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AFFECT, MOTIVATION, AND ENGAGEMENT IN THE CONTEXT OF MATHEMATICS EDUCATION: TESTING A DYNAMIC MODEL OF INTERACTIVE RELATIONSHIPS

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AFFECT, MOTIVATION, AND ENGAGEMENT
IN THE CONTEXT OF MATHEMATICS EDUCATION:
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DISSERTATION

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the College of Education at the University of Kentucky

By

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2018

ABSTACT OF DISSERTATION

AFFECT, MOTIVATION, AND ENGAGEMENT IN THE CONTEXT OF MATHEMATICS EDUCATION: TESTING A DYNAMIC MODEL OF INTERACTIVE RELATIONSHIPS

The present study tested the interactive model of affect, motivation, and engagement (Linnenbrink, 2007) in mathematics education with a nationally representative sample. Self-efficacy, self-concept, and anxiety were indicators of pleasant and unpleasant affect. Intrinsic and extrinsic motivation were indicators of mastery and performance approach. Persistence and cognitive activation were indicators of behavioral and cognitive engagement. The 2012 Programme for International Student Assessment (PISA) supplied a sample of 4,978 students from the United States for structural equation modeling. The results indicated that PISA data overall supported the interactive model. Specifically, PISA data completely supported the specification of the relationship between motivation and affect, largely supported the specification of the relationship between affect and engagement, but failed to support the specification of the relationship between motivation and engagement. Finally, PISA data largely supported the specification of the mediation effects of affect on the relationship between motivation and engagement.

KEYWORDS: Affect, Motivation, Engagement, Mathematics, Structural Equation Modeling

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Date

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This was a tremendous journey and what I have learned has been invaluable.

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Chapter 1: Statement of the Problem

Introduction

There is a strong recognition of the importance of the triangular effect of affect, motivation, and engagement in mathematics education (Linnenbrink, 2007). The role of affect in mathematics has received considerable attention (Carter & Norwood, 1997; Goldin, Epstein, Schorr, & Warner, 2011; Leder & Grootenboer, 2005; McLeod, 1994; Tapia & Marsh, 2000; Underhill, 1988). Although mathematics is considered to be the most objective and logical discipline, mathematical thinking as purely logical reasoning is not immune to the affective domain, which typically includes the emotions, attitudes, beliefs, and values connected with mathematics (DeBellis & Goldin, 1997; McLeod, 1992). Many studies have indicated that mathematics education faces a major problem in that many students and adults have negative attitudes and feelings about the subject (Nardi & Steward, 2002). Mathematics education researchers have demonstrated that positive emotions enhance students' positive beliefs about themselves as mathematics learners (Hart, 1989; McLeod, 1992; Stipek et al., 1998), while negative emotions have been connected with poorer mathematical performance (Hembree, 1990; Ma, 1999; Pajares & Miller, 1994; Pietsch, Walker, & Chapman, 2003). The U.S. reform movement in mathematics education clearly identified affective factors as important, needing substantial change, and having the potential to lead to considerable improvements in student performance (McLeod, 1994). The National Council of Teachers of Mathematics (NCTM) reaffirmed the centrality of affective issues in its standards for curriculum and evaluation (1989). For example, two of the major educational goals stated in the NCTM standards (1989) dealt with helping students understand the value of mathematics and with developing students' confidence. Later, the updated NCTM standards (2000) emphasized that students' confidence in and disposition toward mathematics are critical in mathematics education.

Motivation has traditionally been a major concern among mathematics educators (Ames, 1992; Kloosterman, 1997; Keys, Conley, Duncan, & Domina, 2012; Niepel, Brunner, & Preckel, 2014; Wolters & Pintrich, 1998). In general, motivation has been considered as consisting of intrinsic and extrinsic motivation (Deci & Ryan, 2002) or as consisting of a mastery goal orientation and a performance goal orientation (Eccles & Wigfield, 2002). It is well known that motivated students show interest in activities, feel confident about learning, demonstrate persistence in difficult tasks, and perform well, whereas unmotivated students are likely to be inattentive during lessons and fall behind in their studies (Aunola, Leskinen, & Nurmi, 2006; Schunk, Pintrich, & Meece, 2008). Students with an intrinsic motivation toward mathematics often achieve well in mathematics, whereas students with an extrinsic motivation toward mathematics tend to demonstrate low mathematics achievement (Deci & Ryan, 2002).

Student engagement is the most immediate and persistently identified factor for improving students' mathematical achievement (Boekaerts, Pintrich, & Zeidner, 2000; Kuh, Kinzie, Schuh, & Whitt, 2005; Ladd & Dinella, 2009; Martin & Rimm-Kaufman, 2015; Sciarra & Seirup, 2008). Engagement has frequently been described as having behavioral (e.g., participation and effort), emotional (e.g., a positive attitude about learning), and cognitive (e.g., elaboration and self-regulation) components (Finn, 1989; Fredricks, Blumenfeld, & Paris, 2004). Students who are engaged in mathematics tend to have positive learning outcomes in mathematics, while students with evidences of academic disengagement, such as disruptive behavior, poor attendance, and negative dispositions toward school, often have a negative academic performance in mathematics (Finn, 1993; Kuh, Kinzie, Schuh, & Whitt, 2005; Lee & Burkam, 2003; McCluskey, Bynum, & Patchin, 2004; Sciarra & Seirup, 2008; Valeski & Stipek, 2001).

Because of the importance of affect, motivation, and engagement in mathematics, many studies have investigated the relationships and interactions between these three factors. Some studies have examined the impact of affect on motivation (Erez & Isen, 2002; Gendolla, 2000; Gendolla & Krusken, 2002; Hall, Sampasivam, Muis, & Ranellucci, 2016; Linnenbrink & Pintrich, 2002; Rhoades, Rhoades, Arnold, & Jay, 2001). For example, Erez and Isen (2002) found that a positive affect improved performance by increasing perceptions of expectancy, valence, and instrumentality. Fredrickson (1998, 2001) proposed that a positive affect broadens the scope of attention and facilitates motivation by enhancing holistic attentional processes, cognitive resources, and academic performance. Overall, considerable evidence indicates that affect enhances motivation (Erez & Isen, 2002; Meyer & Turner, 2002).

There also has been a growing interest in how affect shapes engagement in the learning experience (Gendolla & Krusken, 2002; Linnenbrink-Garcia & Pekrun, 2011; Linnenbrink-Garcia, Roga, & Koskey, 2011; Linnenbrink & Pintrich, 2003). For example, Gendolla and Krusken (2002) reported that possessing a mood that encouraged cognitive evaluation contributed to the amount of effort used to perform a task. A positive mood could lead to greater effort or persistence on a task, whereas a negative mood could lead to lower effort or to terminating the task altogether (Gendolla, 2000). Linnenbrink-Garcia, Roga, and Koskey (2011) assessed how, during small group instruction, affect was associated with engagement in small group learning in mathematics among upper-elementary students, demonstrating a reciprocal relationship between affect and engagement. Linnenbrink and Pintrich (2003) found that a pleasant affect is positively correlated with behavioral engagement, whereas an unpleasant affect is negatively correlated with behavioral engagement.

Although one-on-one research, such as that presented above, is abundant in the mathematics education literature, little is known about whether interactions between affect, motivation, and engagement occur when students are learning mathematics. Given how closely related these factors are to each other both conceptually and practically, the paucity of investigations into their interaction in mathematics' education is quite surprising. To fill this gap in the literature, Linnenbrink (2007) developed a concept-based, dynamic model of affect, motivation, and engagement. However, to my knowledge, no study has tested this interactive model nor, even more importantly, utilized a nationally representative large-scale database. As a result, such a significant theoretical advancement remains largely a conceptual hypothesis. The current study used nationally representative data from PISA 2012 to investigate the interactions between affect, motivation, and engagement in mathematics. Its purpose was to explore the extent that the PISA data support Linnenbrink's (2007) dynamic model of affect, motivation, and engagement. In addition, based on an assessment of the model data-fit information, the present study will be in a sound position to test and modify, if necessary, the dynamic model of affect, motivation, and engagement, providing the basis for further testing and refinement.

Definition of Terms

To better understand the key variables used in the present study, operational definitions of the key terms are discussed below. Because the goal of this study was to test the dynamic model of Linnenbrink (2007), the definitions were kept as close as possible to those in Linnenbrink (2007).

Affect. Affect is a general term that encompasses three constructs: affective traits, emotions, and moods (Linnenbrink, 2006; Murphy & Alexander, 2000; Rosenberg, 1998). Affective traits are relatively stable across the lifespan and are pervasively associated with a person's disposition. Emotions, in contrast, are intense affective experiences that are relatively short in duration and are tied to specific events (Watson, Clark, & Tellegen, 1988). Moods are less intense affective experiences that are relatively long in duration compared to emotions, but less enduring than affective traits. According to McLeod (1992), affect in mathematics education is measured by beliefs, attitudes, and emotions. Of these, belief is the most stable and least intense, emotions are the most intense and least stable, and attitudes are intermediate on both dimensions. DeBellis and Goldin (1997) added a fourth element of values. Overall, how mathematics educators see the affective domain in mathematics is presented in Table 1.1.

Linnenbrink (2007) defined affect as possessing affective states that encompass moods and emotions but emphasized that affect should also be considered to be broad and global (see also Linnenbrink & Pintrich, 2002). To capture affective states, many empirical studies have measured very general beliefs and emotions, including self-efficacy, self-concept, and anxiety (opposite of self-confidence) (Bandura, 1994, Ho et al., 2000; Lebens, Graff, & Mayer, 2010; Linnenbrink & Pintrich, 2002; Ma, 1999; Meyer &Turner, 2006; Reyes, 1984).

Table 1.1

Affective Domain in Mathematics Education

Category	Definition	Example
Beliefs	Attribution of trueness to systems of propositions or other cognitive configurations; highly stable, cognitive, and structured; includes beliefs about mathematics, self, mathematics teaching and learning, and social contexts of mathematics (McLeod, 1992).	Mathematics is based on rules (about mathematics). I am able to solve problems (about self). Teaching is telling (about mathematics teaching). Learning is competitive (about the social context).
Attitudes	Affective responses that involve positive or negative feelings; moderately intense and reasonably stable (McLeod, 1992).	Dislike of geometric proof. Enjoyment of problem-solving. Preference for discovery learning.
Emotions	Rapidly-changing states of feeling experienced during mathematical activity; most intense and least stable (McLeod, 1992).	Joy (or frustration) in solving non-routine problems. Aesthetic response to mathematics.
Values	Deeply held ethics and morals as personal "truths" that help motivate priorities in mathematics; highly stable and affective (DeBellis & Goldin, 1997).	Commitment to mathematics learning.

This current study examined affectivity in mathematics, including mathematics self-efficacy, mathematics self-concept, and mathematics anxiety, by means of affective constructs created by PISA 2012. In PISA, mathematics self-efficacy is described as "the extent to which students believe in their own ability to handle mathematical tasks effectively and overcome difficulties" (OECD, 2013, p. 80). Mathematics self-concept is defined as "students' beliefs in their own mathematics abilities" (OECD, 2013, p. 80). Mathematics self-concept differs from mathematics self-efficacy in that mathematics self-efficacy is a context-specific assessment of the competence to perform mathematics, whereas mathematics self-concept is more general and includes beliefs about the self-worth associated with a person's perceived competence (Pajares & Miller, 1994). Mathematics anxiety is defined as "students' feelings of helplessness and stress when dealing with mathematics" (OECD, 2013, p. 80). Thus, the PISA 2012 affective constructs appear to be able to capture what Linnenbrink (2007) refers to as affective states and seem to be in line with the conventional approach in mathematics education to understanding the affective domain in mathematics (see discussion above).

Motivation. Motivation is the psychological feature that arouses a person to act in a way that moves that person toward a desired goal (Sansone & Harackiewicz, 2000). Self-determination theory (SDT) and achievement goal theory are the two best-known theories about motivation. The SDT focuses on the dialectic between the active growth-oriented human organism and social contexts that either support or undermine an individual's attempt to master and integrate their experiences into a coherent sense of self (Ryan & Deci, 1985). According to the SDT, competence (Harter, 1978; White, 1963), relatedness (Baumeister & Leary, 1995; Reis, 1994), and autonomy (de Charms, 1968; Deci, 1975) are three essentials for facilitating the optimal functioning of the natural propensity for growth, integration, and personal well-being. According to the second theory, achievement goal orientations, including affect and engagement, are useful for predicting school-related outcomes (Dweck & Leggett, 1988). This theory attempts to explain cognitive, affective, and behavioral responses to achievement situations by examining the interaction between dispositional and situational variables. Achievement goal theorists (Ames, 1992; Dweck & Leggett, 1988; Elliot & Dweck, 1988; Nicholls, 1984, 1989) have indicated that all individuals strive to demonstrate competence in achievement contexts. This desire motivates them to participate in activities. Nicholls's work (1984, 1989) established the foundation for the use of the achievement goal perspective. Individuals are motivated by a desire to demonstrate competence. Thus, a person's cognitive, affective, and behavioral responses are related to the way in which that individual defines competence.

In the dynamic model of affect, motivation, and engagement, Linnenbrink (2007) used achievement goal theory as the theoretical basis for motivation. She identified two primary goal orientations: a mastery goal orientation, which focused on developing one's competence, and a performance goal orientation, the focus of which was to demonstrate one's competence. A performance goal is also known as an ability-focused goal (Ames, 1992) or an extrinsic goal (Pintrich et al., 1993), whereas a mastery goal is also called a learning goal (Dweck, 2000) or an intrinsic goal (Pintrich et al., 1993).

Both theoretical and empirical evidences indicate that SDT and achievement goal theory share many similarities. When individuals have a high-performance orientation, they are more interested in the anticipated outcomes rather than in the activity itself (Nicholls, 1989). In contrast, a mastery goal orientation facilitates autonomous behavior and promotes intrinsic motivation by fostering challenge-seeking behaviors and task persistence (Butler, 1987). The present study uses the PISA 2012 definition of motivation, conceptualizing motivation in learning mathematics as both intrinsic and instrumental (extrinsic). Students may learn mathematics because they enjoy it or because they perceive learning mathematics as useful. In conclusion, it seems clear that the PISA motivation constructs adequately reflect Linnenbrink's (2007) perception of motivation.

Engagement. In the learning process, engagement is an active behavior that can be defined as the amount of time and effort students put into their studies and activities (Gonyea & Kuh, 2009). Some researchers define engagement as having two components: behavioral (e.g., participation, effort) and emotional (e.g., positive attitude about learning, interest) (Finn, 1989). Behavioral engagement has been referred to as effort and persistence, while emotional engagement has been referred to as positive and negative reactions, including interest, boredom, happiness, sadness, and anxiety, in the classroom (Connell & Wellborn, 1991; Skinner & Belmont, 1993). Behavioral engagement has primarily been measured by students' persistence in, avoidance of, ignoring of, and participation in their schoolwork (Gonida, Voulala, & Kiosseoglou, 2009; Shih, 2008). In general, emotional engagement has been measured by a student's identification, sense of belonging, and positive attitude about learning (Finn, 1989; Marks, 2000; Newmann, Wehlage, & Lamborn, 1992; Willms, 2003). Engagement has also been conceptualized as comprising three components: behavioral, emotional, and cognitive (Fredricks, Blumenfeld, & Paris, 2004; Jimerson, Campos, & Greif, 2003). Cognitive engagement stresses an investment in learning and instruction that involves self-regulation or being strategic in learning (Fredricks, Blumenfeld, & Paris, 2004).

In the dynamic model of affect, motivation, and engagement, Linnenbrink (2007) defined engagement from behavioral and cognitive perspectives. She used behavioral engagement to emphasize effort and persistence and cognitive engagement to emphasize the quality of a student's thinking in terms of cognitive strategies (e.g., elaboration, rehearsal), metacognitive strategy use, and self-regulated learning.

The present study adopted the framework from PISA and measured engagement as behavioral and cognitive engagement in the learning of mathematics. Behavioral and cognitive engagement in the school context of mathematics learning includes students' persistence on school tasks and cognitive activation. PISA engagement constructs seem to adequately capture the behavioral and cognitive aspects of engagement in Linnenbrink (2007).

Theoretical Framework

Linnenbrink (2007) developed a dynamic model of affect, motivation, and engagement to study the interface between the three constructs. The theoretical basis and the empirical support for this model come from many experimental and correlational studies conducted in laboratory and classroom settings (Linnenbrink & Pintrich, 2002, 2003; Linnenbrink, 2005).

To unfold the complexity of the three components (factors) in this interactive model, Linnenbrink (2007) first discussed the interaction between motivation and affect and then the interaction between affect and engagement. Achievement goal theory was the theoretical basis that she used to explore motivation. Overall, this theory indicates that each person's set of beliefs can explain why they approach and participate in academic tasks. This theory distinguishes two types of goal orientations. People with a mastery goal orientation actively learn and seek self-improvement, and people with a performance goal orientation attempt to demonstrate superior ability, to perform better than others, or to avoid looking dumb. Affect interacts with motivation in that mastery-approach goal orientations are positively associated with pleasant affect and negatively associated with unpleasant affect while performance-approach goal orientations are either unassociated with or positively associated with both pleasant affect and unpleasant affect (see Figure 1.1). With regard to the link between affect and engagement, Linnenbrink (2007) adopted behavior and cognition as engagement and asserted that there is a positive correlation between pleasant affect and increased behavioral engagement and a negative correlation between unpleasant affect and less behavioral engagement. She also found that pleasant affect correlates with more cognitive engagement while unpleasant affect correlates with less cognitive engagement.

Linking motivation, affect, and engagement, Linnenbrink (2007) proposed that pleasant affect has a positive mediating function and negative affect has a negative mediating function between the predictive effects of both mastery and performance achievement goals on behavioral and cognitive engagements. In general, four conditions need to be present for mediation: 1) the predictor must be significantly related to the mediator; 2) the mediator must be significantly related to the outcome; 3) the predictor must be significantly related to the outcome; and 4) the relationship between the predictor and the outcome must be significantly reduced in the presence of the mediator (Baron & Kenny, 1986; Linnenbrink, 2007). In the dynamic model of affect, motivation, and engagement, engagement is the outcome, motivation is the predictor, and affect is the mediator.

Linnenbrink (2007) provided empirical studies that aligned with the four conditions for mediation. She reported that the predictor of motivation (achievement goal theory) was significantly related to the mediator of affect (see solid lines from mastery approach to both pleasant affect and unpleasant affect as well as dashed lines from performance approach to both pleasant affect and unpleasant affect in Figure 1.1) and that the mediator of affect was significantly associated with the outcome of engagement (see dashed lines from pleasant affect to both behavioral engagement and cognitive engagement as well as the solid line from unpleasant affect to behavioral engagement and the dashed line from unpleasant affect to cognitive engagement in Figure 1.1). Therefore, empirical studies supported the first two conditions by linking motivation to affect and affect to engagement. For the third condition, mastery goals were related to higher levels of behavioral and cognitive engagement (see solid lines from mastery approach to both behavioral engagement and cognitive engagement in Figure 1.1); but because of mixed findings for performance approach, the model avoided making clear predictions (see no lines from performance approach to either behavioral engagement or cognitive engagement in Figure 1.1). With respect to the fourth condition, no consistent evidence indicated that affect (either pleasant or unpleasant) mediates the relationship between a mastery approach and engagement (either behavioral or cognitive) (Linnenbrink, 2007, p. 119). Meanwhile, no consistent evidence indicated that affect (either pleasant or unpleasant) mediates the relationship between performance approach and engagement (either behavioral or cognitive) (see p. 120). Linnenbrink (2007) admitted that this condition was the weakest part of the model but believed that, overall, the model had sufficient merit to allow it to be used for potential empirical scrutiny and consideration.

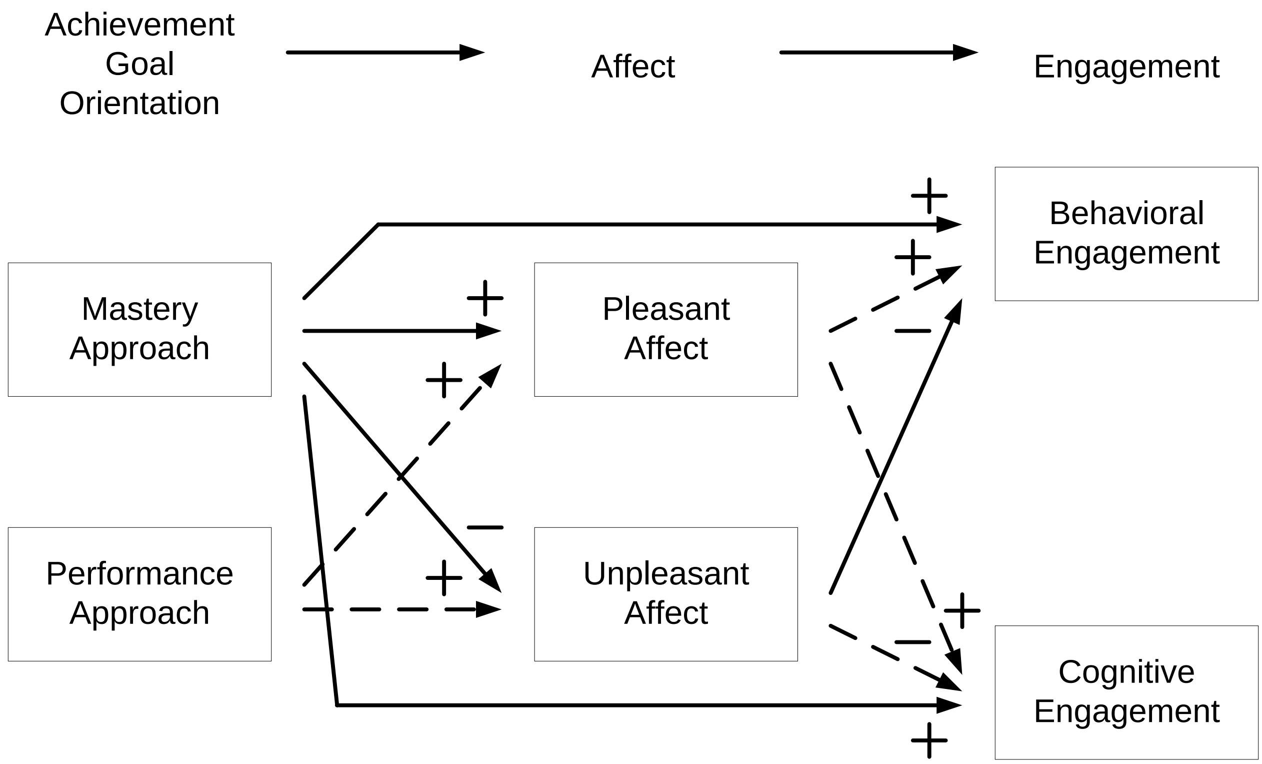

Figure 1.1. Linnenbrink's (2007) interactive model of motivation, affect, and engagement (solid lines indicate consistent findings, dashed lines indicate general patterns based on less consistent findings; + indicates position correlations; – indicates negative correlations).

Research Questions

Using the real world data from PISA 2012, the current study tested the dynamic (interactive) model of affect, motivation, and engagement (Linnenbrink, 2007). The goal of this study was to explore the extent to which the PISA data support the interactive model of affect, motivation, and engagement in mathematics (i.e., the degree to which the data fit the model). Specifically, the following research questions guided this investigation. The first research question tested the extent to which the PISA data support the model. The remaining research questions attempted to understand the nature of the interactions between affect, motivation, and engagement in mathematics.

(1) To what extent do real-world (PISA 2012) data support Linnenbrink’s (2007) dynamic (interactive) model of affect, motivation, and engagement in mathematics?

(2) If they do support Linnenbrink’s (2007) model, how is motivation in mathematics related to affect in mathematics? To what extent do the data patterns (from PISA 2012) match this part of the model specification?

(3) If the data patterns do match this part of the model, how is affect in mathematics related to engagement in mathematics? To what extent do the data patterns (from PISA 2012) match this part of the model specification?

(4) If the data patterns do match the part of the model relating affect to engagement in mathematics, how is motivation in mathematics related to engagement in mathematics? To what extent do the data patterns (from PISA 2012) match this part of the model specification?

(5) If the data patterns also match the part of the model relating motivation to engagement, how does affect in mathematics mediate the relationship between motivation and engagement in mathematics? To what extent do the data patterns (from PISA 2012) match this part of the model specification?

Significance of the Study

Many studies in past decades were dedicated to either motivation or engagement. However, some researchers became aware of the need to examine affect as it relates separately to motivation and engagement. Few studies, however, attempted to capture affect, motivation, and engagement in a dynamic (interactive) environment. Linnenbrink’s (2007) ground-breaking work delivered a theoretical (dynamic) model that was, to our best knowledge, the first attempt to address this issue. Nevertheless, as

Linnenbrink (2007) suggested, there is a strong need to verify and improve this theoretical model. As a pioneering effort, the present study aims to test this theoretical model using a nationally representative sample to fill the void in the existing research and establish an implementable framework of theory and knowledge.

The current study explored the interface of affect, motivation, and engagement in mathematics education. These factors have long been recognized as critical aspects of mathematics education (Leder & Grootenboer, 2005). Information about these critical factors has important implications for educational policies and practices related to the teaching and learning of mathematics. Conclusions about how affect, motivation, and engagement function jointly in mathematics learning can provide references and recommendations for educators and policymakers to better determine strategies, policies, and programs that are designed to promote mathematics learning. For example, from a practical standpoint, the results of this study provide insights into the ways in which educational leaders and policymakers allocate funds for professional development and mathematics instruction.

The present study comes at a time when there is a renewed call for improvement in the mathematics achievement of students in the United States (Ma & Ma, 2014). The results of the current study will contribute directly to the national discussion about ways that mathematics educators can increase the competitiveness of students in the United States in international comparative studies. As the NCTM has stressed many times, affect, motivation, and engagement matter tremendously to the well-being of students in mathematics learning.

Given the fact that Linnenbrink's model was based on a series of experimental and correlational studies, this present study had many significant advantages in that it used structural equation modeling (SEM) to assess the model relationships between affect, motivation, and engagement. This study simultaneously provided overall tests of model fit and individual parameter estimate tests of multiple hypotheses (Schumaker & Lomax, 1996). Multiple indicators from PISA were used to generate each latent variable, the combination of which may provide a more comprehensive perspective on affect, motivation, and engagement in mathematics. Since each latent variable was assessed by multiple observed items, the estimates of the relationships between the latent variables had less measurement error (Schumaker & Lomax, 1996). Additionally, SEM enables examination of both direct and indirect effects between the latent variables. All possible relationships between the predicative variables and the outcome variables, including the mediating effects and the latent compounding variables, were tested simultaneously.

Finally, yet importantly, the current study was an interdisciplinary effort to understand a sophisticated educational issue: the interactive importance of affect, motivation, and engagement in mathematics. This involves an interplay between educational psychology and mathematics education. This interdisciplinary effort can be expected to substantially and methodologically inspire empirical researchers to pursue even more advanced interdisciplinary research.

Chapter 2: Literature Review

The purpose of this study was to test Linnenbrink's dynamic model of affect, motivation, and engagement in mathematics with real world data. Specifically, it investigated the extent to which the PISA data support this dynamic model of affect, motivation, and engagement in mathematics. Chapter 2 consists of three sections: (1) a theoretical framework for affect, motivation, and engagement that will form the basis for this current study and how these three relate to the PISA perspective; (2) the relationships between affect, motivation, and engagement; and (3) the importance of PISA in this study. In the first section, the four components of mathematical affect (beliefs, attitudes, emotions, and values) are reviewed and accepted as forming the main theoretical framework for affect. Self-determination theory (SDT) is reviewed as the major framework for motivation. Behavioral, cognitive, and emotional components are discussed as constituting the chief theoretical framework for engagement. Section 2 discusses Linnenbrink's (2007) model in detail. Section 3 discusses the importance of PISA, which has been used as a decision-making tool for policy and practice in many countries.

Main Theoretical Framework for Affect

The circumplex model was originally proposed by Schlosberg (1941, 1952) and was subsequently most extensively elaborated upon by Russell (1980). This model assumes that all affective states arise from two fundamental neurophysiological systems, one related to valence (a pleasure–displeasure continuum) and the other to arousal or alertness (Russell, 1980). The valence dimension involves a psychological evaluation process that could assign a good or bad, useful or harmful, pleasant or unpleasant, compensating or threatening meaning to a stimulus at a given moment (Barrett, 2006). These evaluations occur along an activation dimension that refers to the mobilization or suspension of energy from low activation, represented by sleep, to high activation, represented by excitement (Barrett & Russell, 1999). For example, joy can be conceptualized as an emotional state that is the product of a strong activation associated with the positive valence, pleasure.

More specifically, Russell (1980) and Russell, Ward, and Pratt (1981) proposed that affective experience can be understood as a circular arrangement of terms around two-dimensional bipolar spaces of an affective valence (pleasure or displeasure) and an arousal or activation dimension (high or low) so that the underlying structure of an affective experience can be characterized as an ordering of affective states on the circumference of a circle (see Figure 2.1). Each affective state can thus be described as a linear combination of valence and activation (Feldman & Russell, 1998). The affective states can be categorized as four variants, the relative relationship of which is illustrated on a circle, as in Figure 2.1: (1) deactivated pleasant affect (on the bottom right of the circle), characterized as relaxation and calmness; (2) deactivated unpleasant affect (on the bottom left of the circle), characterized as boredom, fatigue, or depression; (3) activated unpleasant affect (on the bottom right of the circle), characterized as tension and distress; and (4) activated pleasant affect (on the bottom left of the circle), characterized as energy, excitement, and enthusiasm. Linnenbrink (2007) suggested that "activated unpleasant affect may lead to more intense engagement than deactivated unpleasant affect. Happiness (pleasant, neutral activation) may lead to different patterns of learning and engagement than excitement (activated pleasant)" (p.108).

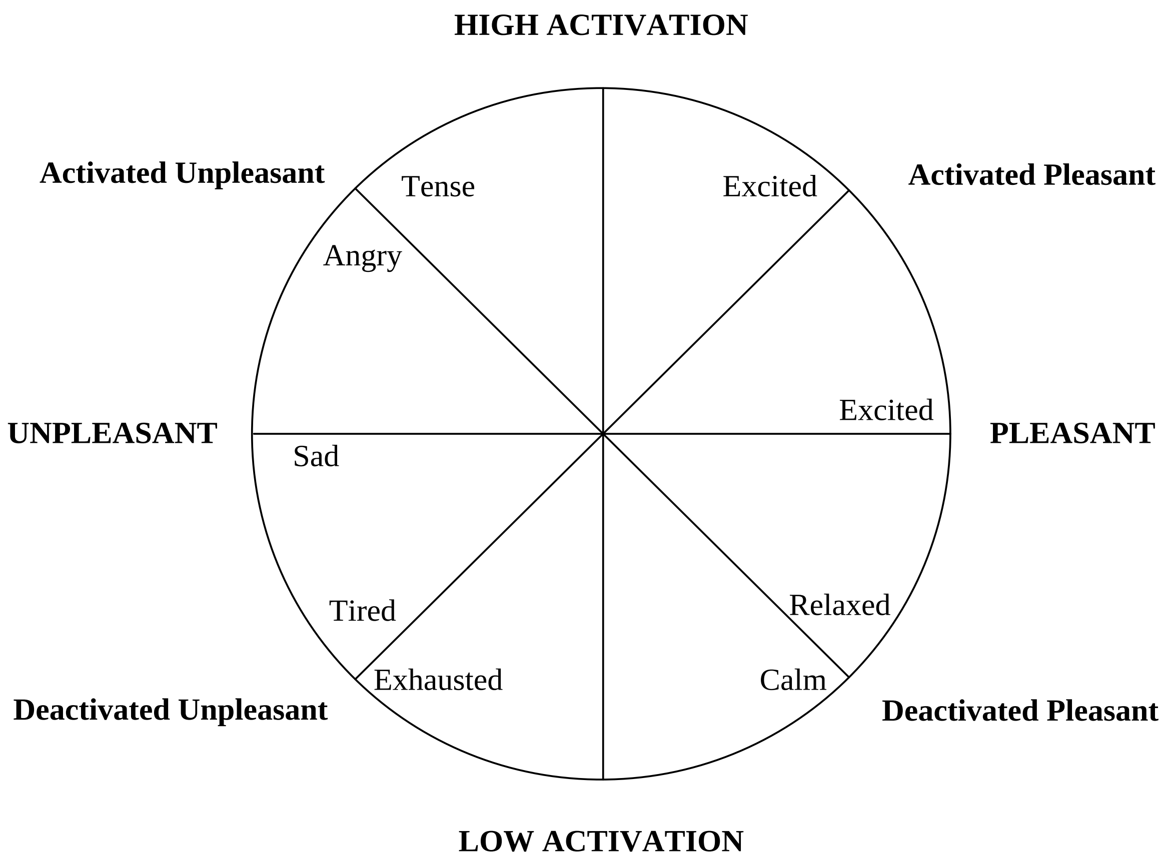

Figure 2.1. The Circumplex Model of Affect (Russell, 1980)

Many researchers have stated that the circumplex model has some merits (Mattila &Wirtz, 2000; Wirtz & Bateson, 1999). First, the circumplex model of affect is convenient because it uses only a few dimensions and scales, and consequently its predictive and explanatory power can result in good external validity (Wirtz & Bateson, 1999). Second, this model separates cognition from affect (Mattila & Wirtz, 2000). The affective component should be separated from the perceptual or cognitive component to aid in understanding how people assess their environment or place (Baloglu & Brinberg, 1997). Linnenbrink (2007) used the circumplex model of affect as her framework for the study of affect, employing the concepts of pleasant and unpleasant affects to connect achievement goal theory and school engagement.

The major disadvantage of the circumplex model of affect is that it is fairly difficult to measure in real life because individuals do not recognize or experience affective states as isolated, discrete entities (Macaulay, 1997). However, indicators of the four affective quadrants (subdomains) are clearly present in mathematics education. With the increasing attention to the affective domain in mathematics in the past couple of decades (NCTM, 1989), McLeod's affective domains have come to be considered to be the most concise and systematic model in mathematics education (Attard, Ingram, Forgasz, Leder, & Grootenboer, 2016; Lomas, Grootenboer, & Attard, 2012; McLeod, 1992, 1994; Zan, Brown, Evans, & Hannula, 2006). The present study used McLeod (1992) as a companion theoretical framework for affect to operationalize (or measure) the circumplex model of affect used in Linnenbrink (2007).

Companion Theoretical Framework for Affect

Affect plays a central role in the social context of the mathematics classroom (McLeod, 1992). Three concepts - beliefs, attitudes, and emotions - constitute the domain of affect and have been used in the research on affect in mathematics education (McLeod, 1992).

Beliefs. According to McLeod (1992), beliefs involve the attribution of some sort of truth to systems of propositions or other cognitive configurations. Of the 3 concepts, beliefs are the most stable, most cognitive, and most structured but the least intense. He described beliefs in terms of beliefs about mathematics, beliefs about self, (student) beliefs about mathematics teaching, and beliefs about the social contexts (i.e., social contexts provided by the school and the home). Beliefs about mathematics refer to students' beliefs about the importance of mathematics. Some researchers have measured beliefs about mathematics by capturing students' perceptions about the usefulness of mathematics. The Mathematics Attitudes Scales measure is a good example of such an attempt (Fennema-Sherman, 1976). Some researchers (Grootenboer, 2003; Kloosterman, 1996; Leder & Grootenboer, 2005; Pehkonen, 1995; Schoenfeld, 1989; Underhill, 1988; Zan, Brown, Evans, & Hannula, 2006) have adopted beliefs about mathematics as part of their analyses of the affective domain in mathematics. For example, Schoenfeld (1989) explored aspects of students' mathematics beliefs (i.e., their sense of mathematics as a discipline) and their relationship to mathematics performance. He also examined students' perceptions about mathematics as a discipline on shaping their engagement in mathematics.

Beliefs about self refer to students' beliefs in their ability or their confidence with regard to mathematics. Many researchers have measured beliefs about self in terms of self-efficacy and self-concept (Bandura, 1977; Dossey et al., 1988; Hackett & Betz, 1989; Reyes, 1984; Weiner, 1986). Indeed, two key self-elements in mathematics education are mathematics self-efficacy and mathematics self-concept. These are quite strongly related to the ability of students to learn new topics in mathematics, perform well in mathematics classes, and score well on mathematics tests (Bandura & Schunk, 1981; Pajares & Miller, 1994; Reyes, 1984). Some researchers (Boruchovitch, 2004; McLeod, 1994; Pehkonen, 1995) have adopted beliefs about self as part of their analyses of the affective domain in mathematics. For example, Nicholls, Cobb, Wood, Yackel, and Patashnick (1990) indicated that students' beliefs about success in mathematics are related to their effort in mathematics education. Mathematical self-efficacy is defined as students' confidence in their ability to do mathematics (Bandura, 1986; Bandura, 1997). Many researchers have measured mathematics self-efficacy by capturing students' perceptions of their performance capability in relation to mathematics problems (i.e., those problems similar to standardized tests of mathematical aptitude and achievement), everyday mathematics tasks, and good grades in mathematics courses (Hackett & Betz, 1989). The Mathematics Self-Efficacy Scale (MATH, also known as MSES) is a good example of such an attempt. Some researchers (Bandura, 1997; Pajares & Miller, 1994; Pajares & Graham, 1999; Pajares & Urdan, 2006; Zimmerman, 2000) have adopted mathematics self-efficacy as their major affective variable in mathematics. An evidence for this is that stronger mathematics self-efficacy has been found to be predictive of higher performance in mathematics and mathematics problem solving (Hackett, 1985; Pajares & Miller, 1994). Men report higher mathematics self-efficacy than women (Pajares & Miller, 1994). The other key self-element, mathematics self-concept, refers to individuals' beliefs about their mathematics ability supplemented by behavioral and emotional reactions to the value of mathematics and the mathematical way of thinking as well as their confidence in and motives for learning mathematics (Opachich & Kadijevich, 1997). Many researchers have measured mathematics self-concept by capturing students' perceptions of their abilities in mathematics (e.g., "Mathematics is one of my best subjects") (Pajares & Miller, 1994; Marsh, Parker, & Barnes, 1985; Marsh, 1992). The math subscale of the Self-Description Questionnaire (SDQ) is a good example of such an attempt (Marsh, 1992). Some researchers (Pajares & Miller, 1994; Pietsch, Walker, & Chapman, 2003) have adopted mathematics self-concept as their major affective variable in mathematics. Evidence that supports this usage can be seen in findings that students' mathematics self-concepts are significantly related to mathematics performance (Pietsch, Walker, & Chapman, 2003; Marsh, Walker, & Debus, 1991).

The affective concept termed “beliefs about mathematics teaching” primarily refers to students’ beliefs that determine their responses to mathematics instruction. Some researchers measured student beliefs about mathematics instruction by capturing students’ task orientation in mathematics (e.g., “I really feel pleased in math when I keep busy.”), ego orientation (e.g., “I feel really pleased when I am the only one that can answer a question.”), work avoidance (e.g., “I feel really pleased in math when I don’t have to work hard.”) , interest and effort (e.g., “Students do well in math if they are interested in learning.”), competitiveness (e.g., “Students do well in math if they get more answers right than others.”), and extrinsic (e.g., “Students do well in math if they behave nicely.”) (Carter & Norwood, 1997). Some researchers (Carter & Norwood, 1997; Kloosterman & Cougan, 1994; Kloosterman, 1996; Pehkonen, 1995; Underhill, 1988; Viholainen, Asikainen, & Hirvonen, 2014) have adopted (student) beliefs about mathematics instruction as part of their analyses of the affective domain in mathematics. For example, students’ working hard to solve problems and striving for understanding is significantly related to their success in mathematics (Carter & Norwood, 1997).

Beliefs about the social contexts of mathematics education refer to students’ views about and perceptions of the social-historical context in which they discuss and experience mathematics, both inside and outside of the classroom. Perspectives inside the classroom include: the social-mathematical norms and practices in their class, the role and functioning of their mathematics teachers, and the role and functioning of the students in their own class (e.g., students’ perception of appropriate behavior in class).

Similarly, perspectives outside the classroom include the influence of their parents (Op't Eynde & De Corte, 2003; Physick, 1998). Many researchers have measured beliefs about the social context by assessing the explicit teaching of social norms and finding a supportive classroom environment (Cole & Griffin, 1987; Cobb, Yackel, & Wood, 1989; Grouws & Cramer, 1989). Other researchers measured mathematics beliefs about the social context by querying the students about the role and functioning of their mathematics teacher and the influences of their parents (Hannula et al, 2005; Op't Eynde & De Corte, 2004). Many researchers (Goldin, Epstein, Schorr, & Warner, 2011; Hannula et al, 2005; McLeod, 1992; Op't Eynde & De Corte, 2004; Underhill, 1988) have adopted belief about the social context as part of their analyses of the affective domain in mathematics. Evidences in favor of using this are that student perceptions of positive support from their teacher, the classroom environment, and their parents are significantly related to better performance in mathematics (Goh & Fraser, 1998; McMahon, Wernsman, & Rose, 2009; Yan & Lin, 2005).

Attitudes. Attitudes are moderately stable and moderately intense orientations or predispositions toward certain sets of feelings in particular contexts such as mathematics (McLeod, 1992). Attitudes toward mathematics manifest in a variety of ways. First, attitude may come from the automatization of a repeated emotional reaction to mathematics. For example, the emotional impact will become less intense over time if a student has repeated experiences with geometric proofs. The emotional reaction to geometric proofs will become more automatic with less physiological arousal, thus forming an attitude. At that point, the response will become a stable one that can be measured through questionnaires (McLeod, 1992). Second, attitude is an existing response to a new but related task. For example, a student who has a negative attitude toward geometric proofs may have the same attitude toward proofs in algebra.

Attitudes are a very broad concept in mathematics education with these key elements: enjoyment (i.e., the degrees to which students enjoy working with mathematics), difficulty or anxiety (i.e., the stress students feel when doing mathematics), and importance (the perceived impact on the future of students) (Ma, 1997). Some researchers have measured attitude by capturing the degree to which students enjoy mathematics class, students' feelings of helplessness, and students' beliefs about the usefulness, relevance, and value of mathematics. The Attitudes toward Mathematics Inventory (ATMI) is a good example of such an attempt (Tapia, 1996; Tapia & Marsh, 2000; Tapia & Marsh, 2005). Many researchers (Fennema & Sherman, 1976; Tapia, 1996; Tapia & Marsh, 2000) have adopted attitude towards mathematics as their major affective variable in mathematics education. For example, some researchers have examined the reciprocal relationships between attitude toward mathematics and achievement in mathematics (Ma & Kishor, 1997; Ma, 1997).

Emotions. Emotions (emotional feelings) represent the rapidly-changing states of feeling experienced during mathematics activities and are the most intense, most local and contextual, but least stable of the theoretical components of affect (McLeod, 1992). Little research has measured emotion directly, in part because it is considerably easier and more possible to measure affective factors that are stable, such as attitudes and beliefs. Emotional responses to mathematics have been used in the literature on affective domain in mathematics because emotion is short and unreliable if measured after the fact. Researchers do know that typical emotional responses to mathematics include joy and excitement when positive things about mathematics learning are present and panic and frustration when negative things about mathematics learning are present (Hembree, 1990; Pekrun et al., 2007; Ma, 1999; Ma & Xu, 2004). When emotional response becomes habitual or fixed, they may function like attitude (McLeod, 1992).

Values. DeBellis and Goldin (1997) added values as a fourth element. Values refers to ethics and morals that are deep personal truths held by individuals that help motivate their priorities and are both stable and structured as well as affective and cognitive. Goldin (2002) considered values as ethics or morals that are deeply-held preferences, stable, and possibly characterized as "personal truths, highly affective as well as cognitive, may also be highly structured" (p.61). Values are closely related to attitudes, but values are held in a deeper and more central position. Values are also close to beliefs, but values are enduring beliefs and are organized in sets or clusters (see Figure 2.2). The distinction between beliefs and values was made clear by Clarkson et al. (1999) when they wrote that, "values are demonstrated in the actions carried out by a person, whereas beliefs can be verbally assented to, but do not necessarily lead to observable behavior in public" (p.3). Some researchers measured values by capturing student perceptions of the usefulness of mathematics and their attitudes toward success in math (Fennema & Sherman, 1976). The Mathematics Attitude Scales (i.e., attitude towards success in mathematics, confidence in mathematics, usefulness of mathematics) is a good example of such an attempt (Fennema & Sherman, 1976). Some researchers (DeBellis & Goldin, 1997; Leder & Grootenboer, 2005) have adopted values as part of their analyses of the affective domain in mathematics. For example, some of them have found that differences in students' values with respect to mathematics significantly contribute to differences in students' learning of mathematics between different regions of the world (Seah, Zhang, Barkatsas, Law, & Leu, 2014).

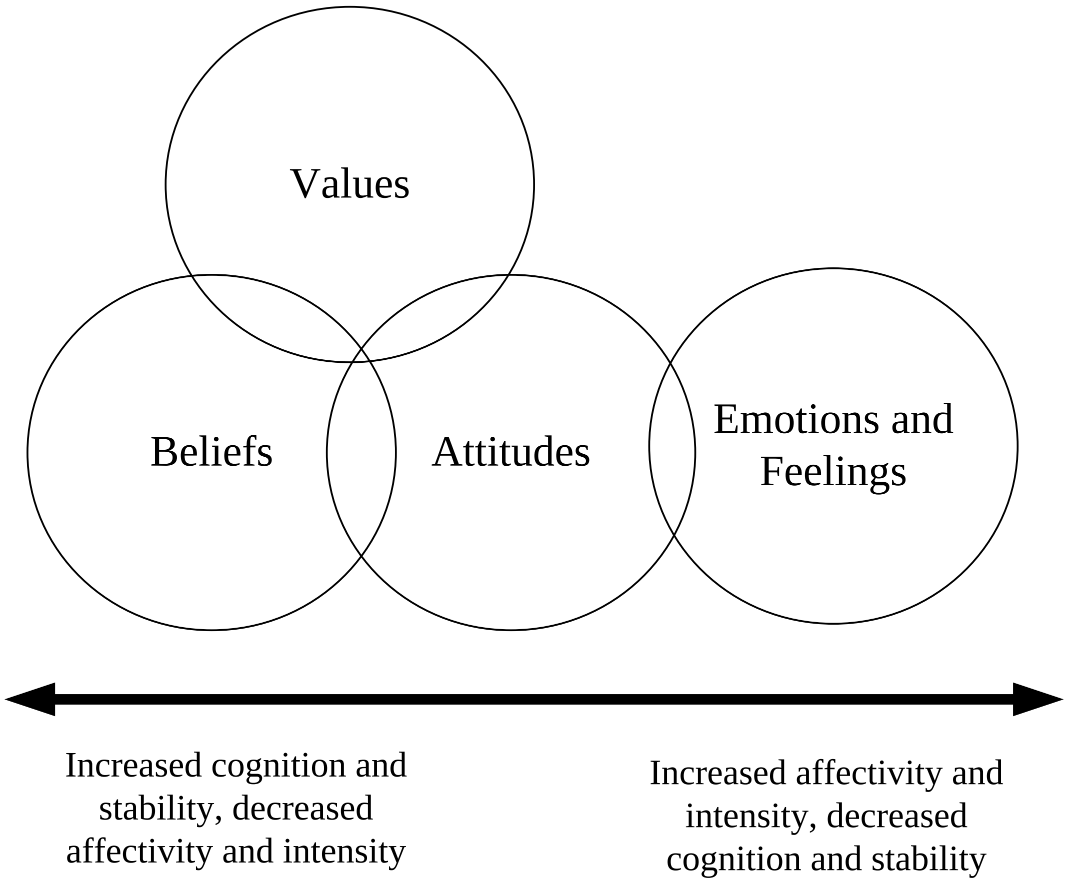

Figure 2.2. A model of conceptions of the affective domain in mathematics (Grootenboer, 2003; Leder & Grootenboer, 2005)

The above various components form a complex, inter-related framework for the affective domain in mathematics (see Figure 2.2). In general, emotions are conceptualized as distinct from beliefs, attitudes, and values; whereas beliefs, attitudes, and values are inter-related and are somewhat loosely inter-changeable (Grootenboer, 2003; Leder & Grootenboer, 2005; McLeod, 1992). The current study will examine affectivity in mathematics in terms of beliefs and attitudes (i.e., mathematics self-efficacy, mathematics self-concept, and mathematics anxiety in connection with a circumplex model of affect, in particular with Linnenbrink's multi-dimensional model of affect, motivation, and engagement (2007) (see Appendix B).

The literature often relates mathematics self-efficacy and mathematics self-concept to a positive or pleasant affect toward mathematics. Mathematics self-concept is a generalization of a person's confidence in learning mathematics (Reyes, 1984), whereas mathematics self-efficacy addresses a person's confidence in his/her ability to successfully perform mathematics tasks. Taken together, they represent a student's confidence about mathematics from general and specific perspectives, and confidence is often considered a positive or pleasant thing. This emphasis is apparently similar to beliefs about self as expressed previously in connection with Mcleod's (1992) affective domain in mathematics. Many researchers have adopted mathematics self-efficacy and self-concept to discuss the connection between affect and learning mathematics from a positive perspective (Pajares & Miller, 1994; Pajares & Urdan, 2006; Pajares & Graham, 1999; Pietsch, Walker, & Chapman, 2003; Reyes, 1984; Tapia, 1996; Tapia & Marsh, 2000; Zimmerman, 2000). Meanwhile, the literature often relates mathematics anxiety to negative or unpleasant traits of affect in mathematics (Fennema &Sherman, 1976; Hembree, 1990; Reyes, 1984; Ma, 1999). Mathematics anxiety is often related to poor performance in mathematics, dislike of mathematics, and avoidance of mathematics (Hembree, 1990; Reyes, 1984; Ma, 1999). Many researchers have used mathematics anxiety to discuss affect in the learning of mathematics from a negative perspective (e.g., the need to reduce mathematics anxiety (Hembree, 1990; Reyes, 1984).

The PISA Perspective for Affect

PISA 2012 provides many indicators for measuring affect in mathematics from the perspectives of belief and attitude in mathematics (see Appendix B). These are similar to Mcleod's affective domain in mathematics. In PISA, beliefs about mathematics are measured by mathematics self-efficacy and mathematics self-concept. Mathematical self-efficacy refers to students' convictions that they can successfully perform mathematical tasks (OECD, 2013). Mathematical self-concept refers to students' responses about their perceived competence in mathematics (OECD, 2013). Self-concept implies a more general perspective and includes beliefs about self-worth that are associated with a person's perceived competence (Pajares & Miller, 1994), whereas self-efficacy is a more context-specific assessment of competence as the belief in one's ability to handle mathematical tasks effectively and overcome difficulties. In PISA, mathematics anxiety was measured by a person's feelings of helplessness and stress when dealing with mathematics (OECD, 2013).

The available PISA affective measures (i.e., mathematics self-efficacy, mathematics self-concept, and mathematics anxiety) seem to adequately represent the key elements (relevant to the present study) of the affective domain in mathematics as depicted in the companion theoretical framework for affect (McLeod, 1992). The PISA measures also capture the essence of the circumplex model of affect as applied by Linnenbrink (2007) (i.e., pleasant or unpleasant affects). Indeed, the concepts of mathematics self-efficacy, mathematics self-concept, and mathematics anxiety in PISA 2012 are similar to the key components of belief and attitude in mathematics, which

McLeod proposed as the affective domain in mathematics. Of all the existing survey studies, PISA provides the data that best captures the affective domain in mathematics.

Theoretical Framework for Motivation

Motivation is the psychological feature that arouses a person to act toward a desired goal (Harackiewicz & Sansone, 2000). Motivation influences what, when, and how people learn (Schunk, 1995). The most direct way to measure motivation is through assessing behaviors, with motivated students showing interest in activities, attending carefully to instruction, taking notes to facilitate study, working diligently to learn new material, feeling confident about learning, showing persistence in difficult tasks, and performing well in school (Schunk, Pintrich, & Meece, 2008). However, there are more systematic ways to measure (and understand) motivation. Achievement goal theory and self-determination theory have been used as the main theoretical frameworks for motivation (in relation to mathematics).

Achievement Goal Theory. Achievement goal theory has received considerable attention (Eccles & Wigfield, 2002; Harackiewicz, Barron, Pintrich, Elliot, & Thrash, 2002; Pintrich, 2000a, 2000b). The key concept is achievement goal orientation which is a social-cognitive approach to motivation that emphasizes students' perception of and the interactions between cognition, affect, and behavior (Dweck & Leggett, 1988; Maehr & Zusho, 2009). According to achievement goal theory, students' academic motivation is defined as their attempts to achieve goals. What learners believe about their abilities and the goals they intend to pursue impacts how they approach learning and how they react to success and failure. Researchers generally measure achievement motivation by assessing the energization and direction of competence-related behaviors, for example, by evaluating competence relative to some standard of excellence (Elliot, 1997). Others have measured motivation by assessing the purpose or reason why students pursue an achievement task as well as by the standards or criteria they construct to evaluate their own competence or success on the task (Urdan, 1997). Some researchers have adopted achievement goal theory as part of their basis for understanding motivation in mathematics (Anderman & Midgley, 1997; Bong, 2004). As an example of the application of this theory, achievement goals have been found to be significantly related to academic achievement in mathematics (Awan, Noureen, & Naz, 2011; Church, Elliot, & Gable, 2001).

Achievement goal theorists (Ames, 1992; Dweck & Leggett, 1988; Elliot & Dweck, 1988; Nicholls, 1984, 1989) hold that all individuals strive to demonstrate competence in achievement contexts. Early on, achievement goals were divided into two dimensions: a mastery goal orientation and a performance goal orientation (Ames, 1992; Dweck, 1996). The mastery goal perspective refers to the fact that the purpose of learning is to grow in competence, master a task, improve in some way, and enjoy a challenge. The performance goal perspective is that the purpose of learning is to show one's ability, look competent, get recognition, and perform better than others or avoid looking dumb. A performance goal is also known as an ego goal (Nicholls, 1984), an ability-focused goal (Ames, 1992), an extrinsic goal (Pintrich et al., 1993), or a competitive goal (Roberts, Treasure, & Kavussanu, 1996), while a master goal is also called a learning goal (Dweck, 1999), a task goal (Nicholls, 1984), or an intrinsic goal (Pintrich et al., 1993).

Some researchers have measured mastery goals by assessing whether student's learning goals are based on interests, learning content, gaining broader knowledge, mastering the materials, learning for curiosity, and learning a challenging task (Elliot & Church, 1997; Elliot & Murayama, 2008; Midgley, et al., 2000). The Achievement Goal Orientation Questionnaire is good example of such an attempt. Some researchers who have adopted the Patterns of Adaptive Learning Survey (PALS) (Midgley et al., 1996, 2000) have measured mastery goals by assessing a student's outlook toward goals set by the teacher, goals set in the classroom or by the parents or in their home life, and personal achievement (Midgley et al., 1998; Patrick, Anderman, Ryan, Edelin, & Midgley, 2001). The adoption of a mastery goal is assumed to predict adaptive outcomes regardless of success or failure (Dweck & Leggett, 1988). Some researchers have adopted mastery goals as a part of motivation in mathematics (Chiang & Lin, 2014; Linnenbrink, 2005). As an example of how this has been used, Linnenbrink (2005) found that mastery goals are positively related to students' achievement in mathematics.

Somewhat more recently, researchers have further divided performance goals into performance-approach and performance-avoidance goals (Elliot & Church, 1997). Performance-approach goals are extrinsically driven goals and focus on the external benefits of achievement (i.e., appearing to have more knowledge than others). Some researchers have measured performance-approach goals by assessing student's endorsements about the importance of a task, about getting better grades, about demonstrating one's ability relative to others, about outperforming one's peer, and about showing one's ability to family, friends, advisors, or others (Elliot & Church, 1997). Performance-avoidance goals are actions taken to withdraw from and avoid academic tasks in an effort to avoid demonstrating a lack of knowledge or skill. Many researchers measure performance-avoidance goals by whether individuals worry about bad grades, fear poor performance, avoid working poorly, or try to look smart (Elliot & Church, 1997). The Achievement Goal Orientation Questionnaire is a good example of an effort to understand both performance-approach goals and performance-avoidance goals. Overall, the adoption of performance goals has been hypothesized to predict maladaptive outcomes (i.e., negative affect and poor performance) and helpless patterns of achievement behavior (i.e., choosing easy tasks, withdrawing effort, or showing lower enjoyment of learning tasks), particularly after experiencing failure (Elliot & Church, 1997). Some researchers have adopted performance-approach and performance-avoidance goals as part of their understanding of motivation in mathematics (Keys, Conley, Duncan, & Domina, 2012; Luo, Paris, Hogan, & Luo, 2011; Magi, Lerkkanen, Poikkeus, Rasku-Puttonen, & Kikas, 2010; Wolters, 2004; Niepel, Brunner, & Preckel, 2014). For example, Luo, Paris, Hogan and Luo (2011) found that students with performance-approach goals were more likely to make an effort when encountering difficulties in learning mathematics and performed better in mathematics.

Elliot and McGregor (2001) extended the subdivision of performance goals to mastery goals, suggesting a 2x2 achievement goal framework involving mastery goals (mastery-approach goals and mastery-avoidance goals) and performance goals (performance-approach goals and performance-avoidance goals). Mastery-approach goals refer to attaining positive possibilities, such as acquiring new skills or improving one's intrapersonal competence. Mastery avoidance goals refer to avoid negative possibilities, such as losing one's skills and abilities, failing to learn, misunderstanding the material, or leaving a task incomplete. They measured this 2x2 achievement goal model using the Achievement Goal Questionnaire (Elliot & MacGregor, 2001) and the Achievement Goal

Questionnaire Revised (Elliot & Murayama, 2008) (see Table 2.1). The adoption of mastery-approach and mastery-avoidance goals was assumed to focus on developing competence or avoiding self-referential or task-referential incompetence, respectively. The adoption of performance-approach and performance-avoidance goals was hypothesized to focus on demonstrating competence relative to others or avoiding a demonstration of incompetence relative to others, respectively (Elliot & McGregor, 2001).

Table 2.1

Achievement Goal Questionnaire (AGQ, Elliot & MacGregor, 2001) and Achievement Goal Questionnaire Revised (AGQ-R, Elliot & Murayama, 2008)

	AGQ	AGQ-R
Performance approach	1) It is important for me to do better than other students. 2) It is important for me to do well compared to others in this class. 3) My goal in this class is to get a better grade than most of the other students.	1) My aim is to perform well relative to other students. 2) I am striving to do well compared to other students. 3) My goal is to perform better than the other students.
Performance avoidance	1) I just want to avoid doing poorly in this class. 2) My goal in this class is to avoid performing poorly. 3) My fear of performing poorly in this class is often what motivates me.	1) My aim is to avoid doing worse than other students. 2) I am striving to avoid performing worse than others. 3) My goal is to avoid performing poorly compared to others.
Mastery approach	1) I want to learn as much as possible from this class. 2) It is important for me to understand the content of this course as thoroughly as possible. 3) I desire to completely master the material prepared in this class.	1) My aim is to completely master the material presented in this class. 2) I am striving to understand the content of this course as thoroughly as possible. 3) My goal is to learn as much as possible.

Table 2.1 (continued)

Mastery avoidance	1) I worry that I may not learn all that I possibly could in this class. 2) Sometimes I'm afraid that I may not understand the content of this class as thoroughly as I'd like. 3) I am often concerned that I may not learn all that there is to learn in this class.	1) My aim is to avoid learning less than I possibly could. 2) I am striving to avoid an incomplete understanding of the course material. 3) My goal is to avoid learning less than it is possible to learn.

Self-Determination Theory (SDT). Self-determination theory (SDT) has received considerable attention in the field of motivation. SDT suggests that learning occurs when an individual is cognitively and emotionally engaged. According to SDT, the needs for competence (people's perceptions of their capabilities and accomplishment), relatedness (learners' perceptions of how they interact with others and how others view them), and autonomy (how much volition or choice a person believes they have) have been identified as three essentials for facilitating the optimal functioning of the natural propensities for growth, integration, and personal well-being. SDT focuses on the degree to which human behaviors are volitional or self-determined, specifically the degree to which people endorse their actions at the highest level of reflection and engage in actions with a full sense of choice.

SDT generally defines motivation as the way that an individual's experiences of autonomy, competence, and relatedness foster their actions. Autonomous motivation (self-driven) and controlled motivation (externally driven) are the key components in SDT. Autonomy involves acting with a sense of volition and experiencing choice. Intrinsic motivation, as an example of autonomous motivation, refers to a situation in which individuals engage in an activity because they find it interesting and engage in activities wholly volitionally. In contrast, control involves acting with a sense of pressure, a sense of having to engage in an action. Extrinsic motivation, as an example of controlled motivation, refers to a person's feeling coerced or seduced into behaving, as a result of experiencing pressure and obligation (Deci, 1971).

Some researchers have measured autonomy motivation (self-driven) by asking the reasons why people act, whether as a result of external pressures or of personal values or interests (deCharms, 1968; Ryan & Deci, 2000). Some scholars have measured controlled motivation (externally driven) by asking whether a person initiated their own behavior or whether it was governed by external factors (Ryan & Deci, 2000). Other researchers have measured both autonomous motivation and controlled motivation through the adapted version of the Self-Regulation Questionnaire (Ryan, Rigby, & King, 1993). Autonomous and controlled motivations are different in their underlying regulatory processes and their accompanying experiences. When externally regulated, people usually respond by intending to obtain a desired consequence or avoid an undesired one.

Specifically, intrinsic motivation is measured through self-reports of interest and enjoyment of the activity (Harter, 1981; Ryan & Deci, 1985), and extrinsic motivation is measured through self-reports of external reasons of the activity (Conti, Amabile, & Pollak, 1995; Harter, 1981; Ryan & Deci, 1985). Harter (1981) distinguished intrinsic versus extrinsic motivations using three subscales: 1) the desire for challenging work versus a preference for assignments that can be accomplished easily; 2) motivation based on curiosity or interest versus motivation based on pleasing the teacher or receiving good grades; 3) independent mastery versus dependence on the teacher. However, these distinctions are not always necessary or appropriate in the average classroom. Some researchers (Hagger, Sultan, Hardcastle, & Chatzisarantis, 2015; Stanko-Kaczmarek, 2012) have adopted intrinsic and extrinsic motivation as their perspective on motivation in mathematics. For example, students with an intrinsic motivation were found to be more likely to pursue a similar mathematics project in the near future than students with an extrinsic motivation (Stanko-Kaczmarek, 2012).

To expand on the extrinsic-intrinsic motivation contrast, the role of internalization refers to the process of transferring behavioral regulation from outside to inside the individual (Deci & Ryan, 2000). This internalization process includes four primary levels: (1) the external level in which behavioral regulation comes from outside the individual; (2) the introjected level that focuses on internal regulation that is based on feelings that the person must do the behavior; (3) the identified level in which behavioral regulation is internal but is based on the perceived benefit of the behavior; and (4) the integrated level, which addresses regulation based on what the individual thinks is valuable and important to the self. Researchers measure the role of internalization through the perceived locus of causality (PLOC), which assesses individuals' self-reported reasons for acting (Ryan & Connell, 1989).

SDT consists of five inter-related theories (Deci & Ryan, 2002): cognitive evaluation theory, organismic integration theory, causality orientations theory, goal contents theory, and basic needs theory. Cognitive evaluation theory explains the effects of extrinsic factors or social contextual events (e.g., competition, deadlines, evaluations, imposed goals, praise, and rewards) on intrinsic motivation, behavior, and experience (Deci, 1975; Deci & Ryan, 1985). This theory is useful when studying people who show some interest or motivation (Ryan & Deci, 2000). Organismic integration theory holds that externally regulated behaviors can be transformed to self-regulated behaviors (Deci & Ryan, 2002). It is often used in the context of internalization, especially with respect to the development of extrinsic motivation (Deci & Ryan, 2002). Causality orientations theory addresses individual differences in global (personality-level) motivational orientations and describes how people incorporate social influences into their motivational styles (Deci & Ryan, 1985, 2002). Goal contents theory deals with the impact of intrinsic and extrinsic goals on human motivation and wellness (Kasser & Ryan, 1996). Finally, basic needs theory suggests that human are motivated to learn and develop because of a drive to satisfy three core psychological needs: autonomy, competence, and relatedness (Deci & Ryan, 2000). Autonomy implies that individuals have a need for autonomy or a desire to do things for personal reasons (Ryan & Connell, 1989). Perceived competence can facilitate intrinsic motivation because of a need to get satisfaction by improving one's abilities (Deci & Ryan, 1985, 2000). The need for relatedness is the need to feel related to significant others, such as peers and teachers (Deci & Ryan, 1985, 2000). Satisfying these three needs is indispensable for facilitating self-determined motivation.

Therefore, self-determination theory seems to provide the overarching theoretical framework that umbrella these five mini-theories because they all are related to the concept of basic needs. As mentioned above, cognitive evaluation theory explains the effects of social contexts on intrinsic motivation. Organismic integration theory addresses the concept of internalization with respect to the development of extrinsic motivation. Causality orientation theory focuses on individual differences in people's tendencies toward self-determined behaviors (Deci & Ryan, 1985, 2002). Goal contents theory explains the impact of intrinsic and extrinsic goals on human motivation and wellness (Kasser & Ryan, 1996). Basic needs theory explains the concept of basic needs and its relationship to life goals and daily behaviors. These theories are discussed in the present study to gain a better understanding of self-determination theory by considering it from a variety of angles. More importantly, intrinsic motivation and extrinsic motivation are the key components in all these mini-theories, thus making self-determination theory a sound "summary" of them all, especially for the purposes of this current study.

Alternative Frameworks for Motivation

Expectancy-value theory, as developed and researched by Eccles, Wigfield, and their colleagues, suggested that individuals' choice, persistence, and performance can be explained by their beliefs about how well they will do on the activity and the extent to which they value the activity (Atkinson, 1957; Eccles et al., 1983; Wigfield, 1994; Wigfield & Eccles, 1992). Motivation is defined as an orientation to the world that is based on a person's expectations and evaluations. According to expectancy-value theory, a direct causal relationship exists between task value and academic achievement. There are four components of task values: attainment value or importance, intrinsic motivation, utility value or usefulness of the task, and cost (Eccles et al., 1983; Wigfield & Eccles, 1992). Attainment value is defined as the importance of doing well on a given task (Eccles et al., 1983). Intrinsic value is defined as the enjoyment a person gains from doing the task. Utility value is defined as the usefulness or relevance a person gains from doing the task. Cost refers to how the decision to engage in an activity limits access to other activities as well as to an assessment of how much effort will be taken to accomplish the activity and its emotional cost (i.e., loss of time, overly-high effort demands, loss of valued alternatives, or negative psychological experiences such as stress). Intrinsic value overlaps the construct of intrinsic motivation defined by Deci and his colleagues (Deci & Ryan, 1985; Deci, Vallerand, Pelletier, & Ryan, 1991) and by Harter (1981), because they both focused on the enjoyment or interest value of the task. Utility value is similar to extrinsic motivation in that both emphasize the external reasons for engaging in a task or the relevance to a larger goal (Deci & Ryan, 1985; Harter, 1981). The importance of cost has barely been studied empirically.

Self-theories are another set of popular theories about motivation that focus on people's ideas about competence or intelligence, that is, what competence is and what it means about the self (Dweck & Molden, 2005). Self-theories are implicit beliefs that people have about their intelligence and can either be regarded as incremental or as traits. Individuals who subscribe to incremental theories believe that intelligence is malleable and can be changed with effort over time. In contrast, individuals who subscribe to trait theories believe that a person's intelligence is fixed and does not change over time (Dweck & Molden, 2005). In self-theories, motivation can be understood as an individual's striving for competence. The Implicit Theories of Intelligence Questionnaire (Dweck, 1999) measures students' general belief about the fixedness or the malleability of intelligence. There are two corresponding motivational reactions to failure that are associated with self-theories. On the one hand, when individuals with a fixed intelligence orientation confront failure, they tend to show defensive reactions such as avoiding challenges or engaging in more handclapping behaviors (e.g., when an individual fails to understand something, s/he becomes discouraged to the point of wanting to give up) (Elliot & Dweck 1988; Rhodewalt, 1994). These reactions are related to performance goals (Chen & Pajares, 2010; Dweck, 2000; Dweck & Molden, 2005; Dweck & Leggett, 1988). On the other hand, when individuals with an incremental orientation face failure, they may immediately begin to consider various strategies to solve the difficult tasks. They believe that their effort will change the circumstances (e.g., When something is difficult, one tries harder”). These reactions are related to mastery goals (Dweck, 2000; Dweck & Molden, 2005; Dweck & Leggett, 1988).

This current study adopted self-determination theory over achievement goal theory, expectancy-value theory, and self-theories. Many researchers have studied the integration of these theories and have tested elements of this integrative model (Anderson, 2015; Cho, Weinstein, & Wicker, 2011; Ciani, Sheldon, Hilpert, & Easter, 2011; Drylund, 2008). According to this integrative model, achievement goal theory can be explained through self-determination theory. Similarly, intrinsic value and utility value from expectancy value theory overlap with the constructs of intrinsic and extrinsic motivations from self-determination theory. In fact, intrinsic and extrinsic motivations are central to expectancy value theory. Researchers who subscribe to self-theories have studied people’s beliefs about their competence. This is less comprehensive than the three components (autonomy, competence, and relatedness) in self-determination theory. Therefore, because of all these comparisons, the framework of the self-determination theory for motivation was adopted in this present study.

Linnenbrink (2007) measured students’ motivation by assessing their mastery goal orientation and performance goal orientation. Although achievement goal theory was expanded to add approach and avoidance dimensions (Elliot, 1999; Pintrich, 2000b), her empirical studies only focused on approach goal orientation (i.e., mastery-approach goal and performance-approach goal) without considering avoidance goal orientation.

The PISA Perspective for Motivation

Although PISA 2012 did not provide any indicators to measure motivation through performance and mastery goals, it did measure motivation as intrinsic and extrinsic motivations. Based on the current trend toward integrating achievement goal theory and self-determination theory, PISA 2012 can still be viewed as a valuable tool for building a theoretical model for the current study. In PISA, intrinsic motivation refers to the drive to do mathematics purely for the joy gained from the activity itself, and extrinsic motivation refers to the drive to learn mathematics because students perceive it as useful to them and to their future studies and careers. PISA 2012 measures intrinsic motivation in mathematics by whether students perform an activity purely for the joy gained from mathematics itself (see Appendix B). This emphasis is similar to intrinsic motivation in the self-determination approach (Harter, 1981; Ryan & Deci, 1985). PISA 2012 measures extrinsic motivation in mathematics by whether students perceive mathematics to be useful to them and to their future studies and careers (see Appendix B). This emphasis is similar to extrinsic motivation in the self-determination approach (Ryan & Deci, 1985; Harter, 1981).

Theoretical Framework for Engagement

Traditionally, engagement is defined as the amount of time and effort students put into their learning activities (Gonyea & Kuh, 2009). Some researchers measure engagement through behaviors (e.g., participation, effort) (Finn, 1989). Others measure engagement through emotions (e.g., passion, interest) (Appleton, Christenson, & Furlong, 2008). Some researchers measure engagement through measuring cognition (e.g., cognitive and metacognitive strategies such as rehearsal, elaboration, organization, and self-regulation) (Karabenick, Pintrich, & Wolters 2003). Currently, engagement has been conceptualized as having behavioral, affective, and cognitive components (Fredricks, Blumenfeld, & Paris, 2004; Jimerson, Campos, & Greif, 2003). The current study adopted the framework of Fredricks, Blumenfeld, and Paris (2004) to approach the construction of engagement. Table 2.2 presents a summary of the components and measures identified in Fredricks, Blumenfeld, and Paris (2004).

Behavioral Engagement. Behavioral engagement is defined as a student's conduct that is beneficial to psychosocial adjustment and achievement at school (Fredricks, Blumenfeld, & Paris, 2004). Some researchers have measured behavioral engagement by identifying the presence of positive conduct such as following rules and adhering to classroom norms and the absence of negative (disruptive) conduct such as skipping school and getting in trouble (Finn, 1993; Finn, Pannozzo, & Voelkl, 1995; Finn & Rock, 1997; Finn, 1989). Other researchers have measured behavioral engagement by assessing involvement in academic learning and tasks, such as persisting when facing difficulties, demonstrating enthusiasm, making effort, asking questions, and contributing to class discussion (Birch & Ladd, 1997; Finn, Pannozzo, & Voelkl, 1995; Skinner & Belmont, 1993; Skinner, Furrer, Marchand, & Kindermann, 2008). Yet other researchers have measured behavioral engagement by investigating participation in school-related activities, such as extra-curricular activities, athletic events, and school governance (Finn, 1993; Finn, Pannozzo, & Voelkl, 1995; Finn, 1989). Others have adopted behavioral engagement as part of their understanding of engagement in mathematics (Alexander, Entwisle, & Dauber, 1993; Lan, et al., 2009; Sciarra & Seirup, 2008). For example, behavioral engagement has been found to significantly predict students' long-term consequences with respect to school performance in mathematics (Alexander, Entwisle, & Dauber, 1993).

PISA 2012 provides one indicator to measure behavioral engagement based on students' persistence on school tasks. Obviously, the PISA items on behavioral engagement emphasize the second of the measurement approaches discussed in the paragraph above, that is, the one that focuses on persisting when facing difficulties.

Emotional Engagement. Traditionally, emotional engagement refers to both positive and negative reactions in the classroom (e.g., interest, boredom, happiness, sadness, and anxiety) (Connell & Wellborn, 1991; Skinner & Belmont, 1993). Some researchers have assessed emotional engagement by measuring emotional reactions to teachers and classmates as well as to academic and school environments (Ladd, Buhs, & Seid, 2000; Ladd & Dinella, 2009; Lee & Smith, 1995; Stipek, 2002). Others have assessed emotional engagement by measuring whether students commit to learning and participate in the academic activities necessary for their schooling (Finn, 1989; Skinner, Furrer, Marchand, & Kindermann, 2008; Wilson & Beard, 2003). Still other researchers have assessed emotional engagement by measuring identification with school, characterized as levels of attachment to school and appreciation of success in school-related outcomes (Christenson et al., 2001; Finn, 1989; Voelkl, 1997). Attachment to school means that students feel embedded in and a part of their school community (Spencer & Markstrom-Adams, 1990). Valuing success in school-related tasks refers to the extent to which students emphasize success in school-related outcomes or the degree to which they perceive education as benefiting them economically or in other ways (Mickelson, 1990). Some other researchers have assessed emotional engagement by measuring the degree to which students feel academically or intellectually challenged (Lee & Smith, 1995). Finally, yet other researchers have adopted emotional engagement as part of their perspective about the engagement domain in mathematics (Barkatsas, Kasimatis, & Gialamas, 2009; Ladd & Dinella, 2009; Martin, Rimm-Kaufman, 2015). An example that demonstrates the impact of emotional engagement is that students' emotional engagement, as evidenced by enthusiasm, pride, and satisfaction, has been found to significantly contribute to their effortful involvement and high achievement in mathematics (Barkatsas, Kasimatis, & Gialamas, 2009; Ladd & Dinella, 2009).

Unfortunately, PISA does not integrate emotion into its theoretical framework for engagement. Nonetheless, PISA items on behavioral engagement reflect work done by Finn (1989), who assessed emotional engagement by measuring whether students participate in the academic work necessary for their schooling.

Cognitive Engagement. Cognitive engagement is defined as a student's level of psychological investment in learning (Fredricks, Blumenfeld, & Paris, 2004). Some researchers have measured cognitive engagement by assessing an individual's desire to go beyond the requirements and their own preferences to take on challenges such as flexibility in problem solving, positive coping in the face of failure, and a preference for challenge (Boekaerts, Pintrich, & Zeidner, 2000; Connell & Wellborn, 1991; Fredricks, Blumenfeld, Friedel, & Paris, 2005; Newmann, Wehlage, & Lamborn, 1992; Nystrand & Gamoran, 1991). Other researchers have measured cognitive engagement by investigating metacognitive and volitional strategies that help promote understanding, such as self-regulated learning (i.e., individuals plan their learning, showing control and autonomy) (Fredricks, Blumenfeld, & Paris, 2004), learning strategies when dealing with failure (i.e., rehearsal, summarizing, elaboration, and organization) (Weinstein & Mayer, 1986), and high level thinking skills when they encounter challenging problems (i.e., task mastery, information-seeking, monitoring, and evaluation of responses and experimentation) (Caraway, Tucker, Reinke, & Hall, 2003; Meece, Blumenfeld, & Hoyle, 1988; Stoney & Oliver, 1999; Yazzie-Mintz, 2007). Another group of researchers have adopted cognitive engagement as part of their perception of the engagement domain in mathematics (Archambault, Janosz, Morizot, & Pagani, 2009; Boekarts, Pintrich, & Zeidner, 2000; Metallidou & Vlachou, 2007; Nystrand & Gamoran, 1991). An example that illustrates the importance of measuring cognitive engagement is that it has been found to significantly predict student achievement in mathematics (Metallidou & Vlachou, 2007; Sciarra & Seirup, 2008).

Table 2.2

Key Aspects of Student Engagement

Behavioral	Emotional	Cognitive
Characteristics		
Positive conduct • Follows rules • Adheres to classroom norms • Involvement in academic tasks • Effort • Completing tasks • Participation, questions Extra-curricular participation	Affective reactions • Feelings of interest, boredom • Feelings about teachers and peers • Feelings about schoolwork • Feelings of being valued at school • Identification with school	Investments and strategy • Preference for challenge • Effort directed toward learning • Motivation to learn • Learning goals vs. performance • Meta-cognitive • Strategic learning

Table 2.2 (continued)

Typologies		
• Cooperative participation • Autonomous participation	• Values; interest, attainment, cost, value • Flow	• Surface-level strategies • Deep-level strategies
Measures		
Conduct • Completing work • Compliance with rules • Tardiness Persistence • Effort Participation • Participation in discussion • Participation in class work • Ask questions	Emotions related to school • Interest, happiness • Identification with school Emotions related to schoolwork • Value of schoolwork Emotions related to people • Relationships with teachers • Relationships with peers Experience sampling	Motivation and Efficacy • Substantive engagement • Procedural engagement • Authentic learning goal • Metacognition

Source: Fredricks, Blumenfeld, & Paris (2004).

PISA 2012 provides one indicator that measure cognitive engagement through cognitive activation. Cognitive activation is about promoting students to use strategies such as summarizing, questioning, and predicting when solving mathematics problems (see Appendix B). Cognitive activation in PISA 2012 is apparently similar to the second approach above in which cognitive engagement is measured through metacognitive and volitional strategies (high level thinking skills in this case) (Stoney & Oliver, 1999; Weinstein & Mayer, 1986; Yazzie-Mintz, 2007).

Alternative Frameworks for Engagement

The theories developed by Finn (1993) and Fredricks, et al., (2004) have framed a majority of the research on student engagement. In particular, Fredricks et al. (2004)

proposed that there are three components of engagement (behavioral, emotional, and cognitive). Appleton, Christenson, Kim, and Reschly (2006) proposed a four-factor model of student engagement that included affective engagement, cognitive engagement, behavioral engagement, and academic engagement. This approach added academic engagement the previous three factors model. Academic engagement can be defined as activities and goals, such as course credits, homework completion, and the length of time in which the student remains on task and is not distracted (Appleton, Christenson, Kim, & Reschly, 2006). Appleton et al. implied that academic engagement could be measured through (a) attendance and lack of suspensions and (b) voluntary classroom participation and extra-curricular participation. Academic engagement overlaps with behavioral engagement because they both emphasize learning-oriented behaviors in school settings.

Hazel et al. (2008) formed a three-factor model of student school engagement that included aspirations, belongingness, and productivity. Aspiration is defined as a student's interest and investment in their education. It can be measured by assessing the student's intention to enroll for an advanced degree. Belongingness is defined as a student's identification with school values and having positive relationships with adults and peers at school. It can be measured through a student's sense that s/he is a member of the school community, as well as by his/her commitment to the school's norms. Productivity is defined as a student's effort, persistence, concentration, attention, and willingness to work on academic tasks. It can be measured by seeing whether the student uses cognitive strategies that monitor and maximize learning.

Obviously, the Fredricks et al. (2004) model is both comprehensive and concise compared with the four factors of student engagement model (Appleton, Christenson, Kim, & Reschly, 2006) or the students school engagement model (Hazel et al., 2008). Although the four types of engagement (i.e., academic, behavioral, cognitive, and affective engagement) are quite comprehensive in theory, qualitative differences between these engagements have not been clearly defined. In particular, the differences between academic and behavioral engagement have not been clearly defined because there are overlaps between academic engagement and behavioral engagement in terms of their emphasis on learning oriented behaviors at school. On the other hand, the students school engagement model (Hazel et al., 2008) appears to oversimplify the concept of engagement because aspirations, belongingness, and productivity obviously fit into the three dimensions of Fredricks et al. (2004). Finally, Fredricks et al. (2004) posited that the patterns of engagement across these three dimensions have long-term effects on students' academic success. The positive correlations between school engagement and school success have been identified by many studies (Appleton, Christenson, Kim, & Reschly, 2006; Wang, Selman, Dishion, & Stormshak, 2010; Wentzel, Battle, Russell, & Looney, 2010). Therefore, the adoption of the framework of Fredricks et al. (2004) in this present study can be expected to produce empirical evidence that is directly useful for school reform and improvement.

Importance of PISA

PISA is an international, large-scale standardized assessment that measures 15-year-old students in the domains of reading, mathematics, and science in a large number of countries. Rather than being limited to measuring the curriculum content that students have learned, the purpose of the PISA is to measure the yield of different education systems and to determine how well students who are approaching the end of mandated education are prepared to meet challenges in the real world. PISA is administrated every three years to assess the students' level of knowledge and skills essential for full participation in society near the end of compulsory schooling. In each cycle, one aspect has been addressed as the major domain while the other two domains are addressed in less detail.

PISA Promotes Educational Improvement. PISA is an ongoing program that provides insights for educational policy and practice and that helps monitor students' acquisition of knowledge and skills across countries and in different demographic subgroups within each country. The PISA results supply important data for politicians so they can know how their country is doing in global knowledge (Donlin, 2007). The PISA results help determine how a country's education system performs in comparison with other education systems nationally and from around the world (Mislevy, 1995; Provasnik et al., 2012). PISA functions as a new form of governance (Saraisky, 2015). Some researchers highlight PISA as a "new mode of global education governance in which state sovereignty over educational matters is replaced by the influence of large-scale international organizations" (Meyer & Benavot, 2013, p.10). The PISA results have received significant attention from the public and from educators by generating data that enhances the ability of policy makers to make evidence-based decisions (Bussiere, Cartwright, & Knighton, 2004). ACT (2011) argued that active participation in PISA promotes the successful implementation of the Common Core State Standards (CCSS). Shivraj (2014) found that PISA can be used to assess the outcome of attempts to increase educational fairness.

PISA Promotes Methodological Advancement. PISA is effective and efficient at addressing educational questions in a historical context (Frankfort-Nachmias & Nachmias, 1996) and providing a basis for assessing comparative change over time (Brooks-Gunn, Elder, & Phelps, 1991). Large sample-size comparisons within and between countries enlarge the scope of generalization and provide global perspectives. PISA avoids many data collection problems, such as appropriate respondents and small sample size. Thus, PISA can monitor change over extended periods. The psychometrically sound instruments developed in PISA have a wide range of applicability to educational research (McQueen & Mendelovits, 2003; Turner & Adams, 2007). PISA has an economic rationale because it is less expensive than alternatives that have a more formal design, collection, and analysis (Hofferth, 2005). Therefore, it is ideal for researchers who are conducting studies at institutions with limited resources (Friedman, 2007). PISA is an innovative assessment instrument because it assesses students on their application of knowledge to real world problems and situations (Saraisky, 2015). PISA focuses on the survival skills of students in modern society, with various readiness measures that can be linked to the socioeconomic and sociocultural wellbeing of the society. This opens the door for many interdisciplinary research-based advancements (Dundas, 2009).

Relationships between Affect, Motivation and Engagement

Overall View. There is a growing interest in studying the role of affect in educational settings. Many scholars have used different definitions of affect, which can be seen through different theoretical and methodological lenses. Providing a clear definition of affect in educational settings by considering the theoretical similarities and differences between the various perspectives is important. Linnenbrink (2007) defined affect in a broader sense as affective traits, including pleasant affect and unpleasant affect. She also expanded the notion of affect in education to include motivation and engagement. Overall, she developed a conceptual model linking affect, motivation, and engagement in classroom settings in Linnenbrink (2007). Realizing that the non-cognitive attributes share common traits and may thus interact, Linnenbrink (2007) proposed the mediational model of affect, motivation, and engagement. Specifically, pleasant and unpleasant affects mediate the relationship between an achievement goal orientation and behavioral and cognitive engagement.

Affect. Linnenbrink (2007) used a circumplex model as the theoretical basis for affect. She further captured affect as an ordering of affective states on the circumference of a circle around two dimensional bipolar spaces of an affective valence (pleasure or displeasure) and an arousal or activation dimension (high or low). Affect was categorized into four conditions: deactivated pleasant affect (i.e., relaxed and calm), deactivated unpleasant affect (i.e., sad, tired, and exhausted), activated unpleasant affect (i.e., tense and angry), and activated pleasant affect (i.e., excited and happy). In doing so, Linnenbrink (2007) used pleasant affect and unpleasant affect to “operationalize” the circumplex model. Linnenbrink (2007) adopted this framework of affect to accommodate the complexity of a student’s emotional life in the classroom. In general, this framework acknowledges the critical role of affect in a students’ academic career. The use of affect allowed Linnenbrink to connect with subject-specific affective domains for the operationalization of affect. In mathematics, this operationalization connects well with beliefs and attitudes perspectives, such as mathematics self-efficacy, mathematics self-concept, and mathematics anxiety, which can adequately capture the key elements of the affective domain in mathematics.

Motivation. Linnenbrink (2007) used achievement goal theory as the theoretical base for motivation. She adopted two primary goal orientations that can motivate students' efforts in achievement behavior: a mastery goal orientation and a performance goal orientation. She measured motivation through mastery and performance approaches. In particular, a mastery goal orientation focuses on developing competence, whereas a performance goal orientation focuses on demonstrating competence. Linnenbrink (2007) justified the use of this framework for motivation by showing that motivation is highly contextual. "These goal orientations are thought to emerge and develop in response to one's schooling experiences; as such the context has an important influence on the goal orientations that students endorse in any particular setting" (Linnenbrink, 2007, p110). As mentioned above, performance goals have also been referred to as ability-focused goals (Ames, 1992) and extrinsic goals (Pintrich et al., 1993), whereas master goals have also been referred to as learning goals (Dweck, 2000) and intrinsic goals (Pintrich et al., 1993).

Engagement. Linnenbrink (2007) measured engagement from behavioral and cognitive perspectives. Behavioral engagement is defined as effort and persistence, whereas cognitive engagement is defined as cognitive strategies (i.e., elaboration, rehearsal), metacognitive strategy use, and self-regulated learning. Behavioral engagement is distinct from cognitive engagement in that the emphasis is on the amount or quantity of a student's engagement, while cognitive engagement emphasizes the quality of thought or the type of engagement (Fredricks, Blumenfeld, & Paris, 2004;

Linnenbrink, 2007). Linnenbrink (2007) attempted to understand student engagement by observing the students' reactions to various real-life and challenging activities. According to Linnenbrink (2007), behavioral engagement has to do with students' effort and persistence, whereas cognitive engagement has to do with learning strategies, metacognitive strategies, or self-regulated learning, which can be reasonably captured by learning strategies.

Linnenbrink's Dynamic Model of Affect, Motivation, and Engagement

Linnenbrink (2007) used a multi-dimensional approach to considering the interactions between affect, motivation, and engagement. As is shown in Figure 1.1, Linnenbrink discussed the interactions between affect, motivation, and engagement in terms of the mediation by affect on the relationship between motivation and engagement. In this figure, a solid line indicates consistent findings, while a dotted line indicates inconsistent findings. Also in this figure, a positive (+) sign indicates a positive relationship, while a negative (-) sign indicates a negative relationship. Linnenbrink began by describing the relationship between motivation and affect based on a series of correlational studies by assessing the upper elementary, middle school, or college students, as in Figure 1.1, by saying

> Mastery-approach goal orientations are associated with higher levels of pleasant affect and lower levels of unpleasant affect. The findings for performance-approach goal orientations are rather mixed, with performance-approach goals either unrelated or positively related to both pleasant and unpleasant affect (Linnenbrink, 2007, p.118).

Linnenbrink (2007) then described the relationship between motivation and engagement based on a series of correlational studies by pointing out that

> Mastery-approach goals are generally associated with higher levels of behavioral and cognitive engagement, although the findings are less consistent when learning/achievement is the outcome. The findings for performance-approach goals are quite mixed, making it difficult to make clear predictions regarding the proposed model (Linnenbrink, 2007, p.119) (see Figure 1.1).

Finally, Linnenbrink (2007) described the relationship between affect and engagement based on a variety of correlational studies conducted with children, adolescents, or young adults. As illustrated in Figure 1.1, Linnenbrink (2007) discussed behavioral engagement and affect as well as cognitive engagement and affect. With respect to behavioral engagement, "pleasant affect does not undermine behavioral engagement and may even enhance it" (Linnenbrink, 2007, p.118). With respect to cognitive engagement,

> The relation for cognitive engagement is more complex; however, we generally found no relation between unpleasant affect and cognitive engagement, including elaborative or metacognitive strategy use as well as learning. Of note, however, were a few studies suggesting that unpleasant affect undermined learning and working memory functioning
>
> (Linnenbrink, 2007, p.119).

Mediation

As discussed above, Linnenbrink discussed the interaction between affect, motivation, and engagement in terms of the mediation of affect on the relationship between motivation and engagement. For the mediation of affect to occur, Linnenbrink (2007) discussed four conditions:

> (1) the predictor variable must relate significantly to the mediator, (2)the mediating variable must relate significantly to the dependent variable, (3) the predictor variable must relate significantly to the dependent variable, and (4)the relation between the predictor variable and the dependent variable must be significantly reduced when the mediating variable is included in the regression equation (p.119).

The current research attempted to test Linnenbrink's dynamic model of affect, motivation, and engagement. One critical aspect of this task was to test the fulfillment of the four conditions. If the four conditions were fulfilled, then there was a full mediation of affect on the relationship between motivation and engagement. If not all four conditions were fulfilled, then there may be some partial mediation effects of affect on the relationship between motivation and engagement. Another critical aspect of this task was to test the signs of the paths in Linnenbrink's dynamic model of affect, motivation, and engagement, and another critical aspect was to examine the paths themselves. If some additions of paths or some reductions in paths indicate a better fit of the data to the model, the present research is in a position to suggest alternatives to Linnenbrink's dynamic model of affect, motivation, and engagement.

Chapter 3: Methodology

Data and Sample

Conducted by the Organization for Economic Cooperation and Development (OECD), the Program for International Student Assessment (PISA) is a systematic, international assessment that measures 15-year-old youths every three years in the domains of reading, mathematics, and science in 65 countries and regions. Although each triennial administration of the PISA assesses achievement in these three content areas, each cycle has a specific focus on one of the three. The PISA 2012 was the programme's 5th survey. It assessed the competencies of 15-year-olds in reading, mathematics, and science with a focus on mathematics in 65 countries and economies. In 44 of those countries took part in an optional assessment of creative problem solving; and in 18 countries and economies, students were assessed in financial literacy (OECD, 2013). A total of about 510,000 students between the ages of 15 years 3 months and 16 years 2 months participated in the PISA 2012, representing about 28 million 15-year-old old youths globally.

The PISA 2012 was designed to have two stages of stratified samples. The first stage sampled individual schools with probabilities proportional to their (enrollment) sizes, with the measure of size being a function of the estimated number of eligible students. The second stage sampled 35 randomly-selected, eligible students at selected schools who were chosen using random selection techniques. The students took paper-based tests that lasted two hours, with different students taking different combinations of test items. This so-called matrix sampling involves dividing a test into subsets of questions (possibly overlapping) and then administering these subsets to different subsamples of an initial sample. The tests were organized around passages describing real-life situations and included multiple choice, short answer, and extended response questions. In addition, the students answered a background questionnaire about their homes, schools, and learning experiences that took 30 minutes to complete. There were two optional questionnaires for the students: one asked students about their familiarity with and use of information and communication technologies, and the other asked students about their education to date, including interruptions in their schooling and whether and how they are preparing for a future career. School principals also answered a questionnaire to provide information about their schools. In some countries and economies, optional questionnaires were also given to parents, who were asked to provide information on their perceptions of and involvement in their child's schooling, their support for learning in the home and their child's career expectations, particularly in mathematics-based occupations.

This current study retrieved the American sample of 4,978 students from the student survey of PISA 2012.This nationally-representative sample of American students contained 2,453 girls, representing 50% of the participants, and 2,525 boys, representing the remaining 50% of the participants (gender was recoded as 0 = female, 1 = male). The sample also indicated that 79% (n = 3,828) of the participants were native students (where at least one parent was born in America), 21% (n = 1002) were immigrant students including first-generation or second-generation students where both parents were born outside America, or where neither the parents nor the student were born in the America (immigrant status was recoded as 0 = native, 1 = immigrant student either first or second generation). The sample also indicated that 22% (n = 982) of students lived with only one parent or guardian, while 78% (n = 3484) of students lived with either parents or guardians (family structure was recoded as 0 = single parent or guardian, 1 = two parents or guardians). The majority spoke English (86%, n = 4196) at home most of the time, and 14% (n = 670) of the students spoke another language at home most of the time (language at home was recoded as 0=English, 1=other language). Table 3.1 presents descriptive statistics of these student background variables which were used in data analysis as control variables. Missing data was handled by using auxiliary correlates during data analyses (see discussion later).

Table 3.1

Descriptive Statistics of the Student-Level Variables Functioned as Gender, Immigrant Status, Family Structure, and Language at home

Variables	*n*	%
Gender		
Male	2,453	50
Female	2,525	50
Immigrant Status		
Native	3,828	79
Immigrant	1,002	21
Family Structure		
Two parents or guardians	3,484	78
One parent or guardian	982	22
Language at home		
English	4,196	86
Another language	670	14

Measures and Variables

To test the dynamic model in which affect, motivation, and engagement interact in the learning of mathematics, measures or variables were sought as indicators that would represent these factors as latent constructs (not directly measured factors that comprise multiple observable variables). Three indicators were selected for the construct of affect: mathematics self-efficacy, mathematics self-concept, and mathematics anxiety (see Appendix B). Mathematics self-efficacy and mathematics self-concept were used as indicators of a pleasant affect, while mathematics anxiety was used as an indicator of an unpleasant affect (see Appendix B).

Eight items were used to assess the level of mathematics self-efficacy, evaluating whether the students were confident about solving a range of pure and applied mathematics tasks involving algebra (see Appendix B). Four-point Likert-type responses of “very confident,” “confident,” “not very confident,” and “not at all confident” were provided to the respondents. Five items that dealt with the students’ belief in their own mathematics abilities were used to measure the students’ mathematics self-concept. Five items that dealt with whether the students experience feelings of helplessness and stress when dealing with mathematics was used to measure the students’ mathematics anxiety (see Appendix B). For the mathematical self-concept and mathematics anxiety items four-point Likert-type responses of “strongly agree,” “agree,” “disagree”, and “strongly disagree” were provided to the respondents. One item (e.g., I am just not good at mathematics) for mathematics self-concept was recoded and the items on the whole mathematics anxiety scale were recoded to keep a consistent format. As a result, these scores are in a negative format; that is, a higher score indicates a lower pleasant affect and a higher unpleasant affect.

Two indicators were used for the construct of motivation: intrinsic motivation and extrinsic (instrumental) motivation in mathematics (see Appendix B). Intrinsic motivation to learn mathematics was used as an indicator of having a mastery-goal orientation, while extrinsic motivation to learn mathematics indicated a performance-goal orientation (see Appendix B). Four items were used to measure the students' intrinsic motivation, showing whether they performed an activity purely for the joy gained from the activity itself. Four items were used to measure the students' instrumental (extrinsic) motivation, showing whether they perceived mathematics as being useful to them and to their future studies and careers (see Appendix B). The response options included "strongly agreed," "agreed," "disagreed", and "strongly disagreed." These scores are in a negative format; that is, a higher score indicates a lower motivation.

The latent construct of student engagement within the school context was measured by two indicators: behavioral engagement and cognitive activation (see Appendix B). Five items were used to measure behavioral engagement based on students' responses about their persistence on school tasks. Students were asked whether they give up or put off difficult problem. Student also were asked whether they continue to working when confronted with a problem in school. Student responses range from: "very much like me", "mostly like me", "somewhat like me", "not much like me", to "not at all like me". Two items (e.g., when confronted with a problem, I give up easily and I put off difficult problems.) for behavioral engagement were recoded. Overall, these scores are in a negative format; that is, a higher score indicates a lower behavioral engagement.

Cognitive activation is about teaching students strategies, such as summarizing, questioning, and predicting, all of which can be used to solve mathematics problems (see Appendix B). Nine items were used to measure how frequently their mathematics teachers have asked them to use a number of specific cognitive activation strategies to solve mathematics problems. The students were asked how often their teachers asked them to reflect on problems, solve complex problems, and apply knowledge to new contexts, etc. Four-point Likert-type responses of “never or rarely,” “sometimes,” “often,” and “always or almost always” were provided to the respondents (OECD, 2013). The cognitive activation was scaled in a negative way so that lower scores indicated more advanced positions. In particular, lower values suggest that students reported that their mathematics teacher more frequently used cognitive activation strategies compared with those mathematics teachers of the average student in OECD countries.

Model and Structure

This study tested Linnenbrink’s (2007) dynamic model of affect, motivation, and engagement in the learning of mathematics using PISA 2012 data. Figure 1.1 in Chapter 1, which showed Linnenbrink’s (2007) model was modified below to form Figure 3.1. Figure 1.1 shows a conceptual model that integrates affect, motivation, and engagement in mathematics, whereas Figure 3.1 depicts a structural equation model (SEM) model that puts into operation Linnenbrink’s (2007) dynamic framework of affect, motivation, and engagement in mathematics.

In the measurement model, the composite scores for mathematics self-efficacy and math self-concept were used as indicators for the latent variable, pleasant affect. The five items of math anxiety (i.e., feel worry, get tense, get nervous, feel helpless, and worry about poor grades) were used as indicators for the latent variable, unpleasant affect. The four items of intrinsic motivation to learn mathematics (i.e., enjoy reading math, look forward to math lessons, enjoy math, and interested in math) were used as indicators for the latent variable, mastery approach in math. The four items of extrinsic motivation to learn mathematics (i.e., help in work, improve career prospects, help for study, and get a job) were used as indicators for the latent variable, performance approach in math. The five items of perseverance was used as indicators for the latent variable, behavioral engagement. The nine items of cognitive activation were used as indicators for the latent variable, cognitive engagement. This measurement model also included measurement errors for each observed variable (indicator).

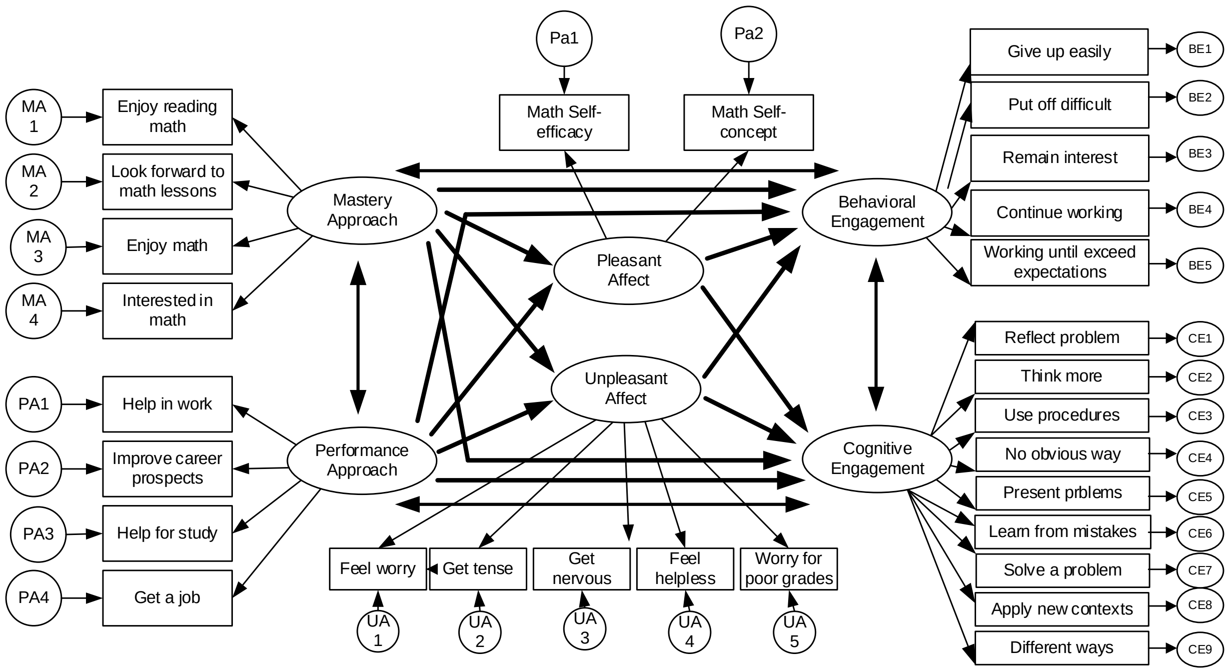

Figure 3.1. Structural model reflecting the relationships between students' affect, motivation, and engagement

This structural model includes both directional (in the form of regression) and nondirectional (in the form of correlation) relationships among latent variables of affect, motivation, and engagement in mathematics (see Figure 3.1). According to the structural model, mastery and performance approaches should respectively affect pleasant and unpleasant affects. Additionally, the mastery and performance approaches should respectively affect behavioral and cognitive engagement. In addition, pleasant and unpleasant affects should influence both behavioral and cognitive engagement. This structural model includes the encircled measurement errors in circles correlated with each indicator for each observed variable.

Procedures and Analyses

As stated earlier, variables that are relevant to the dynamic model in which affect, motivation, and engagement interact in the learning of mathematics (Linnenbrink, 2007) were obtained from the PISA 2012, a cycle of administration that concentrated on mathematics (with more comprehensive and detailed measures of many aspects critical to mathematics education.) The variables served as indicators to represent affect, motivation, and engagement (all considered as latent variables) to operationalize Linnenbrink’s (2007) model. Because the logic behind testing the model involved examining the fit of the data to the model, the present study used structural equation modeling (SEM) to test the model (i.e., to estimate the interactive effects of affect, motivation, and engagement in the learning of mathematics) by examining the extent to which the model fit the data (Kaplan, 2008). If the fit between the model and the data was good, the interpretation of the SEM path coefficients elucidated the structure of the model, providing information for the potential improvement of the model. If the model did not fit the data, an effort was made to improve the model-data fit. The results were able to be used to suggest alternative ways of specifying the model.

Specifically, the data analysis consisted of a three-stage process: a) tests of statistical assumptions; b) confirmatory factor analysis (CFA); and c) SEM. The purpose of stages a and b was to determine if the data was sufficient to conduct the SEM, and the SEM was to determine how well the dynamic model of affect, motivation, and engagement fit the data. To test the statistical assumptions, the Statistical Package for the Social Sciences (SPSS) was used to examine multivariate normality, linearity, and multicollinearity, which are considered critical statistical assumptions pertinent to the SEM (Vogt, 2007). To assess univariate and multivariate normality, the skewness and kurtosis were analyzed. According to Curran, West, and Finch (1996), for univariate normality, skewness ranging from 0 to 2 and kurtosis ranging from 0 to 7 can indicate sufficient normality. Linearity was assessed by evaluating the shape of the scatterplots within the scatterplot matrices. Given an assumption of linearity, that is that a straight-line relationship exists between the independent and dependent variables, the shape of the scatterplots should be elliptical (Mertler &Vannata, 2010). Multicollinearity was assessed by analyzing the correlations between the latent variables. Multicollinearity was assumed to exist if the correlation approached one (Muthén & Muthén, 2007).

Once the relevant statistical assumptions were sufficiently supported, M*plus* version 6 was used to perform SEM with the missing data and produce indices for the model-data fit (Muthén & Muthén, 2007). CFA is an a priori modeling technique that allows the researcher to test the underlying structure of latent variables by testing whether theoretical latent variables account for the correlations between the multiple observed variables (Brown, 2006). Specifically, CFA was performed separately on each construct in terms of affect, motivation, and engagement.

The CFA model was tested for the model identification standards (Byrne, 2013) before conducting SEM. Model identification refers to whether the number of degrees of freedom in the model is sufficient. If the model is over-identified, it means that the number of parameters in the model is less than the number of sample moments (i.e., sample variances and covariance) then the model is sufficient. A maximum likelihood (ML) estimation was undertaken to test the fit of the hypothesized model. ML is appropriate when the variables in the model approximate normality. A robust maximum likelihood (MLR) estimator was undertaken to test the model fit to determine whether the variables were non-normal (Byrne, 2013).

Indices that indicate the fit between the model and the data are critical for testing theories by employing SEM. Multiple indices need to be considered to obtain a good triangulation between the data and the model (Browne & Cudeck, 2003; Byrne, 2013). The probability value associated with χ^2 indicates the fit between a hypothesized model and the corresponding model obtained from a sample population. This probability value represents the likelihood that the χ^2 statistic is greater than the χ^2 value when the null hypothesis is true. Thus, a high *p*-value indicates a closer fit between the two types of models. Importantly for this study, χ^2 can be affected by a large sample size (Cheung & Rensvold, 2002), but the effect of large sample sizes can be reduced by dividing the χ^2 index by the degrees of freedom (Kline, 2005). High correlations between observed variables also increase the probability of rejecting the null hypothesis, because high correlations between the variables increase the power of the tested model, causing an increase in the χ^2 fit index (Miles & Shevlin, 2007).

Given that the χ^2 test is affected by large sample size (Cheung & Rensvold, 2002), the overall fit of the model was also evaluated using indices, specifically, the comparative fit index (CFI), Tucker-Lewis index (TLI), root mean square error of approximation (RMSEA), and standardized root mean square residual (SRMR), which were more robust to sample size (Hu & Bentler, 1999). Many studies concluded that CFI, RMSEA, and

SRMR are sufficient for measuring the fitness of models obtained by SEM (Byrne, 1998, 2013; Hu & Bentler, 1999). The CFI and TLI are incremental indices of fit that measure the relative improvement in fit of a hypothesized model in comparison to the corresponding baseline model (often referred to as the null or independence model) that assumes zero covariance among the observed variables (Byrne, 2013; Kline, 2005). The CFI is the percentage of the observed measure covariance explained by a structural model and tends to be more accurate than other goodness of fit indices. The TLI compares a proposed model against a null model. The CFI measures the same thing as the TLI except that the CFI uses the non-centrality parameter as the measure of misfit. The values of the CFI and TLI lie between 0 and 1, with a value greater than .90 indicating that the population matrix fits the hypothesized model closely (Byrne, 2013; Browne & Cudeck, 2003; Hu & Bentler, 1999). The RMSEA measures the error of approximation and is sometimes considered to be a population-based index. It estimates the amount of error of approximation per model degree of freedom, taking sample size into account. RMSEA, unlike the χ^2 test, is not sensitive to large sample size. A value of 0 in the RMSEA indicates the best fit. Normally, RMSEA values less than .05 are considered a good fit, values in the range of .06 and .08 are considered a moderate fit, and values greater than .10 indicate a poor fit (Browne & Cudeck, 2003). Additionally, the RMSEA and 90% CI for RMSEA were both below .05, and p for close fit is 1, suggesting a close fitting model (Kenny, 2005; MacCallum, Browne & Sugawara, 1996). The SRMR is defined as the standardized difference between an observed correlation and a predicted correlation. It may be biased when a sample has a small N and a low degree of freedom. The SRMR is an absolute measure of fit, so a value of zero indicates a perfect fit. A value of less than .08 is generally considered a good fit (Browne & Cudeck, 2003). Model modification generally occurs when the original model does not fit the data. It involves adding or removing a statistical path as suggested by the residuals and the modification indices (MI) obtained by running the original model (Hoyle, 1995).

An adequate sample size is critical for SEM to produce valid results (Brown, 2006). Meeting the criteria for the minimum sample size decreases the probability of committing a type II error (failing to detect relationships between the variables when they actually exist) and increases the power of a study. Analyses involving various methods have suggested that SEM requires a minimum sample size of at least 100 to 200 (Brown, 2006). In addition, some studies have estimated the acceptable sample size by using the N:q rule, where N is the number of participants and q is the number of parameters included in the statistical model, or by conducting power analyses (Jackson, 2003; Kline, 2005). When researchers determined the minimum sample size using the N:q rule, some suggested that at least five to 10 cases per each freed parameter seem to be appropriate. Additionally, the sample size is adequate if the number of freed parameters is less than or equal to 47 (Brown, 2006). Obviously, minimum sample size was not an issue in the present study because the large-scale PISA dataset is sufficient for the CFA.

Important Statistical Issues

Item Parceling. Using parcels as indicators of constructs in structural equation models, (SEMs) has been common in psychological and educational research. Item parceling is a measurement practice that involves summing or averaging two or more items and using the result as the basic unit of analysis in the SEM model (Bandalos, 2002; Little, Cunningham, Shahar, & Widaman, 2002).

There are many benefits to using item parcels, including that they are more reliable than individual items and have more definitive rotational results (Cattell & Burdsal, 1975; Kishton & Widaman, 1994). A considerable number of studies have also found that parceling contributes to increased reliability (Bandalos & Finney, 2001; Cattell & Burdsal, 1975; Kishton & Widaman, 1994). Another advantage of item parceling is that parcels have distributions that are more continuous and normally distributed than individual items, which are aligned with the assumptions of common normal theory-based estimation methods such as maximum likelihood (ML) (Bridgeman & Rock, 1993).

Although parcels have many advantages, aggregating items to manufacture indicators of a certain construct may lead to misleading results if the parcels are not constructed carefully. A variety of problems can potentially occur when aggregating items into parcels. The most problematic issues are the number and the coherence of the items within each parcel and the method by which the parcels are created. Some researchers have argued that item parcels only work under certain limited conditions, specifically when: 1) the intrafactor parceled items are unidimensional and 2) unique factors within the items do not correlate with unique or common factors of other items in other parcels (Bandalos & Finney, 2001; Enders & Bandalos, 1999; Hall, Snell, & Foust, 1999).

To avoid these potential problems, certain guidelines should be considered before parceling items: 1) items must be valid, individual measures of the construct of interest; 2) items must be at the same level of specificity both within and across parcels (i.e., items and scales or subscales should be parceled together); and 3) items within a parcel must be unidimensional (Bagozzi & Heatherton, 1994; Hall, Snell, & Foust, 1999). Using an isolated uniqueness parceling strategy could increase the unidimensionality of a factor by forcing the influence of a secondary factor into the error term. This strategy works best when the second factor has a relatively weak influence on the items (Hall, Snell, & Foust, 1999). In the current study, this strategy was employed for both practical and theoretical reasons. All things being equal, SEM operates better with single-indicator variables or with three or more indicator variables (Bandalos, 2002; Hall, Snell, & Foust, 1999). Finally, the use of item parcels rather than individual items should be expected to result in the largest improvement in model fit for situations in which the influence of secondary factors is strong and the communalities between the items are low (Bandalos, 2002). In the present analysis, self-efficacy and self-concept in mathematics were parceled as indicators for pleasant affect.

Internal Reliability. Instead of Cronbach's alpha (α), the internal consistency estimate of reliability (ω) is assessed by means of the coefficient alpha function in M*plus* version 6.0. There are two major problems with Cronbach's alpha (α): 1) It is unrealistic to assume that all items have the same item-construct relation and equal item covariances (tau-equivalence). 2) Cronbach's alpha underestimates the population reliability coefficient. The internal consistency estimate of reliability (ω) has many advantages compared with Cronbach's alpha: 1) ω does not assume that all items have the same item-construct relations and equal item covariance. 2) ω is a more consistent (precise) estimator of reliability. And 3) ω is easy to estimate (Crutzen, 2007; Dunn, Baguley, & Brunsden, 2014; Geldhof, Preacher, & Zyphur, 2014; Peters, 2014; Sijtsma, 2009). In the present analysis, ω was employed to estimate the internal consistency of all the measurement scales, including mathematics self-efficacy, mathematics self-concept, mathematics anxiety, behavioral engagement, cognitive engagement, and mastery and performance approaches.

Uniformity of Measurement Scales. Confirmatory factor analysis (CFA) or structural equation modeling (SEM) ia often used to deal with the issue of measurement equivalence (Byrne & Campbell, 1999; Cheung & Rensvold, 1999; Little, 1997; Rensvold & Cheung, 1998; van de Vijver & Leung, 1997). Unified measurement scales are not necessarily required for an SEM model, and some researchers have included SEM variables with different measurement scales (Patrick, Ryan, & Kaplan, 2007; Yoon & Uysal, 2005). The present analysis contained variables with different measurement scales.

Missing Data. Missing data in this study ranged from a low of 2% for language at home to a high of 36% for behavioral engagement. Simply deleting or removing any cases with missing data would have resulted in a loss of important information and, thus, could have caused bias in point estimates, standard errors, the nonpositive covariance matrix, and heteroscedastic error. Ultimately these could lead to inaccurate conclusions (Graham, 2003, 2009; Schumaker & Lomax, 1996).There are multiple ways to handle missing data, including listwise deletion, pairwise deletion, mean substitution, regression substitution, single imputation, multiple imputation, and model-based methods (Bennett, 2001; Roth, 1994; Pampaka, Hutcheson, & Williams, 2016; Pigott, 2001; Schlomer, Bauman, & Card, 2010). Auxiliary correlates, one of the prevailing model based approaches, was adopted in this study rather than common approaches (i.e., data editing or deletion) or single imputation approaches (i.e., mean and regression substitutions). An auxiliary variable is a variable that is highly correlated with the variables in the substantive model, although this variable may not be of substantive interest. Auxiliary variables are useful in the missing data handling method because they include variables that account for the pattern of missing data (Schafer & Graham, 2002; Graham, 2003; Schlomer, Bauman, & Card, 2010). This approach can reduce estimation bias due to missing not at random (MNAR) and can partially restore lost power due to missingness or reduced sample size (Collins, Schafer & Kam, 2001; Graham, 2003, 2009; Schafer & Graham, 2002).

The missing data pattern can be described based on the input data matrix and the values that are missing. There are three patterns of missingness: missing completely at random (MCAR), missing at random (MAR), and not missing at random (NMAR) (Acock, 2005; Bennett, 2001; Schafer & Graham, 2002; Schlomer, Bauman, & Card, 2010).There are no patterns in the MCAR data, and the missing values are not related to any variable under study (Acock, 2005; Bennett, 2001; Schlomer, Bauman, & Card, 2010). There are also no patterns in the MAR data since the missing values are related to other observed variables but not to its own unobserved values (Schafer & Graham, 2002; Schlomer, Bauman, & Card, 2010). There is a pattern of missing data in the NMAR data such that the likelihood of missingness is related to the variables that are missing (Schlomer, Bauman, & Card, 2010). The final SEM model (examining both direct and indirect effects) suggested that there were 220 missing data patterns in this study, indicating that the data was NMAR data. Therefore, the present study used gender, family structure, immigrant status, and language at home as auxiliary variables in the analysis models by manually allowing each to be correlated with 1) other auxiliary variables and themselves, 2) all independent variables and covariates (including mediators), and 3) with all dependent variables (Graham, 2003; Schlomer, Bauman, & Card, 2010). This approach is beneficial for improving the precision of an imputation model by including the above four demographic variables that account for the 220 pattern of missing data, and including variables that are correlated with the variables that have missing data (Schlomer, Bauman, & Card, 2010). Auxiliary correlates were conducted in M*plus* version 6 (Muthén & Muthén, 1998-2010).

Sampling Weight. To account for differences in the probabilities of students selected in the stratified random sampling process, a sampling weight from PISA 2012 was used in the current study. Many factors can lead to systematic differences in the random sampling groups including missing data, non-response, or some other unexpected factors (e.g., subpopulation oversampling, and designed unequal probability sampling (Asparouhov, 2005). PISA data have a two-stage sampling procedure, and the sample size varies between schools and between countries. There were different probabilities in schools and students chosen in the sampled countries, which create overrepresentation or underrepresentation of certain individuals in the sample (Deaton, 1997). To avoid potential problems, a sampling weight at the student level was incorporated into each analysis to ensure that each sampled student is representative of the target population of 15-year-olds. At the student level, the PISA 2012 data has 81 weights, including both final weight and replicate weights. Replicate samples are formed through transformations of the actual sample, and these transformations included obtaining weights for each of the replicate samples. There are also within-school-weights related to student final weights and rescaled to sum up within each school to the school sample size. Between-school weights are related to the sum of student final weights (W_FSTUWT) within each school (OECD, 2014a, 2014b). Student final weight (W_FSTUWT) at the student level was used in this present study.

Mediation. To investigate any mediation by affect, first a full mediation of affect on the relationship between motivation and engagement was tested based on the four conditions for mediation specified by Linnenbrink (2007):

> (1) the predictor variable must relate significantly to the mediator, (2) the mediating variable must relate significantly to the dependent variable, (3) the predictor variable must relate significantly to the dependent variable, and (4) the relation between the predictor variable and the dependent variable must be significantly reduced when the mediating variable is included in the regression equation. (p.119)

Specifically, pleasant affect and unpleasant affect were expected to have mediating functions when it comes to the predictive effects of both mastery and performance approaches on behavioral and cognitive engagements. In particular, both intrinsic motivation and extrinsic motivation were expected to be significantly related to the mediator of affect (both pleasant and unpleasant affect) (see Figure 3.2). The mediator of affect (both pleasant and unpleasant affect) was also expected to be significantly associated with the outcome of engagement (both behavioral and cognitive engagement) (see Figure 3.2).

If the full mediation of affect was not identified or confirmed, the possibility of partial mediation effects of affect on the relationship between motivation and engagement were investigated by removing the conditions that did not occur in the model testing. If a partial mediation of affect was not identified or confirmed, further exploratory data analyses were used to attempt to modify Linnenbrink's (2007) dynamic model.

SEM simultaneously provides overall tests of model fit and individual parameter estimate tests. SEM allows for the simultaneous testing of multiple hypotheses (Kline, 1998) and examines the relationships between latent variables and observed variables. In addition, SEM enables the examination of both direct and indirect effects between latent variables. Thus, SEM tests the mediating effect between independent variables and dependent variables.

The importance of mediating variables has long been recognized by psychologists. In Woodworth's (1928) S-O-R model, an active organism intervenes between a stimulus and a response. This intervention is the most generic formulation of a mediation hypothesis. In general, a given variable may function as a mediator in the relationship between a predictor and a criterion. In particular, this model assumes a three-variable system in which two causal paths affect the outcome variables. These paths are 1) the direct impact of the independent variable (Path c); 2) the impact of the mediator (Path b); and 3) the path from the independent variable to the mediator (Path a) (see Figure 3) (Baron & Kenny, 1986). A variable is considered to be a mediator if it fulfills the following conditions: 1) variations in the levels of the independent variable significantly account for variations in the presumed mediator (i.e., Path a); 2) variability in the mediator significantly contributes to variation in the dependent variables (i.e., Path b); and 3) when Paths a and b are controlled, a previously significant relationship between the independent and dependent variables is no longer significant because of the mediation occurring when Path c is zero (Baron & Kenny, 1986). One of the goals of this current study was to test whether the mediators of the affective variables would affect the relationship between the students' motivation and their engagement in mathematics.

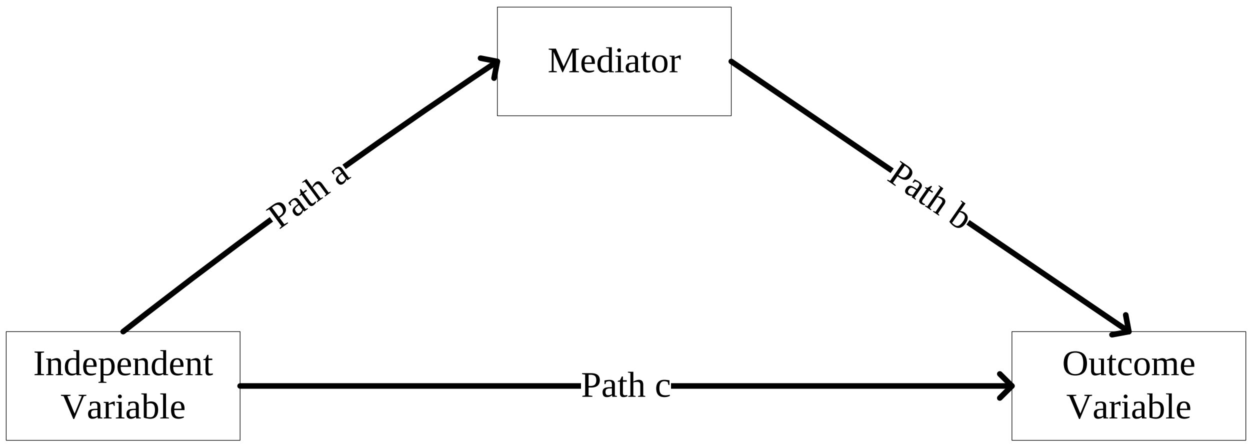

Figure 3.2. Basic mediator model

There are many differences between mediators and moderators. A mediator represents a generative mechanism by which a focal independent variable is able to influence the dependent variable of interest. A moderator functions as a focal independent variable in subgroups that establish its domains of maximal effectiveness with regard to a given dependent variable. A moderator affects the direction and/or strength of the relationship between an independent or predictor variable and a dependent or criterion variable. In a correlation analysis framework, a moderator is a third variable that affects the zero-order correlation between two other variables. Moderator variables always function as independent variables and are uncorrelated with both the predictor and the dependent variables. In contrast, mediating events shift the roles between effects and causes (Baron & Kenny, 1986).

Effect Size for Mediation Models. There are two typical ways to report effect size for mediation models: K^2 (the ratio of the observed indirect effect to the maximum possible indirect effect) and P_M (the ratio of the indirect effect to the total effect)

(Preacher & Kelley, 2011; Wen & Fan, 2015). Wen and Fan (2015) indicated that K^2 is not an appropriate effect size measure for mediation models because it lacks the property of rank preservation. In general, any effect size measure should ensure that a large effect size (in absolute term) always indicates a stronger effect, or vice versa. The total mediation effect size is larger than any of its subparts. However, the magnitude of K^2 may decrease when the mediation effect that K^2 represents increases. In addition, K^2 may lead to paradoxical results in multiple mediation models when it involves multiple mediators. The mediation effect size for each subpart may be larger than the total mediation effect size. Another issue is that there may be smaller mediation effects with larger mediation effect sizes, or vice versa. Therefore, this study used the traditional mediation effect size index P_M, which is calculated by relating the indirect effect to the total effect (i.e., total effect = direct effect + indirect effect) (Wen & Fan, 2015). This approach is meaningful when accompanied by the total effect from a basic mediation model where the indirect effect and the direct effect have the same sign (Wen & Fan, 2015). Caution must be taken when the indirect effect and the direct effect have opposite signs, P_M may not be appropriate as a mediation effect size measure because it is not bounded (e.g., it could be any huge number) (Preacher & Kelley, 2011; Wen & Fan, 2015). For the basic mediation model where the indirect effect and the direct effect have the same sign, it is meaningful to report the magnitude of the mediation effect. The maximum value of the indirect effect is the total effect, in which case P_M would be 1; which is also the ratio of the indirect effect to the total effect. For an inconsistent mediation model in which the indirect effect and the direct effect have opposite signs, it may not be meaningful to report the magnitude of the mediation effect, as it could be 1, 10, 100, or 1,000. The maximum possible value of the indirect effect is greater than the total effect and might be infinite. For example, the magnitude of the mediation effect would be 1000 when the total effect is 0.03 and the indirect effect is 30.

Chapter 4: Results

This chapter consists of four sections. The first section includes descriptive statistics for items measuring affect, motivation, and engagement. The second section checks the degree of item-level normality in the study variables. The third section presents findings from the confirmatory factor analyses (CFAs) used to test the hypothesized measurement models. Finally, the last section describes a series of structural equation models (SEM), which were examined for fit, compared, and summarized.

Item-Level Descriptive Statistics

Prior to conducting CFAs and SEMs, data screening and descriptive statistics were calculated to examine the characteristics of the following variables: affect, motivation, and engagement in mathematics. Missing data were excluded when examining the distribution of the study items of affect, motivation, and engagement in mathematics.

Table 4.1 provides descriptive statistics for items associated with each of the primary study variables. Part of the table indicates descriptive statistics (means and standard deviations). With respect to affect, the items under pleasant affect, as represented by mathematics self-efficacy, were measured on a scale of 1 to 4 and registered the students' responses about their perceived ability to solve a range of pure and applied mathematics problems. For example, the first item, using a train timetable to work out how long it would take to get from one place to another, had a mean of 3.07 (out of 4), indicating that on average the students showed high mathematics self-efficacy on this item. The items under pleasant affect, as represented by mathematics self-concept, were measured on a scale of 1 to 4 and registered the students' responses about their perceived competence in mathematics. For example, the first item, "I am just not good at mathematics" (whose response options were reversed for data analysis so that a higher value indicated more of the corresponding behavior), had a mean of 2.77 (out of 4), indicating that on average the students showed low mathematics self-concept on this item. The items under unpleasant affect, as represented by mathematics anxiety, were measured on a scale of 1 to 4 and registered the students' responses about their feelings of stress and helplessness when dealing with mathematics. For example, the first item, "I often worry that mathematics classes are difficult," had a mean of 2.64 (out of 4), indicating that on average the students showed high mathematics anxiety on this item.

With respect to motivation, the items under mastery-approach goal, as represented by intrinsic motivation, were measured on a scale of 1 to 4 and registered the students' drive to perform an activity purely for the joy gained from the activity itself. For example, the first item, "I enjoy reading about mathematics", had a mean of 2.19 (out of 4), indicating that on average the students showed somewhat high mastery-approach goals in mathematics on this item. The items under performance-approach goal, as represented by extrinsic motivation, were measured on a scale of 1 to 4 and registered the students' drive to learn mathematics because of external reasons other than the activity itself. For example, the first item, "Making an effort in mathematics is useful for future work", had a mean of 3.07 (out of 4), indicating that on average the students showed high performance-approach goals in mathematics on this item.

With respect to engagement, the items under behavioral engagement, as represented by perseverance, were measured on a scale of 1 to 5 and registered the students' response about their willingness to work on difficult problems. For example, the second item (the first one was deleted during the CFA to achieve a better model-data-fit), "I put off difficult problems", whose response options were reversed for data analysis so that a higher value indicated a more presence of the corresponding behavior. This item had a mean of 3.43 (out of 5), indicating that on average students had high levels of behavioral engagement in mathematics on this item. The items under cognitive engagement, as represented by cognitive engagement, were measured on a scale of 1 to 4 and registered the students' cognitive strategies such as summarizing, questioning, and predicting, when solving mathematics problems. For example, the first item, "The teacher asks questions that make us reflect on the problem", had a mean of 2.92 (out of 4), indicating that on average the students had a high cognitive engagement in mathematics on this item.

Table 4.1

Descriptive Statistics for Items Measuring Affect, Motivation, and Engagement

Variable	*M*	*SD*	Skewness	Kurtosis
Pleasant affect (mathematics self-efficacy)				
1. Using a train timetable to work out how long it would take to get from one place to another.	3.07	.77	-.50	-.18
2. Calculating how much cheaper a TV would be after a 30% discount.	3.09	.84	-.55	-.48
3. Calculating how many square meters of tiles would be needed to cover a floor.	3.01	.84	-.44	-.56
4. Understanding graphs presented in newspapers.	3.22	.77	-.77	.15
5. Solving equations like 3x+5=17.	3.63	.64	-1.84	3.41
6. Finding the actual distance between two places on a map with a 1:10 000 scale.	2.68	.92	-.06	-.91
7. Solving equations like 2(x+3) = (x+3)(x-3).	3.29	.84	-1.03	.32
8. Calculating the petrol-consumption rate of a car.	2.92	.85	-.33	-.65

Table 4.1 (continued)

Pleasant affect (mathematics self-concept)				
1. I am just not good at mathematics.	2.77	.95	-.44	-.68
2. I get good grades in mathematics.	2.98	.77	-.50	.00
3. I learn mathematics quickly.	2.72	.89	-.21	-.71
4. I have always believed that mathematics is one of my best subjects.	2.50	1.07	.02	-1.24
5. In my mathematics class, I understand even the most difficult work.	2.47	.91	.01	-.79
Unpleasant affect (mathematics anxiety)				
1. I often worry that it will be difficult for me in mathematics classes.	2.64	.89	-.13	-.74
2. I get very tense when I have to do mathematics homework.	2.29	.90	.32	-.62
3. I get very nervous doing mathematics problems.	2.15	.83	.45	-.24
4. I feel helpless when doing a mathematics problem.	2.01	.84	.66	.02
5. I worry that I will get poor grades in mathematics.	2.48	1.01	.04	-1.09
Behavioral engagement				
1. When confronted with a problem, I give up easily.	3.86	1.00	-.84	.45
2. I put off difficult problems.	3.43	1.10	-.35	-.50
3. I remain interested in the tasks that I start.	3.59	.99	-.48	-.06
4. I continue working on tasks until everything is perfect.	3.60	1.08	-.38	-.57
5. When confronted with a problem, I do more than what is expected of me.	3.37	1.09	-.17	-.62
Cognitive engagement				
1. The teacher asks questions that make us reflect on the problem.	2.92	.88	-.36	-.72
2. The teacher gives problems that require us to think for an extended time.	2.94	.85	-.31	-.70
3. The teacher asks us to decide on our own procedures for solving complex problems.	2.46	.98	.08	-1.01
4. The teacher presents problems for which there is no immediately obvious method of solution.	2.65	.93	-.08	-.88

Table 4.1 (continued)

5. The teacher presents problems in different contexts so that students know whether they have understood concepts.	2.91	.89	-.36	-.75
6. The teacher helps us to learn from mistakes we have made.	3.05	.93	-.61	-.63
7. The teacher asks us to explain how we have solved a problem.	3.16	.88	-.71	-.45
8. The teacher presents problems that require students to apply what they have learned to new contexts.	3.10	.86	-.56	-.57
9. The teacher gives problems that can be solved in several different ways.	2.94	.86	-.35	-.71
Mastery-approach goal orientations				
1. I enjoy reading about mathematics.	2.19	.83	.26	-.53
2. I look forward to my mathematics lessons.	2.43	.89	.09	-.73
3. I do mathematics because I enjoy it.	2.27	.94	.32	-.76
4. I am interested in the things I learn in mathematics.	2.51	.89	.01	-.73
Performance-approach goal orientations				
1. Making an effort in mathematics is worth it because it will help me in the work that I want to do later on.	3.07	.79	-.67	.20
2. Learning mathematics is worthwhile for me because it will improve my career prospects and chances.	3.04	.82	-.73	.26
3. Mathematics is an important subject for me because I need it for what I want to study later on.	2.90	.89	-.46	-.54
4. I will learn many things in mathematics that will help me get a job.	3.05	.81	-.71	.18

Item-Level Normality Assessment

Univariate normality was assessed by inspecting univariate skewness and kurtosis statistics. Missing data were excluded when examining the univariate normality of the study items of affect, motivation, and engagement in mathematics. Table 4.1 provides skewness and kurtosis statistics for items associated with each of the primary study variables. Item-level skew ranged from -1.84 to 0.66, and kurtosis ranged from -1.24 to 3.41. The majority of items had a negative skew, but the items for the mastery-approach goal showed a positive skew. As indicated in Kline (2016), when the absolute values of the skewness and the kurtosis equal 0, the scores are normally distributed. In general, the measured items for affect, motivation, and engagement presented a large degree of deviation from normality and, as a result, the shape of the item distributions was not normal. Therefore, a robust maximum likelihood (MLR) estimator was utilized in the structural equation model (SEM). An MLR estimator is commonly applied when the assumption of normality is violated.

Adequacy of Measurement Models

SEM is a procedure for analyzing structural models containing latent variables. It is composed of two models: a measurement model and a structural model. Prior to SEM, the measurement models were assessed to establish the relationships between the observed variables (indicators) and latent variables. The purpose of performing the CFAs was to determine whether the observed items measured the corresponding latent factors in affect, motivation, and engagement. It is critical to ascertain acceptable fit of the measurement of the latent variables that represent the constructs of multiple indicators prior to test the hypothesized relations among the latent variables in the full structural model. Then the structural equation modeling (SEM) procedures were used to test the validity of the hypothesized structural model between affect, motivation, and engagement in mathematics.

The initial hypothesized measurement model had seven latent factors and their respective observed variables: mathematics self-efficacy with eight indicators, mathematics self-concept with five indicators, mathematics anxiety with five indicators, mastery-approach goal with four indicators, performance-approach goal with four indictors, behavioral engagement with five indicators, and cognitive engagement with nine indicators.

A series of confirmatory factor analyses (CFAs) were conducted in M*plus* version 6.0 using the robust maximum likelihood (MLR) estimator (Muthén & Muthén, 1998-2007). A CFA was used to test the underlying structure of the latent variables in terms of mathematics pleasant affect (self-efficacy), mathematics pleasant affect (self-concept), unpleasant affect, behavioral engagement, cognitive engagement, mastery-approach goal, and performance-approach goal. Missing data was imputed by using gender, family structure, immigrant status, and language at home as auxiliary variables in each CFA analysis model. The internal consistency of the reliability (ω) estimate was assessed in M*plus* version 6.0.

Table 4.2 contains standardized factor loadings (coefficients) and standardized residual variances for each CFA. In the table, standardized factor loading refers to the correlation between the observed variable and a latent construct. The standardized residual variance refers to the variance of the observed variables that is not explained by the latent factors of interest (Bowen & Guo, 2011). Residual correlations are the unexplained correlations that were not reproduced by the estimated model (Bowen, & Guo, 201). In common statistical practice, a standardized factor loading is considered high when its magnitude is larger than .70, considered moderate when its magnitude is larger than .50, and, considered low when its magnitude is lower than .30 (Brown, 2006; Saris et al., 2009). In addition, according to Kline (2016), the standardized factor loading will require further inspection when the standardized residual variance are not statistically significant and the residual correlations are greater than |.10|. Specifically, further inspection should consider whether a factor is missing or whether the items are redundant.

In the table, for example, under pleasant affect, as represented by mathematics self-efficacy, the first item, “using a train timetable to work out how long it would take to get from one place to another”, had a standardized factor loading of 0.64 (out of 1) and a standardized residual variance of 0.59 (out of 1). This indicates that the mathematics self-efficacy has a moderate correlation and a low residual variance. Under the pleasant affect represented by mathematics self-concept, the first item, “I am not good at mathematics”, had a standardized factor loading of 0.77 (out of 1) and a standardized residual variance of 0.40 (out of 1), indicating that the mathematics self-concept has a high correlation and a low residual variance. Under the unpleasant affect represented by mathematics anxiety, the first item, “I often worry that it will be difficult for me in mathematics classes”, had a standardized factor loading of 0.79 (out of 1) and a standardized residual variance of 0.38 (out of 1), indicating that mathematics anxiety had a high correlation and a low residual variance (see Table 4.2).

With respect to the items under the mastery-approach goal represented by intrinsic motivation, the first item, “I enjoy reading about mathematics”, had a standardized factor loading of 0.77 (out of 1) and a standardized residual variance of 0.40 (out of 1), indicating that the mastery-approach goal had a high correlation and a low residual variance. For the items under performance-approach goal represented by extrinsic motivation, the first item, “Making an effort in mathematics is useful for future work", had a standardized factor loading of 0.82 (out of 1) and a standardized residual variance of 0.34 (out of 1), indicating that the performance-approach goal had a high correlation and a low residual variance (see Table 4.2).

Of the items under behavioral engagement, the last item, "When confronted with a problem, I do more than what is expected of me", had a standardized factor loading of 0.72 (out of 1) and a standardized residual variance of 0.48 (out of 1), indicating that the behavioral engagement had a high correlation and a low residual variance. With respect to the items under cognitive engagement, the first item, "The teacher asks questions that make us reflect on the problem", had a standardized factor loading of 0.71 (out of 1) and a standardized residual variance of 0.50 (out of 1), indicating that the cognitive engagement had a high correlation and a low residual variance (see Table 4.2).

Table 4.2

Confirmatory Factor Analysis Standardized Factor Loadings of Each Scale for the Total Sample (N = 4,987)

Variable	*Standardized coefficients*	*Standardized residual variance*
Pleasant affect (mathematics self-efficacy)		
1. Using a train timetable to work out how long it would take to get from one place to another.	.64	.59
2. Calculating how much cheaper a TV would be after a 30% discount.	.71	.49
3. Calculating how many square meters of tiles would be needed to cover a floor.	.75	.44
4. Understanding graphs presented in newspapers.	.68	.54
5. Solving equations like 3x+5=17.	.49	.76
6. Finding the actual distance between two places on a map with a 1:10 000 scale.	.68	.54
7. Calculating the petrol-consumption rate of a car.	.66	.57
Pleasant affect (mathematics self-concept)		

Table 4.2 (continued)

1. I am just not good at mathematics.	.77	.40
2. I get good grades in mathematics.	.75	.44
3. I learn mathematics quickly.	.87	.24
4. I have always believed that mathematics is one of my best subjects.	.83	.31
5. In my mathematics class, I understand even the most difficult work.	.79	.37
Unpleasant affect (mathematics anxiety)		
1. I often worry that it will be difficult for me in mathematics classes.	.79	.38
2. I get very tense when I have to do mathematics homework.	.82	.32
3. I get very nervous doing mathematics problems.	.80	.37
4. I feel helpless when doing a mathematics problem.	.74	.46
5. I worry that I will get poor grades in mathematics.	.76	.43
Behavioral engagement		
1. I put off difficult problems.	.32	.90
2. I remain interested in the tasks that I start.	.67	.55
3. I continue working on tasks until everything is perfect.	.82	.34
4. When confronted with a problem, I do more than what is expected of me.	.72	.48
Cognitive engagement		
1. The teacher asks questions that make us reflect on the problem.	.71	.50
2. The teacher gives problems that require us to think for an extended time.	.63	.61
3. The teacher presents problems in different contexts so that students know whether they have understood concepts.	.71	.50
4. The teacher helps us to learn from mistakes we have made.	.73	.47
5. The teacher asks us to explain how we have solved a problem.	.66	.56
6. The teacher presents problems that require students to apply what they have learned to new contexts.	.71	.49
7. The teacher gives problems that can be solved in several different ways.	.65	.58

Table 4.2 (continued)

Mastery-approach goal orientations		
1. I enjoy reading about mathematics.	.77	.40
2. I look forward to my mathematics lessons.	.87	.24
3. I do mathematics because I enjoy it.	.90	.19
4. I am interested in the things I learn in mathematics.	.85	.28
Performance-approach goal orientations		
1. Making an effort in mathematics is worth it because it will help me in the work that I want to do later on.	.82	.34
2. Learning mathematics is worthwhile for me because it will improve my career prospects and chances.	.85	.28
3. Mathematics is an important subject for me because I need it for what I want to study later on.	.85	.27
4. I will learn many things in mathematics that will help me get a job.	.85	.28

**$p < .01$; *$p < .05$.

Table 4.3 presents the model fits results for all the confirmatory factor analyses for the seven measurement scales measuring affect, motivation, and engagement. The item-level confirmatory factor analysis of the mathematics pleasant affect (self-efficacy) suggested a good fit to the sample data, $\chi^2(34) = 686.446$, $p < .001$, CFI = .941, TLI = .905, SRMR = .050, RMSEA = .049, and RMSEA with a 90% CI [.045, .054], after the exclusion of one item: “Solving equations like 2(x+3) = (x+3)(x-3)”. This item was phrased similarly to another item, “Solving equations like 3x+5=17”. All seven remaining items in the mathematics self-efficacy scale had statistically significant standardized factor loadings ($p < .001$), and all standardized residual correlations were less than |1|, indicating a good local fit (Kline, 2016). The internal consistency of the reliability estimate (ω) for mathematics self-efficacy was .99 (see Table 4.3).

The results from the CFA for mathematics self-concept also suggested a good fit to the sample data, $\chi^2(13) = 158.288$, $p < .001$, CFI = .983, TLI = .953, SRMR = .034, RMSEA = .047, and RMSEA with a 90% CI [.041, .054]. All five items in the mathematics self-concept scale had statistically significant standardized factor loadings ($p < .001$), and all standardized residual correlations were less than |1|, indicating a good local fit (Kline, 2016). The internal consistency of the reliability estimate (ω) for mathematics pleasant affect (self-concept) was .97 (see Table 4.3).

The results from the CFA for unpleasant affect (mathematics anxiety) suggested a good fits to the sample data: χ^2 (13) = 86.745, $p < .001$, CFI = .980, TLI = .944, SRMR = .025, RMSEA = .048, and RMSEA with a 90% CI [.039, .058]. All five items in the unpleasant affect (mathematics anxiety) scale had statistically significant standardized factor loadings ($p < .001$), and all standardized residual correlations were less than |1|, indicating a good local fit (Kline, 2016). The internal consistency of the reliability estimate (ω) for unpleasant affect (mathematics anxiety) was .99 (see Table 4.3).

The results from the CFA for behavioral engagement suggested good fits to the sample data: $\chi^2(2) = 25.882$, $p < .001$, CFI = .994, TLI = .914, SRMR = .010, RMSEA = .049, and RMSEA with a 90% CI [.033, .067], after the exclusion of one item: "When confronted with a problem, I give up easily". All four items in the behavioral engagement scale had statistically significant standardized factor loadings ($p < .001$), and all standardized residual correlations were less than |1|, indicating a good local fit (Kline, 2016). The internal consistency of the reliability estimate (ω) for behavioral engagement in mathematics was .97 (see Table 4.3).

The results from the CFA for cognitive engagement suggested a good fit to the sample data: $\chi^2(22) = 274.717$, $p < .001$, CFI = .968, TLI = .920, SRMR = .026, RMSEA = .048, and RMSEA with a 90% CI [.043, .053], after the exclusion of two items with a poor fit: “The teacher asks us to decide on our own procedures for solving complex problems” and “The teacher presents problems for which there is no immediately obvious method of solution”. All remaining seven items in the cognitive engagement scale had statistically significant standardized factor loadings ($p < .001$), and all standardized residual correlations were less than |1|, indicating a good local fit (Kline, 2016). The internal consistency of the reliability estimate (ω) for cognitive engagement in mathematics was .86 (see Table 4.3).

The results from the CFA for mastery-approach goal suggested a good fit to the sample data: $\chi^2(2) = 14.116$, $p < .001$, CFI = .998, TLI = .979, SRMR = .004, RMSEA = .035, and RMSEA with a 90% CI [.019, .053]. All four items in the mastery-approach goal scale had statistically significant standardized factor loadings ($p < .001$), and all standardized residual correlations were less than |1|, indicating a good local fit (Kline, 2016). The internal consistency of the reliability estimate (ω) for mastery-approach goal in mathematics was .99 (see Table 4.3).

The results from the CFA for performance-approach goal suggested a good fit to the sample data: $\chi^2(14) = 156.800$, $p < .001$, CFI = .980, TLI = .961, SRMR = .037, RMSEA = .045, and RMSEA with a 90% CI [.039, .052]. All five items in the performance–approach goal scale had statistically significant standardized factor loadings ($p < .001$), and all standardized residual correlations were less than |1|, indicating a good local fit (Kline, 2016). The internal consistency of the reliability estimate (ω) for performance-approach goal in mathematics was .91 (see Table 4.3).

In general, all the factors loadings of each observed variable (indicator) to the underlying latent variables were significant ($p < .05$). The results of the CFAs and the values of standardized factor loadings indicate that the measurement model for each latent variable is reasonable. This laid the foundation for the subsequent structural equation models, which were used to test the dynamic model of affect, motivation, and engagement in mathematics.

Table 4.3

Model Fit Results of all Confirmatory Factor Analyses (N = 4,987)

Model	χ^2	*df*	CFI	TLI	SRMR	RMSEA [90% CI]
Mathematics self-efficacy	686.446	34	.941	.905	.049	.050 [.045, .054]
Mathematics self-concept	158.288	13	.983	.953	.034	.047 [.041, .054]
Mathematics anxiety	86.745	13	.980	.944	.025	.048 [.039, .058]
Mastery approach goal	14.116	2	.998	.979	.004	.035 [.019, .053]
Performance approach goal	156.800	14	.980	.961	.037	.045 [.039, .052]
Behavioral engagement	25.882	2	.994	.914	.010	.049 [.033, .067]
Cognitive engagement	274.717	22	.968	.920	.026	.048 [.043, .053]

Note. χ^2 is calculated as maximum likelihood chi-square; CFI = comparative fit index; TLI = Tucker-Lewis index; SRMR = standardized root mean square residual; RMSEA = root mean square error of approximation.

Structural Equation Models Testing Linnenbrink's Dynamic Model

The results of the SEM model were used to address the research questions proposed in Chapter 1. The following sections address each of the research questions.

(1) To what extent do real-world (PISA 2012) data support Linnenbrink's (2007) dynamic (interactive) model of affect, motivation, and engagement in mathematics? To address this research question, a baseline model (M0) was used to compare differences in the fit of the full model. In this null model, all the structural (regression) paths were assumed to be zero, and all measurement paths from the latent variables to the observed indicators were 1. Next, a full SEM (M1) model, which added paths connecting the latent variables, as shown in Figure 2.1, was established to assess how well the predicted interrelationships between affect, motivation, and engagement matched the hypothesized structural model. The full model tested: 1) the direct effects between affect, motivation, and engagement; 2) the mediation of affect on the relationship between engagement (behavioral and cognitive engagement) and motivation (performance and mastery approach goal). Missing data were imputed by using gender, family structure, immigrant status, and language at home, as auxiliary variables in the analysis model (see Chapter 3). Results from the full model suggested a reasonable fit to the sample data, $\chi^2(590) = 3879.381$, $p < .001$, CFI = .931, TLI = .909, SRMR = .061, RMSEA = .033, and RMSEA with a 90% CI [.032, .034] (see Table 4.4). The majority of the relationships had statistically significant standardized factor loadings ($p < .001$) and all the standardized residual correlations were less than |1|, indicating good local fit (Kline, 2016) (see Table 4.4).

The full model (M1) fit the data much better than did the baseline model (M0) (see Table 4.4). The chi-square difference test ($\Delta\chi^2(190) = 44623.083$, $p < .001$) indicated that the full model was significantly different from the null model. In addition, MacCallum, Browne, and Sugawara (1996) used 0.01, 0.05, and 0.08 to indicate excellent, good, and mediocre fit, respectively. The RMSEA of 0.033 and 90% CI for RMSEA were both below .05, and *p* for close fit is 1, suggesting that my full model is better fitting than a close fitting model when the population RMSEA was .05 (Kenny, 2005; MacCallum, Browne & Sugawara, 1996). A mega CFA model (M2) was then established to compare this nested model with the full model by analyzing seven latent variables in one mega model. Gender, family structure, immigrant status, and language at home were used as auxiliary variables to deal with missing data. Results from the mega CFA model suggested a reasonable fit to the sample data, $\chi 2(585) = 2781.872$, $p < .001$, CFI = .954, TLI = .939, SRMR = .039, RMSEA = .027, and RMSEA with a 90% CI [.026, .028] (see Table 4.4). The comparison between M1 and M2 indicates reasonable similarity. This model (M2) therefore provides more evidence that the full model imposes a more parsimonious structure to the path coefficients. This result, in conjunction with the increase in fit for all the goodness of fit indicators, suggests that the full model explained the data much better than the previous null model and that Linnenbrink's (2007) dynamic (interactive) model of affect, motivation, and engagement in mathematics was supported by the PISA 2012 data.

Table 4.4

Model Fit Results for all Competing Models (N = 4,987)

Model	χ^2	*df*	CFI	TLI	SRMR	RMSEA [90% CI]
M0	48502.464	780	.000	.000	.273	.111 [.110, .112]
M1	3879.381	590	.931	.909	.061	.033 [.032, .034]
M2	2781.872	585	.954	.939	.039	.027 [.026, .028]

Note. χ^2 is calculated as maximum likelihood chi-square; CFI = comparative fit index; TLI = Tucker-Lewis index; SRMR = standardized root mean square residual; RMSEA = root mean square error of approximation. M0 = null model. M1 = full model. All the χ^2 statistics are significant at the level of .001. M2 = Mega CFA model.

The estimates of the direct effects are factor loadings (coefficients) and can be interpreted as regression coefficients in standardized forms (see Table 4.5). As mentioned earlier, in common statistical practice, a standardized factor loading is considered high when its magnitude is larger than .70, considered moderate when its magnitude is larger than .50, and, considered low when its magnitude is lower than .30 (Brown, 2006; Saris et al., 2009). Most of these coefficients were statistically significant ($p < .05$), and ranged from .14 to .85, indicating low to high strength. Coefficients with a high magnitude included the direct effects of mastery-approach goal on pleasant affect and the direct effects of mastery-approach goal on unpleasant affect. Coefficients with a moderate magnitude included the direct effects of pleasant affect on behavioral engagement and the indirect effect of pleasant affect on the relationship between mastery-approach goal and behavioral engagement. Coefficients with a low magnitude included the direct effects of unpleasant affect on behavioral engagement, the direct effects of pleasant affect on cognitive engagement, the direct effects of performance-approach goal on behavioral engagement, and the direct effects of performance-approach goal on cognitive engagement. Coefficients with a low magnitude also included the indirect effect of pleasant affect on the relation between mastery-approach goal and cognitive engagement and the indirect affect of unpleasant affect on the relation between mastery-approach goal and behavioral engagement (see Table 4.3).

The fact that the PISA data adequately supported the dynamic model of affect, motivation, and engagement of Linnenbrink (2007) provided a basis for examining the specific relationships in the interplay between affect, motivation, and engagement in mathematics. These relationships are represented as paths or more precisely path coefficients in Figure 4.1. The following sections discuss each important pathway in terms of motivation in mathematics related to affect in mathematics, affect in mathematics related to engagement in mathematics, motivation in mathematics related to engagement in mathematics, and affect as a mediator on the relationship between motivation and engagement in mathematics.

(2) How is motivation in mathematics related to affect in mathematics? To what extent did the data patterns from PISA 2012 match this part of the model specification? To address this research question, the standardized estimates of the path coefficients were calculated (see Table 4.5). The numbers in the single-headed arrows are the standardized path coefficients, and only significant path coefficients are presented in Figure 4.1. This study indicated that mastery-approach goal orientation was significantly related to affect (both pleasant affect and unpleasant affect) ($p < .05$), but the relationship between performance-approach goal orientations and affect (both pleasant affect and unpleasant affect) was not significant (see Table 4.5 or Figure 4.1). In particular, students with a higher mastery-approach goal in mathematics tended to have a higher pleasant affect and a lower unpleasant affect in mathematics ($p < .05$). That is, a one standard deviation increase in mastery-approach goal was associated with an increase of .85 standard deviation in (latent) pleasant affect (based on mathematics self-efficacy and mathematics self-concept), while a one standard deviation change in mastery-approach goal was associated with a decrease of .78 standard deviation in (latent) unpleasant affect (based on mathematics anxiety).

This result has fully supported Linnenbrink's model, in that she stated, "Mastery-approach goal orientations are associated with higher levels of pleasant affect and lower levels of unpleasant affect" (Linnenbrink, 2007, p.118). This study did not find any relationship between performance-approach goal and affect (both pleasant and unpleasant affect), which correlates with Linnenbrink's (2007) claim that, "The findings for performance-approach goal orientations are rather mixed, with performance-approach goal either unrelated or positively related to both pleasant and unpleasant affect" (Linnenbrink, 2007, p.118).

(3) How is affect in mathematics related to engagement in mathematics? To what extent do data patterns (from PISA 2012) match this part of the model specification? To address this research question, the standardized estimates of the path coefficients were calculated (see Table 4.5). The numbers in the single-headed arrows are the standardized path coefficients, and only significant path coefficients are presented in Figure 4.1. This study demonstrated that affect (both pleasant affect and unpleasant affect) was significantly related to engagement (both behavioral engagement and cognitive engagement) ($p < .05$) with the exception of unpleasant affect on cognitive engagement. In particular, students' who expressed a pleasant affect in mathematics tended to have better behavioral engagement and cognitive engagement in mathematics. Students who expressed a more unpleasant affect tended to have high behavioral engagement in mathematics (see Figure 4.1). That is, a one standard deviation increase in (latent) pleasant affect (a combined scale of mathematics self-efficacy and mathematics self-concept) was associated with an increase of .62 standard deviation in behavioral engagement and an increase of .16 standard deviation changes in cognitive engagement. A one standard deviation change in (latent) unpleasant affect (based on mathematics anxiety) was associated with an increase of .14 standard deviation in behavioral engagement. This result correlates with Linnenbrink's model, in that she stated, "With respect to engagement, we found that pleasant affect does not undermine behavioral engagement and may even enhance it" (Linnenbrink, 2007, p.118).

This study showed that unpleasant affect was also positively significantly related to behavioral engagement in mathematics, which is contradictive to Linnenbrink's findings, in that she stated, "Unpleasant affect was negatively correlated with behavioral engagement" (Linnenbrink, 2007, p.114). However, my current findings aligned with the affect-as-information model that states that when a person is in an unpleasant mood, he is motivated to pay attention to the detail in the situation. Thus unpleasant affect contributes to behavioral engagement (Schwarz, 1990; Schwarz & Clore, 1996). Meanwhile, this study found no relation between unpleasant affect and cognitive engagement. These results are consistent with Linnenbrink's findings (2007), in that she stated, "We generally found no relation between unpleasant affect and cognitive engagement" (Linnenbrink, 2007, p.119).

(4) How is motivation in mathematics related to engagement in mathematics? To what extent do data patterns (from PISA 2012) match this part of the model specification? To address this research question, the standardized estimates of the path coefficients were calculated (see Table 4.5). The numbers in the single-headed arrows reflect the standardized path coefficients, and only significant path coefficients are presented in Figure 4.1. This study showed that performance-approach goal orientation was significantly related to both behavioral and cognitive engagement, but no significant relation between mastery-approach goal and engagement (both behavioral engagement and cognitive engagement) was observed. This result did not support Linnenbrink's model, in that she stated, "Mastery-approach goals are generally associated with higher levels of behavioral and cognitive engagement" (Linnenbrink, 2007, p.119). Specifically, students with a higher performance-approach goal tended to have high behavioral engagement and cognitive engagement in mathematics (see Figure 4.1). That is, a one standard deviation change in performance-approach goal was associated with an increase of .17 standard deviation in behavioral engagement and an increase of .21 standard deviation in cognitive engagement. This result provided new evidence for the relationship between motivation and engagement, given that Linnenbrink stated, “The findings for performance-approach goal orientations are rather mixed, making it difficult to make clear predictions regarding the proposed model” (Linnenbrink, 2007, p.119).

(5) How does affect in mathematics mediate the relationship between motivation and engagement in mathematics? To what extent do data patterns (from PISA 2012) match this part of the model specification? According to Baron and Kenny (1986), there are two types of mediation: complete mediation and partial mediation. Complete mediation occurs when the effect of *X* on *Y* decreases to zero with the inclusion of mediator (Preacher, & Hayes, 2004). Partial mediation occurs when the effect of *X* on *Y* decreases by a nontrivial amount, but not to zero. These concepts are in line with the idea of the four conditions underlying mediation effects, as discussed in Chapter 1, and they work together with the four conditions idea to emphasize the nature of mediation.

There were significant indirect effects of affect (both pleasant affect and unpleasant affect) on the relationship between mastery-approach goal and behavioral engagement (see Table 4.5). The direct effect of mastery-approach goal on behavioral engagement was -.12. The direct effects of mastery-approach goal on behavioral engagement were that a one SD increase in mastery-approach goal was associated with a .12 SD decrease in behavioral engagement. The indirect effect of mastery-approach goal on behavioral engagement through pleasant affect was .53 and through unpleasant affect was -.11. The indirect effects of mastery-approach goal on behavioral engagement through pleasant affect showed that a one SD increase in pleasant affect increased the effects of mastery-approach goal on behavioral engagement by .53 SD. Similarly, the indirect effects of mastery-approach goal on behavioral engagement through unpleasant affect showed that a one SD increase in unpleasant affect decreased the effects of mastery-approach goal on behavioral engagement by .11 SD. The total effect of mastery-approach goal on behavioral engagement was .30. The total effects were a combination of direct and indirect effects. A one SD increase in both mastery-approach goal and affect (both pleasant and unpleasant affect) was associated with a .30 SD increase in behavioral engagement through both direct and indirect effects.

Overall, the relationship between mastery-approach goal orientations and behavioral engagement was significant when the mediators of affect (both pleasant affect and unpleasant affect) were included in this model. This indicated that both pleasant affect and unpleasant affect had a complete mediation on the relationship between behavioral engagement and mastery-approach goal. Additionally, the complete mediation of affect on the relationship between mastery-approach goal orientations on behavioral engagement was affirmed, in that the previous four condition for mediation in Chapter 1 were fully tested. This result not only fully supported Linnenbrink's model that unpleasant affect mediated the relationship of mastery-approach goal on behavioral engagement, but also provides new evidence that pleasant affect also significantly mediated the relationhip of mastery-approach goal on behavioral engagement. Linnenbrink stated, "We found no evidence, however, that pleasant affect mediated the relation between mastery-approach goal and learning, behavioral engagement, or cognitive engagement" (Linnenbrink, 2007, p.119). In addition, Linnenbrink stated, "We found that unpleasant affect partially mediated the relation between mastery-approach goal and learning" (Linnenbrink, 2007, p.119).

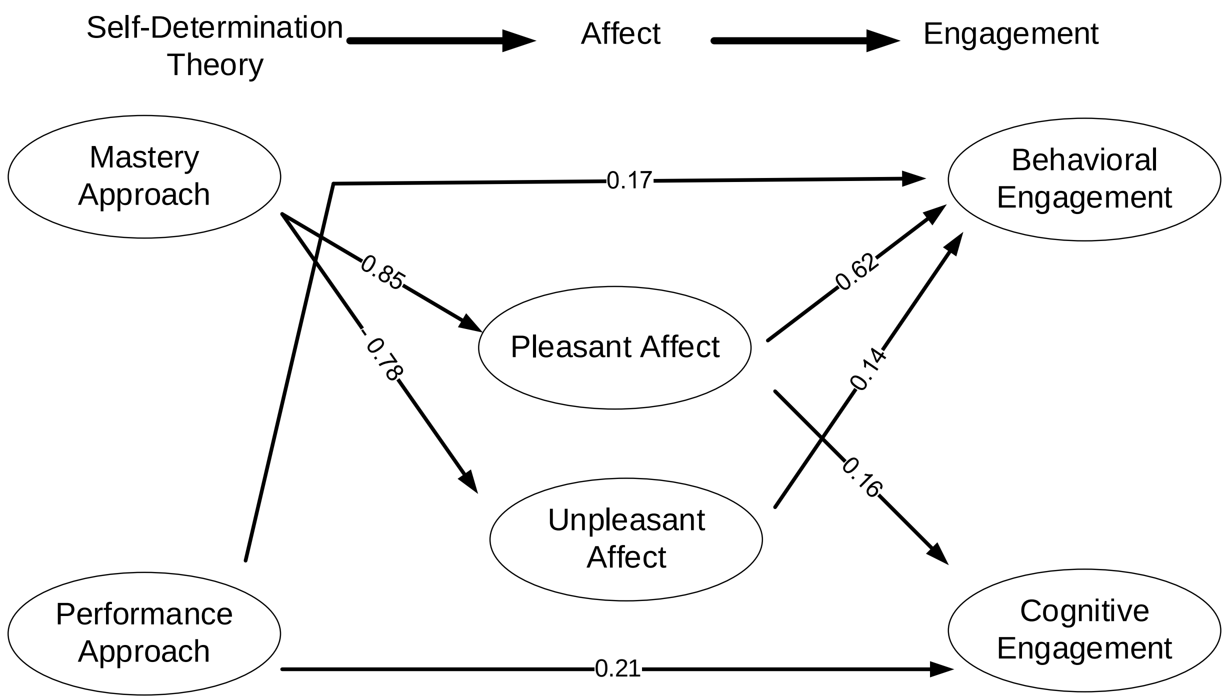

Figure 4.1. Final structural equation model testing Linnenbrink's (2007) interactive model of motivation, affect, and engagement. The values in the model are all statistically significant standardized path coefficients.

With regard to the effects from mastery-approach goal and cognitive engagement, there was a significant indirect effect for pleasant affect on the relationship between mastery-approach goal and cognitive engagement. The direct effect of mastery-approach goal on cognitive engagement was .05. The direct effect of mastery-approach goal on cognitive engagement showed that a one SD increase in mastery-approach goal was associated with a .05 SD increase in cognitive engagement. The indirect effect of mastery-approach goal on cognitive engagement through pleasant affect was .14. The indirect effects of mastery-approach goal on cognitive engagement through pleasant affect showed that a one SD increase in pleasant affect increased the effect of mastery-approach goal on cognitive engagement by .14 SD. The total effect of mastery-approach goal on cognitive engagement was .12. The total effects were a combination of direct and indirect effects. A one SD increase in both mastery-approach goal and pleasant affect was associated with a .12 SD increase in cognitive engagement through both direct and indirect effects. Overall, similar to the situation above, pleasant affect completely mediated the relationship between mastery approach goal and cognitive engagement as well. This complete mediation of affect on the relationship between mastery-approach goal and cognitive engagement was affirmed in that the four condition for mediation in Chapter 1 were fully tested. In addition to fully supporting Linnenbrink's model that pleasant affect mediated the relation between mastery-approach goal on cognitive engagement, this result provided new findings about the relationship between mastery-approach goal and cognitive engagement through the mediator of pleasant affect.

No significant indirect effects for affect (i.e., pleasant affect and unpleasant affect) on the relationship between performance-approach goal and behavioral engagement were observed ($p > .05$) (see Table 4.5). In addition, no significant indirect effects for affect (i.e., pleasant affect and unpleasant affect) on the relationship between performance-approach goal and cognitive engagement were observed ($p > .05$) (see Table 4.5). This result fully supported Linnenbrink's model (2007) that states that there is no clear mediating effect for affect when it comes to performance-approach goal orientation. Linnenbrink (2007) stated, "It is more difficult to test for mediation, as the findings relating performance-approach goals to affect and engagement are less consistent" (p.120).

Table 4.5

Standardized Estimate of Path Coefficients in Final Structural Equation Model (N = 4,987)

Parameter	Standardized Estimate	SE	Z	*p*
Mastery approach ON pleasant affect	.85	.05	18.67	< .001
Performance approach ON pleasant affect	-.01	.06	-.14	.89
Mastery approach ON unpleasant affect	-.78	.05	-14.77	< .001
Performance approach ON unpleasant affect	.09	.07	-1.29	.20
Pleasant affect ON behavioral engagement	.62	.09	6.85	< .001
Unpleasant affect ON behavioral engagement	.14	.06	2.57	.01
Pleasant affect ON cognitive engagement	.16	.07	2.40	.02
Unpleasant affect ON cognitive engagement	.08	.04	1.89	.06
Mastery approach ON behavioral engagement	-.12	.08	-1.46	.14
Performance approach ON behavioral engagement	.17	.04	4.28	< .001
Mastery approach ON cognitive engagement	.05	.07	.63	.52
Performance approach ON cognitive engagement	.21	.05	4.37	< .001
Pleasant affect IND mastery approach and behavioral engagement	.53	.09	6.16	< .001
Pleasant affect IND performance approach and behavioral engagement	-.01	.04	1.14	.89
Pleasant affect IND mastery approach and cognitive engagement	.14	.06	2.39	.02
Pleasant affect IND performance approach and cognitive engagement	-.01	.01	-.14	.89
Unpleasant affect IND mastery approach and behavioral engagement	-.11	.04	-2.50	.01
Unpleasant affect IND performance approach and behavioral engagement	.01	.01	1.14	.25

Table 4.5 (continued)

Unpleasant affect IND mastery approach and cognitive engagement	-.06	.03	-1.90	.06
Unpleasant affect IND performance approach and cognitive engagement	.01	.01	1.14	.26

Note. ON: direct effect. IND: indirect effect

As discussed in Chapter 3, this study used a traditional approach to calculate the mediation effect size by the ratio of the indirect effect to the total effect (i.e., total effect = direct effect + indirect effect). This approach is meaningful when accompanied by the total effect from a basic mediation model in which the indirect effect and the direct effect have the same sign. The mediation size for affect (both pleasant affect and unpleasant affect) was (.53 − .11) /.30 = 1.40 on the relation between mastery-approach goal and behavioral engagement, indicating that the indirect effect of mastery-approach goal on behavioral engagement through affect was approximately 1.4 times the size of the direct effect. This is the complete mediation effect size for affect on the relation between mastery-approach goal and behavioral engagement. The mediation effect size for pleasant affect on the relation between mastery-approach goal and cognitive engagement could not be calculated due to opposite coefficient signs.

Chapter 5: Discussion

This chapter consists of five sections. The first section presents a summary of the principal findings obtained from testing Linnenbrink's (2007) interactive model between affect, motivation, and engagement in mathematics. The second section reviews whether the PISA 2012 data support Linnenbrink's (2007) model. The third section discusses the theoretical and practical implications. The fourth section points out the limitations of the present study. Finally, the last section offers recommendations for future research.

Summary of Principal Findings

The goal of this study was to address five essential research questions related to the interactive model of affect, motivation, and engagement theorized in Linnenbrink (2007). This study applied the model in the domain of mathematics education. To facilitate the summary, a path-to-path comparison was designed to summarize and compare the results from this study with the interactions between affect, motivation, and engagement specified in Linnenbrink (2007) (see Table 5.1).

To what extent do real-world (PISA 2012) data support Linnenbrink's (2007) dynamic (interactive) model of affect, motivation, and engagement in mathematics? To test the dynamic model of affect, motivation, and engagement from Linnenbrink (2007), multiple indicators from PISA 2012 were used to generate each latent variable. The affective domain in mathematics was adopted as the theoretical framework for affect, so the available PISA affective measures (i.e., mathematics self-efficacy, mathematics self-concept, and mathematics anxiety) were used to capture the affective domain in mathematics (McLeod, 1992). Self-determination theory was adopted as the theoretical framework for motivation, so the intrinsic and extrinsic motivation measures from PISA

2012 were used in this study (Ryan & Deci, 1985). To align with Fredricks, Blumenfeld, and Paris's framework for engagement, the available behavioral engagement and cognitive engagement measures from PISA 2012 were adopted in this study. By implementing structural equation modeling (SEM) techniques, all possible relationships between the predictive variables and the outcome variables, including mediating effects and latent confounding variables, were tested simultaneously. Overall, this study found that Linnenbrink's (2007) dynamic (interactive) model of affect, motivation, and engagement in mathematics was supported by the PISA 2012 data. This support was demonstrated by the model-data-fit statistics from the SEM model. Specifically, all the comparative fit indexes, including the CFI, TLI, RMSEA, and SRMR, suggested a good fit of the model to the data.

How is motivation in mathematics related to affect in mathematics? To what extent do data patterns (from PISA 2012) match this part of the model specification? As shown in Table 5.1, Linnenbrink (2007) stated that mastery-approach goals have positive effects on pleasant affect. This path was supported in this study; that is, the effects of mastery-approach goals on pleasant affect were statistically significant and positive.

As shown in Table 5.1, Linnenbrink (2007) specified a lack of effects of performance-approach goal on pleasant affect. This specification was supported in this study; that is, the effects of performance-approach goals on pleasant affect were not statistically significant.

As shown in Table 5.1, Linnenbrink (2007) established the negative effects of mastery-approach goals on unpleasant affect. This path was supported in this study; that is, the effect of mastery-approach goals on unpleasant affect was statistically significant and negative.

As shown in Table 5.1, Linnenbrink (2007) specified a lack of effects of performance-approach goal on unpleasant affect. This specification was supported in this study; that is, the effects of performance-approach goals on unpleasant affect were not statistically significant.

Therefore, the PISA data supported 4 out of 4 specifications concerning the relationship between motivation and affect in Linnenbrink (2007). This is a complete support of Linnenbrink's (2007) model with respect to motivation as related to affect (in the domain of mathematics education).

In addition, this present study is consistent with previous studies that found a significant relationship between motivation and affect in mathematics (Erez & Isen, 2002; Hall, Sampasivam, Muis, & Ranellucci, 2016; Linnenbrink & Pintrich, 2002; Pekrun, Elliot, & Maier, 2009; Meyer & Turner, 2002). In particular, they found positive reciprocal pathways between motivation and pleasant affect and negative reciprocal pathways between motivation and unpleasant affect (Hall, Sampasivam, Muis, & Ranellucci, 2016; Pomerantz & Qin, 2014; Wang, Shakeshaft, Schofield, & Malanchini, 2018).

How is affect in mathematics related to engagement in mathematics? To what extent do data patterns (from PISA 2012) match this part of the model specifications? As shown in Table 5.1, Linnenbrink (2007) established the positive effects of pleasant affect on behavioral engagement. This path was supported in this study; that is, the effects of pleasant affect on behavioral engagement were statistically significant and positive.

As shown in Table 5.1, Linnenbrink (2007) established the positive effects of pleasant affect on cognitive engagement. This path was supported in this study; that is, the effects of pleasant affect on cognitive engagement were statistically significant and positive.

As shown in Table 5.1, Linnenbrink (2007) established the negative effects of unpleasant affect on behavioral engagement. This path was denied in this study; that is, the effect of unpleasant affect on behavioral engagement was statistically significant and positive.

As shown in Table 5.1, Linnenbrink (2007) specified a lack of effect of unpleasant affect on cognitive engagement. This specification was supported in this study; that is, the effect of unpleasant affect on cognitive engagement was not statistically significant.

Therefore, the PISA data confirmed 3 out of 4 specifications concerning the relation between affect and engagement in Linnenbrink (2007). This is a nearly complete support of Linnenbrink's (2007) model with respect to affect related to engagement (in the domain of mathematics education).

In addition, this present study is consistent with previous studies indicating that there was a significant relationship between affect and engagement in mathematics (Caraway, Tucker, Reinke, & Hall, 2003; Gendolla & Krusken, 2002; Linnenbrink-Garcia, Roga, & Koskey, 2011; Linnenbrink & Pintrich, 2003; King, McInerney, Ganotice & Villarosa, 2015; Pardos, Baker, San Pedro, Gowda, & Gowda, 2014; Pekrun, Titz, & Perry, 2002). In particular, pleasant affect enhanced students' engagement in mathematics (Goldin, Epstein, Schorr, & Warner, 2011; King, McInerney, Ganotice, & Villarosa, 2015; Martin, & Rimm-Kaufman, 2015). The positive effect of unpleasant affect on behavioral engagement was supported by many previous studies, indicating that when a student is in an unpleasant mood, he or she is motivated to respond to and pay attention to the situation. Thus, unpleasant affect may lead to prolonged engagement (Schwarz, 1990; Schwarz & Clore, 1996).

How is motivation in mathematics related to engagement in mathematics? To what extent do data patterns (from PISA 2012) match this part of the model specification? As shown in Table 5.1, Linnenbrink (2007) established the positive effect of mastery-approach goal and behavioral engagement. This specification was denied in this study; that is, the effect of mastery-approach goals on behavioral engagement was not statistically significant.

As shown in Table 5.1, Linnenbrink (2007) established the positive effect of mastery-approach goal on cognitive engagement. This specification was denied in this study; that is, the effect of mastery-approach goals on cognitive engagement was not statistically significant.

As shown in Table 5.1, Linnenbrink (2007) specified the lack of effect of performance-approach goal on behavioral engagement. This specification was rejected in this study; that is, the effects of performance-approach goal on behavioral engagement were statistically significant and positive.

As shown in Table 5.1, Linnenbrink (2007) specified the lack of effect of performance-approach goals on cognitive engagement. This specification was rejected in this study; that is, the effect of performance-approach goals on cognitive engagement was statistically significant and positive.

Therefore, the PISA data confirmed 0 out of 4 specifications about the relationship between motivation and engagement in Linnenbrink (2007). Thus, this study did not support Linnenbrink’s’s (2007) model about the way that motivation is related to engagement (in the domain of mathematics education).

In addition, this present study is consistent with previous studies that found that educational correlates were conceptually and empirically relevant to motivation and engagement in mathematics (Gonida, Voulala, & Kiosseoglou, 2009; Plenty & Heubeck, 2013; Skinner, & Belmont, 1993). In particular, students’ engagement was significantly predicated by students’ motivation in mathematics.

How does affect in mathematics mediate the relationship between motivation and engagement in mathematics? To what extent do data patterns (from PISA 2012) match this part of the model specification? As shown in Table 5.1, Linnenbrink (2007) specified that pleasant affect does not mediate the effect of mastery-approach goal on behavioral engagement. This specification was rejected in this study; that is, the mediation effect of pleasant affect on the relationship of mastery-approach goal to behavioral engagement was statistically significant and positive.

As shown in Table 5.1, Linnenbrink (2007) specified that pleasant affect does not mediate the effects of performance-approach goal on behavioral engagement. This specification was supported in this study; that is, the mediation effect of pleasant affect on the relationship of performance-approach goal on behavioral engagement was not statistically significant.

As shown in Table 5.1, Linnenbrink (2007) specified that pleasant affect does not mediate the effect of mastery-approach goal on cognitive engagement. This specification was rejected in this study; that is, the mediation effect of pleasant affect on the relationship of mastery-approach goal to cognitive engagement was statistically significant and positive.

As shown in Table 5.1, Linnenbrink (2007) specified that pleasant affect does not mediate the effects of performance-approach goal on cognitive engagement. This specification was supported in this study; that is, the mediation effect of pleasant affect on the relationship of performance-approach goal on cognitive engagement was not statistically significant.

As shown in Table 5.1, Linnenbrink (2007) specified that unpleasant affect negatively mediates the effects of mastery-approach goals on behavior engagement. This specification was supported in this study; that is, the mediation effect of unpleasant affect on the relationship of mastery-approach goals on behavior engagement was statistically significant and negative.

As shown in Table 5.1, Linnenbrink (2007) specified that unpleasant affect does not mediate the effects of performance-approach goals on behavioral engagement. This specification was supported in this study; that is, the mediation effect of pleasant affect on the relationship of performance-approach goal on behavioral engagement was not statistically significant.

As shown in Table 5.1, Linnenbrink (2007) specified that unpleasant affect does not mediate the effects of mastery-approach goal on cognitive engagement. This specification was supported in this study; that is, the mediation effect of pleasant affect on the relationship of mastery-approach goal on cognitive engagement was not statistically significant.

As shown in Table 5.1, Linnenbrink (2007) specified that unpleasant affect does not mediate the effects of performance-approach goal on cognitive engagement. This specification was supported in this study; that is, the mediation effect of pleasant affect on the relationship of performance-approach goal on cognitive engagement was not statistically significant.

Table 5.1

Summary of Findings in Comparison with the Interactive Model of Affect, Motivation, and Engagement in Linnenbrink (2007)

Parameter	Current Results	Linnenbrink (2007)
Mastery approach ON pleasant affect	Yes (+)	Yes (+)
Performance approach ON pleasant affect	No (0)	No (0)
Mastery approach ON unpleasant affect	Yes (–)	Yes (–)
Performance approach ON unpleasant affect	No (0)	No (0)
Pleasant affect ON behavioral engagement	Yes (+)	Yes (+)
Unpleasant affect ON behavioral engagement	Yes (+)	Yes (–)
Pleasant affect ON cognitive engagement	Yes (+)	Yes (+)
Unpleasant affect ON cognitive engagement	No (0)	No (0)
Mastery approach ON behavioral engagement	No (0)	Yes (+)
Performance approach ON behavioral engagement	Yes (+)	No (0)
Mastery approach ON cognitive engagement	No (0)	Yes (+)
Performance approach ON cognitive engagement	Yes (+)	No (0)
Pleasant affect IND mastery approach and behavioral engagement	Yes (+)	No (0)
Pleasant affect IND performance approach and behavioral engagement	No (0)	No (0)

Table 5.1 (continued)

Pleasant affect IND mastery approach and cognitive engagement	Yes (+)	No (0)
Pleasant affect IND performance approach and cognitive engagement	No (0)	No (0)
Unpleasant affect IND mastery approach and behavioral engagement	Yes (–)	Yes (–)
Unpleasant affect IND performance approach and behavioral engagement	No (0)	No (0)
Unpleasant affect IND mastery approach and cognitive engagement	No (0)	No (0)
Unpleasant affect IND performance approach and cognitive engagement	No (0)	No (0)

Note. Under "Current Results," Yes = statistically significant, and No = Not statistically significant. Under Linnenbrink (2007), Yes = Specified, and No = Not specified. In both columns, (+) = Positive relationship, (–) = Negative relationship, and (0) = No relationship.

Therefore, the PISA data supported 6 out of 8 specifications concerning the mediation effects of affect on the relationships of motivation and engagement in Linnenbrink (2007). This is nearly complete support of Linnenbrink's (2007) model with respect to affect mediating the relation between motivation and engagement (in the domain of mathematics education).

In addition, this present study found that affect mediated the relationships between motivation and engagement, which provides further evidence to consider the important role of affect in mathematics learning and instruction (Gillet, Vallerand, Lafrenière, & Bureau, 2013; McLeod, 1992; McLeod, 1994). Affect may trigger, sustain, or reduce academic motivation and related volitional processes (Pekrun, Titz, & Perry, 2002). Some studies have found positive and negative affect to be mediators of the situational motivation – performance relationship (Gillet, Vallerand, Lafrenière, & Bureau, 2013).

Revisiting the Literature

The literature relating each topic associated with affect, motivation, engagement is plentiful, but Linnenbrink's (2007) dynamic model of affect, motivation, and engagement is a great step forward in connecting these important forces of influence. There is a great need to test this model. The current study is likely the first attempt to test the interactions between affect, motivation, and engagement as specified in Linnenbrink (2007). Specifically, Linnenbrink's (2007) model was tested in the field of mathematics education, using (real-world) national representative data from PISA 2012.

Give that Linnenbrink's (2007) dynamic model of affect, motivation, and engagement is already a theoretical synthesis of the research literature, this revisit to the literature will focus on Linnenbrink's (2007) model. The operationalization of Linnenbrink's (2007) model produced a total of 20 paths or specifications (see Figure 4.1). Four of them pertain to the effects of motivation on affect. These four paths were all supported in mathematics education using the PISA data in this study. This study provides strong support for the effects of motivation on affect. Linnenbrink (2007) was thus validated in terms of the specifications of the relationship between motivation and affect.

Four of the paths pertain to the effects of affect on engagement. These four paths were nearly completely supported in mathematics education using the PISA data. This study provides considerable support for the effects of affect on engagement. Linnenbrink

(2007) was thus basically validated in terms of the specifications of the relationship between affect and engagement.

Four of the paths pertain to the effects of motivation on engagement. None of them supported the effects of motivation on engagement specified in Linnenbrink (2007). This indicated a lack of support for this relationship in mathematics education using the PISA data. This study provided no support for the effects of motivation on engagement. Linnenbrink (2007) was thus partially validated in terms of the specifications of the mediation of pleasant affect on the relationship between motivation and engagement.

Four of the paths pertain to the mediation of pleasant affect on the relationship from motivation to engagement. These four paths were partially supported in mathematics education by the PISA data (i.e., two out of the four were supported). This study provided moderate support for the mediation of pleasant affect on the relationship from motivation to engagement. Linnenbrink (2007) was thus partially validated in terms of the specifications of the mediation of pleasant affect on the relationship between motivation and engagement.

Four of the paths pertain to the mediation of unpleasant affect on the relationship from motivation to engagement. These four paths were completely supported by the PISA data in mathematics education. This study provided strong support for the mediation of unpleasant affect on the relationship from motivation to engagement. Linnenbrink (2007) was thus validated in terms of the specifications about the mediation of unpleasant affect on the relationship between motivation and engagement.

Theoretical and Practical Implications

Theoretical Implications. Because this study utilized a nationally representative dataset that provides results that relate to the relationships between affect, motivation, and engagement in the field of mathematics education, it is in a very good position to modify Linnenbrink's (2007) dynamic model of affect, motivation, and engagement. Among the specifications in Linnenbrink (2007), the relationship between unpleasant affect and behavioral engagement as well as the relationship between mastery-approach goals and behavioral engagement and cognitive engagement were not supported by the real-world PISA data. Thus, modifications of Linnenbrink's (2007) model can be made by forming re-specifications of these paths.

Linnenbrink (2007) specified that unpleasant affect was negatively correlated with behavioral engagement. This path from this study showed that unpleasant affect was positively correlated with behavioral engagement. Students who experience unpleasant affect may still be persistent and effortful in their learning tasks (and thus engaged).

Linnenbrink (2007) specified that mastery-approach goals were positively associated with behavioral engagement and cognitive engagement. These paths from this study were not statistically significant. Students who learn mathematics because of their interests, growth in competence, or enjoyment of a challenge may not engage either behaviorally or cognitively in their learning tasks.

Linnenbrink (2007) specified that performance-approach goals were not associated with either behavioral or cognitive engagement. These paths from this study were statistically significant. Students who perceive mathematics to be useful to them and to their future studies and careers may engage both behaviorally and cognitively in their learning tasks.

Linnenbrink (2007) specified that pleasant affect does not mediate the effects of mastery-approach goals on engagement (neither behavioral engagement nor cognitive engagement). This study showed that pleasant affect positively mediated the effects of mastery-approach goals on both behavioral and cognitive engagement. Students who experience a more pleasant affect may show stronger effects of motivation on both behavioral and cognitive engagement.

Finally, based on the results from this study, Linnenbrink's (2007) interactive model of affect, motivation, and engagement can be tentatively revised as shown in Figure 5.1. In this figure, positive signs indicate positive effects, and negative signs indicate negative effects. The paths or specifications that differed from Linnenbrink (2007) are shown by dotted lines, and the solid lines indicate agreement. Thus, again, this study did not support the paths from unpleasant affect to behavioral engagement, mastery-approach goal to behavioral engagement, performance-approach goal to behavioral engagement, mastery-approach goal to cognitive engagement, and performance-approach goal to cognitive engagement.

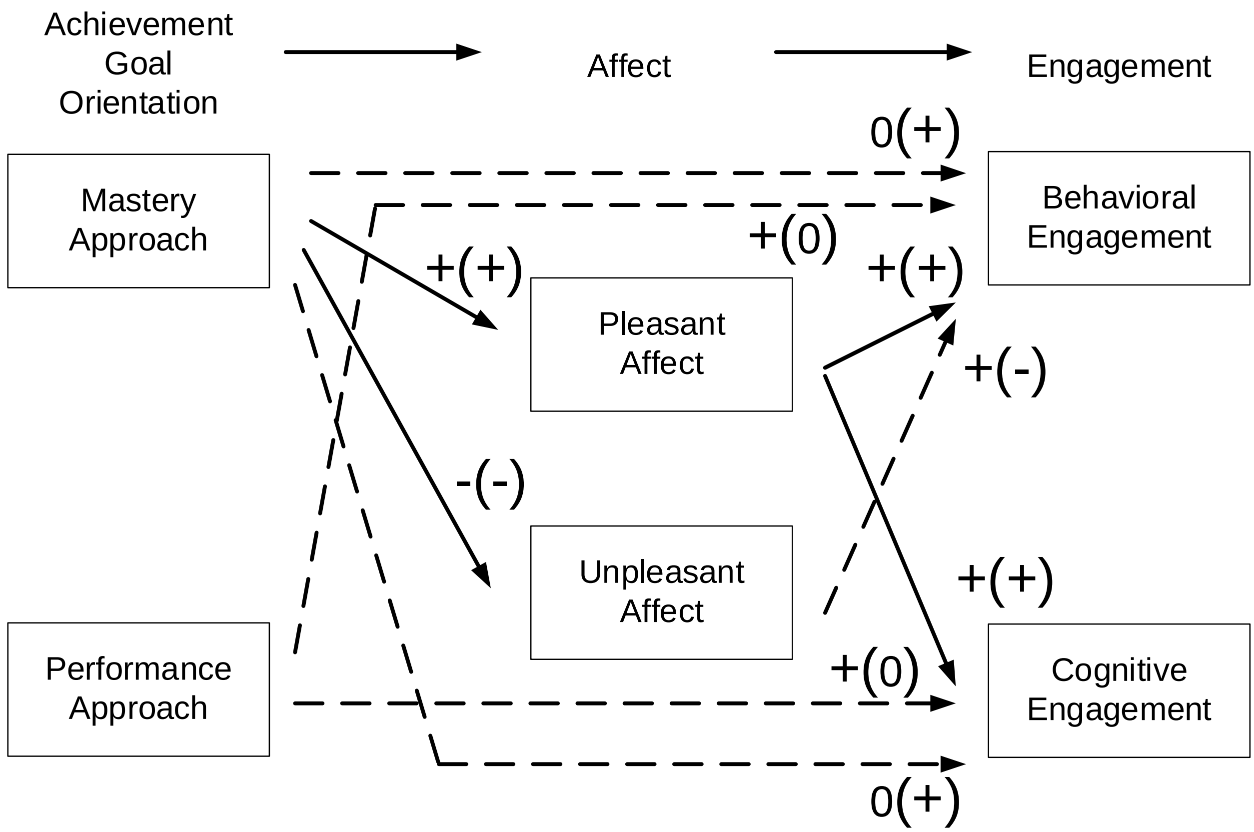

Figure 5.1. Revised mediational model linking affect, motivation, and engagement from Linnenbrink (2007). Dotted lines indicate disagreement with Linnenbrink's model. Solid lines indicate agreement with Linnenbrink's model. Positive signs (+) indicate positive relationships from Linnenbrink's model, and negative signs (–) indicate negative relationships from Linnenbrink's model. Positive signs + indicate positive relationships from the present study, and negative signs – indicate negative relationships from the present study. 0 indicates no relationship in this path.

Implications for Practice. The Common Core State Standards for Mathematics (CCSSM) are the foundation for mathematical thinking and practice for students as well as guidance that helps teachers modify their strategies to develop a more advanced mathematics understanding. Because many mathematics teachers work closely with the common core mathematics education standards (e.g., Illustrative Mathematics 6–8 Math), this present study provides empirical insights for their classroom practices. This is because PISA and CCSSM share many similarities in that they both focus on real-world mathematical problems and emphasize similar standards (OECD, 2013). Thus, this study provides additional direction for mathematics educators by considering the interactive roles of affect, motivation, and engagement simultaneously in mathematics.

This study completely confirmed Linnenbrink's (2007) specifications about the effects of motivation on affect, implying that improving motivation can improve affect. Specifically, mathematics educators may be able to use mastery-approach goals to improve a student's pleasant affect and to reduce an unpleasant affect. Given that pleasant affect was measured by self-efficacy and self-concept and that unpleasant affect was measured by mathematics anxiety, mathematics educators may use their daily interactions with students to purposefully boost students' motivation to learn mathematics based on their interests, growth in competence, and enjoyment of a challenge to help the students experience a more pleasant affect (i.e., belief in their own ability to handle mathematics tasks effectively or belief in their own mathematics ability) and less mathematics anxiety (i.e., helplessness and stress).

This study nearly completely confirmed Linnenbrink's (2007) specifications about the effects of affect on engagement, implying that improving affect can improve behavioral engagement and cognitive engagement (unpleasant affect could also improve behavioral engagement). Specifically, mathematics educators may be able to use pleasant affect to improve a student's behavioral and cognitive engagement. Surprisingly, mathematics educators may also find that unpleasant affect may improve a student's behavioral engagement to some extent. Mathematics educators should strive to help students enhance their positive belief in mathematics. Given that unpleasant affect was measured by mathematics anxiety; mathematics educators could use some appropriate level of mathematics anxiety to improve persistence and effort in learning mathematics. In general, these efforts may contribute to students' engagement in mathematics.

This study completely confirmed Linnenbrink's (2007) specifications about the indirect effects of unpleasant affect on the relationship between mastery-approach goals and behavioral engagement, implying that students' motivation to learn mathematics based on their interests, growth in competence, and enjoyment of a challenge influence students' persistence by reducing their mathematics anxiety. Students who experience a more unpleasant affect may show weaker effects of motivation on behavioral engagement. Mathematics educators should work to reduce unpleasant affect in students' mathematics learning. Strategies for this purpose may include educational intervention and training. For example, mathematics educators may be able to use treatment programs, such as teaching self-management of emotional stress and systematic desensitization, for students with mathematics anxiety. Mathematic educators may also help students experiencing mathematics anxiety build their skill in mathematics by demonstrating what these students can already do and what they need to do next.

Limitations of the Study

Measurement Limitations. Because PISA was not designed with Linnenbrink's (2007) dynamic model of affect, motivation, and engagement in mind, the main limitations of the current study come from the characteristics of the PISA data. The measurements used in PISA may not exactly match Linnenbrink's constructs related to affect, motivation, and engagement. This issue is evident with respect to almost every major construct. First of all, this study employed McLeod's perspective about the affective domain in mathematics (including beliefs and attitudes measured through mathematics self-efficacy, mathematics self-concept, and mathematics anxiety). This measurement of affect approximates, but does not exactly match Linnenbrink (2007), who used the circumplex model as the theoretical framework for affect. For example, mathematics anxiety was used as unpleasant affect in this study, but this measure of unpleasant affect may lack a multidimensional unpleasant affect compared with the activated or deactivated level of unpleasant affect in Linnenbrink's model (2007). The current study used mathematics anxiety to indicate unpleasant affect and correspond to activated unpleasant affect; however there was no adequate measure that could capture deactivated unpleasant affect.

This study was also unable to test motivation in a way that was exactly consistent with the mastery-approach and performance-approach goals from Linnenbrink (2007). Linnenbrink (2007) used achievement goal theory as the theoretical basis for motivation with two primary goal orientations: a mastery goal orientation, which focuses on developing a person's competence, and a performance goal orientation, which focuses on demonstrating a person's competence (Elliot & Church, 1997). The present study used self-determination theory as the theoretical framework to capture motivation from intrinsic and extrinsic perspectives. Although these theoretical perspectives do intertwine considerably, unfortunately, this operationalization does not exactly correspond to Linnenbrink's (2007) theoretical specifications.

This study was also unable to measure engagement exactly and consistently with Linnenbrink (2007). Linnenbrink (2007) used two types of engagement as the theoretical basis. Although this study closely aligned with Linnenbrink's behavioral engagement in that both measures focused on students' persistence or effort on school tasks, this study was weak with respect to cognitive engagement. Linnenbrink (2007) measured cognitive engagement through the quality of thinking, including cognitive strategies (e.g., elaboration, rehearsal), metacognitive strategy use, and self-regulated learning. The present study used cognitive activation from PISA 2012 to measure students' cognitive strategies used, such as summarizing and questioning, which may be close to, but not as comprehensive as those in Linnenbrink (2007).

Generalization Limitations. Generalizing the results from this study should be approached cautiously because of the specific characteristics of the participating students. PISA works with 15-year-old students. Although this sample was nationally representative of the population, it is limited to this specific age group. Linnenbrink's model was not specific for any particular age group. Although her discussion of goal orientations pertains to a wide range of students (upper elementary, middle school, and college students), PISA works with 15-year-olds, who may be too young to precisely express their affect, motivation, and engagement in mathematics. For example, a 15-year-old may have a hard time telling the difference between intrinsic and extrinsic motivation in the real world if the student simultaneously enjoys both mathematics itself (intrinsic motivation) and receiving tangible rewards (extrinsic motivation). This fact may limit the application of the findings of this study to other age groups. It is likely that the findings of this study may not fully or equally apply to elementary, middle, high school, and college students.

Causality Limitations. The last limitation relates to the nature of a cross-sectional study. Causal processes and relationships between factors cannot be verified when cross-sectional data is used. SEM analyses provide suggestive support for putative causal models, but ultimately longitudinal research will be needed to delineate more clearly the processes that link affect, motivation, and engagement in mathematics. For example, a longitudinal design that collects data across multiple distinct time points could aid in elucidating the complex interrelationships between affect, motivation, and engagement in mathematics.

Suggestions for Future Study

Based on the results of the current study, it is apparent that there are many opportunities for future research examining the relationships between affect, motivation, and engagement. Thus, this study has provided references and can make recommendations for further study.

First, to align with Linnenbrink's model (2007), the current study did not include student characteristics in the SEM model. Nonetheless, the constructs for affect, motivation, and engagement do have significant differences by gender, family structure, immigrant status, and language at home (see Appendix C and Table 5.2 -Table 5.5). In particular, male students reported a higher mathematics pleasant affect (measured by mathematics self-concept) and a lower unpleasant affect compared with female students. Students from two-parent families had a higher pleasant affect (measured by mathematics self-concept), lower unpleasant affect, and lower mastery-approach goals than students from one-parent families. Native students had a lower unpleasant affect and lower performance-approach goals compared with immigrant students. Students who spoke English at home reported lower performance-approach goals in mathematics than students who spoke a different language at home. Given the significant individual differences in this study, these variables have the potential to function as confounding factors that may affect the interactive model of affect, motivation, and engagement. Future research should control or control for these confounding factors, including gender, family structure, immigrant status, and language at home, when investigating the interactive model of affect, motivation, and engagement in mathematics (i.e., To control for individual differences, future studies should take into account student characteristics in their SEM model).

Second, future studies should use more comprehensive measures for affect and motivation to fully operationalize the constructs in Linnenbrink (2007). In particular, such studies should measure affect from a multifaceted construct that includes deactivated pleasant affect, deactivated unpleasant affect, activated pleasant affect, and activated unpleasant affect (Feldman & Russell, 1998; Russell, 1980; Russell, Ward, &Pratt, 1981). Many researchers have noted that achievement goal theory has been extended to form a 2x2 achievement goal framework involving mastery goals (mastery-approach goals and mastery-avoidance goals) and performance goals (performance-approach goals and performance-avoidance goals) (Elliot & McGregor, 2001). This speaks to the need to conduct research studies that are specifically designed to test Linnenbrink's (2007) interactive model of affect, motivation, and engagement.

Third, future studies need to confirm the model using data collected from different age groups to allow for further generalizability of the model. Further analyses ideally should fit the model to more than one dataset by comparing two or more populations or cross-validating within the same population. Further studies need to analyze the variability or invariability of the structural paths between affect, motivation, and engagement in mathematics across different age groups.

Table 5.2

Independent t-tests Results of Study Variables by Gender

Variable	Female			Male			*t*	Cohen's *d*
	M	*SD*	*n*	*M*	*SD*	*n*		
Self-efficacy	3.05	.55	1596	3.19	.57	1600	-7.07	.25
Self-concept	2.60	.81	1558	2.78	.74	1643	-6.65**	.23
Unpleasant affect	2.40	.77	1561	2.23	.69	1630	6.82**	.23
Mastery approach	2.29	.79	1619	2.42	.78	1611	-4.55	.17
Performance approach	2.99	.73	1619	3.05	.72	1606	-2.18	.08
Behavioral engagement	3.55	.76	1576	3.59	.73	1570	-1.74	.05
Cognitive engagement	2.88	.63	1552	2.92	.62	1628	-1.96	.06

**$p < .01$.

Table 5.3

Independent t-test Results of Study Variables by Family Structure

Variable	Single Parent			Two Parents			*t*	Cohen's *d*
	M	*SD*	*n*	*M*	*SD*	*n*		
Self-efficacy	3.06	.60	612	3.16	.55	2241	-3.95	.17
Self-concept	2.63	.83	574	2.72	.77	2291	-2.37*	.11
Unpleasant affect	2.36	.78	572	2.28	.72	2279	2.06*	.11
Mastery approach	2.37	.83	613	2.36	.77	2271	.31*	.01
Performance approach	3.00	.74	611	3.04	.72	2267	-1.41	.05
Behavioral engagement	3.52	.77	599	3.60	.74	2208	-2.16	.11
Cognitive engagement	2.82	.65	571	2.93	.62	2275	-3.55	.17

*$p < .05$.

Table 5.4

Independent t-test Results of Study Variables by Immigrant Status

Variable	Native			Immigrant				
	M	*SD*	*n*	*M*	*SD*	*n*	*t*	Cohen's *d*
Self-efficacy	3.12	.57	2498	3.07	.57	434	1.50	.09
Self-concept	2.69	.78	2492	2.67	.76	448	.45	.03
Unpleasant affect	2.31	.75	2485	2.33	.70	448	-.62**	.03
Mastery approach	2.29	.79	2520	2.51	.75	446	-5.35	.29
Performance approach	2.99	.75	2515	3.07	.64	446	-2.24**	.11
Behavioral engagement	3.57	.75	2456	3.54	.72	432	.69	.41
Cognitive engagement	2.90	.63	2477	2.86	.61	447	1.20	.06

**p < .01; *p < .05.

Table 5.5

Independent t-test Results of Study Variables by Language at Home

	English			Other Language				
Variable	*M*	*SD*	*n*	*M*	*SD*	*n*	*t*	Cohen's *d*
Self-efficacy	3.13	.57	2731	3.07	.56	421	2.04	.11
Self-concept	2.69	.78	2722	2.71	.77	438	-.64	.03
Unpleasant affect	2.30	.74	2713	2.36	.70	439	-1.46	.08
Mastery approach	2.31	.78	2763	2.60	.75	422	-6.93	.37
Performance approach	3.00	.74	2757	3.16	.62	423	-4.25*	.23
Behavioral engagement	3.57	.75	2694	3.58	.72	411	-.23	.01
Cognitive engagement	2.90	.63	2708	2.92	.61	434	-0.86	.03

*$p < .05$.

Appendices

Appendix A

Matching of Measurements between Linnenbrink (2007) and PISA 2012

Latent variables in L (2007)	Latent variables from PISA 2012
Affect	
Pleasant affect (excited, happy, relaxed, and calm)	Mathematics self-efficacy: 1) Using a train timetable to work out how long it would take to get from one place to another. 2) Calculating how much cheaper a TV would be after a 30% discount. 3) Calculating how many square meters of tiles would be needed to cover a floor. 4) Understanding graphs presented in newspapers. 5) Solving equations like 3x+5=17. 6) Finding the actual distance between two places on a map with a 1:10 000 scale. 7) Solving equations like 2(x+3) = (x+3)(x-3). 8) Calculating the petrol-consumption rate of a car. Responses: *"very confident"," confident"*, *"not very confident"*, *"not at all confident"*
	Mathematics self-concept: 1) I am just not good at mathematics. 2) I get good grades in mathematics. 3) I learn mathematics quickly. 4) I have always believed that mathematics is one of my best subjects. 5) In my mathematics class, I understand even the most difficult work. Responses: *"agree"*, *"strongly agree"*, *"disagree"*, *"strongly disagree"*
Unpleasant affect (tense, angry, sad, tired, and exhausted)	Mathematics anxiety: 1) I often worry that it will be difficult for me in mathematics classes. 2) I get very tense when I have to do mathematics homework. 3) I get very nervous doing mathematics problems. 4) I feel helpless when doing a mathematics problem. 5) I worry that I will get poor grades in mathematics. Responses: *"agree"*, *"strongly agree"*, *"disagree"*, *"strongly disagree"*
Motivation	
Mastery-approach goal orientations	Intrinsic motivation to learn mathematics: 1) I enjoy reading about mathematics. 2) I look forward to my mathematics lessons. 3) I do mathematics

	because I enjoy it. 4) I am interested in the things I learn in mathematics. Responses: *"strongly agree", "agree", "disagree", "strongly disagree"*
Performance-approach goal orientations	Extrinsic (Instrumental) motivation to learn mathematics: 1) Making an effort in mathematics is worth it because it will help me in the work that I want to do later on. 2) Learning mathematics is worthwhile for me because it will improve my career prospects and chances. 3) Mathematics is an important subject for me because I need it for what I want to study later on. 4) I will learn many things in mathematics that will help me get a job. Responses: *"strongly agree", "agree", "disagree", "strongly disagree"*
Engagement	
Behavioral engagement (effort and persistence)	Behavioral engagement: 1) When confronted with a problem, I give up easily. 2) I put off difficult problems. 3) I remain interested in the tasks that I start. 4) I continue working on tasks until everything is perfect. 5) When confronted with a problem, I do more than what is expected of me. Responses: *"very much like me", "mostly like me", "somewhat like me", "not much like me", "not at all like me"*
Cognitive engagement (metacognitive strategy use, and self-regulated learning)	Cognitive Activation: 1) The teacher asks questions that make us reflect on the problem. 2) The teacher gives problems that require us to think for an extended time. 3) The teacher asks us to decide on our own procedures for solving complex problems. 4) The teacher presents problems for which there is no immediately obvious method of solution. 5) The teacher presents problems in different contexts so that students know whether they have understood the concepts. 6) The teacher helps us to learn from mistakes we have made. 7) The teacher asks us to explain how we have solved a problem. 8) The teacher presents problems that require students to apply what they have learned to new contexts. 9) The teacher gives problems that can be solved in several different ways. Responses: *"always or almost always", "often", "sometimes", "never or rarely"*

Appendix B

Use of Data from PISA

Latent variables	Indicators	PISA scales	Final scales
Pleasant affect	1) Using a train timetable to work out how long it would take to get from one place to another. 2) Calculating how much cheaper a TV would be after a 30% discount. 3) Calculating how many square meters of tiles would be needed to cover a floor. 4) Understanding graphs presented in newspapers. 5) Solving equations like 3x+5=17. 6) Finding the actual distance between two places on a map with a 1:10 000 scale. 7) Solving equations like 2(x+3) = (x+3)(x-3). 8) Calculating the petrol-consumption rate of a car.	4 = not at all confident 3 = not very confident 2 = confident 1 = very confident	1 = 4 2 = 3 3 = 2 4 = 1
	1) I am just not good at mathematics. 2) I get good grades in mathematics. 3) I learn mathematics quickly. 4) I have always believed that mathematics is one of my best subjects. 5) In my mathematics class, I understand even the most difficult work.	4 = strongly disagree 3 = disagree 2 = agree 1 = strongly agree	1 = 4 2 = 3 3 = 2 4 = 1 No reverse for 1
Unpleasant affect	1) I often worry that it will be difficult for me in mathematics classes. 2) I get very tense when I have to do mathematics homework.	4 = strongly disagree 3 = disagree 2 = agree 1 = strongly agree	1 = 4 2 = 3 3 = 2 4 = 1 Reverse for all items

	3) I get very nervous doing mathematics problems. 4) I feel helpless when doing a mathematics problem. 5) I worry that I will get poor grades in mathematics.		
Mastery-approach goal orientations	1) I enjoy reading about mathematics. 2) I look forward to my mathematics lessons. 3) I do mathematics because I enjoy it. 4) I am interested in the things I learn in mathematics.	4 = strongly disagree 3 = disagree 2 = agree 1 = strongly agree	1 = 4 2 = 3 3 = 2 4 = 1
Performance-approach goal orientations	1) Making an effort in mathematics is worth it because it will help me in the work that I want to do later on. 2) Learning mathematics is worthwhile for me because it will improve my career prospects and chances. 3) Mathematics is an important subject for me because I need it for what I want to study later on. 4) I will learn many things in mathematics that will help me get a job.	4 = strongly disagree 3 = disagree 2 = agree 1 = strongly agree	1 = 4 2 = 3 3 = 2 4 = 1
Behavioral engagement	1) When confronted with a problem, I give up easily. 2) I put off difficult problems. 3) I remain interested in the tasks that I start. 4) I continue working on tasks until everything is perfect. 5) When confronted with a problem, I do more than what is expected of me.	1 = very much like me 2 = mostly like me 3 = somewhat like me 4 = not much like m 5 = not at all like me	1 = 5 2 = 4 3 = 3 4 = 2 5 = 1 No reverse for 1) and 2)
Cognitive engagement	1) The teacher asks questions that make us reflect on the problem.	4 = never or rarely 3 = sometimes 2 = often	1 = 4 2 = 3 3 = 2

	2) The teacher gives problems that require us to think for an extended time.	1 = always or almost always	4 = 1
	3) The teacher asks us to decide on our own procedures for solving complex problems.		
	4) The teacher presents problems for which there is no immediately obvious method of solution.		
	5) The teacher presents problems in different contexts so that students know whether they have understood the concepts.		
	6) The teacher helps us to learn from mistakes we have made.		
	7) The teacher asks us to explain how we have solved a problem.		
	8) The teacher presents problems that require students to apply what they have learned to new contexts.		
	9) The teacher gives problems that can be solved in several different ways.		

Appendix C

Individual Differences in Affect, Motivation, and Engagement

Seven *t*-tests were conducted on all the measured variables to determine if there were group mean differences by gender, family structure, immigrant status, or language spoken at home.

Gender Differences

The results of *t*-tests for gender differences along with the descriptive statistics for all the affect, motivation, and engagement variables in both the female and male groups are reported in Table 5.2. For the independent samples *t*-test, Cohen's d was determined by the ratio of the mean difference between the two gender groups to the pooled standard deviation. Overall, the males and females had significant differences in the mean for mathematics pleasant affect (self-concept) ($p < .001$), and unpleasant affect ($p < .001$). In particular, the male students reported a higher mathematics self-concept and a lower mathematics anxiety. The effect sizes for the mathematics pleasant affect (self-concept) , and unpleasant affect (mathematics anxiety) both had a Cohen's *d* of 0.23, indicating small differences between females and males in the means for mathematics pleasant affect (self-efficacy) and unpleasant affect (mathematics anxiety). No significant differences between males and females were observed for mathematics pleasant affect (self-efficacy), mastery-approach goal, performance-approach goal, behavioral engagement, and cognitive engagement.

Family Structure Difference

Table 5.3 shows the means and standard deviations for students with different family structures on all the measured variables to test whether there were group mean differences between different family structures. Prior to the analysis, family structure was recoded as 0 = single parent or guardian, 1 = two parents. Overall, students with different family structures (one parent and two parents) showed significant differences in the mean for pleasant affect (mathematics self-concept) (p = .02), unpleasant affect (p = .01), and mastery-approach goal (p = .01). In particular, students with two parents had higher pleasant affect (mathematics self-concept), lower unpleasant affect, and lower mastery-approach goal. The effect sizes for pleasant affect, unpleasant affect, and mastery-approach goal, were Cohen's *d*s of 0.11, 0.11, and 0.01, respectively, indicating small differences between different family structures on mathematics pleasant affect and unpleasant affect, and trivial differences (likely due to chance) in mastery-approach goal. There were no significant differences in the mean for mathematics pleasant affect (self-efficacy), performance-approach goal, and behavioral engagement between students with different family structures (p > .05).

Immigrant Status Difference

Table 5.4 shows the means and standard deviations for students with different immigrant status on all the measured variables to test whether there were group mean differences across different immigrant status. Prior to the analysis, immigrant status was recoded as 0 = native, 1 = immigrant student (i.e., first or second generation). Overall, students with different immigrant status reported significant differences in the mean for unpleasant affect (p < .001) and performance-approach goal in mathematics (p <.001). In particular, immigrant students had a higher unpleasant affect and a higher performance-approach goal in mathematics. The effect size for unpleasant affect and performance-approach goal were small with Cohen's *ds* of .03, and 0.11, respectively, indicating from trivial (likely due to chance) to small differences between different family structures on mathematics unpleasant affect and performance-approach goal in mathematics. No significant differences between different immigrant status were observed on mathematics self-efficacy, mathematics self-concept, mastery-approach goal, behavioral engagement, and cognitive engagement in mathematics ($p > .05$).

Language at Home Difference

Table 5.5 shows the means and standard deviations for students with different languages on all measured variables to test if there were group mean differences between students who spoke a different language at home. Language at home was recoded as 0 = English, 1 = other language. Overall, student with a different language at home reported significant differences in the mean for performance-approach goal in mathematics ($p = .03$). In particular, students who spoke English at home had lower scores in the performance-approach goal in mathematics. The effect size for the performance-approach goal was small with a Cohen's *d* of .23, indicating small differences between different languages at home on performance-approach goal in mathematics. There were no significant differences in the mean for mathematics pleasant affect (self-efficacy), mathematics self-concept, unpleasant affect, behavioral engagement, and cognitive engagement between students with different language at home.

In sum, male students reported a higher mathematics pleasant affect (measured by mathematics self-concept), and a lower unpleasant affect compared with female students. Students from two-parent families had a higher pleasant affect (measured by mathematics self-concept), lower unpleasant affect, and lower mastery-approach goal than students from one-parent families. Native students had a lower unpleasant affect and lower performance-approach goal compared with immigrant students. Students who spoke English at home reported lower performance-approach goal in mathematics compared with those students who spoke another language at home.

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PUBLICATIONS

Ma, X., Shen, J., Krenn, H. Y., Yuan, J., & **Hu, S**. (2015). The role of system alignment in care and education of children from birth to Grade 3. *Early Child Development and Care*, 185, 1067-1087.

Ma, X., & **Hu, S**. The Effects of Teacher Professional Development on Child Learning Outcomes during Early Childhood Education and Early Elementary Education: A Meta-Analysis, *Journal of Teacher Education*. Under Review

Ma, X., McGee, D., & **Hu, S**. Rethinking Teacher Leadership: An Update Review, *International Review of Education.* Under Review

PRESENTATIONS

Hu, S (2014, March). *Do we have gender difference in Metacognitive Strategies Instrument?: Testing measurement invariance among American youth*. Paper published in 2014 UK Appalachian Research Community Symposium.

Hu, S (2014, April). *Exploring how self-beliefs influence motivation in science among American Youth*. Paper published to the annual convention of the Spring Research Conference, Cincinnati, OH.

Hu, S., Toland, M. D., & Ma, X. (2014, August). *Measurement invariance of metacognitive strategies instrument across ethnic, gender, and SES groups*. Paper published in the annual convention of the American Psychological Association, Washington, DC.

Ma, X., Shen, J., Krenn, H. Y., Yuan, J., & **Hu, S**. (2014, September). *The role of system alignment in care and education of children from birth to Grade 3*.Paper presented at the annual meeting of the European Educational Research Association.Porto, Portugal.

Hu, S. (2014, November). *Do we have gender difference in mathematics self-efficacy?: Testing measurement invariance in gender groups*. Paper published in the annual convention of the Mid-South Educational Research Association in Knoxville, TN.

Hu, S., &Ma, X. (2015, April). *Measurement Invariance of Mathematics Self-Efficacy between American and Chinese Students*. Paper published in the annual convention of the American Educational Research Association, Chicago, Illinois.

Ma, X., Shen, J., Krenn, H. Y., Yuan, J., & **Hu, S**. (2015, April). *Effects of teacher professional development on child learning outcomes: A meta-analysis*. Paper presented at the annual meeting of the American Educational Research Association. Chicago, IL.

Hu, S. (2015, August). *The Relationship between Mathematics Self-Efficacy and Opportunity to Learn* Mathematics. Paper published in the annual convention of the American Psychological Association, Toronto, Canada.

Hu, S., &Ma, X. (2015, August). *An Examination of Measurement Invariance of Mathematics Motivation across Gender, Racial and Family Structural Groups*. Paper published in the annual convention of the American Psychological Association, Toronto, Canada.

Ma, X., Shen, J., Krenn, H. Y., **Hu, S**., & Yuan, J. (2015, September). *A meta-analysis of the relationship between learning outcomes and parental involvement during early childhood education and early elementary education*. Paper presented at the annual meeting of the European Educational Research Association. Budapest, Hungary.

Hu, S. (2016, August).*The Dynamic Effects of Affect, Motivation, and Engagement on Mathematics Behavior.* Paper published in the annual convention of the American Psychological Association, Denver, Colorado.

AWARDS AND DISTINCTIONS

Doris Nowak and William E. Stilwell, III Graduate Fellowship	2014
International Student Tuition Scholarship	2015
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CERTIFICATION

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