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Marek Molenda

Polish Academy of Sciences, Poland

Jozef Horabik

Polish Academy of Sciences, Poland

I. Joe Ross

University of Kentucky

Michael D. Montross

University of Kentucky, michael.montross@uky.edu

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FRICITION OF WHEAT: GRAIN-ON-GRAIN AND ON CORRUGATED STEEL

M. Molenda, J. Horabik, I. J. Ross, M. D. Montross

ABSTRACT. *Coefficients of friction of wheat for grain-on-grain and on galvanized corrugated steel sheet were investigated using a modified direct shear apparatus. Tests were conducted under a normal pressure of 20.7 kPa using soft red winter wheat at a moisture content of 11.2% (w.b.) and an uncompressed bulk density of 740 kg/m³. Three consolidation procedures and three methods of deposition of grain in the test chamber were used. Test results of grain-on-grain friction showed that consolidation procedure markedly influenced the force-displacement relationship, while its influence on the coefficients of friction were small. Shearing to peak strength as a consolidation method erased all effects of loading history and resulted in the highest values of the coefficient of friction. Grain-on-grain coefficients of friction were in a range from 0.47 ± 0.007 to 0.56 ± 0.004 depending on the method of grain deposition.*

Friction on two dimensionally different samples of corrugated steel sheet was examined using three methods of grain deposition. Corrugation depths were 13 mm on both samples, while their periods were 67.5 mm (short) and 104 mm (long). Coefficients of friction on the short-period corrugated samples were in a range from 0.42 ± 0.0 to 0.46 ± 0.004 and were significantly higher ($\alpha = 5\%$) than those on the long-period corrugated sample, which ranged from 0.36 ± 0.003 to 0.39 ± 0.003 . The method of grain deposition significantly ($\alpha = 5\%$) influenced the coefficients of friction of wheat on both types of corrugated steel sheet.

Keywords. *Wheat grain, Coefficients of friction, Deposition method, Load history, Direct shear test.*

Physical properties of grain are used in the design of storage structures and handling equipment. Properties such as the load-deformation behavior and coefficients of friction are influenced by numerous factors. Knowledge about the role of many of these factors is still incomplete, and additional experimental work is needed to determine the limits of uncertainty and to describe the behavior of grain in various conditions. Janssen's equation likely will remain the basis for the majority of standards used for the calculation of pressures in bins, and the coefficient of wall friction will remain, next to the bulk density, the most important design parameter (Wilms, 1991). For rough bin walls, such as concrete or pitted carbon steel, shearing takes place within the grain, and the coefficient of wall friction is close to the coefficient of grain-on-grain friction. Another specific property necessary for calculating grain loads on bin walls is the pressure ratio (k). There are many recommendations for calculating this

property from angles of internal and wall friction (Drescher, 1991).

The coefficients of both grain-on-grain and wall friction can be derived from shear tests. Munch-Andersen (1987) observed that a boundary layer forms between grain and a rough bin wall and dilation takes place, resulting in overpressure. His experiments on small model bins showed that the thickness of this layer was independent of the bin diameter, which contributed to scale errors. According to Oda (1997), when the interface layer between a bin wall and stored material is sheared, large relative displacements between the particles and rotation of the particles take place. Large voids develop in the shear-zone that are associated with dilatancy. It can be expected that higher stress would develop in a more dilatant material. Upon initiation of shear displacement, the state of stress in the material gradually changes from compression to shear. To achieve fully developed shear conditions of stress and strain, sufficiently large deformation of bulk material must take place. Therefore, the experimental investigation of shearing in a material layer requires large displacement to model the actual behavior.

According to Feise (1998), during particle rearrangement from the consolidation to the shear structure, the sample can show a stress decrease, stress increase, or constant stress levels, depending on the actual structural changes of the granular particles. For most granular materials, the way the sample was formed and consolidated influences the yield locus, as it introduces anisotropy into an initially isotropic sample. *Induced anisotropy* is the material anisotropy introduced into the sample by the consolidation procedure, while *inherent anisotropy* stems from the shape of the particles and the way the bedding is formed (Wong and

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The authors are **Marek Molenda**, Assistant Professor, and **Josef Horabik**, Assistant Professor, Institute of Agrophysics, Polish Academy of Sciences, Lublin, Poland; and **I. Joe Ross**, ASAE Fellow Engineer, Professor Emeritus, and **Michael D. Montross**, ASAE Member, Assistant Professor, Biosystems and Agricultural Engineering Department, University of Kentucky, Lexington, Kentucky. **Corresponding author:** Michael D. Montross, Biosystems and Agricultural Engineering Dept., University of Kentucky, Lexington, KY 40546-0276; phone: 859-257-3000, ext. 106; fax: 859-257-5671; e-mail: montross@bae.uky.edu.

Arthur, 1985). The anisotropy introduced during consolidation depends on the type of test procedure used. For the Jenike et al. (1958) method, the sample is first pre-sheared to a steady-state flow and later sheared to failure at a lower stress level. In the research reported here, filling the test apparatus and applying the consolidation procedure are conducted in a way to model conditions that may develop in a bin as a result of filling method or condition of operation.

No standard method currently exists for determining the coefficient of friction of bulk solids on corrugated materials. The behavior of the coefficient of friction between wheat and corrugated steel has been investigated by various authors (Moore et al., 1984; Versavel and Britton, 1986; Zhang et al., 1994) using different instrumentation. Based on these studies, it may be concluded that the coefficient of friction of wheat on corrugated steel is mainly comprised of grain-on-grain friction. The coefficient of friction was found to vary with moisture content, pressure, and velocity. Corrugations on galvanized steel applied in these investigations were 67 to 70 mm long and 13 to 19 mm high. For wheat of approximately 12% moisture content (w.b.) and a range of normal pressure from 30 to 70 kPa, the average values of coefficient of friction were reported in a range from 0.40 (Moore et al., 1984) to 0.48 (Versavel and Britton, 1986). ASAE Standard EP 433 (ASAE Standards, 1997) recommends using a coefficient of friction of 0.37 for calculations of loads in corrugated walled bins.

The purpose of this study was to examine the shear deformation of a wheat layer using a modified shear tester. The specific objectives were:

1. To examine the force response to shear deformation of wheat samples using different consolidation procedures and filling methods.
2. To compare friction of wheat on corrugated galvanized steel of two profiles and grain-on-grain friction of wheat.

EQUIPMENT AND PROCEDURES

This study utilized a modified direct shear apparatus (Molenda et al., 2000) shown schematically in figure 1, to measure the force of friction of wheat grain-on-grain and on corrugated galvanized steel sheets. The body of the apparatus was made of 13 mm thick steel and designed to limit load deflections to less than 0.2 mm. The test specimen tray contained the corrugated steel test specimen or vertical blades to determine grain-on-grain friction. It was supported on six pairs of roller bearings that allowed the test specimen to move in the longitudinal direction of the apparatus with the coefficient of friction negligible as compared to that of grain. Two sidebars were attached to the frame adjoining the long walls of the apparatus with ultra-high-density plastic (UHDP) plates attached to the top surface of each side bar. A flexible diaphragm mounted on the bottom of the cover of the apparatus was used to exert a known normal pressure on the grain mass located in the grain compartment. Pressure was applied by compressed air.

During testing, wheat was filled to the top of the grain compartment and leveled. Two end plates were attached horizontally across the plastic plates adjoining the front and back walls of the grain compartment. The role of the plates was to eliminate contact between the grain and specimen tray in the regions of most non-uniform pressure resulting from

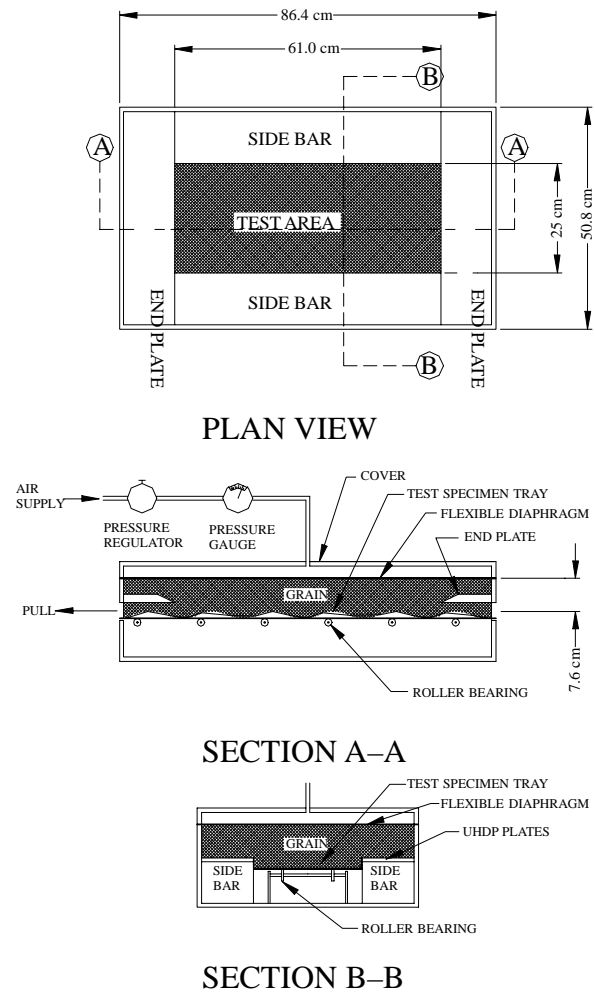


Figure 1. Schematic drawing of the modified direct shear apparatus (friction tester).

compaction of grain near the front wall and dilation of grain near the back wall. These plates also reduced the total effective length of the test specimen exposed to grain within the test apparatus. The sample tray was 25 cm wide with 61 cm of its length exposed to grain pressure. The sample tray was attached through a chain system and a gear arrangement to an Instron 5500 universal test machine to measure the force required to pull the tray located under the pressurized grain mass. A shearing speed of 50 mm/min was used in all experiments. Three replications were made for each measurement. The force of friction was measured with a calibrated electronic load cell having an accuracy of ± 5 N.

Soft red winter wheat with a moisture content of 11.2% (w.b.) and an uncompacted bulk density of 740 kg/m^3 was used in the experiments. For each test, 24.9 kg of grain was placed into grain compartment, which resulted in mean uncompacted bulk density of grain before each test of 740 kg/m^3 . For all tests, except those comparing filling methods, the test chamber of the friction tester apparatus was filled with a hand-held scoop (approximately 1.3 kg of grain). Grain was slowly poured into the apparatus, forming a thin layer on the bottom or on the layer that was already deposited. This method of filling is referred to in this article as "standard" filling.

The influence of normal load on the coefficient of grain-on-grain friction was examined in a series of prelimi-

nary tests. The coefficient of grain-on-grain friction was found to be 0.51 ± 0.004 and independent of normal pressure in the range from 6.9 to 34.5 kPa. A normal pressure of 20.7 kPa was applied in all further experiments. Coefficient of friction between wheat and smooth galvanized steel was also determined and, after wear-in, found to be 0.135 ± 0.002 and independent of normal pressure in the range from 6.9 to 34.5 kPa. This value should be treated as an extremely low coefficient of friction that would occur after prolonged sliding between grain and galvanized steel. ASAE Standard EP 433 recommends a coefficient of friction of wheat against flat steel of 0.3, and this value is appropriate for design situations.

Two series of tests were conducted with the friction tester apparatus (fig. 1). The first series of tests was conducted to determine the effects of consolidation procedures and method of grain deposition on grain-on-grain friction of wheat. To compare the influence of consolidation procedure, three methods were applied before shearing the sample. In the first method, herein referred to as “standard,” the sample was sheared under a 20.7 kPa normal load with no additional pretreatment. The second method, herein referred to as “compacted,” consisted of a 15-minute consolidation period under a normal pressure of 48.2 kPa. The time of consolidation was chosen based on Clower et al. (1973). These authors found that the change in volume of grain decreased to a constant level after a 15-minute application of constant pressure. With the third method, the sample was first pre-sheared to steady-state flow and later sheared to failure at a lower stress level, as recommended by Jenike et al. (1958). A normal pressure of 48.2 kPa was applied for this initial shearing, and this method is herein referred to as “pre-sheared.”

Four methods of grain deposition in the grain compartment of the friction tester were used. The first method was the “standard” filling method described earlier. The second method was “sprinkle” filling. This technique used a hand-held scoop to transfer grain from a batch into a hand-held No. 4 sieve with 4.75 mm openings. Grain was transferred to the sieve at a rate to maintain a head of grain 2 to 3 cm thick above the wire mesh. The wire mesh was held approximately 10 cm above the test surface of the grain in the grain compartment. The grain flowed through the openings in the wire mesh and “sprinkled” onto the surface. The sieve was moved around the grain surface to maintain an approximate level surface.

Two methods of inclined filling were tested using the device shown schematically in figure 2. For both methods, the device rested on top of the friction tester and was filled with a hand-held scoop. The grain was allowed to slide down its plane of natural repose to the front edge of the device. The grain was contained on the other three sides by a back and side panel. Generally, the wheat kernels would orient themselves with their longer axes along their path of travel as they moved down the angle of repose formed by the grain in both the fill device and the grain chamber of the friction tester, as suggested by the insert diagrams in figure 2. They bedded themselves at their natural angle of repose of approximately 27° . For inclined filling forward (IFF), the fill device was moved along the grain chamber of the friction tester in the direction of pull (see fig. 2a) at 4 cm increments to fill the entire 86 cm long grain chamber. Inclined filling reverse (IFR) was the same as IFF except the direction of

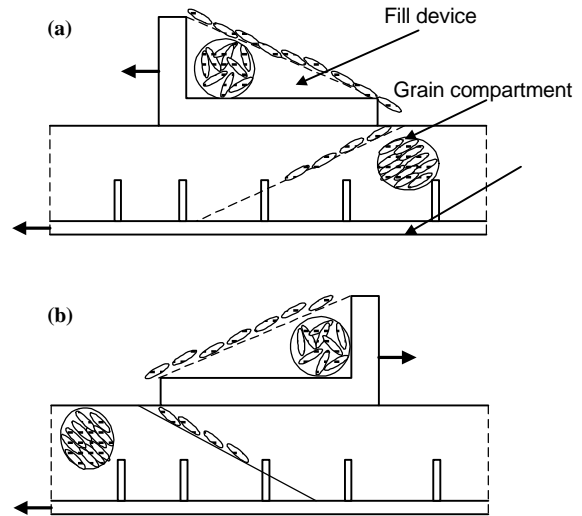


Figure 2. Schematic of sample deposition with the inclined filling device: (a) inclined filling forward (IFF), and (b) inclined filling reverse (IFR).

filling was reversed. For the IFF method, filling was started from the back wall of grain compartment (with respect to the direction of pull), which resulted in the preferred orientation of the long axes of grains at an obtuse angle to the direction of shearing. For the IFR method, filling started from the front wall of grain compartment. Grains tended to bed with their long axes oriented at an acute angle to the direction of shearing.

The second series of experiments compared the frictional behavior of wheat on two samples of galvanized corrugated steel of different corrugation periods. The height of corrugations of both steel samples was 13 mm. Test samples had corrugation periods of 67.5 mm (“short period”) and 104 mm (“long period”). The four filling methods described above were used on both types of test samples.

RESULTS

CONSOLIDATION PROCEDURE

The variation in the force-displacement characteristics with the consolidation procedure was relatively large, as shown in figure 3. The initial part of the curve was characterized by a secant modulus of stiffness calculated as the value of force at 5 mm of displacement divided by 5. The consolidation had a considerable effect on the rate of force increase. The lowest value of the modulus was found to be 182 N/mm for the unconsolidated sample, while a value of 300 N/mm was found for the pre-sheared sample. The method of consolidation changed the secant modulus of deformation by 164%. After reaching 50% of maximum force, the characteristics of consolidated samples differentiated. Friction force for the pre-sheared sample increased to the maximum almost linearly, while the force for the compacted sample increased more slowly and asymptotically. Observed variations in initial moduli are probably caused by differences in the initial angular distribution of grain contacts.

Force-displacement characteristics of the sample that was sheared prior to actual determination of the coefficient of friction show a form of induced anisotropy characteristic of this type of deformation. Its degree depends on the deforma-

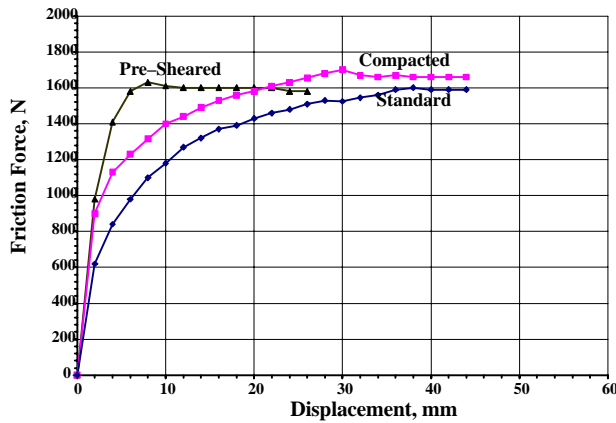


Figure 3. Friction force–displacement characteristics as dependent on loading histories at 20.7 kPa of normal pressure.

tion caused by initial shearing. If the sample was pre-sheared to steady-state flow, then all stress history was erased. In these tests, the failure of the pre-sheared sample took place at a displacement several times lower than for the other two types of samples. Thus, the deformation work required to produce failure of the pre-sheared sample was distinctly lower. Similar effects take place during the first opening of a discharge gate in a bin when the flow channel forms along the wall and inside the grain mass. The material experiences a change in major principal stress direction, and an oriented structure is formed that corresponds to the new state of stress. Subsequent flow initiations do not require these changes in bedding structure and do not produce significant overpressure.

Applied differences in stress histories did not cause any large variations in the coefficients of grain-on-grain friction. The values of the coefficient were found to be 0.50 ± 0.010 , 0.52 ± 0.016 , and 0.52 ± 0.010 for standard, compacted, and pre-sheared samples, respectively.

METHOD OF SAMPLE DEPOSITION

The method of sample deposition had a strong influence on both the force–displacement characteristics and coefficients of grain-on-grain friction. The four curves in figure 4 represents the characteristics for the deposition methods. For the standard filling, inclined reverse filling, and sprinkle filling methods, values of force stabilized after a displacement of approximately 40 mm. The curve for inclined forward filling did not stabilize within the available range of displacement of the test apparatus. As shown in figure 4, after 46 mm of displacement, the force of friction was still increasing, which made the determination of the coefficient of friction impossible.

The secant moduli that reflect initial tangent stiffness were calculated for 5 mm of displacement. Values of tangent stiffness moduli were 144, 164, 171, and 216 kN/mm for the standard filling, IFF, IFR, and sprinkle filling samples, respectively. The probable reason for the differences in these moduli is the initial angular distribution of particle contacts. Oda (1972) suggested that, for a two-dimensional system, the distribution of contact normals may be approximated by an ellipse. The major axis of such an ellipse is initially oriented normal to the bedding plane. The preferred orientation of the long axes of the particles is parallel to the bedding plane. In the process of biaxial deformation, the distribution

of the contact normals changes such that a greater number of contact normals tend to orient themselves in the direction of maximum compressive force (Oda et al., 1982). The principal axes of the contact distribution ellipse tend to become coaxial with the principal axes of stress.

The friction force for sprinkle filling passed through a maximum value after approximately 27 mm of displacement. Such a mechanical behavior is typical for dense granular materials. The strength at peak stress is referred to as the maximum shear resistance, while the condition when force become constant is called steady-state flow (Jenike et al., 1958). For sprinkle filling, the peak coefficient of friction was found equal to 0.56 ± 0.004 , while the steady-state values equaled 0.53 ± 0.008 . The method of sample deposition had considerable influence on the coefficients of grain-on-grain friction. The values of the coefficient for standard filling and IFR were found to be 0.52 ± 0.0 and 0.47 ± 0.007 , respectively. A maximum difference of 0.09, or 19%, in grain-on-grain coefficients of friction was found in these experiments. ASAE Standard EP 433 does not address internal friction. Australian Standard AS 3774 (1996) recommends using two values for each property, an upper characteristic value and a lower characteristic value. These two values represent the range in property variability that can exist in material. Angles of internal friction (ϕ) for wheat range between 26° and 32° and, in terms of the coefficient of friction ($\mu = \tan\phi$) correspond to a range of 0.49 and 0.62. The results shown in table 1 are close to the lower limit of this range.

FRICTION ON CORRUGATED STEEL

A preliminary series of 15 tests on both samples of steel was performed with a normal load of 35 kPa to determine possible change in surface conditions. No wear-in effects

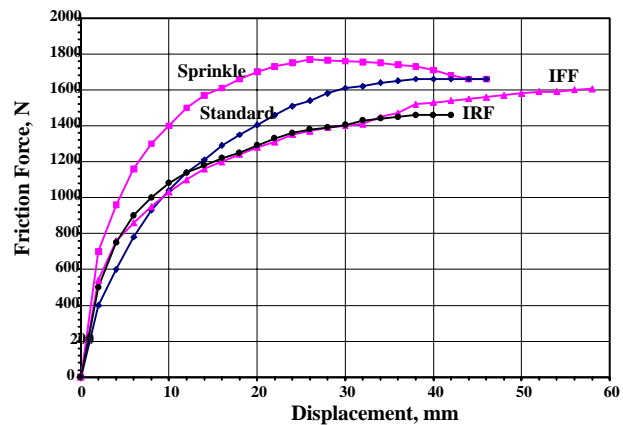


Figure 4. Friction force–displacement characteristics as dependent on deposition methods at 20.7 kPa of normal pressure.

Table 1. Coefficients of friction of wheat (with standard deviations) on two types of corrugated steel and grain-on-grain as affected by filling method after wear-in period.

Filling Method	Test Configuration		
	Long-period corrugated ^[a]	Short-period corrugated	Grain on grain
IFR	0.36 ± 0.003	0.42 ± 0.000	0.47 ± 0.007
Standard	0.37 ± 0.000	0.43 ± 0.010	0.52 ± 0.000
Sprinkle	0.39 ± 0.003	0.46 ± 0.004	0.53 ± 0.008
Average	0.37 ± 0.012	0.44 ± 0.018	0.50 ± 0.029

^[a] Values found after wear-in period.

were observed in the tests on the short-period corrugated steel. The coefficient of friction on long-period corrugated steel decreased from 0.44 for the first test to 0.37 for 12th test and remained approximately stable in subsequent tests, as shown in figure 5.

Values of the coefficients of friction on corrugated steel and grain-on-grain, with standard deviations, are given in table 1. An analysis of variance performed on the data showed that for all filling methods, the coefficients of friction on short-period corrugated sheets were significantly ($\alpha = 0.05$) lower than grain-on-grain coefficients of friction. For all filling methods, coefficients of friction on long-period corrugated sheets were significantly lower ($\alpha = 0.05$) than coefficients of friction on short-period corrugated sheets. The relation of the coefficient of friction on corrugated sheets to the coefficient of grain-on-grain friction appeared dependent on the profile of the corrugations. The average coefficient of friction for all filling methods on short-period corrugated sheets was found to be 87% of the coefficient of grain-on-grain friction, while the coefficient on long-period corrugated sheets was 74% of the coefficient of grain-on-grain friction. Australian Standard AS 3774 (1996) states that the wall friction on profile sheeting with horizontal ribs is a function of internal friction, friction against the wall, and the profile of the sheeting. The recommended formula for such a combination is:

$$\mu_{eff} = u_2 \mu_i + u_3 \mu_w \quad (1)$$

where

μ_{eff} = effective wall friction coefficient

μ_i = internal friction coefficient

μ_w = wall friction coefficient (flat wall surface)

u_2 and u_3 represent the proportion of the bulk solid moving against itself and moving against the wall, which is determined by the profile of the sheeting. Formulae for u_2 and u_3 are given as:

$$u_2 = \frac{y_1}{(x_2 + y_1)} \quad (2)$$

$$u_3 = \frac{x_2}{(x_2 + y_1)} \quad (3)$$

where x_2 is the length of the profile period along which grain moves, and y_1 is the length of the profile period along which grain remains stagnant.

The method of determining x_2 and y_1 is not given precisely in the standard. Zhang et al. (1994) estimated the lower

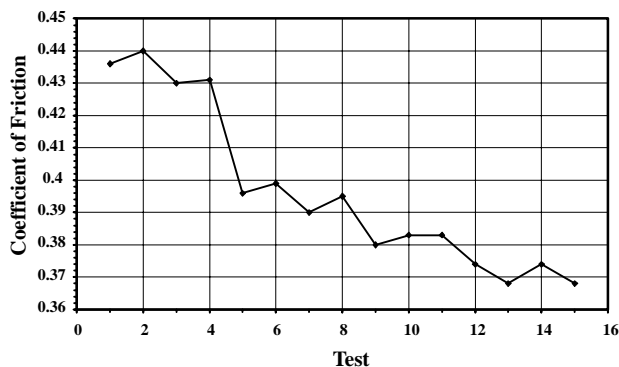


Figure 5. Wear-in graph for long-period corrugated steel.

boundary of the shearing zone in wheat as 4.5 mm below the corrugation peaks. Using this value for x_2 , values of 27.5 mm and 42 mm were obtained for short-period and long-period corrugated profiles, respectively. Corresponding y_1 values were 40 mm and 62 mm, which resulted in an effective coefficient of friction of 0.354 for both profiles. This value is distinctly lower than the measured values obtained in these experiments. The real course of deformation is more complex (Feise, 1988, Oda, 1997) than the sliding of two rigid blocks of material against each other along the plane of rupture. The pattern of deformation is also pressure dependent. Until the deformation of granular materials is better understood, the most reliable method to estimate the coefficient of internal friction of grain and against corrugated steel is experimental methods.

SUMMARY

The force-displacement behavior of bulk wheat depends on the method of sample deposition and, to a lesser extent, on the consolidation procedure. Consolidation of the sample with pre-shear resulted in a secant-modulus value that was 163% that of unconsolidated sample. Samples of wheat formed with different filling procedures showed large differences in stiffness, as expressed by their secant moduli. Coefficients of grain-on-grain friction were strongly influenced by the filling procedures. Their values ranged from 0.47 for inclined filling to 0.56 for sprinkle filling. Variations in the coefficient of friction resulted from variations in orientation of grain against the bin wall (inherent anisotropy) that may result in an uneven distribution of lateral pressure around the bin circumference.

The effect of stress history (induced anisotropy) on the coefficient of grain-on-grain friction was less evident. Various consolidation procedures resulted in different force-displacement characteristics, while the coefficient of internal friction did not change significantly (in the range of 0.50 to 0.52). Consolidation deformation in the form of shearing to steady-state flow resulted in a sample of maximum strength. This type of consolidation erased the effect of any previous loading history.

The effect of filling method on the coefficient of friction was also observed in tests of friction on corrugated steel sheets. The highest coefficients of friction were found for sprinkle filling, while the lowest were those obtained for inclined filling. For the same height of corrugations (13 mm), the coefficient of friction was approximately 0.44 on short-period (67.5 mm) corrugated samples, and 0.37 on long-period (104 mm) corrugated samples, a difference of approximately 19%.

Large force-displacement anisotropy and considerable strength anisotropy shows nonhomogeneity as a fundamental feature of the flow properties of bulk materials. The present state of knowledge precludes a complete interpretation of deformation of wheat in shear tests. Efficient methods are required to describe bedding structure and monitor its changes during shear deformations. Further research is necessary to systematically describe changes in bedding structure during shearing. Grain, being more uniform than many bulk solids, may serve as a model material for investigations of the interactions of properties of individual particles and properties of material in bulk.

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