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COMMISSIONER OF HIGHWAYS

COMMONWEALTH OF KENTUCKY
DEPARTMENT OF HIGHWAYS
FRANKFORT

November 29, 1956

ADDRESS REPLY TO
DEPARTMENT OF HIGHWAYS
MATERIALS RESEARCH LABORATORY
132 GRAHAM AVENUE
LEXINGTON 29, KENTUCKY

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MEMORANDUM

TO: D. V. Terrell
Director of Research

For the past two years the Research Division has directed a large part of its effort in the field of drainage to model studies of culvert hydraulics. This has been in response to questions concerning relative culvert capacity and particularly the retarding influence of entrance conditions - a natural outgrowth of the more comprehensive studies of rainfall-runoff relations and attendant culvert requirements on small watersheds.

Equipment for making the model studies was designed, built and placed in operation in the hydraulics laboratory of the College of Engineering, University of Kentucky. Many engineers in the Department have become familiar with the equipment and its uses through displays and discussions during the Highway Conference last March*, and more recently through Engineering Experiment Station Bulletin No. 41 which was widely distributed in September.

As a result of the previously published materials, most of the information in the attached Report No. 1 on "Hydraulic Model Studies of Culvert Operation," by E. M. West, is available elsewhere. However, it is our intent to present and make record of our past and current work along these lines in three logical steps of testing on models representing the following:

1. Current Standard boxes having 30° and 45° wingwalls.
2. Current Standard boxes modified to include hooded openings.
3. Boxes and circular culverts having drop inlets.

* See "Model Study of Flow Through Culverts." Some Technical Papers, Kentucky Highway Conference, Engr. Exp. Sta. Bull. No. 40, pp 40-46, June, 1956.

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The attached report deals exclusively with detailed descriptions of the equipment, methods of operation, and data from tests in Series 1, while the Experiment Station Bulletins cover parts of Series 1 and 2. Subsequent reports will give specific treatment of Series 2 and Series 3 separately.

You will note there are no conclusions as such in the report, since most of the results are comparative. After a greater variety of data is available we hope to establish some numerical values for design of full-scale culverts of many sizes and shapes, operating under different conditions.

Respectfully submitted,



L. E. Gregg
Assistant Director of Research

LEG:dl

cc: Research Committee Members
J. C. Cobb (3)

Commonwealth of Kentucky
Department of Highways

Report No. 1

on

HYDRAULIC MODEL STUDIES OF CULVERT OPERATION

by

E. M. West
Research Engineer

Highway Materials Research Laboratory
Lexington, Kentucky

November, 1956

INTRODUCTION

Since 1951 the Division of Research has been engaged in a comprehensive study of highway drainage problems. This study has included special investigations of rainfall and runoff, drainage structures in use and the practices used in their design, culvert hydraulics, and other relevant investigations. To date, reports have been published concerning runoff and rainfall variables (1 and 2)* and the effects of barrel roughness upon culvert operation (3), together with the preparation of a drainage manual for the use of Highway Department engineers and consultants.

The most recent special project, part of the over-all study, has dealt with the effects of inlet geometry upon the operation of culverts under entrance control. The procedures of this study have been somewhat unique in that they have made use of a scale model of a box culvert, set up in such a way that its operation could be closely observed and accurate readings could be made of water pressures, discharge quantities and the like. Although rather summarily reported on previously (4), the methods, underlying theories and results of this study are the subject of this report.

The project itself has developed from an attempt to overcome certain serious difficulties long inherent in the problem of culvert design. Primary among these has been the necessity of being able to predict accurately the head loss at the entrance for a given inlet design

* Numbers in parantheses refer to the list of references at the back of this report.

operating under given conditions. This loss, a direct function of inlet geometry, has been virtually impossible to evaluate solely by mathematical means. Since the flow patterns of water moving through any opening are complexly affected by slight variations in the shape of that opening, and since these effects vary with varying conditions of slope, headwater depth and the like, there is no formula which can be used accurately to predict the effects of all the variables for any given opening. But by constructing a scale model and by observing its operation under these varying conditions, certain dimensionless relationships may be set-up to provide an accurate means for predicting the hydraulic operation of an opening of the same geometry and of virtually any size.

In 1951, the Bureau of Public Roads contracted with Oregon State College to study and develop an improved box culvert inlet. This study was carried out with considerable success by the use of scale models and an inlet was designed which, under certain conditions of flow, increased efficiency by as much as 100 percent.

The Oregon procedures and results (5) were used as a guide and as a basis for comparative evaluation by the Kentucky Department of Highways, Division of Research, in carrying out its model study, although there were variations in the testing procedures and in the use of the data. At present, the Kentucky model is being used to test inlet modifications, with the intention of improving efficiency. Tests have already been completed on models of two standard inlets for box culverts -- a 30° wingwall and a 45° wingwall inlet -- commonly

in use in Kentucky, and from the data taken it is now possible to predict entrance losses for inlets of these types, as well as other significant factors which affect their operation.

Since the operation of the model has proved thus far to be quite successful, it is intended that its use be continued in order to provide, for the future, more accurate criteria for efficient culvert design.

THEORY OF CULVERT OPERATION

Since the principal objective in culvert design is to provide the most economical means, within specific limits of headwater elevation and velocity, of transmitting a given discharge from one side of the roadway to the other, it is necessary to evaluate the headwater-discharge relationship before the over-all design situation can be analyzed.

In order to determine this relationship it is convenient first to make a general classification of the types of culvert operation; i. e., of the various conditions of flow. These may be classified as four primary conditions: (1) full flow, (2) part-full flow, (3) flow with inlet submerged, and (4) flow with inlet non-submerged. Through hydraulic analysis it is possible to predict the condition for a given culvert under given sets of variables, such as slope, size, shape, length, roughness, headwater elevation, tailwater elevation and inlet geometry.

These variables, through their magnitudes and relationships, combine in different ways to form controlling conditions at different

locations along the culvert. A convenient method of finding the maximum discharge possible with a given culvert under given conditions of flow is through the location of the control section - the critical point, or "bottleneck" of the entire hydraulic system - since the principal flow characteristics are determined by that section and by its position, at the inlet, outlet, or in the barrel. With the inlet as the control section the head-discharge relationship is not affected either by friction in the barrel or by conditions at the outlet. Conversely, when the control is at the outlet, the inlet does not influence the head-discharge relationship. In all cases where a culvert flows full, except for very short structures, this relationship depends on the inlet conditions, the barrel, and all of the design variables. Thus, when the control is at the inlet, the geometry of the inlet is very significant; but when the control is in the barrel or at the outlet the inlet geometry is much less so. Therefore, for a study of the characteristics of culverts of various types of inlet geometry it seems most logical to conduct tests under conditions of inlet control. Also, a study under such conditions deals with what is probably the most normal of the three types, since inlet control ordinarily occurs when the culvert is on a steep grade and the flow in the upstream channel is subcritical. In such an instance the critical depth occurs in the region of the culvert entrance, accompanied by a sharp increase in velocity. The location of this depth is near the entrance to the barrel when the slope of the flow line is constant; and in cases where there is a downward break in the slope of the flow line the critical depth occurs near the break.

In a study of culvert inlets it is also advantageous to give preference to the conditions of inlet control since this permits the roughness characteristics of the barrel to be ignored. This is of particular advantage since the roughness effect would be virtually impossible to simulate or evaluate.

The operation of a culvert with inlet control will be in one of two categories, depending upon the head-discharge relationship. The culvert will be flowing either with the inlet submerged or not submerged. These categories will be dealt with individually in this report.

Non-Submerged Operation

Since for inlet control the flow in the barrel must be supercritical and the effects of roughness and slope can not be reflected upstream to the entrance, the geometry of the inlet determines the head-discharge relationship -- or, more specifically, the discharge that the structure will carry for a given head. When a structure operates in this manner it is operating under what is termed critical depth control. Thus, the width of the inlet at the point of critical depth determines the discharge for a given head.

Since the critical depth for a rectangular section occurs when the velocity head is equal to one-half the depth of the moving water, this may be expressed in terms of discharge per unit width:

$$D_c = \left(\frac{q^2}{g} \right)^{1/3}$$

Then the total energy head may be found as follows:

$$H_E = D_m + \frac{V^2}{2g} \text{ where: } \frac{V^2}{2g} = \frac{D_m}{2}$$

Since the critical depth D_c = the mean depth D_m :

$$H_E = D_c + \frac{D_m}{2}$$

$$H_E = D_c + \frac{D_c}{2} = 1.5 D_c$$

or
$$H_E = 1.5 \left(\frac{q^2}{g} \right)^{1/3} \text{ and } Q = w(g)^{1/2} \left(\frac{H}{1.5} \right)^{3/2}$$

From this equation the headwater elevation above the flow line at the critical section for a given discharge per foot of width of the barrel can be computed. This equation can be used to analyze the conditions when the culvert is flowing part full with entrance control.

Submerged Inlet Operation

When a culvert is operating with headwater level above the top of its inlet and the barrel is not flowing full, its operations are similar to those of an orifice. If the entrance is square-edged the operation is like that of a sharp-edged orifice discharging horizontally, assuming that the momentum of the fluid approaching the entrance non-axially will cause a contraction in the area of flow downstream from the opening (5).

For an orifice, the vena contracta, the section where the contraction caused by the converging paths of the moving particles of water ceases, controls the discharge. In the case of circular, sharp-edged orifices with a diameter D , the vena contracta has been found to occur at a distance of about $1/2 D$ from the plane of the orifice (6). In the case of a supported jet the energy available for producing flow

is a function of the head measured between the centerline of the orifice and the upstream water level. This relationship is expressed in the general orifice formula:

$$Q = C_v A_j (2gH)^{1/2} \quad \text{Equation No. 1}$$

where

A_j = area of the jet at the vena contracta

C_v = coefficient of velocity of the orifice.

It is assumed that a culvert with a square-edged entrance will have approximately the same relationships as an orifice with a supported jet. In the case where the barrel slope is more-or-less a continuation of the upstream channel slope and where wingwalls are provided, the contraction will be at the top and the operation analogous to that of a sluice gate. The energy head (energy available for conversion to velocity head) is measured from the energy line of the upstream pool to the water surface at the vena contracta. In the case of a supported jet the water surface at this point is taken as the pressure line.

Substituting in equation 1 for area (A) this equation becomes:

$$Q = C_v W D (2gH)^{1/2} \quad \text{Equation No. 2}$$

where

C_v = coefficient of velocity

W = width at the vena contracta

d = depth at the vena contracta

H = energy head producing flow

The ratio between the area at the vena contracta and the total area of the orifice is generally referred to as the coefficient of contraction and expressed as a decimal fraction from the following formula:

$$C_c = \frac{A_j}{A}$$

where A_j = area at the vena contracts

A = total area of the opening

thus $A_j = C_c A$

and $Q = C_v C_c A (2gH)^{1/2}$

Equation No. 3

The product of the coefficient of velocity and the coefficient of contraction is usually referred to as the coefficient of discharge (C_d), or:

$$C_d = C_v C_c$$

Submerged Inlet with Full Flow

When the culvert barrel is flowing full with the inlet submerged the area of flow is obviously greater than the contracted area of a sluice gate type of operation. In addition to the energy from the head-water elevation above the entrance, the culvert also utilizes the additional energy head due to the fall in the barrel from inlet to outlet.

It is convenient to compare this type of operation to short tube operation. When the inlet is square-edged, the top contraction will occur when the barrel is flowing full. Although the contraction is not reflected in the cross sectional area there is a pressure drop near the vena contracta along the barrel. If air is admitted to this area and the pressure neutralized, the flow will again become of sluice gate type, provided that there is no obstruction downstream, and the outlet is free.

With the culvert barrel flowing full the energy available for producing flow is measured from the upstream energy grade line to the pressure line at the outlet. When the outfall is free and the velocity

heat 0.8 times the height of the culvert or greater, the pressure line is at or near the center of the jet. If the outfall is not free and the jet is supported, the pressure line is at the water surface. Hence, in all cases of full flow, the discharge is a function of entrance geometry, barrel friction, and the slope and length of the culvert.

When a conduit is flowing full with the outlet submerged the difference between the headwater and tailwater elevations, expressed as total head, is the sum of the head losses. These losses are velocity head loss, entrance head loss and friction head loss due to roughness of the conduit. It seems reasonable to assume for purposes of calculation that the head loss is the same whether the conduit discharges freely or discharges into a submerged outlet (7).

Thus the headwater elevation for the case of full flow can be calculated by the energy equation for steady flow:

$$H = \frac{V^2}{2g} + K_e \frac{V^2}{2g} + \frac{fL}{4R} \frac{V^2}{2g} + \frac{P}{W} + Z \quad \text{Equation No. 4}$$

where

H = energy of the upstream pool above a common datum plane

L = length of structure

V = velocity in the pipe

K_e = entrance loss coefficient

f = friction factor for Darcy-Weisbach equation

R = hydraulic radius

$\frac{P}{W}$ = pressure energy

Z = elevation above a common datum plane

In the above equation $\frac{P}{W} + Z$ represents the potential energy head due to pressure elevation and $\frac{V^2}{2g}$ represents the kinetic energy head due to flow.

DESCRIPTION OF THE MODEL TESTING DEVICE

The model testing apparatus was designed to simulate, as nearly as possible, an actual box culvert installation. The layout used was quite like those used by others conducting similar studies, particularly that of the model research at Oregon State College. This layout was decided upon in order to obtain a device that would be workable to begin with, eliminating certain problems in design and in testing procedure.

A model-to-prototype scale ratio of 1:12 was chosen and was considered quite conservative for studies of the type. This scale was chosen on the basis of certain laws of hydraulic similitude (see Appendix) and the fact the ratio had been quite commonly used in similar studies conducted by other organizations. The one limiting factor in the selection of the model-scale ratio was the laboratory facilities for delivering and circulating the water supply.

The apparatus (see Fig. 1) consisted of a diffuser tank to dissipate the energy and turbulence in the water from the supply line, a trapezoidal approach channel with a plexiglass end section and flanges to accommodate various types of inlets, a plexiglass culvert barrel section with peizometer connectors located at frequent intervals along the bottom, a receiving tank with a V-notch weir to measure the discharge, and a series of manometers, mounted on boards, to show the head at the various peizometer locations.

The water supply for the tests was taken from the supply pit in the Hydraulics Laboratory by the laboratory pumping system, through a 4-in. diameter line, to the diffuser tank. The piping system

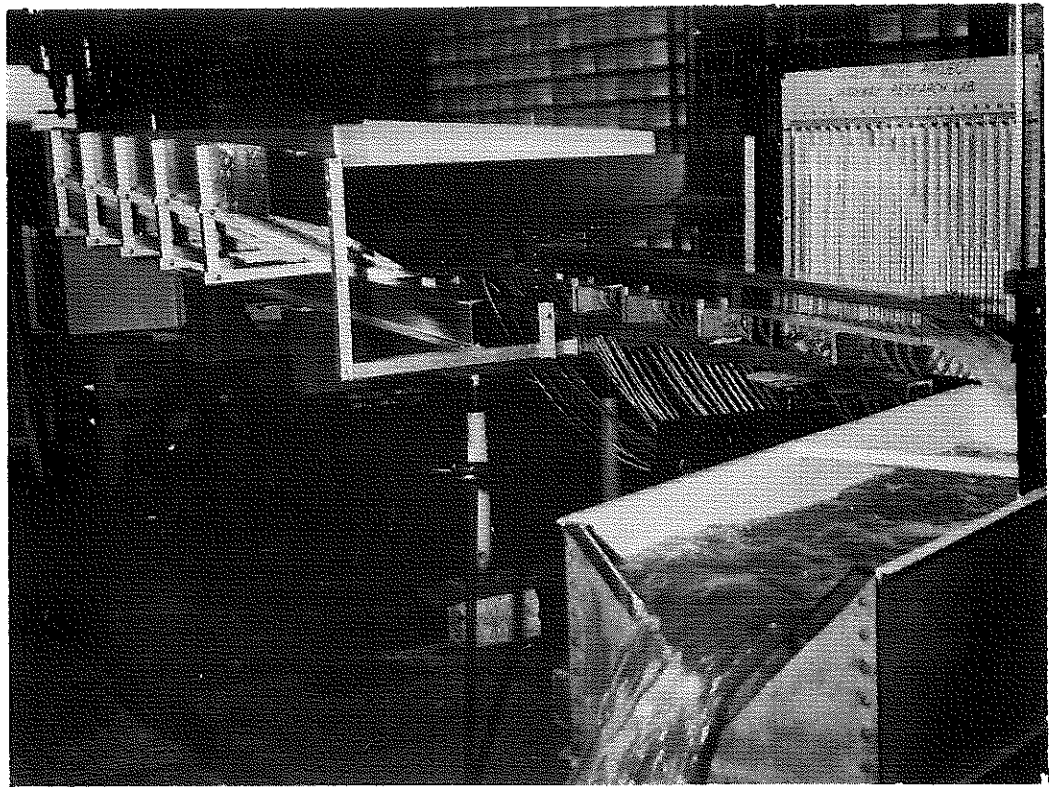
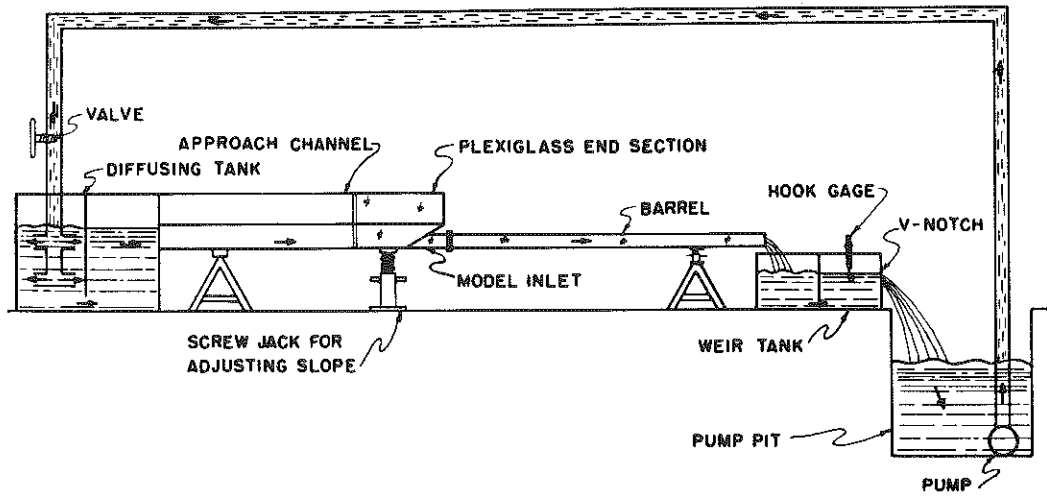


Fig. 1. Schematic and Over-all Views of Model Testing Apparatus.

included a series of plumbing fixtures made up of tee's and ell's to form an H-type arrangement. With this design, the water discharging from the supply line and controlled by the valve was diffused by jetting against opposing jets and the sides of the tank. The tank itself, 5 ft. wide by 4 ft. long and 3 ft. deep, was constructed of No. 14 gauge steel. Stiffeners were used to prevent buckling, and the approach channel served to dampen any existing vibrations. Further quieting of turbulence was gained by designing the tank with baffle boards and allowing a sump in the bottom. With this arrangement, no distinguishable turbulence from the supply was carried to the approach channel and there was no definite velocity at the channel's upstream end.

The approach channel itself contained two parts. The first, constructed from 1/2 in. exterior plywood, was 9 ft. long, 16 in. deep, and 26 in. wide at the bottom, with the lower half of each side at a 2:1 slope, the top 8 in. being vertical. The second section of the channel was made entirely of 3/8 in. plexiglass, with the same side slopes as the first section and with the end sloped in the same manner as the sides, representing a typical embankment slope of 2:1. Part of the sloping end was cut out and adapters were added to receive a flange constructed on each of the inlet models and fitted to give a smooth, watertight joint.

The inlet sections to be tested (see Fig. 2) were constructed entirely of 3/16 in. plexiglass, the details and dimensions varying with the type tested. Flanges on both ends of the model provided a bolt connection to the apparatus.

The plexiglass culvert barrel had inside dimensions of 4 in. by 4 in. and was 72 in. long. Piezometer connectors, made from 3/8 in. round plexiglass stock with a No. 40 drill hole, were welded to the

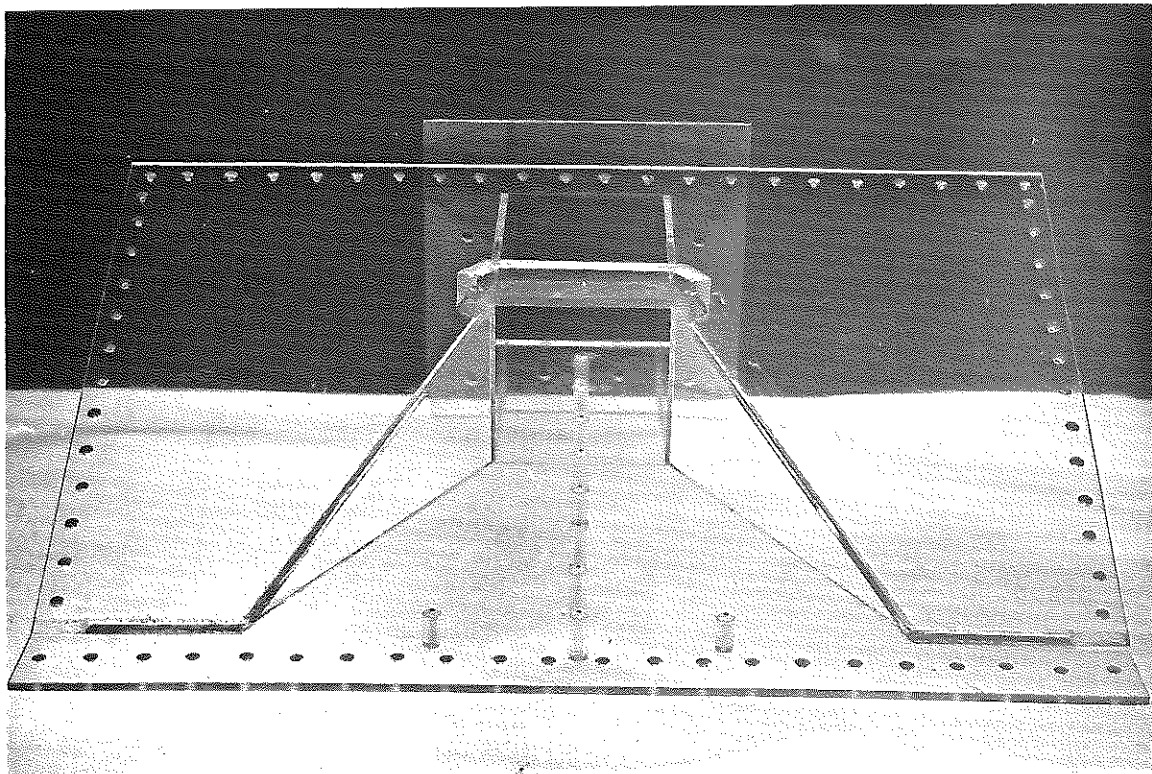
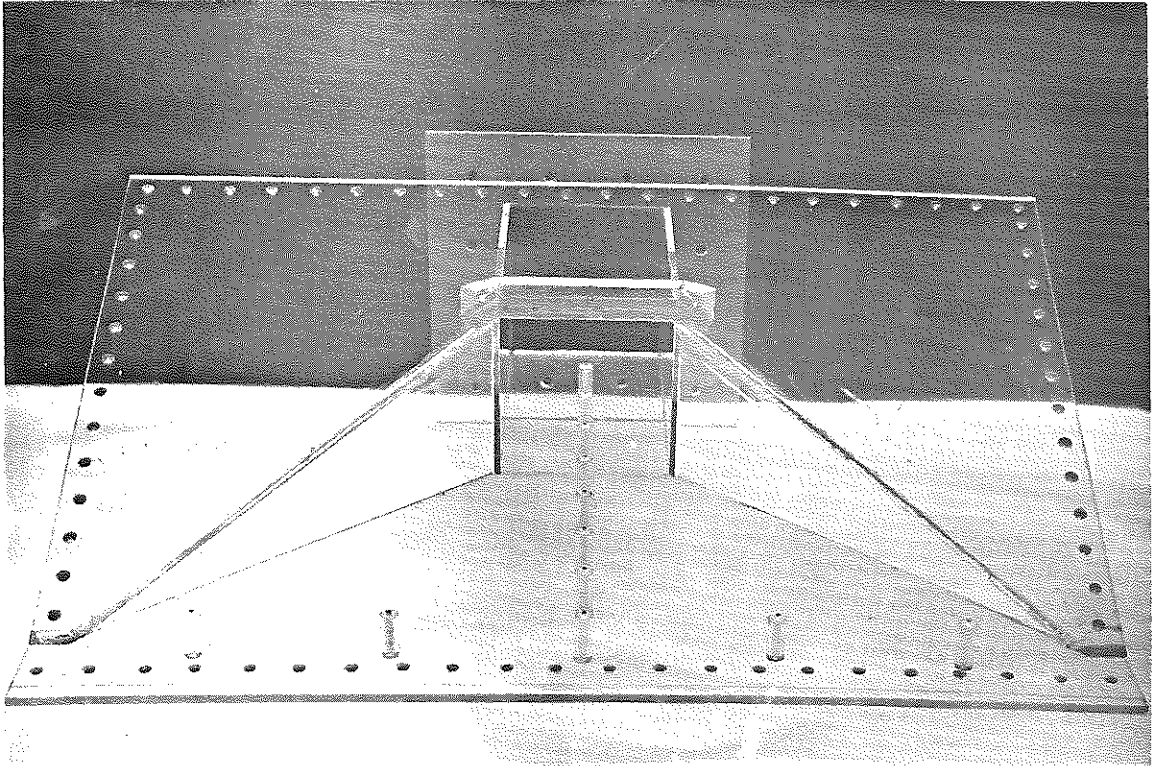


Fig. 2. Plexiglass Models of Standard Inlets Tested.

bottom. The outlet end of the barrel discharged freely with an unsupported jet into a receiving tank. This tank, 2 ft. wide, 5 ft. long and 4 ft. deep, was constructed of 14 gauge sheet metal with an expanded metal mesh diffuser in the first half and a baffle in the center. A hook gauge was connected to the side of the second half with a small stilling well around the hook. The end of the tank was cut out and flanged to receive a V-notch wier plate, calibrated by means of the laboratory's weighing tank and installed so as to permit the flow to discharge directly into the supply pit.

Provisions for changing the approach channel and culvert barrel slope were made by placing screw jacks at the end of the channel and a small machinist's jack under the culvert barrel at the outlet end. The entire length of the approach channel was supported by two continuous aluminum I-beams and the culvert barrel supported by an aluminum channel beam.

The manometer boards were made of 1/4 in. plywood with places for 44 glass tubes of 1/2 in. inside diameter (see Fig. 3). These were backed with white cardboard graduated with India-inked lines, then sprayed with clear lacquer to prevent water damage. Leveling adjustments were made possible by slots in the manometer boards and leveling screws in the bases. For the connections between the piezometers and manometers, 1/4 in. inside diameter clear Tygon tubing was used.

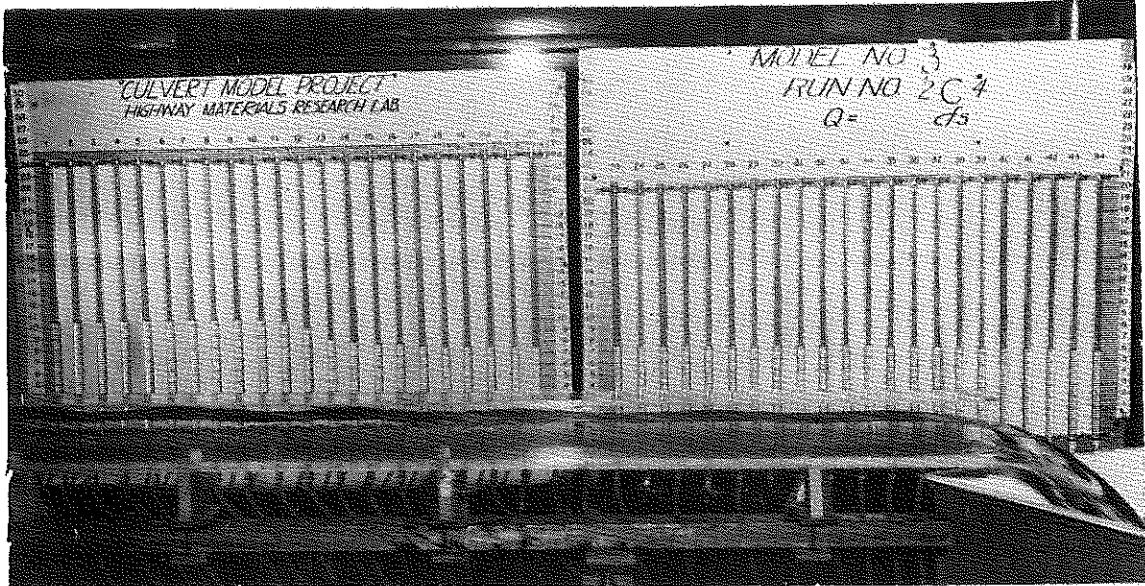


FIG. 3: MANOMETER BOARDS, SHOWING PIEZOMETER CONNECTIONS TO BARREL.

TESTING PROCEDURES AND RESULTS

As a matter of convenience the model of the standard 45° wing-wall inlet has been designated as Model A, and the 30° wingwall model is called Model B. Future models or revisions will be designated alphabetically in order of their construction.

The study of these two models was made for two basic reasons. First, to verify this method of conducting model tests, and second, to evaluate the performance of these most commonly used types of culvert inlets. It was visualized that an evaluation of the data from a study of these standard types would serve later in comparing other types and modifications to be tested as well as furnishing basic data for designs using the standard types.

This particular portion of the over-all model research program, testing of Models A and B, was conducted as a joint endeavor with the Highway Graduate Scholarship Program. The experimentation was done by R. W. Hodges and J. A. Wells, graduate scholarship students, under the supervision of the Drainage Section of the Highway Research Laboratory. It has been the basis for a joint thesis to be submitted as part of their requirements for the MS degree in Civil Engineering.

Experimental Procedure

The testing procedure was identical for both models; therefore, for convenience "the model" is used to refer to either Model A or Model B.

In order to cover the normal range of slopes on which culverts would be constructed, tests were run on slopes set at 0, 1, 2, 3, 4, 5, 6 and 7 percent. For each setting, the model was tested for a range in quantity varying from that which would only partially fill the inlet to a quantity which would completely submerge it and give a headwater depth approaching overflow. The supply valve was regulated to give test runs on four quantities before the inlet became submerged and four after submergence. This procedure, then, permitted four tests for unsubmerged inlet operation and four for submerged inlet operation at each of the eight slopes.

The sequence of testing was as follows: The slope was set at zero and tests were run for the eight discharge quantities, four below and four above submergence. When this test run was completed the pumps were shut off, the slope changed to one percent and the

same procedure repeated. This sequence was carried out through the range of slopes. The entire test was then repeated for the other inlet model.

The desired slope for each run was set by the leveling jacks. To permit accurate slope settings, a wye-level was used; and this also permitted the elevation of the outlet of the culvert barrel to be indexed with reference to the manometer boards. The supply valve was then opened, thus permitting the flow of water through the model. The valve was adjusted, by trial, to give the desired headwater depth at the inlet. With this setting and after waiting a sufficient time for the pool level to become constant, usually about 10 or 15 minutes, successive hook gauge readings were taken a minute apart, until equilibrium was reached. A photograph was then made and the following measurements were noted and recorded:

Hook gauge reading (to nearest 1/1000 ft.)

Headwater depth at piezometer No. 7 (measured with steel rule to nearest 1/16 in.)

Depth at the vena contracta (steel rule to nearest 1/16 in.)

Location of the vena contracta (measured to the nearest piezometer connection)

Depth of flow at the inlet (steel rule to nearest 1/16 in.)

Any associated phenomena were recorded, such as position of the hydraulic jump, presence of any vortices, turbulence anywhere in the system, type of flow in the barrel, presence of standing waves and their position, and any irregularities in the complete system.

The photographic recording of the manometer board data for each discharge (valve setting) was accomplished by floodlighting the boards and using a Kodak Medalist camera with Panatomic-X, 120 roll film. Proper arrangement of the lighting gave sharp delineation in the filled portion of the glass piezometer tubes. The photographs were enlarged to 8-1/2 x 11 in. to facilitate reading of the data.

Interpretation of Data

From the photographic enlargements the headwater pool elevation and headwater depth were computed for each run. The depth at the vena contracta and the average depth of flow were read and noted. There was some difficulty in determining the exact location of the former, both by direct observations during the run and from the photographs later. This difficulty also made the reading of the depth at the vena contracta somewhat indefinite; however, it was believed that the reliability of these measurements would be adequate for their intended use.

The discharge was computed for each of the tests by use of the weir rating table prepared during construction of the model apparatus. Since the weir was rated by a weighing system the discharge values were considered quite reliable. The only exception to the accuracy of the method of measuring the discharge was the impossibility of completely dampening the turbulence in the weir tank; however, the discharge measurements were believed to be well within the degree of accuracy of the other measurements taken. For instance, the discharge was computed from readings to 1/1000 ft., but the closest the manometer readings could be read was within 1/100 ft.

The measurements taken with the steel rule for the depth at piezometer No. 7 and at the vena contracta were made and recorded to be used as checks for major errors in computations and not to be used in the final analysis.

Analysis of Data

The results of the tests were first analyzed for the effects of slope, headwater depth (i. e. on the inlet), and discharge. This analysis was made by plotting the headwater depth as ordinate and discharge as abscissa, for each of the eight slopes tested (Figs. 4 and 5).

Inspection of these curves indicates that for heads up to and a little above 4 in.; that is, up to submergence or slightly above, there is an appreciable increase in the discharge with increase in slope, following the expected open channel performance as predicted by Manning's Formula*. However, when the inlet becomes sufficiently submerged the effect of slope is almost completely negligible and the structure operates with entrance control. Within the range of slopes tested this condition began at heads in excess of 5 in. at the lesser slopes and at nearly 4 in. for the steeper.

Theoretically, if a structure operates with entrance control, an increase in slope will not be reflected in the headwater-discharge relationship as it would be if the structure operated as an open channel. In open channel flow the discharge varies with the square root of the slope (Manning's Formula); but for entrance control, where the orifice analogy is utilized, slope does not enter the equation (see Equation No. 1).

$$* V = \frac{1.486}{n} R^{2/3} S^{1/2}$$

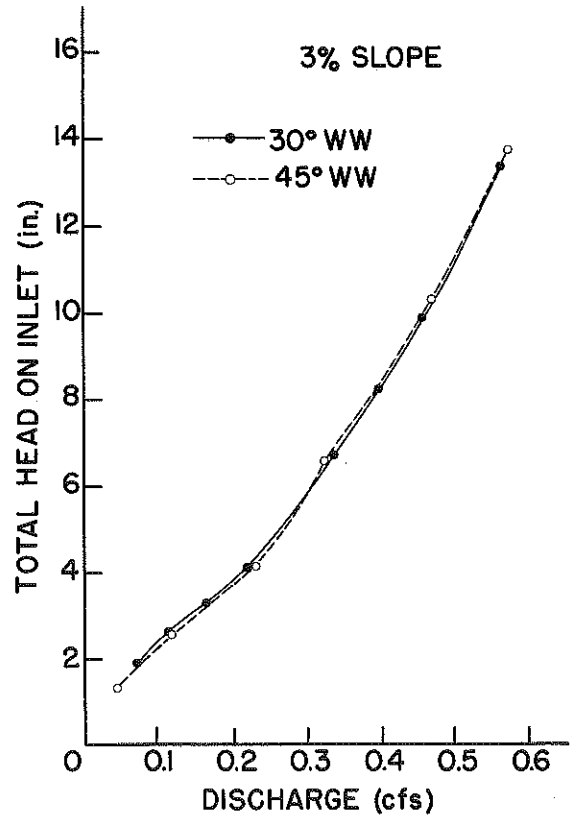
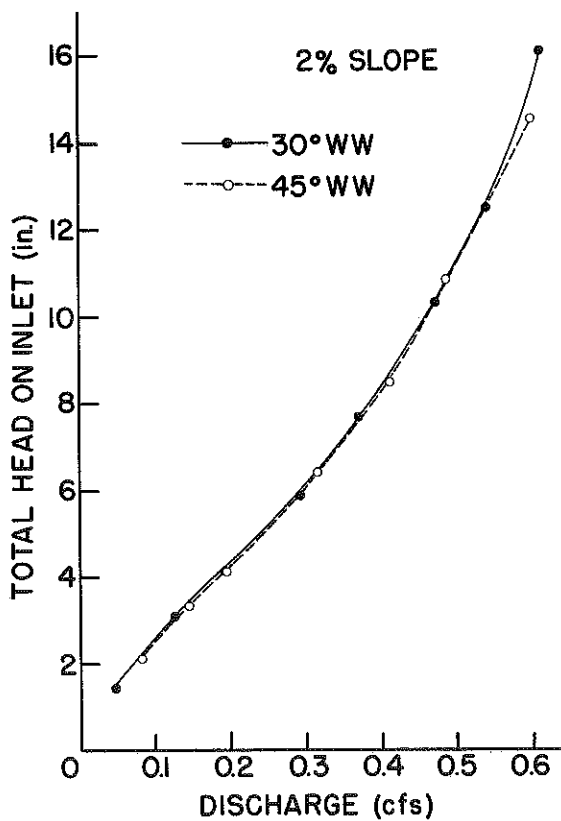
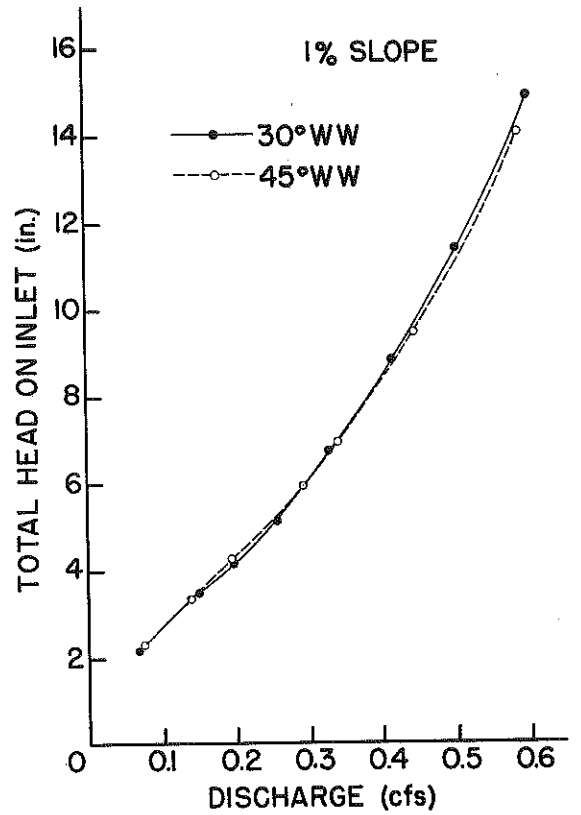
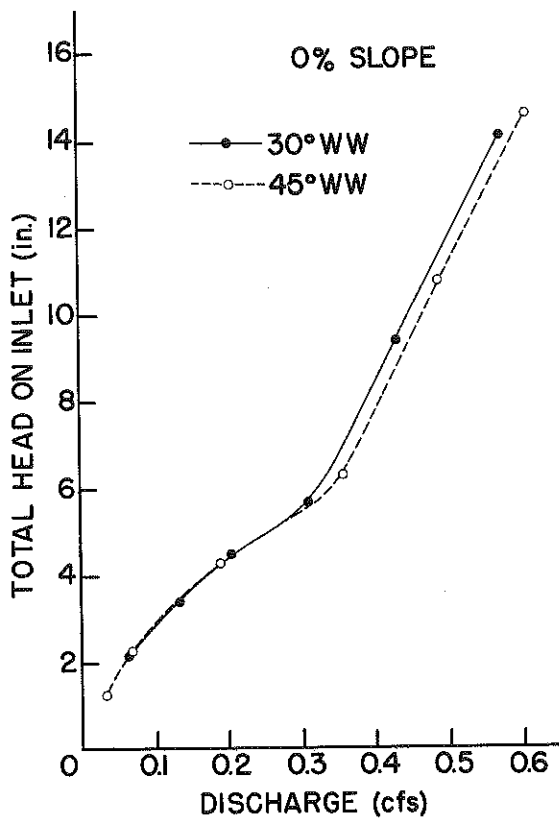


Fig. 4. Headwater-Discharge Relationships for Models A and B at Slopes of 0, 1, 2 and 3 Percent.

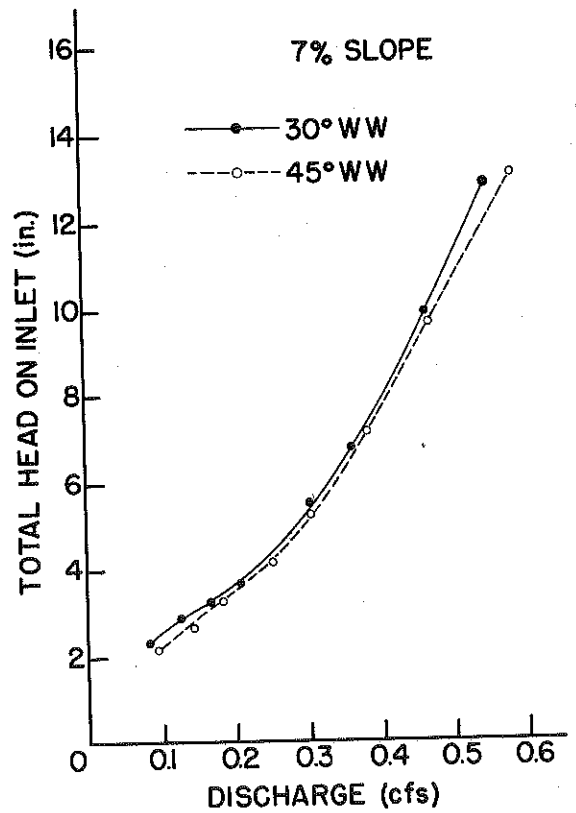
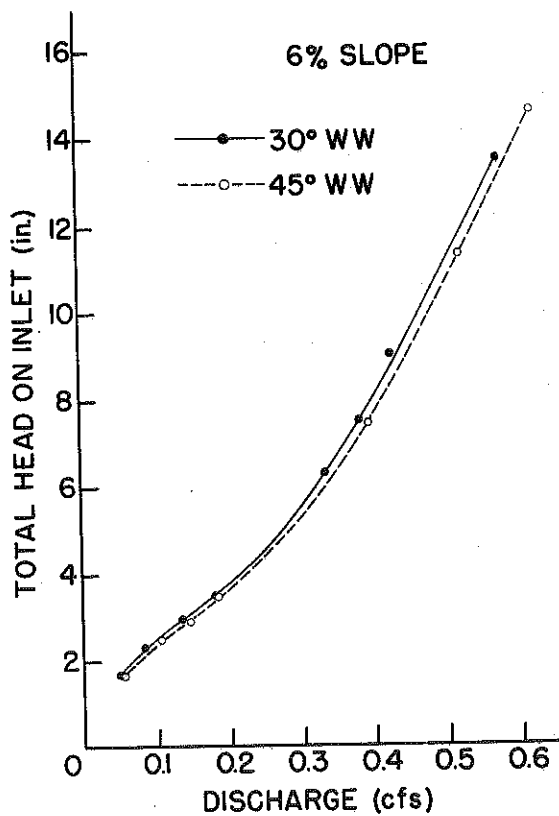
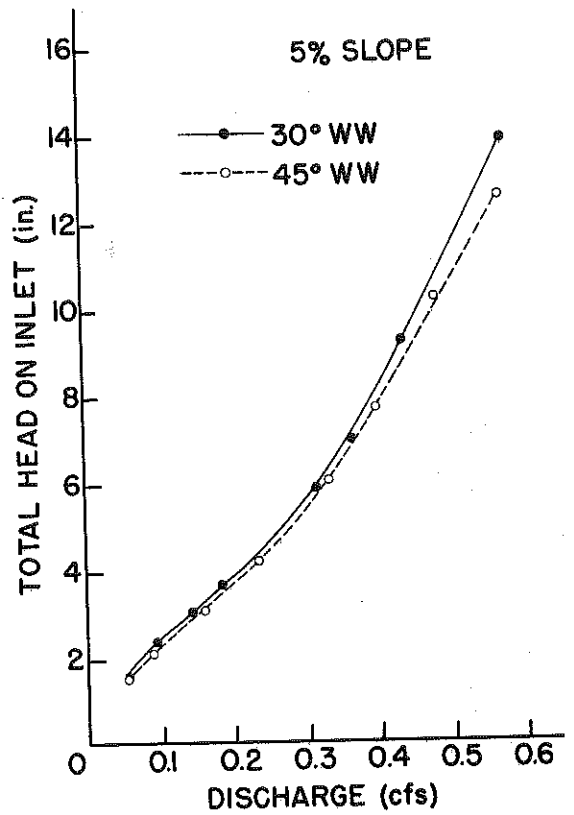
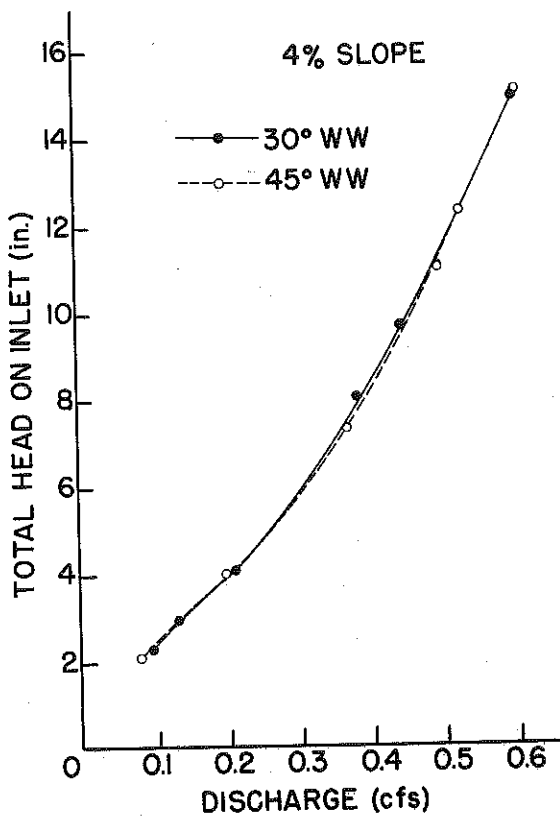


Fig. 5. Headwater-Discharge Relationships for Models A and B at Slopes of 4, 5, 6 and 7 Percent.

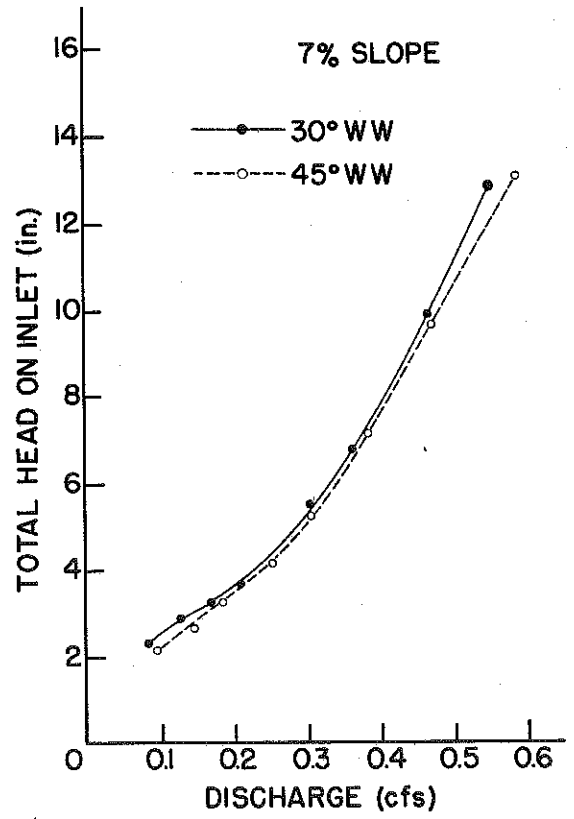
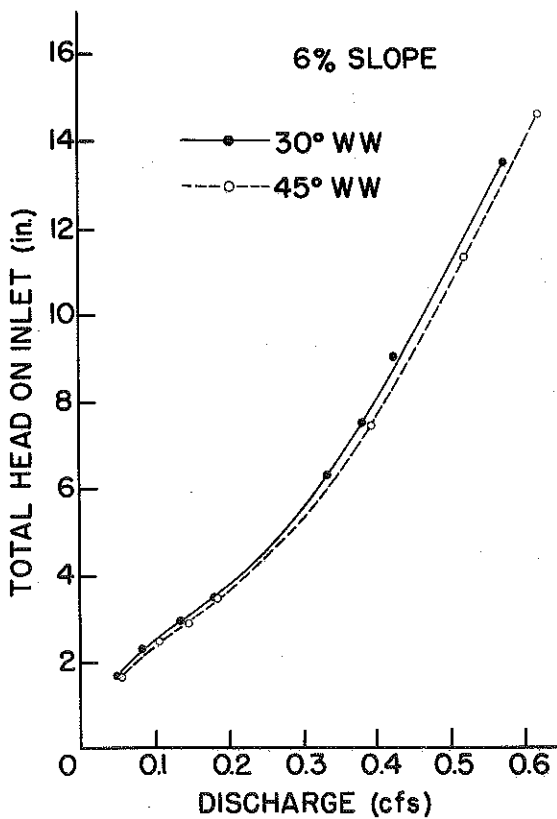
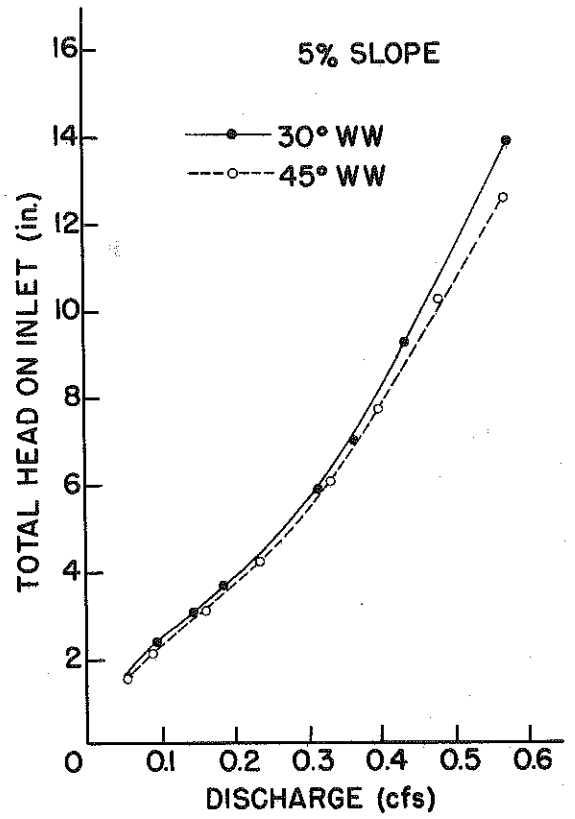
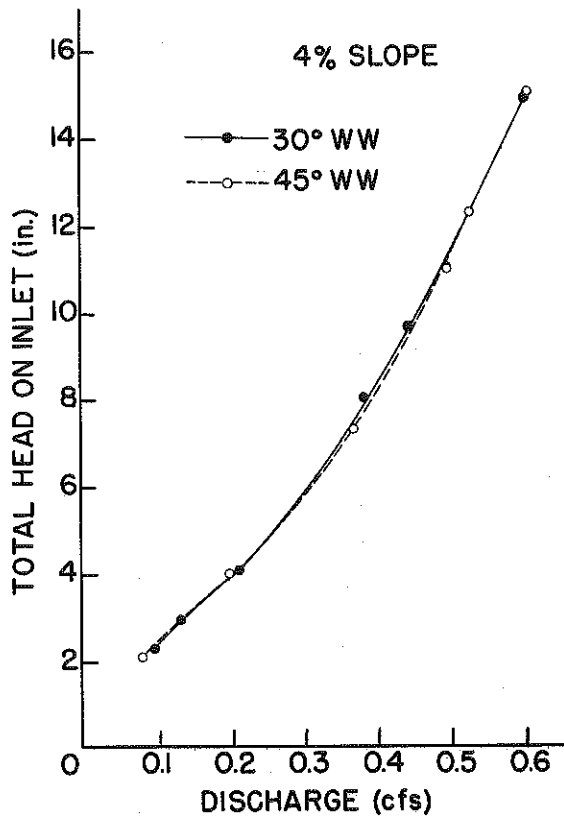


Fig. 5. Headwater-Discharge Relationships for Models A and B at Slopes of 4, 5, 6 and 7 Percent.

In comparing these curves, however, some effect of slope can be seen in the portion of the curves above submergence, although it has been assumed that this is clearly a case of entrance control. Changes in slope appeared to have a minor effect on the discharge value for a given head. This effect could be attributed to minor variation in the discharge coefficient (in the orifice formula) with changes in slope. It is also visualized that the velocity of approach, although assumed negligible in effect, could be increased with increased slope sufficiently to have a minor effect on discharge.

A comparison of the headwater-discharge relationships between the two models shows that the curve for Model A is to the right of the Model B curve on all slopes and for the complete range of submerged operation except on the three percent slope. It is believed that this particular run may have had some irregularities in the Model A portion, since the shape of the curve is slightly different from the pattern established by the other seven runs. Neglecting the three percent slope irregularities, it is evident that the efficiency of Model A is slightly greater than that of the other for any given head after submergence, since the wider wingwall inlet carries a slightly greater discharge. The efficiency differential between Models A and B is slight, however, and their performance is very similar.

In order to analyze further the effect that headwater depth has upon the discharge capacity, the coefficient of discharge (C_d) was computed for each of the test runs. These were plotted as abscissae, with headwater depth (head on inlet) as ordinates, and corresponding

curves were drawn for each slope of 1, 2, 3 and 4 percent. Curves were not drawn for the 5, 6 and 7 percent slopes due to the scattering of the points in this range.

Analysis of these curves indicates that there is little variation in the coefficient of discharge with increase in headwater depth. A slight increase is distinguishable for the lower slopes (See Fig. 6); however, there is no pronounced increase with headwater for the higher slopes. For the higher slopes, the plotted points line up more-or-less vertically.

The variation in slope has some effect upon the coefficient of discharge. There appears to be a general tendency for this coefficient to decrease with increase in slope, particularly at the lower slopes. At higher slopes the variation is not pronounced.

The curves in Figs. 7 and 8 were drawn with a ratio of the head to the height of the culvert barrel (H/D) as ordinate and ratio of the discharge per unit area to height of the culvert barrel ($Q/D^{5/2}$) as abscissa. In this manner the relationships are analyzed using dimensionless terms, thus making it possible to use these curves to predict the performance of other sizes of square conduit barrels. This method also provides a more linear plot which makes for greater ease of analysis.

The same relationships are expressed in these curves as were pointed out in the analysis of the curves in Figs. 4 and 5 since they represent the same data, expressed as dimensionless in this case.

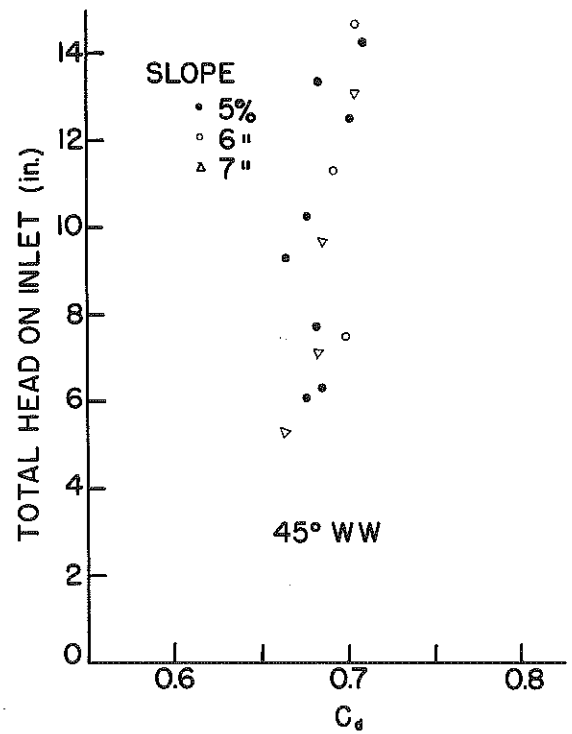
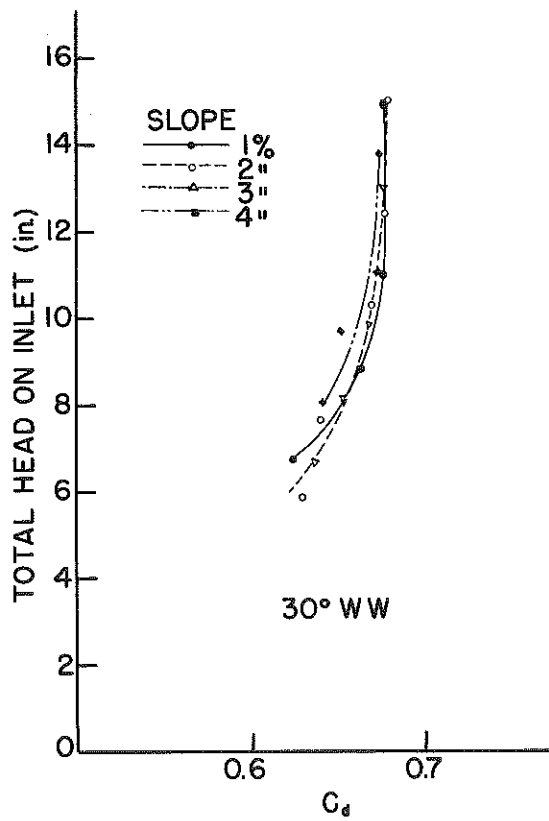
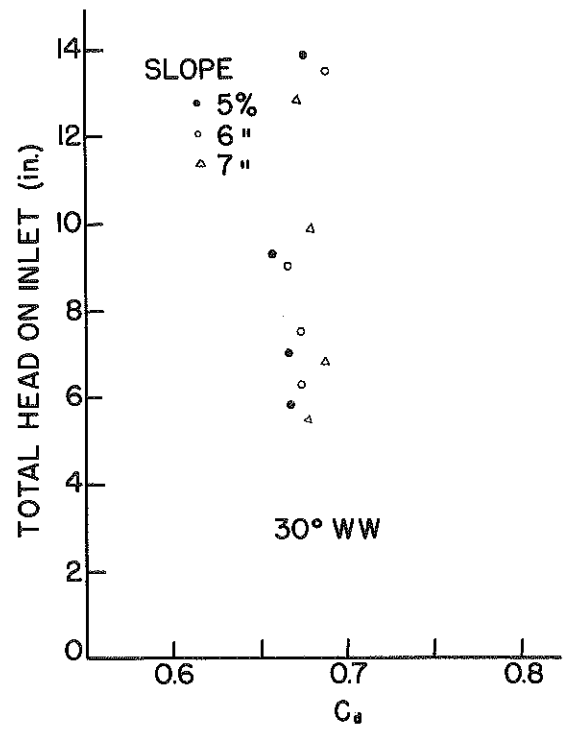
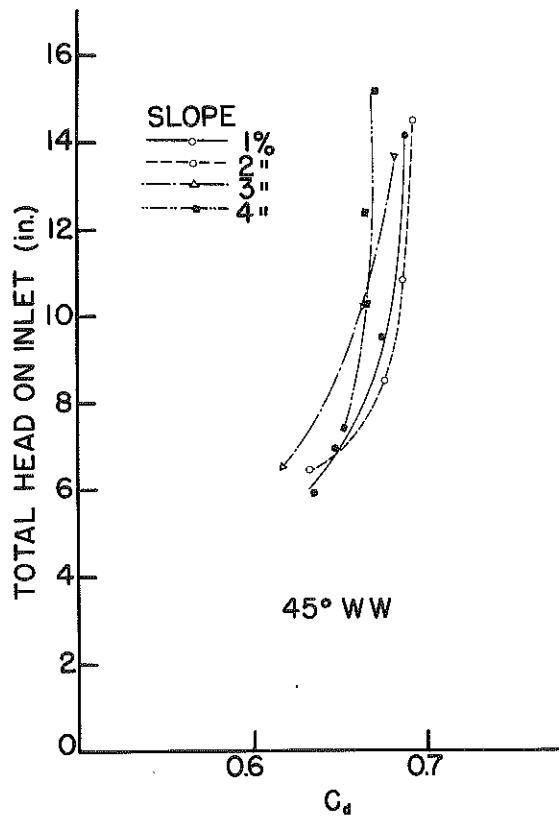


Fig. 6. Relationships of Total Head on Inlet Versus Coefficient of Discharge for Models A and B at Seven Slopes.

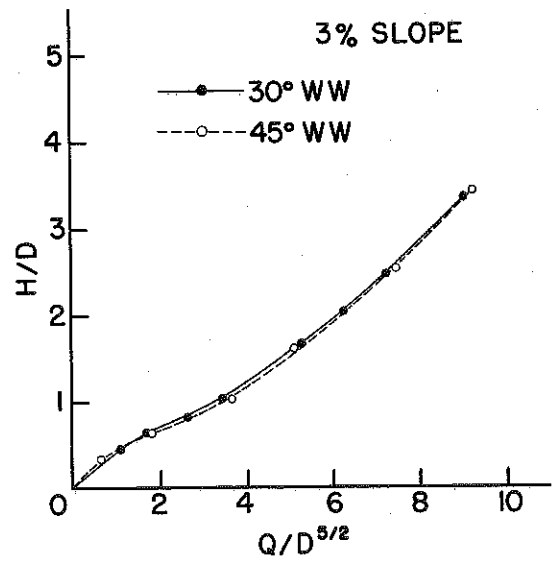
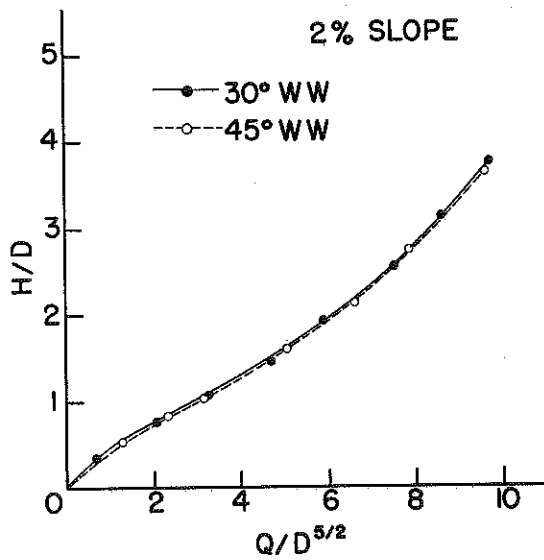
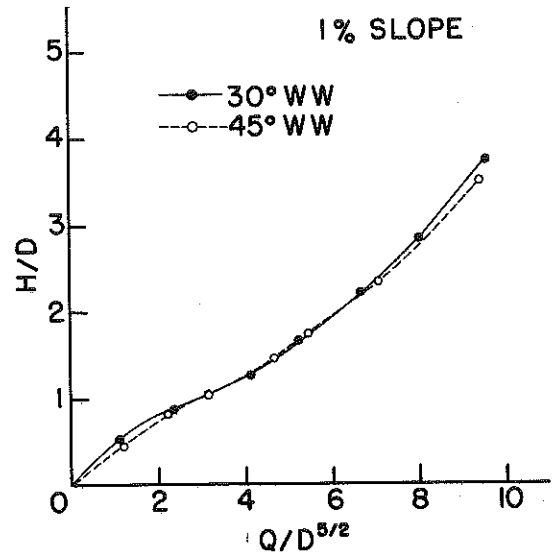
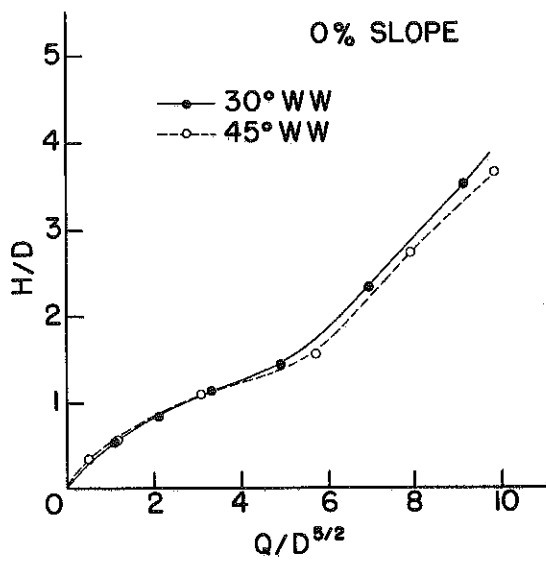


Fig. 7. Relationships of Head Ratios Versus Discharge Factor for Models A and B at Slopes of 0, 1, 2 and 3 Percent.

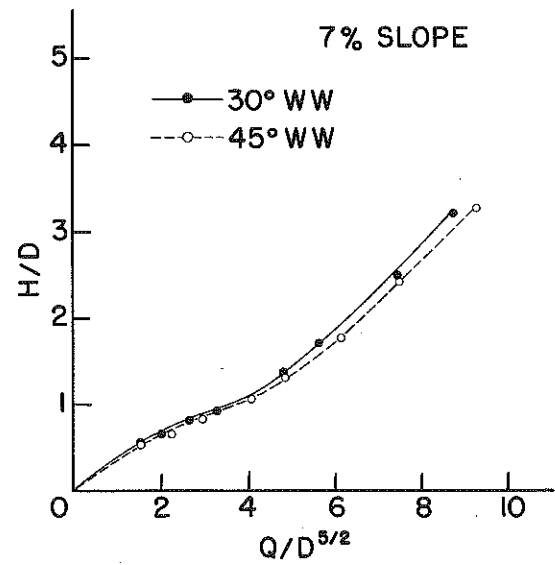
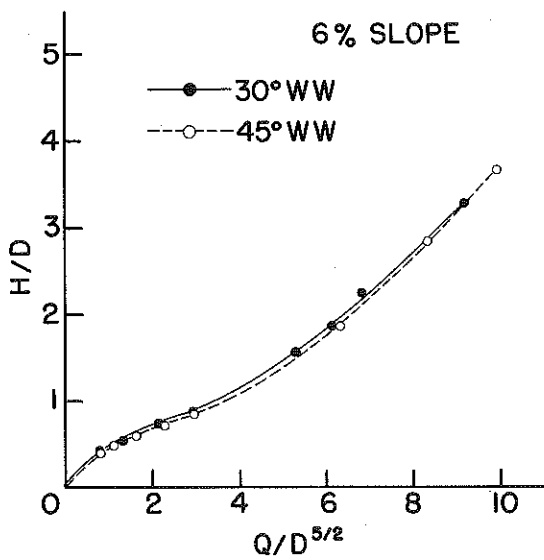
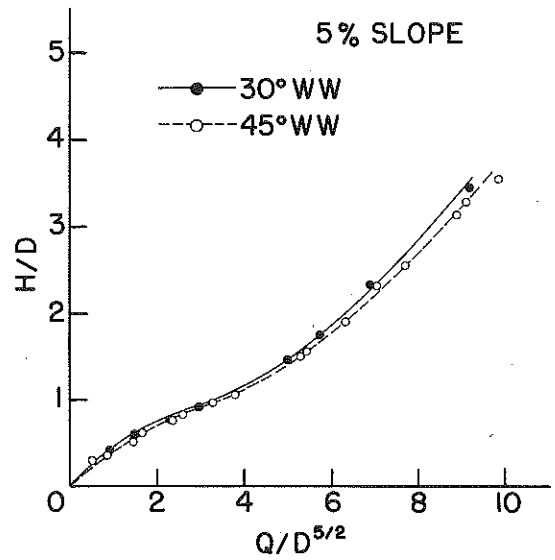
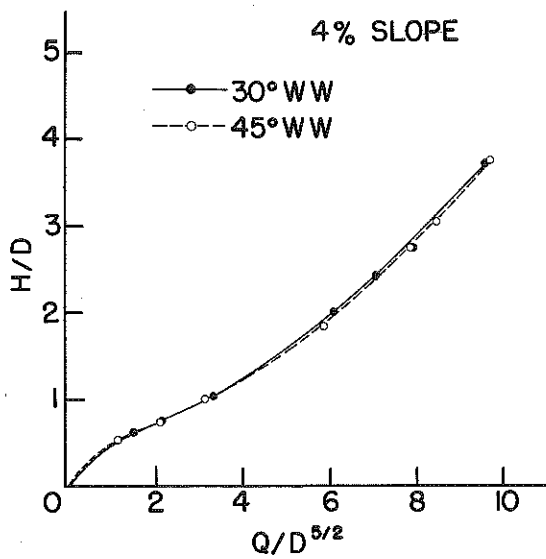


Fig. 8. Relationships of Head Ratio Versus Discharge Factor for Models A and B at Slopes of 4, 5, 6 and 7 Percent.

APPENDIX: HYDRAULIC SIMILITUDE*

The principles of hydraulics are based on mathematical theory; however, in the application of these principles to practical engineering problems the accuracy of the results frequently depends on experimental data, both from the field and from laboratory studies.

Originally, studies of the principles of hydraulic design were usually conducted at full scale on weirs, channels, existing dams, pipes and the like; but in recent years methods have been developed for predicting the behavior of full size structures from scale models.

The basis of model study prediction of prototype behavior - the prototype is the full-scale structure which the model represents - is the theory of hydraulic similitude. The analysis of the relationships of the physical quantities involved in the motion and dynamic action of a fluid is referred to as dimensional analysis.

There are three types of similarity to be considered in analyzing the relationship between a model and its prototype. These types are Geometric, Kinematic and Dynamic similarities, with the following definitions:

Geometric Similarity implies similarity of form. A model is geometrically similar to its prototype if the ratios of all homologous lengths in model and prototype are equal.

* For the equations and much of the other material in this Appendix, the author is indebted to King, Wisler and Woodburn, Hydraulics (6).

Kinematic Similarity implies similarity of motion. Kinematic similarity of model to prototype is attained if the paths of homologous moving particles are geometrically similar and if the ratios of velocities of the various homologous particles are equal.

Dynamic Similarity implies similarity of forces. A model is dynamically similar to the prototype if it is kinematically similar, and if the ratios of homologous moving masses and of the forces producing motion are respectively equal.

Geometric Similarity

For geometric similarity the ratio of homologous lengths in the model and prototype is expressed as $\frac{L_m}{L_p} = L_r^*$

Since area (A) is equal to the square of a characteristic length, the ratio of homologous areas is expressed as

$$\frac{A_m}{A_p} = \frac{L_m^2}{L_p^2} = L_r^2$$

Likewise, volume being the cube of a characteristic length, the ratio of homologous volumes is expressed as

$$\frac{\text{Vol. } m}{\text{Vol. } p} = \frac{L_m^3}{L_p^3} = L_r^3$$

Kinematic Similarity

For kinematic similarity between model and prototype, time is introduced in addition to length, which was considered in geometric similarity. The ratio of the times required for homologous particles to travel homologous distances in model and prototype is $\frac{T_m}{T_p} = T_r$.

* Subscript m denotes model, r denotes ratio, and p denotes prototype.

The kinematic quantities involved in a model study of this type are principally linear velocity (V) and discharge (Q).

Since linear velocity is expressed in terms of length per unit time, thus:

$$\frac{V_m}{V_p} = \frac{L_m/T_m}{L_p/T_p} = \frac{L_m/L_p}{T_m/T_p} = \frac{L_r}{T_r}$$

Discharge Q is expressed in terms of volume per unit time, thus:

$$\frac{Q_m}{Q_p} = \frac{\text{Vol}_m/T_m}{\text{Vol}_p/T_p} = \frac{L_m^3/L_p^3}{T_m/T_p} = \frac{L_r^3}{T_r}$$

Linear acceleration (a) is expressed as length per unit time squared; therefore:

$$\frac{a_m}{a_p} = \frac{L_m/T_m^2}{L_p/T_p^2} = \frac{L_m/L_p}{T_m^2/T_p^2} = \frac{L_r}{T_r^2}$$

Dynamic Similarity

For dynamic similarity the ratios of homologous forces in the model and prototype must be equal.

$$\frac{F_m}{F_p} = F_r$$

Force = mass (M) times acceleration (a)

$$F_r = \frac{F_m}{F_p} = \frac{M_m a_m}{M_p a_p} = M_r \frac{L_r}{T_r^2}$$

The Froude Model Law

The Froude Model Law, expressed in the equation $T_r = \sqrt{\frac{L_r}{g_r}}$ was derived for the conditions under which it can be assumed that the forces of inertia and gravity control the flow. Ordinarily the value of g_r is 1 and the equation becomes $T_r = \sqrt{L_r}$.

Substituting for T_r in the basic equations for similarity the various scale ratios for amplifying quantities such as depth, velocity and discharge to the prototype can be derived.

$$L_p = 12 L_m$$

$$A_p = 144 A_m$$

$$Vol_p = 1728 Vol_m$$

$$Q_p = 498.8307 Q_m$$

$$V_p = 3.4641 V_m$$

For complete similarity the Froude Number, a dimensionless ratio derived from the general expression $\frac{V}{\sqrt{Lg}}$ must be the same for the model as for the prototype. For all practical purposes, however, it is found that g_r will be unity for these culvert model studies, since the force due to centrifugal motion of the water is negligible compared to the force due to gravity. Tests indicate that the equations developed herein provide quite accurate prototype operation estimations if the model used has a scale ratio between 1:10 and 1:25. It has been found that a scale ratio of 1:12 is quite conservative for tests of this nature. (7), (8), (9).

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