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## INCORPORATING TRAVEL TIME RELIABILITY INTO TRANSPORTATION NETWORK MODELING

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Dr. Mei Chen, Major Professor

Dr. Yi-Tin Wang, Director of Graduate Studies

INCORPORATING TRAVEL TIME RELIABILITY INTO  
TRANSPORTATION NETWORK MODELING

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DISSERTATION

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A dissertation submitted in partial fulfillment of  
requirements for the degree of Doctor of Philosophy in the  
College of Engineering  
at University of Kentucky

By

Xu Zhang  
Lexington, Kentucky

Director: Dr. Mei Chen,  
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2017

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## ABSTRACT OF DISSERTATION

### INCORPORATING TRAVEL TIME RELIABILITY INTO TRANSPORTATION NETWORK MODELING

Travel time reliability is deemed as one of the most important factors affecting travelers' route choice decisions. However, existing practices mostly consider average travel time only. This dissertation establishes a methodology framework to overcome such limitation.

Semi-standard deviation is first proposed as the measure of reliability to quantify the risk under uncertain conditions on the network. This measure only accounts for travel times that exceed certain pre-specified benchmark, which offers a better behavioral interpretation and theoretical foundation than some currently used measures such as standard deviation and the probability of on-time arrival.

Two path finding models are then developed by integrating both average travel time and semi-standard deviation. The single objective model tries to minimize the weighted sum of average travel time and semi-standard deviation, while the multi-objective model treats them as separate objectives and seeks to minimize them simultaneously. The multi-objective formulation is preferred to the single objective model, because it eliminates the need for prior knowledge of reliability ratios. It offers an additional benefit of providing multiple attractive paths for traveler's further decision making.

The sampling based approach using archived travel time data is applied to derive the path semi-standard deviation. The approach provides a nice workaround to the problem that there is no exact solution to analytically derive the measure. Through this process, the correlation structure can be implicitly accounted for while simultaneously avoiding the complicated link travel time distribution fitting and convolution process.

Furthermore, the metaheuristic algorithm and stochastic dominance based approach are adapted to solve the proposed models. Both approaches address the issue where classical shortest path algorithms are not applicable due to non-additive semi-standard deviation.

However, the stochastic dominance based approach is preferred because it is more computationally efficient and can always find the true optimal paths.

In addition to semi-standard deviation, on-time arrival probability and scheduling delay measures are also investigated. Although these three measures share similar mathematical structures, they exhibit different behaviors in response to large deviations from the pre-specified travel time benchmark. Theoretical connections between these measures and the first three stochastic dominance rules are also established. This enables us to incorporate on-time arrival probability and scheduling delay measures into the methodology framework as well.

**KEYWORDS:** Travel Time Reliability, Transportation System Uncertainty,  
Route Choice, Traffic Assignment, Stochastic Dominance

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To my beloved family

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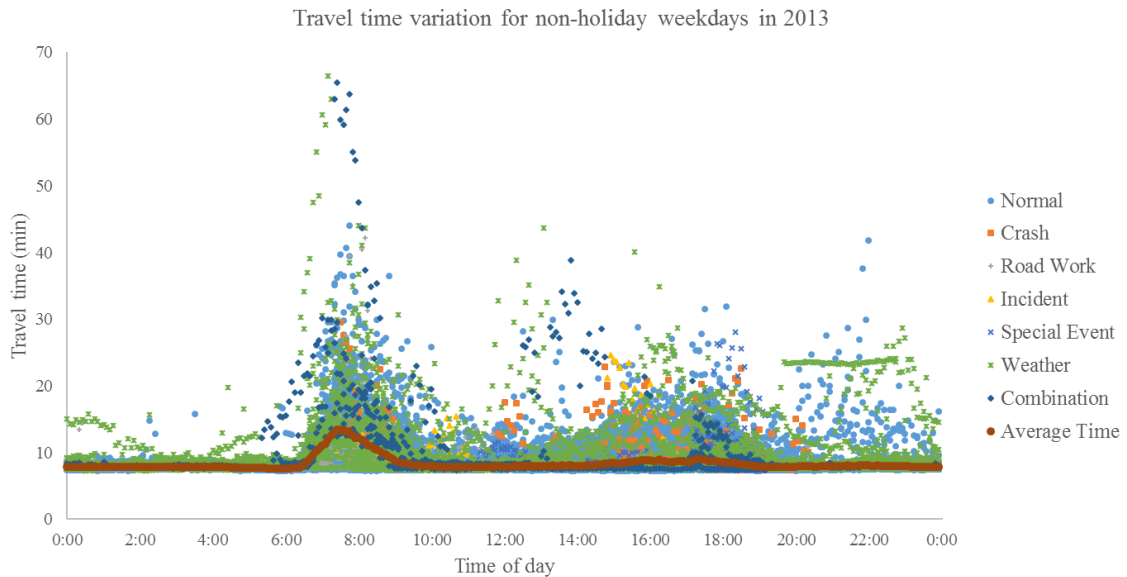
# CHAPTER 1 INTRODUCTION

## 1.1 BACKGROUND

The efficiency of a transportation system in moving people and goods around is of critical importance to societal vitality and development. However, such efficiency is greatly undermined as the system is often faced with uncertainty resulted from a variety of non-recurring events. Aside from extreme cases like floods, tornados, and hurricanes that can be disastrous to the regular transportation activities, more common events including traffic incidents, work zones, inclement weather, control device malfunctions, festival and sport events, and so forth could also have apparent negative impacts on the system. These factors disrupt or interfere with normal traffic operation and lead to unstable system conditions over the course of time. In order to better investigate and understand the uncertainty and reliability aspects of road network, many approaches have been proposed from different perspectives. For instance, the connectivity reliability models the probability of successful connection between an origin and destination (OD) pair in a network(1; 2). The capacity reliability concept has also been developed to model the probability of roadway network handling a certain level of travel demand during capacity-degrading circumstances(3). Another important concept that has been heavily investigated and will be the focus of current research is travel time reliability (TTR).

As one of the most common performance measures, travel time represents the amount of time a traveler spends traveling from an origin to a destination. It is a direct reflection of the traffic condition at the time when the observation is made. For instance, when the traffic volume is low and there is no delay on the road, the experienced travel time would be considered as free-flow travel time. However, when the volume is close to the capacity and a crash occurs on the road, the resulting travel time would be significantly higher compared to that during the free-flow condition. Due to the impact of those factors mentioned before, travel times actually experienced on a link or a route are always fluctuating at different times of day and from day to day.

Conventionally, only average travel time is measured and communicated to show the system performance by transportation agencies. In the meantime, it is traditional practice during the network modeling process to assume travelers are risk neutral and only consider average travel time when making route choice decisions. It is then assumed travelers always select the route with minimum travel time between specified origin and destination, regardless of how variable travel times could be. However, the limitations underlying these assumptions can be easily illustrated using real-world travel time observations, as displayed in Figure 1.1. The data is collected from an 8-mile segment on Interstate 71/75 in Northern Kentucky area. The travel times on non-holiday weekdays are broken down with respect to each event type. It is clearly shown that travel times are dramatically variable throughout the day, especially during morning and afternoon peak periods. In addition, average travel time would significantly misrepresent travel conditions under many occasions. Thus, it is not able to reflect the whole story of actual travel conditions on the corridor of interest.



**Figure 1.1 Travel Time Variation on I-71/75**

Under this notion, the travel time reliability concept has been proposed and applied by transportation agencies and practitioners to provide additional perspective on the variation aspect of travel times. According to the SHRP2 L03 project, which evaluates various procedures for analytically determining the impacts of congestion and reliability mitigation strategies, the TTR is “*the level of consistency in travel time conditions represented by the distribution of travel times that occur over a substantial period of time*”(4). Another research effort that incorporates the travel time reliability concept into the Highway Capacity Manual provides a much thorough definition as “*travel time reliability aims to quantify the variation of travel time. It is defined using the entire range of travel times for a given trip, for a selected time period (for example, the pm peak hour during weekdays), and over a selected horizon (for example, a year). For the purpose of measuring reliability, a “trip” can occur on a specific segment, facility (combination of multiple consecutive segments), any subset of the transportation network, or can be broadened to include a traveler’s initial origin and final destination. Measuring travel time reliability requires that a sufficient history of travel times be present in order to track travel time performance; this history is described by the travel time distribution for a given trip*”(5).

Recently, transportation agencies have recognized that TTR provides a new means to measure the deficiency of the system, and thus devoted numerous efforts to incorporate it into performance management and project appraisals. The previously enacted law, the Moving Ahead for Progress in the 21st Century Act or MAP-21, requires the State Departments of Transportation (DOT) and Metropolitan Planning Organization (MPO) to shift their focus toward establishing performance based transportation system management programs. In particular, travel time reliability is one of seven national-level performance measures to be included in the performance reporting(6). Authorized by Congress, the second Strategic Highway Research Program or SHRP2 has also been



established to conduct in-depth research on four imperative areas including system reliability on national highway system, aiming to address the negative impacts resulting from a wide range of abnormal traffic events(7). In addition, a number of DOTs and MPOs have also put significant effort into integrating travel time reliability in their network monitoring, reporting, and planning processes(8-10).

Many empirical studies have also confirmed that TTR is one of the most important elements in travelers' route choice decisions. Jackson and Jucker find that some travelers are willing to choose the more reliable route over the less reliable one to avoid the risk of being delayed, even when the former may take longer than the latter(11). Through a stated preference survey, Small et al find that both passenger travelers and freight shippers value travel time predictability during congested periods and they are willing to pay to increase the reliability of travel and avoid the cost of late arrival(12). They also indicate that the value of reliability depends on trip purpose. For example, commuters and business travelers usually put a higher value on trip reliability than recreational or casual travelers. Another study conducted by Abdel-Aty et al indicates that travel time reliability is a significant factor in route choice among other factors like travel time and traffic safety(13). Senna shows that the value of reliability is close to or even higher than the value of travel time under investigated scenarios. He also indicates there is a need to incorporate the reliability into travel choice models(14). Similarly, using both revealed preference and stated preference data on State Route 91 in Los Angeles, Small et al apply the difference between the 80<sup>th</sup> and 50<sup>th</sup> percentile of travel times as reliability measure and estimate that the median value of time is \$21.46/hour while the median value of reliability is \$19.56/hour(15). Carrion and Levinson summarize the state of the practice on valuation of travel time reliability (Figure 1.2) and find that the value of reliability is typically 0.5 to 2 times the value of travel time(16).

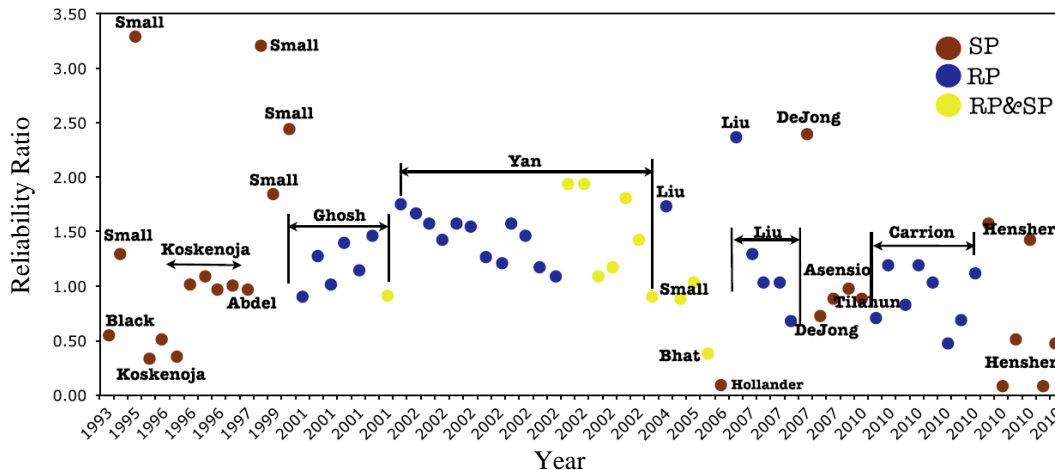


Figure 1.2 Valuation of Travel Time Reliability(16)

## 1.2 PROBLEM STATEMENT

Despite the significance of TTR in influencing travelers' decision-making behaviors, the consideration of reliability is not included in the conventional route choice and traffic assignment models(17). These models are developed under the assumption that travelers

only consider average travel time in their utility function and accordingly select paths with minimal travel time between OD pairs. Then, according to the Wardrop's Principle, equilibrium flow pattern on the network can be obtained when travel times on all used paths are equal to the minimum travel time between each origin and destination pair.

To obtain equilibrium, the minimum travel time between each OD pair has to be known beforehand. Therefore, the shortest path problem has to be solved iteratively during the procedure. Since travel time is included in the cost function, classical algorithms, e.g. Dijkstra's algorithm, can be used to find the optimal path.

However, as many empirical studies have shown, travelers also take travel time reliability into consideration when making trip decisions, in addition to average travel time. In fact, under some circumstances, they are even more concerned with their knowledge of travel time variation, which is gradually built based on their past experiences. As a result, the identified optimal path from traditional models may fail to represent most travelers' risk-averse behaviors(16).

The traditional travel demand forecasting model is a vital tool for many transportation agencies in their planning process. It would be of great value to incorporate the reliability component into the model so that it is more realistic and comprehensive. Many studies have been conducted on this front and contributed to the advance of current modeling techniques. A variety of models including on-time arrival probability(18), travel time budget(19), stochastic dominance(20), etc. have been proposed and investigated in previous research. Although they provide valuable insights into the topic, the drawbacks in these models are also apparent. For example, many of them assume travel times are normally distributed and associated distributions among links are mutually independent, which is not the case based on real-world observations(4).

### **1.3 OBJECTIVES**

The objective of this research is to extend the travel forecasting model by taking travel time reliability into consideration and evaluate the corresponding effects. To better understand the current issue and prepare for the research, existing literature will be extensively reviewed. A realistic reliability measure that is different from those already studied will be proposed in the research. The measure should have better behavioral representation and be readily incorporated into the model.

With incorporation of the reliability component, it will be shown later that the cost function would be no longer additive, i.e. the total cost on the path cannot be obtained from the direct summation of cost on links comprising the path. As a result, traditional deterministic shortest path algorithms are no longer applicable. To overcome this challenge, two metaheuristic algorithms will be explored in current research, one of which has not been previously evaluated in the reliability modeling context.

In addition, Stochastic Dominance Theory has been recently introduced to the transportation modeling field. It has been shown to be effective in prioritizing alternative paths in stochastic setting and representing various risk-taking preferences as well. This

research will intend to further its application by evaluating the relationship between the first three stochastic dominance criteria and existing reliability models. The results from this effort will provide behavioral foundations for reliability models and facilitate the development of a more effective approach.

Last, with the understanding of specific behaviors of the proposed route choice model, traditional traffic assignment model will be extended so that the travel time reliability component can be accounted for in achieving new equilibrium. The model developed should be solvable with effective solution methods and implementable in practical applications.

## **1.4 OUTLINE OF DISSERTATION**

In this section, the specific organization of the dissertation is provided as follows.

In Chapter 1, the background of research is first introduced to provide a broad context for this dissertation. The statement of the problem is then elaborated, followed by the objectives that are intended to be achieved in the following chapters.

In Chapter 2, findings from extensive review on the existing literature are provided. The review covers a broad range of topics, involving travel time distributions, correlations of link travel times, and extension of traditional path finding and traffic assignment models by integrating travel time reliability. The limitations underlying previous studies are also discussed.

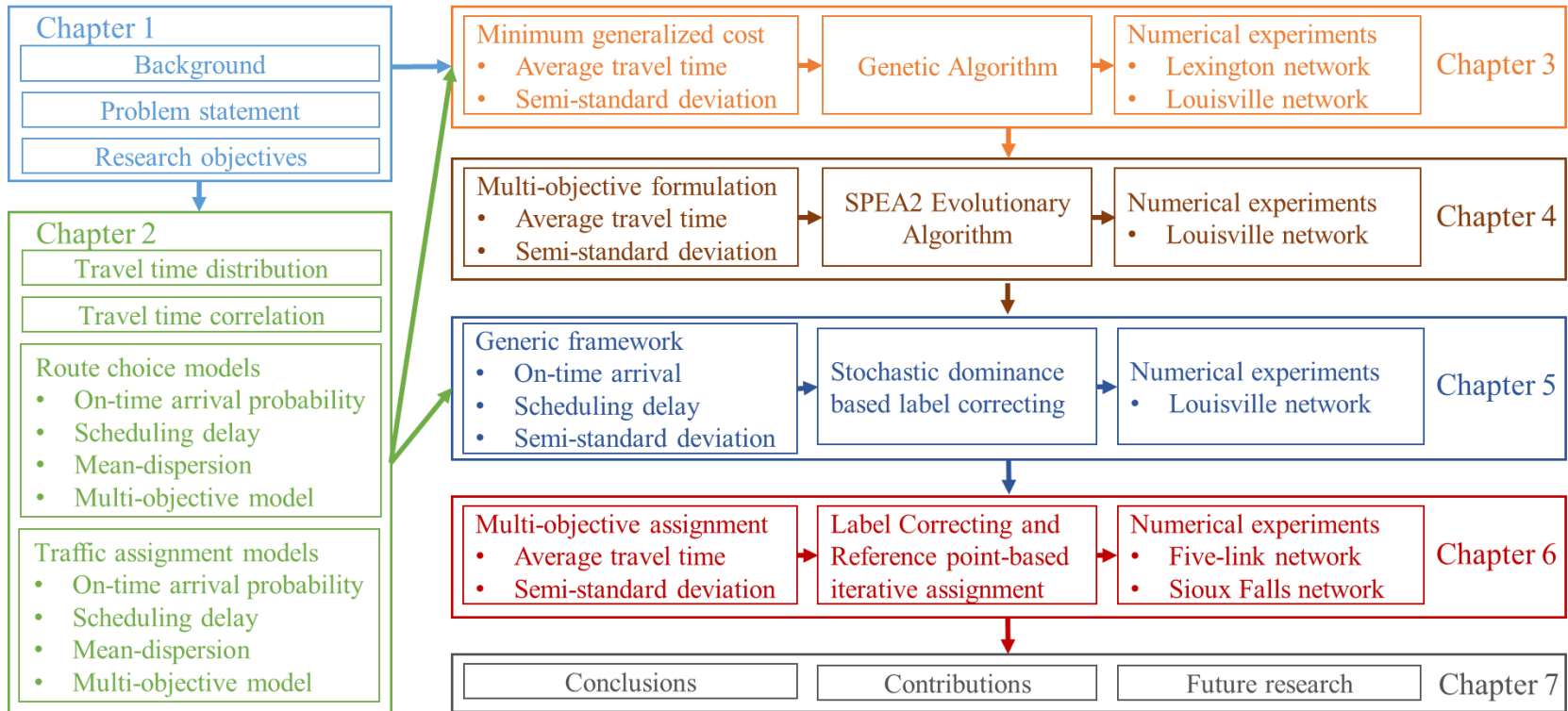
In Chapter 3, the semi-standard deviation is proposed as new reliability measure and the route choice model is reformulated by using a sampling based approach. To solve the new model, the Genetic Algorithm based approach is adapted. In addition, a Label Correcting algorithm is applied to calibrate Genetic Algorithm parameters in order to achieve better performance. Numerical experiments based on real-world data are conducted.

In Chapter 4, the single objective route choice model is transformed into the multi-objective counterpart, after recognizing the limitations underlying the former model. A multi-objective evolutionary algorithm is applied to solve the reformulated model. To reduce the objectivity in choosing algorithm parameters, the second-order stochastic dominance criterion is implemented and obtained non-dominated paths are used as ground truth for assessing the performance of proposed solution approach. Numerical experiments based on Louisville urban network and collected GPS speed data are conducted to evaluate the model.

In Chapter 5, a more in-depth study based on Stochastic Dominance Theory is conducted. The specific risk-taking behaviors corresponding to each of the dominant rules are discussed in this chapter. Later, the relationships between three reliability models and stochastic dominance rules are established, which in turn provide theoretical foundations for reliability models of interest. The extended Label Correcting algorithms are implemented to find Pareto optimal solutions for different models.

In Chapter 6, the research is further enriched by looking into a multi-objective traffic assignment model, so that both average travel time and reliability can be simultaneously minimized. Next, the solution algorithm taking advantage of the Label Correcting algorithm and Reference Point-based Method of Successive Averages approach is developed. At last, numerical experiments are conducted on two hypothetical networks to assess the effectiveness of the proposed model.

In Chapter 7, an overall summary of this dissertation and future research directions are provided.



**Figure 1.3 Research Flowchart**

## CHAPTER 2 LITERATURE REVIEW

### 2.1 INTRODUCTION

The transportation community has increasingly recognized the importance of travel time reliability and numerous efforts have been devoted to this area, aiming to advance the understanding and implementation of the reliability component in various transportation applications. In this chapter, existing literature on four main aspects that are directly related to TTR concept are carefully reviewed to better understand the state of the art. In particular, Section 2.2 summarizes findings regarding travel time distribution, which is often regarded as the building stone of TTR quantification. Section 2.3 provides discussions on the correlation of travel times, which is another important element in reliability modeling area. Furthermore, route choice models involving TTR considerations are extensively reviewed and discussed in Section 2.4. Section 2.5 is dedicated to extended traffic equilibrium models by taking travel reliability into account. At last, the findings from literature review are discussed and summarized in Section 2.6.

### 2.2 TRAVEL TIME DISTRIBUTION

With the advancement of data collection and communication technologies, travel time data can now be consistently obtained by a myriad of means at large spatial and temporal scales. This offers a valuable opportunity to quantify travel time variability over time across the network. As there is no consensus on the performance measure that is suitable to quantitatively represent TTR, different indicators have been proposed to assess reliability condition from different perspectives(21). In contrast, there is no dispute that the travel time distribution is the starting point to reliability quantification. Therefore, it is of practical value to appropriately construct travel time distributions.

SHRP2 L03 Analytic Procedures for Determining the Impacts of Reliability Mitigation Strategies project finds out that distributions on different types of facility, including urban freeways, rural freeways, and urban arterials, are all having longer tails at right side of the distribution(4). The observation indicates excessively long travel times exist across facility types, regardless of the fact that traffic flow patterns can be very different on those facilities. In addition, it is reported that the distribution of travel times during peak periods has a broader and less sharp shape than that during off-peak periods.

Another SHRP2 study establishes a reliability monitoring system using data from various sources to construct probability density functions and cumulative density functions under different traffic operating regimes(22). Each regime represents a combination of prevalent traffic operating condition (e.g. uncongested, moderately congested, highly congested) along with the occurrence of traffic influencing events (e.g. accidents, no-accident incidents, road work, weather, special events. etc.). It is shown that obtained distributions under different regimes are often skewed to the right.

In another study, Arroyo and Kornhauser look into normal, gamma, weibull, and lognormal distributions on the basis of GPS-based data using a maximum likelihood

technique, and they find that the lognormal distribution provides the best performance in fitting the observed data(23).

Li et al find that travel times during weekdays have a highly skewed distribution with a long right tail. Depending on time periods, the travel time distribution curve during uncongested period has the shortest right tail whereas the distribution during afternoon peak is mostly skewed(24). They also show that the normal distribution best describes travel times when a shorter time window is used while lognormal distribution has the best fitness as time window increases.

Recognizing the non-symmetrical feature of travel time distribution, Emam et al evaluate weibull, exponential, lognormal, and normal distributions using dual loop detector data on I-4 in Orlando and discover that lognormal distribution works best in terms of the goodness-of-fit(25).

Similarly, Rakha et al analyze observed automatic vehicle identification data from San Antonio, and conclude that lognormal distribution best describes the travel time, rendering the normal distribution assumption frequently assumed in previous research inappropriate(26).

Considering traffic flow typically experiences congested and uncongested conditions, multistate distribution models have also been evaluated by a few studies. For example, Part et al fit the travel time data with a bimodal model by assuming each component followed normal distribution(27). In order to take the non-symmetrical property of travel times into account, Guo and Rakha use skewed distributions for components of the multistate model, and demonstrate that multi-lognormal model has the best performance among models investigated during peak hours. While during non-peak period, the single state model could still be a suitable option(28).

Based on these studies, it is apparent that travel times are not normally distributed as often assumed in previous studies; therefore, applying normal distribution in reliability problems may not represent the real-world situation. Instead, travel times are often observed to be asymmetrically distributed with a long fat tail at the right side. This phenomenon can be easily explained with the occurrence of non-recurring events which often result in long delays. Furthermore, by assuming the symmetrical distribution, previous studies overlook the skewness of the distribution, which has been considered as an important property of reliability analysis(29).

### **2.3 TRAVEL TIME CORRELATION**

Travel time distribution evaluation is often conducted on individual segments due to lack of trip level data. However, it is often required to obtain travel time distributions at path or trip level. Therefore, travel times on component links have to be combined in some manner to obtain such information. On the other hand, travel times between links are usually correlated, especially during congested time periods (30; 31). This is because the queue on a congested upstream link can propagate backwards to downstream links and cause congestion on those links as well. The phenomenon often can be seen when a crash

or severe weather event happens. Accounting for the correlation between links ensures the accuracy of estimated path travel time distribution, whereas ignoring the correlation altogether would lead to inaccurate estimation of true distribution and consequently reliability measures. To incorporate the correlation into path travel time estimation, three alternatives exist in current literature, including variance-covariance matrix, joint distribution function, and sampling-based approach.

Recognizing the importance of spatial correlations of travel times in journey travel time estimation, Chan et al apply a variance-covariance matrix by time of day, day of week, and week of month in their real-time estimation of arterial travel times(30). Horowitz and Granato state that omitting the correlation would cause imprecise calculation of path travel time measures(32). Therefore, they consider the correlation when computing standard deviation of path travel times. However, this is done only for adjacent links while correlations between non-adjacent links are ignored. Unlike previous work, Shababi et al assume travel times on one link is correlated with travel times on all other links on the network and Hardin's method is adopted to generate valid correlation matrices in their shortest path problem(33). Yet, it should be pointed out that the correlation information is introduced into the model by randomly generated numbers, instead of estimation from real-world observations.

Another means to consider the correlation is through joint distribution functions. Polychronopoulos and Tsitsiklis apply this method for a dependent case of recursive shortest path problem(34). However, as argued by Xing and Zhou, when the size of network becomes significant, the joint distribution based method would become computationally intractable for each path between OD pairs(35). As a result, its applicability is rather limited.

The third option is to apply a sampling based approach, which has been proved effective in several studies(36). The method works based on the idea that the path travel time at a certain time interval is equal to the sum of travel times on component links at the same time interval. When travel time observations are available for a long time period, a relatively complete set of path travel times can be obtained, from which the distribution function can be derived with ease. This way the correlation of travel times between links can be implicitly considered during the procedure.

Based on these studies, the correlation information is an essential part in network modeling process. Among three available options, covariance matrix and joint distribution methods suffer from their computational complexity, especially in large-scale networks. In contrast, the concept underlying the sampling-based approach is rather straightforward. It circumvents the need to directly estimate the covariance matrix or joint distribution functions, and therefore provides a practical means to account for travel time correlations across network. It should be noted that no matter which method is used, a relatively large dataset should be collected over a long time period. This is to reduce sampling errors and ensure the accuracy of derived path travel time distributions.



## 2.4 ROUTE CHOICE MODELS

The route choice model has been extensively studied in transportation field due to its practical importance to many applications, such as routing navigation, traffic assignment, and network design. Traditionally, only a single deterministic criterion, for example the physical distance or average travel time, is considered in classical models(37). However, such practice is unable to account for the risk-averse behaviors when travelers are faced with travel time uncertainty. In order to incorporate the impact of travel time reliability, many research attempts have been made to develop routing models with integration of the reliability term. After extensive review, previous studies can mainly be classified into three groups, i.e. distributional approach, centrality-dispersion approach, and multi-objective modeling approach.

### 2.4.1 Distributional Approach

The group of distributional approaches is conducted on the basis of convolution of known link travel time distributions to obtain path travel time distribution. Within this group, one direction is to apply Stochastic Dominance Theory (SDT) to compare and rank routes with random travel times in terms of known cumulative distribution functions. Assuming independence of travel times on links, Miller-Hooks and Mahmassani investigate three path comparison schemes involving expected value comparison, deterministic dominance, and first-order stochastic dominance under stochastic and time-varying settings(38). Wu and Nie apply first-, second-, and third-order stochastic dominance criteria to model heterogeneous risk-taking route choice decisions with respect to insatiable, risk-averse, and ruin-averse behaviors, respectively(20). Later, a Label Correcting technique is developed to determine all admissible paths in terms of each dominance criterion. It is also assumed that travel times on links are independent, which is not realistic based on studies reviewed in last section. Nonetheless, the SDT approach fits well with the optimal path finding problem under the stochastic travel time setting, and will be utilized later in current study as well.

Another direction of study is to apply the concept of on-time arrival probability (OTAP). Recognizing travel times on a link are random, Frank studies the probabilistic shortest path problem in finding the optimal path, which maximizes the probability of arriving at destination within a certain amount of time(39). Following Frank's work, Fan et al look into a similar problem but in a dynamic and adaptive environment(40). They propose that it would be a success if travelers arrive at the destination within a given time. Otherwise, it would be considered a failure. Also, at every location, travelers want to choose next node based on current location in order to obtain maximum probability of on-time arrival with the remaining travel time budget. The Bellman Principle of Optimality is applied to formulate the model and then the Picard Method of Successive Approximation is adopted to solve the problem. The limitations of this study are: (a) it assumes travel time distributions at any given time are known across network; (b) the applicability of proposed method in real-world applications requires further assessment.

Similar to Fan's work, Nie and Wu's proposition incorporates OTAP as a priori shortest path problem, assuming travel time distributions on links are already known as well(18).

They first demonstrate that the optimal solution to the problem can be obtained from the local-reliable path set which is generated by the first-order stochastic dominance. A Label-Correcting algorithm is then developed to solve the model. Although proven to be effective, the solution algorithm requires a complicated distribution convolution process, which may limit its practical implementation.

Later, Zockaie et al put forth a Monte Carlo Simulation method to approximately solve the a priori problem defined in the previous study(41). They assume that specific link travel time distributions and corresponding joint distribution functions are already known beforehand during the simulation process. For each simulation run, the shortest path is obtained and stored in the candidate set. After the first round of simulations, the second round is conducted on paths in the candidate set to get their travel time distributions, from which the optimal path can be eventually determined. Even though the simulation method avoids the need to convolve link distributions, the assumption of available joint distribution functions across network is not valid.

To account for travel time uncertainty, Chen and Ji provide three optimal path models(42). The expected value model tries to find the path with minimum expected travel time, which is essentially equivalent to traditional models. The maximum probability model aims to find the path that is attached with highest probability of arriving at the destination within specified time interval. While  $\alpha$ -reliable model tries to find the path with minimum effective travel time while ensuring the on-time arrival within certain level of confidence. A simulation-based Genetic Algorithm is used to find optimal paths for proposed models. Similar to this study, the Genetic Algorithm will also be adopted to solve the extended route choice model proposed in current work.

In addition to OTAP, the travel time budget (TTB) or effective travel time model has also been studied (18; 42). The model tries to minimize the budgeted travel time given a pre-specified on-time arrival probability and makes use of inverse of cumulative probability function, which is directly utilized by OTAP model. It should be noted that the TTB and OTAP models are considered equivalent when the travel times are assumed to follow normal distribution. Under normal distribution assumption, Chen et al first introduce the first-order stochastic dominance and mean-variance dominance conditions, and then develop an  $A^*$  algorithm to find the most reliable path in a transformed two-level hierarchical network, where correlations are represented by a covariance matrix(43). Yet, it is already shown that travel times are often asymmetrically distributed.

Zhou and Chen argue that TTB model is only able to account for the reliable part of travel time distribution while unable to account for the unreliable part where travel times exceed the budgeted travel time(44; 45). They develop a mean-excess travel time (METT) based model and incorporate it into the route choice decision process. Later, based on the finding that travel time distributions on urban interrupted facilities are bi-modal, instead of normally assumed unimodal, Yang et al evaluate the most reliable, TTB, and METT models on an urban arterial network under bi-modal distribution circumstance(46).

Based on these studies, in order for this group to work, travel time distributions at path level between OD pairs need to be constructed(47). Different approaches have been investigated for this purpose, yet most of them are too theoretically complex to be readily adapted to existing MPO models. To simplify the process, it is usually assumed that travel times on links are normally distributed and mutually independent. However, travel times on links are usually correlated and travel time distributions are typically asymmetrical with a positive skewness value. Moreover, OTAP has been criticized for its inability to fully account for the reliability(20). Based on reviews so far, a simpler method based on a more realistic measure is still desired.

#### **2.4.2 Centrality-Dispersion Approach**

Many studies have directly used a simple statistical measure as the reliability term and then incorporated the measure into routing models. The centrality-dispersion based trade-off model is the most commonly studied in current literature. First introduced in the portfolio theory by Markovitz, the mean-dispersion type of criteria have been extensively used in a variety of applications to accommodate uncertain and risky circumstances(14). Unlike former group of research, such formulation eliminates the requirement that travel time distributions on the network should be known a priori and the need to assume distributions between links are mutually independent to reduce the complexity involving distribution convolution. As a result, it looks more promising to be adapted into travel demand models.

Using standard deviation (STD) as the travel time reliability term, Xing and Zhou reformulate the traditional shortest path problem into a most reliable path finding problem. Two formulations are proposed depending on whether correlation structure is considered by the model. Due to the quadratic form of standard deviation, the cost function that is to be optimized is no longer additive. A Lagrangian substitution method is adopted to approximate the optimal solution for problems both under independent and correlated assumption of link travel times(35). However, the method is not guaranteed to always find the optimal solution. It should be noted that the sampling based method is used by the authors to account for the correlation and proved to be effective. Therefore, this approach will also be adapted in the current work.

Prakash et al also suggest to use a sampling based approximation method to circumvent the hard to obtain correlation matrix(36). They state that under current conditions, travel times could be continuously collected and thus path travel times could be obtained from the links that make up of the path and then the mean and variability, which is represented by standard deviation, could be determined. With this reformulation, a sub-path elimination procedure is developed to eliminate suboptimal sub-paths and a network pruning algorithm is further applied to find the optimal solution to the problem.

In another study, Shahabi et al also use standard deviation to represent travel time uncertainty in a shortest path problem(33). In their research, the original formulation is transformed into a mixed integer conic quadratic program, which is easier to solve. An outer-approximation solution algorithm is developed to first decompose the problem into a sub-problem and a master problem and then solve them respectively to find the optimal

solution when solutions for two decomposed problems converge. However, as pointed out by the authors, accurate covariance matrix for the whole network may be hard to obtain, and therefore, more robust estimation methods are needed to relieve this restriction.

Khani and Boyles provide a different perspective to solve above problem. They first prove that the optimal solution of the mean-standard deviation problem is a subset of the mean-variance problem, and then develop an exact algorithm based on the Label Correcting and bisection-type line search approach to solve the model(48). To simplify the problem, travel times are assumed to be independent during the model development, which is an unrealistic assumption as have been discussed before.

Despite being widely accepted and applied, utilization of variance or standard deviation has some limitations(49-51). First, it is a symmetrical measure. Therefore, it gives the same emphasis on travel times that are both below and above the mean(52). In other words, travel times that are shorter than the expected value will also be viewed as undesirable by travelers, which is usually not the case in reality. In addition, studies have shown that travel time distribution is usually asymmetrical and has a longer tail on the right side of the distribution(26). Standard deviation, however, fails to take into account the skewness of the distribution, which is considered to be an important aspect of reliability. As a result, it may not be consistent with how travelers consider the reliability factor (49; 52). In this dissertation, a new statistical measure that is similar to standard deviation but has a better behavioral implication will be proposed and used to extend traditional travel models.

### **2.4.3 Multi-Objective Shortest Path Problem**

Instead of having only one objective, actual route decision often involves a trade-off process that takes multiple incommensurable and usually conflicting factors into consideration. As a result, a single route that is optimal in every respect usually doesn't exist. In this regard, a multi-objective formulation and optimization seems more suitable for route choice problems.

Many studies have been conducted from the multi-objective perspective. As one of the first to study the bi-criteria shortest path problem, Hansen evaluates ten related problems and their associated computational complexities, and provides several algorithms that can be used for different kinds of bi-criteria problems(53). In addition to travel time, Gwo-Hsiung and Chien-Ho argue that travel distance and air pollution are also important factors considered by other stakeholders(54). Then they extend the conventional single-objective model to a multi-objective traffic assignment model. Li and Leung develop a route planning model for dangerous goods transportation by simultaneously considering expected travel time, probability of an accident, population exposure, and negative economic effect(55).

In order to incorporate the impact of travel time reliability, Sen adopt the variance into the route choice decision process by using a bi-criteria model(56). Ji et al state that travelers can have multiple requirements of travel time reliability to evaluate and select

routes between an OD pair and accordingly formulate a chance constrained multi-objective programming model to find paths that simultaneously satisfy more than one confidence requirements of on-time arrivals(57). The study also indicates that meta-heuristic algorithms can be effective in finding the Pareto optimal paths.

As a more realistic approach, the multi-objective formulation will also be adopted in the current work to further develop the route choice model so that more than one path that are attractive in some aspect to some travelers can be identified.

## **2.5 TRAFFIC EQUILIBRIUM MODELS**

Similar to route choice models, the consideration of reliability is not included in conventional traffic assignment models as well(17; 58). As a result, the identified optimal paths from these models may not represent travelers' complete decision making processes under uncertain travel time conditions. Acknowledging the limitation rooted in traditional assignment models in terms of the reliability concern, many models that incorporate TTR have also been proposed and developed recently. The grouping scheme used in the route choice section are also applicable in categorizing those assignment models here, as built-in cost functions in those models are directly reliant on associated route choice models.

### **2.5.1 Distributional Approach**

The on-time arrival probability based traffic assignment models have been developed in several studies. Asakura and Kashiwadani examine the network reliability under fluctuated demand condition in terms of connectivity and travel time reliability(1). An iterative traffic assignment simulation model is used in their research due to the data availability issue. Each time, OD demand is obtained from a distribution and a traffic assignment procedure is called to assign the obtained demand to the network. After enough iterations, the distribution of travel time could be obtained and then the on-time arrival probability measure could be derived for each OD pair. Although this application is not requiring any modification of underlying model structure, the traffic assignment procedure is still based on the minimum average travel time decision principle.

Shao et al extend traditional user equilibrium by considering the on-time arrival probability(59) where the uncertainty of travel times is resulted from day-to-day demand fluctuation. The traffic demand between each OD pair is assumed to follow a given probability distribution, and the flow distribution at both link and path level could then be derived if independent assumption is made between path flows. In the meantime, link travel times are assumed to be independent during the model development. The equilibrium conditions are converted to the variational inequality formulation and a heuristic algorithm based on the method of successive averages is proposed to solve the model.

Following the same concept, Lo et al and Siu et al both develop a multiclass mixed equilibrium model to account for different risky attitudes among travelers(19; 60). The flows on used paths are determined to be positive if the travel time required to ensure on-

time arrival within certain confidence level for each class of travelers with same risk-coping behaviors is equal to the minimum. The model is then reformulated as the complementary conditions and solved by an unconstrained mathematical program.

In another study, Nie propose a percentile user equilibrium model which minimizes the travel time budget to ensure certain level of probability of on-time arrival(61). The stochasticity of travel time, which is represented by a distribution function, is derived from the probability density function of the service flow rate. The link travel time distributions are assumed to be independent in the model. The route travel time distribution is then derived by convolving link travel time distributions using a numerical evaluation method. The equilibrium conditions are reformulated as variational inequality problem and solved by a gradient project algorithm.

Lo and Tung extend traditional deterministic user equilibrium by accounting for travel time variability in a probabilistic user equilibrium model(62). The variability is considered as a consequence of capacity degradation due to stochastic events like incidents. During the model development, link capacity distributions are assumed to be independent, therefore so are link travel time distributions. At last, the equilibrium state is obtained with following conditions: a) the traffic flow on a path is positive if the mean travel time is minimum; b) the travel time distribution of the used path satisfies the travel time reliability requirement.

Chen and Zhou propose a mean-excess traffic equilibrium (METE) model by incorporating the mean-excess travel time based route choice model(63). The equilibrium condition is obtained when no traveler could reduce their path mean-excess travel time by unilaterally changing the route. The travel times on the links were assumed to be mutually independent between each other. With formulation of METE, the additive assumption no longer applies; therefore, the authors reformulate the model to a VI problem and solve it with a modified alternating direction algorithm. Along this line, Xu et al extend Chen's METE model by taking the perception errors into consideration(64). In their model, the assumption that travelers have perfect information about the travel time distribution on the routes is relaxed and the perceived travel time distribution is used to formulate the model. Next, the corresponding impacts from the stochastic version of METE model on the obtained equilibrium results are investigated. However, detailed survey data is required to have an accurate representation of travelers' perception errors.

Although extending traditional traffic assignment models by integrating OTAP measure have been proved doable, the limitations such as normal and independent distribution assumption underlying the route choice model still exist.

### **2.5.2 Centrality-Dispersion Approach**

Under this group, one direction of studies have applied the scheduling delay (SD) concept which is commonly used in travelers' departure time choice model to determine the optimal departure time given the preferred arrival time by trading off the cost associated with possible early and late arrival(16). A preferred arrival time point or window is pre-defined such that the arrival time occurs before the desired arrival time or at the left side

of the desired arrival time window is considered as early arrival. Similarly, arrival time happens after the preferred arrival time or exceeds the right boundary of the desired arrival time window is considered as late arrival. The logic behind the model is that both early arrival and late arrival can cause some negative impacts on travelers, like less time at home or late penalty at work. A utility function which explicitly combines those considerations has been proposed and widely used in existing literature(65).

Following this line, Watling proposes a route disutility function that comprises the generalized travel cost plus the late penalty(66). The author then develops a late arrival penalized user equilibrium model so that the disutility on all used paths is equal, and less than that on any unselected paths. Due to the nonlinear relationship of utilities between a path and its component links, the shortest path algorithms developed under additive assumption could not be used here. Therefore, the model is reformulated as a complementarity problem and solved by a route-based algorithm proposed by Lo and Chen(67).

To model travelers' risk-taking behaviors on congested network, Yin et al also apply the SD approach in their modeling framework(68). Similarly, they propose a generalized cost function and assume travelers are to minimize the disutility associated with the routes under consideration. However, some assumptions that have been criticized are also present during the model development, such as Normal distribution of link travel times and independence of travel times between links. Then, the model is transformed into an equivalent non-linear complementary problem, which is solved by two algorithms, including a gap-function and iterative heuristic algorithm.

The scheduling delay is considered as a more realistic measure dealing with uncertainty, and has been widely used in behavioral preference surveys. However, it also suffers from some drawbacks that greatly limit its practical applicability. First, effective solution algorithms tailored for this kind of model are lacking in current literature. Second, it requires detailed survey data, which is hard to obtain and thus usually unavailable, to derive travelers' preferred arrival times at destinations for different departure times(69). In addition, the scheduling delay has been criticized for its inability to fully capture travel time reliability(70).

Another direction within the group is to use standard deviation as reliability measure, as have been discussed in the route choice model section. In a real-world application, Horowitz and Granato integrate standard deviation into the cost function in a dynamic traffic assignment based travel forecasting model(32). The coefficient of variation regression equation derived by Black(71) is adopted to relate travel time variability to average condition. The correlations only between adjacent links that are also not separated by turns at controlled intersections are considered. A Dijkstra-type and shortest marginal path finding algorithms are developed to find paths with minimum cost between OD pairs. The model is tested on a metropolitan network and outputs shown that inclusion of reliability would produce more realistic path choice modeling.

SHRP2 C04 project, which aims to improve the understanding of how highway congestion and pricing affect travel demand, recommends a general highway utility

function to account for additional effect from travel time reliability(52). Particularly, the reliability term is represented by day-to-day standard deviation of travel times divided by distance. Accordingly, the total cost on a path is derived based on the monetary cost, travel time and travel time reliability, assuming the linear relationship between standard deviation and mean travel time per unit distance. To overcome the non-additive difficulty, a heuristic procedure is developed, which first constructs a path set with minimum cost omitting standard deviation in the cost function. Next, the reliability component is put back to the model via the linear equation developed beforehand, and the final optimal path can be determined. This procedure has been implemented in a large-scale network, which proves its practical applicability, yet, the heuristic method may not guarantee the final path is always the global optimal. Also, the standard deviation, as discussed in last section, may not represent a traveler's actual perspective on travel time reliability. As a result, a more realistic measure and model are still desired.

### **2.5.3 Multi-Objective Traffic Assignment Model**

Compared to rich research on single objective based traffic assignment models in previous literature, the multi-objective counterparts in reliability based modeling practice are rather limited. Wang et al revisit two popular user equilibrium models, i.e. travel time budget and scheduling delay model(72). They first show that the single objective formulation would omit some reasonable choices that may still be attractive to rational decision makers. In addition, it is pointed out that with the single objective model, travelers are indifferent to any paths as long as the sum of mean and reliability measure on those paths are equal, which in turn contradicts the behavioral assumption made at first for the model development. To deal with these issues, a bi-objective equilibrium model that integrates travel time reliability represented by standard deviation is proposed. It is shown that the proposed bi-objective equilibrium model is a generalized framework for TTB and SD models after some transformations under Normal distribution assumption. A numerical example based on a three link network is used to demonstrate the applicability of the model.

Similarly, Tan et al also put forth a bi-objective traffic equilibrium model with standard deviation as the reliability indicator and then conduct a pareto efficiency analysis as well as risk-taking behavior evaluation(73). The non-dominated path concept in terms of mean and standard deviation, and pareto efficient flow pattern are defined to facilitate the model development. Based on the geometric condition of the mean-standard deviation indifference curve, it is found that it must be downward sloping at equilibrium when travelers are risk averters. The authors further investigate specific risk averse behaviors of other reliability models, including OTAP, TTB, METT, and quadratic disutility function. The models are later tested on a two-link network with a single OD pair.

Based on above studies, standard deviation is still a popular choice to represent reliability consideration in existing multi-objective traffic assignment models. Therefore, these models still suffer from the undesirable properties resulted from standard deviation's mathematical formulation when the underlying distribution is not symmetrical. Also, as they provide valuable theoretical contributions to the subject of interest, neither of them offer an effective solution that can be readily applicable in real-world applications.



Nonetheless, the studies show that the Pareto efficiency is an important property of network reliability analysis; hence, the multi-objective formulation will also be adapted in current work to develop a new user equilibrium model that involves a more realistic measure. More details will be presented in following chapters in this dissertation.

## **2.6 CONCLUSIONS**

Travel time reliability recently has attracted significant attention from transportation profession. Empirical studies have shown that the reliability of trip is one of the most important components in traveler's route choice decision when facing uncertain situations. This is due to negative consequences that may be resulted from late arrivals. In order to advance the state of the art, numerous studies have been conducted on relevant topics, including theoretical definitions and concepts, quantitative reliability measures, and extension of traditional route choice and traffic assignment models.

Through extensive literature review, it is found that the travel time distribution is often asymmetrical with a longer tail at the right side of the distribution. Many studies find that the lognormal distribution works best with respect to the goodness of fit among many tested distribution functions. The finding directly contradicts the normal distribution assumption in many previous studies, and as a result makes them unrealistic. In addition, it is also shown that travel times between links on the network are often correlated. Therefore, it must be incorporated into the modeling process when deriving path travel time distributions. Ignoring this important element may result in biased outcomes from route choice and traffic assignment models.

On the route choice and traffic assignment modeling front, many different ways have been explored in current literature to account for the impact of travel time reliability. The reliability measures proposed in different studies have their own merits in interpreting traveler's behavioral reaction to travel time uncertainty. However, limitations with respect to each group are also apparent and have restricted their practical applicability. Those limitations include but not limited to theoretical complexity of distribution convolution, unrealistic behavioral representation of travelers' actual behaviors, and lack of effective solution algorithms. Therefore, there is still a gap to be filled between the behavioral importance of travel time reliability and practical implementation in real-world applications. To achieve this goal, a more appropriate reliability measure that has a better behavioral representation and can be readily incorporated into existing travel models should be proposed. The extended model after incorporating the reliability measure should also be solvable with effective solution methods and implementable in practical applications.

## CHAPTER 3 A MINIMUM WEIGHTED COST PATH FINDING MODEL

### 3.1 INTRODUCTION

Literature review shows that the mean-dispersion model is one of the most commonly adopted approaches in optimal path finding problem involving travel time reliability. Also, standard deviation has been widely used as the quantitative measure in many existing models. However, there have been concerns regarding its behavioral interpretation and theoretical limitations when the underlying travel time distribution is asymmetrical, which has been proved to be the case based on real-world observations. In this chapter, we propose a new reliability measure based on semi-standard deviation (SSD). SSD not only has the same advantages that STD has, but also has some desirable features that overcome the limitations of STD. A traveler usually remembers unexpectedly long travel times from his/her past experiences and will budget a buffer time to deal with the uncertainty of travel conditions to ensure on-time arrival at the destination. SSD only considers travel times that exceed certain threshold as undesirable and applies this part into the calculation procedure. Furthermore, the skewness is accounted for by SSD since only the right part of the distribution, which usually has a longer tail, is taken into consideration.

The semi-variance, i.e. the square of semi-standard deviation, has recently been used in several SHRP 2 research projects as one of primary measures of travel time reliability. List et al indicate that semi-variance is a better measure because it is sensitive to travel times above the mean(22). Potts et al. state that it is more useful to use semi-standard deviation to describe how travel times are deviating from the pre-specified threshold(74). The proposed SSD measure places more emphasis on larger deviations from the mean and therefore can better assess the reliability.

The semi-variance and semi-standard deviation have been widely used in economic and financial applications, such as in portfolio selection model that trades off the return and risk to find optimal combination of securities or assets(66-69). In those financial models, large values of the return variable are preferred, and accordingly they try to minimize the downside risk that is considered as the loss at the left side of the distribution when the return is less than a predefined reference value. In our proposed model, larger travel times are deemed undesirable and travelers will try to minimize the risk at the right side of travel time distribution.

The rest of the chapter is organized as follows. The following section provides the definition of semi-standard deviation and the reformulation of the routing model using semi-standard deviation as the reliability indicator. Due to the complexity of deriving the correlation structure, the sampling-based approach is adopted here. The approach has been used in existing literature(35). Section 3.3 describes a Genetic Algorithm based solution approach to solve the proposed model. Section 3.4 provides numerical examples on two real-world networks to test the proposed model and solution algorithm. The findings are summarized and discussed in the final section.

### 3.2 PROBLEM STATEMENT

Consider a directed network  $G = (N, A, D)$  where  $N$  is the set of nodes,  $A$  is the set of links, and  $D$  is the set of probability distributions of travel times associated with individual links. Let  $r \in N$  and  $s \in N$  represent the origin and destination node respectively. Let  $t_k^{rs}$  be the random travel time variable on path  $p_k^{rs} \in P^{rs}$ , where  $P^{rs}$  denotes the set of paths connecting the origin  $r$  and destination  $s$ . Let  $\mu_k^{rs}$ ,  $\sigma_k^{rs}$  and  $\tau_k^{rs}$  represent the mean, STD, and SSD of  $t_k^{rs}$  along  $p_k^{rs}$ , respectively.

#### 3.2.1 Semi-Standard Deviation as Reliability Measure

The semi-standard deviation only accounts for travel times that are above the reference value. Therefore, the reference value plays an important role in determining SSD. In this chapter, we choose the average travel time as the benchmark to maintain consistency with the standard deviation counterpart. A more detailed discussion on the impact of benchmark can be found in following chapters, especially Chapter 5.

Accordingly, the STD  $\sigma_k^{rs}$ , and SSD  $\tau_k^{rs}$  of  $t_k^{rs}$  can be respectively defined as

$$\sigma_k^{rs} = \sqrt{E[(t_k^{rs} - \mu_k^{rs})^2]} \quad (3.1)$$

$$\tau_k^{rs} = \sqrt{E[(t_k^{rs} - \mu_k^{rs})_+^2]} \quad (3.2)$$

where  $z_+ = \max(z, 0)$ ,  $E[\cdot]$  is the expectation operator.

If  $t_k^{rs}$  is a discrete random variable and takes  $n$  random values, the semi-standard deviation can be calculated with the following equation.

$$\tau_k^{rs} = \sqrt{\frac{1}{n} \sum_{i=1}^n [(t_{k_i}^{rs} - \mu_k^{rs})_+^2]} \quad (3.3)$$

where  $\mu_k^{rs} = \frac{1}{n} \sum_{i=1}^n t_{k_i}^{rs}$ .

If  $t_k^{rs}$  is a continuous random variable and has a probability density function of  $f(t)$ , the semi-standard deviation can then be calculated as:

$$\tau_k^{rs} = \sqrt{\int_{\mu_k^{rs}}^{+\infty} (t_k^{rs} - \mu_k^{rs})^2 f(t) dt} \quad (3.4)$$

where  $\mu_k^{rs} = \int t f(t) dt$ .

Based on SSD equations, we can see that unlike STD, which considers a path to be unreliable if the travel time varies a lot over time, SSD considers a path to be unreliable if travel time beyond traveler's expected travel time varies a lot. In other words, STD considers travel times below and above the mean as equally undesirable, while SSD only considers travel times that are above the mean as undesirable. As a result, when the travel

time distribution is asymmetrical, which is usually the case, SSD incorporates the skewness of the distribution implicitly, while STD does not.

A simple hypothetical three-path network is used to illustrate above idea. Again, different benchmark values can be used in SSD calculation, which will lead to different SSD values, but the mean is chosen here to be consistent with STD calculation. The sampled travel time data and summarized statistics are shown in Table 3.1.

Table 3.1 Path Travel Times on Illustrative Network

Path	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10	Mean	STD	SSD	Skewness
A	7	7	2	7	7	2	7	7	7	7	6	2	0.89	-1.78
B	6	8	6	4	2	6	6	6	10	6	6	2	1.41	0
C	5	5	5	10	5	5	10	5	5	5	6	2	1.79	1.78

The mean and STD of travel times on three different paths are identical based on 10-day samples. However, Path A has a negatively skewed distribution of travel times, Path B has a symmetrical distribution, and Path C has a positively skewed distribution. Therefore, it is clear that STD is unable to account for the asymmetry of the distribution and ineffective in differentiating the variability as a reliability measure. In contrast, if SSD is used as the reliability measure, Path A will be the most reliable route, while Path C will be the most unreliable.

### 3.2.2 Model Formulation

Now consider a path  $p_k^{rs}$  connecting the OD pair between  $r$  and  $s$ . The travel cost on the path is consisting of the mean and semi-standard deviation of travel times. Accordingly, it can be formulated as

$$\eta_k^{rs} = \mu_k^{rs} + \lambda \tau_k^{rs} \quad (3.5)$$

where:

$\eta_k^{rs}$  is the cost on path  $p_k^{rs}$  between OD pair rs;

$\mu_k^{rs}$  is the expected travel time on path  $p_k^{rs}$  between OD pair rs;

$\tau_k^{rs}$  is the semi-standard deviation of travel time on path  $p_k^{rs}$ ;

$\lambda$  is the sensitivity to the unreliability. It is indicative of the weight travelers place on travel time reliability relative to the mean travel time when making route choice decisions. The more weight travelers place on the potential delay to reduce or avoid the probability of late arrival, the larger the  $\lambda$  value will be.

When deriving the path level reliability, correlation between links should be considered. Unlike standard deviation that can make use of the covariance matrix, due to the special

formulation of semi-standard deviation, there is no closed form equation to come up with a similar semi-covariance matrix. In other words, the exact semi-covariance of link travel times can only be derived on the basis of path travel times. However, the path travel time distribution is unknown in advance and therefore needs to be derived from link travel times. Although approximation methods have been proposed as an alternative(75), preliminary analysis indicates the resulting errors from those methods are too large for them to be applicable in current study. On the other hand, it is cost prohibitive to derive the correlation structure on the whole network.

Besides the covariance matrix, the sampling-based approach has also been applied in current literature to account for the correlation(35). Compared to the covariance method, the sampling based approach directly circumvents the extensive estimation process and is able to incorporate the correlation inherently in the samples. With the recent advancement of data collection technologies, we are provided with an unprecedented opportunity to cost-effectively collect continuous data on a large-scale network for transportation system monitoring and management. As the availability and quality of such data improves, it also provides a valuable opportunity for route choice modeling. Therefore, the sampling based approach is adopted in this study as well.

Assume field collected travel time data is continuously available across links on the network for a period that is long enough to accurately measure the mean and travel time reliability. Now suppose there are  $w$  discrete travel time realizations for each link, and let  $r_m^{ij}$  denote the travel time realization on link  $a_{ij}$  at time interval  $m$ . Let  $x_{ij}^{rs}$  be the binary variable where  $x_{ij}^{rs} = 1$  if link  $a_{ij}$  is a member link of path  $p_k^{rs}$ , and  $x_{ij}^{rs} = 0$  otherwise. Accordingly, we can determine the mean and SSD of path travel time  $t_k^{rs}$  as follows.

$$\mu_k^{rs} = \frac{1}{w} \sum_{m=1}^w \sum_{a_{ij} \in A} r_m^{ij} x_{ij}^{rs} \quad (3.6)$$

$$\tau_k^{rs} = \left( \frac{1}{w} \sum_{m=1}^w \left( \sum_{a_{ij} \in A} r_m^{ij} x_{ij}^{rs} - \mu_k^{rs} \right)_+^2 \right)^{0.5} \quad (3.7)$$

### 3.2.3 Routing Model Considering Travel Unreliability

The travel cost from the Equation (3.5) is essentially the sum of average path travel time and the extra amount of time allocated by travelers to cope with travel time variability. It is assumed that travelers want to minimize the disutility associated with their trip and always choose the path with the minimum cost from the path set connecting the origin and destination. The route choice model considering the travel unreliability is given below.

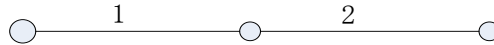
$$\text{Min } \eta_k^{rs} = \frac{1}{w} \sum_{m=1}^w \sum_{a_{ij} \in A} r_m^{ij} x_{ij}^{rs} + \lambda * \left( \frac{1}{w} \sum_{m=1}^w \left( \sum_{a_{ij} \in A} r_m^{ij} x_{ij}^{rs} - \mu_k^{rs} \right)_+^2 \right)^{0.5} \quad (3.8)$$

$$\text{s. t. } \sum_{j: a_{ij} \in A} x_{ij}^{rs} - \sum_{i: a_{ji} \in A} x_{ji}^{rs} = \begin{cases} 1, & \text{if } i = r \\ -1, & \text{if } i = s \\ 0, & \text{otherwise} \end{cases} \quad (3.9)$$

$$x_{ij} \in \{0,1\} \forall (i,j) \in A \quad (3.10)$$

It is important to recognize that complications arise due to incorporation of SSD into the model. This is because the path cost is no longer the direct summation of the cost of links that comprise the path. Consequently, the classic algorithms developed to solve the traditional shortest path problems, such as Dijkstra's algorithm, are no longer applicable to solve this problem because they are based on the additivity assumption.

The non-additive concept can be illustrated with a simple two-link example below. The travel time observations on each individual link are also listed in the following table. Based on the semi-standard deviation equation which uses the mean as the benchmark, the SSD on link 1 and 2 is 1.63 and 1.76, respectively. If SSD were additive, then the path SSD would be  $1.63+1.76=3.39$ . However, the actual SSD based on obtained path travel times is 2.53 instead. By comparing those values, we can easily conclude that the SSD on a path is not equal to the summation of that on component links, attesting to the non-additive property of SSD.



**Figure 3.1 Two-Link Example**

**Table 3.2 Two-Link Travel Time Samples**

Travel Time	Link 1	Link2	Path
1	6	5	11
2	8	4	12
3	12	8	20
4	7	10	17
5	9	5	14
SSD	1.63	1.76	2.53

### 3.3 SOLUTION ALGORITHM

#### 3.3.1 Overview

A variety of algorithms have been applied in the existing literature to the routing problem with consideration of travel time reliability. They can be grouped into three main categories:

- Mathematical programming based algorithms(33; 56)
- Simulation based methods(41; 76)
- Genetic Algorithm (GA) based approaches(42; 57)

GA is an iterative process, which mimics the natural selection and evolution scheme(77). The algorithm starts with an initial population that consists of a pre-determined number of chromosomes and iterates through the GA operators to generate a new population each

time until the optimal solution or the specified stop criterion is achieved. It is a widely applied stochastic and heuristic technique to solve many complex optimization problems(78). It is easy to understand and apply, and effective to solve non-continuous and non-differentiable problems. Also, the parallel searching ability helps to keep the solutions from being trapped at the local optima. Because of those advantages, GA is also adapted in this research to solve the proposed shortest path model.

The overview of the GA-based solution procedure is as follows.

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### **Genetic Algorithm**

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Step 1: Algorithm Input. Specify the number of population, crossover probability, mutation probability, and number of iterations.

Step 2: Population Initialization. Generate an initial population with chromosomes representing feasible paths in the route network.

Step 3: Fitness Assignment. Calculate the fitness of each chromosome on path in the current generation using the travel time matrix.

Step 4: Selection Operation. Use the binary tournament selection without replacement method to choose the more fit chromosomes from the current generation to take into next generation.

Step 5: Crossover. Apply the crossover operator to previously selected parent chromosomes to generate new offspring chromosomes and ensure the newly generated chromosomes are acyclic.

Step 6: Mutation. Apply the mutation operator to genetically modify chromosomes in current generation and ensure modified chromosomes are acyclic.

Step 7: Termination. Stop the procedure and output the final optimal path, if the stopping criteria are met. Otherwise, go back to Step 3.

---

### **3.3.2 Chromosome Encoding and Population Initialization**

Genetic algorithm is an iterative process which mimics the natural selection and evolution scheme. The algorithm starts with an initial population which is consisted of a pre-determined number of chromosomes and iterates through GA operators to generate a new set of population each time until the stop criterion is satisfied. Each chromosome in the population set represents a potential solution to the problem being solved. Therefore appropriately encoding the chromosome serves a critical role during the evolutionary process. In particular, to apply GA in the routing problem, each chromosome constructed from the encoding procedure should represent a feasible path which actually exists in the network and at the same time doesn't contain any loop.

To construct a feasible path, the procedure below is followed. At first, the first gene on the chromosome is set to be the origin node of the path. Then based on the topological and connectivity information of the transportation network, all the successor nodes that are connected to the origin node are identified and stored in a temporary vector. Next, a random integer number that is between 1 and the number of previously found successor nodes is generated and used to reference the node that will be selected and encoded at the second gene. In the meantime, all the nodes that are connected to this node are eliminated from the scan list. This is to ensure the same node will not be selected again in the

remaining process, therefore preventing the loop from the path. The node selection process is repeated until the destination node is reached.

The above procedure is repeated until pre-specified population size is reached and then we will have a randomly generated initial population.

### 3.3.3 Fitness Assignment

Fitness assignment is a process to quantitatively evaluate the fitness associated with each chromosome in current generation. The more fit individuals are more likely to be selected and used to reproduce next generation, while less fit ones are more likely to be eliminated from current population. The fitness of a chromosome usually can be calculated directly based on the objective function. The fitness function on the particular path  $p_k^{r,s}$  is defined as

$$Fit_k^{r,s} = \frac{1}{\eta_k^{r,s}} \quad (3.11)$$

Based on the sequential order of nodes in each chromosome, the corresponding total cost in terms of the mean and semi-standard deviation can be obtained, and then associated fitness can be calculated by taking the reciprocal. A more fit chromosome means less travel cost along that path.

### 3.3.4 Selection Operation

After evaluation of each solution in the fitness assignment step, a selection operator is developed to choose the more fit chromosomes from current generation to move into next generation. Various approaches have been proposed in existing literature(79). The first option is a completely random procedure, which is called roulette wheel selection. It is developed in a way that the probability of a chromosome getting selected is proportional to its fitness compared to the total fitness from all the chromosomes. Another option is through elitist selection, which is a deterministic scheme. It involves ranking all the chromosomes in the population by their respective fitness and only individuals with highest fitness are selected. The third alternative is the tournament selection, which involves both random and deterministic process. Each time a fixed number of chromosomes, which is called the tournament size, are randomly selected from the population and the best ones are picked out. Based on whether chromosomes are placed back to the original population, the method can be further specified as tournament selection without replacement and tournament selection with replacement. In this study, the binary tournament selection with replacement is used.

### 3.3.5 Crossover Operation

Crossover operator is one of the schemes used to generate new offspring chromosomes from previously selected parent chromosomes(79; 80). In order to ensure the feasibility of produced child chromosomes, some restrictions are applied during the process. For two selected parent chromosomes, they have to have at least one common node excluding the origin and destination nodes so that the crossover operation can be performed. If more

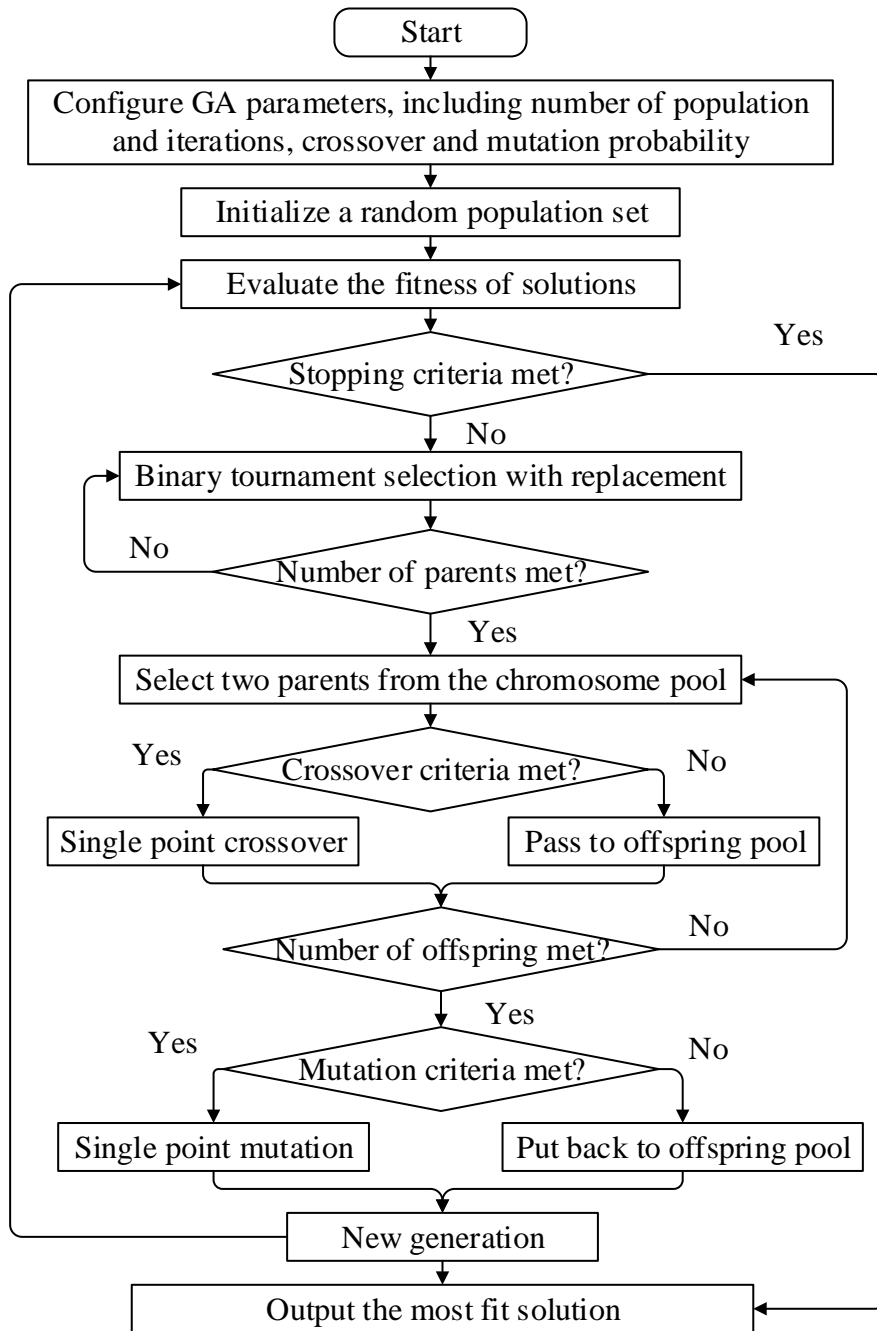


than one set of shared nodes are present, one of them is randomly selected as the crossover node. New offspring is generated by integrating the partial sequence from beginning node to the selected node from one parent and the partial sequence from the selected node to the destination node from the other parent. In addition, the sequential order of nodes in each newly generated chromosome should be examined to ensure they are acyclic. If a loop is present, all the nodes except for the first one are removed.

### **3.3.6 Mutation Operation**

Mutation operator is the other scheme adopted in this study to genetically modify chromosomes in current population and therefore ensure genetic diversity(79). During the mutation operation, a node located in between the origin and destination node is randomly selected. Then the selected mutation node is serving temporarily as the starting node and the same procedure developed in the population initialization section is applied to form the remaining partial path. It is important that existing nodes preceding the mutation node shall not be selected again to eliminate possible loops in the final path. Therefore, similar to the population generation procedure, starting from the origin node, the initial scan list is first enumerated to exclude any nodes that are connected to the node under evaluation. This step is iterated until the mutation node is reached.

Combining all the steps, the complete GA-based solution flow chart is shown in Figure 3.2.



**Figure 3.2 Genetic Algorithm Flowchart**

### 3.4 TUNING ALGORITHM PARAMETERS

As GA is a stochastic method in nature, it cannot guarantee to always find the global optimal solution. The performance of the algorithm is greatly related to the parameters implemented, including number of population, number of generation, crossover probability, and mutation probability. For instance, a larger number of generation would expand the searching space and increase the probability of finding the optimal path, yet it will be done in the expanse of longer executing time. In contrast, a smaller number of

generation would expedite the evolutionary process, however, the algorithm may suffer from not having a good coverage of searching space.

Hence, it is critical to quantitatively assess the performance of GA on finding the optimal path for the proposed approach with various combinations of parameters so that the best combination can be determined for implementation. In order to do so, the global optimal has to be acquired beforehand as ground truth. Due to the special model structure, it is hard to analytically solve the optimization problem and find the global optimal. However, it has been a mature practice to find the shortest path in the deterministic context, for example, only looking at average travel time in this case. In this study, the classical Label Correcting (LC) algorithm is adopted to find the shortest path in terms of average travel time(81). Then, GA is implemented and the identified optimal path is compared to that from LC algorithm. The relative percentage error, i.e.  $100 \times (\text{actual objective value} - \text{minimum objective value}) / \text{minimum objective value}$ , and computing time are selected as performance metrics to evaluate GA solutions.

The implementation procedure of the LC algorithm is provided as follows.

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### **LC Algorithm**

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Step 1: Initialization. Let  $p^{ss}$  be the path from  $s$  to itself and  $t_0^{ss}$  be the average travel time which is first set to zero. Initialize the scan list  $Q = \{p^{ss}\}$ .

Step 2: Select the first path from  $Q$ , and denote it as  $p_k^{js}$ , then delete it from  $Q$ .

Step 3: For any predecessor node  $i$  of  $j$  and  $i$  is not contained in current  $p_k^{js}$ , create a new path  $p_k^{is} = p_k^{js} + a^{ij}$ , and update the path travel time  $t_k^{is} = t_k^{js} + r^{ij}$  and the average travel time  $\mu_k^{is}$ .

Step 4: Compare the updated path travel time  $\mu_k^{is}$  of path  $p_k^{is}$  to  $\mu_l^{is}$  of path  $p_l^{is}$ , which is the existing shortest travel time path. If  $\mu_k^{is} < \mu_l^{is}$ , replace  $p_l^{is}$  with  $p_k^{is}$ ; otherwise, drop  $p_k^{is}$ .

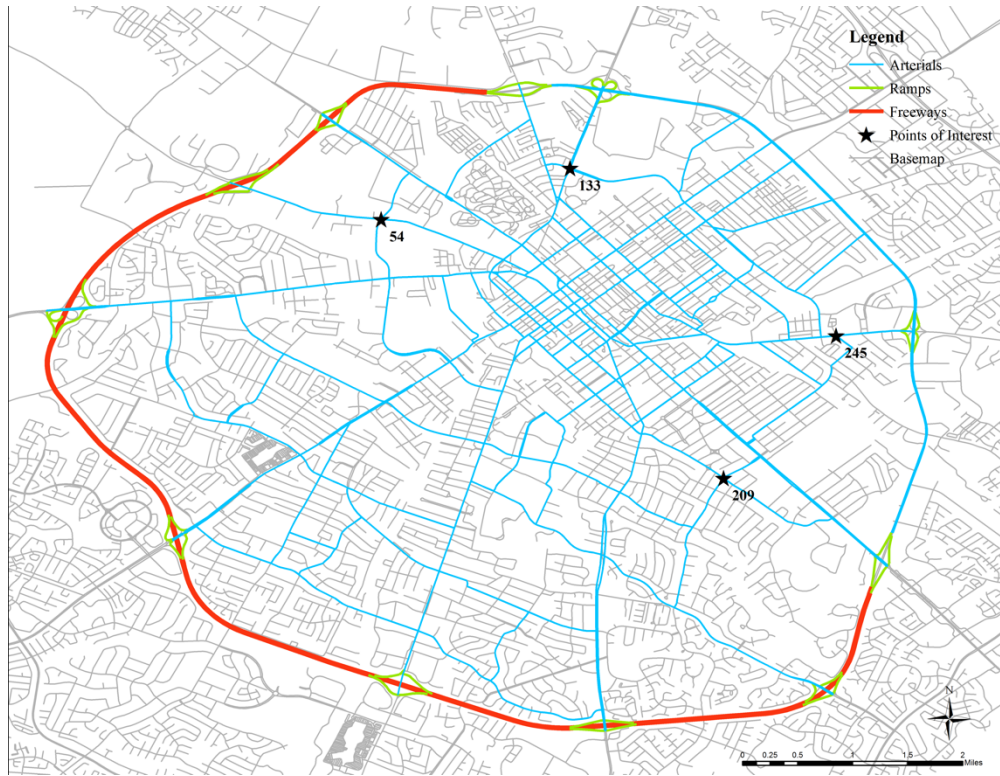
Step 5: If  $Q$  is empty, terminate the algorithm; otherwise go to step 2.

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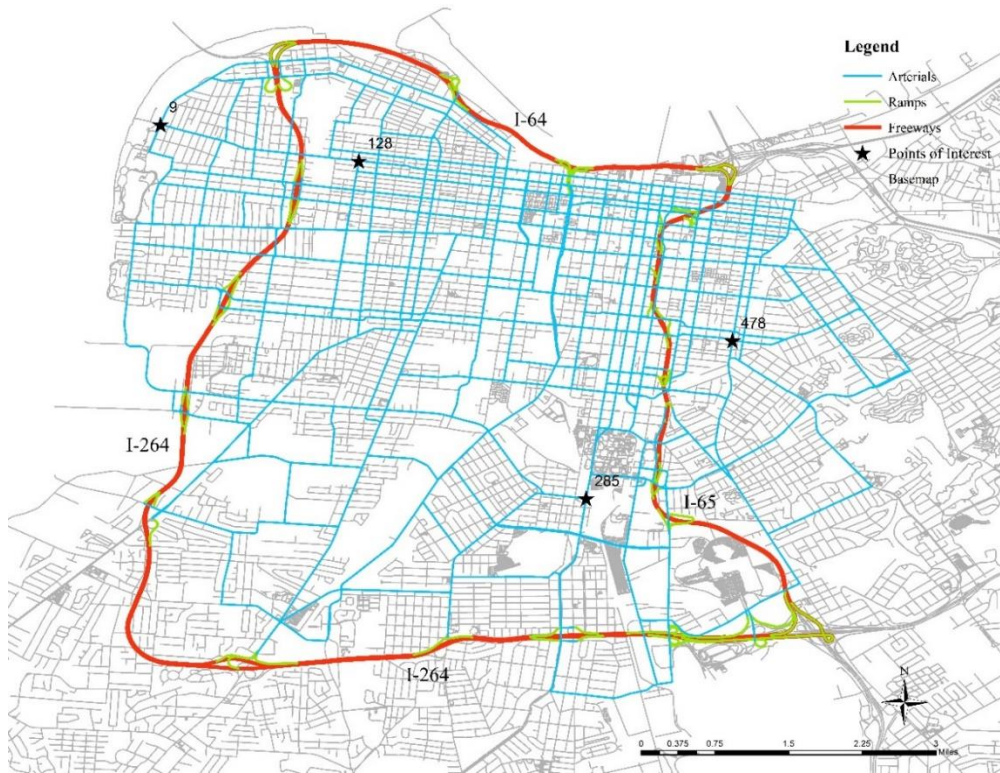
## **3.5 NUMERICAL EXPERIMENTS**

### **3.5.1 Network and Travel Time Data**

In this section, numerical experiments are designed to evaluate the proposed routing model and the effectiveness of GA-based solution method. Two real-world networks from Lexington and Louisville urban areas in Kentucky, as shown in Figure 3.3 and Figure 3.4, are selected. They are obtained from a navigation map provided by a private company, where large number of GPS-based probe data are also available. The original network comes in as a shapefile with granularity at block-by-block level as shown by basemaps in following figures. Due to the penetration rate of probe vehicles on lower functional class roads is relatively low, there may not be enough samples to infer credible travel time distribution over the whole year. Thus, some pre-processing work is required to reduce the network size to only include higher functional class roads and then obtain the machine-readable topology information such as the link-node incidence table for following route choice analysis.



**Figure 3.3 Lexington Area Route Network**

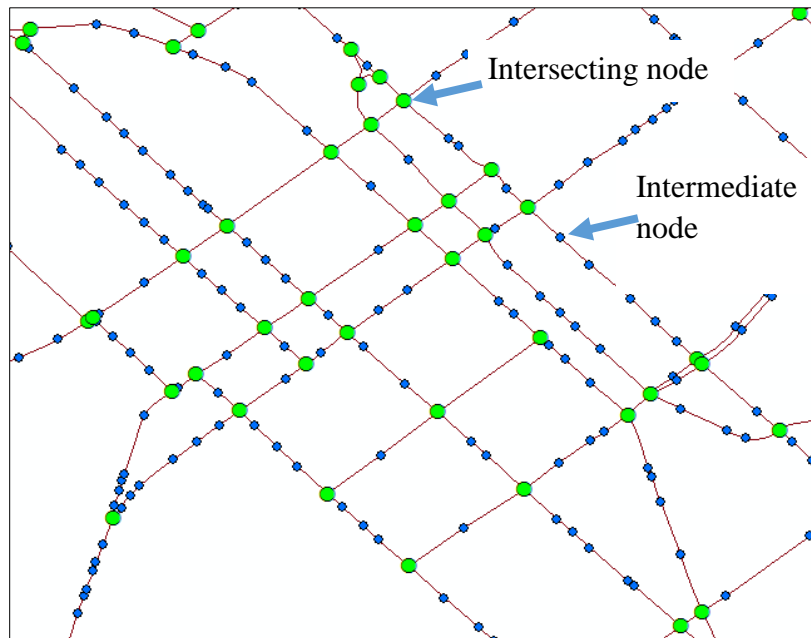


**Figure 3.4 Louisville Area Route Network**

At first, only higher functional class roads (colored lines in above maps) are selected, and then ArcGIS software is used to generate end nodes for all the links on each network and count number of links that intersect at each end node. To further reduce the size of networks under study, following definitions are first provided for different node types to facilitate the discussion.

- A node is an intersecting node where more than two links intersect.
- A node is an intermediate node where only two links intersect.

An example of different types of nodes is shown in Figure 3.5. Since the turning option is restricted at intermediate nodes and on the links in between any two intermediate nodes, those nodes and links have no impact on the final route choice between a selected OD pair. Therefore, they should be aggregated into one longer segment, bounded by intersecting nodes.



**Figure 3.5 Network Topology Example**

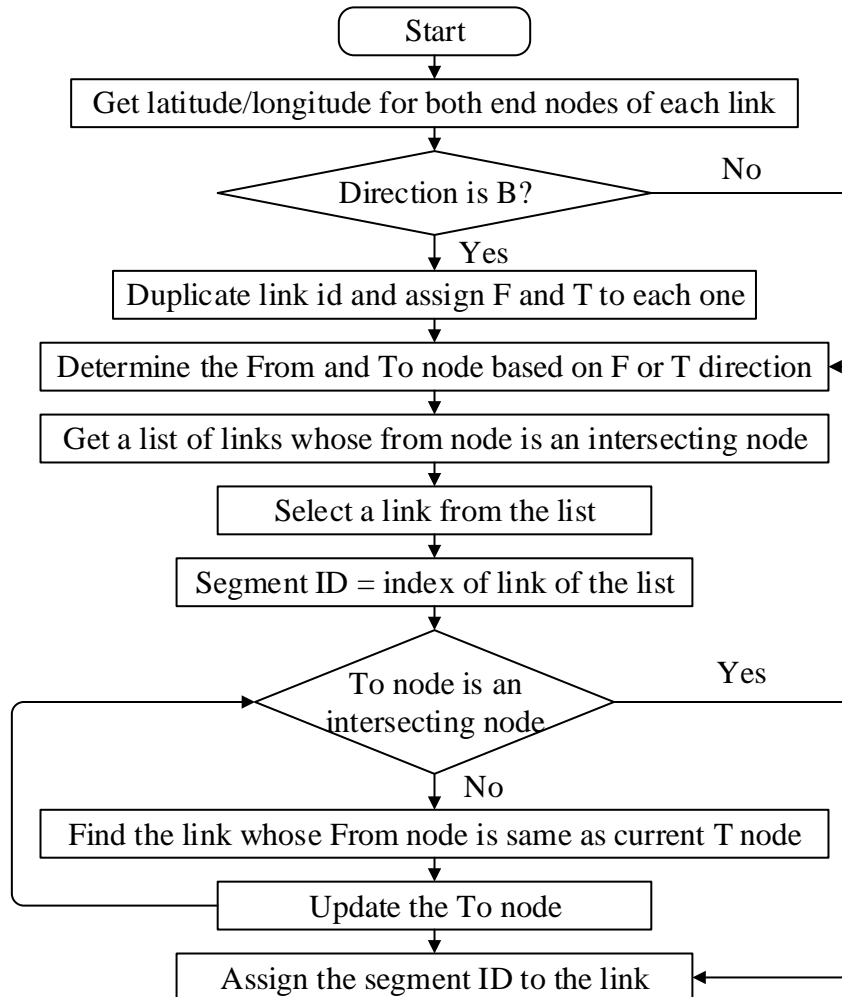
On difficulty arises due to links are directional on the network while most links share a single line and same link ID for both directions in the provided shapefile. Therefore, the directional information has to be accounted for during the aggregating process. Different from standard linear reference system that has been used by public transportation agencies, the navigation map provider uses a reference node based definition to infer the direction information.

- A node is the reference node of a link if its latitude is lower.
- If the latitudes of both nodes are identical, the node with a lower longitude is the reference node.

Based on reference node definition, the direction of a link is then defined as follows.

- The direction of a link is from the reference node or F if the link traverses from the reference node to the non-reference node.
- The direction of a link is to the reference node or T if the link traverses from the non-reference node to the reference node.

According to above definitions, the latitude and longitude of each node have to be known. Such information is obtained by ArcGIS Add Coordinates tool. Since our goal is to figure out which links belong to the same longer segment, the following logic is developed.



**Figure 3.6 Link Aggregation Procedure**

After aggregation process, the Lexington network consists of 263 nodes and 636 segments while Louisville network consists of 528 nodes and 1269 segments. In particular, there is a ring road which is partially freeway in the Lexington area and there are three interstate corridors in the Louisville area, as displayed on the maps. An example of obtained link-node incidence table is shown in Table 3.3.

**Table 3.3 Link-Node Incidence Table**

Segment ID	From Node	From Long	From Lat	To Node	To Long	To Lat	Length (ft)
1	1265	-84.470	37.997	997	-84.486	38.007	5630.2
2	297	-84.521	37.998	252	-84.525	37.998	1274.4
3	1327	-84.464	37.998	1307	-84.466	37.996	1083.8
4	1295	-84.468	37.997	1319	-84.465	37.995	941.3
5	1295	-84.468	37.997	1284	-84.469	37.994	1079.5
6	1322	-84.465	37.995	1328	-84.464	37.998	989.8
7	1319	-84.465	37.995	1322	-84.465	37.995	57.2
8	1278	-84.469	37.994	1319	-84.465	37.995	1235.2
9	912	-84.491	37.991	820	-84.495	37.992	1121.8
10	912	-84.491	37.991	660	-84.499	37.991	2452.5

The GPS-based speed data is also obtained to provide link travel times across network. The speed data is organized by 15-minute increments and available by time of day and day of week in each month on every link. Based on previously obtained topological relationship, the speed data at the shorter link level is converted to the longer segment level using following equations(82).

$$t_{seg} = \sum_{i=1}^n \frac{l_i}{v_i} \tag{3.12}$$

where  $t_{seg}$  represents the travel time on the segment;  $n$  is the number of links belong to the segment;  $l_i$  and  $v_i$  are the length and speed of  $i$ th link, respectively.

The aggregated travel time table is shown as follows.

**Table 3.4 Segment Travel Time**

Segment ID	Month	Day	Time	Travel Time (sec)
1	January	Monday	15:00	115.28
1	February	Tuesday	15:15	102.37
1	March	Wednesday	15:30	91.18
1	April	Thursday	15:45	80.31
1	January	Friday	17:45	95.02
1	February	Monday	17:30	95.02
1	March	Tuesday	16:30	106.04
1	April	Wednesday	16:45	106.04
1	May	Thursday	17:00	96.45

It should be noted that only the afternoon peak period (3-6pm) during weekdays is used in the experiments. Therefore, each segment has 12 months\*5 weekdays\*3 hours\*4 15-minute intervals=720 individual travel time realizations. The preliminary analysis indicates the skewness values for individual segments vary from -0.36 to 26.5, with only three segments from two networks having negative values and the rest having positive values; indicating the travel time distributions on the network are asymmetrical.

### 3.5.2 Model Calibration

Both GA and LC algorithms are coded in Matlab on a Windows 7 personal computer equipped with a 3.30 GHz Intel i5 CPU and 8 GB RAM. At first, it is important to evaluate the performance of GA approach under different parameter settings. As it is impractical to test every combination of input parameters, a wide range of input values are selected for testing, as shown in Table 3.5 -Table 3.6.

**Table 3.5 Options of GA Parameters**

<b>Level</b>	<b>1</b>	<b>2</b>	<b>3</b>
<b>Generation</b>	200	400	600
<b>Population</b>	20	40	80
<b>Crossover</b>	0.25	0.5	0.75
<b>Mutation</b>	0.1	0.3	0.5

**Table 3.6 Combination of GA Parameters**

<b>Combination</b>	<b>Generation</b>	<b>Population</b>	<b>Crossover</b>	<b>Mutation</b>
<b>1</b>	1	1	1	1
<b>2</b>	2	1	2	2
<b>3</b>	3	1	3	3
<b>4</b>	1	2	2	3
<b>5</b>	2	2	3	1
<b>6</b>	3	2	1	2
<b>7</b>	1	3	3	2
<b>8</b>	2	3	1	3
<b>9</b>	3	3	2	1

The LC algorithm is also applied to find the shortest path whose travel time serves as the baseline for comparison between each OD pair under study. The relative percentage error, i.e.  $100 \times (\text{actual objective value} - \text{minimum objective value}) / \text{minimum objective value}$ , and computing time are recorded for each test run. Note that although the model is not intended for real-time applications, a more computationally efficient algorithm is desired because once implemented it will allow transportation agencies to more quickly run travel models and thus greatly improve their productivity. This is especially essential for large scale networks where hundreds of thousands of OD pairs are under evaluation. The average percent error (APE) and average computing time are then obtained by



averaging over the corresponding values from five repeated experiments for each set of input values.

In this experiment, two OD pairs are selected from each network to conduct the performance evaluation. For Lexington network, one is from node 209 to node 133 and the other is from node 54 to node 245. Similarly, for Louisville network, one is from node 128 to node 478, and the other is from node 285 to node 9. Locations of those points of interest are illustrated in Figure 3.3 and Figure 3.4. According to LC, the shortest travel time paths for these four OD pairs take 21.5, 17.7, 19.6, and 22.2 minutes, respectively. Next, the GA approach with each combination of input parameters as specified in above tables is implemented and run 5 times. Therefore, 45 trial runs in total for each OD are conducted. The results after averaging quality measures from 5 separate runs for each combination are summarized in Table 3.7 and Table 3.8 below.

**Table 3.7 GA Performance on Lexington Network**

Combination	OD 209-133		OD 54-245	
	APE (%)	CPU Time (sec)	APE (%)	CPU Time (sec)
1	17.5	23.7	72.4	24.7
2	11.9	44.6	20.8	43.5
3	2.9	52.9	11.3	53.0
4	1.4	37.8	2.1	35.7
5	1.5	99.7	28.1	101.9
6	14.6	141.3	10.1	139.4
7	6.6	92.9	5.0	95.4
8	7.2	158.8	5.7	165.3
9	4.3	309.5	13.9	325.9

**Table 3.8 GA Performance on Louisville Network**

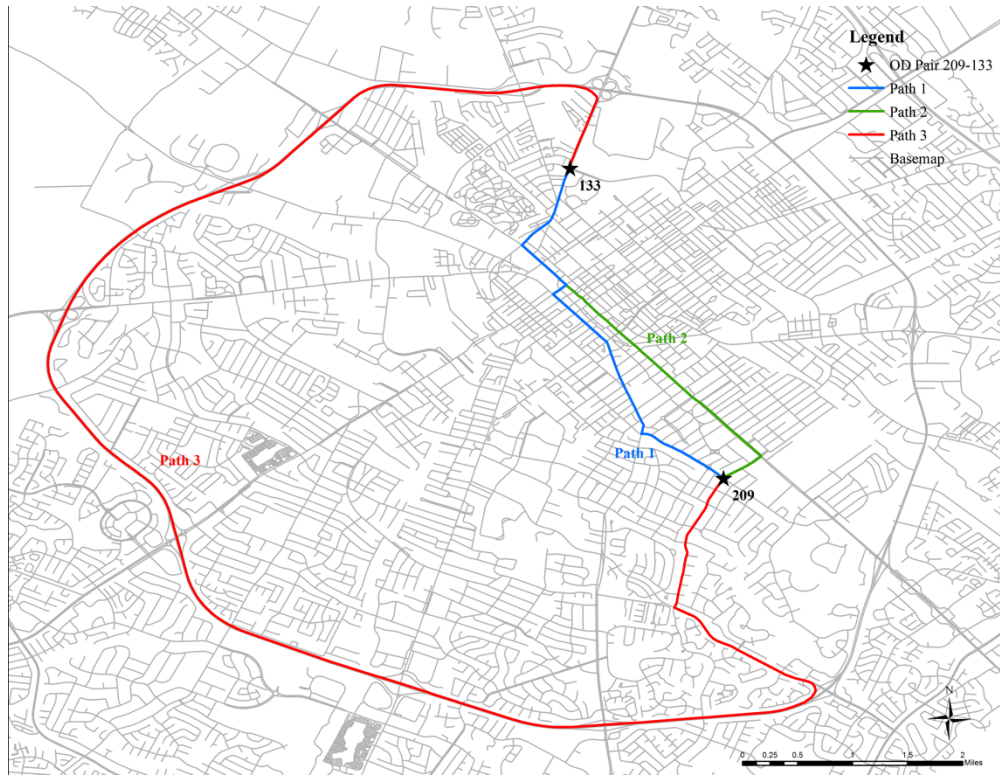
Combination	OD 128-478		OD 285-9	
	APE (%)	CPU Time (sec)	APE (%)	CPU Time (sec)
1	42.5	120.6	48.6	101.3
2	6.4	218.6	10.1	186.7
3	11.6	264.1	11.4	211.1
4	4.1	175.8	7.7	139.9
5	4.6	505.2	10.2	404.1
6	4.6	709.6	4.0	553.0
7	2.3	436.9	5.7	363.5
8	4.0	739.3	5.2	611.5
9	5.5	1507.1	7.6	1250.2

Based on above tables, it is clearly seen that the performance of GA is significantly affected by input variables. The average percentage error can vary from as low as 1.4% to

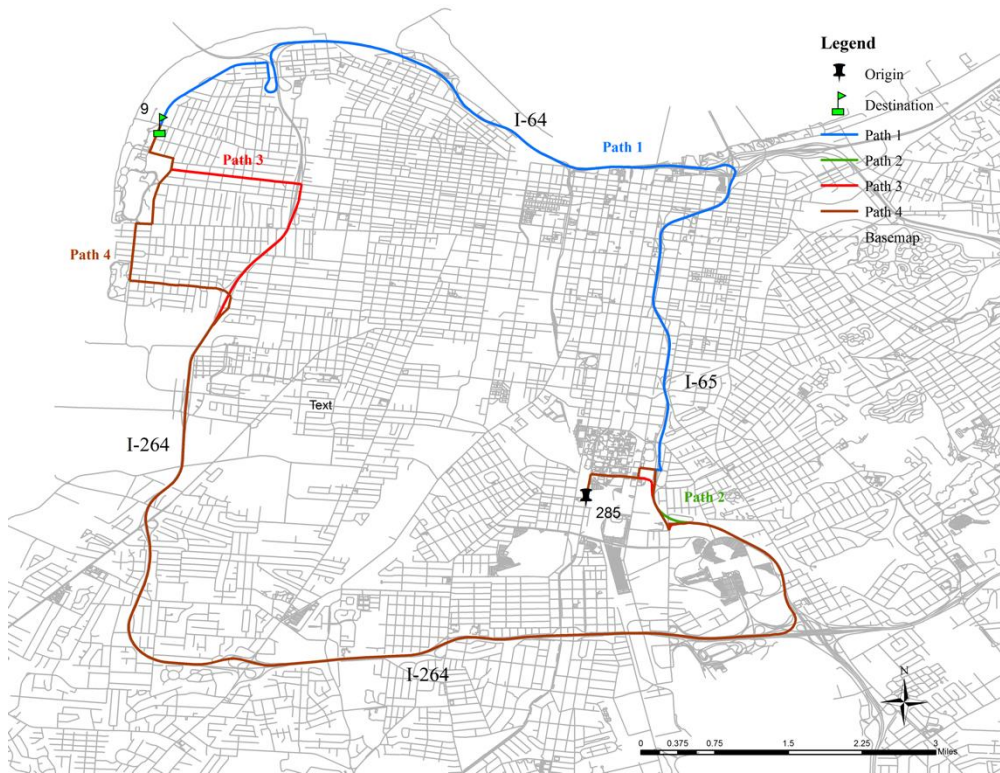
as high as 72.4%, depending on specific values implemented. At the same time, the computing time are also significantly different among combinations tested and between networks. It can also be observed that although the size of Lexington network is half of that of Louisville network, the program running time is only 1/5-1/4 of the time took by the Louisville network when using same parameter setting, indicating the non-linear impact of network size on the algorithm performance. Combining results from both networks, it is determined that GA with combination 4 and 7 have relatively better performance in that they have lower percentage errors while take less amount of time as well. From the perspective of individual network, combination 4 performs best on Lexington network while combination 7 works best on Louisville network. Thus, they are selected for implementation in further analysis for respective networks.

### **3.5.3 Model Implementation and Discussion**

In the following analysis, the proposed route choice model is evaluated with more details for OD 209-133 and OD 285-9 from Lexington and Louisville network, respectively. In addition to the proposed mean-semi-standard deviation (MSSD) model, the minimum expected travel time (ETT) and mean-standard deviation (MSTD) models are also investigated. It is expected that different travelers may have different attitudes towards the risk of being late and such attitudes can also vary depending on their specific purpose of trip. In order to understand the impact of traveler's sensitivity to the risk on the routing results, different  $\lambda$  values varying from 0.5 to 4 are analyzed in the experiments. Although travelers may adopt a different  $\lambda$  value with respect to SSD in the MSSD model from that in the MSTD model, the same value is used here when comparing two models. The results are summarized in Figure 3.7-Figure 3.8 and Table 3.9-Table 3.10. The optimal objective values of the models are marked in bold in Table 3.10.



**Figure 3.7 Routing Options for OD Pair 209-133**



**Figure 3.8 Routing Options for OD Pair 285-9**

**Table 3.9 Descriptive Statistics of Identified Path Travel Times**

OD Pair	Path ID	Mean (min)	STD (min)	SSD (min)	Skewness	95 <sup>th</sup> travel time (min)	99 <sup>th</sup> travel time (min)
209-133	1	21.54	15.27	13.87	2.84	53.26	83.59
	2	21.67	11.71	10.08	1.78	37.74	64.06
	3	24.15	8.42	7.89	4.48	38.29	64.23
285-9	1	22.18	8.50	7.36	1.96	39.15	51.95
	2	24.21	6.28	5.73	3.56	34.34	56.73
	3	24.49	6.27	5.72	3.56	34.58	56.96
	4	27.04	6.32	5.63	3.14	36.99	56.40

**Table 3.10 Objective Values of Identified Paths**

OD Pair	Path ID	ETT	0.5		1		2		4		
			MSTD	MSSD	MSTD	MSSD	MSTD	MSSD	MSTD	MSSD	
209	1	<b>21.54</b>	29.18	28.48	36.81	35.41	52.07	49.28	82.60	77.02	
-	2	21.67	<b>27.53</b>	<b>26.71</b>	33.38	<b>31.75</b>	45.09	41.83	68.51	61.98	
133	3	24.15	28.36	28.09	<b>32.57</b>	32.03	<b>40.99</b>	<b>39.92</b>	<b>57.83</b>	<b>55.69</b>	
285-9	1	<b>22.18</b>	<b>26.43</b>	<b>25.86</b>	30.68	<b>29.54</b>	39.19	36.90	56.19	51.62	
	2	24.21	27.35	27.08	<b>30.50</b>	29.95	<b>36.78</b>	<b>35.68</b>	<b>49.34</b>	<b>47.14</b>	
	-	3	24.49	27.63	27.35	30.76	30.21	37.03	35.94	49.57	47.38
	9	4	27.04	30.20	29.85	33.35	32.67	39.67	38.30	52.30	49.57

Based on the test results, three distinct paths are found as optimal depending on the specific model of interest for OD pair 209-133. Similarly, four different paths are identified for OD pair 285-9. The average travel time and travel time reliability both in terms of STD and SSD vary noticeably among candidate paths connecting each OD pair. The positive skewness values of all the paths indicate route travel times are also asymmetrically distributed with a longer right tail on both networks examined. From Table 3.9, Path 1 provides the shortest average travel time and hence, will be chosen as optimal by ETT model for both OD pairs. However, corresponding travel time variations in terms of either STD or SSD are also the highest.

If both average travel time and travel time reliability are included in the routing process, the identified optimal path may be different. For OD pair 209-133, Path 1 is no longer optimal under all four evaluated scenarios when either STD or SSD is considered in the model. Path 2 becomes the most appealing route based on both MSTD and MSSD models when  $\lambda$  is 0.5. In general, paths with less variable travel time would be identified as the optimal routes as travelers become increasingly intolerant to travel time variation, which is marked by the increasing  $\lambda$  values. This appears to be the case regardless of whether SSD or STD is used as the reliability metric. At some point (when  $\lambda$  is set to 1 in this example), MSTD and MSSD models give different optimal paths. The MSTD reports Path 3 as the most attractive choice while the MSSD model prefers Path 2. This

difference can be explained by the distribution of travel times on these two paths. Historical data indicates travel times on Path 3 have a more skewed distribution with a longer right tail compared to that on Path 2. This is consistent with the characteristics of an expressway type facility where while day-to-day variation in travel times might be rather small, major incidents can cause excessive delays due to limited access to other routes. Since SSD focuses on the right part of the distribution only, it would be more sensitive to the long tail or, in other words, excessive delays. As a result, when SSD is applied in the routing process, the MSSD model tends to avoid such route.

A similar occurrence can be observed for OD pair 285-9 in that different optimal paths may be resulted depending on the model and reliability metric applied in the experiments. Since the largest difference of average travel time on the four paths is approximately 5 minutes, compared to that of STD and SSD with roughly 2 minutes, it will take a higher  $\lambda$  value to shift away from the ETT-optimal path (i.e. Path 1). When the shift occurs at  $\lambda = 1$ , Path 2 becomes optimal. It can be seen that Path 2 and Path 3 are essentially the same, except for where they take ramps on to I-65. Under this situation, when the optimal path switches to Path 2, it will remain the best choice even when  $\lambda$  increases hereafter. When the disagreement happens at  $\lambda = 1$ , the MSTD model identifies Path 2 as the optimal while MSSD model still suggests Path 1. This can be explained based on the skewness statistics and higher percentile travel times as they clearly indicate that Path 2 travel time is more skewed and has a longer tail at the right end of its travel time distribution. This indicates long delays on Path 1 are less excessive than those on Path 2. Because of its tendency to penalize paths with long tails, the MSSD model identifies Path 1 as optimal.

### **3.6 CONCLUSIONS**

In this chapter, a new routing strategy is proposed to take travel time reliability into consideration when the underlying travel time distribution is asymmetrical. It is argued that under this circumstance, SSD is a better measure of reliability due to its more intuitively meaningful explanation with respect to traveler's decision-making behavior and accountability for the asymmetry of the probability distribution. A sampling based method is adopted to incorporate correlations among link travel times and to circumvent the complicated and time consuming procedure to derive the correlation matrix for the whole network. Then, a Genetic Algorithm based method is developed to solve the problem as it is deemed suitable to solve the nonlinear models. Numerical experiments using real-world networks with varying sizes and field-collected speed data are conducted to test the proposed models.

Since the GA approach relies on heuristic search, implemented parameters will have a collective impact on its performance. In order to enhance the algorithm, a parameter tuning process is first conducted on two real-world networks. The Label Correcting algorithm is implemented to find the ground truth for selected OD pairs. The finding from the analysis confirms that the number of population and generation, crossover probability, and mutation probability collectively have a significant impact on final results. In addition, it is found that the optimal parameter combination can be different depending on the specific network under study.

After model calibration, proposed path finding models are evaluated. In general, when the mean travel time among all alternatives is similar, more reliable path (i.e. those marked by lower STD and/or SSD values) will be chosen as optimal. Those travelers who place more emphasis on travel time reliability tend to gravitate toward the route with the least variability. However, it also depends upon the relative scale of the average travel time and travel time variability, as well as the degree of sensitivity to the reliability. When the mean travel times among all alternative paths are similar, slight difference in the reliability measure could change the suggested optimal route. Otherwise, a more substantial difference in the reliability metric or a traveler population that puts more weight on travel time reliability, or both, will be needed to effect a change.

In most cases in the test, the MSTD and MSSD models gave the same suggested optimal route. This is hardly surprising because STD and SSD are highly correlated, and the distribution of travel times is not heavily skewed for all paths according to the field data. The scale of the mean travel time, reliability measure, and  $\lambda$  also play a role. In this study, MSSD suggests a different optimal route than MSTD at certain points. In those cases, the MSSD model considers the route with less excessive delays as the most attractive choice.

## CHAPTER 4 A MULTI-OBJECTIVE PATH FINDING MODEL

### 4.1 INTRODUCTION

In the previous chapter, the optimal path finding model focusing on minimizing the travel cost that consists of average travel time and travel time reliability is studied. The semi-standard deviation is proposed as the reliability measure and integrated into the cost function via a weighting parameter, namely the reliability ratio. Although it is relatively easy to formulate and solve, the model suffers from several weaknesses. First, it is practically difficult to determine the actual reliability ratio travelers would use in reality. This is because for the present there is a lack of detailed survey data to empirically derive those values for different groups of travelers. They may have different views on travel time uncertainty and even the same traveler may change their attitude under different circumstances. For instance, catching a flight certainly requires allocating more time for the trip than going shopping. Second, even if the ratio could be determined, the linear combination of two components can also lead to some unrealistic outcomes. This is due to the single objective model cannot differentiate paths with equal objective values, even though average travel time and SSD are significantly different among those paths. However, different mean and SSD represent distinct reliability ratios and hence, contradict to the initial assumption of prior knowledge of reliability ratio. Furthermore, there may be paths that are potentially attractive to rational travelers but ignored by the single objective model.

With regard to above concerns, the multi-objective formulation seems more suitable to account for the route choice decision, which involves a trade-off process that takes multiple conflicting factors into consideration. In this case, instead of finding a single optimal route with minimum cost, the multi-objective optimization will optimize both average travel time and reliability simultaneously. This usually results in an optimal frontier that includes many Pareto-optimal alternatives that can be provided for travelers' further route choice decisions.

In this chapter, the single objective problem studied in previous chapter is expanded to a multi-objective problem. The model integrates advantages of appealing characteristics underlying SSD and multi-objective formulation. The rest of the chapter is organized as follows. In section 4.2, the multi-objective model is applied to reformulate the routing problem with SSD as the reliability indicator. The sampling-based approach directly using field collected travel time data is also adopted to account for correlations. Section 4.3 introduces an multi-objective evolutionary algorithm to solve the proposed model. Due to the stochastic nature of the search heuristic, the performance of algorithm is evaluated in Section 4.4 based on the established relationship between the mean-semi-standard deviation (MSSD) dominance rule and second-order stochastic dominance (SOSD) rule. Section 4.5 provides numerical examples using Louisville network to test proposed models and solution algorithms. The final section summarizes the analysis and concludes the chapter.

## 4.2 PROBLEM STATEMENT

### 4.2.1 Definitions

Consider a directed network  $G = (N, A, D)$  where  $N$  is the set of nodes,  $A$  is the set of links, and  $D$  is the set of probability distributions of travel times associated with individual links. Let  $r \in N$  and  $s \in N$  represent the origin and destination node respectively. Let  $t_k^{rs}$  be the random travel time on path  $p_k^{rs} \in P^{rs}$ , where  $P^{rs}$  denotes the set of paths connecting the origin and destination. Let  $\mu_k^{rs}$ ,  $\sigma_k^{rs}$  and  $\tau_k^{rs}$  represent the mean, STD, and SSD of  $t_k^{rs}$  along  $p_k^{rs}$ , respectively. Easily, the semi-standard deviation of  $t_k^{rs}$  can be derived as

$$\tau_k^{rs} = \sqrt{E[(t_k^{rs} - b)_+^2]} \quad (4.1)$$

where  $z_+ = \max(z, 0)$ ,  $E[\cdot]$  is the expectation operator, and  $b$  is the benchmark value specified by decision makers.

One difference from above equation compared to Equation (3.2) in Chapter 3 is the benchmark value. In this chapter, the use of average travel time as benchmark is relaxed. In other words, different threshold values can be specified in the model to offer more flexibility in SSD calculation. With such relaxation, SSD is able to reflect different behaviors from travelers with different degrees of sensitivity concerning the uncertainty as the benchmark value can be easily shifted. A lower benchmark indicates a lower degree of tolerance of unreliability, thus a more risk-averse attitude from the traveler.

With the multi-objective formulation of the routing problem involving stochastic travel times, the model tries to minimize average travel time and travel time unreliability simultaneously. The multi-objective model incorporating SSD is formulated as shown below.

$$\begin{cases} \min \mu^{rs} \\ \min \tau^{rs} \end{cases} \quad (4.2)$$

The model with respect to STD is also formulated so that two models can be compared.

$$\begin{cases} \min \mu^{rs} \\ \min \sigma^{rs} \end{cases} \quad (4.3)$$

In order to solve the models, following path dominance rules in terms of mean and dispersions including STD and SSD are first defined.

**Definition 1** A path  $p_k^{rs} \in P^{rs}$  dominates another path  $p_l^{rs} \in P^{rs}$  by the mean-standard deviation dominance rule (MSTD) or  $p_k^{rs} \succ_{MSTD} p_l^{rs}$ , if  $\mu_k^{rs} \leq \mu_l^{rs}$  and  $\sigma_k^{rs} \leq \sigma_l^{rs}$  with at least one inequality holds.



**Definition 2** A path  $p_k^{rs} \in P^{rs}$  dominates another path  $p_l^{rs} \in P^{rs}$  by the mean-semi-standard deviation dominance rule (MSSD) or  $p_k^{rs} \succ_{MSSD} p_l^{rs}$ , if  $\mu_k^{rs} \leq \mu_l^{rs}$  and  $\tau_k^{rs} \leq \tau_l^{rs}$  with at least one inequality holds.

**Definition 3** A path  $p_k^{rs} \in P^{rs}$  is a MSTD non-dominated path, if and only if no such a path  $p_l^{rs} \in P^{rs}$  exists that  $p_l^{rs} \succ_{MSTD} p_k^{rs}$ .

**Definition 4** A path  $p_k^{rs} \in P^{rs}$  is a MSSD non-dominated path, if and only if no such a path  $p_l^{rs} \in P^{rs}$  exists that  $p_l^{rs} \succ_{MSSD} p_k^{rs}$ .

#### 4.2.2 Model Formulation

To further formulate the model, it also assumes field collected travel time samples are continuously available across links on the network for a temporal period that is long enough to accurately measure the mean and travel time reliability. This approach is particularly advantageous to SSD formulation because there is no closed form equation available to come up with a semi-covariance matrix similar to the covariance matrix in the STD case(51). Now suppose there are  $w$  discrete travel time realizations for each link, and let  $r_m^{ij}$  denote the travel time realization on link  $a_{ij}$  at time interval  $m$ . Let  $x_{ij}^{rs}$  be the binary variable where  $x_{ij}^{rs} = 1$  if link  $a_{ij}$  is a member link of path  $p_k^{rs}$ , and  $x_{ij}^{rs} = 0$  otherwise. Accordingly, we can determine the mean and SSD of path travel time  $t_k^{rs}$  as follows.

$$\mu_k^{rs} = \frac{1}{w} \sum_{m=1}^w \sum_{a_{ij} \in A} r_m^{ij} x_{ij}^{rs} \quad (4.4)$$

$$\tau_k^{rs} = \left( \frac{1}{w} \sum_{m=1}^w \left( \sum_{a_{ij} \in A} r_m^{ij} x_{ij}^{rs} - b \right)_+^2 \right)^{0.5} \quad (4.5)$$

In order to understand the impact of traveler's risk-taking behavior on the routing results, both average travel time, which represents his/her expectation about the traffic condition, and the 15th percentile travel time that represents a more desirable traffic condition are respectively used as benchmark  $b$  and analyzed in the study. Let  $\rho_k^{rs}$  be the 15th percentile travel time on path  $p_k^{rs}$  and it is derived using the following equation.

$$\rho_k^{rs} = f \cdot t_{k_{g+1}}^{rs} + (1 - f) \cdot t_{k_g}^{rs} \quad (4.6)$$

where  $f$  is the fractional part of  $(0.15 * w + 0.5)$ ,  $g$  is the integer part of  $(0.15 * w + 0.5)$ , and  $t_k^{rs}$  is travel time on path  $p_k^{rs}$  that has been sorted in ascending order.

According to above equations, the multi-objective path finding model can now be reformulated as follows.

$$\text{Min } \frac{1}{w} \sum_{m=1}^w \sum_{a_{ij} \in A} r_m^{ij} x_{ij}^{rs} \quad (4.7)$$

$$\text{Min } \left( \frac{1}{w} \sum_{m=1}^w \left( \sum_{a_{ij} \in A} r_m^{ij} x_{ij}^{rs} - b \right)_+^2 \right)^{0.5} \quad (4.8)$$

$$s. t. \sum_{j:a_{ij} \in A} x_{ij}^{rs} - \sum_{i:a_{ji} \in A} x_{ji}^{rs} = \begin{cases} 1, & \text{if } i = r \\ -1, & \text{if } i = s \\ 0, & \text{otherwise} \end{cases} \quad (4.9)$$

$$x_{ij} \in \{0,1\} \forall (i,j) \in A \quad (4.10)$$

Equation (4.7) is to minimize the average travel time travelers experienced which represents their expectation under normal traffic conditions. Equation (4.8) is to minimize the travel time variability with regard to the pre-specified benchmark which can be average travel time or the 15<sup>th</sup> percentile travel time. Equation (4.9) ensures all the links on the path are feasible. Equation (4.10) defines a binary link-path incidence variable.

### 4.3 SOLUTION ALGORITHM

#### 4.3.1 Overview

To solve multi-objective problems, a variety of algorithms have been developed or applied in the existing literature (83; 84). In particular, as a widely applicable stochastic search heuristic and optimization technique, the multi-objective evolutionary algorithm has been proved to be a suitable and effective option(84). Among different evolutionary algorithms, the improved Strength Pareto Evolutionary Algorithm (SPEA2) is one of the most widely applied methods and considered to have better performance(85). Therefore, the algorithm is also adopted here and modified for solving the proposed model. For more algorithmic details, interested readers are referred to (85-87).

The overview of SPEA2 procedure is as follows.

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#### **SPEA2 Algorithm**

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Step 1: Algorithm Input. Specify the population size  $nPop$ , archive size  $nArchive$ , and number of generations  $nGen$ . Set generation counter  $c = 0$ .

Step 2: Initialization. Generate an initial population  $Pop_0$  and obtain the associated travel time matrix  $M_0$  during the process. Also, create an empty archival set  $\overline{Pop}_0$ .

Step 3: Fitness assignment. Calculate the fitness of each chromosome or path in  $Pop_c$  and  $\overline{Pop}_c$  based on  $M_c$ .

Step 4: Environmental selection. Select all the non-dominated paths in  $Pop_c$  and  $\overline{Pop}_c$  to  $\overline{Pop}_{c+1}$ . If number of paths in  $\overline{Pop}_{c+1}$  exceeds  $nArchive$ , then execute the truncating operation until the number is equal to  $nArchive$ . If the number of paths is less than  $nArchive$ , then fill  $\overline{Pop}_{c+1}$  with dominated paths in  $Pop_c$  and  $\overline{Pop}_c$ .

Step 5: Mating selection. Apply the binary tournament selection with replacement on chromosomes in  $\overline{Pop}_{c+1}$  until the mating pool is filled.

Step 6: Variation. Apply crossover and mutation operators to the mating pool and the resulted population is set to be  $Pop_{c+1}$ .

Step 7: Termination. If  $c \geq nGen$ , then terminate the procedure and output the final Pareto optimal paths, otherwise, increment  $c$  by 1 and go to step 3.

---

It should be noted that the population initialization process, mating selection, and crossover and mutation operations all follow same procedures as in GA implementation

in the previous chapter. The new components in SPEA2 including fitness assignment and environmental selection will be discussed in detail as follows.

### 4.3.2 Fitness Assignment

Fitness assignment is a process to quantitatively evaluate the fitness associated with each chromosome in current generation. The more fit individuals are, the more likely they are selected and used to reproduce the next generation, while less fit ones are more likely to be eliminated from current population. The fitness of a chromosome in SPEA2 is essentially calculated based on the dominance relationship with others.

- Sub-step 1: Obtain the vector  $Z_c$  consisting two elements including average travel time and travel time reliability which are computed based on path travel time matrix  $M_c$ .
- Sub-step 2: Assign a strength value to each chromosome in  $Pop_c$  and  $\overline{Pop}_c$ . The strength value of a chromosome is equal to the number of individuals it dominates. It can be calculated as  $S(k) = |\{l | l \in Pop_c + \overline{Pop}_c \wedge k \succ l\}|$ , where  $|\cdot|$  means the number of chromosomes in the set and  $+$  means the union operation.
- Sub-step 3: Calculate the raw fitness value. The raw fitness value  $R(k)$  is computed based on derived  $S$  values from previous step with  $R(l) = \sum_{k \in Pop_c + \overline{Pop}_c, k \succ l} S(k)$ . In other words, the raw fitness of a chromosome is determined by the sum of strength values of dominating chromosomes in the population and archive sets.
- Sub-step 4: Calculate the density value using k-th nearest neighbor method. The density is considered to differentiate chromosomes that may have same raw fitness values. The standardized Euclidean distance, instead of Euclidean distance in the original algorithm, is applied here to balance out the contribution from variables with different scales of values. So the distance between two individuals can be calculated as  $d(k, l) = (Z_c(k) - Z_c(l))V^{-1}(Z_c(k) - Z_c(l))'$ , where  $V$  is a two-by-two diagonal matrix whose first and second diagonal element is the variance of average travel time and travel time reliability on paths contained in  $Pop_c$  and  $\overline{Pop}_c$ . Once all the distances between path  $k$  and all other paths in  $Pop_c$  and  $\overline{Pop}_c$  are calculated, they are sorted in ascending order, and then the distance  $d_k^\theta$  at  $\theta$ th point is selected, where  $\theta = \sqrt{nPop + nArchive}$ . Then, the density  $Den(k)$  is computed as  $Den(k) = \frac{1}{d_k^{\theta+2}}$ , where 2 is added to ensure the density value is less than 1.
- Sub-step 5: Calculate the final fitness value  $Fit(k)$  with  $Fit(k) = R(k) + Den(k)$ .

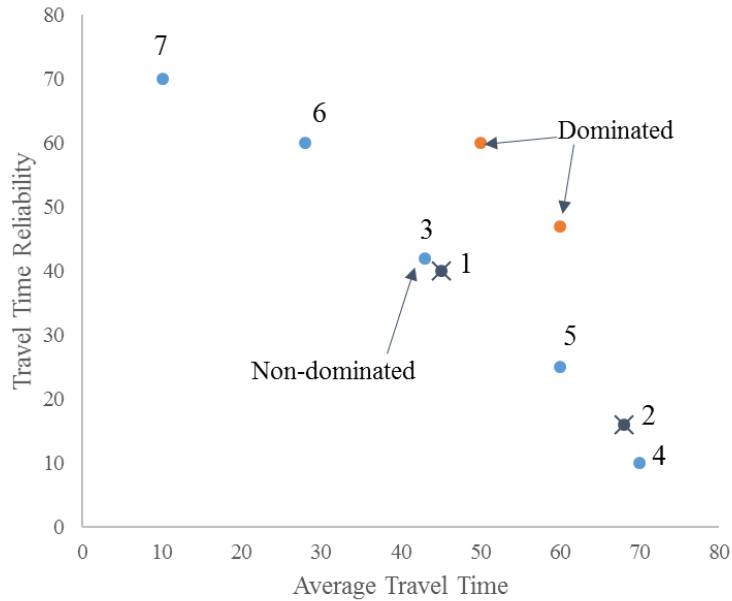
### 4.3.3 Environmental Selection

The environmental selection operation is to update the archive set with chromosomes that have higher fitness values. For a non-dominated path, its fitness value should be smaller than 1. Therefore, all the non-dominated paths can be identified and then copied to  $\overline{Pop}_{c+1}$  as  $\overline{Pop}_{c+1} = \{k | k \in Pop_c + \overline{Pop}_c \wedge Fit(k) < 1\}$ . If the number of chromosomes

that are selected into  $\overline{Pop}_{c+1}$  is equal to  $nArchive$ , then the task is completed. If the number is smaller than  $nArchive$ , then the next best ( $nArchive - |\overline{Pop}_{c+1}|$ ) chromosomes in  $Pop_c$  and  $\overline{Pop}_c$  in terms of fitness values which have been sorted in increasing order are selected to fill the archive set. On the other hand, if the number is larger than  $nArchive$ , the iterative truncating operation is triggered and executed until the number of remaining chromosomes is equal to  $nArchive$ . Following condition is evaluated to identify the path to be removed from  $\overline{Pop}_{c+1}$ :

$\forall 0 < \theta < |\overline{Pop}_{c+1}|: d_k^\theta = d_l^\theta \vee \exists 0 < \theta < |\overline{Pop}_{c+1}|: [(\forall 0 < \vartheta < \theta: d_k^\vartheta = d_l^\vartheta) \wedge d_k^\theta > d_l^\theta]$ . In other words, the path with the minimum distance to another path in the archive set is selected at each iteration and then removed. If there are more than two paths with same minimum distance, the second smallest distance is used and the comparison is continued until the tie is broken. The purpose of this step is to exclude the solutions that are close to each other and keep those that are far away from each other, so that the diversity of solutions can be maintained.

The following example is used to illustrate the idea of truncating operation. Here,  $nArchive$  is set to 5, i.e. the ideal number of non-dominated paths is five. Yet as can be seen in Figure 4.1, there are seven paths in the external archive. Therefore, the truncating operation needs to be executed. At first, the distance between each non-dominated path to all the other non-dominated paths is calculated and then sorted by increasing order. Since Path 1 and 3 have the minimum distance, and the distance between Path 1 and Path 5 is smaller than that between Path 3 and Path 6, Path 1 is first removed from the external set. Following the same procedure, Path 2 can also be identified and eliminated from the set.



**Figure 4.1 Environmental Selection Example**

#### 4.4 TUNING ALGORITHM PARAMETERS

Similar to GA, the SPEA2-based solution algorithm is also a heuristic method and its performance is greatly dependent on the proper choice of the implemented parameters. Although model calibration has been conducted on GA in previous chapter, the underlying structure of SPEA2 has been fundamentally changed, therefore there is still a need to quantitatively assess the performance and quality of the adopted approach and understand the impact from different parameter combinations. There have been a variety of performance metrics to evaluate the multi-objective evolutionary algorithms, many of which require the true global optimal paths to be known beforehand. In order to determine the combination of parameters that generate the best performance, it is critical to find the global optimal path set first. In this study, an alternative solution based on the relationship that will be established between the MSSD dominance rule and SOSD rule is adopted. Note that the calculation of SSD here uses a constant benchmark value across different paths, which is a minor adjustment to the proposed model in which the benchmark is actually path-specific. Since the structure of the models are still identical, the parameter values calibrated in this section can be directly applied to solve proposed multi-objective models with respect to STD and SSD.

Let  $\Gamma_{MSSD}^{rs}$  and  $\Gamma_{SOSD}^{rs}$  represent the sets of non-dominated paths under MSSD and SOSD dominance rules, respectively.

**Proposition**  $\Gamma_{MSSD}^{rs} \subseteq \Gamma_{SOSD}^{rs}$  except for the scenario where paths have identical mean and SSD.

**Proof.** To show the relationship holds, consider two paths  $p_k^{rs}$  and  $p_l^{rs}$  with cumulative travel time distribution function  $F_k$  and  $F_l$ . Then, based on the definition of SSD and using integration by parts twice, here we can have

$$(\tau_k^{rs})^2 = \int_{b_0}^T (x - b_0)^2 dF_k(x) = 2 \int_{b_0}^T \left[ T - t - \int_t^T F_k(x) dx \right] dy \quad (4.11)$$

$$(\tau_l^{rs})^2 = \int_{b_0}^T (x - b_0)^2 dF_l(x) = 2 \int_{b_0}^T \left[ T - t - \int_t^T F_l(x) dx \right] dy \quad (4.12)$$

Where  $b_0$  and  $T$  are the uniform threshold and upper bound travel time value for both paths, respectively. Accordingly,

$$(\tau_k^{rs})^2 - (\tau_l^{rs})^2 = -2 \int_{b_0}^T \int_t^T [F_k(x) - F_l(x)] dx dy \quad (4.13)$$

In addition, Based on the definition of SOSD rule, if  $p_k^{rs} \succ_{SOSD} p_l^{rs}$ , then  $\int_t^T [F_k(x) - F_l(x)] dx \geq 0$ , for all values of  $t \in [0, T]$ , with the inequality holds for at least one  $t$ . Therefore, we can get  $(\tau_k^{rs})^2 - (\tau_l^{rs})^2 \leq 0$ , then  $\tau_k^{rs} \leq \tau_l^{rs}$ .

At the same time, as proved in (50), if  $p_k^{rs} \succ_{SSD} p_l^{rs}$ , then  $\mu_k^{rs} \leq \mu_l^{rs}$ . ■

It should be noted that the exception condition noted in the proposition only occurs if a path is MSSD non-dominated yet dominated by another path in  $\Gamma_{SOSD}^{rs}$ . Even though the

scenario is possible to happen, its impact on the obtained non-dominated paths is negligible thanks to at least one path with identical mean and SSD is already included in  $\Gamma_{SOSD}^{rs}$ . If the included path is not ultimately chosen by the decision maker, the omitted path would also not be chosen anyway; if the included path is chosen as the desired path to travel, there would be no difference between the chosen and omitted paths as their attributes are exactly the same. Therefore, the relationship of  $\Gamma_{MSSD}^{rs} \subseteq \Gamma_{SOSD}^{rs}$  is considered true and used in finding MSSD non-dominated paths.

The problem of finding SOSD non-dominated paths has been extensively studied and an extended Label Correcting algorithm based on the Bellman's Principle of Optimality has been developed and proved to be effective in solving the problem(20). Based on above proposition, we first adopt the SOSD-based Label Correcting algorithm (SOSD-LC) to find all the non-dominated paths. Next, from the identified candidates we can determine final non-dominated paths based on the MSSD dominance rule. Interested readers are referred to (20; 50; 88) for detailed implementation of the SOSD-LC algorithm. A brief description of the algorithm is provided here for the sake of reading continuity.

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### SOSD-LC Algorithm

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Step 1: Initialization. Let  $p^{ss}$  be the path from  $s$  to itself and  $t_0^{ss}$  be the discrete travel time which are zero. Initialize the scan list  $Q = \{p^{ss}\}$ .

Step 2: Select the first path from  $Q$ , and denote it as  $p_k^{js}$ , then delete it from  $Q$ .

Step 3: For any predecessor node  $i$  of  $j$  and  $i$  is not contained in current  $p_k^{js}$ , create a new path  $p_k^{is} = p_k^{js} + a^{ij}$ , and update the path travel time  $t_k^{is} = t_k^{js} + r^{ij}$ .

Step 4: Compare the travel time  $t_k^{is}$  of path  $p_k^{is}$  to  $t_l^{is}$  of path  $p_l^{is} \in \Gamma_{SOSD}^{is}$ , where  $\Gamma_{SOSD}^{is}$  is the existing SOSD non-dominated path set. If  $E(t_k^{is} - \eta)_+ \leq E(t_l^{is} - \eta)_+$  for all  $\eta \in \Psi^{is}$ , where  $\Psi^{is}$  is the set of unique travel time realizations of path  $p_k^{is}$  and

$E(t_k^{is} - \eta)_+ < E(t_l^{is} - \eta)_+$  for at least one  $\eta$ , drop  $p_l^{is}$  and update  $\Gamma_{SOSD}^{is} = \Gamma_{SOSD}^{is} \cup p_k^{is}$  and  $Q = Q \cup p_k^{is}$ ; otherwise, drop  $p_k^{is}$ .

Step 5: If  $Q$  is empty, go to step 6; otherwise go to step 2.

Step 6: Identify paths in  $\Gamma_{SOSD}^{rs}$  that are MSSD non-dominated and generate  $\Gamma_{MSSD}^{rs}$ .

---

Once  $\Gamma_{MSSD}^{rs}$  is obtained, we now have the true Pareto-optimal paths to evaluate the performance of SPEA2. In this study, the cardinality of actual Pareto-optimal solutions found by SPEA2 and the closeness of SPEA2 solutions to  $\Gamma_{MSSD}^{rs}$  are regarded as two quality measures. In addition, the computing time required by SPEA2 and SOSD-LC for generating those solutions are also recorded and reported as additional evaluation criterion. Detailed analysis are conducted in the following section.

## 4.5 NUMERICAL EXPERIMENTS

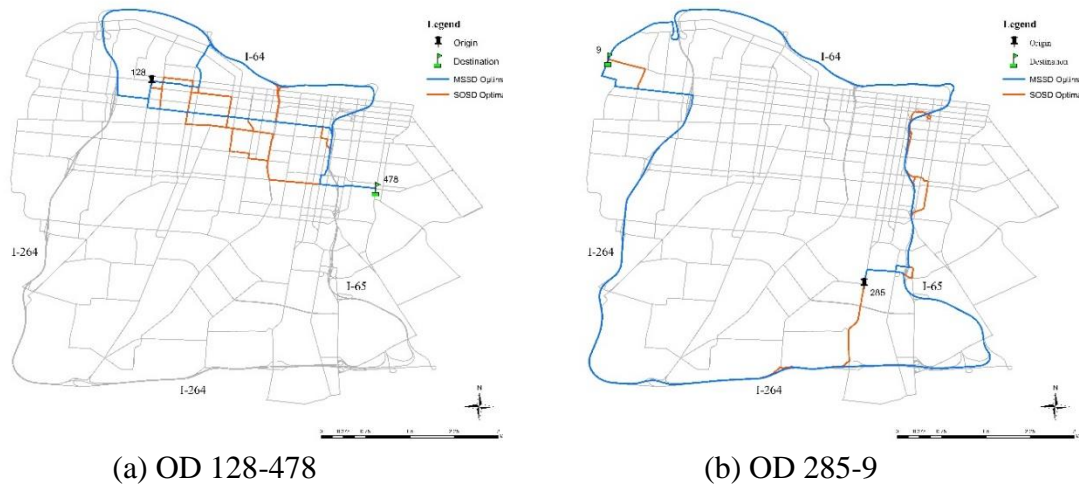
### 4.5.1 Model Calibration

In this section, numerical experiments are designed to test the proposed multi-objective models and solution algorithms. The same test network in the Louisville, Kentucky area is selected here. Both SPEA2 and SOSD-LC algorithms are coded and executed in

MATLAB. Two OD pairs are selected for detailed analysis. One is from node 128 to node 478, and the other is from node 285 to node 9. The SOSD-LC algorithm is first executed for selected OD pairs and results are reported in Table 4.1 and illustrated in Figure 4.2. The average and 15th percentile travel time for each of SOSD non-dominated paths are calculated and comparison between paths based on the MSSD dominance rule is conducted. The final non-dominated paths in terms of the MSSD rule are marked in bold in Table 4.1.

**Table 4.1 SOSD and MSSD Non-Dominated Paths**

OD 128-478						OD 285-9		
Path ID	Mean	SSD	Path ID	Mean	SSD	Path ID	Mean	SSD
<b>1</b>	<b>19.83</b>	<b>9.55</b>	<b>14</b>	<b>19.63</b>	<b>10.12</b>	<b>1</b>	<b>22.18</b>	<b>10.77</b>
<b>2</b>	<b>19.60</b>	<b>10.45</b>	15	21.32	11.23	2	22.89	11.21
3	20.03	11.33	16	20.94	10.83	3	22.83	12.15
4	20.05	11.10	17	21.06	11.11	4	22.97	11.62
5	21.49	11.06	18	20.08	10.12	5	24.76	12.36
6	20.08	10.05	19	21.93	10.85	6	25.35	12.56
7	21.68	11.62	20	21.61	11.37	7	24.30	11.86
8	20.70	10.70	21	21.70	11.40	8	22.81	11.29
9	21.48	11.86	22	22.17	11.29	9	24.07	11.35
10	20.51	11.05	23	23.60	12.75	<b>10</b>	<b>24.21</b>	<b>10.70</b>
11	20.58	11.09	24	23.03	12.13	11	27.47	14.06
12	19.62	10.46	25	24.46	13.56	12	25.04	11.77
13	20.28	10.62						



**Figure 4.2 SOSD Non-Dominated Paths**

Based on the results, it is first observed that the number of non-dominated paths according to the SOSD rule is fairly large. There are 25 and 12 paths found for OD pair 128-478 and 285-9, respectively. In particular, compared to OD 285-9, OD 128-478 is

within the downtown area, whose network has a gridiron type structure with a higher density of intersections and links. As a result, more paths using local streets are found. In contrast, OD 285-9 is further apart and across the city, and trips are heavily reliant on the interstates, which will very likely dominate local streets and thus provide fewer options. In addition, it is seen that there are only 3 and 2 non-dominated paths found for OD pair 128-478 and 285-9, respectively. This finding indicates fixed benchmark based MSSD rule is more stringent than the SOSD rule. As a result, applying the MSSD rule significantly reduces the number of non-dominated paths.

Next, we proceed to assess the solution quality of the adopted SPEA2 algorithm and tune its parameters to achieve desired performance. In order to do so, the minimum 15<sup>th</sup> percentile travel time among all identified SOSD non-dominated paths for each OD pair is first determined and then used as the uniform benchmark to implement in SPEA2. At the same time, it is acknowledged that the algorithmic performance relies significantly on the proper selection of input parameters. As shown in Table 4.2, we consider three different values for each variable. Nine different combinations of those values are then considered and listed in Table 4.3. Five trials are repeated for each combination. The approximate solutions from each trial are obtained and evaluated against the ground truth found by the SOSD-LC algorithm. The computing time required to finish each trail is monitored as well. At last, the performance of each combination of parameters is determined by averaging corresponding metric values from those five repeated trails. The experiment results are reported in Table 4.4.

**Table 4.2 Options of EA Parameters**

<b>Level</b>	<b>1</b>	<b>2</b>	<b>3</b>
<b>Generation</b>	200	400	600
<b>Population</b>	50	100	150
<b>Archive</b>	50	100	150
<b>Crossover</b>	0.25	0.5	0.75
<b>Mutation</b>	0.1	0.3	0.5

**Table 4.3 Combination of EA Parameters**

<b>Combination</b>	<b>Generation</b>	<b>Population</b>	<b>Archive</b>	<b>Crossover</b>	<b>Mutation</b>
<b>1</b>	1	1	1	1	1
<b>2</b>	2	1	1	2	2
<b>3</b>	3	1	1	3	3
<b>4</b>	1	2	2	2	3
<b>5</b>	2	2	2	3	1
<b>6</b>	3	2	2	1	2
<b>7</b>	1	3	3	3	2
<b>8</b>	2	3	3	1	3
<b>9</b>	3	3	3	2	1



**Table 4.4 EA Performance Measures**

Combination	OD 128-478			OD 285-9		
	Number	Percentage	CPU Time	Number	Percentage	CPU Time
1	1.2	40.0	37.9	0.4	20.0	37.4
2	2	66.7	204.2	1.2	60.0	177.1
3	1.8	60.0	507.4	1.2	60.0	416.3
4	2	66.7	356.4	1.4	70.0	292.1
5	1.8	60.0	178.1	1	50.0	153.8
6	2.2	73.3	655.8	1.2	60.0	541.7
7	2.2	73.3	355.9	1.8	90.0	304.4
8	1.8	60.0	1108.2	0.8	40.0	936.1
9	2.4	80.0	450.3	1.4	70.0	401.2
Global Optimal	3	100	663.6	2	100	458.9

Based on above table, it can be seen that the performance of SPEA2 is directly correlated with different combinations of parameter values while no dominating combination, in terms of every performance measure, is observed. It is interesting to note that the running time may not positively correlate with the distance of OD as we can see that OD 128-478 requires more executing time than OD 285-9 which is further apart than the former. In general, quality of solutions from combination 2, 4, 6, 7, 9 appear to be better than that from other combinations. Among those five combinations, combination 2 is the most efficient choice in terms of the computing time. However, the quality of solutions is inferior to the other four combinations. After combining all the considerations together, combination 7 is chosen to implement SPEA2: the population and archive size are 150, the number of generations is 200, crossover probability is 0.75, and mutation probability is 0.3.

#### 4.5.2 Model Implementation and Discussion

In the following experiments, different models involving two categories are evaluated with more details for OD 285-9 whose origin is from the University of Louisville and destination is Shawnee Park, located at the northwest side of the city. To distinguish two proposed asymmetrical reliability measures, hereafter the SSD with regard to average travel time is referred to as  $SSD_{\mu}$ , while SSD in terms of the 15<sup>th</sup> percentile travel time is represented by  $SSD_{\rho}$ . Also, the multi-objective model involving  $SSD_{\mu}$  or  $SSD_{\rho}$  is respectively referred to as  $MSSD_{\mu}$  or  $MSSD_{\rho}$  model.

Single-objective optimization scenarios:

- Minimum expected travel time (ETT) path;
- The  $STD$ -based most reliable path;
- $SSD_{\mu}$ -based most reliable path;

- $SSD_\rho$ -based most reliable path;

Multi-objective optimization scenarios:

- The  $MSD$  model;
- The  $MSSD_\mu$  model;
- The  $MSSD_\rho$  model.

The test results for the selected OD pair are discussed. For single-objective scenarios, the following observations are made. As depicted in Figure 4.3, the optimal paths based on different objectives are quite different, except for the most reliable paths based on  $STD$  and  $SSD_\rho$ . Those two paths only differ at one link as to when to take I-65. It can be observed that if only average travel time is used as the objective, the ETT path will rely heavily on I-65 and I-64 and take 22.18 minutes to arrive at the destination. However, the fastest path does not take travel time reliability into consideration. In fact, it would be the worst or most unreliable path among paths identified here in terms of selected reliability measures. The  $STD$ ,  $SSD_\mu$ , and  $SSD_\rho$  are 8.5, 7.36, 10.78 minutes respectively on this route as shown in Table 4.5.

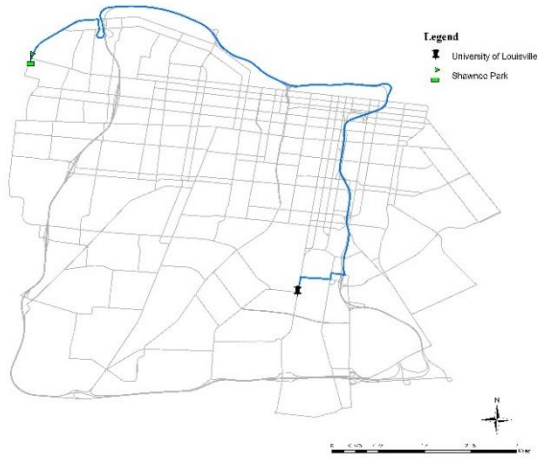
In contrast, if only travel time reliability is taken into consideration, the optimal path would be completely different from previously identified ETT path. Regardless of which reliability measure is used, it seems the travel time would be less variable if travelers take I-65 southbound and then take I-264 for most of the remaining trip. Although the optimal routes with respect to  $STD$  and  $SSD_\rho$  are very similar, the difference between optimal solutions from  $STD$  and  $SSD_\mu$  at the end of the trip are obvious. It can be observed that the  $SSD_\mu$  based path will get off I-264 earlier and then rely on arterials to get to the destination. It should be noted that average travel time on this path will be 27.04 minutes, which is the longest among those identified paths.

From the above analysis, different paths will be selected with respect to different criteria and there isn't a single path that is optimal with respect to all the examined criteria. Instead, the objectives may be in conflict and an optimal path under one objective is often obtained at the expense of other objectives. Therefore, without knowing the decision maker's specific risk preference, it is premature to exclude all the other paths. Unlike single objective optimization, when both the average travel time and travel time reliability are optimized simultaneously, multiple paths will be consequently identified between the same OD pair. The obtained Pareto optimal path set can then be used for decision-making based on users' preferences.

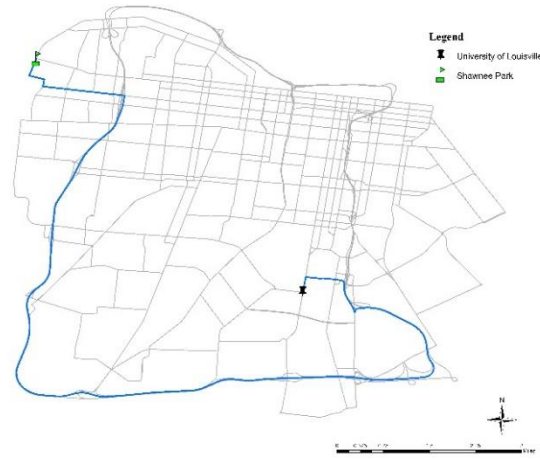
It can be seen if both mean and  $STD$  are used as objectives, five paths that are mutually non-dominant are generated with average travel time ranging from 22.18 to 24.49 minutes and  $STD$  ranging between 6.27 and 8.5 minutes. In particular, two paths will mainly use I-65 northbound and I-64 and the three other paths will mainly use I-65 southbound and then I-264. In contrast, if  $SSD_\mu$  is used to substitute  $STD$ , the non-dominated solutions largely remain the same but include two additional paths, which suggest to leave I-264 early and then turn on to Hale Ave to finish the trip. The average

travel time of this non-dominated path set ranges from 22.18 to 27.04 minutes, while the  $SSD_{\mu}$  ranges from 5.63 to 7.36 minutes. Furthermore, if the desirable travel time is instead chosen as the benchmark and resulted  $SSD_{\rho}$  is used as the reliability measure, an apparent change can be seen with the addition of path 3 which is not identified by either  $MSTD$  or  $MSSD_{\mu}$  model. The new route avoids I-65, and instead uses Southern Parkway first and then turns on to I-264 to continue the trip. It is almost equally as attractive as path 1 in that the mean and  $SSD_{\rho}$  of two routes are very close.

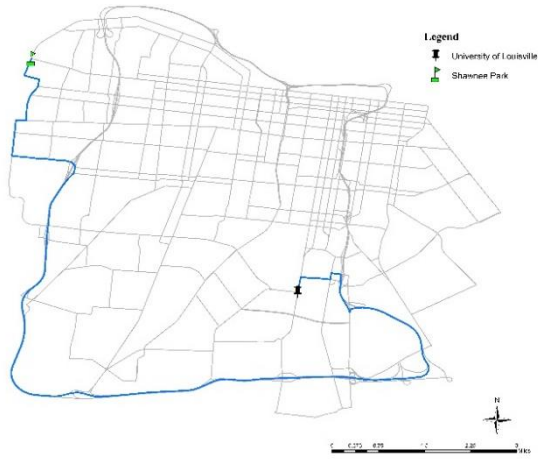
The Pareto optimal paths from evaluated multi-objective models are also compared to those obtained from the SOSD dominance rule. The last column in Table 4.5 marks those paths identified by multi-objective models that are also SOSD non-dominated. First, it is observed that the  $MSSD_{\rho}$ -based paths are completely contained in the SOSD non-dominated path set. However, this is not true for  $MSTD$  and  $MSSD$  non-dominated paths. In fact, the  $STD$ -based and  $SSD_{\mu}$ -based most reliable paths are not even included in the SOSD optimal path set. Therefore, it is empirically sufficient to say that  $MSTD$  and  $MSSD$  dominance rules are not compatible with SOSD rule. The findings confirm with observations made in (20; 50). In addition, these results indicate more caution should be used when applying SOSD non-dominated paths as a basis to generate the minimum TTB path or  $MSTD/MSSD$  non-dominated paths due to some highly potential paths may not be identified by SOSD rule.



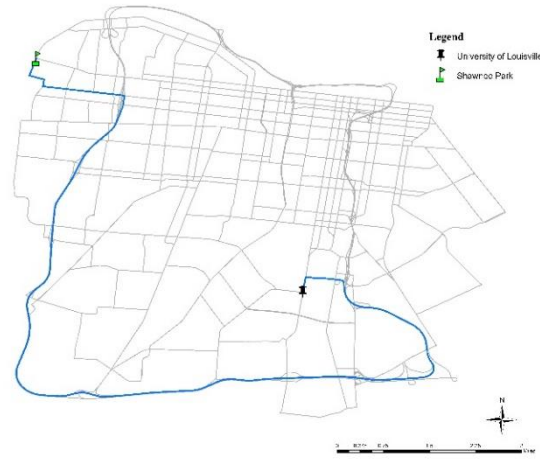
(a) ETT Path



(b)  $SD$ -Most Reliable Path

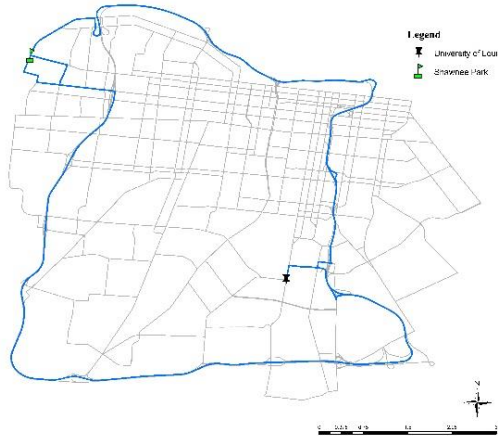


(c)  $SSD_\mu$ -Most Reliable Path

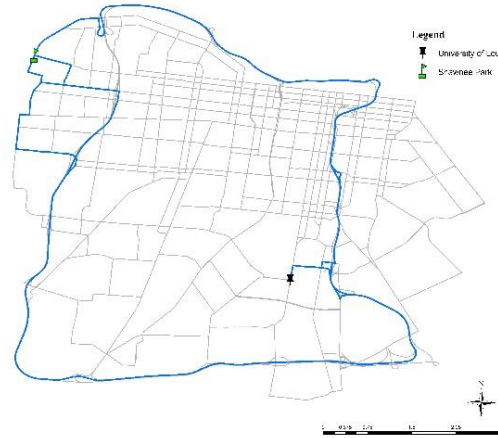


(d)  $SSD_\rho$ -Most Reliable Path

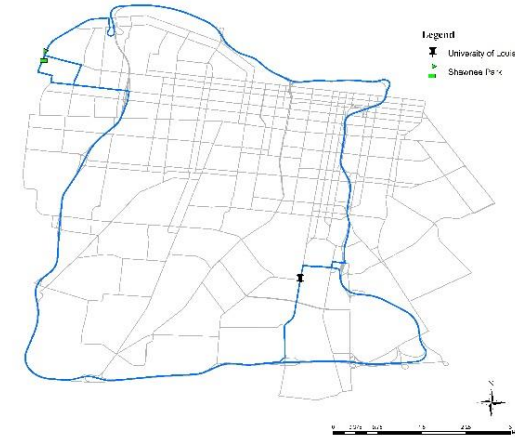
Figure 4.3 Single-Objective Optimal Path



(a)  $MSTD$  Optimal Paths



(b)  $MSSD_{\mu}$  Optimal Paths



(c)  $MSSD_{\rho}$  Optimal Paths

**Figure 4.4 Multi-Objective Non-Dominated Paths**

**Table 4.5 Attributes of Selected Paths**

Path ID	$\rho$	$\mu$	$\sigma$	$SSD_{\mu}$	$SSD_{\rho}$	MTT	Most Reliable Path			Pareto Optimal Paths			SOSD Path
							STD	$SSD_{\mu}$	$SSD_{\rho}$	$MSD$	$MSSD_{\mu}$	$MSSD_{\rho}$	
1	15.55	22.18	8.50	7.36	10.78	√				√	√	√	√
2	16.14	22.89	8.47	7.32	10.83					√	√		√
3	17.67	22.97	8.94	8.21	10.39							√	√
4	20.19	24.07	7.50	7.06	8.44					√	√	√	√
5	20.36	24.21	6.28	5.73	7.36				√	√	√	√	√
6	20.66	24.49	6.27	5.72	7.36		√			√	√		
7	22.20	26.69	6.33	5.65	7.74						√		
8	22.61	27.04	6.32	5.63	7.70						√		

## 4.6 CONCLUSIONS

In this chapter, a multi-objective model is developed aiming to simultaneously minimize the expected travel time and travel time reliability. The improved Strength Pareto Evolutionary Algorithm is adapted in the study to solve the proposed models. In order to quantitatively assess the performance of SPEA2, the relationship between MSSD and SOSD dominance rules is first established and a Label Correcting algorithm is applied to generate the ground truth Pareto optimal paths. The SPEA2 parameters are then tuned so the algorithm can achieve desired performance both in solution quality and required computational resource. The calibrated algorithm is later applied to solve multiple multi-objective models with regard to STD and SSD. The findings from the analysis are summarized as follows.

First, the SOSD rule may not be able to effectively determine the dominance relationship among candidate paths. This leads to a relatively large number of non-dominated paths. As pointed out by Messac and Mattson, too many solutions would be undesirable to decision makers(89); it is valuable to augment the SOSD rule with additional criteria. As shown in the study, when a proper constant is applied to the SSD calculation for different paths, the resulting MSSD non-dominated paths would become a subset of the SOSD non-dominated paths. In fact, the empirical results indicate the MSSD rule can eliminate a significant portion of paths from the SOSD path set for two OD pairs investigated.

In addition, input parameters have considerable impact on the performance of the adopted metaheuristic algorithm. Based on the paths generated by SOSD-LC algorithm, the quality of solutions from a wide range of parameter values are evaluated. It is found that the solution quality and computing time are conflicting objectives for the implemented algorithm as certain combination of values can produce better quality of solutions, but require longer time to run. Therefore, the trade-off of performance metrics is necessary to choose the best combination.

Furthermore, when different criteria are used in the routing model, different alternative paths are identified. The optimal paths in terms of average travel time, STD, and SSD alone are all different from each other. Even though these different criteria can be combined into a single objective, many potentially attractive paths would be consequently omitted. Thus, the multi-objective formulation is more appealing in that it provides more options for further trade-off and decision-making. Based on the identified paths from the proposed models, it is shown that SSD not only can account for the asymmetry and skewness of the distribution, but also offer more flexibility for users with different risk preferences. Additional paths as attractive as MSTD non-dominated paths are identified by the MSSD model.

Through this study, it is empirically shown that there exists inconsistency among MSTD,  $MSSD_{\mu}$  and SOSD dominance conditions. In other words, at least one of the non-dominated paths in terms of MSTD or  $MSSD_{\mu}$  dominant rule is dominated by SOSD non-dominated paths. Likewise, there are path(s) that are SOSD non-dominated but not MSTD or  $MSSD_{\mu}$  non-dominated. As an effective and tractable approach, the SOSD-based LC algorithm can still be utilized to find paths that approximate the true optimal

solutions to the multi-objective model involving STD and SSD. However, the inconsistent relationship should still be kept in mind even when such compromise is deemed acceptable in the application.

In the future study, it would be valuable to extend the range of parameter values considered in the study and apply experimental design techniques such as Taguchi method to objectively determine the optimal input values. Also, as a number of novel genetic operators have been proposed in many applications, it will be interesting to see how its implementation into the proposed algorithm can lead to performance improvement. In addition, it is postulated that the size of travel time samples considered during the model development could affect the identified optimal paths. Therefore, it is a practically important topic worth more research efforts. Finally, the mutual relationships among MSTD, MSSD, and SOSD dominance rules and the necessary conditions required to ensure compatibility among them is also a continued area of interest.

## CHAPTER 5 A GENERALIZED ROUTE CHOICE MODELING FRAMEWORK

### 5.1 INTRODUCTION

It has been shown in previous chapters that using SSD as the reliability measure has advantages of being more realistic in representing traveler's attitude toward uncertain travel times. Also, if the benchmark is set to a constant across paths connecting the same OD pair, the obtained non-dominated paths according to the MSSD rule would be a subset of non-dominated paths based on SOSD criterion. In this chapter, more general relationships between existing reliability models and stochastic dominance rules are investigated.

Using a constant benchmark in SSD calculation actually offers realistic meaning. In previous chapters, either the average or 15<sup>th</sup> percentile travel time on each path is used as the threshold for that path. As a result, each path will have its own benchmark to compute the reliability measure. However, as shown in (16; 90), instead of having path specific values, travelers often have fixed departure time and acceptable travel time when making trip decisions. Applying the OD specific benchmark can offer additional benefits. For instance, consider two paths where one path has travel times of 1, 3, and 5 minutes, whereas the other has travel times of 6, 7, and 8 minutes. If average travel time is used as the threshold, SSD would be 1.15 and 0.58 minutes for respective paths. According to previously defined MSSD rule, both paths are non-dominated. However, the first path looks much more attractive to a rational traveler, because experienced travel times will always be shorter. If a uniform value, e.g. 5 minutes, is used instead, the first path would have no reliability issue, and consequently, dominate the second path. It is under this notion that we decide to use the uniform benchmark across paths from hereafter for semi-standard deviation calculation.

In addition to SSD, reliability measures, including on-time arrival probability and scheduling delay have also been proposed. In particular, the OTAP model tries to find the path that maximizes the probability of arriving at the destination on time, whereas the SD model intends to minimize the penalty resulted from late arrivals. Both measures have realistic meaning in interpreting travelers' route choice behaviors. Therefore, they are revisited and discussed in more details in this chapter.

Meanwhile, the Stochastic Dominance Theory has received extensive application in measuring risk and ranking uncertain prospects in finance and economics. It is designed to establish partial orders over a set of alternatives under consideration based on associated distributions, even with a limited knowledge of specific utility function that a decision maker may possess. Recently, SDT has also been introduced to the transportation network modeling field(38), as it fits well with the optimal path finding problem under the stochastic travel time setting. Another appealing advantage of SDT is its compliance with the Bellman's Principle of Optimality which makes it efficient in finding non-dominated paths. Also, traveler's risk taking behavior can be accounted for by different ordering SDT rules when ranking different routes with stochastic travel times



in terms of known cumulative distribution functions. Therefore, a few studies have been conducted on its application in the transportation network modeling problem involving time-adaptive path comparison(38), heterogeneous risk taking behavioral modeling(91), travel time and reliability valuation(92), and traffic assignment(93).

While different approaches have their own merits in representing decision making behaviors under uncertain conditions, there is a lack of consensus as to which measure is more reflective of traveler's actual perception of reliability. There have been studies investigating relationships among different measures. For instance, under the assumption that travel time distribution is independent of the departure time and there is no discrete lateness penalty, there is a theoretical equivalence between scheduling delay and mean-standard deviation approaches (94). However, the underlying assumption is often violated in reality. Wu and Nie demonstrate that the optimal OTAP and METT path can be found by applying the first-order stochastic dominance rule, while it is not true for mean-variance/standard deviation model(20). Similar to the work done by Ogryczak and Ruszczyński (95), Wu has proved the incompatibility between the mean-standard deviation model and SDT rules(50). Also, he points out there is a consistent relationship between semi-standard deviation and second-order stochastic dominance rule, yet the reliability ratio should not exceed one.

As the accuracy of route choice and demand forecasting models is heavily reliant on the proper incorporation of the reliability component, it continues to be an important area to understand travelers' risk-taking behaviors under uncertainty and corresponding quantitative measures. The objective of this chapter is to advance the understanding of existing reliability models by investigating their underlying attributes and behavioral implications as well as the relationships between each other. This will help facilitate the proper selection of the reliability measure that is more consistent with the attitude of a certain group of travelers towards uncertainty. More specifically, a generic formulation is proposed based on the similar structural properties of three measures, including OTAP, SD, and SSD. This is similar to the lower partial moment concept applied in financial risk analysis(96), however, our study regards long travel times as undesirable and concentrate on the right side of the distribution. In addition, as Pareto efficiency is an important property of reliability analysis, the study also presumes the decision process involves trading-off of the mean and reliability requirements and applies the multi-objective formulation in route choice models. Furthermore, consistent relationships between the generalized reliability model and SDT criteria are established. This not only provides theoretical foundation regarding risk preferences for the reliability measures under study, but also makes it possible to analytically and efficiently solve route choice models involving those measures.

The outline of the chapter is as follows. The Section 5.2 introduces the three reliability models of particular interest in our study and a generic multi-objective formulation is developed in the following section. The definitions associated with the first three orders of SDT are provided in Section 5.4, and relationships between reliability models and stochastic dominance rules are examined. The results facilitate the development of

solution algorithm in the fifth section based on the Label Correcting algorithms. Next, numerical experiments are conducted on the real-world network using field travel time data to evaluate the proposed models and algorithms. The findings from the study are summarized in the final section.

## 5.2 ALTERNATIVE RELIABILITY MODELS

To facilitate the discussion, following notations are used. First consider a directed network  $G = (N, A, D)$  where  $N$  is the set of nodes,  $A$  is the set of links, and  $D$  is the set of probability distributions of travel times associated with individual links. Let  $r \in N$  and  $s \in N$  represent the origin and destination node respectively. Let  $t_k^{rs}$  be the random travel time on path  $p_k^{rs} \in P^{rs}$ , where  $P^{rs}$  denotes the set of paths connecting the origin and destination and follow a probability distribution  $f_k^{rs}$  with a cumulative function  $F_k^{rs}$ . Let  $\mu_k^{rs}$ ,  $\alpha_k^{rs}$ ,  $\beta_k^{rs}$ , and  $\gamma_k^{rs}$  represent the mean, on-time arrival probability, schedule delay, and semi-standard deviation of  $t_k^{rs}$  along  $p_k^{rs}$  with respect to a pre-defined acceptable travel time benchmark  $b$ , respectively. Assume travelers know exactly the travel time and variability on each link and would make rational choices among path alternatives. Accordingly, three route choice models of interest in terms of each reliability measure are respectively introduced in the following sections.

### 5.2.1 On Time Arrival Probability

First applied by Frank in the probabilistic shortest path problem, the OTAP model has become one of the most commonly studied approaches to the reliable routing problem in the stochastic network setting. The model aims to determine the optimal path that maximizes a traveler's chance of arriving at a destination on time given his/her specific travel time constraint for the trip. The OTAP model can be mathematically represented as follows.

$$\text{Max } \alpha_k^{rs}(b) = F_k^{rs}(b) = \int_0^b f_k^{rs}(t)dt, \forall (r, s) \in RS, p_k^{rs} \in P^{rs} \quad (5.1)$$

To transform the OATP model into an equivalent optimization problem, first we define a complementary cumulative distribution function as:

$$\bar{F}(b) = 1 - F(b) = \int_b^T f(t)dt \quad (5.2)$$

where  $\bar{F}(b)$  represents the probability of having a travel time that is above the benchmark value  $b$ . Therefore, contrast to  $F(b)$ ,  $\bar{F}(b)$  is a non-increasing function with  $\bar{F}(0)=1$  and  $\bar{F}(T)=0$ , where  $T$  is the upper bound of the travel time distribution.

Accordingly, the OATP model can be equivalently reformulated as a late arrival probability (LAP) model, which tries to minimize the probability of late arrival at the destination.

$$\text{Min } \alpha_k^{rs}(b) = [\bar{F}_k^{rs}(b)] = \int_b^T f_k^{rs}(t)dt = \int_b^T (t_k^{rs} - b)^0 dF_k^{rs}, \forall (r, s) \in RS, p_k^{rs} \in P^{rs} \quad (5.3)$$

### 5.2.2 Scheduling Delay

Scheduling delay approach is commonly used in traveler's departure time choice model to cope with travel time uncertainty. Travelers usually have a preferred arrival time in mind when making their trip decisions and accordingly determine the departure time according to the travel time allocated for the trip. When faced with the variability of travel times, in addition to choose the most reliable route, travelers can also adjust their departure time as traffic conditions on the same route can be significantly different during different time periods. Building on the previous work, following formulation is proposed by Small et al to simultaneously account for the scheduling and travel time reliability(65):

$$U(t_d) = \gamma_1 t + \gamma_2 SDE + \gamma_3 SDL + \gamma_4 DL \quad (5.4)$$

where  $t_d$  represents the traveler's departure time,  $U(t_d)$  represents the utility associated with the particular  $t_d$ ,  $\gamma_i$  is the estimated coefficient and expected to be negative,  $t$  is the trip travel time; SDE is schedule delay early; SDL is the schedule delay late; DL indicates whether the trip is late or not, and takes 1 if yes, and 0 otherwise. In particular, the SDE and SDL can be respectively represented by

$$SDE = (PAT - [t + t_d])_+ \quad (5.5)$$

$$SDL = ([t + t_d] - PAT)_+ \quad (5.6)$$

where  $z_+ = \max(z, 0)$ .

Argued that above formulation is only suitable under certainty situation, as the travel time is essentially a constant, Noland and Small further extend the scheduling model by considering the travel time as a random variable that follows a probability density function. As a result, above equation can be rewritten as

$$E[U(t_d)] = \gamma_1 E[t] + \gamma_2 E[SDE] + \gamma_3 E[SDL] + \gamma_4 p_L(t_d) \quad (5.7)$$

where  $E[\cdot]$  is the expectation operator and  $p_L(t_d)$  represents the probability of late arrival given the departure time  $t_d$ .

Following this vein, Watling proposes a late arrival penalty model, assuming the departure time is fixed and the penalty only incurs when the trip travel time exceeds the pre-defined benchmark or acceptable travel time as stated in his work(66). The adapted disutility function, which comprises the generalized travel cost plus late penalty, can be represented as follows:

$$u_r = \theta_0 d_r + \theta_1 E[C_r] + \theta_2 E[(C_r - \tau_k)_+] \quad (5.8)$$

The author then incorporates the function to extend the traditional user equilibrium model, so that the disutility on all used paths is equal, and less than that on any unselected paths. However, no specific solution method is provided in the study. In this

study, an efficient algorithm based on SDT is adopted after the consistent relationship between the scheduling delay and stochastic dominance rule is established. Similar as in Equation (5.8), we only focus on the scheduling delay component. Therefore, the route choice model can be reformulated as

$$\text{Min } \beta_k^{rs}(b) = E(t_k^{rs} - b)_+ = \int_b^T (t_k^{rs} - b) dF_k^{rs} \quad (5.9)$$

### 5.2.3 Semi-Standard Deviation

Introduced in the portfolio theory by Markovitz, the centrality-dispersion based trade-off model has been extensively used in a variety of applications to accommodate risky circumstances(14). In the portfolio optimization, investors intend to maximize the return of the investment as often represented by the mean while minimize the potential loss, which is usually represented by the variance or standard deviation. In the route choice context, since travel time is an undesirable feature, travelers attempt to minimize both the average and variability of travel time at the same time. Therefore, the objective of the model is to minimize the utility which is the linear combination of two components. The obtained utility is essentially equivalent to the travel time budget or effective travel time concept in many studies. The general formulation is expressed as follows.

$$U = \mu + \lambda\sigma \quad (5.10)$$

where  $U$  is the utility of the path;  $\mu$  and  $\sigma$  represents the mean and standard deviation of travel times on the path;  $\lambda$  represents the relative importance of the reliability compared to the average travel time.

Under normal distribution assumption, there is one-to-one relationship between the mean-STD and OTAP models. However, such corresponding relationship does not hold when the assumption is violated, which is often the case since travel times are often asymmetrically distributed. On the other hand, the shortcomings of standard deviation are apparent under asymmetrical distribution condition(27). In contrast, SSD not only has the advantages that standard deviation has, but also has the feasibility in accounting for the important rule the benchmark value plays in the route choice model. This property echoes very well with LAP and SD based measures.

$$\text{Min } \gamma_k^{rs}(b) = \sqrt{E[(t_k^{rs} - b)_+^2]} = \sqrt{\int_b^T (t_k^{rs} - b)^2 dF_k^{rs}} \quad (5.11)$$

Despite the flexibility in using different  $b$  values, such as the average and 15<sup>th</sup> percentile travel time, here we will define  $b$  as the acceptable travel time, same as used in LAP and SD models. In contrast to using mean as the benchmark, the constant value is regarded as more realistic as travelers usually compare candidate paths with a universal criterion instead of path-specific.

Since SSD is the square root of semi-variance, the routing model that minimizes the semi-variance of travel times will equally minimize the SSD. Therefore, the formulation can be equivalently rewritten as below.

$$\text{Min } \gamma_k^{rs}(b)^2 = E[(t_k^{rs} - b)_+^2] = \int_b^T (t_k^{rs} - b)^2 dF_k^{rs} \quad (5.12)$$

## 5.3 A GENERALIZED FRAMEWORK

### 5.3.1 Measure Generalization

From the above formulations, we can immediately see that three reliability models essentially follow the same mathematical structure, which only accounts for the distributional portion that exceed the benchmark. The only difference is the power factor applied in the formula. Naturally, we can generalize the reliability measures with following formulation.

$$\tau_\theta^{rs}(b) = \int_b^T (t_k^{rs} - b)^\theta dF_k^{rs} \quad (5.13)$$

where  $\theta$  is the reliability parameter that governs how the deviation from the benchmark travel time is treated in the calculation.

Based on the generalized formulae above, the reliability measurement is determined by two parameters. The benchmark  $b$  reflects at what extent beyond which travelers would consider the trip to be unreliable, therefore represents their tolerance of unreliability. A small value indicates travelers are intolerant of relatively long travel time whereas a very large value indicates only under highly congested conditions would the traveler consider the impact of reliability on their trip planning. An extreme scenario exists if  $b$  takes the upper bound of travel time distribution or even a larger value so that any travel time realizations would be below the benchmark. Under this situation, travelers are insensitive to any travel time variation. In other words, they are not concerned with the reliability condition and do not include it in their route choice decisions. In this case, the reliability model degenerates to the traditional minimum expected travel time model.

The reliability parameter  $\theta$ , on the other hand, determines the behavior of the reliability measure in response to the degree of deviation of travel time from the specified benchmark. If  $\theta = 0$ , the deviation, no matter small or large, has no impact on the quantity of the measure; therefore, the generalized measure simply degenerates to the probability measure, i.e. LAP, and it is only affected by the benchmark value. If  $0 < \theta < 1$ , the small deviation has a relatively larger impact on the reliability measure, however, such contribution gradually decreases as  $\theta$  increases. If  $\theta = 1$ , the generalized measure becomes SD and the small and large deviations have the same weight in calculating the measure. In contrast, if  $\theta > 1$ , the large deviation will have a larger weight in quantifying the reliability condition and the impact increases exponentially as  $\theta$  increases. This is consistent with the understanding that travelers are more concerned with excessive delays than the average condition. A special case would be when  $\theta = 2$ , the generalized measure becomes SV. As it is critical to choose an appropriate reliability parameter, the

behavioral implication corresponding to each value will be further investigated through the congruent relationship between the generalized measure and stochastic dominance criteria.

### 5.3.2 Model Formulation

With the multi-objective formulation of the routing problem involving stochastic travel time, travelers are assumed to try to minimize the expected travel time, besides the travel time reliability consideration. Accordingly, the multi-objective shortest path problem that seeks to optimize both objectives simultaneously can be developed into following mean-travel time reliability (MTTR)  $\mu - \tau_\theta$  model:

$$\begin{cases} \text{Min } \mu^{rs} \\ \text{Min } \tau_\theta^{rs} \end{cases} \quad (5.14)$$

Then, the following path dominance rules are defined so that the comparison between alternative paths can be made.

**Definition 1** A path  $p_k^{rs} \in P^{rs}$  dominates another path  $p_l^{rs} \in P^{rs}$  by the mean-travel time reliability i.e.  $\mu - \tau_\theta$  decision rule or  $p_k^{rs} \succ_{\mu-\tau_\theta} p_l^{rs}$ , if  $\mu_k^{rs} \leq \mu_l^{rs}$  and  $\tau_{\theta_k}^{rs} \leq \tau_{\theta_l}^{rs}$  with at least one strict inequality holds.

**Definition 2** A path  $p_k^{rs} \in P^{rs}$  is a  $\mu - \tau_\theta$  non-dominated path, if and only if no such a path  $p_l^{rs} \in P^{rs}$  exists such that  $p_l^{rs} \succ_{\mu-\tau_\theta} p_k^{rs}$ .

Since this dissertation takes advantage of the sampling based approach to account for the inherent correlation between travel times on adjacent links, now suppose there are  $w$  discrete travel time realizations for each link, and let  $r_m^{ij}$  denote the travel time realization on link  $a_{ij}$  at time interval  $m$ . Let  $x_{ij}^{rs}$  be the binary variable where  $x_{ij}^{rs} = 1$  if link  $a_{ij}$  is a member link of path  $p_k^{rs}$ , and  $x_{ij}^{rs} = 0$  otherwise. Accordingly, we can determine the mean and SSD of path travel time  $t_k^{rs}$  as follows.

$$\mu_k^{rs} = \frac{1}{w} \sum_{m=1}^w \sum_{a_{ij} \in A} r_m^{ij} x_{ij}^{rs} \quad (5.15)$$

$$\tau_{\theta_k}^{rs} = \frac{1}{w} \sum_{m=1}^w \left( \sum_{a_{ij} \in A} r_m^{ij} x_{ij}^{rs} - b \right)_+^\theta \quad (5.16)$$

According to equations (5.14), (5.15), and (5.16), the multi-objective model can now be reformulated as follows.

$$\text{Min } \frac{1}{w} \sum_{m=1}^w \sum_{a_{ij} \in A} r_m^{ij} x_{ij}^{rs} \quad (5.17)$$

$$\text{Min } \frac{1}{w} \sum_{m=1}^w \left( \sum_{a_{ij} \in A} r_m^{ij} x_{ij}^{rs} - b \right)_+^\theta \quad (5.18)$$

$$s. t. \sum_{j:a_{ij} \in A} x_{ij}^{rs} - \sum_{i:a_{ji} \in A} x_{ji}^{rs} = \begin{cases} 1, & \text{if } i = r \\ -1, & \text{if } i = s \\ 0, & \text{otherwise} \end{cases} \quad (5.19)$$

$$x_{ij} \in \{0,1\} \forall (i,j) \in A \quad (5.20)$$

Equation (5.17) is to minimize the average travel time which represents travelers' expectation under normal traffic conditions. Equation (5.18) is to minimize the travel time variability derived based on their repeated travel experiences. Equation (5.19) ensures all the links on the path are feasible. Equation (5.20) defines a binary link-path incidence variable.

## 5.4 CONNECTIONS WITH STOCHASTIC DOMINANCE THEORY

### 5.4.1 Stochastic Dominance Definitions

In this section, we proceed to show that there is a consistent relationship between multi-objective models under study and the classical stochastic dominance theory so that the risk taking behaviors corresponding to the reliability measures can be derived accordingly. To introduce the stochastic dominance to the shortest path domain and assist the route choice decision making, following notions with regard to different ordering rules are first defined. As travel time is an undesirable feature, travelers are assumed to always prefer less travel time to more.

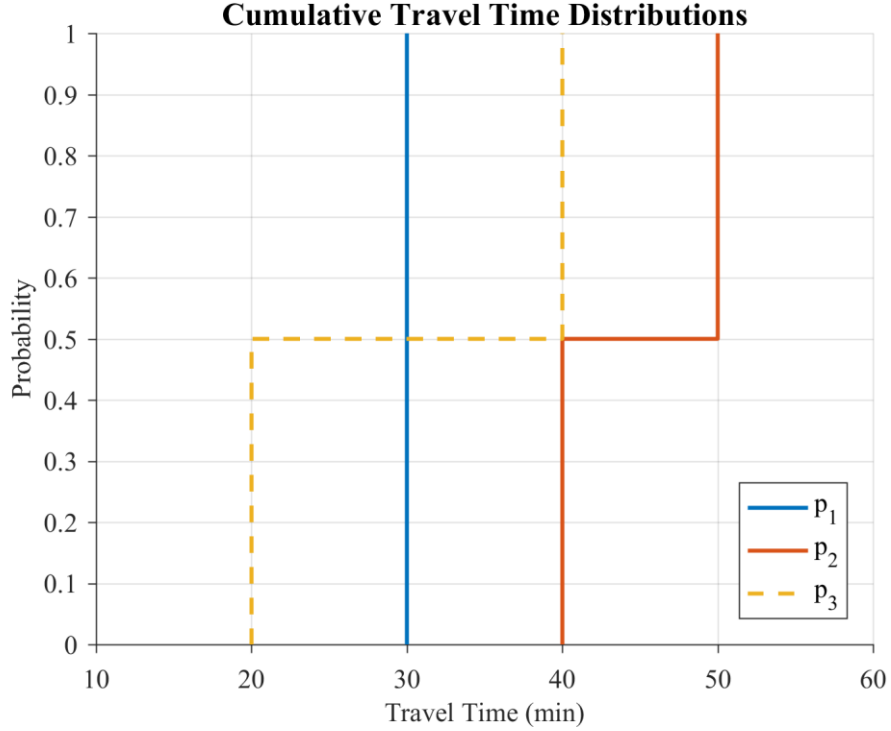
**Definition 3** A path  $p_k^{rs} \in P^{rs}$  dominates another path  $p_l^{rs} \in P^{rs}$  in the sense of the first-order stochastic dominant (FOSD) rule or  $p_k^{rs} \succ_{FOSD} p_l^{rs}$ , if  $F_k(t) \geq F_l(t)$ , for all values of  $t \in [0, T]$ , with the strict inequality holds for at least one  $t$ .

Based on the FOSD definition, it indicates the cumulative distribution curve of travel times on one path lies on or at the right of the distribution curve on the other path. In other words, the cumulative distribution curves of these two paths do not cross throughout the travel time range. Under this condition, the cumulative probability of having travel times that are less than any specific value on the first path is always equal to or larger than that on the second path and for at least one value, the former is strictly larger than the latter. In this sense, any travelers that prefers less to more, which is generally true in the real world, would choose the first path over the second one.

The implication of FOSD can be illustrated with following simple example. If there are three paths, where

- Path 1 takes a constant travel time of 30 minutes;
- Path 2 takes 40 and 50 minutes each with 50% of chance;
- Path 3 takes 20 and 40 minutes each with 50% of chance.

The corresponding cumulative distributions are shown in Figure 1.



**Figure 5.1 Cumulative distributions on three example paths**

Therefore, it is clear that Path 1 and 3 are at the right side of Path 2, except that Path 3 and 2 are overlapping when travel time is 40 minutes. In spite of that, Path 2 will be dominated by the other two paths according to the FOSD definition. However, other than preferring less to more, FOSD does not necessarily involve any risk taking preference. Consequently, it may not be able to differentiate a more reliable path from a riskier path. Therefore, the risk-taking behavior corresponds to the FOSD rule is generally considered as risk-neutral(97). As shown in above example, both Path 1 and 3 have a mean travel time of 30 minutes, but Path 3 has a much higher chance to take a longer travel time than Path 1, which makes Path 3 riskier. However, this cannot be reflected by the FOSD rule. In reality, many travelers are risk averse, as they try to avoid the risky route even at the expense of taking longer travel time. In order to deal with the risk averse behavior, the second-order stochastic dominance criterion is introduced below.

**Definition 4** A path  $p_k^{rs} \in P^{rs}$  dominates another path  $p_l^{rs} \in P^{rs}$  in the sense of the second-order stochastic dominant (SOSD) rule or  $p_k^{rs} \succ_{SOSD} p_l^{rs}$ , if  $\int_t^T F_k(x) dx \geq \int_t^T F_l(x) dx$ , for all values of  $t \in [0, T]$ , with the strict inequality holds for at least one  $t$ .

Based on the definition above,  $p_k^{rs}$  would be preferred to  $p_l^{rs}$ , if the area enclosed by the cumulative distribution curve of former path is always equal to or greater than that of the latter path, for any travel time in the whole range. Still use Figure 5.1 as example. Since there are only three discrete values, we can analyze each of them individually. When the travel time is 20 minutes, the area under Path 1 is  $(30 - 20) \cdot 0 + (40 - 30) \cdot 1 = 10$ , while the area under Path 3 is  $(40 - 20) \cdot 0.5 = 10$  as well. Now if the travel time is 30

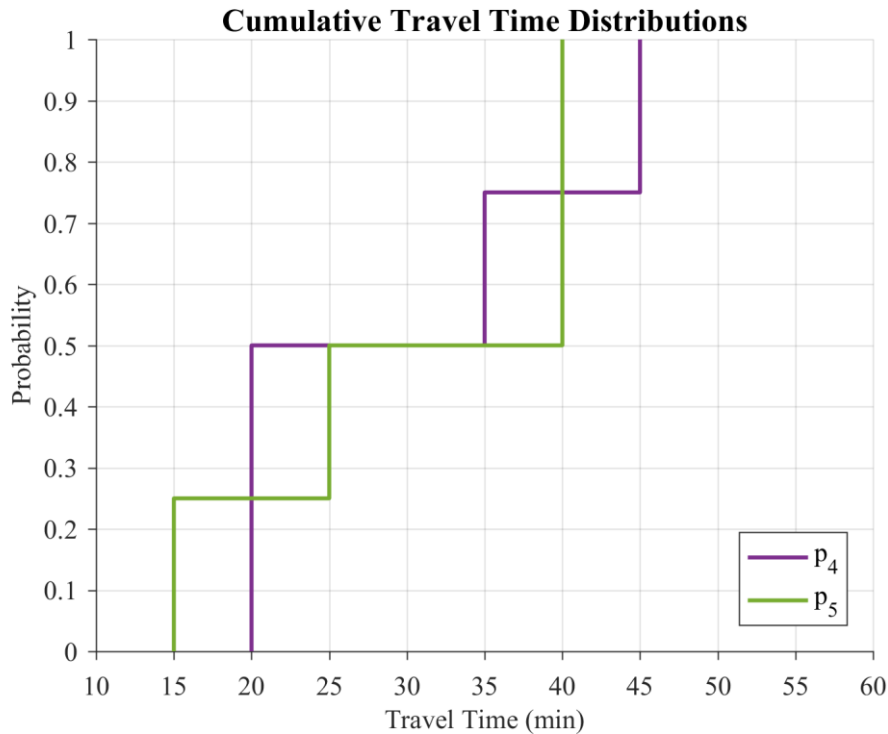


minutes, the area under Path 1 is still 10, while the area under Path 3 becomes  $(40 - 30) \cdot 0.5 = 5$ , which is smaller. It is also easy to see the area under both cumulative curves is 0 when travel time takes 40 minutes. Under this circumstance, we can see Path 1 dominates Path 3 according to the SOSD rule.

Now assume there is additional variability in travel times in which there are 50% of chance of reducing 5 minutes and 50% of chance of increasing 5 minutes and such variability component can be added to either lower or higher travel time values on Path 3. Accordingly, this results in two new scenarios with following travel time distributions:

- Path 4 takes 20, 35, and 45 minutes with probability of 50%, 25%, and 25%, respectively;
- Path 5 takes 15, 25, and 40 minutes with probability of 25%, 25%, and 50%, respectively.

The corresponding cumulative distribution curves are illustrated in Figure 5.2.



**Figure 5.2 Cumulative distributions with additional variability**

In this case, FOSD rule would be indifferent to both paths as their cumulative curves are intersecting multiple times. Meanwhile, we can observe that Path 4 has a larger area enclosed by the cumulative curve when travel time is 20 minutes. However, when travel time is 40 minutes, the area under Path 4 becomes smaller than that under Path 5. Therefore, the SOSD rule becomes ineffective in determine the dominance relationship between them. In this regard, the third order stochastic dominance (TOSD) rule can be applied. In particular, the TOSD rule corresponds to the ruin averse behavior, which tries

to avoid the possibility of encountering heavily congested conditions, even though such possibility is very small. A ruin-averse traveler is definitely risk-averse, but the reverse is not true.

**Definition 5** A path  $p_k^{rs} \in P^{rs}$  dominates another path  $p_l^{rs} \in P^{rs}$  by the TOSD rule or  $p_k^{rs} \succ_{TOSD} p_l^{rs}$ , if  $\mu_k^{rs} \leq \mu_l^{rs}$  and  $\int_t^T \int_y F_k(x) dx dy \geq \int_t^T \int_y F_l(x) dx dy$ , for all values of  $t \in [0, T]$ , with the strict inequality holds for at least one  $t$ .

The definition implies travelers whose behaviors are pertinent to the TOSD rule prefer the route with less travel time variability at the upside of travel time distribution. In last example, the path with variability added to the lower end will dominate the path that having additional variability at the higher end, which makes it more unreliable as travelers have a higher chance of have longer travel time. Such behavior can also be reflected by looking at the skewness of distributions. As shown in Figure 5.2, Path 4 has a positive skewness with 0.54 whereas Path 5 has a negative skewness with -0.54. As Path 5 is considered superior to Path 4, the TOSD rule is able to take the skewness into account, a property that is also reflected in SSD.

#### 5.4.2 Theoretical Connections

With understandings of the stochastic dominance rules, we proceed to examine the relationship between the generalized reliability formulation and stochastic dominance criteria. To facilitate discussion, here we denote  $\Gamma_{FOSD}$ ,  $\Gamma_{SOSD}$ , and  $\Gamma_{TOSD}$  as non-dominated path set with respect to FOSD, SOSD, and TOSD decision rules, respectively. At first, it is easy to obtain the relationship among three stochastic dominance rules.

**Proposition 2** The non-dominated path set in terms of FOSD rule contains that in terms of SOSD rule, which contains that in terms of TOSD rule, or  $\Gamma_{FOSD} \supseteq \Gamma_{SOSD} \supseteq \Gamma_{TOSD}$ .

**Proof.** Firstly,  $\int_t^T F_k(x) dx - \int_t^T F_l(x) dx = \int_t^T [F_k(x) - F_l(x)] dx$ . Based on the definition of FOSD rule, if  $p_k^{rs} \succ_{FOSD} p_l^{rs}$ , then  $F_k(x) - F_l(x) \geq 0$ . Accordingly,  $\int_t^T [F_k(x) - F_l(x)] dx \geq 0$ . Therefore,  $\Gamma_{FOSD} \supseteq \Gamma_{SOSD}$ . Similarly, we can get  $\Gamma_{SOSD} \supseteq \Gamma_{TOSD}$ . ■

**Proposition 3** if  $p_k^{rs} \succ_{FOSD/SOSD/TOSD} p_l^{rs}$ , then  $\mu_k^{rs} \leq \mu_l^{rs}$  where the strict inequality holds for FOSD condition.

**Proof.** First we can get  $\mu_k^{rs} - \mu_l^{rs} = \int_0^T t f_k^{rs}(t) dt - \int_0^T t f_l^{rs}(t) dt = \int_0^T t [f_k^{rs}(t) - f_l^{rs}(t)] dt$ . Then by applying the integration by parts, we can accordingly have  $\mu_k^{rs} - \mu_l^{rs} = - \int_0^T [F_k^{rs}(t) - F_l^{rs}(t)] dt$ . With FOSD rule,  $F_k(x) - F_l(x) \geq 0$  and the inequality holds for at least one value, therefore the integral part is negative, which leads to  $\mu_k^{rs} < \mu_l^{rs}$ . Under SOSD and TOSD definitions, we can directly know  $\mu_k^{rs} \leq \mu_l^{rs}$ . ■

**Proposition 4** The optimal solutions to the  $\mu - \tau_\theta$  model for any value of  $\theta \geq 0$  is a subset of FOSD non-dominated paths.

**Proof.** To show the relationship holds, consider two paths  $p_k^{rs}$  and  $p_l^{rs}$  with cumulative travel time distribution function  $F_k$  and  $F_l$ . Then, based on the generalized reliability measure we can have

$$\Delta\tau = \tau_k^{rs}(b) - \tau_l^{rs}(b) = \int_b^T (x - b)^\theta d(F_k^{rs} - F_l^{rs}) \quad (5.21)$$

$$\text{If } \theta = 0, \text{ then } \Delta\tau = F_l^{rs}(b) - F_k^{rs}(b) \quad (5.22)$$

Based on the definition of FOSD rule, if  $p_k^{rs} \succ_{FOSD} p_l^{rs}$ , then  $F_k(x) - F_l(x) \geq 0$ , for all values of  $t \in [0, T]$ , with the inequality holds for at least one  $t$ . Therefore,  $\Delta\tau \leq 0$ .

If  $\theta > 0$ , then by applying the integration by parts, we can accordingly obtain

$$\Delta\tau = -\theta \int_b^T (x - b)^{\theta-1} (F_k^{rs} - F_l^{rs}) dt \quad (5.23)$$

Because  $x \geq b$ , we can know  $(x - b)^{\theta-1} \geq 0$ . In the meantime, if  $p_k^{rs} \succ_{FOSD} p_l^{rs}$ , then  $F_k(x) - F_l(x) \geq 0$ . Combining these together, we can know  $\Delta\tau \leq 0$ , i.e.  $\tau_k^{rs} \leq \tau_l^{rs}$ . In addition, based on Proposition 3, if  $p_k^{rs} \succ_{FOSD} p_l^{rs}$ , then naturally we can have  $\mu_k^{rs} < \mu_l^{rs}$ . ■

Above proposition is critical in a sense that finding FOSD non-dominated paths will provide a unanimous approach to solving three reliability models all at once. In previous studies, the label correcting algorithm developed based on the Bellman's Principle of Optimality has been validated to be effective in identifying optimal paths(20). This approach will be adapted in the study as well. Before elaborate on the algorithm, following propositions can also be obtained for higher dominance orders.

**Proposition 5** The optimal solutions to the mean-SD (MSD) and mean-SSD (MSSD) model is a subset of SOSD non-dominated paths, except for the scenario where paths have identical mean and SD/SSD.

**Proof.** Based on the definition of SOSD rule, if  $p_k^{rs} \succ_{SOSD} p_l^{rs}$ , then  $\int_t^T F_k(x) dx - \int_t^T F_l(x) dx \geq 0$ . Then based on Proposition 4,  $\Delta\tau \leq 0$ . At the same time, if  $p_k^{rs} \succ_{SOSD} p_l^{rs}$ , then  $\mu_k^{rs} \leq \mu_l^{rs}$ . This means the optimal solutions to the  $\mu - \tau_\theta$  model for any value of  $\theta > 0$  is a subset of SOSD non-dominated paths, including the MSD and MSSD models. ■

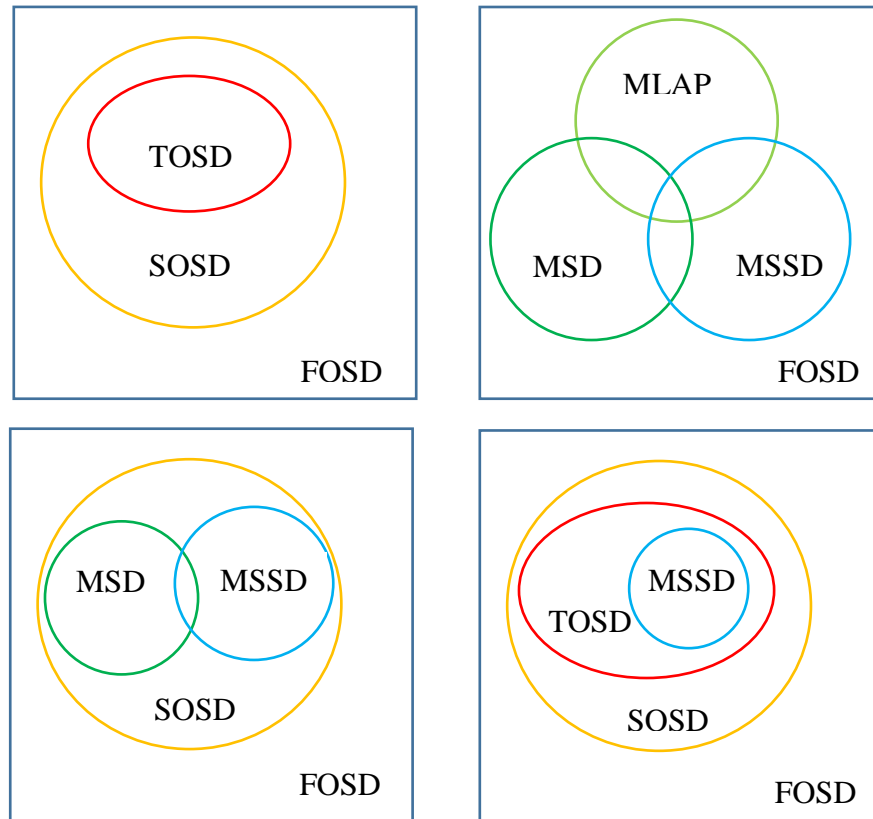
**Proposition 6** The optimal solutions to the MSSD model is a subset of TOSD non-dominated paths, except for the scenario where paths have identical mean and SSD.

**Proof.** Now with  $\theta > 1$ , apply integration by parts once again, we can easily have

$$\Delta\tau = -\theta(\theta - 1) \int_b^T \int_y^T (x - b)^{\theta-2} (F_k^{rs} - F_l^{rs}) dx dy \quad (5.24)$$

First, it is easy to know  $\theta(\theta - 1) > 0$  and  $(x - b)^{\theta-2} \geq 0$  since  $x \geq b$ . Also, based on the TOSD definition,  $\int_t^T \int_y^T [F_k(x) - F_l(x)] dx dy \geq 0$ . As  $F_k^{rs} - F_l^{rs}$  is non-negative, we now have  $\Delta\tau \leq 0$ . Similarly, if  $p_k^{rs} >_{TOSD} p_l^{rs}$ , then  $\mu_k^{rs} \leq \mu_l^{rs}$ . Therefore, if travelers are more concerned with the possible occurrence of long travel time, the exponential reliability factor should be set to above one so that the model gives larger weight to longer travel time to reflect such behavior. If it is chosen to be 2, then the model will become the MSSD model. ■

It should be noted that the exception condition noted in the propositions only occurs if a path is MSD or MSSD non-dominated yet dominated by another path in  $\Gamma_{SOSD}^{rs}$  or  $\Gamma_{TOSD}^{rs}$  (for MSSD non-dominated case only). The probability of encountering this condition is rather low. However, even though it is possible to happen, the impact on the obtained optimal paths is negligible thanks to the path with identical mean and SSD is already included in the SOSD or TOSD non-dominated path set. If the included path is not ultimately chosen by the decision maker, the omitted path would also not be chosen anyway; if the included path is chosen as the optimal path to travel, there would be no difference to the omitted path as two considered attributes are the same. Therefore, the propositions can be considered true under this circumstance without the noted exception condition and the SOSD or TOSD based LC algorithms can be used in finding the MSD or MSSD non-dominated paths. The various relationships can be illustrated as follows.



**Figure 5.3 Consistency between MTTR and SDT Rules**

## 5.5 SOLUTION ALGORITHM

Many previously developed algorithms that are used to solve classical models are no longer feasible for the proposed model because the assumption that path cost is the direct summation of the cost of links that comprise the path is no longer held with incorporation of the travel time reliability measures. A variety of new solutions including but not limited to mathematical programming, simulation based methods, and evolutionary algorithms have been proposed in existing literature to specially deal with such challenge(18; 42; 76). In particular, the stochastic dominance theory based approach has recently been investigated and shown to be efficient in solving transportation models in both continuous and discrete travel time distribution scenarios in relatively large network settings(98). This is because of the appealing advantage of SDT in compliance with the Bellman's principle of optimality, which makes it possible to adapt the traditional label correcting algorithm developed for the deterministic shortest path problems.

In last section, we has proven the congruent relationship between reliability models and the FOSD dominance rule. Hence, the FOSD-based label correcting algorithm (FOSD-LC) can be adapted to find all non-dominated paths, from which the final non-dominated MTTR non-dominated paths can be determined. In the meantime, the label correcting algorithm based on the SOSD and TOSD rules can also be adopted for determining MSD and MSSD non-dominated paths.

To determine the dominance relationship between paths in the solution procedure, it is inevitable to examine the corresponding distributions associated with each path. This is computationally costly because theoretically for every pair of distributions, two comparisons may have to be made in order to know the final dominant relationship. That is to say, we still need to determine if the second path dominates the first one, after we know the first path does not dominate the second one. This is especially burdensome when the number of paths under consideration is large. In order to improve the efficiency of the algorithm, the following proposition is first provided.

**Proposition 7** If  $p_k^{rs} \succ_{FOSD/SOSD/TOSD} p_l^{rs}$ , then  $t_{k_{min}}^{rs} \leq t_{l_{min}}^{rs}$  where  $t_{k_{min}}^{rs}$  and  $t_{l_{min}}^{rs}$  represent the minimum travel time on  $p_k^{rs}$  and  $p_l^{rs}$ , respectively.

**Proof.** Suppose there is a travel time observation  $t_0$  such that  $t_{l_{min}}^{rs} < t_0 < t_{k_{min}}^{rs}$ , then we can easily get  $F_l(t_0) > F_k(t_0) = 0$ . As a result,  $F_k(t_0) - F_l(t_0) = -F_l(t_0) < 0$ . First, this directly contradicts to the FOSD rule. Second, integrating  $-F_l(t_0)$  over the range  $[t_{l_{min}}^{rs}, t_0]$  generates a negative value, which also violates the SOSD and TOSD requirements. ■

Implications from proposition 2 and 6 are valuable in that some unnecessary comparisons can be pre-eliminated by first examining the minimum and average of travel times on two paths. Suppose there are three alternative paths connecting the OD pair. In order to obtain the non-dominated paths, maximum number of pairwise evaluations can be six times. Now assume the minimum travel time on those paths are now known and ordered as  $t_{k_{min}} < t_{l_{min}} < t_{m_{min}}$ . Based on Proposition 6, we could theoretically eliminate half of

the number of comparisons required. For instance, it is clear  $p_l$  would not be preferred  $p_k$ ; otherwise it is a violation to the proposition. In addition, if the average travel time are obtained and suppose the order follows  $\mu_m < \mu_k < \mu_l$ . This additional condition further reduces the number and only one comparison becomes necessary, i.e. whether  $p_k$  dominates  $p_l$ . As a result, the efficiency of the solution procedure is significantly improved. The improved procedure for finding non-dominated paths is provided below.

---

### SDT-LC Algorithm

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Step 1: Initialization. Let  $p^{ss}$  be the path from  $s$  to itself and  $t_0^{ss}$  be the discrete travel time which are zero. Initialize the scan list  $Q = \{p^{ss}\}$ .

Step 2: Select the first path from  $Q$ , and denote it as  $p_k^{js}$ , then delete it from  $Q$ .

Step 3: For any predecessor node  $i$  of  $j$  and  $i$  is not contained in current  $p_k^{js}$ , create a new path  $p_k^{is} = p_k^{js} + a^{ij}$ , and update the path travel time  $t_k^{is} = t_k^{js} + r^{ij}$  and then obtain the set of unique travel time realizations  $\Psi^{is}$  of path  $p_k^{is}$ .

Step 4: Pre-eliminating condition check. Compare the  $t_{k \min}^{is}$  and  $\mu_k^{is}$  of path  $p_k^{is}$  to those of path  $p_l^{is} \in \Gamma_{SDT}^{is}$ , where  $\Gamma_{SDT}^{is}$  is the existing non-dominated path set according to the particular dominance rule of interest. If  $t_{k \min}^{is} < t_{l \min}^{is}$  and  $\mu_k^{is} < \mu_l^{is}$ , check if path  $p_k^{is}$  dominates path  $p_l^{is}$ ; else if  $t_{k \min}^{is} > t_{l \min}^{is}$  and  $\mu_k^{is} > \mu_l^{is}$ , check if path  $p_l^{is}$  dominates path  $p_k^{is}$ ; Otherwise, keep  $p_l^{is}$  and update  $\Gamma_{SDT}^{is} = \Gamma_{SDT}^{is} \cup p_k^{is}$  and  $Q = Q \cup p_k^{is}$ , then go to Step 2.

Step 5: Dominance condition evaluation. Suppose we now need to check whether  $p_k^{is}$  dominates  $p_l^{is}$ . Depending on the particular dominance rule under evaluation:

- (a) FOSD dominance: if  $F_k^{is}(\eta) \geq F_l^{is}(\eta)$  for all  $\eta \in \Psi^{is}$ , and  $F_k^{is}(\eta) > F_l^{is}(\eta)$  for at least one  $\eta$ ;
- (b): SOSD dominance: if  $E(t_k^{is} - \eta)_+ \leq E(t_l^{is} - \eta)_+$  for all  $\eta \in \Psi^{is}$ , and  $E(t_k^{is} - \eta)_+ < E(t_l^{is} - \eta)_+$  for at least one  $\eta$ ;
- (c): TOSD dominance: if  $E[E(t_k^{is} - \eta)_+ - \gamma]_+ \leq E[E(t_l^{is} - \eta)_+ - \gamma]_+$  for all  $\eta, \gamma \in \Psi^{is}$ , and  $E[E(t_k^{is} - \eta)_+ - \gamma]_+ < E[E(t_l^{is} - \eta)_+ - \gamma]_+$  for at least one  $\eta, \gamma$ .

Drop  $p_l^{is}$  and update  $\Gamma_{SDT}^{is} = \Gamma_{SDT}^{is} \cup p_k^{is}$  and  $Q = Q \cup p_k^{is}$ . Otherwise, drop  $p_k^{is}$  and go to Step 6.

Step 6: If  $Q$  is empty, go to step 7; otherwise go to step 2.

Step 7: Identify and output the MTTR non-dominated path set  $\Gamma_{MTTR}^{rs}$ .

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## 5.6 NUMERICAL EXPERIMENTS

In this section, numerical experiments are designed to assess various MTTR models and Label Correcting based solution algorithms. Louisville urban network and probe GPS-based travel time data are used for experiments. Same OD pairs used in previous chapters are also selected for detailed analysis here. To determine the non-dominated paths, the desirable reference travel time is set to be the minimum average travel time for each OD pair, which is obtained from the deterministic LC algorithm implemented in Chapter 3.

### 5.6.1 OD Pair 128-278

The non-dominated paths with regard to each dominance rule for OD 128-478 are presented and discussed in detail as follows.

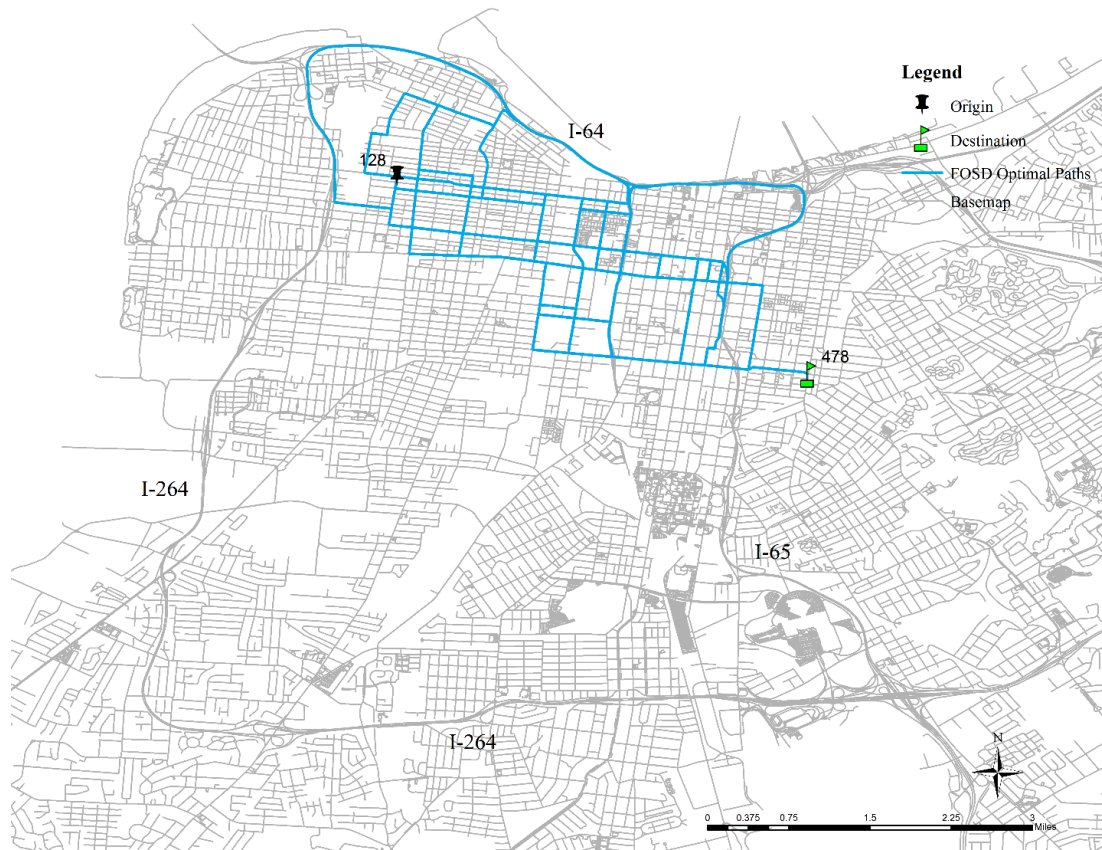
**Table 5.1 SDT Non-Dominated Paths for OD 128-478**

Dominance Rule	Number of Paths	CPU Time (sec)
FOSD	148	1010.6
SOSD	25	159.0
TOSD	7	281.6

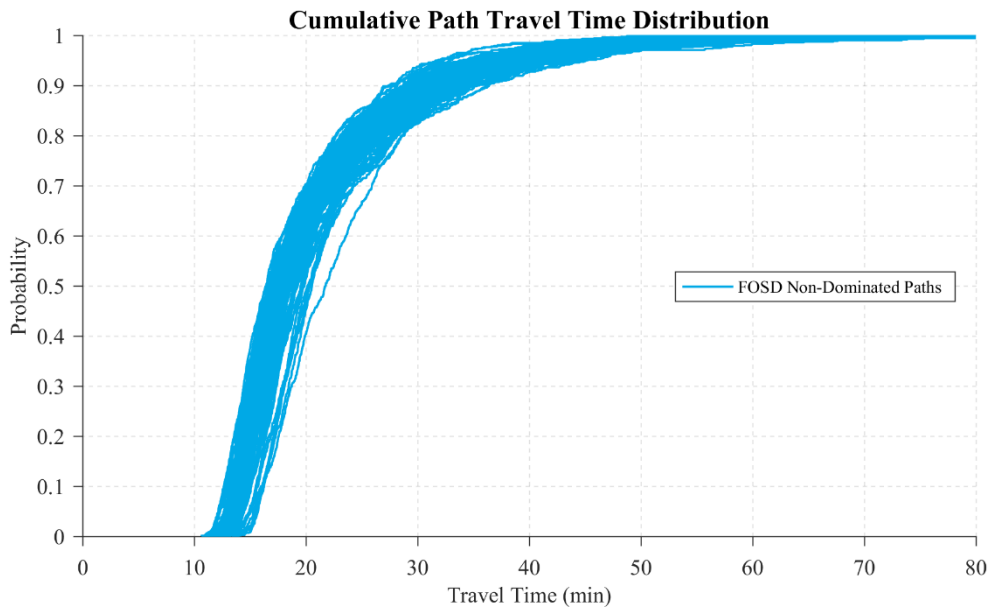
It first can be seen that the FOSD-LC algorithm requires a significantly longer computing time and generates a much larger size of non-dominated paths. This is mainly due to the relatively strict requirement that travel time distribution on a path has to be on the right side of another path for every travel time realization in order to satisfy the FOSD criterion. Since the condition can be easily violated, especially in congested networks, a considerably larger amount of paths in the scan list and accordingly pair-wise comparisons are resulted during the evaluation procedure. This in turn causes a much longer computing time for the FOSD-LC algorithm. The details of routes included in the non-dominated path set are shown in Figure 5.4 and Figure 5.5. It can be observed that there is no single path offering systematically lower travel times compared to other alternatives. This is mainly due to long travel times observed in all the non-dominated paths. The results indicate the FOSD rule alone may not be an effective strategy as it finds too many options for further decision-making.

Compared to the FOSD rule, the SOSD rule significantly reduces the number of non-dominated paths. This is straightforward as the non-dominated paths based on the SOSD rule are part of those found by the FOSD rule, and SOSD involves additional risk-averse behavior when evaluating alternatives. As the number of paths reduces during the path evaluation procedure, the time needed to determine the dominant relationship between newly added path and current non-dominated paths also significantly drops. The obtained non-dominated paths in terms of the SOSD rule are shown in Figure 5.6 and Figure 5.7. It can be seen that only part of links on the map are used and only those paths whose cumulative travel time distributions have smaller portion of long travel times are selected by the SOSD rule. This is consistent with risk-averse travelers avoiding those routes with higher probability of having longer travel times.

As to the TOSD rule, the number of obtained non-dominated paths are further reduced and only 7 paths are included in the final path set. This is understandable as TOSD non-dominated paths are a subset of those obtained from SOSD or FOSD rules. However, TOSD-LC takes longer to run than SOSD-LC. Closer inspection shows that TOSD needs additional expectation operation in determining the dominance relationship between any two paths, and thus requires more time to execute. The travel time distributions of those obtained non-dominated paths also have smaller portion of excessively long travel times.

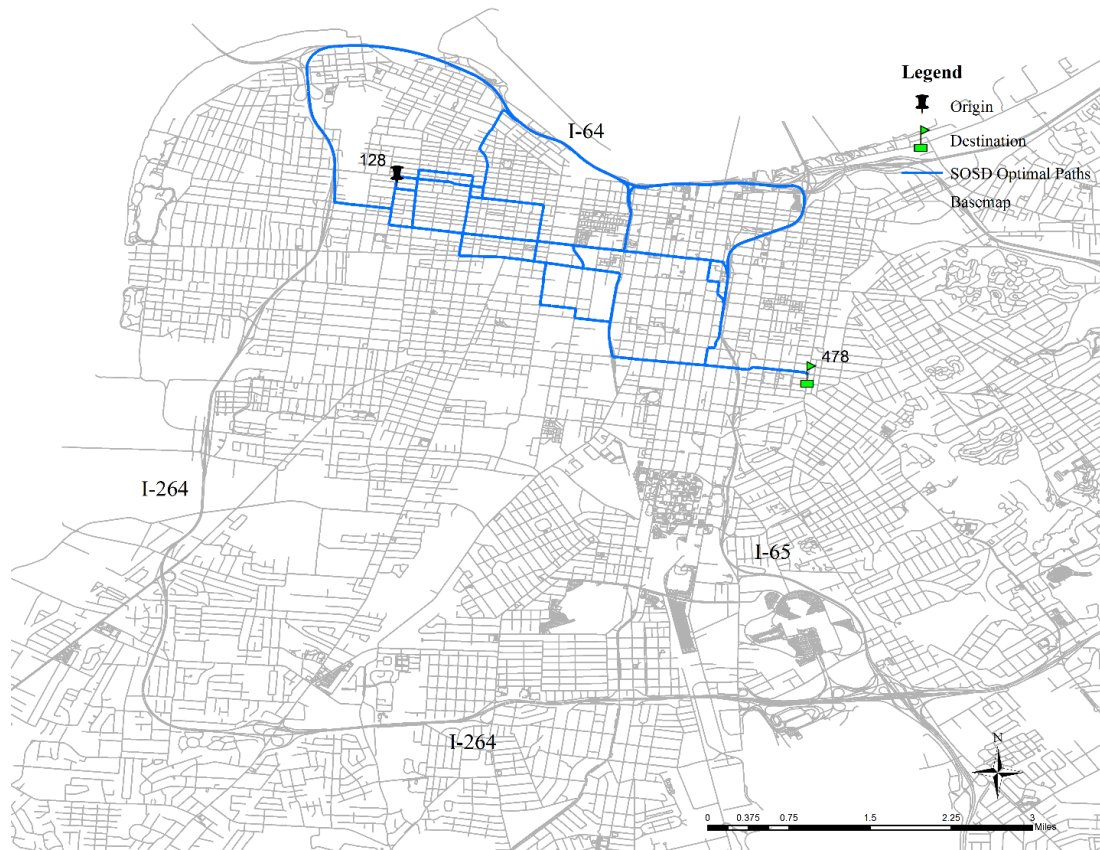


**Figure 5.4 FOSD Non-Dominated Paths for OD128-478**

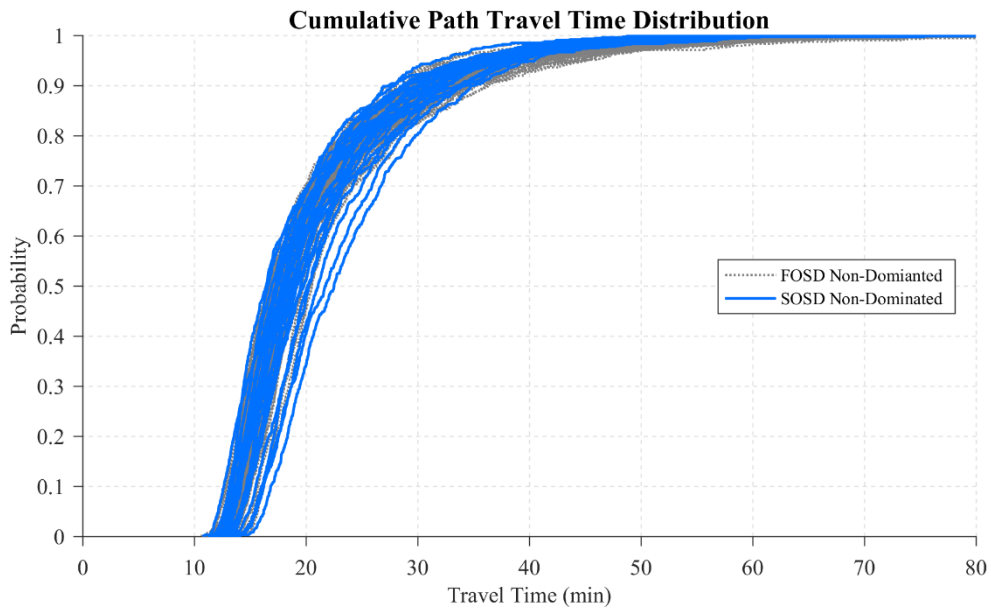


**Figure 5.5 Cumulative Distributions of Travel Time on FOSD Non-Dominated Paths for OD128-478**

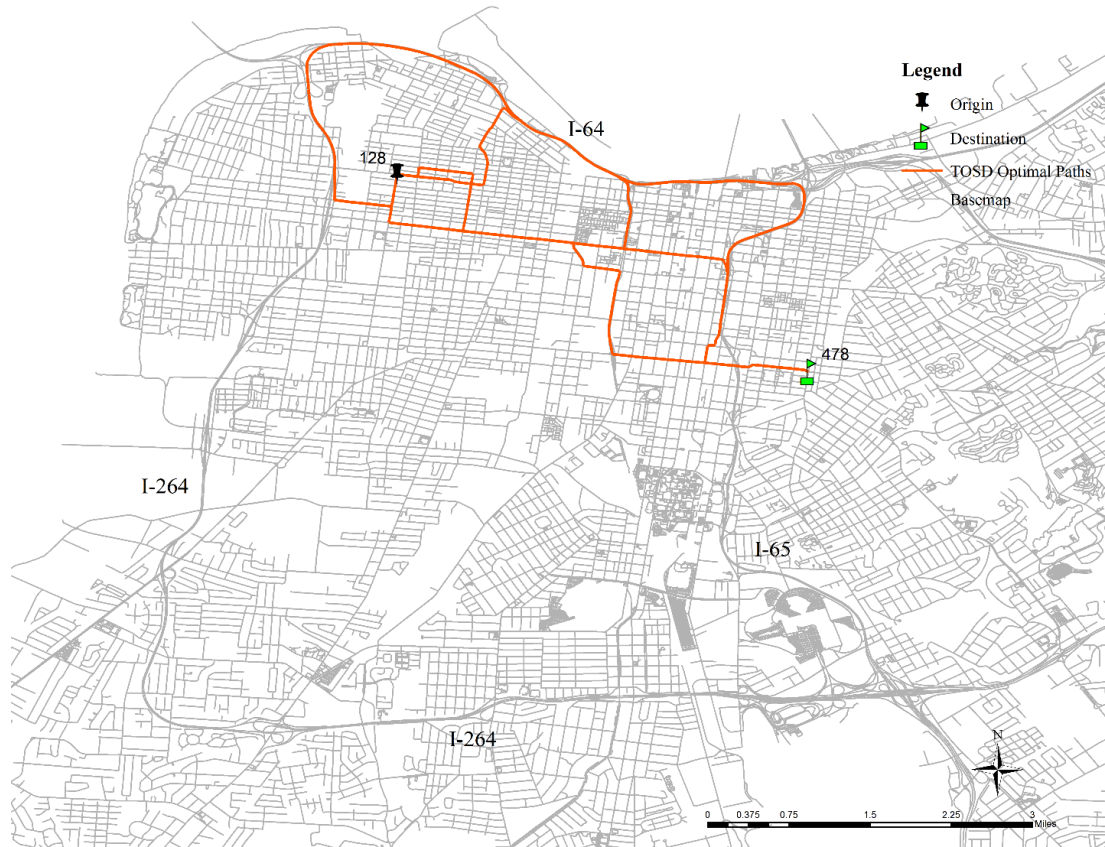




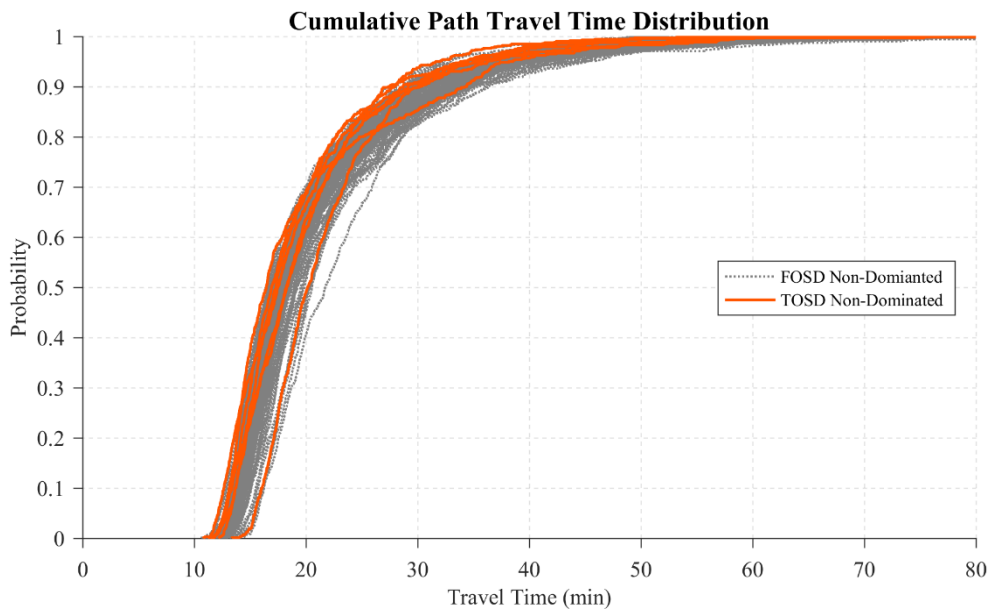
**Figure 5.6 SOSD Non-Dominated Paths for OD128-478**



**Figure 5.7 Cumulative Distributions of Travel Time on SOSD Non-Dominated Paths for OD128-478**



**Figure 5.8 TOSD Non-Dominated Paths for OD128-478**



**Figure 5.9 Cumulative Distributions of Travel Time on TOSD Non-Dominated Paths for OD128-478**

Next, we look at the three reliability models under consideration here. According to the deterministic LC algorithm, the minimum travel time between OD 128-478 is 19.6041 minutes. It is then used as the benchmark to derive LAP, SD, and SSD on all the non-dominated paths obtained from the FOSD-LC algorithm. The final paths in terms of each MTTR dominance rule are summarized in Table 5.2 and Table 5.3. The paths with respect to the minimum average travel time, LAP, SD, and SSD are highlighted with bold in the table. It can be seen that when a single criterion is used, the optimal paths can be different from each other. Path 1 takes the least amount of time on average to arrive at the destination. However, the travel times are more variable compared to other available paths. In contrast, Path 6 has the lowest probability of being late with 31.25 percent of the time. However, this comes in at an expense of taking a longer time. In fact, its average travel time is the longest among the six paths. Meanwhile, the minimum SD and SSD path share the same path, i.e. Path 4. The 95<sup>th</sup> and 99<sup>th</sup> percentile travel times are also provided as indicators of prolonged travel times that can be experienced on each path. Interestingly, Path 4 has the smallest 95<sup>th</sup> and 99<sup>th</sup> percentile travel times, indicating smallest chance of experiencing excessively long travel times; an observation demonstrates risk-averse behaviors underlying the SD and SSD measures.

**Table 5.2 Statistics of MTTR Non-Dominated Paths for OD128-478**

Path ID	Mean	Probability	SD	SSD	Skewness	95th	99th
1	<b>19.6041</b>	35.83	2.82	7.17	2.69	34.99	51.83
2	19.6043	33.47	2.77	7.23	2.46	36.26	55.21
3	19.6287	33.47	2.59	6.90	2.57	35.60	56.37
4	19.8322	39.44	<b>2.46</b>	<b>5.95</b>	2.93	31.63	42.48
5	19.8788	31.67	3.05	8.31	2.83	37.31	60.98
6	20.3032	<b>31.25</b>	3.70	9.12	2.39	40.00	58.31

**Table 5.3 Non-Dominated Paths Regarding Each MTTR Rule for OD128-478**

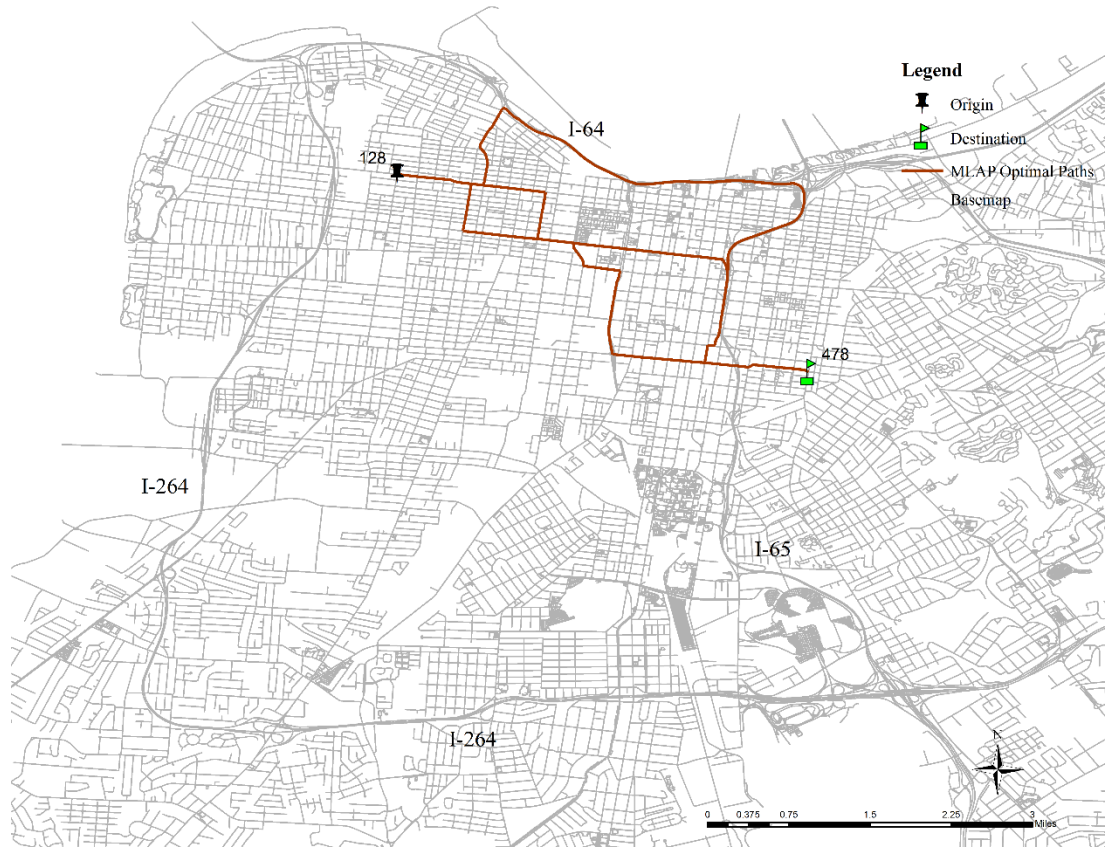
Path ID	MOTAP	MSD	MSSD
1	√	√	√
2	√	√	
3		√	√
4		√	√
5	√		
6	√		

If both mean and LAP are considered simultaneously, four different paths would be identified, as shown in Figure 3. The average travel time varies from 19.6 to 20.3 minutes while the late arrival probability varies from 31.25% to 35.83%. Also, the cumulative travel time distribution curves of four MLAP non-dominated paths are at the right of most FOSD non-dominated paths before the benchmark travel time. After exceeding the

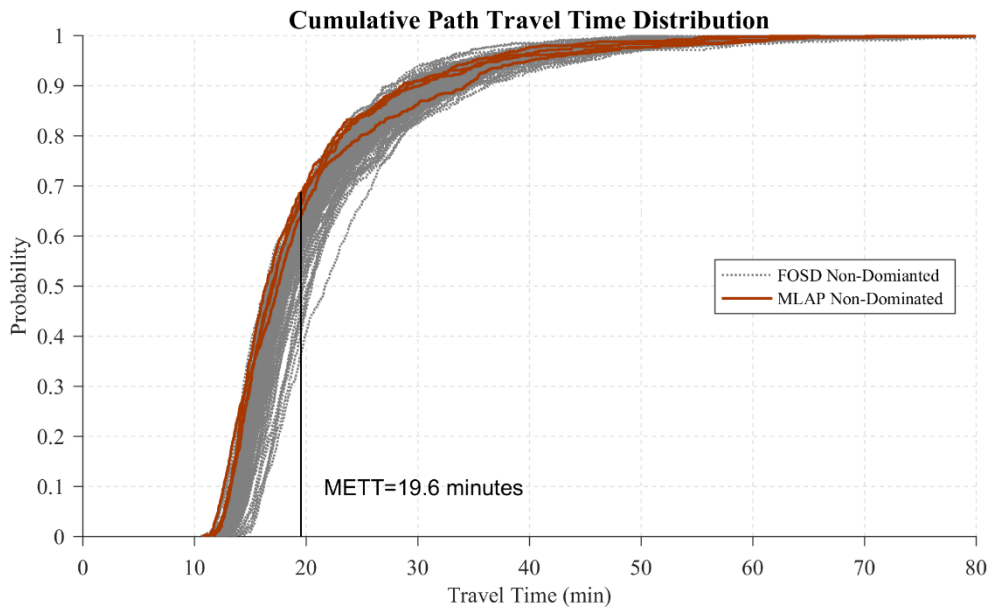
benchmark, especially above the 90<sup>th</sup> percentile line, the cumulative curves on these four paths are below many other paths. This indicates the LAP measure is not able to take into account the distribution of travel times that exceeds the benchmark value. This finding is consistent with the notion that FOSD does not necessarily involve risk-taking behavior as it can only account for the reliability portion of the whole distribution.

If both mean and SD are taken into account, four different paths would be resulted from MSD multi-objective model. Two paths will mainly take interstates whereas two other paths will heavily rely on urban arterial roads. Two other paths which are MLAP non-dominated are now dominated under the MSD dominant rule. The 95th and 99th percentile travel times indicate there is a higher probability of having significant delay on these two paths, and this situation is accounted for by MSD criterion.

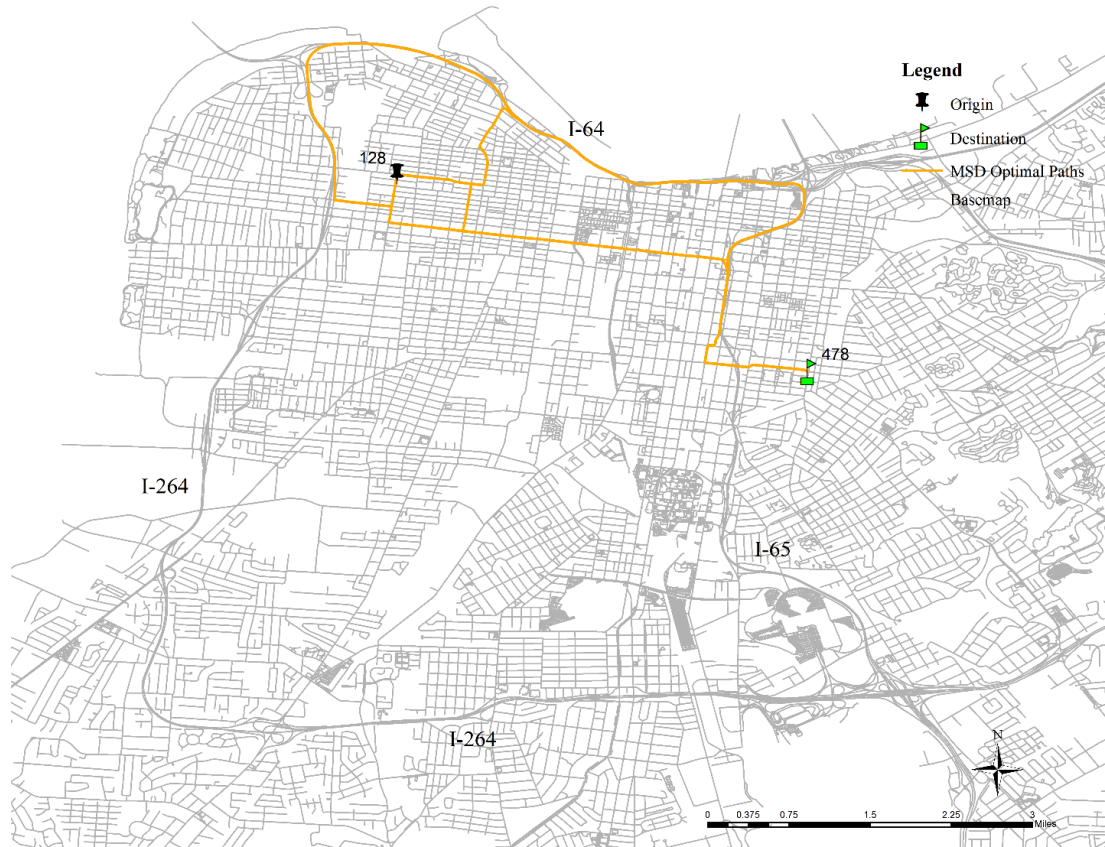
Now take SSD as the reliability indicator, and the MSSD model accordingly generates three non-dominated paths. One of those will heavily rely on urban arterials while the other two will seek to use Interstates, which are more reliable than arterials. Path 2, which is previously MSD non-dominated is avoided by the MSSD rule, as the delays are more significant than other paths according to higher percentile travel time values.



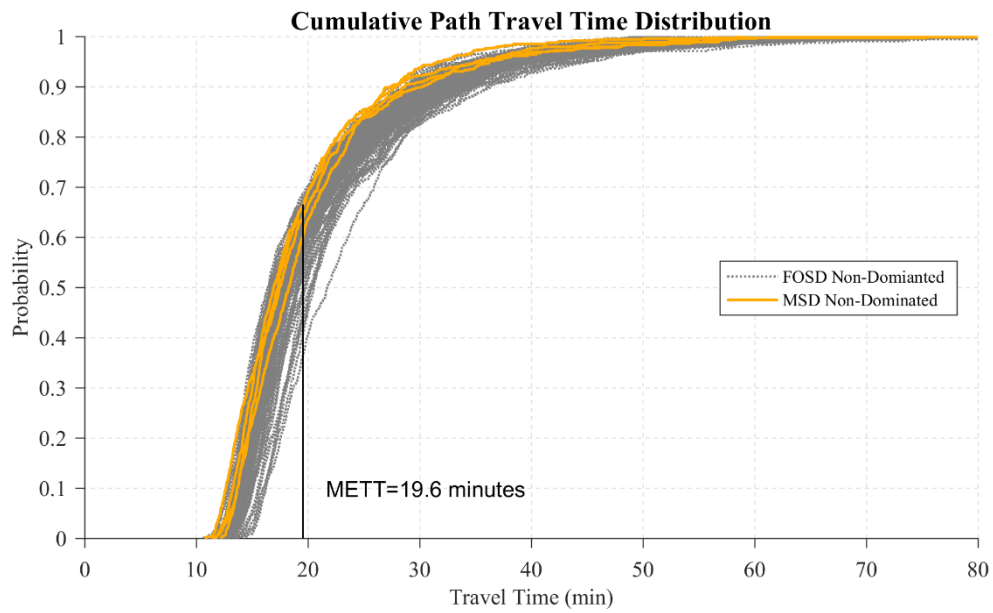
**Figure 5.10 MLAP Non-Dominated Paths for OD128-478**



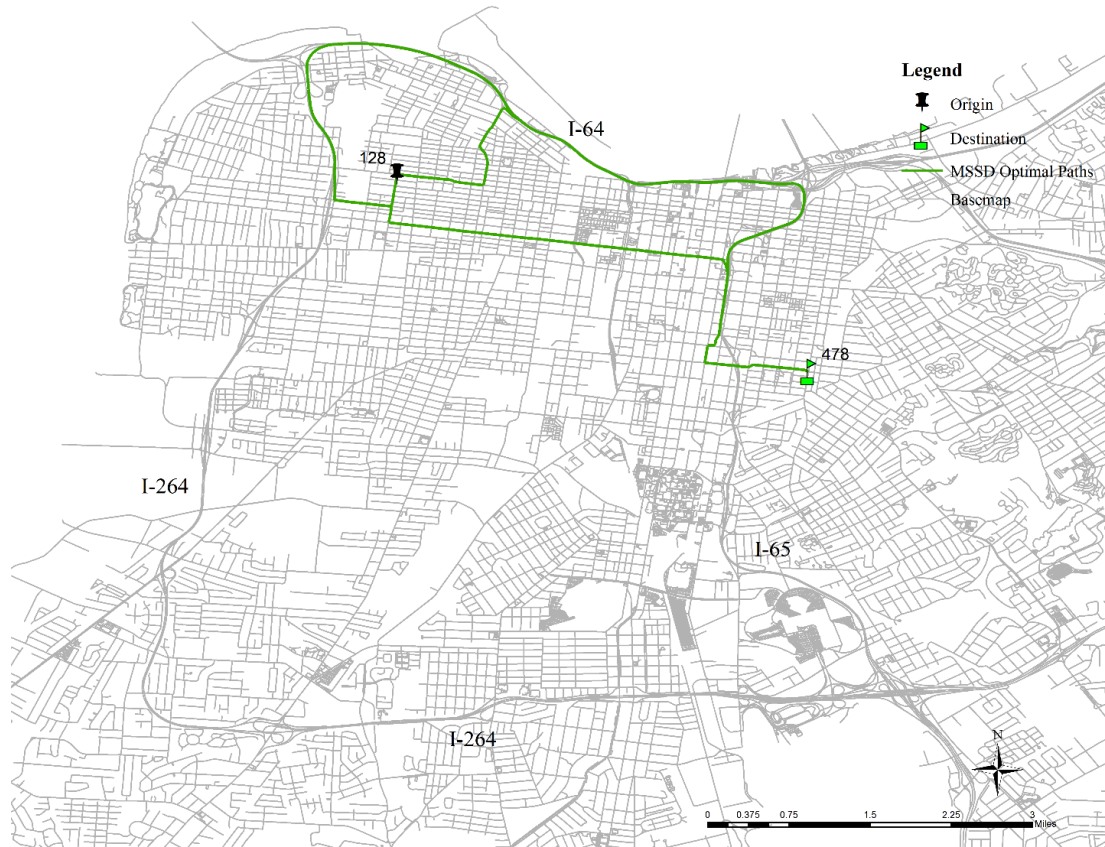
**Figure 5.11 Cumulative Distributions of Travel Time on MLAP Non-Dominated Paths for OD128-478**



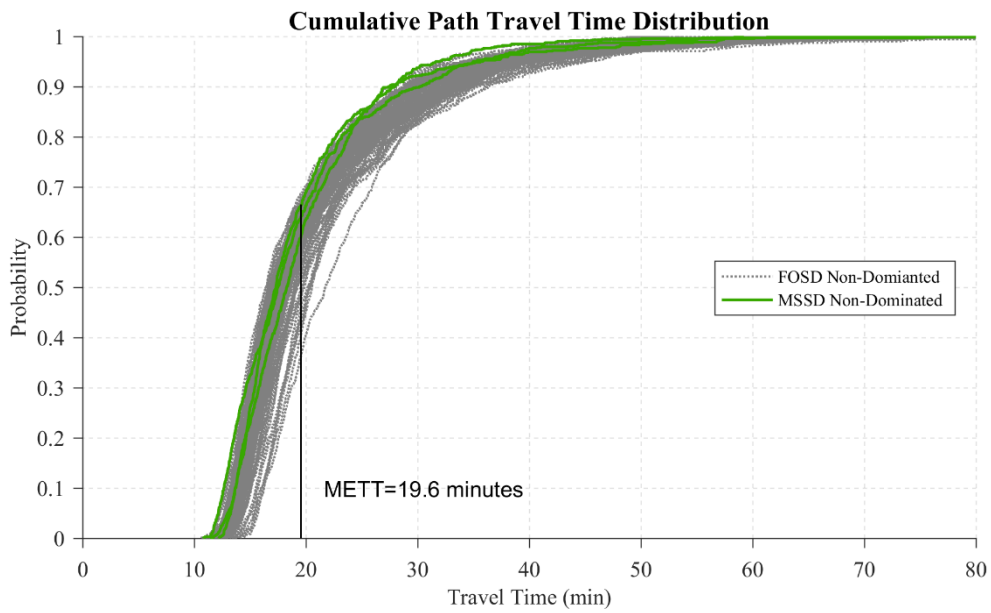
**Figure 5.12 MSD Non-Dominated Paths for OD128-478**



**Figure 5.13 Cumulative Distributions of Travel Time on MSD Non-Dominated Paths for OD128-478**



**Figure 5.14 MSSD Non-Dominated Paths for OD128-478**



**Figure 5.15 Cumulative Distributions of Travel Time on MSSD Non-Dominated Paths for OD128-478**

### 5.6.2 OD Pair 285-9

Similar analysis using SDT and MTTR dominant rules on OD pair 285-9 is also conducted. The results are summarized as follows. At first, the summary of three stochastic dominance rules in terms of the number of optimal paths and computing time is shown in Table 5.4.

**Table 5.4 SDT Non-Dominated Paths for OD 285-9**

Dominance Rule	Number of Paths	CPU Time (sec)
FOSD	42	596.9
SOSD	12	98.3
TOSD	6	263.0

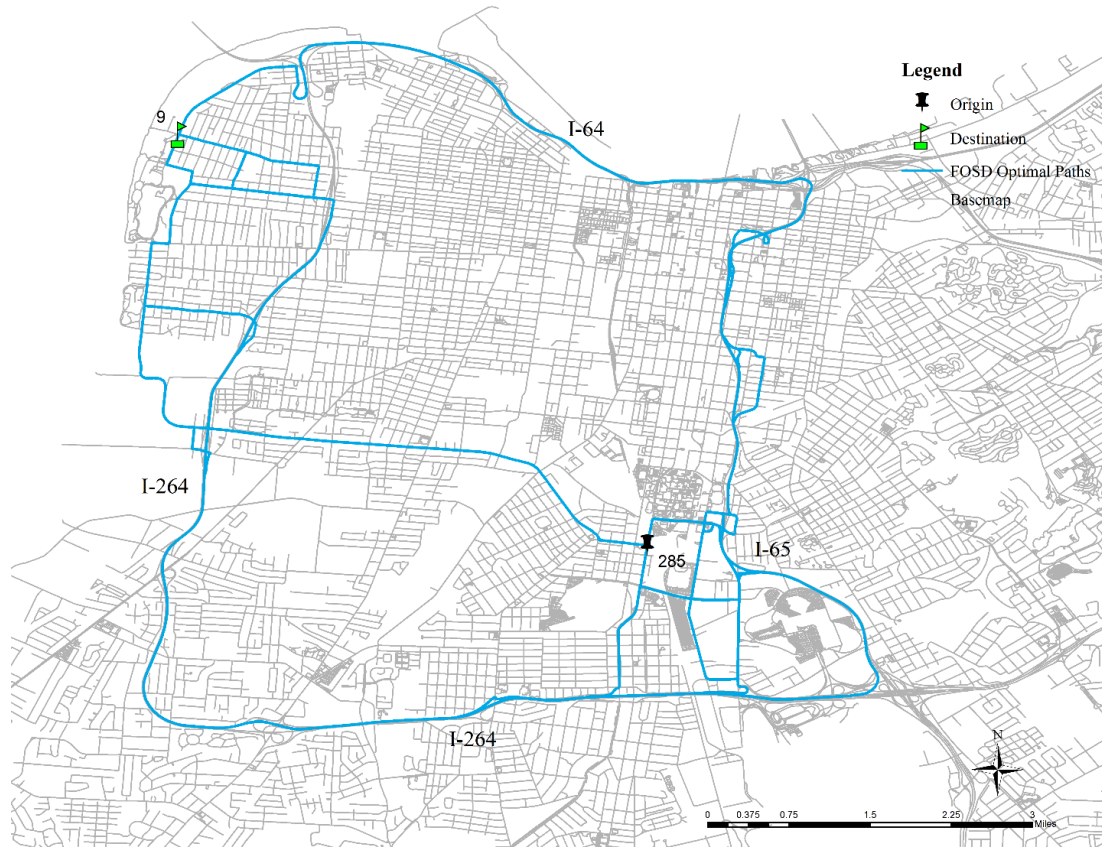
Based on the results, it is first observed that the number of non-dominated paths gradually decreases as the order of dominance rule increases, which is consistent with the observations from the first OD pair. One noticeable difference is that due to the origin and destination being across the city, the identified non-dominated paths are highly reliant on interstate highways, which very likely dominate local streets and thus provide fewer choices. Compared to OD 285-9, OD 128-478, however, is within Louisville downtown area, whose road network has a gridiron type structure with a higher density of intersections and links, which provide more options to maneuver around. In addition, it is shown that the computing time required for OD 285-9 is also less than that for OD 128-478 which is also understandable. The finding from the analysis indicates the effectiveness and efficiency of LC algorithms are also greatly dependent on the spatial structure of the regional network under investigation.

The analysis results with regard to the FOSD rule for OD 285-9 are presented in Figure 5.16 and Figure 5.17. Some observations can be made as follows. First, many different options are recommended and most of them heavily rely on the interstates in the area. This makes sense since interstates are generally more reliable than urban arterials where traffic is periodically interrupted by signals. Second, although some paths are included in the final non-dominated path set, they are not necessarily practically feasible to travelers, because it is highly unlikely for travelers to get off an interchange and then immediately take next ramp to get back to the interstate again.

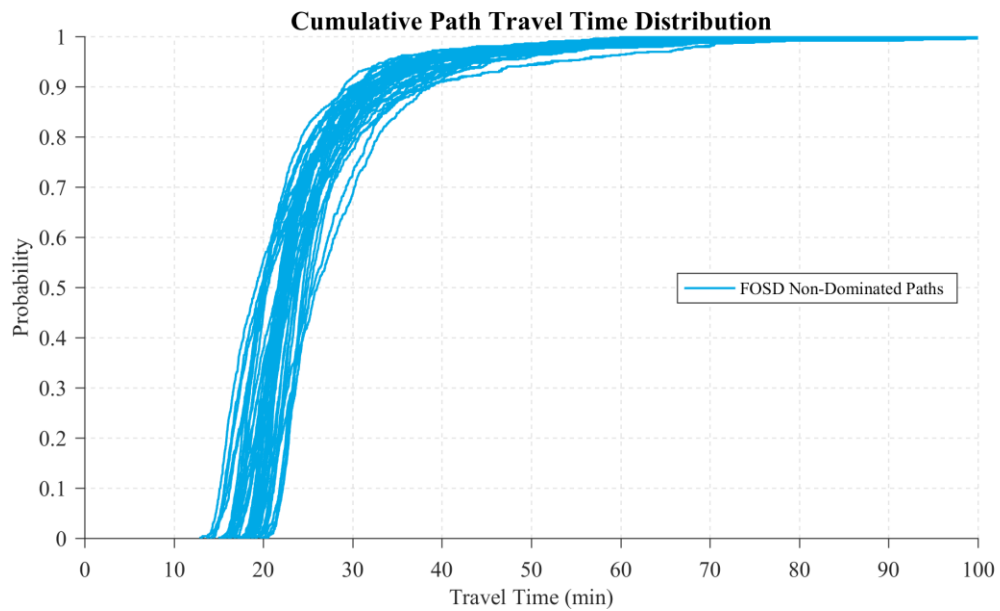
The resulting non-dominated paths from the SOSD criterion are illustrated in Figure 5.18 for spatial display and Figure 5.19 for detailed cumulative distribution curves. With the risk averse implication, the number of paths recommended are significantly reduced. There are three main options. The first option mainly relies on I-64, and the second takes I-65 first and then goes on to I-264, whereas the third option avoids I-65 completely, and instead uses Southern Parkway first and then switches to I-264 to continue the trip. The unfeasibility of some paths as discovered in the FOSD non-dominated paths is also present in the SOSD path set.

By assuming travelers are ruin averse, i.e. more concerned with longer delays, the final non-dominant set further decreases to only 6 paths, as shown in Figure 5.20.

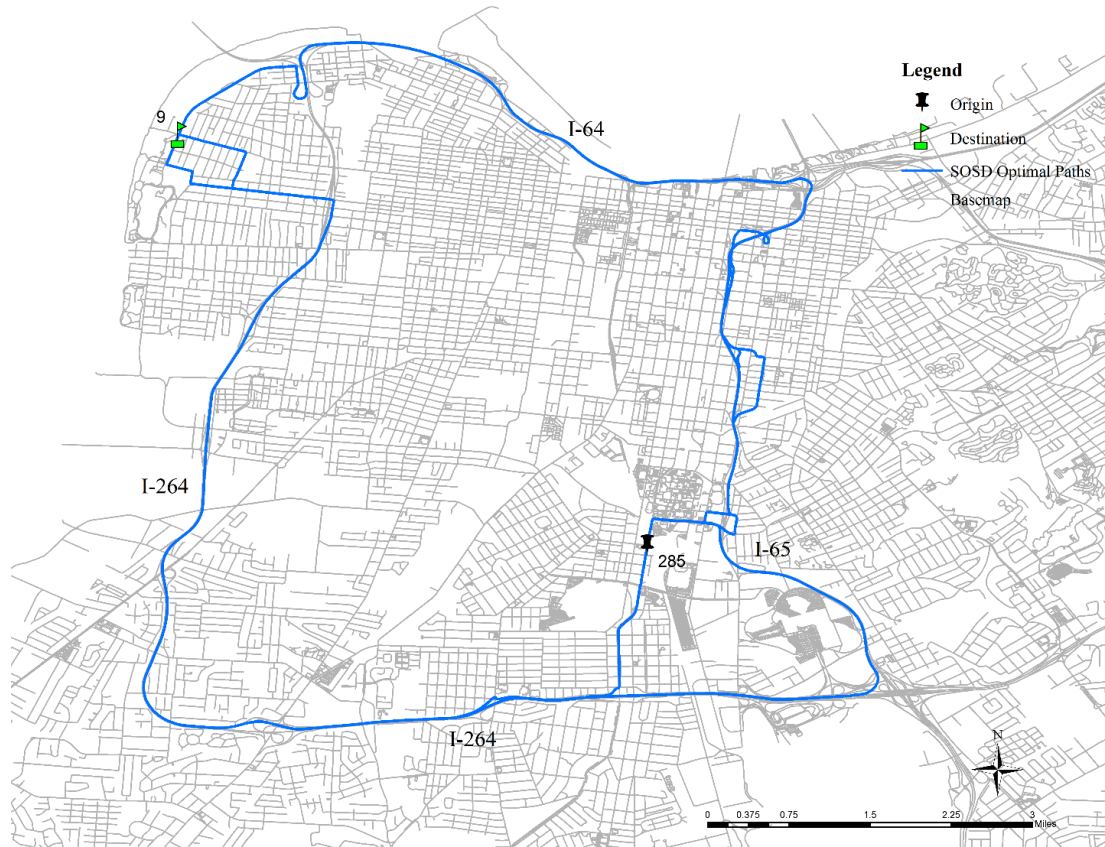




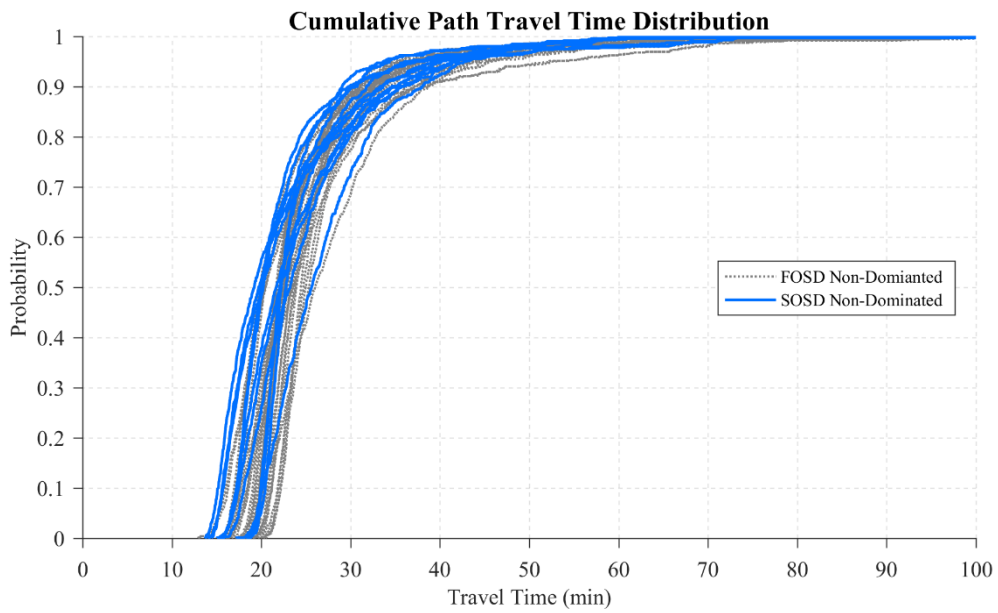
**Figure 5.16 FOSD Non-Dominated Paths for OD285-9**



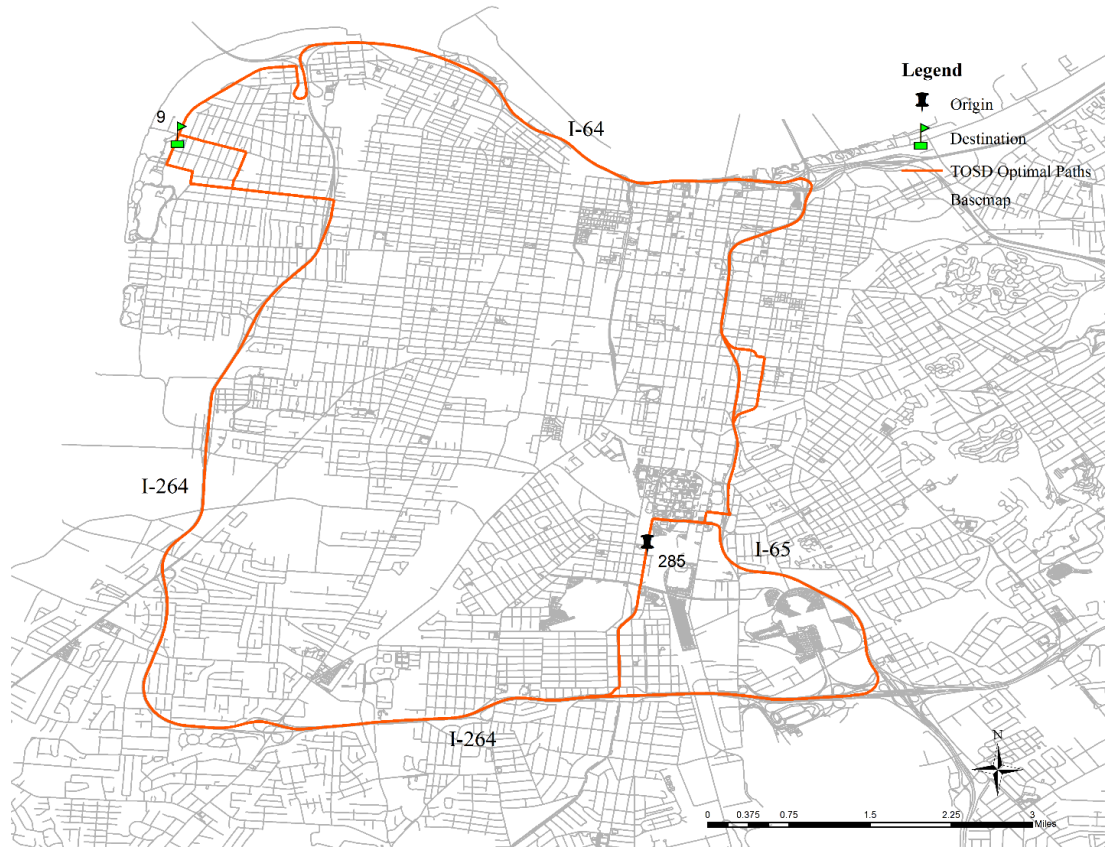
**Figure 5.17 Cumulative Distributions of Travel Time on FOSD Non-Dominated Paths for OD285-9**



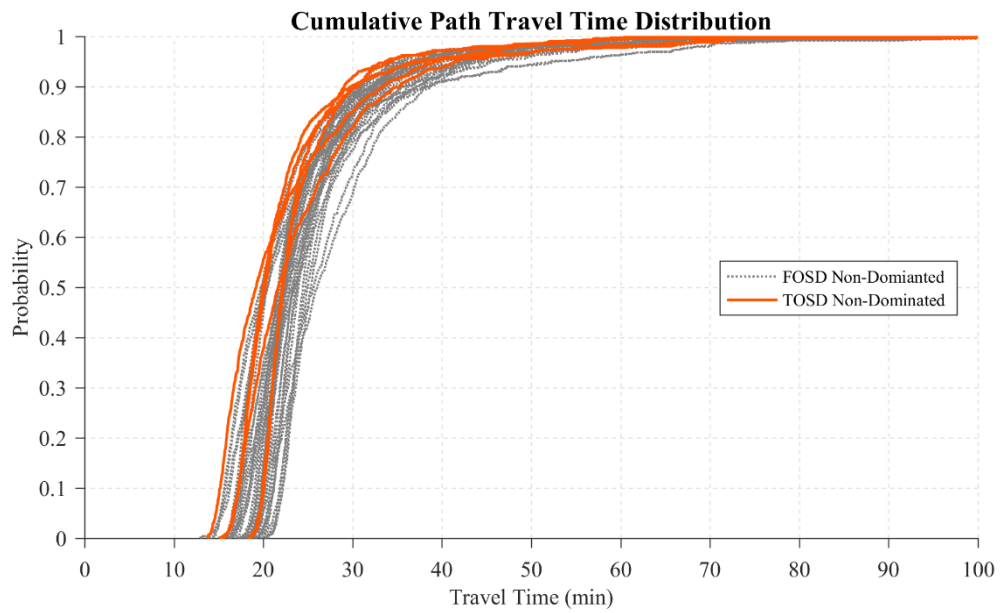
**Figure 5.18 SOSD Non-Dominated Paths for OD285-9**



**Figure 5.19 Cumulative Distributions of Travel Time on SOSD Non-Dominated Paths for OD285-9**



**Figure 5.20 TOSD Non-Dominated Paths for OD285-9**



**Figure 5.21 Cumulative Distributions of Travel Time on TOSD Non-Dominated Paths for OD285-9**

Next, we proceed to evaluate the application of reliability models on the same OD pair. According to the deterministic LC algorithm, the shortest average travel time between OD 285-9 is 22.18 minutes. Adopting it as the benchmark to calculate reliability measures, the corresponding non-dominated routes with regard to each MTTR dominance rule are summarized in Table 5.5 and Table 5.6. Similar to OD 128-478, it is clearly shown that when only optimizing one single objective, different optimal paths can be identified, depending on the particular measure to be minimized.

**Table 5.5 Statistics of MTTR Non-Dominated Paths for OD285-9**

Path ID	Mean	Probability	SD	SSD	Skewness	95th	99th
1	<b>22.18</b>	35.00	3.13	7.36	1.96	39.15	51.95
2	22.83	<b>31.25</b>	2.90	9.27	4.20	36.82	70.45
3	24.07	49.17	<b>2.63</b>	7.64	6.59	33.63	56.00
4	24.21	50.83	2.73	<b>6.50</b>	3.56	34.34	56.73

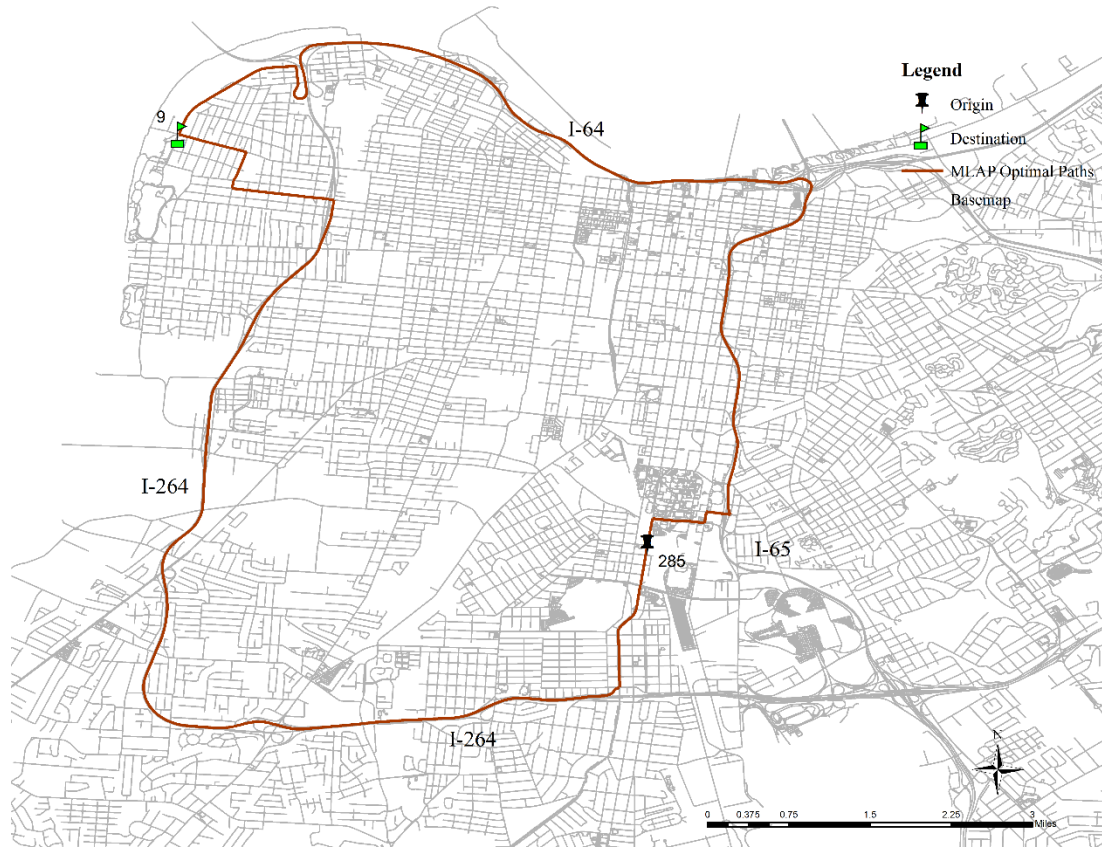
**Table 5.6 Non-Dominated Paths Regarding Each MTTR Rule for OD285-9**

Path ID	MLAP	MSD	MSSD
1	√	√	√
2	√	√	
3		√	
4			√

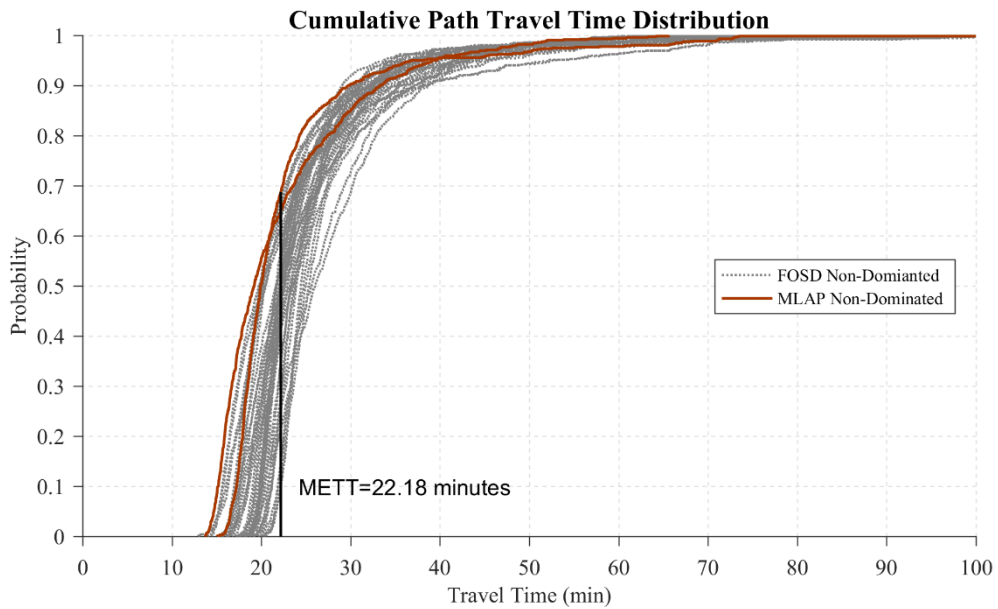
In particular, if the model tries to minimize the mean and late arrival probability at the same time, two distinct paths are more attractive to travelers. The first path takes I-65 first and then switches to I-64 for the most of remaining trip. In comparison, the second path would choose Southern Parkway first and then rely on I-264 for a significant portion of trip before getting off the interstate and travelling on local streets.

If travelers possess a risk averse preference, SD can be used to replace LAP in the routing model. In this case, an additional Path 3 will become MSD non-dominated attributing to its SD value is smaller than the other two paths previously identified by the MLAP dominance rule.

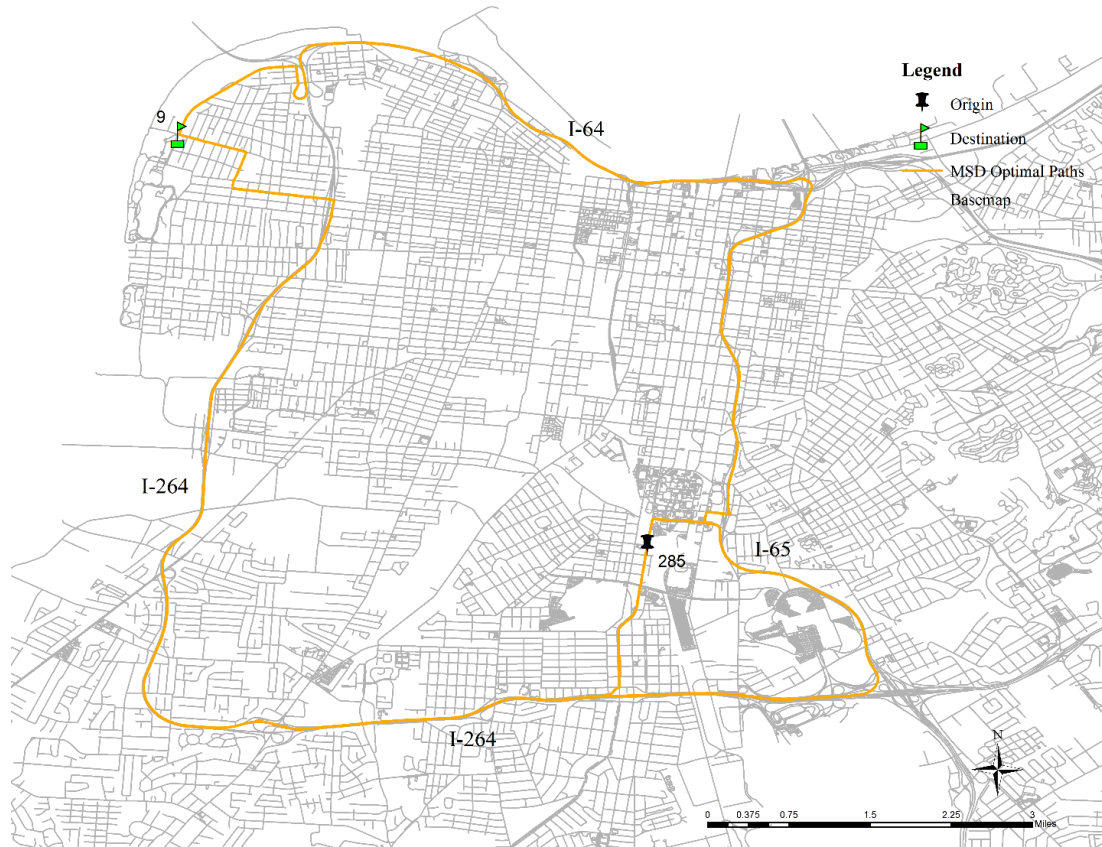
In the situation where travelers are not only risk averse but even ruin averse, the SSD measure is a more appropriate indicator as it gives disproportionate emphasis on larger deviations. With the MSSD model, Path 1 and Path 4 are identified as the non-dominated paths. Closer analysis on the distributional statistics reveals that these two paths both have lowest skewness value and 95<sup>th</sup> and 99<sup>th</sup> percentile travel times; this indicates travel time distributions on these two paths are less skewed to the right with a much shorter right tail. As a result, the MSSD non-dominated paths have less probability of encountering significantly long delays.



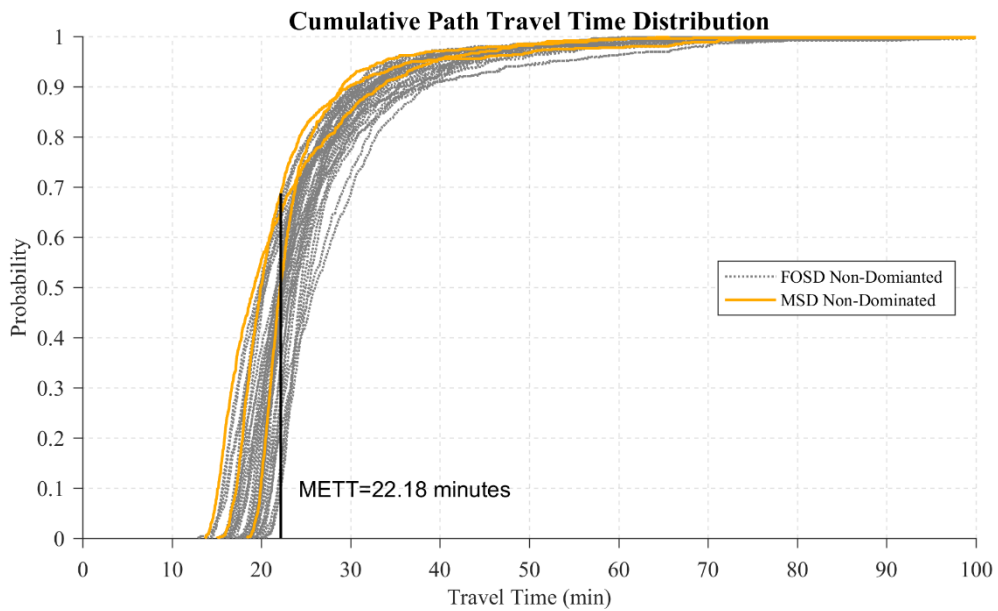
**Figure 5.22 MLAP Non-Dominated Paths for OD285-9**



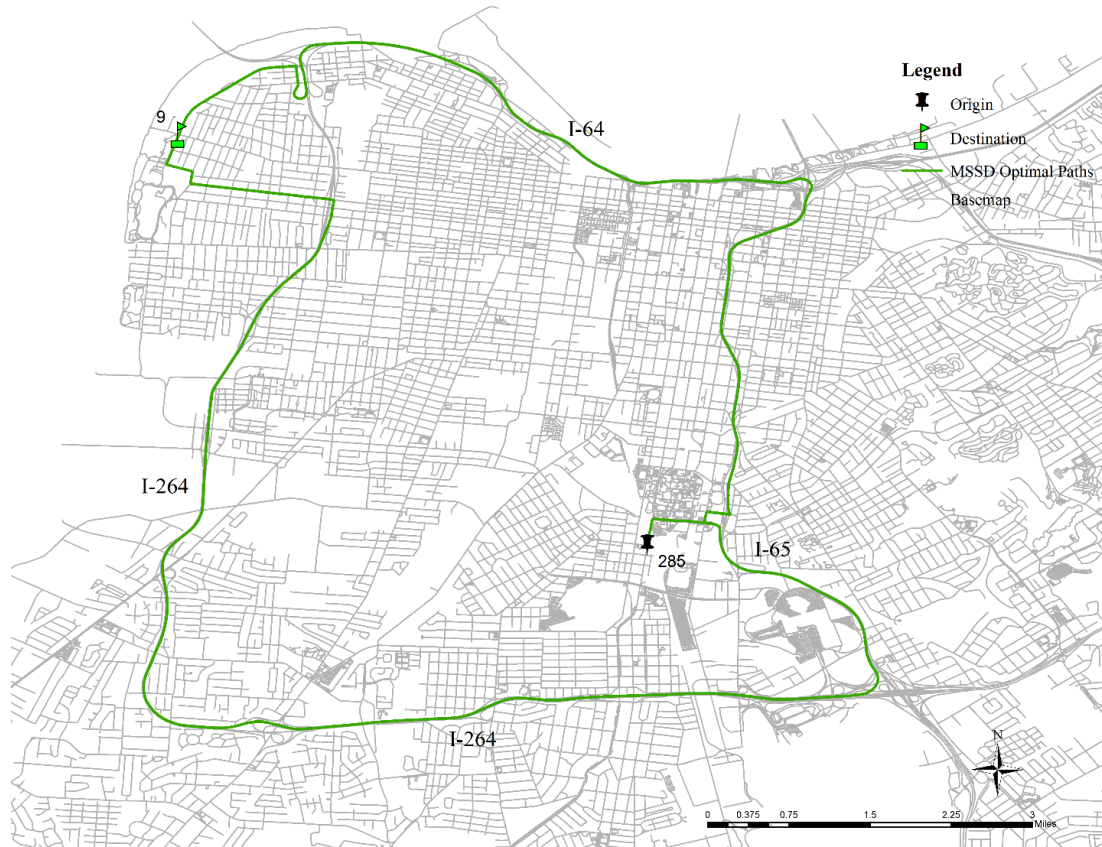
**Figure 5.23 Cumulative Distributions of Travel Time on MLAP Non-Dominated Paths for OD285-9**



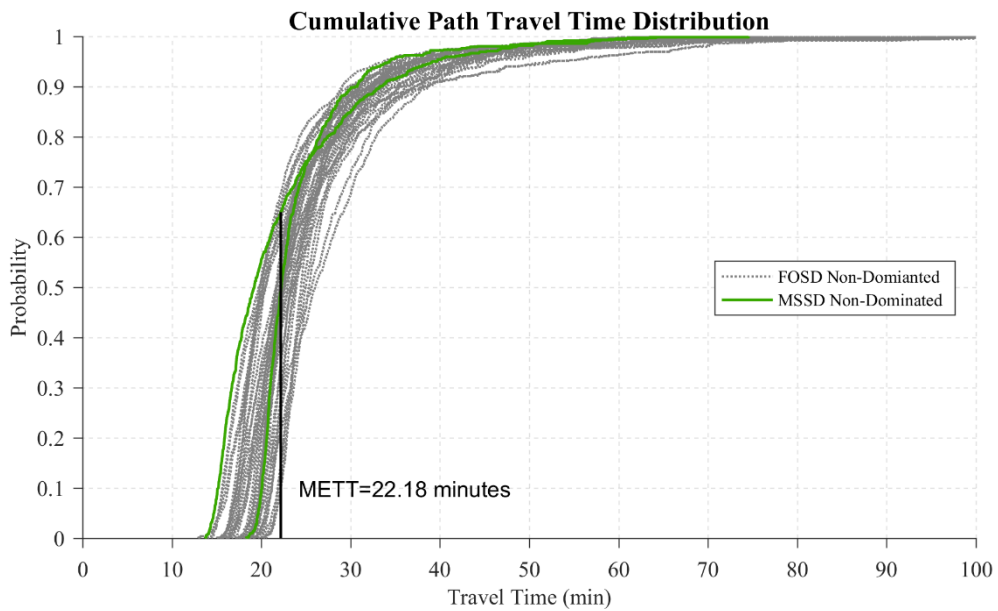
**Figure 5.24 MSD Non-Dominated Paths for OD285-9**



**Figure 5.25 Cumulative Distributions of Travel Time on MSD Non-Dominated Paths for OD285-9**



**Figure 5.26 MSSD Non-Dominated Paths for OD285-9**



**Figure 5.27 Cumulative Distributions of Travel Time on MSSD Non-Dominated Paths for OD285-9**

## 5.7 CONCLUSIONS

A variety of route choice models have been studied in transportation modeling community, in response to the emerging recognition of the importance of travel time reliability in travelers' route choice decision making process. In this paper, the on-time arrival probability, scheduling delay, and semi-standard deviation measures are selected for further examination of their properties. It is found that all three measures base their calculations solely on the right side of the distribution; therefore, they belong to a more generic upper partial moment formulation. Depending on the value of exponential factor, the three measures under study express different behaviors with respect to the travel time above the benchmark. In particular, OTAP corresponds to a single point on the distribution, therefore is indifferent to the magnitude of variation of travel time over the benchmark. In contrast, SD and SSD both focus on unreliability part of the distribution, but SD assigns equal weights to the deviations whereas SSD is more affected by the larger deviations.

In addition, the consistent relationships between the generalized measure and SDT decision rules are established. This provides two important implications. First, according to the risk-taking behavior of each SDT decision rule, it is deduced that OTAP, SD, and SSD are risk-neutral, risk-averse, and ruin-averse, respectively. This finding is consistent with how the deviations are treated by the three measures. Therefore, if a traveler's attitude towards uncertainty or large deviations is known, a more representative reliability measure can be chosen to reflect his/her behavior. Second, the established relationships facilitate the development of analytical solution algorithms for reliability models by taking advantage of currently available LC algorithms. Since there is lack of effective algorithms for SD and SSD models, this alternative approach offers a great value to apply the reliability measures to practical applications, such as travel demand forecasting and network design models.

Numerical experiments based on a real-world urban network and GPS-based data are conducted to evaluate the reliability models and SDT rules. Unlike observations made in previous studies that only a small size of paths are found to be non-dominated with respect to SDT rules, a much larger number of paths are included in the study, especially for the FOSD rule. This is due to the more variable and skewed travel time distribution on the paths, making the FOSD condition easily violated at the right tail of the distributions. This observation indicates the SDT rules alone may not be efficient enough for highly unreliable networks. In this regard, the multi-objective formulation involving the reliability consideration is preferred. It can significantly reduce the size of non-dominated set to only 2-4 paths, which is desirable to decision makers. The results also show that the non-dominated paths are distinct from the three reliability models, attesting the importance to incorporate travelers' risk-taking preferences into route choice models.

In future studies, empirical surveys on the stated or revealed preferences are necessary to understand travelers' actual decision-making behaviors under uncertain conditions. It is also important to understand how travelers set their benchmark travel time to make



departure time and route choices. In addition, since the travel time variation is caused collectively by various non-recurring events, the degree of variation varies at different times of day and incident conditions. As a result, the identified optimal path could be different under varying scenarios. In this regard, there is still a need to evaluate the travel time reliability separately at different time periods and travel conditions.

## CHAPTER 6 A MULTI-OBJECTIVE USER EQUILIBRIUM MODEL

### 6.1 INTRODUCTION

The traditional travel demand forecasting model has been extensively used by state DOTs, MPOs and other transportation agencies for decades in their planning process. By comparing predictions from the model with current infrastructure condition, transportation planners are able to identify inadequacies in current transportation system, and therefore make better informed decisions on project prioritization to accommodate foreseeable challenges. In this process, the travel demand model serves a crucial role in linking travelers' travel behaviors and transportation network performance. As one of the critical components in the traditional "four-step" model, the traffic assignment determines how travelers use the road network. Traditionally, the equilibrium flow pattern is obtained when the travel time on all used paths is equal to the minimum travel time whereas the travel time on all unused paths is either equal to or longer than that minimum value between OD pairs. To achieve equilibrium, the original problem is often transformed into a mathematical program which can take advantage of the fact that the path travel time is the linear summation of travel times on links comprising the path(99).

Similar to the route choice model discussed in previous chapters, the fundamental assumption underlying the traditional traffic assignment model also involves only considering the travel time between OD pairs. However, it has been empirically found that travel times on the network can be unreliable, and travelers are well aware of such uncertainty from their daily experiences and factor it into their decision making process. Consequently, travel time reliability should also be incorporated into the user equilibrium model as well; otherwise, the procedure may not represent traveler's actual perspective, and lead to biased results.

Based on the work in previous chapters, we know that SSD and multi-objective formulation are more appropriate in representing travel time reliability and reconstructing the route choice model. As the multi-objective route choice model incorporating SSD has been developed and analytical approach has been proven effective in large-size networks, the next step is to reformulate the traditional traffic assignment model to account for the travel time reliability consideration. Therefore, the objective of this chapter is to propose a new multi-objective user equilibrium (MOUE) model in which SSD is applied as the reliability objective to extend traditional deterministic user equilibrium (DUE) model, and then develop an effective solution algorithm to obtain the equilibrium condition under the multi-objective setting for practical applications.

The rest of the chapter is organized as follows. Section 6.2 provides the problem statement, which applies the multi-objective approach to reformulate the route choice and traffic assignment models. A solution algorithm is proposed in Section 6.3 to solve the new user equilibrium model based on previously implemented FOSD-LC algorithm and method of successive averages approach. Numerical experiments are conducted on two test networks with varying sizes to demonstrate the applicability of the proposed model

and algorithm in Section 6.4. The findings from the study are summarized in the final section.

## 6.2 PROBLEM STATEMENT

Assume travelers know exactly the travel time and variability on each link and would make rational choices among path alternatives. Accordingly, with the multi-objective path finding formulation involving stochastic travel times, travelers try to minimize the average travel time and travel time unreliability at the same time when making route choice decisions. Based on previous chapter, the corresponding route choice model can be described as shown below.

$$\begin{cases} \min \mu^{rs} \\ \min \tau^{rs} \end{cases} \quad (6.1)$$

In order to solve the multi-objective routing model, previously defined path dominance rules in terms of the mean and SSD still hold here. After incorporating dominance definitions with regard to the route choice decision making process into the traffic assignment model, the multi-objective user equilibrium is reached when the following conditions are met.

**Definition 1** Multi-objective user equilibrium conditions are achieved such that no traveler on a path can be better off in terms of either criterion, whether the average travel time or variability, without worsening the other criterion by unilaterally switching to other routes.

In other words, the non-dominated paths connecting each OD pair should have positive traffic flows, whereas the dominated paths should carry no traffic. Now let  $\Gamma_{MSSD}^{rs}$  represent the set of all non-dominated paths under the MSSD dominance rule. Accordingly, above condition can be expressed mathematically as

$$f_k^{rs} > 0, \forall (r, s) \in RS, k \in \Gamma_{MSSD}^{rs} \quad (6.2)$$

$$f_k^{rs} = 0, \forall (r, s) \in RS, k \notin \Gamma_{MSSD}^{rs} \quad (6.3)$$

In addition, the flow conservation requirement should be satisfied as follows.

$$\sum_{k \in \Gamma_{MSSD}^{rs}} f_p^{rs} = q^{rs}, \forall (r, s) \in RS \quad (6.4)$$

Meanwhile, let  $v_a$  denote the traffic flow on the link  $a$ . Therefore,

$$v_a = \sum_{r \in N} \sum_{s \in N} \sum_{k \in P^{rs}} f_k^{rs} \delta_{a,k}^{rs}, \forall a \in A \quad (6.5)$$

where  $\delta_{a,k}^{rs}$  is link-path incidence indicator. It is 1 if link  $a$  is on the path  $p$ , and 0 otherwise.

Here we adopt the widely used Bureau of Public Road (BPR) function to model the link travel time which is flow-dependent and increases as traffic flow on the link increases.

$$\mu_a = t_a^0 \left( 1 + \alpha \left( \frac{v_a}{c_a} \right)^\beta \right), \forall a \in A \quad (6.6)$$

where  $t_a^0$  is the free flow travel time on link  $a$ ;  $c_a$  is the capacity on link  $a$ ;  $\alpha$  and  $\beta$  are function parameters and are chosen to be 0.15 and 2, respectively.

Accordingly, the average path travel time  $\mu_k^{rs}$  can be derived directly from summation of the travel time from links that comprise the path, which can be expressed as

$$\mu_k^{rs} = \sum_{a \in A} \mu_a \delta_{a,k}^{rs}, \forall (r, s) \in RS, k \in P^{rs} \quad (6.7)$$

Since the sampling based approach has been used to account for the correlation structure, a simulation-based approach based on joint travel time distributions is adopted to generate random samples in each iteration, so that  $\tau_k^{rs}$  can be constantly updated in response to the newly generated  $\mu_k^{rs}$ . This way the previously implemented FOSD-LC algorithm is still applicable. Now suppose during each assignment iteration, a traffic simulation module is employed to generate  $w$  discrete travel time realizations for each link on the network, and let  $r_i$  denote the  $i$ -th travel time realization on link  $a$ . Then, we can determine SSD of path travel time  $t_k^{rs}$  as

$$\tau_k^{rs} = \left( \frac{1}{w} \sum_{i=1}^w (\sum_{a \in A} r_i \delta_{a,k}^{rs} - b)_+^2 \right)^{0.5} \quad (6.8)$$

The multi-objective traffic assignment model based on the above equations will be iteratively implemented until the MOUE condition is reached when the difference between input and updated link flows on the network becomes insignificant.

### 6.3 SOLUTION ALGORITHM

In this chapter, the FOSD-based all-to-one approach is adapted to find the non-dominated paths during the iterative traffic assignment process. Reflecting on last chapter, the relationship between MSSD and FOSD dominance rules has been established. Therefore, we first adapt the FOSD-LC algorithm to find all the non-dominated paths for every OD pair on the network. From these paths, we can then determine the non-dominated paths  $\Gamma_{MSSD}^{rs}$  based on the MSSD dominance rule. This path finding procedure will be called periodically during the traffic assignment process.

Once  $\Gamma_{MSSD}^{rs}$  is obtained, we now have the non-dominated paths on which travel demand can be assigned. In next step, an approach based on the method of successive averages and reference point assignment (MSA-RPA) is applied in order to solve the multi-objective user equilibrium condition. In this method, a reference point with regard to the best scenario in terms of average travel time and variability that travelers may encounter from their past experiences is defined beforehand. In other words, travelers will have an ideal or imagined path that requires least travel time and possesses best reliability in mind

and will compare current alternatives to the ideal path when selecting a path. A similar method has also been discussed in (100). Based on this idea, the attractiveness of each path can then be determined based on the distance between the actual path and reference point. Suppose  $C_\Gamma$  and  $C_r$  are the cost vector consisting of average travel time and SSD for a non-dominated path set and reference point, respectively. The standardized Euclidean distance is applied here to balance out the contribution from variables with different scales of values. Accordingly, the distance between two individuals can be calculated as

$$d_k = (C_\Gamma(k) - C_r) V^{-1} (C_\Gamma(k) - C_r)' \quad (6.9)$$

where  $V$  is a two-by-two diagonal matrix whose first and second diagonal element is the variance of the average travel time and SSD on respective values contained in  $C_\Gamma$  and  $C_r$ .

Therefore, the smaller the distance between the path of interest and reference point, the more appealing the path is to travelers. As a result, more travelers are expected to select such path. To ensure the portion of travel demand assigned to the path is proportionate to its attractiveness, following route choice probability equation is defined:

$$\gamma_k = \frac{1/d_k}{\sum_{k \in \Gamma_{MSSD}^{rs}} (1/d_k)}, \forall (r, s) \in RS \quad (6.10)$$

Therefore,

$$f_k^{rs} = \gamma_k q^{rs}, \forall (r, s) \in RS, k \in \Gamma_{MSSD}^{rs} \quad (6.11)$$

Accordingly, the traffic assignment procedure based on the method of successive averages can be developed as follows.

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### MSA-RPA Algorithm

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Step 1: Initialization. Specify the number of iteration  $n_{max}$  and convergence criterion  $\varepsilon$ . Set iteration counter  $n = 1$ . Perform an initial simulation run based on the average travel time and variance-covariance matrix  $\Omega$  to generate the travel time matrix  $M_1$ .

Step 2: Path selection.  $\forall (r, s) \in RS$ , call procedure FOSD-LC to generate  $\Gamma_{MSSD}^{rs}$ .

Step 3: Traffic assignment. Based on equation (12)-(14), determine the portion of  $q^{rs}$  to assign to path  $k \in \Gamma_{MSSD}^{rs}$  and obtain path flow vector  $F_n$ . Update the link flow vector  $\hat{V} = F_n \cdot \Delta$ , where  $\Delta$  is the link-path incidence matrix.

Step 4: Network reloading. Update link flows as  $V_n = \left(1 - \frac{1}{n}\right) V_{n-1} + \frac{1}{n} \hat{V}$  where  $n > 1$  and average link travel time vector  $\mathcal{T}_n$  based on equation (6). Run the simulation module again using  $\mathcal{T}_n$  and  $\Omega$  to obtain  $M_n$ .

Step 5: Converge test. Calculate the gap function  $Gap_n = \frac{\sum_{a \in A} |v_{a_{n-1}} - \hat{v}_a|}{\sum_{a \in A} v_{a_{n-1}}}$ ,  $n > 1$ . If

$Gap_n \leq \varepsilon$  or  $n = n_{max}$ , then stop; otherwise go to step 2 and set  $n = n + 1$ .

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## 6.4 NUMERICAL EXPERIMENTS

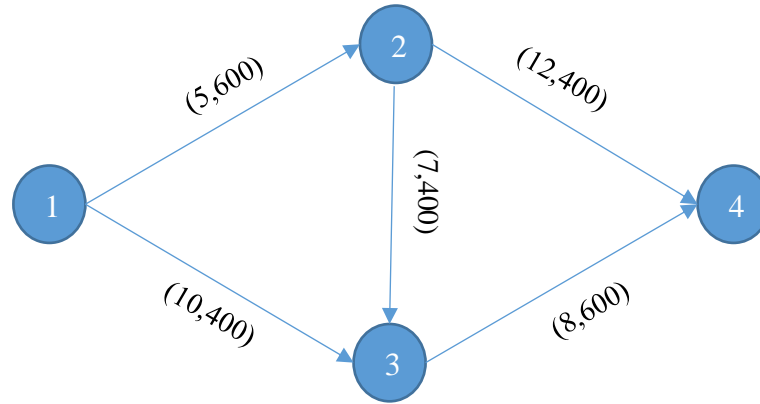
In this section, numerical experiments are carried out on two networks to test the effectiveness of the proposed simulation-based solution approach in solving the multi-objective route choice and traffic assignment model. Particularly, the travel time on links on two networks are all assumed to follow the log-normal probability distribution, which is deemed suitable with the underlying properties such as non-negativity and asymmetry. Many studies have shown log-normal distribution fits best with empirical data(25). Furthermore, the variance-covariance matrix stays the same throughout the modeling process. It is postulated that since travelers build the concept of variability from their day-to-day experiences, such long term variation is independent of short-term flow fluctuation when the reliability is factored into their route choice decision.

### 6.4.1 Small Network

A small network consists of four nodes, five links, and three paths as shown in Figure 6.1 is first analyzed(63). The first number in the parentheses next to each link represents the free-flow travel time while the second number means the capacity associated with the link. The node sequence for each link and path are shown in Table 6.1. The demand between the origin and destination is 1000 units. The travel time threshold for calculating SSD uses the following equation.

$$b = \vartheta * FFTT \quad (6.12)$$

where *FFTT* stands for the free-flow travel time, and  $\vartheta$  is the reliability parameter and the lower the value, the more risk averse the traveler will be. The above equation will enable us to adjust the parameter value to evaluate its impact on the equilibrium state later. During each traffic assignment iteration, 10,000 travel time realizations are simulated simultaneously for all the links on the network with a correlation coefficient of 0.5 between any two links. The first element of the reference point is chosen to be shortest FTTT, which is 17 in this case. The second element is the travel time reliability, where 0 is used to indicate no variation in travel time is desired. Meanwhile, the deterministic user equilibrium model which only considers average travel time in the cost function is also solved and used as a benchmark to compare with solutions from the proposed model. In particular, the obtained link travel times under DUE condition can be considered as a long-term travel time pattern that is finally optimal to travelers on the network. In order to obtain the additional day-to-day or long-term travel time reliability pattern, the study simply assumes the v/c ratio to represent coefficient of variation, which can be directly used to derive the variation of travel time on the link of question. The idea behind the assumption is that links with higher level of congestion tend to be less reliable. Note that link 3 carries no flow when DUE is obtained. Therefore, the variation is assumed to be 2.



**Figure 6.1 Small Network**

The solution algorithm is coded and executed in MATLAB environment. In addition to DUE and MOUE, the use equilibrium model that only considers the travel time reliability component (RUE) is also analyzed for comparison purposes. This can be easily done since the variance-covariance stays constant. For MOUE, it takes 193 iterations to achieve the equilibrium condition with the convergence error less than 0.002%. A closer inspection of the converging trend indicates that the traffic flows on three paths quickly approach the equilibrium state and stay stable after 15 iterations. The CPU running time is about 1.1 minutes. The results from three models are reported in Table 6.1.

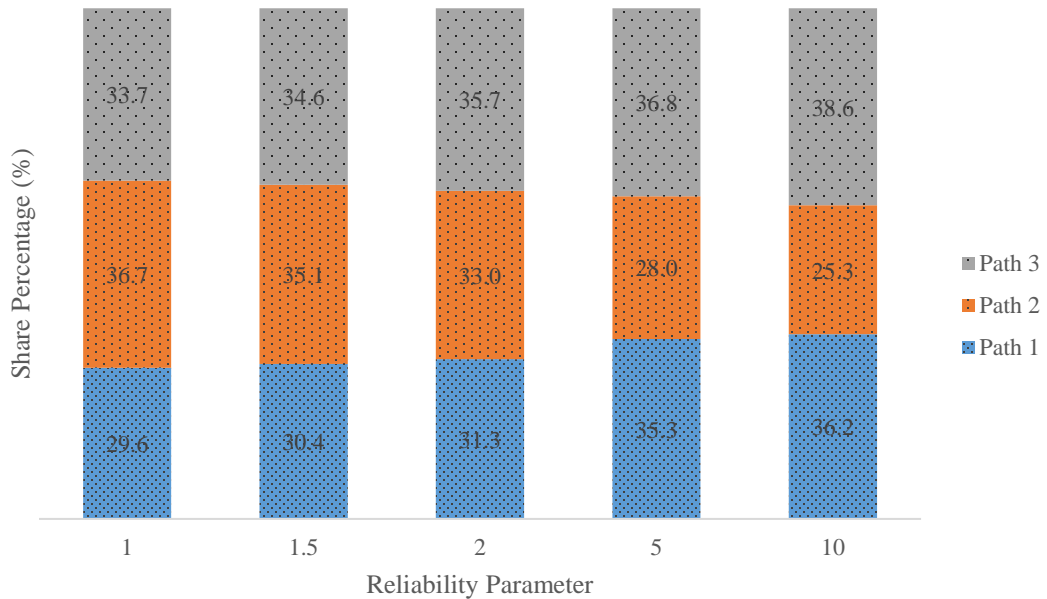
**Table 6.1 Network Performance under User Equilibriums**

Scenario	ID	Node Sequence	DUE			RUE			MOUE		
			Flow	Mean	SSD	Flow	Mean	SSD	Flow	Mean	SSD
Link	1	1-2	532.4	5.59	5.51	1000	6.65	5.06	662.5	5.91	4.68
	2	2-4	532.4	15.19	30.25	0	12.00	18.09	295.8	12.98	19.32
	3	2-3	0	7	1.55	1000	12.19	5.54	366.7	7.88	2.04
	4	1-3	467.6	12.05	17.84	0	10.12	13.21	337.5	11.07	12.50
	5	3-4	467.6	8.73	6.85	1000	11.33	7.41	704.2	9.65	6.63
Path	1	1-2-4	532.4	20.78	33.41	0	18.65	21.15	295.8	18.90	21.99
	2	1-2-3-4	0	21.32	13.12	1000	30.17	17.47	366.7	23.45	12.93
	3	1-3-4	467.6	20.78	22.49	0	21.45	18.32	337.5	20.72	17.36

Based on the result, when only considering the average travel time regardless of the variability in the DUE model, Path 2 would not be selected due to its travel time of 21.32 minutes, which is the longest among the three paths. However, with no traffic on link 3, the reliability condition is the best among all links, which makes Path 2 the most reliable path also. Therefore, when travelers only consider travel time reliability as the determining factor, all travelers will choose Path 2 over the other two paths. Even with all the demand assigned to Path 2, the SSD of 17.47 minutes is still the smallest, even though the mean travel time become 30.17 minutes, which is the longest among the three

paths. In contrast, when both average travel time and reliability are integrated into the assignment model, a dramatically different pattern in the equilibrium pattern is found. As we can see, in contrast to DUE, 36.7% travelers will now switch to Path 2 in MOUE model in seeking to achieve a higher reliability to ensure a higher probability of arriving at the destination. Meanwhile, Path 1, which attracts most travelers under DUE condition, now is less appealing to drivers because it contains link 2, which has the most unreliable travel time with its SSD at 19.32 minutes. Also, the mean and SSD on Path 3 are in between that on Path 1 and 2, which is not the most congested nor unreliable, thus attract 33.75% of travelers. Hence, from the multi-objective perspective, all three paths seem to be attractive to some travelers.

With the formulation of SSD, it is clear that different benchmark values directly determine how travelers treat the uncertain conditions and influence their final route choice decisions correspondingly. Therefore, a sensitivity analysis is conducted on this aspect in order to understand its impact on the obtained user equilibrium state (Figure 6.2).



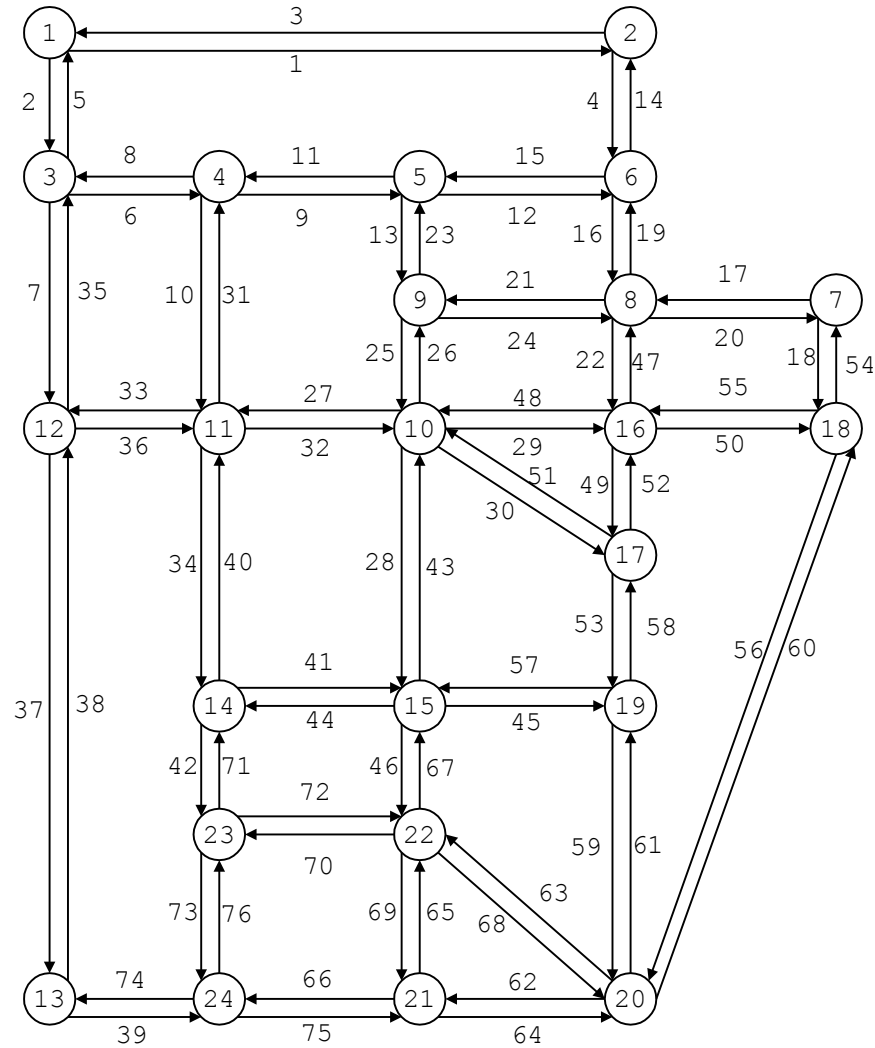
**Figure 6.2 Demand Share on Paths under Equilibrium Condition**

It is observed that as risk parameter value increases, the share of the demand on Paths 1 and 3, which have less reliable travel times, also increase as travelers become more tolerant of the uncertainty thus have less incentive to switch paths. In contrast, the number of travelers choosing Path 2 decreases because its attractiveness as the most reliable path gradually declines. For example, when the reliability parameter is chosen as 10, the SSD on Path 1, 2, 3 are 6.3, 0, and 1.5 minutes, respectively. The difference between them is much smaller than that when just FTT is used as the SSD threshold.



### 6.4.2 Medium-Sized Network

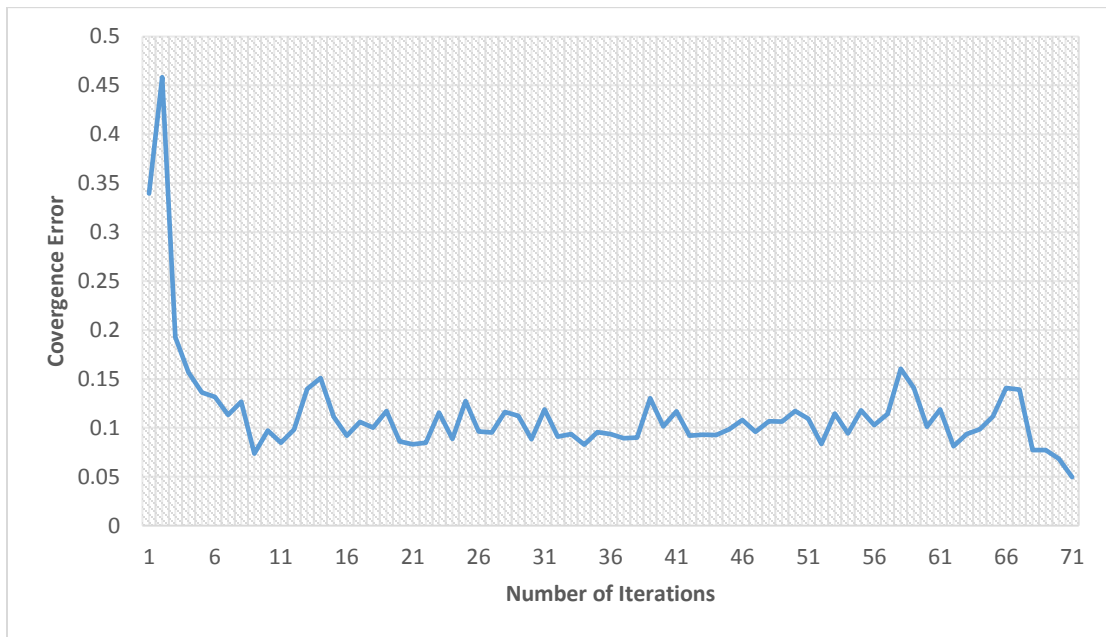
In this section, the proposed model is tested on a medium-sized network to evaluate its capability in solving relatively large-scale network problems. The widely used Sioux Falls network, which consists of 24 nodes, 75 links, and 550 OD pairs (Figure 6.3) is used here. The network data involving FFTT and capacity on links and demands between OD pairs are obtained from (101).



**Figure 6.3 Medium-Sized Network**

The well-known best solutions in terms of the DUE is also obtained from the same link and used as the initial input to implement the MOUE model. Like small network analysis, the d/c ratio under DUE condition is used to derive the variation on links. The approach proposed in (102) is adapted to generate multi-variate log-normally distributed travel time realizations. In this experiment, only travel times from adjacent links are assumed to be correlated with a coefficient of 0.5. The shortest FFTT between each OD pair is also selected as the benchmark for computing SSD for paths connecting the origin and destination.

The convergence performance of the proposed MOUE traffic assignment model is shown in Figure 6.4. The program terminates after 71 iterations when the gap between input and updated link flows is less than a pre-determined threshold, which is set to be 0.05 in this case. Each assignment iteration takes 25 seconds on average to complete. Using link flows under DUE condition as the base scenario, the mean absolute error is 2002.7 and the mean absolute error percentage is 19.95%, indicating a considerable change in final user equilibrium pattern when both mean and SSD are considered by the MOUE model.



**Figure 6.4 Convergence Performance on the Medium-Size Network**

Based on the experiments conducted from this study, it shows the proposed multi-objective model is a viable extension to the traditional models by accounting for travel time reliability. Also, the proposed algorithm is effective in finding solutions for MOUE on a relatively large network.

## 6.5 CONCLUSIONS

In this study, a multi-objective traffic assignment model is proposed to extend the traditional user equilibrium models by incorporating the travel time reliability consideration into the modeling process. Particularly, SSD based on a constant benchmark specified by travelers is chosen as the reliability measure under asymmetric distribution conditions. It has the following benefits. First, it has been shown that SSD has more appealing characteristics over standard deviation in this context and has more intuitively meaningful interpretation of travelers' behaviors. In addition, using a constant benchmark in SSD formulation is similar to the scheduling delay concept, but SSD emphasizes more on the larger deviations than small deviations. As stated in (70), the scheduling delay may not be able to fully capture the travel time uncertainty, which is mostly affected by large deviations at the right tail of the travel time distribution.

Therefore, the proposed approach provides an alternative to evaluate travelers' departure time and route choices under uncertain conditions.

The developed multi-objective model is able to include multiple Pareto-optimal paths; thus, relaxing the limitations underlying the linear combination of the mean and reliability measure, which is widely used in previous studies. Also, the MSSD dominance rule is shown to be contained by the FOSD rule. This finding is particularly important because it enables us to directly take advantage of Bellman's Principle of Optimality and already developed LC algorithm and analytically solve the route choice model. Based on obtained non-dominated paths, a certain proportion of traffic demand is assigned to each path based on travelers' perceived attractiveness. This is also deemed more realistic than traditional single-objective assignment procedure, because in reality different travelers may prioritize and select different paths between the same OD pair.

Numerical experiments are conducted to evaluate the effectiveness of proposed algorithms. It is found that the user equilibrium pattern under the proposed multi-objective formulation is significantly different from that of the traditional DUE model. When both travel time and reliability are considered, a considerable portion of travelers will switch to the path that won't be selected by the DUE model even though it is most reliable among available alternatives. In addition, it is observed that how travelers set the acceptable reference values also affects the assignment results, which of itself is a practical research topic(103). It is also shown that the algorithm can quickly converge to the equilibrium condition, attesting its potential in practical models.

In future studies, it will be interesting to see how the results from the proposed model compare to that from the standard deviation based model. Also, extending the current problem to a dynamic network setting is also worth studying.

## CHAPTER 7 CONCLUSIONS AND FUTURE RESEARCH

### 7.1 CONCLUSIONS

Travel times may be highly variable across the network due to frequent traffic-impacting events. This unreliability may result in late arrivals, which in turn causes negative consequences. It has been established that travel time reliability is an important factor in travelers' route choice decisions. However, the factor is non-present in existing MPO models. In this dissertation, a methodology framework is developed to incorporate travel time reliability into travel demand models. The framework contains four major components that enhance the reliability measurement and the applicability in the operational models.

The first component proposes semi-standard deviation as the reliability measure. Compared to standard deviation that regards travel times both below and above the mean as undesirable, semi-standard deviation only concentrates on the right side of the distribution, i.e. those relatively long travel times. It also has the flexibility in setting the benchmark value. Thus, it is able to account for travelers' different degrees of sensitivity about the uncertainty. In contrast to on-time arrival probability and scheduling delay measures, semi-standard deviation gives disproportionate emphasis on longer travel times, and hence, can better capture the impact of travel time reliability.

The sampling based approach provides many benefits compared to current methods. Deriving path SSD directly from component links can be very difficult. As a non-additive measure, the path SSD is not equal to the linear summation of SSD on component links; a challenge most reliability measures face. In addition, unlike standard deviation and covariance matrix, there is no closed form solution to analytically derive the accurate semi-covariance matrix. A workaround to the problem is using archived travel time data to directly obtain path travel times and SSD. Through this process, the correlation structure can be implicitly accounted for and the complicated link travel time distribution fitting and convolution process can be avoided.

The third key component involves reformulating the model from the multi-objective perspective compared to the widely used single objective formulation. The reformulated model focuses on minimizing the mean and SSD at the same time, thus eliminating the need for reliability ratios to be known beforehand. The multi-objective formulation also offers an additional benefit of providing multiple attractive choices for travelers' further decision making, including the optimal path in single objective case. In reality, this property is highly applicable because it presents multiple paths that can be selected by travelers for the same OD pair. To balance out disproportionate contributions from the mean and SSD with different scales, the standardized distance is applied to determine the attractiveness of each path.

Application of the stochastic dominance based approach to solve the proposed model marks the final integral component of the methodology. Due to the non-additive property

of SSD, traditional shortest path algorithms, such as Dijkstra's algorithm, are no longer applicable. To address this issue, two different approaches are evaluated: the metaheuristic algorithm and the stochastic dominance based approach. The metaheuristic algorithm demonstrates reasonable performance in finding optimal paths. A particular advantage is its ability to not be restricted to certain objective functions. However, the algorithm is stochastic in nature and thus the global optima is not guaranteed. On the other hand, the stochastic dominance ordering criterion is more effective and efficient. It is an all-to-one approach and able to find the true optimal paths. In addition, it can directly take discrete travel time samples as inputs, which fits very well with the adopted sampling based approach.

In addition to these four components, this work also contributes to the reliability modeling field by establishing theoretical connections between the first three stochastic dominance rules and three reliability models. A generic formulation is provided for on-time arrival probability, scheduling delay, and semi-standard deviation measures, as they share common mathematical structures. Through the application of the generalized formulation, theoretical connections between stochastic dominance rules and reliability models under evaluation are established. These findings provide great insight into the behavioral implication with regard to each reliability model. Based on the risk-taking behaviors of stochastic dominance rules, we are able to infer that on-time arrival probability, scheduling delay, and semi-standard deviation correspond to risk-neutral, risk-averse, and ruin-averse behaviors, respectively. The generic formulation and its association with stochastic dominance rules also offer an opportunity to incorporate on-time arrival probability and scheduling delay into the framework.

The results from numerical tests demonstrate the advantages of the proposed methodology. The semi-standard deviation based model shows a better representation of traveler's route choice decision involving skewed travel time distribution with excessively long delays. Observations also indicate no single path is optimal in every criterion, reinforcing the need of a multi-objective model to find multiple non-dominated paths that are attractive to travelers. The overall methodology is effective in finding optimal paths in each assignment iteration and ultimately achieving the equilibrium condition. It is suggested that the traffic flows under equilibrium are sensitive to various benchmarks set by travelers. The impact of travel time reliability on equilibrium is apparent for two tested networks. As a result, the models and algorithms developed in this dissertation are highly applicable to the real-world situations and have great potential to be adapted into current MPO models.

## **7.2 FUTURE RESEARCH**

This work proposes a more representative reliability measure and greatly enhances the applicability of route choice and user equilibrium models in dealing with stochastic travel times. However, there are some limitations in the modeling process that are worth discussing as well as recommendations to further improve the proposed models and algorithms.

The first limitation is that both GA and SPEA2 methods are metaheuristic, whose performance is greatly reliant on proper parameter selections. While only a relatively small range of values are evaluated in this work, it would be valuable to extend the range of parameter values and apply experimental design techniques such as Taguchi method(104) to objectively determine the optimal input values. As many novel genetic operators have been proposed in recent applications, it would also be beneficial to evaluate how their implementation into the proposed models can lead to performance improvement(80; 105). Moreover, a number of new evolutionary algorithms have been developed recently(106; 107). This presents an opportunity for future research to adapt them to solve the proposed models and compare with currently implemented SPEA2 approach.

Second, the sampling-based approach serves an important role in the proposed methodology to implicitly account for the underlying correlation structure between links on the network, circumventing the complex distribution convolution procedure. However, the accuracy of the results from the approach directly depends on the adequate size of travel time samples during the model development. As this research directly makes use of available GPS-based probe vehicle data, it is an important topic worth more efforts to determine the impact of sample size on the identified optimal paths.

In addition, different risk-taking behaviors corresponding to existing reliability models are investigated in this work. As they are all used in the current modeling practice, appropriately choosing the right model that better reflects traveler's perspective on uncertainty remains an area of further research. Therefore, empirical surveys on the stated or revealed preferences are necessary to understand traveler's actual decision making behavior under uncertain conditions. Furthermore, it is important to understand how travelers set their benchmark travel time to make departure time and route choices.

In addition, since travel time variation is caused collectively by various non-recurring events, the degree of variation varies at different times of day and incident conditions. Travel times are expected to be more reliable during the night-time incident free condition than the morning peak period while snowing. As a result, the identified optimal paths could also be distinct under varying conditions. In this regard, there is still a need to evaluate travel time reliability separately at different time periods and travel conditions.

Finally, the models are built on the assumption that travelers have exact information regarding the travel time distribution and always make rational choices during uncertain situations. However, studies have shown bias exists in traveler's perception on traffic conditions(108). In other words, the travel time distribution possessed by travelers are subjective and may be different from the objective distribution based on actual observations. Therefore, the impact of perception error should also be accounted for in the route choice and traffic assignment models in future research.

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### ACDEMIC PUBLICATIONS AND PRESENTATIONS

1. Zhang, X. and M. Chen. Quantifying Effects from Weather on Travel Time and Reliability. Proceedings of the 96th Annual Meeting of the Transportation Research Board, Washington, DC, 2017.
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