September 28, 1961

ADDRESS REPLY TO DEPARTMENT OF HIGHWAYS
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## MEMORANDUM

TO: A. O. Neiser
Assistant State Highway Engineer

The attached report, "Performance of a Reinforced Concrete Pipe Culvert with Standard and B 1 High Fill Bedding under Rock Embankment (Scott County I-75-6(5) 123)", has been prepared by R. R. Taylor under the direct supervision of R. C. Deen。Research Engineer Senior.

During the 1960 construction season, after some 400 feet of the 48 inch reinforced concrete pipe culvert, located at Sta. $37+50$ of the southwest ramp and US-460 Interchange on Interstate 75 in Scott County had been installed, some cracks were observed by the construction crew on routine inspection. The embankment was partially completed and approximately 150 feet of the pipe culvert remained to be placed. The Research Division was requested to make a study of the culvert.

After a review of the construction plans it was found that the design was in error in specifying a standard bedding with Class III pipe for a fill height approaching 36 feet. A comprehensive study, including a very detailed performance survey, was made of the porm tion of the culvert in place. The following recommendations were made to the State Highway Engineer ${ }^{\wedge}$ s Office.

1. Construct the remaining 150 feet of the culvert in accordance with Department design criteria. Class III pipe with $\mathrm{B}_{1}$ high fill bedding is required for 36 feet of embankment.
2. Observe the performance of the 400 -foot section during the 1960-61 winter to determine need for additional construction revisions (patching or lining with corrugated metal pipe).

The recommendations were accepted and the remaining section of the culvert was placed during the early part of October, 1960. The embankment construction proceeded as shown in Fig 9, page 32. The grading operations were completed about June 15, 1961.

Four performance surveys were made with the last two being May 5, and June 16, 1961. The cracking observed is recorded on the sketch in Fig. 10, page 34 of the report. Note that the first 100 sections were constructed without the $\mathrm{B}_{1}$ bedding trench and sections from 100 through 139 were placed in accordance with normal design requirements. The color signifies the date first observed and the designation denotes the size of crack or extent of deterioration. Following the June, 1961, survey it was found that there were numerous hairline cracks, and a few 0.01 inch or larger cracks. There were no serious shear type failures. It was recommended that the pipe not be repaired but be observed for an extended period of time.

The performance to date of this culvert emphasizes the signifies cance of the $B_{l}$ trench, since this is the only difference in the as -built condition of the two sections. We plan to make annual inspection of the performance of this culvert.

Mr. Taylor has requested to use this report for partial fulfillmont of the requirements for a thesis for the degree of Master of Science in Civil Engineering. Because of this, he has made a comprehensive study of the subject literature as noted and referenced in the Bibliography.

WBD:dl
Re spectfully submitted,


Enc.
cc: Research Committee Members
Bureau of Public Roads (3)

## Commonwealth of Kentucky Department of Highways

# PERFORMANCE OF A REINFORCED CONCRETE PIPE CULVERT WITH STANDARD AND Bl HIGH FILL BEDDING UNDER ROCK EMBANKMENT (Scott County, I-75-6(5) 123) 

by
Ralph R. Taylor Research Engineer Assistant

## INTRODUCTION

When taken as a general ciassification, underground conduits in some form have been used by mankind for at least 3000 years for drainage or water supply purposes. In recent years interstate highways with flatter grades and longer radius curves have resulted in the construction of deeper cuts and higher fills. Concrete pipe culverts under these higher fills require either special methods of installation, such as the imperfect trench, or stronger pipe which will carry the required heavier loading.

An extensive research program on pipe problems was inaugurated at Iowa State College in 1908 and has been carried on practically continuously since that time. Through this research a comprehensive theory widely known as 'Marston's Theory of Loads on Underground Conduits" has been developed. At the present time, the greatest amount of research on determining loads carried by underground concrete pipe has been conducted by Messrs. Marston, Schlick, Spangler and their associates at Iowa State College。

Conceivably, it would be possible to compute by modern methods of soil mechanics the various factors that affect the load on a conduit, but such a procedure would involve the expenditure of both time and money that would be prohibitive in relation to the total.
cost of the structure. Therefore, many of these factors may be evaluated by an accumulation of data obtained from measurement and observation of actual installation performances.

With the damage that has occurred to concrete pipe in this and other states in very recent years, the re has been an immediate reaction that the specifications are in error and should be revised. In order to intelligently review the specifications it would be necessary to understand the basis for the requirements. This report attempts to bring together the background and development of the theories as currently used in the determination of fill heights which can be safely placed over concrete pipe culverts.

In comoperation with the Bureau of Public Roads (12), a number of reinforced pipe culvert installations in a number of states have been selected to evaluate the effectiveness of the design and consa truction criteria. One particular pipe has been chosen for this special study because of its installation conditions. This pipe is located on I 75-6 (5) 123 in Scott County at Station $37+50$, U'S 460, SW Ramp, and has been constructed using both type beddings that are specified in the 1956 edition of the Kentucky Department of Highways Standard Specifications for Road and Bridge Construction and amendments thereto. Observation of the performance of this in stallation during
construction and after completion was desirable so as to check the validity of the theory as applied to rock fill material, which was used in this particular installation. The ultimate objective of this study and investigation, the refore, is to align the theory more with actual field conditions. No attempt has been made to discuss the hydraulic or functional phase of culvert design or the structural design of the conduit.

## SURVEY OF LITERATURE

The successful application of the scientific principles of mechanics to the analysis of soil has made possible efforts directed toward the development of rational methods for determining the magnitude and character of the loads on underground conduits. The structural performance of an underground conduit is dependent upon a number of factors and conditions which need to be carefully taken into account in the design of such a structure. Also, because the maximum height of fill which can be safely placed over a culvert pipe depends so intimately upon the installation conditions and because these influencing conditions are usually covered up and obliterated when the embankment is constructed, adequate information is necessary concerning conditions under which the conduit is installed.

The Iowa State Experiment Station for many years has conducted research in the area of load determination and supporting strength of underground conduits and has published a considerable number of bulletins and technical papers dealing with the various phases of this research. A comprehensive theory widely known as "Marston's Theory of Loads on Underground Conduits' (40)(41), named for the late Anson Marston, Dean Emeritus of Engineering at Iowa State College, has evolved from the principles developed in these studies.

A study of Barbour's (41) experiments reported before the Boston Society of Civil Engineers in 1897, with Marston's theory in mind, affords strong confirmation of the correctness and general reliability of the theory. However, some very serious errors in planning and interpretation were made due to the fact that the true mathematical theory of loads on pipes in ditches was not known at that time. Culvert load tests (15)(16) conducted at Farina, Illinois, by the American Railway Engineering Association from 1925 to 1927 verify。 in a general way, the Marston theory. Studies (9)(10) made at the University of North Carolina at Chapel Hill from 1924 to 1927, in which the loads on culverts were weighed with reliable apparatus, supported the Marston theory where sufficient data allowed comparison. Through the years of research at Iowa State College the theory and results for ditch or trench conduits seem to check very closely with experience, but much additional experimentation is necessary on projecting (embankment) conduits.

When considering the loads produced on pipe conduits due te fills, conduits have been grouped into two major classes or groups $(40)(41)$ on the basis of construction conditions which affect the development of loads on them. These classes are trench or ditch conduits and embankment conduits. The essential elements of the se classes of conduits are illustrated in Fig. 1. Some conduits have characteristics of both classes and may or may not fall wholly within one or the other.


Fig. 1. Classes of Conduits.

Embankment conduits, which constitute nearly all underground conduit construction in the highway field, may be either positive prow jecting or negative projecting in relation to the natural or prepared ground surface. Positive projecting conduits are embankment conduits installed in shallow bedding with the top of the conduit projecting above the surface of the natural ground or prepared grade. Negative projecting conduits are those installed in relatively narrow trenches of such depth that the tops of the conduits are below the level of the natural ground surface or prepared grade.

In the consideration of an embankment built over a conduit, the re are three masses or prisms of soil, one of which, called the "interior prism", is directly over the conduit between the vertical planes tangent to the sides of the conduit in the case of positive projecting conduit and, for negative projecting conduit, between the vertical extensions of the sides of the trench in which the conduit is placed (see Figs. 2 and 4). The other two masses of soil, called the "exterior prisms", are those on each side of the structure adjacent to these vertical planes. The height of the interior prism will be different from that of the exterior prisms by the amount that the conduit projects above or below the natural ground. For positive projecting conduits the ratio of the vertical distance which the top of the


Fig. 2. Elements of a Positive Projecting Conduit.
conduit projects above the natural or prepared ground surface adjacent to the conduit to the outside width of the conduit is defined as the projection ratio. For negative projecting conduits, the projection ratio is the ratio of the distance between the natural or prepared ground surface and the top of the conduit to the width of the ditch in which the conduit is placed.

It is recognized that definite shearing planes between the interior and exterior prisms of the soil do not actually exist in an earth embankment and that, in all probability, the shearing stresses are transferred from one prism to another through more or less narrow zones of material. Nevertheless, actual vertical shearing planes are assumed and experiments indicate that this assumption is reasonable.

If the embankment is not very high, the shearing stresses that develop along the planes between the interior and exterior prisms may extend upward from the conduit completely to the top of the embankment. In the case of higher fills they terminate at some horizontal plane, between the top of the conduit and the top of the embank= ment, known as the "plane of equal settlement" (see Figs. 2 and 4). The plane of equal settlement is defined as the horizontal plane in the embankment at and above which the settlements of the interior and exterior prisms of soil are equal. The existence of a plane of equal
settlement was first suggested by Dean Marston in 1922 (40) on the basis of pure mathematical reasoning and a formula for determining its height above the top of the pipe was developed at that time.

Twelve culvert pipe load experiments were made at the University of North Carolina at Chapel Hill from 1924 to 1927. The final detailed report (10) was made by G。M. Braune and H. J. Janda, with appendices by William Cain. Neither Dean Braune nor Professor Cain finally derived a definite comprehensive culvert load theory from the North Carolina results, although they both toyed considerably with Dean Braune's idea (9):
"... the theory on culvert pipes, namely, that the pressures vary inversely as some power of the deflection of the pipe, and values of the ratio, $K$, have been obtained ranging between limits above and below unity"。

The factor, $K$; as used in Dean Braune's discussion was related to the settlement ratio of Iowa State. Because they did not recognize the proven existence of planes of equal settlement in their research ino vestigation, no measurement or discussion of any settlement other than the deflection of the conduit was made.

The basic settlement situation is usually modified by two additional factors which must be considered in the development of a general theory of fill loads. The first of these is the settlement or subsidence of the undisturbed subgrade under the exterior prisms
adjacent to the conduit; and the second, the settlement of the top of the conduit. The settlement of the top of the conduit has a tendency to neutralize the stresses between prisms. If the conduit is sufficiently flexible or if it is placed on a very yielding foundation the direction of the induced shearing forces is reversed and the stresses are subtractive from the weight of the prism of soil over the conduit.

A horizontal plane in the fill material at the top of the conduit at the beginning of construction of the embankment and before settlements have begun to develop has been defined as a "critical plane" (see Figs. 2 and 4). Therefore, when the critical plane settles more than the top of the conduit, the shearing stresses act downward on the interior prism and when it settles less they act upward. A neutral or transition case occurs when the plane of equal settlement and the critical plane coincide, with the result that the load on the conduit is equal to the weight of the prism of soil directly over it (66). The net effect of all the settlement factors, both as to magnitude and direction of relative movements of the three prisms of soils is combined into an abstract ratio known as the "settlement ratio" and is defined as the ratio of the difference between the critical plane and the top of the conduit to the deformation of the fill material adjacent to the conduit within the vertical distance between the top of the conduit and the natural ground line (76).

When the fill along the sides of the pipe tends to settle more than that over the pipe the friction along the vertical faces of the prism over the pipe tends to drag the interior prism down rather than hold it up. Therefore, the load on the culvert is often greater than the weight of the superimposed earth. Complication arises from the fact that the frictional forces may not be in operation to the top of the fill; mathe matically it may be shown that above a certain plane over the culvert the settlement of the fill is uniform. Thus, the culvert is subjected to the weight of a prism of earth dragged down by the friction along its sides plus a load caused by the earth superimposed above the plane of equal settlement.

The settlement ratio is wholly rational in the theoretical analysis of loads on projecting conduits. However, it is not practio cable to predict the value to use in the design of a given structure but to consider the settlement ratio as a semi-empirical factor and to determine proper design values on the basis of observed performance of projecting conduits in service.

Rather extensive field measurements of the settlement characteristics of actual culverts under highways have indicated working values to be used for the settlement ratio (68) (75) (see Table 1).

TABLE 1

For rigid culverts on rock or $\quad r_{s d}=+1.0$ unyielding soil foundation

For rigid culverts on ordinary $\quad r_{\text {sd }}=+0.5$ to +0.8 soil foundation

For rigid culverts on yielding
$r_{s d}=0$ to +0.5 foundation as compared to the adjacent natural ground

For flexible culverts with
$r_{s d}=-0.4$ to 0 poorly compacted sidefills

For flexible culverts with
$r_{s d}=-0.2$ to +0.8 well compacted sidefills

Marston's formula (40)(41) for vertical loads on positive projecting conduits due to earth fill is

$$
\begin{equation*}
\mathrm{w}_{\mathrm{c}}=\mathrm{C}_{\mathrm{c}} \mathrm{wB}_{\mathrm{c}}^{2} \tag{1}
\end{equation*}
$$

in which $W_{c}=$ load per unit length of conduit,
$C_{c}=$ load coefficient (see Equations (7) and (10), Appendix I),
$\mathrm{w}=$ unit weight of embankment material,
$\mathrm{B}_{\mathrm{C}}=$ outside width of the conduit, and
$\mathrm{H}=$ height of fill.

The coefficient, $C_{C}$, in Equation (1) is a function of the ratio of the height of fill to the outside width of the conduit, $H / B_{C}$; of
the projection ratio, $p$; the settlement ratio, $r_{\text {sd }}$; and the coefficient of internal friction of the fill material. The variation in load is relatively small for wide variations of the coefficient of internal friction and it is not necessary or practical to differentiate between the various kinds of soil (40) (66). Working values of the load coefficient in Equation (1) may be obtained from Fig. 3.
M. G. Spangler (65) in 1950 followed the same general principles of Marston's theory (40) (41) of loads on ditch and positive projecting conduits and presented a theory for negative projecting conduits. Loads on a negative projecting conduit, generally speaking, are considerably less than those on a positive projecting conduit and permit construction of higher fills without damage to the structure (54)(65). Work by W. J. Schlick (55) on the effect of construction in wide ditches on pipe loads was presented in 1932. The results showed that as the width of the ditch increases, other conditions remaining constant, the load upon the conduit increases until it equals that obtained by the projecting conduit load theory, and then remains constant for all greater widths.

The following definitions, as used in the case of negative projecting conduits, are slightly modified from those used in the derivation of the load formula for positive projecting conduits (see Fig. 4).


Fig. 3. Load Coefficient Diagram for Positive Projecting Conduits.


Fig. 4. Elements of a Negative Projecting Conduit.

> Critical Plane: The horizontal plane in the embankment material which was originally level with the natural or prepared ground surface before settlements occurred.

> Settlement Ratio: The ratio of the difference between the the settlement of the natural ground surface and the settlement of the critical plane to the compression of the column of soil in the ditch within the distance between the top of the conduit and the natural ground surface.

The relationship developed by Spangler (65) for computing the load on a negative projecting conduit due to earth fill is:

$$
\begin{equation*}
\mathrm{w}_{\mathrm{c}}=\mathrm{C}_{\mathrm{n}} \mathrm{wB}_{\mathrm{d}}^{2} \tag{2}
\end{equation*}
$$

in which $\mathrm{W}_{\mathrm{C}}=$ load per unit length of conduit,
$\mathrm{C}_{\mathrm{n}}=$ load coefficient for negative projecting conduits (see Equations (7) and (10), Appendix I),
$\mathrm{w}=$ unit weight of embankment material,
$\mathrm{B}_{\mathrm{d}}=$ width of the ditch in which the conduit is placed, (If the ditch has sloping sides, use the width at the top of the conduit), and
$\mathrm{H}=$ height of fill.

The values of $\mathrm{C}_{\mathrm{n}}$, load coefficient, are dependent upon the projection ratio, $\mathrm{p}^{8}$; and the settlement ratio, $\mathrm{r}_{\mathrm{sd}}$; as well as on the ratio of the height of fill to the width of the trench, $H / B_{d}$.

The effect of the variation of the coefficient of friction of different fill.
materials is relatively small and need not be assertained for sufficiently accurate design values (65). $C_{n}$ can be read directly from prepared diagrams which make it unnecessary to solve cumbersome equations (see Fig. 5).

Despite the lapse of time and the accumulation of field experien ce, the safe structural design of a negative projecting conduit installation still involves considerable judgement. Spangleris load theory appears to be a logical development based upon correct theoretio cal concepts, but he presents no supporting data. W. . . Schlick (54) in a study at Iowa State College in 1950 demonstrates the effectiveness of negative projection loading as a means of decreasing the load on culverts to be installed under high fills.

With the objective in mind of devising a method construction that would reduce or eliminate shearing forces that add to the weight of the interior prism, or wou\& possibly reverse their direction so that they would act benevolently as is the case in ditch corduits. Marston (40) developed the imperfect trench (ditch) method of con= struction (see Fig. 6). For construction of the imperfect trench the soil on both sides and above the conduit for some distance abve the top is thoroughly compacted by rolling and tamping. The trench is dug in this compacted fill by removing the prism of material directly over the conduit and then is refilled with very loose and compressible


Fig. 5. Load Coefficient Diagram for Negative Projecting or Imperfect Trench Conduits, $p=1.5, K \mu=0.13$.


Fig. 6. Imperfect Trench Condition (Positive Projection).
material after which the embankment is completed in a normal manner.

The theoretical analysis of loads on negative projecting conduits can be used to estimate loads on conduits installed by the imperfect trench method (3). The natural ground surface may be considered to be the top surface of the initially compacted material. The width of the imperfect trench can be made much smaller than that of the negative projecting case and for the most favorable results it should be no wider than the outside width of the conduit.

The load which a conduit will support is determined to a large extent by the way the conduit is bedded; four methods of bedding from the best possible to the worst have been defined (47)(57)(60)(76). Without going into an extensive description of bedqing details, which may have variations from user to user, it can be said that the bedding varies from the construction of a concrete cradle to hardly any preparation at all before placing the pipe (see Fig. 7). Pipe bedding as used by the Kentucky Department of Highways is shown in Fig. 14, Appendix II. Concrete cradles (60) are not used extensively since the additional cost incurred might be applied to insure higher pipe strength.

In the imperfect trench method of installation, the pipe culvert is installed in accordance with the requirements of the bedding


Fig. 7. Types of Bedding in the Embankment Condition.
specified. Then the fill is compacted at each side of the pipe for a desired lateral distance and carried up to an elevation above the pipe. Next, a trench equal in width to the outside pipe diameter is dug in the fill directly over the pipe. After the trench is excavated, it is refilled with loose, highly compressible soil material. Straw, hay, cornstalks, leaves, brush, or sawdust may also be used to fill the lower $1 / 4$ to $1 / 3$ of the trench. After the backfill is completed, the balance of the fill is constructed by normal methods up to the finished grade of the embankment. A reduction in the load on the conduit is accomplished in this method of construction wherein it is certain that the prism of material directly over the conduit will settle more than the adjacent prisms.

In recent years the problem of high overfills has resulted in the recognition of the advantages to be gained by the negative projecting and imperfect trench methods of construction. Qualitative checks on the validity of the Marston theory are of extreme importance at this time because of the requirements of modern highway transportation. Confidence in the principles involved in the theoretical development is reinforced by the close correlation with the measured pressures on a Panama Canal Zone concrete box culvert under 51 feet of fill as described by Wilson V. Binger (7) of the U. S. Corps of Engineers. Furthermore, the principle of linear relationship between the load and height of fill material above the plane of equal settlement was found to
be valid for culvert investigations under 168 feet of fill in North Carolina reported in 1955 by Costes and Proudly (18). Timmer's results (82) for three-parallel 84-inch-diameter, multi-plate pipes under 137 feet of embankment in Cullman County, Alabama, also show agreement with Marston's theory.

Although experience with the imperfect trench method of installation has not been as widespread as with the more common methods, there is some evidence that concrete pipe culverts under embankments with fill heights up to 100 feet may be used without requiring unusually high strength concrete pipe. The imperfect trench condition was given sufficient publicity that the method was reported to have been used in Pennsylvania in the early $1930^{\prime}$ s. It was recognized in 1934 by the Bureau of Public Roads in a memorandum to its district engineers. Under R. Robinson Rowe (11) of California numerous installations (51) have been made which help to confirm the fact that the method is correct in theory and practical in its field application.

All the rational work done in determining loads on concrete pipe culverts has assumed the fill material to be soil. However, in 1929 three 36 -inch cast iron pipe culverts were constructed on the primary road system of Iowa under rock fills and advantage was taken of this opportunity to study the load effect of rock embankments (70). Two of the pipe were considered to have failed (one crushed, one crushed
badly) ; the third carried the load successfully. Marston's theory, broadly speaking, is applicable to this condition.

States that have specifications or standard plans permitting the use of the imperfect trench as proposed by Marston at Iowa State College, or modifications the reof, are Alabama, Arizonia, California, Georgia, Kansas, Kentucky, Minnesota, Montana, Pennsylvania, South Dakota, and Wyoming. In addition to the se states preliminary specifications, which include the imperfect trench method, have been submitted by Missouri and North Carolina. With the exception of California, most of the states have adopted the imperfect trench method of construction within recent years.

Advancement of the present knowledge of loads on pipe culverts is dependent upon an accumulation of field experience and supporting data. The Bureau of Public Roads, with the co-operation of various states, recently inaugurated an investigational program of concrete pipe with the idea of determining the performance of the se pipe culverts. This long range program will be used to attempt an evaluation of cortstruction practices of the participating groups and provide verification of the correctness of methods used. It is hoped that the analysis of a few installations in similar soils will divulge a behavior pattern which might be used to advantage in future designs. Once a behavior pattern
for a given soil under known installation conditions has been established, the theory should provide an accurate and handy tool for design purposes.

## PIPE INSTALLATION AND PERFORMANCE

A pipe installation, shown in Fig. 8, on Interstate 75 near Georgetown, Kentucky, in Scott County, was selected for special study. This study was inaugurated to take advantage of an opportunity to ob= serve both bedding conditions as used in the Kentucky Department of Highways Standard Specifications at one location. The installation conditions and performance of this particular pipe were needed to aid in an evaluation of the effectiveness of design and construction practices.

Distress was noted in the pipe on I 75-6(5)l23 at Station $37+50$, US 460, SW Ramp, near the latter part of August, 1960, before the full design length of pipe ( 560 feet, 140 sections) had been laid. The fill had not reached full design height ( 36 feet) and the pipe had hardly been in place a month at this time. This first portion of the pipe was laid using Standard B Bedding; after a review and check of plans and specifications was made the bedding for the remainder of the pipe was changed to $\mathrm{B}_{1}$ Bedding (imperfect trench). With Class III pipe, which was used at this installation, and Standard B Bedding the allowable fill height is 1.5 feet to 20 feet and for $B_{1}$ Bedding the allowable height is 21 feet to 40 feet (see Fig. 15, Appendix II)。


Fig. 8. Sketch Showing Culvert Location.

A total of 1024 -foot pipe sections manufactured on different dates, from a design length of 140 sections, was placed with Standard B Bedding before the change. Plans specified the Standard B Bedding and it must be assumed that this was obtained. Field data substantiate that the backfill depth requirement above rock (see Fig. 14, Appendix II) was obtained before placing the pipe; the refore, one of the factors, ledge rock in the foundation too close to the bottom of the pipe, conside red to be a major contributing cause to pipe failure, is nonexistent at this location. Distłess was assumed to originate from exogenous causes rather than pipe manufacture as is verified from numerous test results.

For the remaining 37 sections actually placed, a $\mathrm{B}_{1}$ Bedding was used. Inspection of the installation of this portion of the pipe was made with close co-operation between the Resident Engineer on the project and Research Division personnel. At least three feet of unsuitable material were removed from the foundation area below the bottom of the pipe before the backfill was placed (see Fig. 16, Appendix III); the Resident Engineer estimated a 7-foot soil foundation for camber calculations (44). Pans placed the backfill in approximately one foot lifts and a rubber-tired dozer (Michigan) was used to compact it. A sand cushion bedding was prepared in a satisfactory manner except that it was not shaped to perfectly support the pipe.

The pipe was laid in the sand cushion bedding (see Fig. 19, Appendix III) and the backfill brought up uniformly in layers not exm ceeding 6 inches for a minimum height of 0.30 times the outside pipe diameter. Each layer was compacted thoroughly by pneumatic tampers. Care was exercised to thoroughly compact the backfill under the haunches of the pipe with a single action pneumatic tamper to insure that the backfill was in intimate contact with the sides of the pipe. The embankment was then extended upward in a normal manner to a specified elevation above the top of the pipe, which was equal to the over-all height of the pipe plus 12 inches, in preparation for construction of the imperfect trench (see Fig. 20, Appendix III).

A trench equal in width and height to the outside width and height of the pipe was dug in the compacted embankment with a backhoe (see Fig. 21, Appendix III). At this time the alignment of the center-line of the pipe and the center-line of the trench were noted to be off approximately one foot in some places; this was corrected by refilling and compacting the trench in 6 -inch layers and redigging the trench over the center-line of the pipe. The trench was then filled with loose straw to a depth of approximately 24 inches. Loose backfill in the remainder of the trench compressed the straw to a depth of about 16 inches. The trench was then covered and bridged by a 2 -foot layer of compacted soil to a width of 20 feet on each side of the pipe (see Fig. 22,

Appendix III). The pipe was ready for the placement of the rock fill. after completion of the imperfect trench.

By this time, however, it was late in the season and construc. tion equipment had been moved from the general construction area. Consequently, the rock fill over the pipe was not completed until the spring of 1961. A smail volume of earth fill (see Fig, 9) was placed the last part of the year as an approach for construction of a nearby overpass pier. The majority of the last 37 sections placed with $\mathrm{B}_{1}$ Bedding remained in the same state of construction during the winter, 1960, as they were after completion of the imperfect trench in September and early October (see Fig. 23, Appendix III). Therefore, an evaluation of the imperfect trench installation could not be made until some time after completion of the fill in the spring of 1961.

The embankment material other than that used in the immediate backfill near the pipe and imperfect trench consisted for the most part of rock. With the placement of the rock material (see Fig. 9) during the last part of August, 1960, over the 102 sections laid with Standard B Bedding, the re developed a series of hairline cracks. A survey of pipe conditions revealed nearly every section, beginning at section 13 through section 72, had at least one hairline crack in the top and bottom by the last of August. Hairline cracks were visible in section 13 through section 86 by the last of September with a progressive


increase in the size and number of cracks in sections 33 to 76 since the survey made the last part of August.

In January, l961, the sections constructed with Standard B Bedding still exhibited progressive development of hairline cracks under the partial fill load but seemed to be approaching an equilibrium condition with respect to load increase on the pipe. Previous work on load analysis and determination has shown that the initial load is equal to approximately 75 percent of the final ultimate load to be developed some time after construction. By May of 1961 , the approximate ultimate load appeared to have been reached since progressive distress was no longer noted in the form of increased size or number of cracks.

Construction of the rock embankment over the pipe and the 12 inch soil layer above the rock and below the subgrade elevation was completed the last of May, 1961. The additional load on the pipe sec. tions with Standard B Bedding resulted in renewed increase of distress. The amount of additional fill may be seen in Fig. 9. Increased distress in the $B$ Bedding sections due to the additional embankment was most noticeable in sections 75 through 100。 Six hairline cracks were visible in some sections that had previously shown no distress (see Fig, 10). The sectiors with $\mathrm{B}_{1}$ Bedding, now with nearly full embankment design height, remained in good condition and exhibited no distress.

## NEGATIVE PROJECTING CONDUITS

## Complete Ditch Condition

Considering the complete ditch condition of a negative projecting conduit, i.e. $H<H_{e}$, the vertical forces acting on a horizontal element of the interior prism (see Fig, ll) can be given as

$$
\begin{equation*}
V+d V=V+w B_{d} d h-2 K \mu\left(V / B_{d}\right) d h \tag{1}
\end{equation*}
$$

where $V=$ vertical load on any horizontal plane in the interior prism, lb/ft. of length;
$\mathrm{w}=$ unit weight of fill material, lb/cu.ft.;
$B_{d}=$ width of ditch, ft. ;
$h=$ distance from top of fill down to any horizontal plane, ft.;
$K=$ coefficient of active earth pressure;
$\mu=$ coefficient of internal friction of the fill material;
$H=$ height of fill above the top of the conduit, ft.; and
$H_{e}=$ height of the plane of equal settlement above the top of the conduit, ft.

Solving Equation (1) for dV gives

$$
d V=\left(2 K \mu / B_{d}\right) \cdot\left(B_{d} / 2 K \mu\right) w B_{d} d h-\left(2 K \mu V / B_{d}\right) d h
$$



Fig. 1l. Force Diagram for Negative Projecting Condition.
and after rearranging, the result is

$$
\begin{equation*}
\mathrm{dV} /\left[\mathrm{V}-\left(\mathrm{wB}_{\mathrm{d}}^{2} / 2 \mathrm{~K} \mu\right)\right]=-\left(2 \mathrm{~K} \mu / \mathrm{B}_{\mathrm{d}}\right) \mathrm{dh} . \tag{2}
\end{equation*}
$$

Integrating Equation (2)

$$
\begin{equation*}
\ln \left[\mathrm{V}-\left(\mathrm{wB}_{\mathrm{d}}^{2} / 2 \mathrm{~K} \mu\right)\right]=-\left(2 \mathrm{~K} \mu \mathrm{~h} / \mathrm{B}_{\mathrm{d}}\right)+\mathrm{C} \tag{3}
\end{equation*}
$$

Making use of the boundary condition, $V=0$ when $h=0$, the constant of integration $C$ is found to be

$$
\begin{equation*}
\mathrm{C}=\ln \left(-\mathrm{wB}_{\mathrm{d}}^{2} / 2 \mathrm{~K} \mu\right) \tag{4}
\end{equation*}
$$

Substituting Equation(4) into Equation (3) gives

$$
\mathrm{V}-\left(\mathrm{wB}_{\mathrm{d}}^{2} / 2 \mathrm{~K} \mu\right)=-\left(\mathrm{wB}_{\mathrm{d}}^{2} / 2 \mathrm{~K} \mu\right) \cdot \exp \left(-2 \mathrm{~K} \mu \mathrm{~h} / \mathrm{B}_{\mathrm{d}}\right)
$$

or

$$
\begin{equation*}
\mathrm{V}=-\left(\mathrm{wB}_{\mathrm{d}}^{2} / 2 \mathrm{~K} \mu\right)\left[\exp \left(-2 \mathrm{~K} \mu \mathrm{~h} / \mathrm{B}_{\mathrm{d}}\right)-1\right] . \tag{5}
\end{equation*}
$$

Since at the top of the conduit $\mathrm{V}=\mathrm{W}_{\mathrm{c}}$ and $\mathrm{h}=\mathrm{H}$, Equation
(5) becomes

$$
\begin{equation*}
\mathrm{w}_{\mathrm{c}}=\mathrm{C}_{\mathrm{n}} \mathrm{wB}_{\mathrm{d}}^{2} \tag{6}
\end{equation*}
$$

in which $C_{n}=\left[\exp \left(-2 K \mu H / B_{d}\right)-1\right] /(-2 K \mu)=$ load coefficient
and $W_{c}=$ load on the conduit due to the fill materials, $\mathrm{lb} / \mathrm{ft}$. of length.

## Incomplete Ditch Condition

Considering now the incomplete ditch condition, i.e.
$H>H_{e}$, it is seen that Equation (3) is valid for this case (see Fig. 11). In order to evaluate the constant of integration $C$ for this case, use the boundary condition $V=w\left(H-H_{e}\right) B_{d}$ when $h=0$. Thus

$$
\begin{equation*}
C=\ln \left[w_{d}\left\{\left(H-H_{e}\right)-\left(B_{d} / 2 \mathrm{~K} \mu\right)\right\}\right] . \tag{8}
\end{equation*}
$$

Substituting Equation (8) into Equation (3) gives

$$
V-\left(w B_{d}^{2} / 2 K \mu\right)=w B_{d}\left[\left(H-H_{e}\right)-\left(B_{d} / 2 K \mu\right)\right] \exp \left(-2 K \mu h / B_{d}\right)
$$

or

$$
\begin{align*}
V= & -\left({w B_{d}}^{2} / 2 K \mu\right)\left[\exp \left(-2 K \mu h / B_{d}\right)-1\right]+ \\
& {w B_{d}}^{2}\left[\left(H / B_{d}\right)-\left(H_{e} / B_{d}\right)\right] \exp \left(-2 K \mu h / B_{d}\right) \tag{9}
\end{align*}
$$

where $h=$ distance from the plane of equal settlement down to any
horizontal plane, ft.

At the top of the conduit $\mathrm{V}=\mathrm{W}_{\mathrm{c}}$ and $\mathrm{h}=\mathrm{H}_{\mathrm{e}}$ and the load on the conduit is given by

$$
\mathrm{w}_{\mathrm{c}}=\mathrm{C}_{\mathrm{n}} \mathrm{wB}_{\mathrm{d}}^{2}
$$

where $C_{n}=\left\{\left[\exp \left(-2 K \mu \mathrm{H}_{\mathrm{e}} / \mathrm{B}_{\mathrm{d}}\right)-1\right] /(-2 \mathrm{~K} \mu)\right\}+$

$$
\begin{equation*}
\left[\left(H / B_{d}\right)-\left(H_{e} / B_{d}\right)\right] \exp \left(-2 K \mu H_{e} / B_{d}\right) . \tag{10}
\end{equation*}
$$

In order to solve Equation (10) it is necessary to know the value of $\mathrm{H}_{\mathrm{e}}$. An expression for $\mathrm{H}_{\mathrm{e}}$ can be derived by equating the settlement of the plane of equal settlement in the interior prism to
that in the exterior prisms. This equation is

$$
\begin{equation*}
\lambda+s_{d}+s_{f}+d_{c}=\lambda^{\prime}+s_{g} \tag{11}
\end{equation*}
$$

where $\lambda$ = compression of the interior prism between the natural ground line and the plane of equal settlement, $s_{d}=$ compression of the material in the ditch within the distance $\mathrm{p}^{\prime} \mathrm{B}_{\mathrm{d}}$, $s_{f}=$ settlement of the conduit foundation, $\mathrm{d}_{\mathrm{c}}=$ vertical deflection of the conduit, $\lambda^{\prime}=$ compression of the exterior prism between the natural ground line and the plane of equal settlement, $s_{g}=$ settlement of the natural ground line, and $p^{\prime}=$ projection ratio.

Since

$$
\begin{equation*}
r_{s d}=\left[s_{g}-\left(s_{d}+s_{f}+d_{c}\right)\right] / s_{d} \tag{12}
\end{equation*}
$$

where $\mathrm{r}_{\mathrm{sd}}=$ settlement ratio, Equation (11) becomes

$$
\begin{equation*}
\lambda=\lambda^{\prime}+r_{s d}{ }_{d} \tag{13}
\end{equation*}
$$

To derive an expression for $\boldsymbol{\lambda}$ it is necessary to assume that the internal friction of the fill material distributes the small changes of pressure from shear into the interior prism so as to effect settlement
in the same manner as for uniform vertical pressure. The differential equation for $\lambda$ is given by

$$
\begin{equation*}
\mathrm{d} \boldsymbol{\lambda}=\left(\mathrm{V} / \mathrm{B}_{\mathrm{d}} \mathrm{E}\right) \mathrm{dh} \tag{14}
\end{equation*}
$$

where $\mathrm{E}=$ modulus of compression of the fill material and V is given by Equation (9) with $H$ replaced by $H^{\circ}$, the height of the plane of equal settlement above the natural ground line.

Substituting Equation (9) into Equation (14) and integrating between the limits $\mathrm{h}=0$ and $\mathrm{h}=\mathrm{H}_{\mathrm{e}}{ }^{\text {' }}$ gives

$$
\begin{align*}
\lambda= & \frac{w B_{d}{ }^{2}}{E}\left[\frac{H^{2}}{B_{d}}-\frac{H_{e}^{i}}{B_{d}}-\frac{1}{2 \mathrm{~K}_{\mu}}\right] \frac{\exp \left(-2 \mathrm{~K}_{\mu} \mathrm{H}_{\mathrm{e}} / \mathrm{B}_{\mathrm{d}}\right)-1}{-2 \mathrm{~K} \mu}+ \\
& \frac{w B_{d}{ }^{2}}{\mathrm{E}} \cdot \frac{\mathrm{H}_{Q}}{\mathrm{~B}_{\mathrm{d}}} \cdot \frac{1}{2 \mathrm{~K} \mu} . \tag{15}
\end{align*}
$$

The differential equation for $\lambda^{\prime}$ is

$$
\begin{equation*}
\mathrm{d} \lambda^{d} \leftrightharpoons \mathrm{~V}^{\prime} \mathrm{dh} / \mathrm{B}_{\mathrm{d}} \mathrm{E} \tag{16}
\end{equation*}
$$

where $V^{\prime}=$ vertical load on any horizontal plane in the exterior prisms, lb/ft. of length.

It is necessary to assume that the internal friction in the fill material distributes the increments of pressure from the shear into each of the exterior prisms below the plane of equal settlement in such a manner that the effect on settlements is the same as though the pressure were
distributed uniformly over a prism of width $B_{d}$. Thus

$$
\begin{equation*}
V^{v}=w_{d}\left(h+H^{8}-H_{e}^{q}\right)-(F / 2) \tag{17}
\end{equation*}
$$

where

$$
\begin{equation*}
F=V-w B_{d}\left(h+H^{8}-H_{e}{ }^{8}\right) . \tag{18}
\end{equation*}
$$

Substituting Equations (9) and (18) into Equation (17) gives

$$
\begin{align*}
\mathrm{V}^{v} & =\left(3 \mathrm{wB}_{\mathrm{d}} / 2\right)\left(\mathrm{h}+\mathrm{H}^{v}-\mathrm{H}_{\mathrm{e}}{ }^{\imath}\right)+\left(\mathrm{wBd}^{2} / 4 \mathrm{~K} \mu\right)\left[\exp \left(-2 \mathrm{~K} \mu \mathrm{~h} / \mathrm{B}_{\mathrm{d}}\right)\right. \\
& -1]=\left(\mathrm{wB}_{\mathrm{d}}^{2} / 2\right)\left[\left(\mathrm{H}^{v} / \mathrm{B}_{\mathrm{d}}\right)-\left(\mathrm{H}_{\mathrm{e}} / \mathrm{B}_{\mathrm{d}}\right)\right] \exp \left(-2 \mathrm{~K} \mu \mathrm{~h} / \mathrm{B}_{\mathrm{d}}\right) . \tag{19}
\end{align*}
$$

Substituting for $\mathbf{V}^{\mathbf{8}}$ in Equation (16) and integrating between $h=0$ and $h=H_{e}{ }^{8}$ gives

$$
\begin{align*}
& \frac{H_{e}{ }^{\gamma}}{B_{d}}-\frac{w B_{d}{ }^{2}}{2 E}\left[\frac{H^{0}}{B_{d}}-\frac{H_{e}{ }^{0}}{B_{d}}-\frac{1}{2 K \mu}\right] \frac{\exp \left(-2 K \mu H_{e^{8}} / B_{d}\right)-1}{-2 K \mu} . \tag{20}
\end{align*}
$$

Noting that

$$
\mathrm{E}=\mathrm{w}_{\mathrm{n}} \mathrm{~B}_{\mathrm{d}} /\left(\mathrm{s}_{\mathrm{d}} / \mathrm{p}^{9} \mathrm{~B}_{\mathrm{d}}\right)
$$

the expression for $s_{d}$ is given by

$$
\begin{align*}
& s_{d}=\left(w^{3} B_{d}^{2} / E\right)\left\{\left[\left(H^{0} / B_{d}\right)-\left(H_{e}^{0} / B_{d}\right)\right] \exp \left(-2 K \mu H_{e^{8}}^{8} / B_{d}\right)\right. \\
& \left.+\left[\exp \left(-2 \mathrm{~K}_{\mu} \mathrm{H}_{\mathrm{e}}{ }^{\vartheta} / \mathrm{B}_{\mathrm{d}}\right)-1\right] /(-2 \mathrm{~K} \mu)\right\} \text {. } \tag{2I}
\end{align*}
$$

Substituting Equations (15), (20) and (2 1) into Equation (13)
gives

$$
\begin{align*}
& {\left[\frac{\mathrm{H}^{\prime}}{\mathrm{B}_{d}}-\frac{\mathrm{H}_{e}{ }^{2}}{\mathrm{~B}_{\mathrm{d}}}-\frac{1}{2 \mathrm{~K} \mu}\right] \frac{\exp \left(-2 \mathrm{~K} \mu \mathrm{H}_{e}{ }^{\prime} / \mathrm{B}_{d}\right)-1}{-2 \mathrm{~K} \mu}=} \\
& \frac{\mathrm{H}_{e^{\prime}}}{\mathrm{B}_{\mathrm{d}}}\left[\frac{\mathrm{H}^{2}}{\mathrm{~B}_{\mathrm{d}}}-\frac{\mathrm{H}_{e^{\mathrm{a}}}}{\mathrm{~B}_{\mathrm{d}}}+\frac{1}{2} \frac{\mathrm{H}_{\mathrm{e}}{ }^{\mathrm{i}}}{\mathrm{~B}_{\mathrm{d}}}-\frac{1}{2 \mathrm{~K} \mu \mathrm{~m}}\right]= \\
& \frac{2}{3} r_{s d p}\left\{\frac{\exp \left(-2 \mathrm{~K}_{\mu} \mathrm{H}_{\mathrm{e}} \mathrm{i} / \mathrm{Bd}\right)-1}{-2 \mathrm{~K} \mu}+\right. \\
& \left.\left[\frac{H^{s}}{B_{d}}-\frac{H_{e}}{B d}\right] \exp \left(-2 K \mu H_{e^{i}} / B_{d}\right)\right\} \tag{22}
\end{align*}
$$

From Equation (22) it is possible to determine values of $H_{e}{ }^{9} / B_{d}$ for corresponding values of $H^{1} / B_{d}$. Since $H=H^{8}+p^{8} B_{d}$ and $H_{e}=$ $H_{e}{ }^{1}+p^{1} B_{d}$ it is possible to determine $C_{n}$ from Equation (10).

## POSITIVE PROJECTING CONDUITS

Referring to Fig。12, it is noted that the force diagram for positive projecting conduit is similar to that for negative projecting conduit. The force equations thus take the same form as those in the preceeding section and the expression for the load acting on the conduit is therefore found to be

$$
\begin{equation*}
\mathrm{w}_{\mathrm{c}}=\mathrm{C}_{\mathrm{c}} \mathrm{wB}_{\mathrm{c}}^{2} \tag{23}
\end{equation*}
$$

$$
\begin{align*}
\text { where } C_{c}= & {\left[\left(H / B_{c}\right)-\left(H_{e} / B_{c}\right)\right] \exp \left( \pm 2 \mathrm{~K}_{\mathrm{C}} \mathrm{H}_{\mathrm{e}} / \mathrm{B}_{\mathrm{c}}\right)+} \\
& {\left[\exp \left( \pm 2 \mathrm{~K} \mu \mathrm{H}_{\mathrm{e}} / \mathrm{B}_{\mathrm{c}}\right)-1\right] /( \pm 2 \mathrm{~K} \mu) \text { and } } \tag{24}
\end{align*}
$$

$B_{C}=$ outside width of the conduit, $f t$.

The positive signs are applicable to the incomplete projection condition while the negative signs apply to the incomplete ditch condition.

An expression for evaluating $H_{e}$ is found by equating the total settlement in the exterior prism to that in the interior prism.

Thus

$$
\begin{equation*}
\lambda+s_{f}+d_{c}=\lambda^{r}+s_{m}+s_{g} \tag{25}
\end{equation*}
$$

where $s_{m}=$ compression of the material in the exterior prism in a height $\mathrm{pB}_{\mathrm{C}}$.

Top of Embankment


Fig. 12. Force Diagram for Positive Projecting Condition

In a manner similar to that used for the negative projecting conduit, expressions are found for $\lambda$ and $\lambda$ and the resulting equation for $\mathrm{H}_{\mathrm{e}}$ is

$$
\begin{align*}
& {\left[\frac{1}{2 K_{\mu}} \pm\left(\frac{H}{B_{c}}-\frac{H_{\rho}}{B_{c}}\right) \pm \frac{r_{s d} p}{3}\right] \frac{\exp \left( \pm 2 K \mu H_{e} / B_{c}\right)-1}{( \pm 2 K \mu)} \pm} \\
& \pm \frac{r_{s d P}}{3}\left(\frac{H}{B_{c}}-\frac{H_{e}}{B_{c}}\right) \exp \left(+2 \mathrm{~K}_{\mu} H_{e} / B_{c}\right) \pm \frac{1}{2}\left(\frac{\mathrm{H}_{e}}{\mathrm{~B}_{\mathrm{c}}}\right)^{2}- \\
& \frac{1}{2 \mathrm{~K}_{\mu}} \cdot \frac{\mathrm{H}_{e}}{\mathrm{~B}_{\mathrm{c}}} \mp\left(\frac{H}{\mathrm{~B}_{\mathrm{c}}} \cdot \frac{\mathrm{H}_{e}}{\mathrm{~B}_{\mathrm{c}}}\right)= \pm \mathrm{r}_{\mathrm{sd}} \mathrm{P} \underset{\mathrm{~B}_{\mathrm{c}}}{\mathrm{H}} \text {. } \tag{26}
\end{align*}
$$

Here the upper signs apply to the incomplete projection condition and the lower signs to the incomplete ditch condition. Equation (26) provides a means of obtaining values of $\mathrm{H}_{\mathrm{e}} / \mathrm{B}_{\mathrm{c}}$ to be used in Equation (24).

## TRENCH CONDUITS

Referring to Fig. 13, the forces acting on an element of the material in the trench can be summed in the following manner:

$$
\begin{equation*}
V+d V=V+w B_{d} d h-\left(2 K \mu^{i} V / B_{d}\right) d h \tag{27}
\end{equation*}
$$

where $\mu^{i}=$ coefficient of friction between the backfill material and the original soil.

It is noted that Equation (27) has the same form as Equation (1) of this Appendix and thus the desired solution of Equation (27) is given by

$$
\begin{equation*}
\mathrm{W}_{\mathrm{c}}=\mathrm{C}_{\mathrm{d}} \mathrm{wB}_{\mathrm{d}}{ }^{2} \tag{28}
\end{equation*}
$$

where $C_{d}=\left[\exp \left(-2 K \mu^{9} H / B_{d}\right)-1\right] /\left(-2 K \mu^{9}\right)=$ load coefficient.


Fig. 13. Force Diagram for Trench Condition.

## APPENDIX II

Kentucky Department of Highways Standard Drawings Showing Pipe Bedding Details and Allowable Fill Heights


Fig. 15. Kentucky Allowable Fill Heights.

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Fig. 16. Pipe Bedding being Prepared. Undesirable material was removed and replaced with selected compacted backfill. Section with Standard B bedding is in the background, $B_{l}$ bedding in the foreground.


Fig. 17. Patrol Grader with Special Blade Preparing Backfill for Laying Pipe.


Fig. 18. Close-up of Special Blade on Patrol Grader.


Fig. 19. Placing Pipe in Sand Cushion Bedding.


Fig. 20. Placing Backfill before Construction of the Imperfect Trench.


Fig. 21. Construction of the Imperfect Trench.


Fig. 22. Loose Backfilling of Imperfect Trench. Straw placed to a depth of 24 inches.


Fig. 23. Completion of the Imperfect Trench. Embankment conditionduring the winter.

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