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# A FAULT LOCATION ALGORITHM FOR UNBALANCED DISTRIBUTION SYSTEM WITHOUT FAULT TYPE INFORMATION

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Yizhe Li, Student

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Dr. Caicheng Lu, Director of Graduate Studies

# A FAULT LOCATION ALGORITHM FOR UNBALANCED DISTRIBUTION SYSTEM WITHOUT FAULT TYPE INFORMATION

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THESIS

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A thesis submitted in partial fulfillment of the requirement of the
degree of Master of Science in the College of Engineering at
University of Kentucky

By

Yizhe Li

Lexington, Kentucky

Director: Dr. Yuan Liao, Professor of Electrical and Computer Engineering

Lexington Kentucky

2017

# ABSTRACT OF THESIS

## A FAULT LOCATION ALGORITHM FOR UNBALANCED DISTRIBUTION SYSTEM WITHOUT FAULT TYPE INFORMATION

Power system faults normally result in system damage, profit loss and consumer dissatisfaction. Consequently, there is a strong demand on precise and fast fault location estimation for power system to minimize the system restoration time.

This paper examines a method to locate short-circuit faults on a distribution system with unbalanced loads without fault type information. Bus impedance matrix technique was harnessed in the fault location estimation algorithm. The system data including line impedances, source impedance and distribution system layout was assumed to be known factors, hence pre-fault bus impedance can be calculated and implemented into the algorithm. Corresponding methods to derive system matrix information were discussed. Case studies were performed to evaluate the accuracy of the fault location algorithm and illustrate the robust performance under measurements errors influences, load variation impacts and load compensation implementations.

Traditional fault location methods involve current and voltage measurements mandatorily locating at each ends of faulted section to locate the fault. The method examined finds fault location for distribution system utilizing impedance matrix accompanied with sparse measurements in the power network. This method fully considers the unbalance of distribution system.

KEYWORDS: Distribution systems, fault locations, bus impedance matrix, power systems, fault diagnosis.

Yizhe Li

11/19/2017

# A FAULT LOCATION ALGORITHM FOR UNBALANCED DISTRIBUTION SYSTEM WITHOUT FAULT TYPE INFORMATION

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11/19/2017

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*For my family*

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## TABLE OF CONTENTS

# TABLE OF FIGURES

# LIST OF TABLES

# Chapter 1 INTRODUCTION

In the first section of this introduction, there will be a general description of power system faults and distribution systems, followed by a brief discussion on published fault analysis theories along with adopted algorithm and techniques for distribution systems and bus impedance matrix.

## 1.1. POWER SYSTEM FAULT BACKGROUND

A typical power system includes generation, transmission and consumption along with all aspects of fault detection and management systems. Faults which mainly occur as short-circuit faults may take place at power lines, transformers and generation system, causing heavily aggravation in rerouted lines or even immediately leading to electric blackout. Rapid restoration of service and excellent pinpoint accuracy during a fault clearance play an important role in power system regular operation to reduce overall cost and increase safety of the whole system.

A common cause for a power system fault is an unexpected connection between power lines and grounded objectives such as trees. This case tends to happen more frequently in summer because thermal expansion of lines and fast-growing vegetation. Disaster phenomena also contributes greatly system failures since most design of power system would only consider natural factors to a certain degree in order to achieve feasibility in marketing aspect during planning phase of that power system. There has been a snow disaster in south china area causing damage and even collapses of more than one thousand main feeders including 500kV units. One of the analysis shows that the design schemes on power lines and towers were not considering such huge overweight from ice and snow on power cables and towers especially at turning sections, which used to be considered as feasible designs due to the subtropical monsoon climate of the area [1]. Human errors and equipment failure including illegal structure under or close to power line or improper breaker tripping also could lead to faults. Lightning strikes are big portion of current flood and causing breaker to open, although most modern fault detection systems have the re-connection feature to eliminate non-permanent fault [2].

Three phase power line fault can be classified into five types considering number and means of connections of lines possibly including ground [3]. The first type is line to ground fault(LG) where occurs an unexpected connect between one single phase line to the ground. The second type is line to line fault(LL) where either two lines of different phases make connection. The third type is line to line to ground fault (LLG) that involves two lines of different phases and the ground. The fourth type is line to line to line(LLL) faults which is similar to LL fault but includes all three phases. The last type is line to line to line to ground fault(LLLG) which indicates that all three phases are connected to the ground.

## 1.2. CURRENT PUBLISHED TECHNIQUES FOR POWER SYSTEM FAULT.

Fault location algorithms have been under developing for decades, implemented with digital and analog fault detecting instruments to increase the stability and reliability of power system and reduce financial loss. Although significant efforts have been spent on high voltage level transmission system protection, there has been relatively little work in the development of fault location estimation for distribution system until lately.

There have been numerous developed and developing methodologies for fault location estimation in the past decades [4] . There is a potential demand on more means to solve the fault locations. As a matter of fact, several methods would be chosen to provide the estimations of one fault location in reality, due to the natural flaws of certain fault location methods and possible multiple results in some specific cases. Implementing different algorithms could reveal the unlikely fault locations and reinforce the correct ones which could be tricky to identify. The financial consequences for an accurate fault location can be so significant that it can deliver more precise fault location for the ground mechanic team then generally shorten the fault clearance time significantly leading to less complaints from consumers and penalty fee [3]. Therefore, fault analysis has become one of the most important research in electric engineering and new techniques keep coming up [4].

Although the long research history and good amount of experience from practice, fault analysis is still considered as a major issue in long term. The problems engineers and mechanics facing nowadays are generally measurements error during to the nature of instrumentation, synchronization problems of measurements in remote end of fault usually caused by the communication channel distortion, unknown fault resistance influence for some of the algorithms that needs an estimate fault resistance, the influence on power system protection from distributed generations and the difficulty brought by the unbalanced system in distribution network [2] [5] [6]. Furthermore, different types of fault analysis methods generally have their preferred scenarios, which leads to the necessity of utilizing specified or multiple methods for better understanding of the faulted section.

In this paper, fault location related techniques were discussed in three aspects, namely methods involving travelling wave analysis techniques, methods involving fault voltage and current analysis and fault location algorithm for underground distribution system.

### 1.2.1. METHODS INVOLVING TRAVELLING WAVE ANALYSIS TECHNIQUE

Travelling waves techniques utilize the theory of wave propagation through transmission line with the help of modern devices that are capable to harness the high frequency transient wave. Since the fault wave travels at a known speed that is only dependent on the material of the power line [7], the fault location can be calculated through wave analysis. It requires high-quality measurement equipment to capture and record the fault wave and synchronized measurement with GPS technology for wave analysis procedure [6].

High frequency transients and wavelet analysis specifically uses the transients generated by the fault to find out the fault lateral and further analysis on wavelet coefficient to pinpoint the fault location on the highlighted lateral [6]. This method not only identifies the faulted lateral but also has excellent robustness to the initial phase angle and network load.

Distributed generations bring a little more challenge to power system protection. Travelling wave analysis technique shows its strength in this type of scenario due to its insensitive to load and initial phase influence when locating the fault. Since this method is already highly hardware demanded, utilizing deep communication among power system protection devices, it can deal with the fault condition considerably efficiently by correctly isolating the fault area without either damaging the power system or limiting the potential benefit from the distributed generation systems [5].

Article [8] presents an idea for calculating transmission parameters including positive sequence parameters, temperature and sag using voltage and current gathered by Phasor Management Unit (PMU), which is considered to be crucial inputs for power system analysis. The algorithm in this article utilizes non-linear optimal estimation theory with a capability of indicating and removing bad data, reducing measurement error and increasing the estimation accuracy dramatically.

However, majority of current travelling wave methods only focus on a single transmission line, therefore it can be less sensitive and reliable when one of travelling wave fault locator became malfunctional. One solution is adding record of the fault wave arriving time of each measurements in a big region of the grid since the fault wave would travel quite far distance. The fault estimate location can be successfully calculated and remains accurate. Rather than depending on measurement from the both side of the fault condition line, using more data from other substations makes this method more reliable [9].

### 1.2.2. METHODS INVOLVING FAULT VOLTAGE AND CURRENT ANALYSIS

Fault location estimation algorithms with fault voltage and current analysis, as one of the earlies developed power system protection techniques, are the mostly widely applied in the field. Generally, this kind of algorithms use voltage sags created by the faults, sometimes current data instead. Additionally, comparing to travelling wave analysis, which requires excellently synchronized data, unsynchronized data can be used to locate the fault potentially. This kind of methods can be divided into two major types, direct circuit analysis relying on solving polynomial equations and analysis with system impedance matrices with known system impedance in most cases.

Circuit analysis consisting of KVL, KCL and Newtown- Raphson iteration, plays an important role in general fault analysis. With the help of such, an algorithm that only uses local end voltage and current data can be achievable [10]. In this algorithm, local pre-fault data and data during the fault are used and through circuit analysis equations were achieved to solve the unknown fault location. This algorithm is considered especially useful on ultra-high voltage un-transposed parallel transmission lines comparing to other conventional algorithms.

Accurate time domain algorithm methodology utilizing raw sampled data from terminals could provide estimation with zero theoretical error and no influence from the fluctuation of system frequency [11]. This algorithm utilizes differential equations of faulted condition and pure-fault networks to determine fault location.

An algorithm for radial and non-radial ungrounded power distribution systems was developed in [12] aiming for accurate, generalized and robust solution for fault location estimations

A new method for single-ended fault for overhead distribution network based on superimposed phase signals and special filtering techniques for fundamental phasor extraction utilizes interactive estimates equivalent admittance matrices, resulting an excellent robustness to load and remote source capacity [13].

Method for unbalanced distribution network using fundamental frequency phasors assuming the network parameters and topology are known can be highly efficient according to a field test in Brazil. This method allows local measurement data and network information contributes together to an accurate fault estimation regardless of the complexity of local distribution network. [4]

The authors in [14] utilized current measurements from other branches in un-faulted sections filling the gap where there was a lack of measurements on faulted line which conventional fault analysis required. With bus impedance matrix technique accompanied, this fault analysis method achieved relatively good accuracy and decent robustness to bad measurement influences in simulation studies. An algorithm introducing optimal fault-location estimator to enhance the accuracy and reliability of fault location estimations for distribution system with distributed generators was demonstrated in [15] where chi-squared test was implemented to pick out corrupted data。 Article [16] mentioned a fault detection scheme concept cooperating with auto-restoration system involving load forecasting and restoration time reduction logic to improve transmission line reliability. This collaboration work proves to reduce fault restoration time significantly and totally preventing unnecessary outage by taking good use of load trend forecasting and latest also practical techniques.

A method to determine transmission line parameters of electric power grid was presented in [17]. Adaptive software was developed in this paper that is capable to eliminate the tedious and error-prone manually importing data and provide a tool to streamline and update line parameters in databases.

Technique using Unsynchronized data from both side of line allows poor synchronized or unsynchronized protection system to participate together to generate the estimation of fault location [18]. Solving some realistic problem in the field including communication channel fail or protection system failure where little digitalization was applied in some area, which could lead to a theoretically unsolvable situation for most other techniques. Most fault location algorithms require line parameters, but in some cases it could be unavailable or corrupted. [19] introduces a method that needs no pre-fault information or line parameters with shunt capacitances considered, meaning only the voltage and current during the fault are required. This new approach not only gives quite accurate results for both balanced and unbalanced faults.

A technique to manipulate bus voltage for a photovoltaic system is introduced in [20], taking the advantage of performance matrix and reactive power controller to mitigate the fluctuations and improve the overall performance in distribution system with photovoltaic generation.

### 1.2.3. METHODS INVOLVING DIRECT CIRCUIT ANALYSIS

Circuit analysis is a fundamental tool for any type of fault location methods however deriving fault location by solely mastering circuit analysis should be considered remarkable. Complex double fed distribution systems were taken into consideration in [21]. Current and voltage vectors were utilized in this article, as well as taking the advantage of topology concept of distribution systems. Without sacrificing considering unbalanced systems, the developed algorithm employed greatly simplified procedures and involved voltage and current estimations. Apparent impedance approach method was used in [22]. The disturbances of currents and voltages caused by the fault were harnessed and distribution system under unbalanced condition was discussed as well as different phases. This method naturally comes with a remarkable error diminishing feature considering loads forecasting and fault resistance estimation.

### 1.2.4. FAULT LOCATION ALGORITHM FOR UNDERGROUND SYSTEM

Fault location estimation faces more challenges in underground distribution system because the existence of huge shunt capacitance of the line, which can lead to considerable errors in fault location. A method using iterative procedure to find the fault location with data from primary end assuming the system is operating in balanced condition. [23]. One other method to conquer this problem is to iteratively compensate all the capacitance [24]. Ref [25] demonstrates a fault location algorithm with direct circuit analysis where no lateral or tapped loads, only using data gathered at the sending end of the line. An underground cable equivalent circuit model was built with the aid of boundary condition concept which gives this method a promising potential to locate faults on power line cable with multiple sections. The method presented in [19] could reduce the iterative steps for grounded system, at the same time utilizing bus impedance matrixes.

To conclude literature review, existing fault location methods are demanding considerably on iterations upon circuit analysis procedure, targeting on short circuit analysis or voltage and current calculations. This paper demonstrates a method to further introduce the benefits to ungrounded overhead distribution systems with short circuit faults.

# Chapter 2 FAULT LOCATION METHODS FOR UNBALANCED DISTRIBUTION SYSTEMS

This chapter introduces the methodology behind the fault location method. Firstly, the concept of ‘Node’ and bus impedance are going to be introduced along with an overall procedure to calculate fault location. Secondly, the procedure to formulate the pre-fault and during fault bus impedance matrix will be presented as well as according notations used throughout this paper. This part mainly focuses on driving point impedance and transfer impedance since they are the only variables containing the unknowns. Thirdly, it will be introducing bus voltage during faults which is essential to format the relationship between the unknown fault location, fault resistance and other known data such as system parameters and local measurements. Lastly, the procedure to determine fault location single-phase line, two-phase line and three-phase line will be demonstrated. The derivation steps to determine some components involved will be shown in Appendix 5,6.2,6.3. The method described here are from [3] and the objective of the study is to examine the performance of the method without assuming the fault type.

## 2.1. FUNDAMENTALS OF ALGORITHM

It has been assumed that information on generator, namely line currents and line to ground voltages could be measured by local measurements and a neutral point of the source can be achieved. This algorithm tends to find the fault location through such measurements.

A bus, also called bus bar, is a huge conductor physically connecting several power lines with an identical electric potential. In a three-phase electric distribution system it normally comes with one to three phases as the circuit indicates. The term ‘Node’ will be used to indicate a phase joint connection point [26]. Since multi-phase buses normally are simplified as one in circuit drawing, in this paper one of bus phases is represented by the terminology “Node”.

Bus impedance matrix illustrates the inter-impedance between different nodes, can also determine the relation between voltages and current injections throughout the system.

Introducing fault nodes in the system impedance matrix aids to find the fault location. Fault node divide the faulted line into two segments. Utilizing voltage and current injection information gathered from substations, new system impedance matrix with fault nodes can be derived. Generally, fault location combined with fault resistance, impedance matrix and current injection form a solid relationship with the node voltages, from which the fault location can be derived.

The need for fault type classification is eliminated by assuming LG fault on single-phase line, LLG fault on two-phase line and LLLG fault on three-phase line. The algorithm would adaptively generate nearly infinite fault resistance to represent a non-faulted node.

## 2.2. FAULT LOCATION METHOD FOR DISTRIBUTION SYSTEMS

Assuming the fault locates at a section in a distribution system as shown in Figure 2.1, a typical distribution system. Such system consists of three-phase, two-phase and one-phase laterals and loads. A fault location method would be generated merely using local information from CTs and VTs at substations and system information.

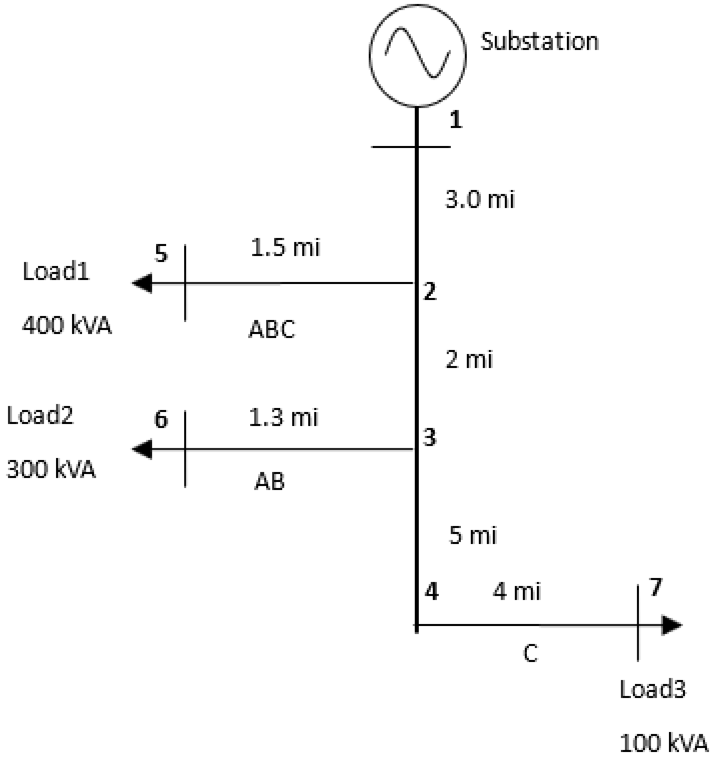

**Figure 2.1Sample Unbalanced Distribution System**

The fundamental idea is to modify a bus impedance matrix of pre-fault condition into one during fault condition, which will be shown with details in this section. Different number of fictitious nodes are added to the pre-fault matrix on fault location considering the number of phases that the fault lines contain, namely one-phase line, two-phase line or three-phase line fault. After adding in fictitious nodes, the transfer impedance and driving point impedance need to be expressed in form of fault location. Voltages at local station nodes then can be represented as a function of bus impedance matrix and current injections at the substation. With the known node voltages at substations, fault location can also be in part of expression consisting of transfer impedance, driving point impedance, node voltages and fault resistance. Then fault resistance and fault location can be calculated. It clarifies that the fault location can be determined by using local measurements and known system information in this algorithm. Note to mention that there were very few iteration calculations involved in this method, which is one of the most distinguishing features of this algorithm. In this article, all voltages and currents refer to 60Hz frequency phasors.

The bus impedance matrix shall be based on three-phase domain due to the nature of the innate unbalances of distribution system. Consequently, the fault location method would not suffer from unbalances in distribution system.

Equivalent impedance models had to be created to equivalently replace loads to simulate load variation impact and load compensation, which will be discussed in case studies.

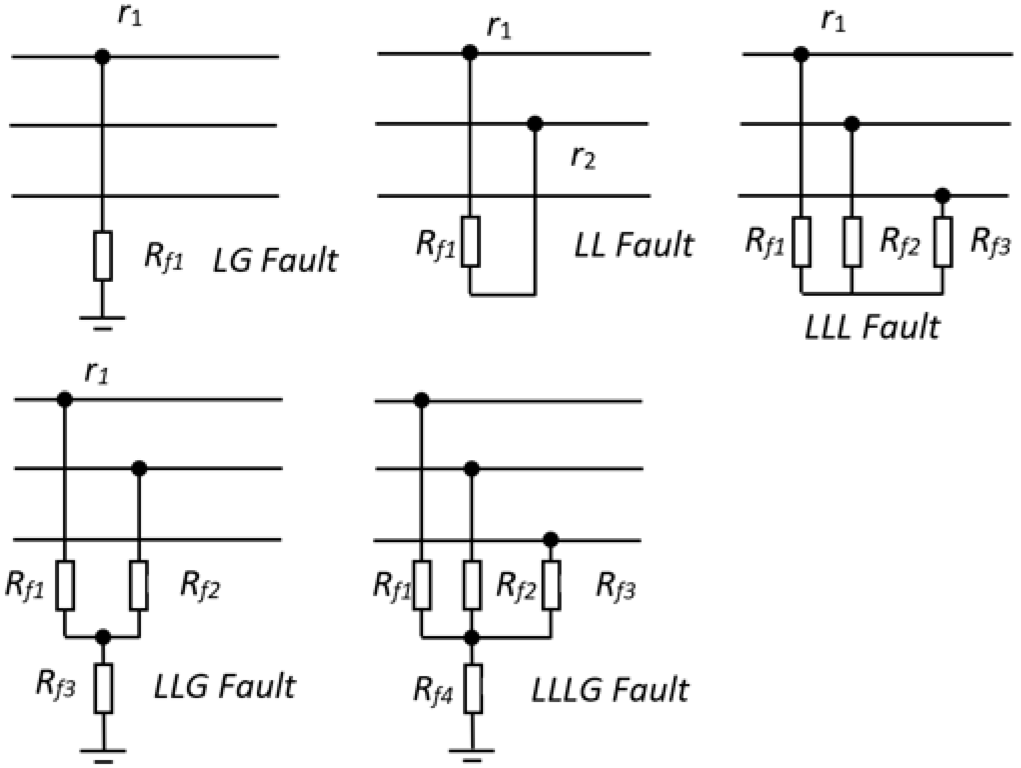

**Figure 2.2Demonstrations of Fault Types**

Different fault types are represented in Figure 2.2 [3], consisting of line to ground faults(LG), line to line faults(LL), line to line to ground faults(LLG), line to line to line faults(LLL) and line to line to line to ground faults(LLLG). It is important to note that there could be all kinds of fault on a three-phase feeder or lateral. For two-phase laterals, only LG, LL, LLG fault could take place. While on a single-phase feeder, there could only be and only be just one kind of fault occurring which is LG fault. Fault location methods will be discussed regarding different type lines respectively.

## 2.3. TRANSFER IMPEDANCE AND DRIVING POINT IMPEDANCE

Figure 2.3 represents the specific section of distribution system where the fault occurs [3]. It is capable to illustrate all kinds of faults. As for notation declarations, please refer to next section.

Throughout the derivation, pre-fault impedance matrix [$Z_0$] consists of *n* rows and *n*

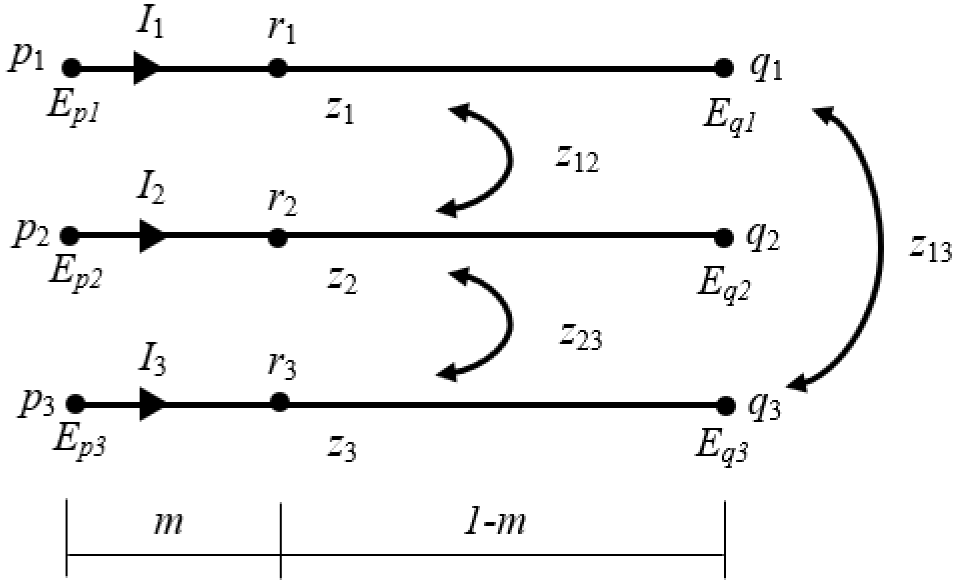

**Figure 2.3 Faulted Section of Distribution System**

columns elements which are identical to the up left part of during fault impedance matrix [$Z$]. The rest part of [$Z$] contains transfer and driving point impedances concerning fault nodes.

$$Z_{kr_i} = B_{ki} + C_{ki}m,\ i = 1, 2, 3 \tag{1}$$

$$Z_{r_ir_t} = A_{it_0} + A_{it_1}m + A_{it_2}m^2,$$
$$i = 1,2,3, t = 1,2,3, and\ i\ \neq t \tag{2}$$

$$Z_{r_ir_i} = A_{ii_0} + A_{ii_1}m + A_{ii_2}m^2,\ i = 1,2,3 \tag{3}$$

where

| | |
|---|---|
| $Z_{kr_i}$ | transfer impedance between node $k$ and node $r_i$; |
| $Z_{r_ir_t}$ | transfer impedance between node $r_i$ and $r_t$; |
| $Z_{r_ir_i}$ | driving point impedance at node $r_i$; |
| $B_{ki}, C_{ki}, A_{it_0}, A_{it_1}, A_{it_2}, A_{ii_0}, A_{ii_1}, A_{ii_2}$ | constants for a known system |

Note that the expression of transfer and driving point impedances are assigned in a function of fault location to generate a solvable equation for fault location.

A basic relation between node voltage vector and current injections forms the fundamental principle for fault location method. Assuming current injections at any nodes were zero except node $r_i$, the voltage at node $k$ should be the result of $Z_{kr_i}$, the transfer impedance between node $k$ and node $r_i$, multiplying current injection at node $r_i$. According to Superposition Theorem, when there were other nodes with current injections the results would be the sum of individual calculations where zero injections at other nodes were assumed.

## NOTATIONS USED IN Figure 2.3 AND BUS IMPEDANCE DERIVATION

| | |
|---|---|
| $n$ | nodes number in the distribution system without fictitious fault nodes; |
| $p_1, p_2, p_3, q_1, q_2, q_3$ | sample feeder nodes: |
| $z_1, z_2, z_3$ | self-impedance between $p_1$and $q_1$, $p_2$ and $q_2$, $p_3$ and $q_3$ respectively; |
| $z_{12}, z_{23}, z_{13}$ | mutual impedance between phases; |
| $m$ | the ratio between fault distance and length of the section, i.e. p.u. fault distance; |
| $[Z_0]$ | bus impedance matrix of prefault system without considering fault nodes; size of $n$ by $n$; |
| $Z_{0,kl}$ | $k$th row and $l$th column of $[Z_0]$; |
| $[Z]$ | bus impedance matrix of faulted system considering fault nodes; size of $(n+3)$ by $(n+3)$; |
| $[Z_{kl}]$ | $k$th row and $l$th column of $[Z]$; |
| $E_{p1}$，$E_{p2}$，$E_{p3}$，$E_{q1}$，$E_{q2}$，$E_{q3}$ | voltages at node $p_1$, $p_2$, $p_3$, $q_1$, $q_2$ and $q_3$, respectively; |
| $I_1, I_2, I_3$ | currents of branch $p_1$-$r_1$, $p_2$–$r_2$, $p_3$-$r_3$; |

## 2.4. BUS VOLTAGE DURING THE FAULT

Considering a LG fault occurring on phase A at node $r_i$ [3], and assign the node on substation corresponding to phase A as node $k$, according to the definition of transfer impedance, fault node voltage can be derived as

$$E_k = E_{k_0} - Z_{kr_1} I_{f_1} - Z_{kr_2} I_{f_2} - Z_{kr_3} I_{f_3} \tag{4}$$

the voltage difference between pre-fault and during fault at node $k$ is thus

$$\Delta E_k = E_{k_0} - E_{k_0} = -Z_{kr_1} I_{f_1} - Z_{kr_2} I_{f_2} - Z_{kr_3} I_{f_3} \tag{5}$$

where

| | |
|---|---|
| $E_k$ | node voltage during fault at node $k$; |
| $E_{k0}$ | node voltage before fault at node $k$; |
| $\Delta E_k$ | voltage difference between pre-fault and fault condition; |
| $I_{fi}$ | fault current from node $i$ to fault resistance; |

Same equations apply to two-phase or three-phase fault where every node voltages are just simply derived individually and then bundled up a vector form. Fault location algorithm would be around equation (5)

According to KVL law, the current of feeder or lateral line can be derived from the product of the inverse of feeder or line impedance matrix and node voltage difference, which ends up being an equation with fault location. This relationship will be very handy for fault analysis.

## 2.5. DERIVING FAULT LOCATION

### 2.5.1. FOR SINGLE-PHASE LINE (LG)

Considering a LG fault occurring on phase A at node(r1) [3], the fault current $I_{f1}$ is expressed as

$$I_{f_1} = \frac{E_{r_1 0}}{Z_{r_1 r_1} + R_{f_1}} \tag{6}$$

where

| | |
|---|---|
| $E_{r10}$ | pre-fault voltage at node $r_1$; |
| $R_{f1}$ | fault resistance. |

In order to express $\mathrm{E}_{\mathrm{r}_1 0}$ in terms of fault location, assign the faulted section with two nodes, $p_1$ and $q_1$, at each end and $\mathrm{E}_{\mathrm{r}_1 0}$can be derived as

$$E_{r_1 0} = E_{p_1 0} - m(E_{p_1 0} - E_{q_1 0}) \tag{7}$$

Where $E_{p_1 0}$ and $E_{q_1 0}$ are pre-fault voltages at node $p_1$ and $q_1$

$I_{f1}$ can be expressed as

$$I_{f_1} = \frac{-E_{p_1 0} + m(E_{p_1 0} - E_{q_1 0})}{Z_{r_1 r_2} + R_{f_1}} \tag{8}$$

For this single-phase LG fault, assign the node of substation on phase A as $k_1$,

$$\Delta E_{k_1} = -Z_{k_1 r_1} I_{f_1} = \frac{-E_{p_1 0} + m\left(E_{p_1 0} - E_{q_1 0}\right)}{Z_{r_1 r_2} + R_{f_1}} Z_{k_1 r_1} \tag{9}$$

According to the transfer and driving point impedance derivation in (7) (8) (9), rewritten as,

$$\Delta E_{k_1} = \frac{-E_{p_1 0} + m\left(E_{p_1 0} - E_{q_1 0}\right)}{A_{11_0} + A_{11_1} m + A_{11_2} m^2 + R_{f_1}} \left(B_{k_1 1} + C_{k_1 1} m\right) \tag{10}$$

Where only $m$ and $R_{f1}$ are the only unknown parts in this quadratic equation for fault location $m$. Since $m$ is not a complex number, we can solve this by generating real and imaginary part of this equation where $R_{f_1}$ can be solved and then *m*.

The voltage change during fault could be considerably less comparing with current change if the fault resistance was large, therefore a technique to avoid using direct voltage measurements was utilized in this paper.

This technique involves current change [$\Delta I$] and local impedance matrix [$z_s$]. Voltage change is given as

$$[\Delta E] = [z_s][\Delta I] \tag{11}$$

Take the advantage of good amount of change in current measurements to generate more accurate fault location result. This will be further demonstrated in case studies. One advantage of avoiding voltage measurements is the improved fault location estimation robustness to voltage measurement errors.

### 2.5.2. FOR TWO-PHASE LINE FAULT (LG, LL, LLG)

The fault current passing the fault resistances should be

$$\begin{bmatrix} I_{f_1} \\ I_{f_2} \end{bmatrix} = \begin{bmatrix} Z_{r_1r_2} + R_{f_1} + R_g & Z_{r_1r_2} + R_g \\ Z_{r_1r_2} + R_g & Z_{r_2r_2} + R_{f_2} + R_g \end{bmatrix}^{-1} \cdot \begin{bmatrix} E_{r_10} \\ E_{r_20} \end{bmatrix} \tag{12}$$

where

| | |
|---|---|
| $R_g$ | fault resistance to the ground; |
| $R_{f_1}$, $R_{f_2}$ | fault resistance on corresponding fault nodes to ground fault resistance. |

Consider the voltage changes at substations node $k_1$ and $k_2$,

$$\Delta E_{k_1} = -Z_{k_1r_1} I_{f_1} - Z_{k_1r_2} I_{f_2} \tag{13}$$

$$\Delta E_{k_2} = -Z_{k_2r_1} I_{f_1} - Z_{k_2r_2} I_{f_2} \tag{14}$$

To find the roots for these equations, Newton-Raphson method shall be utilized at this point.

By dividing above into 4 equations containing real and imaginary elements of fault location in each.

### 2.5.3. FOR THREE-PHASE LINE FAULT (LG, LL, LLG, LLL, LLLG)

For three-phase line fault, fault location is obtained by creating a relationship between fault current and fault node voltages as shown

$$\begin{bmatrix} I_{f_1} \\ I_{f_2} \\ I_{f_3} \end{bmatrix} = \begin{bmatrix} Z_{r_1r_1} + R_{f_1} + R_g & Z_{r_1r_2} + R_g & Z_{r_1r_3} + R_g \\ Z_{r_1r_2} + R_g & Z_{r_2r_2} + R_{f_2} + R_g & Z_{r_2r_3} + R_g \\ Z_{r_1r_3} + R_g & Z_{r_2r_3} + R_g & Z_{r_3r_3} + R_{f_3} + R_g \end{bmatrix}^{-1} \cdot \begin{bmatrix} E_{r_10} \\ E_{r_20} \\ E_{r_30} \end{bmatrix} \tag{15}$$

And voltage change due to the fault at node $k_1$, $k_2$ and $k_3$ of substation is

$$\Delta E_{k_1} = -Z_{k_1r_1}I_{f_1} - Z_{k_1r_2}I_{f_2} - Z_{k_1r_3}I_{f_3} \tag{16}$$

$$\Delta E_{k_2} = -Z_{k_2r_1}I_{f_1} - Z_{k_2r_2}I_{f_2} - Z_{k_2r_3}I_{f_3} \tag{17}$$

$$\Delta E_{k_3} = -Z_{k_3r_1}I_{f_1} - Z_{k_3r_2}I_{f_2} - Z_{k_3r_3}I_{f_3} \tag{18}$$

The pre-fault voltage data of the system was harnessed and complex equations were formed and solved using Least Square (LS) method [29]., the fault location and fault resistance could then be calculated.

While applying this method, all sections of line in the proposed model were attempted. A list of possible fault locations was obtained and analyzed to narrow down and locate the actual fault location based on available information such as consumer complaint reports and weather condition records.

# Chapter 3 SIMULATION MODEL

This section of the paper presents the developed simulation model and key supportive components in evaluation studies to test the discussed fault location methods. The tool used in this paper to aid testing was Simulink SimPowerSys module in Matlab 2017a [30]. A four-bus, 12.47kV, 60Hz unbalanced distribution system, as Figure 3.1, is built. Three-phase, two-phase and single-phase loads and laterals were implemented with a 0.9 lagging power factor for all loads. For per unit system, 12.47kV and 1MVA were chosen to be base value for calculations.

In order to generate inputs for the fault location algorithm in a large variety and furthermore to visualize comparisons on fault locations between model inputs and algorithm results, simulations had been run numerous times. Simulink SimPowerSys has a great potential to simulate a huge variety of electrical power system evaluations and monitor the associating behaviors. Simulink core modules also help significantly due to its powerful mathematical functionalities. Key features which contribute greatly in simulation procedure of this paper will be discussed in this chapter.

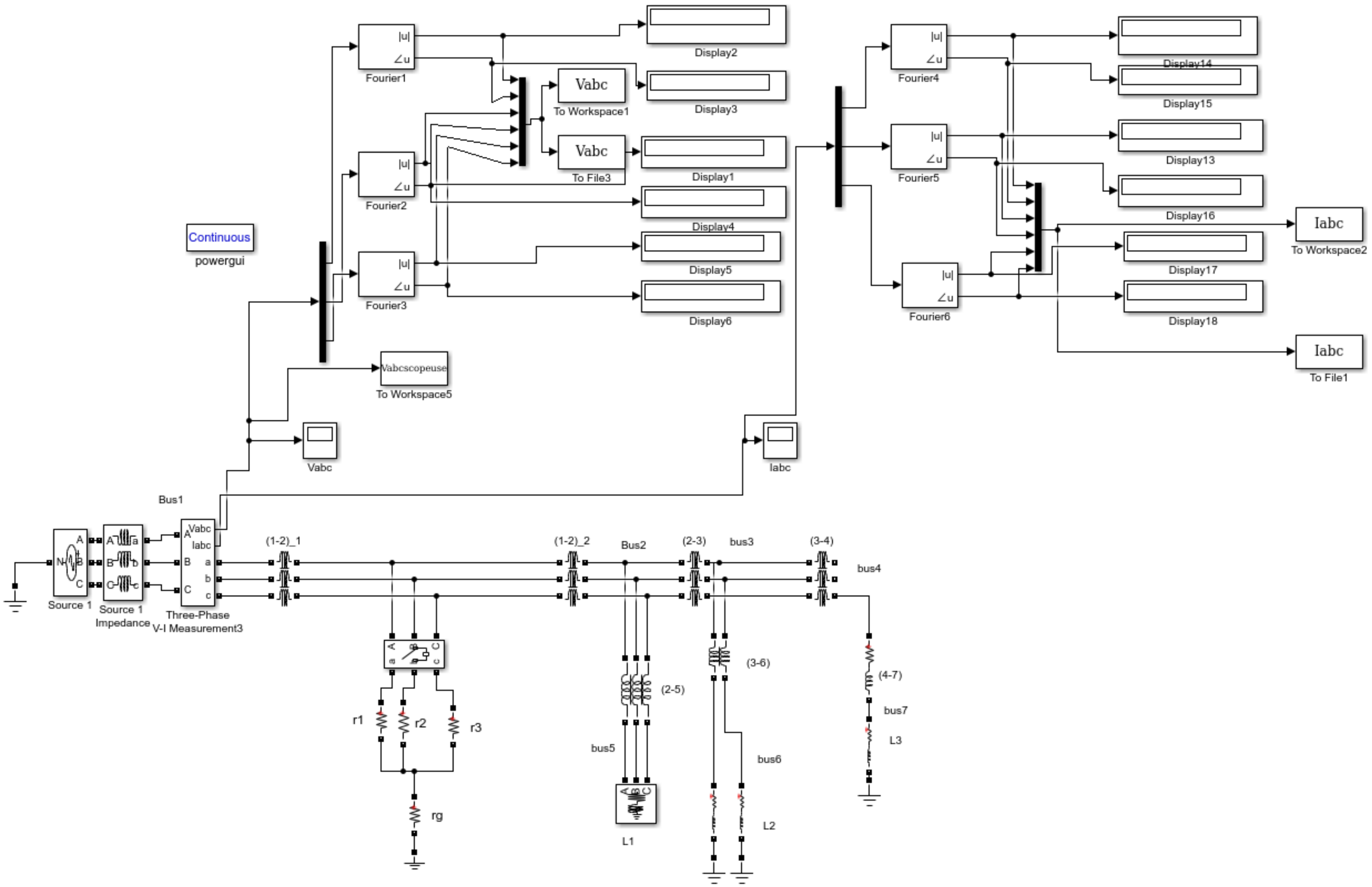

Figure 3.1 Sample Distribution System Diagram

## 3.1. DATA COLLECTING AND RECORDING SECTION

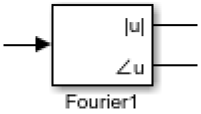

**Figure 3.2 Model of Fourier Transformation Block**

The overview of the fictitious distribution system model is shown Figure 3.1. The lower left part of the model is the main body of the fictitious distribution system. And the upper part plays a role of data collecting and recording throughout the simulation. Fast Fourier Transform (FFT) module has been harnessed to extract instant voltage and current phasers, which were recorded and saved into .m file for further analysis and fault location calculations. The algorithm requires the voltage and current data stream taken before and after the fault. In this paper FFT modules were used to measure voltage and current at fundamental frequency(60Hz). Note that there should be some inaccurate data flow during the first cycle of the simulation. Extra care had been taken to avoid using collected data taken during the first cycle or using non-constant data if newer version of FFT modules in Matlab 2017a were used, Figure 3.2. The Simulink SimPowerSys includes a more advanced FFT module in 2017 that is capable of holding the output at a user determined value for the first cycle of an Fourier Transformation procedure, in this paper it was set to be zero magnitude and zero phase angle. Voltage and current data was loaded to local files after collected from the model. Note in this specific format of file, the pattern of voltage and current data and respective phasor angles saved has a specific form which needs to be matched when extracting data from those files during fault location estimation procedure.

## 3.2. MODELLING THE DISTRIBUTION SYSTEM SECTION

A four-bus fictitious unbalanced distribution system was built in Simulink consisting of a three-phase voltage source, three-phase feeders, loads with three-phase, two-phase and single-phase, and corresponding laterals. A fictitious three-phase ground fault was added into the model made up of programable three phase breakers, according fault resistances and modified feeder/lateral impedance. Define the fictitious fault as Fault Unit. The modified feeder/lateral impedances involve the fault location parameter *m* that makes this specific part of line into two parts with a ratio of *m*/(1-*m*), with *m* being manually assigned every run regarding the test being run. As Figure 3.3, three-phase impedance (1-2)_1 and (1-2)_2 are modified with multiplier *m* or (1-*m*) in both unit inductance matrix and resistance matrix respectively.

By adjusting the Fault Unit as the fault type and fault resistances indicate accordingly, faults with different type, fault type, fault location and fault resistance can be revised. In this research, all the loads are built as equivalent Z models or equivalent impedance models

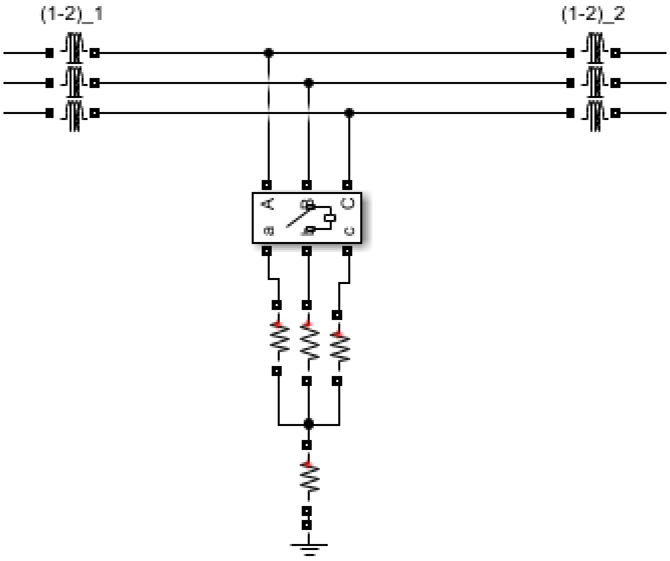

**Figure 3.3 Fault Unit Section in Simulink Model**

for convenience. CT and VT at local substation are represented by three phase voltage and current measurement block placed between source impedance and line impedance to simulate the realistic arrangement in power plant. Measurements were harnessed and delivered to data recording part of the model for further process.

# Chapter 4 CASE STUDIES

In this section, the results of simulations were presented in Figure 3.1.. Different case studies had been gone through including analysis upon fault location estimation accuracy, load variation impact on estimations of fault location and robustness test on voltage and current measurement errors. Some specific algorithm settings will be mentioned. Simulation procedure and Waveforms of voltage and current verse time during several types of faults, fault locations and fault resistances were generated. Using Matlab SimPowerSys in this section to simulate distribution system faults, key characteristics of this algorithm were revealed. During the simulation procedure, Discrete Fourier Transform (DFT) blocks were used to assist the algorithm, where phasers with fundamental frequency 60Hz were harnessed both voltage and current.

As shown in Figure 3.1, a fictitious distribution system with three-phase feeders and three-phase, two-phase and single-phase lateral and loads was implemented into the simulation.

A few assumptions were made as follow. System voltage is based at 12.47 kV and all three loads are assumed to have 0.9 power factor. The laterals have the same number of phases as the load they are connected to. Last not the least, the system impedance information is as Appendix 6.4 including feeder impedances, later impedances, load impedances and source impedances.

As for Matlab iteration procedures, fault location starts at 0.5 p.u., interphase fault resistances have a 1 Ω starting value and 10 Ω for ground resistances. No more than 10 iterations were processed using this algorithm in Matlab and the concept of error was implemented as

$$\%error = \frac{Acutal\ Location - Location\ Estimation}{Total\ length\ of\ main\ feeder} \times 100 \qquad (19)$$

A model of distribution system was created in Simulink and the pre-fault impedance matrix can be generated at the same time in Matlab for fault location algorithm. Different types of faults were imported into Simulink model and then data were recorded and saved in .mat files.

## 4.1. TEST RESULTS

### 4.1.1. SIMULATION RESULTS

Simulation generates the input for the fault location algorithm, meanwhile it is capable to present some key features on the specific types of faults, which helps greatly in locating the fault section and fault type through reverse interpretation. For example, a voltage sag generally indicates a short circuit was created unexpectedly, in which case grounded or interphase connection was built up somehow [5]. A current drop may be introduced by a breaker tripping or an open circuit situation took place somewhere along that cable [4]. In this part of the article, several types of behavior of voltage and current during a fault will be presented and discussed.

Let us assign a short circuit single-phase line to ground fault occurring between bus 1 and bus 2, per unit fault location *m* being 0.2, phase fault resistance being 10 Ω and ground fault resistance being 50 Ω .

Figure 4.1 shows the three phase voltages during a phase A LG fault. It is evidence that there is a voltage drop on phase A at time 0.05s also known as the third cycle of the signal which is the time to trip the breaker that simulates faults. Figure 4.2 is the graph for three-phase current during phase A LG fault. It is apparent that current change is much more dramatical than voltage change in this case. As a matter of fact, this rule applies to all the kinds of faults and furthermore being part of the reason why current measurements are used in fault location estimation algorithm in this paper instead of voltage measurements. Figure 4.4 shows the voltages at substation during a LL fault at section 1-2 with a totally fault resistance being 20 Ω, indicating that the voltage of phase A raised up and voltage of phase B dropped down clearly. Current change under this condition is shown Figure 4.3, where the currents of both phase A and phase B were increased significantly. For LLG fault condition, other parameters remained the same except phase A fault resistance being 2 Ω, phase B fault resistance being 4 Ω, and ground fault resistance being 13 Ω. As Figure 4.5 and Figure 4.6 presents, there have been a voltage drop on phase A and phase B accompanied with line current increases at the same period time. Noticeably phase voltage and line current of phase B received more changes than phase A in this particular system.

To examine the phase voltages and line currents behaviors under three phase LLL fault condition, phase C phase fault resistance was added in the fault unit as 5 Ω and ground fault resistance was eliminated. Simulation results of three phase voltage and current were obtained in Figure 4.7 and Figure 4.8. It is evidential that all three phases have voltage drops and current skyrocketing increases and phase A gained the most amount of changes. Considering the unequal fault resistances in each phase, the result fits expectations.

As for LLLG fault, ground fault resistance was reintroduced into the model with a resistance value of 10 Ω. In some LLLG fault location estimation cases, each single-phase fault resistance was treated as identical hence LLLG faults can be equivalent to a superposition of three LG faults. In this paper, unbalanced three phase fault resistance were taken into consideration and the discussed algorithm has the ability to solve according problems. Figure 4.9 and Figure 4.10 display the three-phase phase voltage and line current behavior under LLLG fault condition with unequal fault resistances on each phase. The results mostly follow the LLL fault condition result, with an exception of a reduced phase A fault voltage.

Initial simulation result fitted the expectation upon the fictitious distribution system and at the same time demonstrated the ordinary behaviors of typical kinds of faults with visualizations. Greater quantity of simulations had been performed and three-phase phase voltage and current data was saved into .mat files for followed algorithm process.

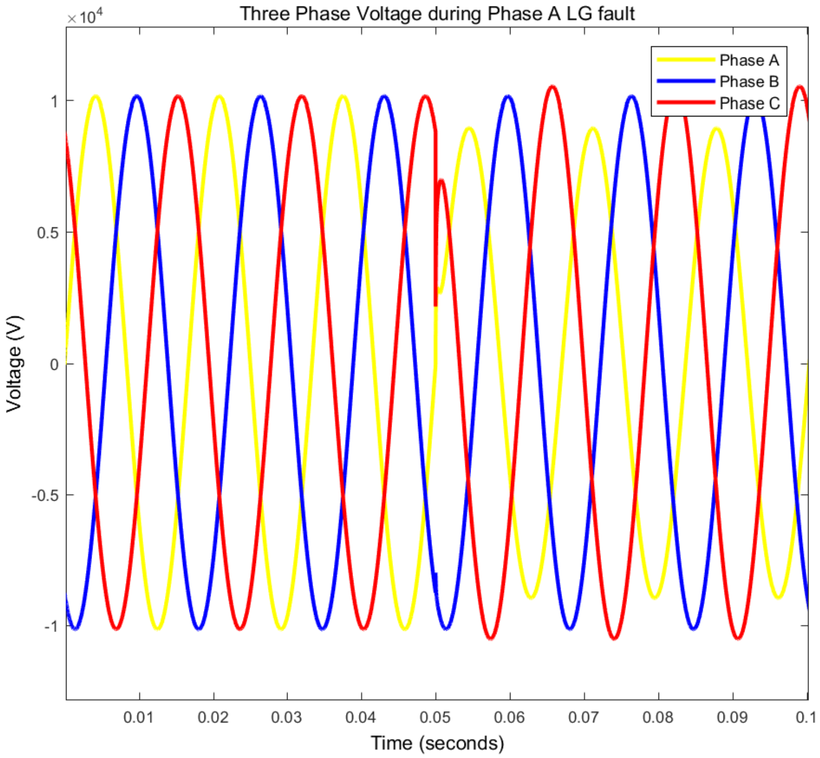

Figure 4.1Three Phase Voltage during Phase A LG Fault

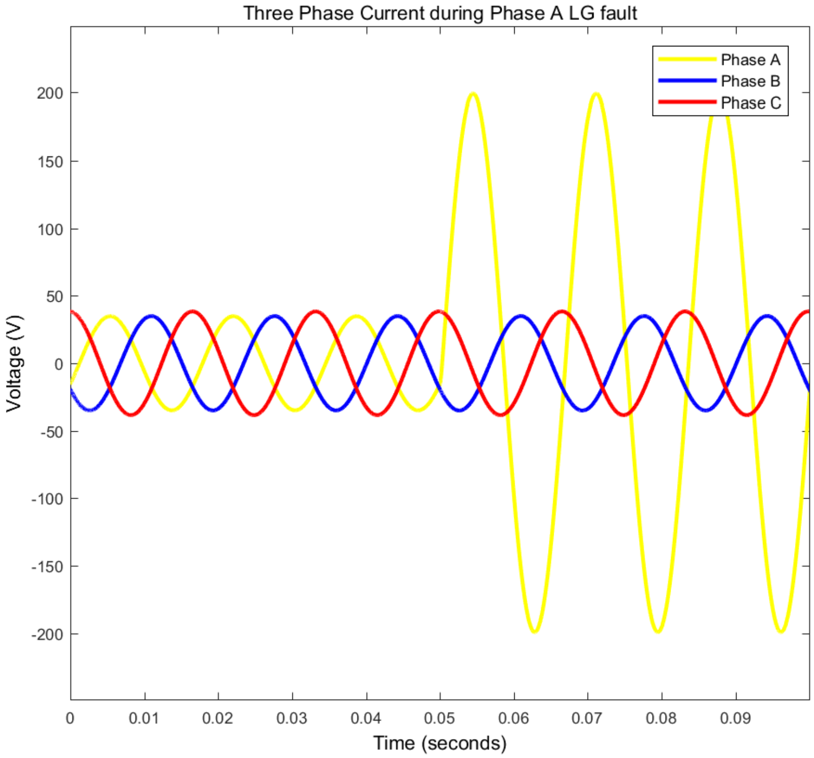

Figure 4.2Three Phase Current during Phase A LG Fault

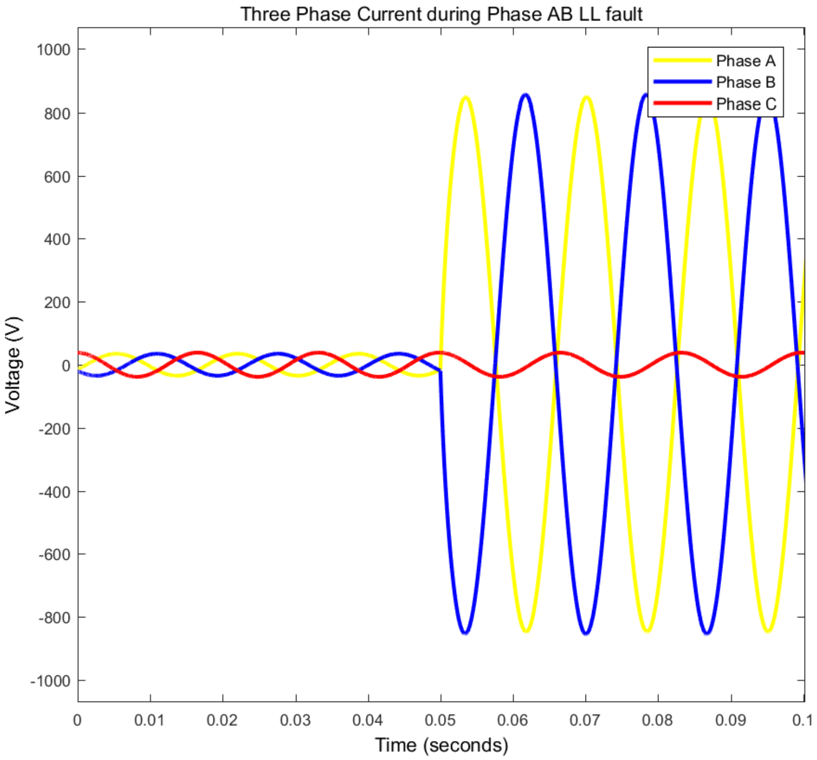

Figure 4.3Three Phase Current during Phase AB LL Fault

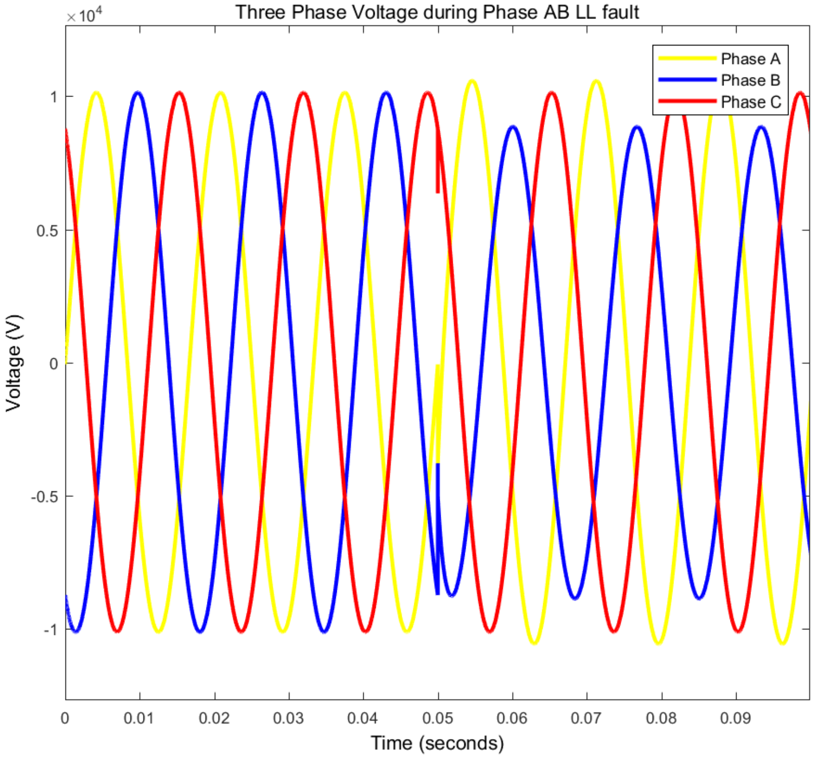

Figure 4.4Three Phase Voltage during Phase AB LL Fault

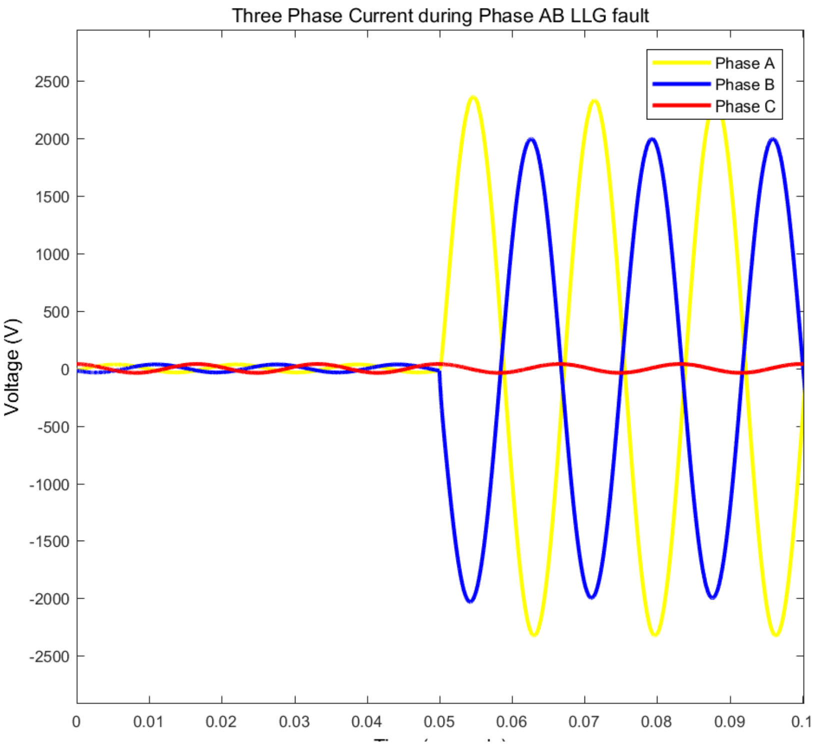

Figure 4.6Three Phase Current during Phase AB LLG Fault

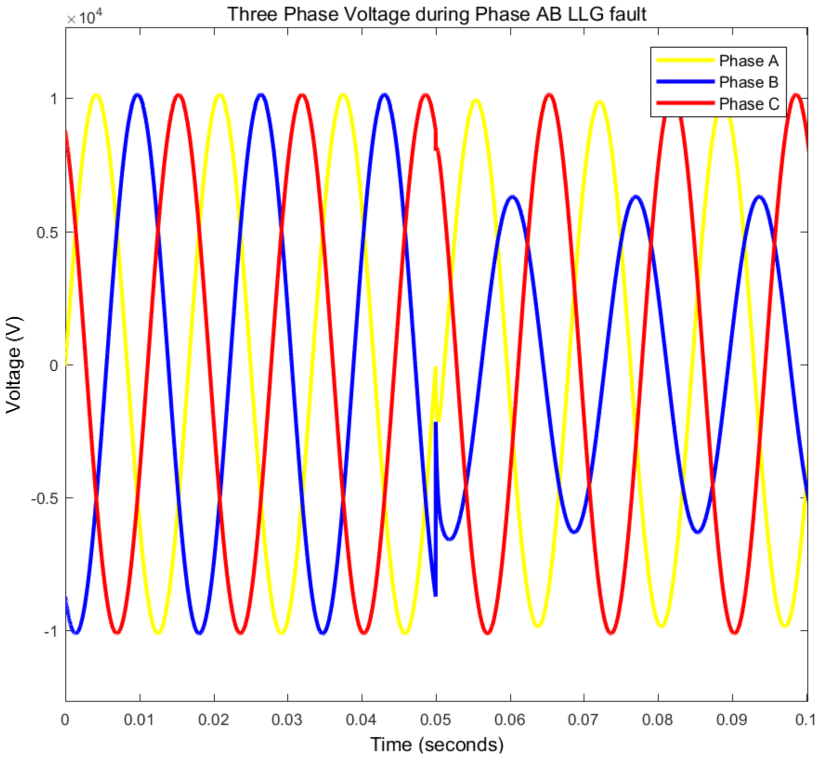

Figure 4.5Three Phase Voltage during Phase AB LLG Fault

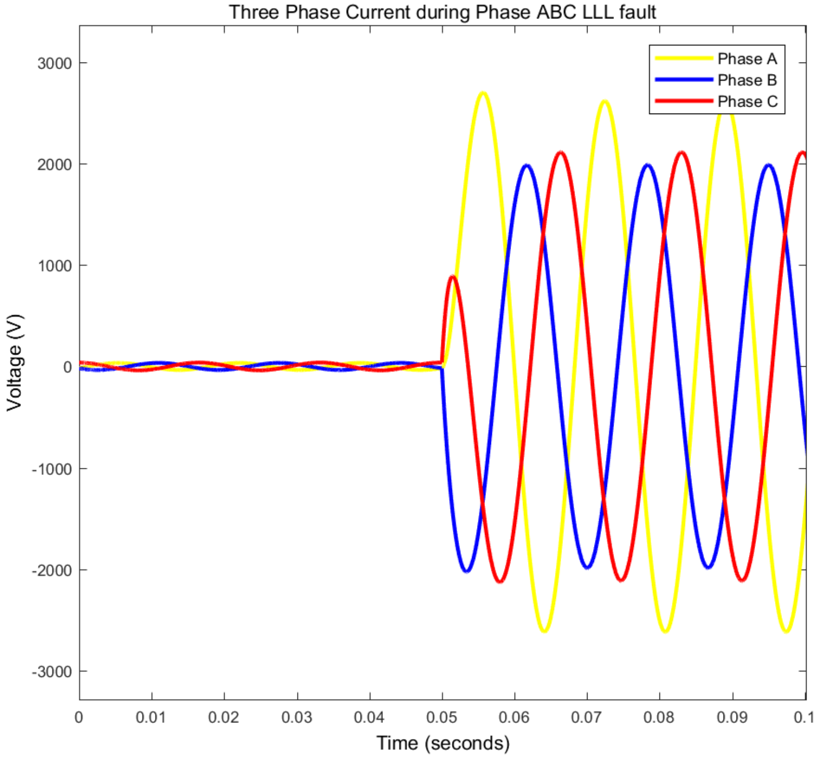

Figure 4.8Three Phase Current during Phase ABC LLL Fault

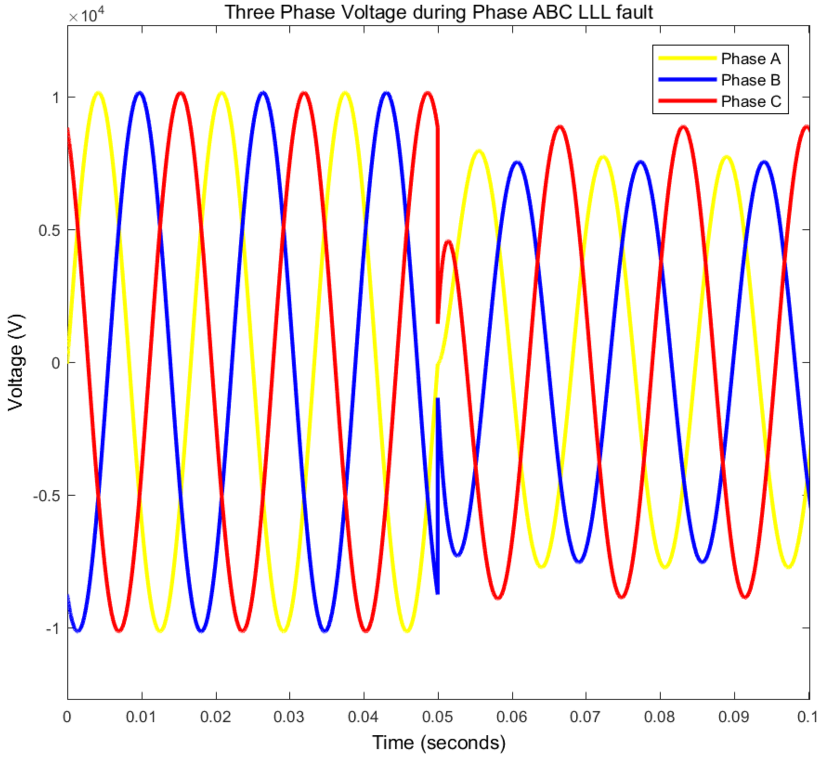

Figure 4.7Three Phase Voltage during Phase ABC LLL Fault

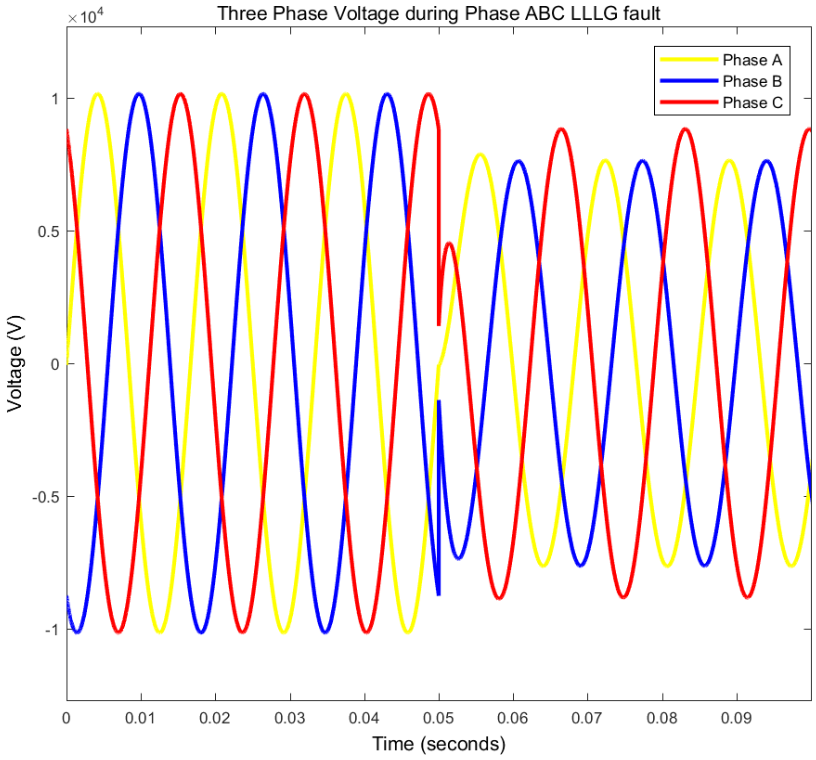

**Figure 4.9Three Phase Voltage during Phase ABC LLLG Fault**

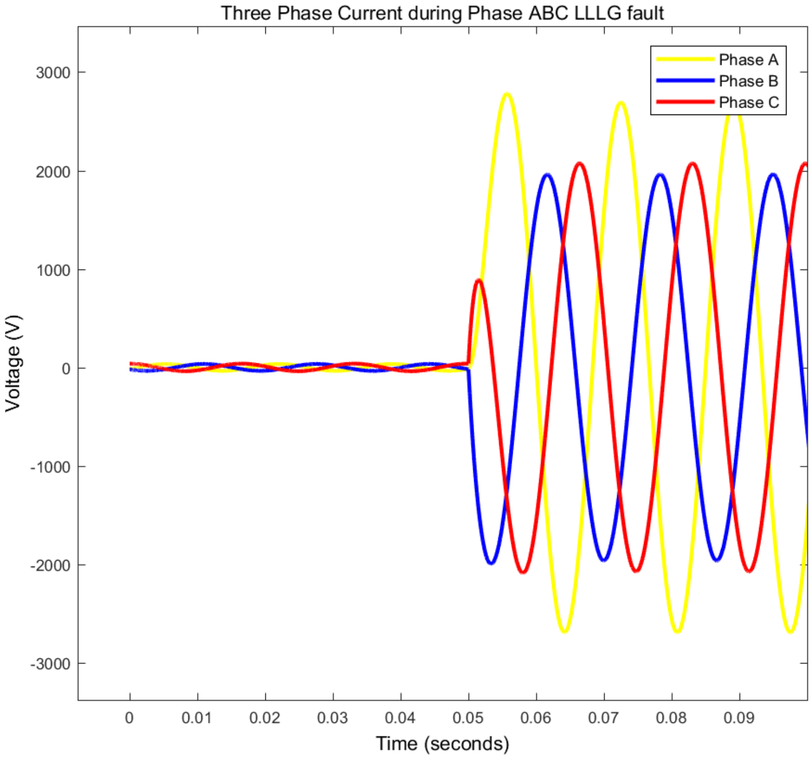

**Figure 4.10Three Phase Current during Phase ABC LLLG Fault**

### 4.1.2. ALGORITHM OVERALL PERFORMANCE

In this part of the paper, the test results of fault location algorithm on faults with different fault type, fault resistances and fault locations were shown in detail in Table 4-1, Table 4-4 and Table 4-3.. Equivalent Z models were used to implement loads into bus impedance matrix following the definition of transfer impedance and transfer impedance. As a result, load variation would lead to a natural inconsistency of this algorithm, which will be discussed in later section.

**Table 4-1 Fault Location and Fault Resistance Estimation Results**

| Fault Section | Fault Type | Fault Location (p.u.) | Fault Resistance (Ω) | F.L Error % | F.R. Estimation (Ω) |
|---|---|---|---|---|---|
| Bus1-2 | LG | 0.2 | 60 | 0.01 | 60.09 |
| | LL | 0.2 | 20 | 0.02 | 20.00 |
| | LLG | 0.4 | [2 4 13] | 0.01 | [2.00 4.00 13.00] |
| | LLL | 0.4 | [2 4 5 ] | 0.01 | [2.00 4.00 5.00] |
| | LLLG | 0.4 | [2 4 5 10] | 0.01 | [2.00 4.00 5.00 10.00] |
| Bus2-5 | LG | 0.7 | 40 | 0.05 | 40.02 |
| | LL | 0.6 | 10 | 0.02 | 10.00 |
| | LLG | 0.6 | [2 3 10] | 0.02 | [2.00 3.00 10.00] |
| | LLL | 0.5 | [2 3 5] | 0.02 | [2.00 3.00 5.00] |
| | LLLG | 0.5 | [2 3 5 50] | 0.02 | [2.00 3.00 5.00 50.00] |
| Bus2-3 | LG | 0.3 | 20 | 0.03 | 20.00 |
| | LL | 0.1 | 8 | 0.02 | 8.00 |
| | LLG | 0.1 | [4 5 40] | 0.02 | [4.00 5.00 40.01] |
| | LLL | 0.4 | [4 5 7] | 0.02 | [4.00 5.00 7.00] |
| | LLLG | 0.4 | [4 4 4 10] | 0.02 | [4.00 4.00 4.00 10.02] |
| Bus3-6 | LG | 0.8 | 40 | 0.06 | 40.00 |
| | LL | 0.9 | 10 | 0.01 | 10.00 |
| | LLG | 0.9 | [5 6 20] | 0.01 | [5.00 6.00 20.00] |
| Bus3-4 | LG | 0.4 | 10 | 0.01 | 10.00 |
| | LL | 0.3 | 2 | 0.01 | 2.00 |
| | LLG | 0.3 | [1 1 10] | 0.01 | [1.00 1.00 10.00] |
| | LLL | 0.5 | [2 3 2] | 0.01 | [2.00 3.00 2.00] |
| | LLLG | 0.5 | [3 4 3 15] | 0.01 | [3.00 4.00 3.00 15.00] |
| Bus4-7 | LG | 0.7 | 20 | 0.02 | 20.00 |

The first four columns indicate the input of the simulation, namely the fault section, fault type, fault location and fault resistances. For LLG, LLL and LLLG faults where different fault impedance between phases could be present, unequal fault impedances have been listed separately. For an instance, the fourth case where it was an LLL fault with a 0.6 fault location, the fault resistances are labeled as [2 4 5] indicating that fault resistances between phases are 2 and 4 ohm and ground fault impedance is 5 ohm. Column 5 and 6 are placed with estimated fault locations and estimated fault impedances respectively, making it straightforward to witness the accuracy of the test results generated in different scenarios.

### 4.1.3. PERFORMANCE UNDER MEASUREMENT ERROR

In Table 4-2, the influence of measurement errors upon estimated fault location are presented. Error of estimated fault location for faults taking place in section 1 to 2 are listed through column 5, 6, 7 and 8, where current measurement errors were set to 0.5%, 1% and voltage measurement errors were set to 0,5% and 1% respectively.  First four column remain the same definitions as in Table 4-1. Current measurement error brings little error in estimated fault location column 5 and 6 shows, fault location errors remain little. In

**Table 4-2 Fault Location Estimations under Measurement Errors**

| Fault Section | Fault Type | Fault Location (p.u.) | Fault Resistance (Ω) | Fault Location Estimation error % | | | |
|---|---|---|---|---|---|---|---|
| | | | | 1% Current Error | 4% Current Error | 1% Current & Voltage Error | 4% Current & Voltage Error |
| Bus1-2 | LG | 0.2 | 60 | 0.02 | 0.02 | 0.02 | 0.02 |
| | LL | 0.2 | 20 | 0.02 | 0.02 | 0.02 | 0.02 |
| | LLG | 0.4 | [2 4 13] | 0.01 | 0.01 | 0.01 | 0.01 |
| | LLL | 0.5 | [1 3 5 ] | 0.01 | 0.01 | 0.01 | 0.01 |
| | LLLG | 0.6 | [4 4 4 10] | 0.01 | 0.01 | 0.01 | 0.01 |

column 6 and 7, voltage and current measurement errors were set to be opposite otherwise they would diminish the overall error. However, in this algorithm voltage measurement error has zero impact on fault location estimation. The rest shows that the demonstrated algorithm has an excellent robustness to current and voltage measurement error.

### 4.1.4. PERFORMANCE UNDER LOAD VARIATION

It is unavoidable to take the algorithm into load variation test since equivalent impedance models were implemented to model the loads. Equivalent impedance method could perfectly demonstrate load under nominal condition, but load does change dramatically during some time of the day in realistic situation. In order to simulate load variation impact, a separate multiplier for equivalent impedance models was introduced in the algorithm Table 4-3 shows the impact of load variation on error of fault location estimation. Column 1 to 4 share the same meaning as in previous tables, and column 5, 6 show the errors of estimated fault location for faults happening in section 1 to 2 while 5% and 10% load variation were implemented into the model. It is evidenced that load variation influences the estimation error in noticeable but acceptable degree.

**Table 4-3 Fault Location Estimation Results under Load Variations**

| Fault Section | Fault Type | Fault Location (p.u.) | Fault Resistance (Ω) | Fault Location Estimation Error % | |
|---|---|---|---|---|---|
| | | | | 5% load variation | 10% load variation |
| Bus1-2 | LG | 0.2 | 60 | 0.67 | 1.31 |
| | LL | 0.2 | 20 | 1.45 | 2.89 |
| | LLG | 0.4 | [2 4 13] | 1.67 | 3.33 |
| | LLL | 0.4 | [2 4 5] | 1.58 | 3.15 |
| | LLLG | 0.4 | [2 4 5 10] | 1.57 | 3.14 |

### 4.1.5. PERFORMANCE UNDER LOAD COMPENSATION

For the purpose of reducing the impact of load variation in the evaluated algorithm, load compensation method was utilized [14]. Bus impedance matrix method for pre-fault condition was utilized in this section. Firstly, according to equivalent impedance models, individual load level can be calculated based on pre-fault voltages and currents at each load. Then regarding on load level, load impedance model reactive power consumption can be determined.

Table 4-4 illustrates the effectiveness of the load compensation technique. The effectiveness on countering errors of estimated fault location when applying the technique of load compensation for faults is presented in column 5 and 6, which display the fault location errors for cases utilizing load compensations corresponding for 5% and 10% load variations. Clearly the accuracy of fault location estimation has been significantly improved after adopting load compensation technique.

**Table 4-4 Fault Location Estimation Results under Load Variation while Implementing Load Compensations**

| Fault Section | Fault Type | Fault Location (p.u.) | Fault Resistance (Ω) | Fault Location Estimation Error % | |
|---|---|---|---|---|---|
| | | | | 5% load variation with load compensation | 10% load variation with load compensation |
| Bus1-2 | LG | 0.2 | 60 | 0.02 | 0.04 |
| | LL | 0.2 | 20 | 0.05 | 0.10 |
| | LLG | 0.4 | [2 4 13] | 0.06 | 0.12 |
| | LLL | 0.4 | [2 4 5 ] | 0.05 | 0.11 |
| | LLLG | 0.4 | [2 4 5 10] | 0.05 | 0.11 |

# Chapter 5 CONCLUSION

Fault location method for unbalanced distribution systems without fault type information was investigated in this work.

Methods involving phase to neutral voltage and line current measurements were demonstrated, thoroughly attending unbalanced system with minimum local measurements. The discussed method uses bus impedance matrix combining with utilizing circuit analysis techniques with minimum iteration steps. Evaluation studies were performed for a variety of faults. Every single kind of fault was taken into consideration including unbalanced interphase faults. The algorithm does not require fault type and fault resistance information. Remarkably, it only requires solving quadratic equation for LG faults and small amount of iterations for others.

Test results have shown that the algorithm is able to achieve accurate fault location estimation and fault resistance with excellent robustness to voltage and current measurement errors along with load variations. Load compensations technique discussed in this paper successfully reduced the impact of load variations. This algorithm has huge potential for distribution system applications in the field.

# Chapter 6 APPENDIX

## 6.1. TRANSFER IMPEDANCE

To find transfer impedance between node $k$ and fictitious fault node $r_i$, $Z_{kri}$, no source is needed or influential in this procedure. And considering injecting 1 A current at node $k$, [3]

$$E_{p_1} - E_{r_1} = m(z_1 I_1 + z_{12} I_2 + z_{13} I_3) \tag{20}$$

$$E_{p_1} - E_{q_1} = z_1 I_1 + z_{12} I_2 + z_{13} I_3 \tag{21}$$

Where $p_1$ and $q_1$ are the ends of the faulted section line.

And the node voltage is

$$E_{r_1} = E_{p_1} - m(E_{p_1} - E_{q_1}) \tag{22}$$

In this case there is no mutual impedance involved so the transfer impedance between node $k$ and fictitious fault node $r_i$, $Z_{kri}$ is

$$Z_{kr_1} = Z_{kp_1} - m(Z_{kp_1} - Z_{kq_1}) \tag{23}$$

For convenience, assigning

$$B_{k1} = Z_{kp_1} \tag{24}$$

$$C_{k1} = -(Z_{kp_1} - Z_{kq_1}) \tag{25}$$

Then

$$Z_{kr_1} = B_{k1} - mC_{k1} \tag{26}$$

This equation applies to all kinds of fault and similar procedure derivation works on $r_2$ and $r_3$

## 6.2. TRANSFER IMPEDANCE BETWEEN FAULT NODE *R* AND FAULT NODE ***R**$_S$*, ***Z**$_{r1r2}$*

Assuming no source was in the system, voltage of node $r_i$ shall be obtained as below after injecting 1 A current at node $r_i$, [3]

$$E_{r_2} = E_{p_2} - m(z_2 I_2 + z_{12} I_1 + Z_{23} I_3) \tag{27}$$

$$E_{p_2} - E_{q_2} = z_2 I_2 + z_{12} I_1 + z_{23} I_3 + (1-m) z_{12} \tag{28}$$

Then we can get

$$E_{r_2} = E_{p_2} - m\left(E_{p_2} - E_{q_2}\right) + m(1-m) z_{12} \tag{29}$$

$E_{p_2}$ and $E_{q_2}$ can be obtained as

$$E_{p_2} = Z_{p_2 p_1} - m(Z_{p_2 p_1} - Z_{p_2 q_1}) \tag{30}$$

$$E_{q_2} = Z_{q_2 p_1} - m(Z_{q_2 p_1} - Z_{q_2 q_1}) \tag{31}$$

There we have node voltage $E_{r_2}$ which equals to transfer impedance $Z_{r_1 r_2}$

$$Z_{r_1 r_2} = Z_{p_1 p_2} + m\left(z_{12} - 2Z_{p1 p_2} + Z_{p_1 q_2} + Z_{q_1 p_2}\right)$$

$$+m^2(Z_{p_1 p_2} + Z_{q_1 q_2} - Z_{p_1 q_2} - Z_{q_1 p_2} - z_{12}) \tag{32}$$

Here let’s define

$$A_{12\_0} = Z_{p_1 p_2} \tag{33}$$

$$A_{12\_1} = z_{12} - 2Z_{p_1 p_2} + Z_{q_1 p_2} \tag{34}$$

$$A_{12\_2} = Z_{p_1 p_2} + Z_{q_1 q_2} - Z_{p_1 q_2} - Z_{q_1 p_2} - z_{12} \tag{35}$$

Then $Z_{r_1 r_2}$ becomes

$$Z_{r_1 r_2} = A_{12\_0} + A_{12\_1} m + A_{12\_2} m^2 \tag{36}$$

where fault location *m* is the only variable.

Similarly, other transfer impedance can be derived and results are shown in previous chapter.

## 6.3. DRIVING POINT IMPEDANCE $Z_{r1r2}$

Assume eliminating all the sources in the system and injecting 1 A current at node $r_1$. We can obtain [3]

$$Z_{r_1r_1} = Z_{p_1p_1} + m\left(z_1 - 2Z_{p_1p_1} + 2Z_{p_1q_1}\right) + m^2(Z_{p_1p_1} + Z_{q_1q_1} - 2Z_{p_1q_1} - z_1) \tag{37}$$

Which is a function of fault location m. For convenience lets define

$$A_{11\_0} = Z_{p_1p_1} \tag{38}$$

$$A_{11\_1} = z_1 - 2Z_{p_1p_1} + 2Z_{q_1p_1} \tag{39}$$

$$A_{11\_2} = Z_{p_1p_1} + Z_{q_1q_1} - 2Z_{p_1q_1} - z_1 \tag{40}$$

There we have driving point impedance $Z_{r_1r_1}$

$$Z_{r_1r_1} = A_{11\_0} + A_{11\_1}m + A_{11\_2}m^2 \tag{41}$$

Similarly, other driving point impedance at other fault nodes can be found with same procedure.

## 6.4. PARAMETERS USED IN SIMULATION AND CASE STUDY

Source impedance of source 1:

positive-sequence: 0.23 + j2.10 ohm

zero-sequence: 0.15 + j1.47 ohm

The feeder series impedance matrix and later impedance matrices in ohms/mile are given as follows [31]

Main feeders impedance matrix:

$$zMFabc = \begin{bmatrix} 0.3465 + 1.0179i & 0.1560 + 0.5017i & 0.1580 + 0.4236i \\ 0.1560 + 0.5017i & 0.3375 + 1.0478i & 0.1535 + 0.3849i \\ 0.1580 + 0.4236i & 0.1535 + 0.3849i & 0.3414 + 1.0348i \end{bmatrix} \tag{42}$$

Three phase lateral impedance matrix:

$$zLFabc = \begin{bmatrix} 0.7526 + 1.1814i & 0.1580 + 0.4236i & 0.1560 + 0.5017i \\ 0.1580 + 0.4236i & 0.7475 + 1.1983i & 0.1535 + 0.3849i \\ 0.1560 + 0.5017i & 0.1535 + 0.3849i & 0.7436 + 1.2112i \end{bmatrix} \quad (43)$$

Two phase lateral impedance matrix:

$$zLF2p = \begin{bmatrix} 1.3294 + 1.3471i & 0.2066 + 0.4591i \\ 0.2066 + 0.4591i & 1.3238 + 1.3569i \end{bmatrix} \quad (44)$$

Single phase lateral impedance matrix:

$$zLF1p = [1.3292 + 1.3475i] \quad (45)$$

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