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THE productive capacity of agricultural lands is enhanced by timely application and removal of water. The characterization of water movement in field soils is required for the efficient design of irrigation and drainage systems. Studies conducted by soil scientists and engineers have led to a significant body of knowledge concerning water movement in soils. For example, subsurface drainage can be characterized by numerically solving nonlinear partial differential equations requiring complex inputs in terms of soil properties and boundary conditions, or by employing one of the several approximate but less sophisticated theories. In most cases, the alternative which provides the best basis for engineering design is not evident. When field variability of the soil regime is considered, the advantages of more sophisticated methods may be negated insofar as engineering design is concerned.

In this study, one-dimensional water movement during drainage was examined experimentally using large field cores. The cores were 51 cm in diameter and were considered large enough to incorporate heterogenities such as worm holes and plant roots, yet small enough to bring into the lab where experimental measurements could be made under controlled conditions. The objective of the study was to evaluate alternate methods of characterizing one-dimensional drainage in natural soils with relatively shallow water tables.

BACKGROUND

The so-called "exact" method of describing vertical water movement in porous media employs the Richards equation which may be written as:

$$C(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left[K\frac{\partial h}{\partial z}\right] - \frac{\partial K}{\partial z} \quad \ldots\ldots [1]$$

where h is the pressure head, z is vertical displacement measured positively downward from the surface and t is time. The hydraulic conductivity, K, and volumetric water content Θ, are functionally related to pressure head and $C(h) \equiv d\Theta/dh$ is defined as the water capacity function.

Solutions to equation [1] require that the hydraulic conductivity function, K(h), and the soil water characteristic, $\Theta(h)$, be specified. The field-effective relationships for these properties are difficult to obtain and may represent a significant cost to the design of a water management system. Furthermore, due to the nonlinearity of these functions, only numerical solutions have been achieved for most cases of interest. Day and Luthin (1956) developed a numerical solution to equation [1] for the case of vertical drainage from an initially saturated soil column and the results were generally consistent with observations using a fine sand. Remson et al. (1965) presented a numerical solution for the case of a specified initial Θ-distribution and specified values of Θ maintained at the vertical boundaries of the medium. Whisler and Watson (1968) solved equation [1] numerically subject to zero flux at the surface and a fixed water table depth. This analysis compared favorably with other published solutions to the Richards equation under comparable conditions.

Less sophisticated approximate methods have been developed to describe the drainage process. Such methods require simpler inputs but are less general in their application. Youngs (1960) developed an approximate equation based on the capillary tube model employed by Green and Ampt (1911) and Philip (1954). The model assumes that a drainage front of constant pressure head proceeds vertically into the soil as it drains from an initially saturated state and that soil voids drain uniformly behind the front. The resulting expression is:

$$1 - D/D_\infty = \exp\left(-\frac{K_s}{D_\infty}t\right) \quad \ldots\ldots\ldots [2]$$

where D is the cumulative drainage volume, D_∞ is the total amount of water which will drain from the soil, and K_S is the saturated hydraulic conductivity of the soil profile.

Jackson and Whisler (1970) extended Youngs' approach to include consideration of non-constant hydraulic conductivity. Solutions analogous to equation [2] were obtained for cases where, (a) conductivity decreases linearly, and (b) conductivity decreases quadratically with increasing depth of the drainage front. In both cases, conductivity is assumed to decrease from a maximum equal to the saturated hydraulic conductivity, K_S. The resulting expressions are:

$$\frac{l_1}{l_2}\left(1-\frac{l_1}{l_2}\right)\left[\frac{D/D_\infty}{1-D/D_\infty} + D/D_\infty\right] - \left[\left(1-\frac{l_1}{l_2}\right)^2 + \left(\frac{l_1}{l_2}\right)^2\right]\ln(1-D/D_\infty) = \frac{K_s t}{D_\infty} \quad \ldots\ldots [3]$$

for the linear case, and

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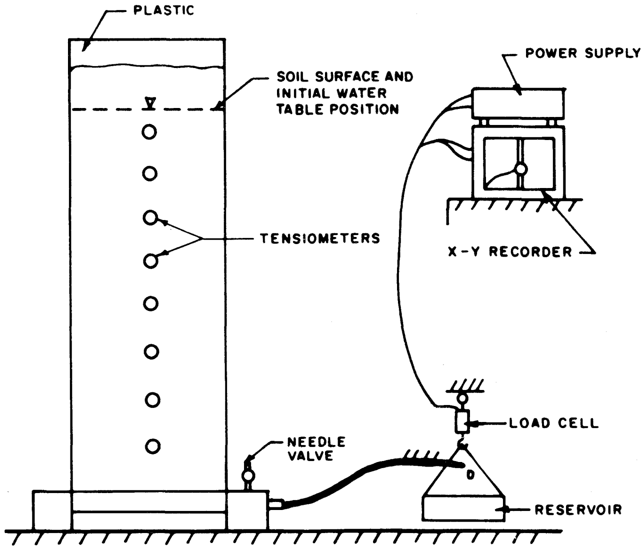

FIG. 1 Illustration of apparatus for measuring drainage volume.

$$\left(1-\frac{l_1}{l_2}\right)^2\left[\frac{D/D_\infty}{1-D/D_\infty}\right]+\frac{1}{2}\frac{l_1}{l_2}\left(1-\frac{l_1}{l_2}\right)\left[\frac{1}{(1-D/D_\infty)^2}+2\,D/D_\infty-1\right]$$

$$-\left(\frac{l_1}{l_2}\right)^2\ln(1-D/D_\infty)=\frac{K_s t}{D_\infty} \qquad [4]$$

for the quadratic case. The parameter l_2 represents the length of the soil profile (measured from the base) which, if saturated would contain all the water initially in the soil profile. Similarly, l_1 represents the length of profile which, if saturated, would contain all the later remaining in the profile at equilibrium. Both l_1 and l_2 can be determined from the soil water characteristic, $\Theta(h)$.

EXPERIMENTAL PROCEDURE

In order to evaluate the various methods of quantifying drainage, a series of experiments were conducted. Large undisturbed soil cores, 51 cm in diameter, were collected from two field soils, a Wagram loamy sand and a Lumbee sandy loam with uniform core depths of approximately 86 and 61 cm, respectively. The cores were obtained by driving 16 gauge galvanized cylinders into the soil with an anchored hydraulic ram device with minimal disturbance of the natural soil profiles. All cores were collected in a field proximity of less than 9 m for each soil. Upon removal they were brought to the laboratory and placed atop gravel-filled metal bases.

Desorption soil water characteristics, $\Theta(h)$, were determined experimentally for each soil type with pressure plates using a method similar to the one described by Richards (1965). Small undisturbed samples collected at various depths and field locations were used for these determinations. The hydraulic conductivity-pressure head relationships were determined for a single core of each soil type using a method similar to that of Nielsen et al. (1973). Also, the apparent saturated hydraulic conductivities of each core used in the experiments was determined by measuring steady-state flux under flooded conditions. Details of the experimental methods related to determination of soil properties are presented by Wells and Skaggs (1976).

Drainage experiments were conducted using four cores of each soil type. The initial condition was achieved by raising the water table to the surface or some specified depth near the surface via subirrigation. With the top of the containers covered to prevent evaporation, water was drained through a non-restrictive opening at the core base and diverted to a collection reservoir suspended on a load cell. The drainage volume was thereby continuously measured and recorded. It was determined that the errors associated with any point on the measurements of the drainage volume-time relationships did not exceed ±0.5 percent for volume and ±0.01 percent for time. The experimental apparatus for the drainage experiments is illustrated in Fig. 1.

During the experiments, pressure head values were recorded continuously by repeated scans of tensiometers emplanted in the soil cores at approximately 10 cm increments. The tensiometers were scanned and recorded automatically at 15-sec intervals during the tests. Static checks indicated the errors associated with these readings did not exceed ±0.5 cm of water. Also, the initial condition for each test was verified using tensiometers. The tests were not begun until the pressure head at ech tensiometer position was within ±1 cm of the appropriate equilibrium value. It should be noted that there was no evidence of seepage along the soil-container interface in any of the experiments.

APPLICATION OF THEORY

The Richards equation was solved subject to the following boundary conditions:

$$\begin{aligned}
h &= z & &;\ t = 0,\ 0 \leq z \leq L\\
q &= -K(h)\left[\frac{\partial h}{\partial z}-1\right]_{0,t} = 0 & &;\ t > 0,\ z = 0\\
h &= d & &;\ t > 0,\ z = L
\end{aligned} \qquad [5]$$

where L is the total depth of the profile, q is flux, and d is the height of the water outlet above z = L. An implicit numerical finite difference scheme (Skaggs et al. 1970) was employed to solve equation [1] subject to conditions 5.

The approximate drainage models represented by equations [2], [3], and [4] were employed to characterize vertical drainage for boundary conditions 5. The parameters associated with the drainage models were estimated from the initial and boundary conditions, length of the soil profiles, and the desorption soil water characteristic, $\Theta(h)$, for each soil type.

RESULTS AND ANALYSIS

Desorption soil water characteristic determinations indicated that variability in the $\Theta(h)$ relationships resulting from different sampling depths as opposed to that resulting from different proximate locations was of the same order of magnitude. Thus the

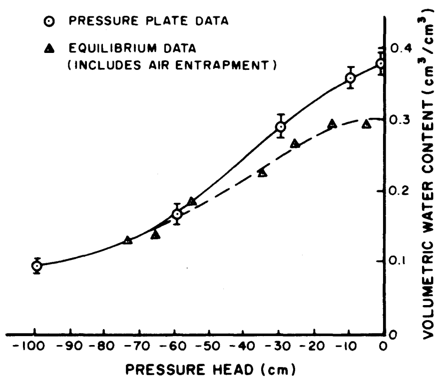

FIG. 2 Soil water characteristic for Wagram loamy sand [bars indicate ± one standard deviation].

Θ(h) relationships for both soils were determined by grouping Θ(h) measurements for all depths and locations; the mean Θ values are plotted for Wagram as the solid line of Fig. 2. The standard deviation was computed for each pressure increment and is also shown in Fig. 2. The average standard deviation for the Wagram soil was 0.0281 cm^3/cm^3 and that for the Lumbee soil was 0.0341 cm^3/cm^3. These values are within the variability range reported by Nielsen et al. (1973) for a Panoche soil.

Preliminary experiments indicated that a significant amount of air was trapped as the cores were saturated from a "drained to equilibrium" condition. Air entrapment reduces the volume of water that can be stored in the soil profile and has a significant effect on infiltration and drainage in shallow water table soils. For example, the total available storage predicted from the "solid-line" Θ(h) curve for Wagram (Fig. 2) was 100 percent higher than the observed when the water table was raised from an equilibrium position near the base to a final position at the soil surface. The effect of this phenomenon was discussed in detail by Wells and Skaggs (1976). An effective Θ(h) relationship which accounts for air entrapment was defined and is shown for Wagram by the broken curve in Fig. 2.

Values of "effective saturated hydraulic conductivities", K_e, are given in Table 1 for all cores used in the experiments. The term "effective saturated hydraulic conductivity" is used because air entrapment during the wetting process causes the hydraulic conductivity for flooded conditions to be somewhat less than would be obtained if the cores were completely saturated. Further, the K_e values were obtained from overall hydraulic gradients and thus represent the core as a whole rather than the conductivity at a given point within the core. The results compiled in Table 1 indicate substantial field variability in K_e within a relatively close proximity for both soils.

The hydraulic conductivity-pressure head relationship determined from core 1 of the Wagram soil is presented in Fig. 3. The shaded area represents ± one standard deviation from the mean K values. The K value at h = 0 in Fig. 3 is 5.92 cm/hr and corresponds to effective saturated conductivity of core 1 (Table 1). The

TABLE 1. MEASURED EFFECTIVE SATURATED HYDRAULIC CONDUCTIVITIES

Wagram loamy sand		Lumbee sandy loam	
Core	K_e(cm/hr)	Core	K_e(cm/hr)
1	5.92	1	21.32
2	7.66	2	13.16
3	8.49	3	11.45
4	4.21	4	1.18

Θ(h) and K(h) relationships for Lumbee as well as details of the variability associated with determinations of these functions and the effects of air entrapment on the relationships are presented elsewhere (Wells and Skaggs 1976).

The cumulative drainage volume, D(t), is plotted for Wagram in Fig. 4. The results show considerable core variability with respect to the drainage volume-time relationship. The variation between cores shown in Fig. 4 appears to be primarily due to two factors, variation in hydraulic conductivity and in the total amount of water that can be drained from each core for the above conditions. The effect of the latter factor can be removed for purposes of analysis by dividing the cumulative volume at any time, D(t), by the total volume drained from the respective core. This reduced drainage volume is plotted versus time in Fig. 5. For Wagram soil the difference at t = 3 hr between reduced volumes for the cores was 34 percent as opposed to a 51 percent difference when the effect of dif-

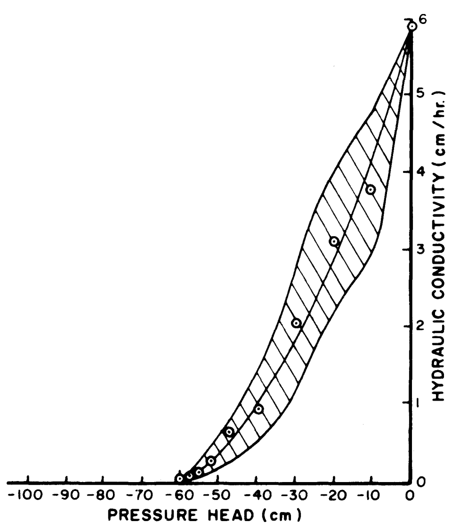

FIG. 3 Hydraulic conductivity-pressure head relationship for Wagram [values of K only approach zero].

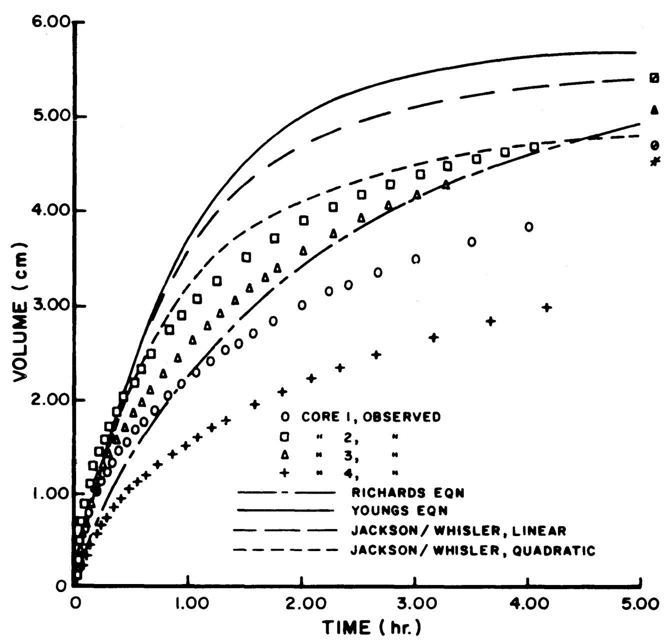

FIG. 4 Cumulative drainage volume versus time for Wagram, initially saturated with water exiting at 76.2 cm [slashed symbols at right margin represent equilibrium values].

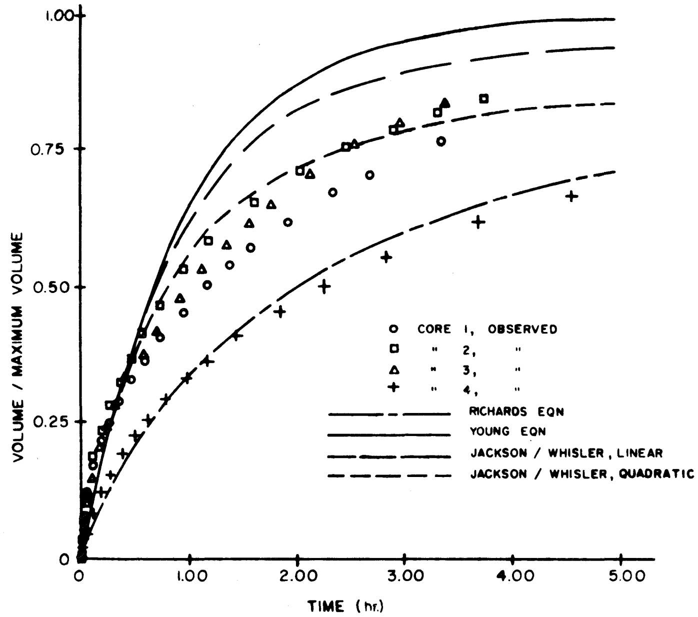

FIG. 5 Reduced drainage volume versus time, Wagram.

ferences in the total drainage volume was not removed. While the variation between cores is reduced in Fig. 5, other factors also contribute to the variation shown in Fig. 4. Differences in K_e, which is two-fold between cores 3 and 4, is a major cause of the observed variation.

The h-based form of the Richards equation (equation [1]) was solved subject to conditions 5. In addition, the approximate drainage model presented by Youngs (1960), equation [2], and the two approximate equations suggested by Jackson and Whisler (1970), equations [3] and [4], were solved for these boundary conditions. With the exception of K_e, the parameters used in these models were determined from the Θ(h) relationship given by the broken curve in Fig. 2. Values of the parameters for the approximate drainage equations are compiled in Table 2 for both soils and for all drainage cases examined in this study. It should be noted that these values were independently determined and are not empirically derived from the drainage experiments illustrated in Fig. 4.

Predicted D(t) relationships for the various theoretical models are presented in Fig. 4 for the Wagram soil. The total drainage volume predicted by each model is 5.7 cm, as determined from the Θ(h) relationsip given in Fig. 2. For this case, the exact solution appears to provide somewhat better agreement with the observations than the other methods. The approximate models agree well with observations for small times but overestimate the drainage volume as time increases. Jensen et al. (1967) proposed that equation [2] should be accurate in describing "primary" drainage, i.e. when the water table falls rapidly due to drainage of large pores. During the remaining or "secondary" drainage, water held in smaller pores is released more slowly than is predicted by the capillary tube model of Youngs. The models proposed by Jackson and Whisler address this problem in that the effective hydraulic conductivity is reduced as water table falls. Among the approximate models, the Jackson-Whisler equation (equation [4]) which assumes a quadratic reduction of K_e provides the best agreement with observations.

To quantify the agreement between the predictions of the various theoretical models and measured results, an estimate of error, Φ, was defined as follows:

$$\Phi = \left[\sum_{i=1}^{N} (D_i - \hat{D}_i)^2 / (N - 1) \right]^{1/2} \qquad [6]$$

where N is the total number of observations and D_i, $\hat{D}_i$ are observed and predicted values of cumulative drainage volume, respectively. For a drainage test involving specific boundary conditions and soil type, i.e. Fig. 4, a value of Φ was computed for each of the theoretical models using the data from each of the soil cores tested. For a specific theoretical model, a mean value, $\bar{\Phi}$, was then computed from Φ-values obtained from all cores tested. Relative agreement among the theoretical models presented in Fig. 4 is summarized by values of $\bar{\Phi}$: 0.49 cm for equation [1], 0.66 cm for equation [4], 0.74 cm for equation [3], and 1.16 cm for equation [2]. It should be noted that the "estimate of error" is biased toward agreement during initial stages because of more frequent observations for small times.

TABLE 2.
SOIL PARAMETERS FOR APPROXIMATE DRAINAGE MODELS

	Wagram loamy sand	Lumbee sandy loam
Initial water table depth	0	0
Final water table depth	76.2 cm	61.0 cm
K_e	5.92 cm/hr	11.45 cm/hr
D_∞	5.7 cm	2.13 cm
l_2	76.2 cm	61.0 cm
l_1	57.6 cm	54.8 cm

The predicted D(t) relationships shown in Fig. 4 were divided by the total predicted drainage volume (5.7 cm) as shown in Fig. 5. The results show improved agreement with the variation of the total drainage volume removed. This is not surprising since the total predicted drainage volume is higher than was observed in any of the tests. Values of $\bar{\Phi}$ are summarized as follows: 0.077 for equation 4, 0.127 for equation [3], 0.116 for equation [1], and 0.155 for equation [2]. It is interesting to note that the relative agreement among the theoretical models is somewhat different when the variability of total drainage volume is removed.

The results of experiments conducted on each core of Lumbee soil are presented in Fig. 6. When reduced drainage volumes were obtained (as in Fig. 5 for Wagram) the maximum difference between cores, for example, at t = 2.0 hr was 29 percent as compared to 62 percent for the volumes as plotted in Fig. 6. Thus a larger part (as compared with Wagram) of the variation in the D(t) relationships is due to differences in the total amount of water drained. However, as was indicated in Fig. 5 for Wagram, differences in other soil properties such as K_e contributes substantially to the variation shown in Fig. 6.

Predicted and observed D(t) relationships are presented in Fig. 6 for the Lumbee soil. The total drainage volume, as determined theoretically from the soil water characteristic and given in Table 2, is 2.13 cm. Thus all of the prediction equations, both exact and approximate, will have a final predicted drainage volume of 2.13 cm. The remaining parameters

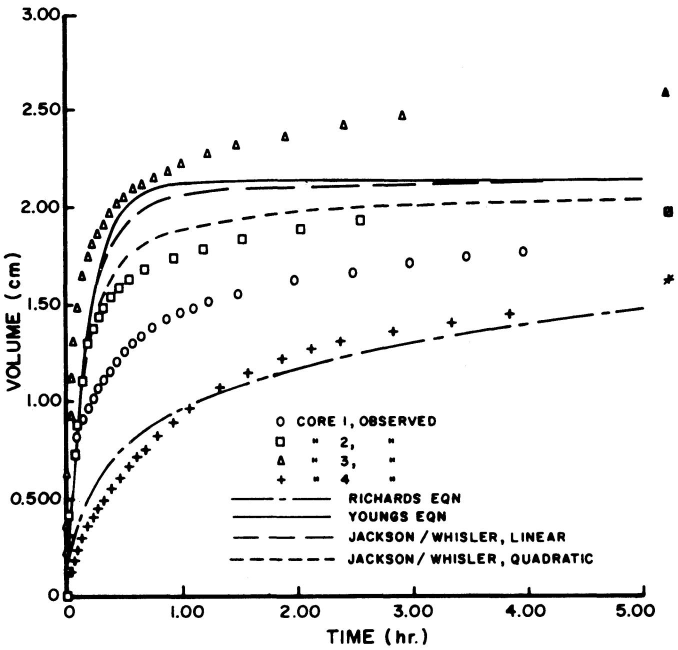

FIG. 6 Cumulative drainage volume versus time for Lumbee, initially saturated with water exiting at 61 cm [slashed symbols at right margin represent equilibrium values].

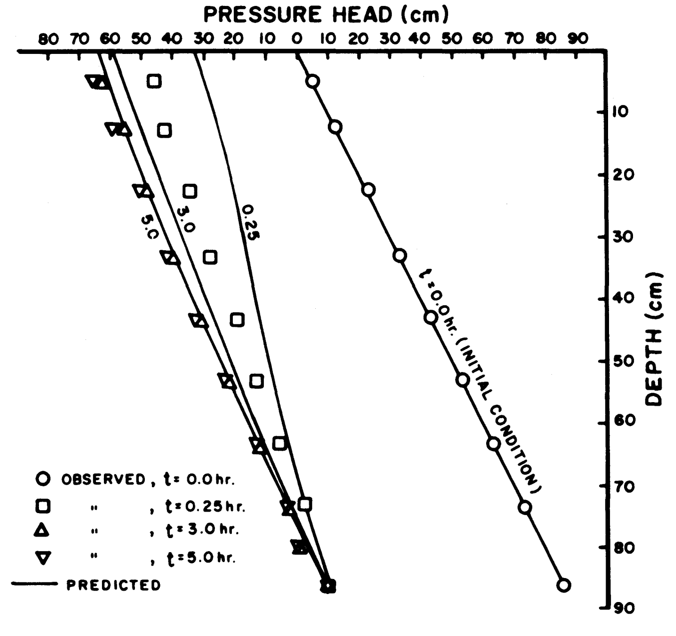

FIG. 7 Observed and predicted pressure head profiles during drainage, Wagram.

(l_1, l_2) used in the Jackson-Whisler equations are listed in Table 2. It is not apparent from Fig. 6 as to which of the prediction methods gives the best agreement with observed values. Again the estimate of error was computed for each combination of observed and predicted D(t) relationships. The resulting values of Φ are as follows: 0.55 cm for equation [1], 0.55 cm for equation [2], 0.53 cm for equation [3], and 0.46 cm for equation [4].

Observed pressure head profiles for core 1 of the Wagram soil and profiles predicted by numerical solutions to equation [1] are presented in Fig. 7. In general, the measured pressure heads were lower than predicted for all times with the difference being greatest for small and intermediate times. This disagreement may be due to the non-uniquenes of Θ(h) during transient drainage as demonstrated by Smiles et al. (1971), in that more water is retained for a given value of h during transient drainage than would be predicted by the statically determined Θ(h) relationship which was used in solving equation [1]. Vachaud et al. (1972) showed that as a draining front moves from a less permeable soil stratum into a more permeable one below, negative air pressures may exist between the interface of the strata and the receding saturated front. However, in order for the disagreement shown in Fig. 7 to be explained by this phenomenon, the soil near the profile surface must be relatively impermeable. Because there was no evidence to support such stratification, this possibility was discounted.

SUMMARY AND CONCLUSIONS

Drainage experiments were conducted in the laboratory using large, undisturbed soil cores collected from two field soils. Field variability among the cores was evaluated by imposing identical initial and boundary conditions for tests involving more than one core of each soil type. The flow volume and soil water pressure head relationships were recorded continuously during each experiment. Conventional methods were used to determine the soil water characteristic, Θ(h), and hydraulic conductivity function, K(h), for each soil.

The Richards equation was solved numerically for the conditions imposed during the experiments. Also, various approximate models were employed to describe vertical water movement during drainage. Observed relationships between cumulative flow volume and time were compared with those predicted by the various theoretical models. The agreement between theoretical predictions and observations was determined by computing an "estimate of error".

The conclusions of the study are as follows:

1 Accurate determination of water storage volume in field soils which correspond to various boundary conditions of interest is essential if any type of theoretical model is to be successful in characterizing the drainage process.

2 Substantial field variability was found in both soils examined in this study. Even though the cores were collected in a relatively small area, results indicate variability similar to that reported by Nielsen et al. (1973) where tests were conducted over a much larger area.

3 The approximate drainage model proposed by Jackson and Whisler (1970), in which the effective conductivity decreases quadratically with water table depth, was found to provide the best agreement with observations among the approximate drainage models considered. This method was as accurate as the exact model for the drainage cases considered.

4 In view of the significant field variability associated with the experiments, it is not evident that sophisticated approaches, such as numerical solutions to the Richards equation which require substantial time and expense for use, are more desirable than approximate models for characterizing vertical drainage in field soils.

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