1 layer	5.92		4.16+	”		
”	25.4	2 layers	3.98	6.49	”	”	13.3	0.05
Lumbee	0.0	1 layer	11.45		2.13	61.0		
”	0.0	2 layers	25.10	9.34	”	”	17.8	0.12
”	25.4	1 layer	11.45		1.83	35.6		
”	25.4	2 layers	25.10	9.34	”	”	17.8	0.12
”	25.4	1 layer	11.45		0.78+	”		
”	25.4	2 layers	25.10	9.34	”	”	17.8	0.12

+ hysteresis effects estimated

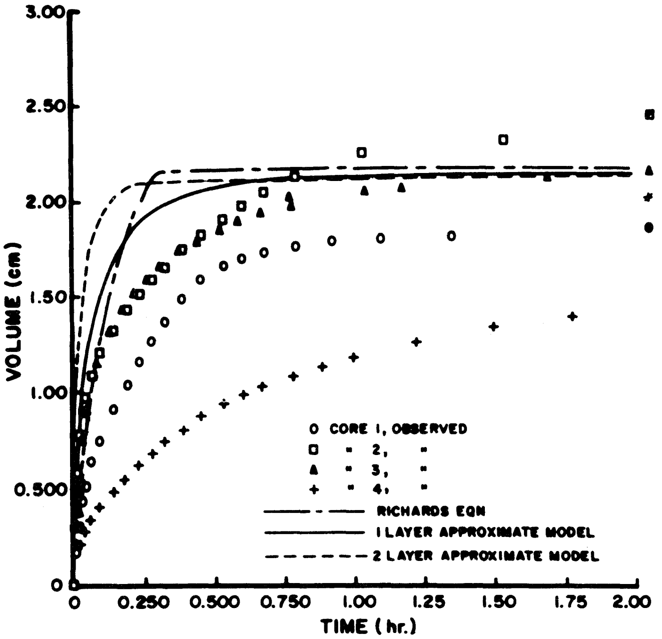

FIG. 9 Cumulative subirrigation volume versus time, Lumbee sandy loam, initial water table depth equal 61 cm, final water table level at soil surface. [Slashed symbols at right margin represent equilibrium values.]

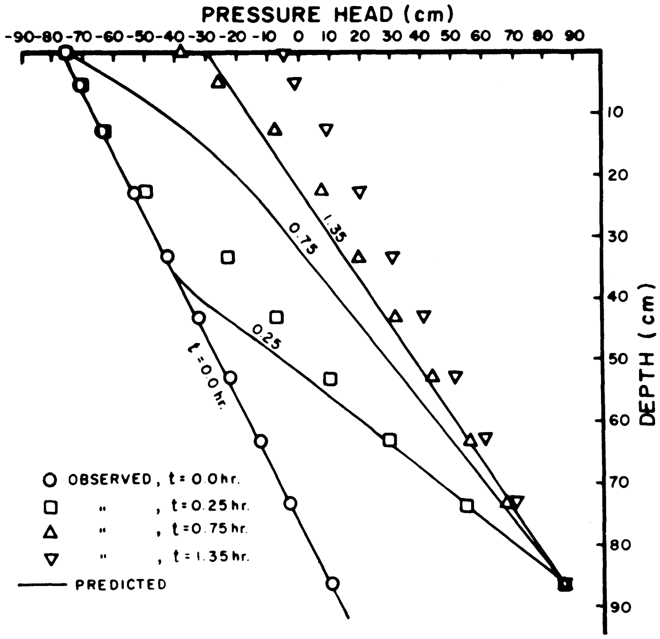

FIG. 10 Observed and predicted pressure head profiles during subirrigation, Wagram loamy sand.

imbibition branch, use of the drainage properties results in over-estimating both total volume and rate of upward water movement. Thus use of the imbibition soil properties would tend to improve the predictions of the Richard's equation in Figs. 7 and 9. However it is important that these properties reflect air entrapment as previously discussed. For example, the proper soil water characteristic for describing the subirrigation process in Wagram would be an imbibition relationship corresponding to the broken curve in Fig. 3. Note that the use of such a relationship would probably also improve the fit of the approximate equation because it would reduce the value of a_1 and therefore the total predicted inflow volume. In general it appears that use of the approximate equations would be acceptable for engineering purposes. While it may be possible to obtain a somewhat better characterization of the subirrigation process by numerical solutions to the Richards equation, the input requirements; i.e., effective relationships for Θ(h) and K(h); make it difficult to use this method. In view of this fact and of the field variability exhibited in Figs. 7 and 9, it appears that use of the Richards equation will not be justified for most field situations.

Observed pressure head profiles during subirrigation are shown for core 1 of Wagram in Fig. 10. Also, predicted profiles obtained from solutions of equation [1] are presented for the same times. Pressure heads at all points tended to rise more rapidly than predicted. Vachaud et al. (1972) showed that positive air pressure may exist as a wetting front approaches a less permeable soil stratum. This would have the effect of increasing the pressure head below such a layer and therefore causing differences of the type shown in Fig. 10. However, there was no evidence of layering that would explain the disagreement between observed and predicted results.

On two cores of each soil the water table was raised to a final position of 25.4 cm below the soil surface. The initial condition for these tests was an equilibrium profile above a water table located 76.2 cm deep for Wagram and 61 cm deep for Lumbee. The tests were continued until inflow ceased and the tensiometer reading indicated that the profile was at equilibrium above the final water table position.

A numerical solution of equation [1] subject to the above conditions was obtained for each soil type. Values of the parameter a_1 used in the approximate subirrigation models were computed from the Θ(h) relationship for each soil by assuming the profile would eventually reach a "drained to equilibrium" condition above the final water table position. These values are listed in Table 3.

The resulting predictions and observations of the $F_S(t)$ relationship for Wagram are shown in Fig. 11. Total predicted subirrigation was 5.46 cm. Similar results were found for Lumbee, where the total predicted subirrigation volume was 1.83 cm. The models substantially overpredict the total observed water influx in both soil types. The predicted final subirrigation volume is 24 percent greater than the mean of the observed values for Wagram and 103 percent greater for Lumbee.

A possible explanation for these differences is a failure to recognize substantial non-zero air entry suction in the Θ(h) relationsips. However, evidence of this was not found in any of the Θ(h) measurements, either by using pressure plates or by sampling from profiles which were drained to equilibrium. Furthermore, Nielsen et al. (1973) reported that field samples studied were not characterized by non-zero air entry suction such as would be expected for packed columns of porous material.

Hysteresis in the Θ(h) relationships provided an explanation of the discrepancies between observed and predicted total inflow volumes. Fig. 12 illustrates the water content distribution in a core drained to equilibrium with the water table initially at z_1. After the water table is raised to z_f and equilibrium is reached, the final Θ(z) distribution predicted by using a Θ(h) relationship determined under drainage conditions (with no provision for hysteresis) is shown by the solid curve. An alternative Θ(z) distribution showing hysteresis effects is illustrated by the broken curve; both profiles are identical below z_f. The total volume of

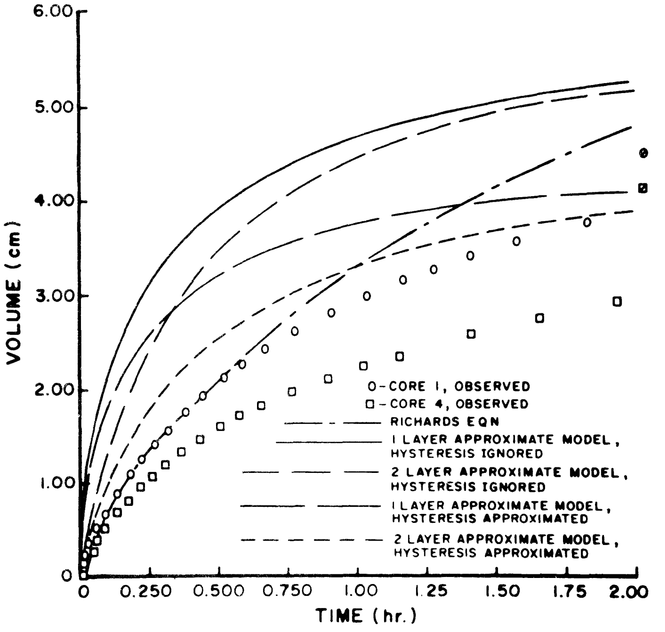

FIG. 11 Cumulative subirrigation volume versus time, Wagram loamy sand, initial water table depth equal 76.2 cm, final water table depth equal 25.4 cm. [Slashed symbols at right margin represent equilibrium values.]

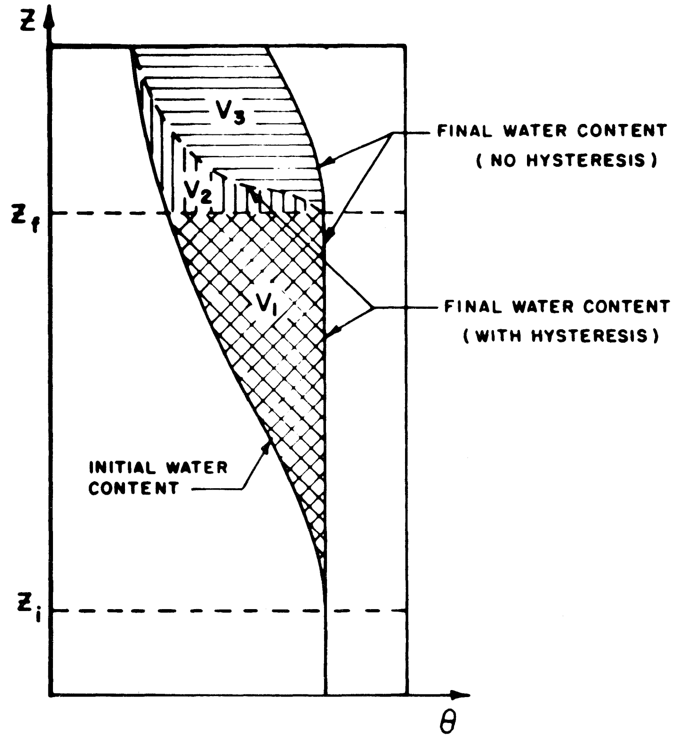

FIG. 12 Illustration of possible hysteresis effect in the Θ [h] relationship upon final water content distributions after subirrigation.

water entering the core predicted by the drainage Θ(h) relationship is represented by $V_1 + V_2 + V_3$, as compared to $V_1 + V_2$ if hysteresis is considered.

The maximum effect of hysteresis under such subirrigation conditions would be to assume that no water enters the profile above the final water table positions, i.e. $V_2 = 0$. The total volume entering the profile shown in Fig. 12 is then represented by V_1. This volume was computed using the Θ(h) relationships for both soils and was found to be equal to 4.16 cm for Wagram and 0.78 cm for Lumbee. These volumes are 6 percent less than the observed mean for Wagram and 13 percent less than the observed mean for Lumbee. Although this approximation overestimates hysteresis effects, the agreement is much better than when such effects are ignored.

The approximate subirrigation models were employed to estimate the effect of hysteresis as discussed above. The values of parameters associated with both 1 and 2 layer models are listed in Table 3 and the predicted relationships are given in Fig. 11. For Wagram the estimate of error was 0.63 cm for the 2-layer model and 1.06 cm for the 1-layer model. The corresponding values for Lumbee were 0.17 cm for both the 2-layer and 1-layer models. The corresponding estimates of error are greater by a factor of approximately 2 for Wagram and 5 for Lumbee when hysteresis is not considered. Despite this improvement in accuracy, the approximate models predict that the process occurs more rapidly than observations indicate.

SUMMARY AND CONCLUSIONS

Subirrigation experiments were conducted under various initial and boundary conditions using large, undisturbed soil cores from two field soils. The pressure head distribution and flow volume were measured continuously during each test. The desorption and imbibition branches of the soil water characteristic were determined using pressure plates. The effect of air entrapment on Θ(h) was determined by collecting gravimetric samples from profiles of each soil type drained to equilibrium above a fixed water table position. The relationship between hydraulic conductivity and pressure head, K(h), was determined for each soil type from transient pressure head measurements during a drainage event.

An approximate model was developed to describe vertical water movement during subirrigation. The model assumed that the water table rises uniformly in the profile, making no provision for water movement in advance of the water table. All of the resulting soil parameters can be measured independently or calculated from the Θ(h) relationship. The model is capable of considering profile stratification. Also the Richards equation was solved numerically for the boundary and initial conditions imposed in the experiments. Predictions of the theoretical models were compared with experimental results for the various subirrigation conditions on both soil types.

The conclusions of the study are as follows:

1 Substantial soil variability was found for both field soils examined in this study. This variability was evident in both measured soil properties and in water movement phenomena observed during the tests. Even though the core samples were collected in relatively close field proximity, results indicate variability similar to that reported by Nielsen et al. (1973) where tests were conducted over a much larger area.

2 The approximate subirrigation model provided acceptable agreement with the observations considering the variability between the soil cores. In general it was in better agreement with observations than solutions to the Richard's equation, although this may have been due to the fact that hysteresis effects were not considered. Consideration of soil stratification generally improved the accuracy of predictions.

3 In view of significant field variability of the soil properties it is

not evident that sophisticated approaches, such as numerical solutions to the Richard's equation which require substantial time and expense, are more desirable than approximate models in characterizing water movement for engineering design purposes.

4 Determination of the total volume of water that will be stored in a profile under specified initial and boundary conditions is essential to the characterization of transient water movement in field soils. Unless this volume is accurately evaluated the choice of a particular model for predicting water movement is of little consequence. Thus it seems that primary attention must be given to evaluating water retention and release in field soil if good engineering designs are to be achieved.

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