SKID TESTING WITH AN AUTOMOBILE

by

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CONTENTS

ABSTRACT

A study was conducted to evaluate the theoretical and practical aspects of using an automobile as a testing device for measurement of pavement slipperiness. Every parameter and significant event in the excursion history of a skidding automobile was measured and recorded. The resultant skid-resistance values, twenty-five in all, were compared and correlated. As a result of the study, the measurement of time in the velocity increment between 30 mph to 20 mph was selected as an interim standard test. A number of experiments were also conducted to aid in the interpretation of test results and to establish control tolerances for the standard test. The British Portable Tester was further evaluated on various roads common to Kentucky and was found to have limited usefulness.

INTRODUCTION

Slippery roads, as all experienced drivers know, are fraught with peril and treachery. The frictional or tractional stability between automobile tires and pavement surfaces have long been important factors in the design of Highways. Improved pavement surfaces and various types of de-slicking treatments have emerged as a result of inquiring studies of the skidding mechanisms and skid testing. Unfortunately, the test methods employed in the past have not been wholly reliable and realistic. Considerable effort has been devoted to the development of better methods of skid-resistance testing and to the standardization of testing devices. The trailer method--which yields a constant velocity measurement of friction--has received a great deal of attention because it is seemingly reliable and safe. However, a standard trailer-type testing device is not yet commercially available.

The Kentucky Department of Highways has been engaged in laboratory skid-resistance studies since 1956 (9)* and in field testing since 1958 (10, 11, 12). In the course of these studies, numerous methods of measurement have been employed in the laboratory as well as on the road. Several methods

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*Numbers in parentheses refer to references at end of report.

have been abandoned because they yielded unreliable results. Some other methods have been continued temporarily until a more reliable test was found. As a consequence, meaningful, accurate, long-term histories of pavement surfaces have not been accumulated. Hence, a standard method of testing has been--and is--needed. Certainly, any method of test should correlate well with the coefficients of friction derived from real skidding excursions of an automobile; skidding deceleration are fundamentally more complex than steady-state friction (constant-velocity friction); it is the fundamental aspects of skidding decelerations with which this report is concerned. The purpose of the study was to develop a standard method of skid test using an automobile.

In the summer of 1964, a test automobile was instrumented to record the various parameters associated with a vehicle in skid. Five pavements having different compositions and skid resistances were selected as test sites to represent a typical array of road surfaces found in Kentucky. The sites were tested in series at three different times. Every possible skid-resistance value and coefficient of friction, twenty-five in all, was then determined from the resultant recordings and "on the spot" measurements. The test results were compared, and regression equations were determined in order to correlate the results. A number of experiments were also conducted to aid in the interpretation of test results and to establish control tolerances for a standard test. In conjunction with the skid test automobile, the British Portable Tester was used to further evaluate the instrument on fourty-five pavement surfaces of varying types and ages.

THEORETICAL CONSIDERATIONS

CLASSICAL LAWS

The study of friction phenomena dates back to 1500, when Leonardo Da Vinci noted that friction between two solid bodies is proportional to the load and independent of the apparent contact area. Additional study was done by Amontons in 1700 and by Coulomb and Morin in 1781. Their findings form the classical laws of dry friction which may be stated as follows:

1. Friction is independent of the apparent contact area and the load.
2. The static coefficient of friction is greater than the kinetic coefficient of friction.
3. The kinetic coefficient of friction is independent of velocity.

Most materials do not obey these classical laws, especially viscoelastic materials. Kummer and Meyer (1) have shown that the coefficient of friction of rubber is dependent on normal pressure (load and contact area), velocity and temperature. Their studies also indicated that the highest coefficient of friction does not occur at rest but at a sliding velocity of 0.1 to 5 in. per second.

MECHANISMS OF FRICTION

Classical laws explain nothing about the mechanism of friction. Generally speaking, friction is regarded phenomenalistically--that is, it can be observed and measured but not explained. Coulomb's law, $F = fN$, is phenomenalogical in that sense; and f is considered to be a phenomenalogical coefficient. However, other physical laws provide additional insight and understanding; in the case of the deceleration of an inertial body for instance, the doctrine of conservation of energies may be invoked:

Kinetic energy (loss) = Mechanical energy (loss) + Heat

Heat arises from inter- and intra-molecular straining (or internal friction); it is irreversible and is known as a hysteretic loss.

Static friction is conceived as the interlocking (or mating) of surface asperities. Sliding friction involves the inherent shear-resistance of the materials and is a function of discrete interfacial pressures and bearing areas. Interfacial welding, or adhesion, has been suggested as a mechanism. Anti-friction mechanisms, such as fluid lubrication of the interface, are infinitely complex.

Tire friction (traction) is thus not altogether definable in terms of discrete mechanistic parameters; however, some general observations may be cited:

1. Coarse wear, or abrasion, is thought to be a combination of ploughing, tearing, and shearing (filing and rasping); this action would account for the deposition of skid marks if the rubber were powdery and non-adherent to the pavement.

2. Adhesion of skid-deposited rubber would strongly indicate melting at the surface of contact. Rubber tends to become tacky from the hysteretic heating accompanying severe abrasion. Melting and tackiness may result from surface heating and drying - even when the surfaces are wet.

3. "Scratching off" burns tire rubber--due to hysteresis heating--and has been observed on ice and under water.

4. Normal wear on tires may involve vaporization of rubber because rubber debris does not accumulate on roads in direct proportion to tire wear.

5. Wet friction is usually less than dry friction due to lubrication and hydrostatic pressures. Captive water in surface cavities interferes with the mating of the surfaces.

COEFFICIENT OF FRICTION EQUATIONS

All coefficients of friction in this study were calculated from three different equations that incorporate various measured physical parameters. These equations were derived on the assumption that Coulomb's Law ($F = fN$)--the friction forces "F" is constant and proportional to the normal force "N"--is applicable. The proportionality is expressed by the coefficient of friction, f. This low, however, applies only to dry surfaces, low speeds, and low contact pressures (6). Visco-elastic materials, such as rubber, do not fully adhere to the classical concept of friction. Many other tire, vehicle, and road characteristics influence f; in fact, Meyer lists over thirty variables (4).

In the application of these equations, the tire and vehicle characteristics, as well as all other influencing variables, are assumed to be constant--except when intentionally varied--and the road surface friction is expressed by f. The resultant f denotes a relative skid-resistance value, but will be referred to in this report as a coefficient of friction. This skid-resistance value may be subject to large "incremental" errors arising from the assumption that a linear relationship exists between velocity and time or velocity and distance while the automobile is skidding. Actually such a relationship does not exist, and f (calculated) is higher than the actual f at the midpoint of the velocity increment; that is, the average f calculated for the increment $V_1 - V_2$ will be greater than f for $V_2 + \frac{V_1 - V_2}{2}$, where V_1 and V_2 are velocities in m.p.h.;

a) <u>Skidding Distance</u>. The work-energy principle of physics states: "The work of the resultant force on a body is equal to the change in kinetic energy of the body"*. For a skidding vehicle, the

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*Richards, Sears, Wehr, and Zemansky, <u>Modern University Physics</u>, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1960, p. 129.

resultant force is the friction force, F_e, and:

$$F_e S = 1/2\ mv_1^2 - 1/2\ mv_2^2 \qquad (1)$$

When S is the distance in feet, the vehicle, skids while decelerating from a velocity of v_1 to a velocity of v_2 ft./sec. and m is the mass of the vehicle.

Coulomb's law of friction defines the friction force as a function of the normal force:

$$F_e = fN \qquad (2)$$

For a skidding vehicle, the normal force is equal to the weight of the vehicle, W, substituting in equation (2):

$$F_e = fW \qquad (3)$$

Combining (1) and (3) equates frictional energy to the change in kinetic energy; then, substituting $\frac{w}{g}$ for m, where g is the acceleration due to gravity:

$$fws = 1/2\ \frac{w}{g}\ v_1^2 - 1/2\ \frac{w}{g}\ v_2^2$$

which simiplifies to:

$$f = \frac{v_1^2 - v_2^2}{2gS} \qquad (4)$$

Multiplying by 1.47^2 to allow substitution of V in mph and substituting 32.2 ft./sec.^{2} for g, equation (4) becomes:

$$f = \frac{V_1^2 - V_2^2}{30S} \qquad (5)$$

b) <u>Skidding Time</u>. The skidding distance, S, can be expressed as the product of the average velocity and the time in skid, or:

$$S = 1/2(v_1 + v_2)(t_2 - t_1) \qquad (6)$$

where t_1 is the time at the start of measurement, in sec., and t_2 is the time at the end of measurement, in sec. The initial and terminal velocities for t_1 and t_2 are v_1 and v_2 respectively and are in ft./sec.

Substituting the above value for S, equation (4) can be written as:

$$f = \frac{(v_1 + v_2)(v_1 - v_2)}{1/2(2g)(v_1 + v_2)(t_2 - t_1)} \qquad (7)$$

For which:

$$f = \frac{v_1 - v_2}{g(t_2 - t_1)} \qquad (8)$$

Multiplying by 1.47 to allow substitution of V in m.p.h. and substituting 32.2 ft./sec.^{2} for g, equation (8) becomes:

$$f = \frac{0.0456\ (V_1 - V_2)}{t_2 - t_1} \qquad (9)$$

c) <u>Deceleration</u>. The equation for a force due to acceleration (or deceleration), a, is:

$$F = ma = \frac{w}{g} a \qquad (10)$$

Combined with equation (3):

$$fW = \frac{w}{g} a$$

or:

$$f = \frac{a}{g} \qquad (11)$$

To determine the average coefficient over a time interval, it is necessary to integrate f with respect to time. Then the effective coefficient of friction, f_e, is:

$$f_e = \frac{\int_{t_1}^{t_2} (a/g)_t \, dt}{\int_{t_1}^{t_2} dt} \qquad (12)$$

CORRELATION STUDY

Many testers are used today to obtain a friction measurement between tire and pavement. The mode of operation of the testers varies, but the various modes can generally be divided into three groups: 1) steady-state sliding, 2) non-steady-state sliding, and 3) steady-state slip (8). The steady-state sliding group includes all testers which measure the sliding coefficient at a constant velocity--such as the towed trailer testers. The non-steady-state sliding group, also referred to as energy devices, operate on the principle of converting kinetic or potential energy into frictional energy during the test. These devices usually measure a mean coefficient over a velocity range. A skidding, decelerating automobile would be included in this category since it is converting the kinetic energy of the automobile into frictional energy. The British Portable Tester, another example of this group, converts potential energy into kinetic energy, then into frictional energy. The steady-state slip group includes testers which operate at a constant rate of slip with respect to the pavement surface--most of these testers are found in Europe.

The coefficients obtained by friction-testing devices are largely dependent on the mode of operation and may not be directly comparable since rubber friction is dependent on speed and accompanying temperature changes. The coefficient of friction is therefore not an absolute number and may best be regarded as a performance value. This does not mean that a specific coefficient of friction does not exist between a given tire and pavement surface for specific test conditions. On a given pavement, under identical test conditions, the coefficient should be reproducible, either by the particular tester involved or by a similar tester.

Coefficients obtained by means of a skidding automobile may be determined by using one of three combinations of measurements--velocity and distance, velocity and time, or deceleration. Using any one of these, several coefficients can be determined: mean f, f for a velocity increment, and f at a specific velocity in the case of a deceleration measurement. To examine the various coefficients and the practical aspects of measuring them, a correlation study was conducted in the summer of 1964. This study had the following purposes:

1) Compare theoretically similar coefficients obtained from measurements of different parameters.

2) Determine repeatability of tests.

3) Correlate dissimilar coefficients.

4) Select a standard test.

TEST SITES

The five pavements selected for use in the correlation study provided a wide range of skid resistance values as shown below and as illustrated in Figure 1. Other criteria used in the selection of the test sites were accessibility, gradient, surface uniformity, and safety to testing personnel.

Pertinent information concerning the test pavements is given in the following tabulation:

TEST SITE	ROUTE NO.	LOCATION	TYPE OF PAVEMENT	f_v(30-20)
1a	KY 89	Winchester-Irvine	Chip-Seal	0.33
1b	I 64	Winchester-Mt. Sterling	Bituminous	0.47
2	US 60	Frankfort-Shelbyville	Bituminous	0.40
3	US 25	Georgetown-Corinth	Bituminous	0.54
4	I 64	NE C. L. of Lexington	Concrete	0.57
5	US 62	Lawrenceburg-Bloomfield	Ky. Rock Asp.	0.70

Each site was tested at 40 mph in three rounds during a two-month period. Site 1a, however, was tested at 35 mph. because of the inherent danger of skidding on a slippery surface having a large cross-slope. Whenever coefficients at a higher test velocity were required, Site 1b was substituted for 1a. Five tests per site were conducted in Round I and ten in Rounds II and III. Unfortunately, Site 3 was resurfaced before testing in Round III could be carried out; therefore, the test results for Stie 3 are based on measurements in Rounds I and II only.

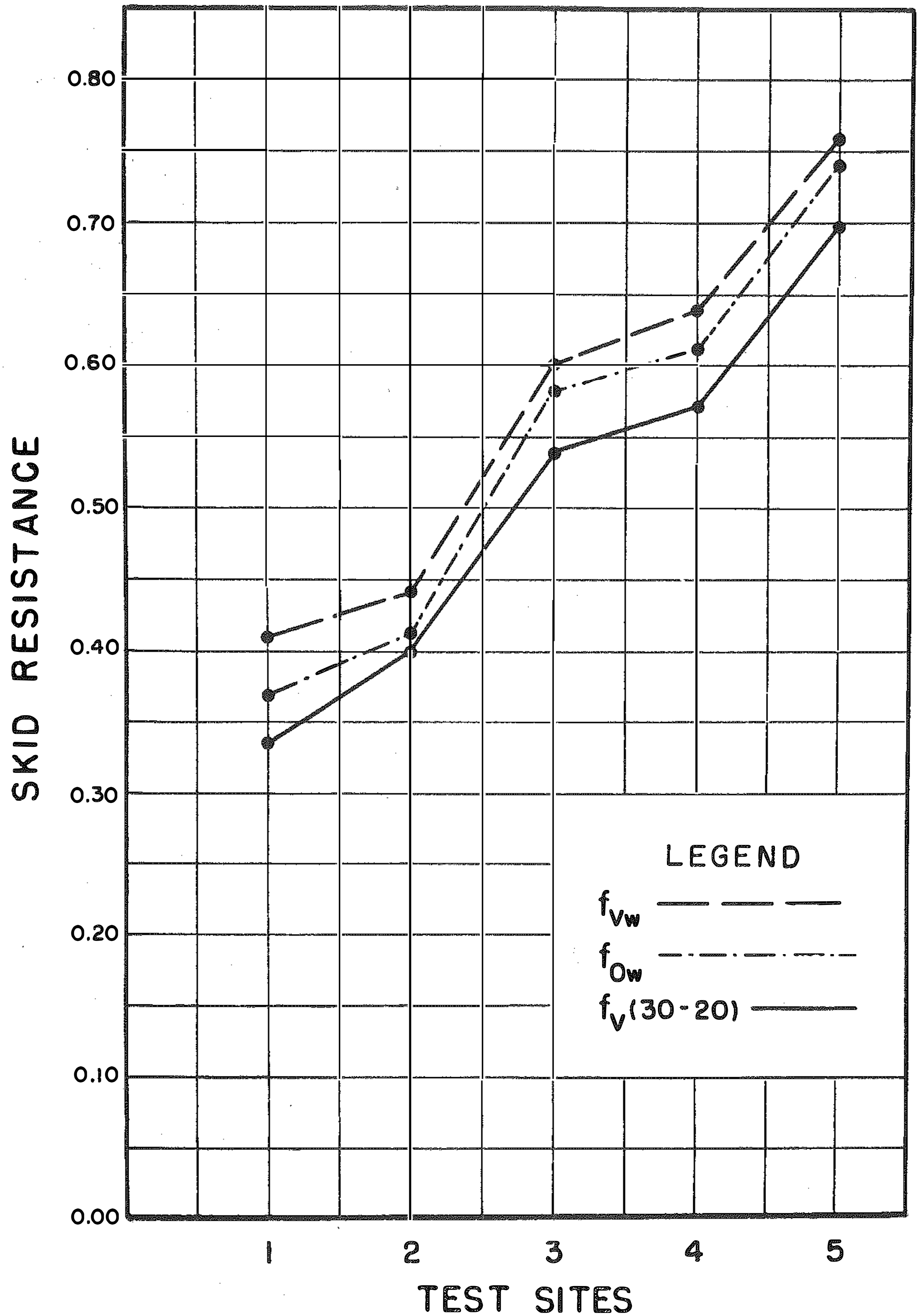

Figure 1. A Plot of Several Skid-Resistance Values vs Test Sites.

INSTRUMENTATION

The test vehicle, a 1962 Ford sedan, was instrumented to record time, distance, velocity and deceleration. Also recorded were: brake application, brake light energization, and wheel rotation. A block diagram of the instrumentation is shown in Figure 2, and accomodation of the equipment in the automobile is shown in Figure 3. A brief description of the manner in which each of the parameters and events were detected and recorded follows:

Time: A Sanborn recorder, operated at a chart speed of 100 mm./sec., permitted measurement of very small increments of time. Since chart speed is inversely proportional to the frequency of the A. C. power supply, it was necessary to monitor the frequency of the inverter and to appropriately correct the measurement of time. A vibratory-reed-type frequency meter was used for this purpose.

Distance: A magnetic counter, cam-operated micro-switch on the fifth wheel, summated the skidding distance. The total count represented the distance measured from the instant power was provided for brake lights to the point where the vehicle came to rest--each full count being equivalent to 1.32ft. The position of rotation

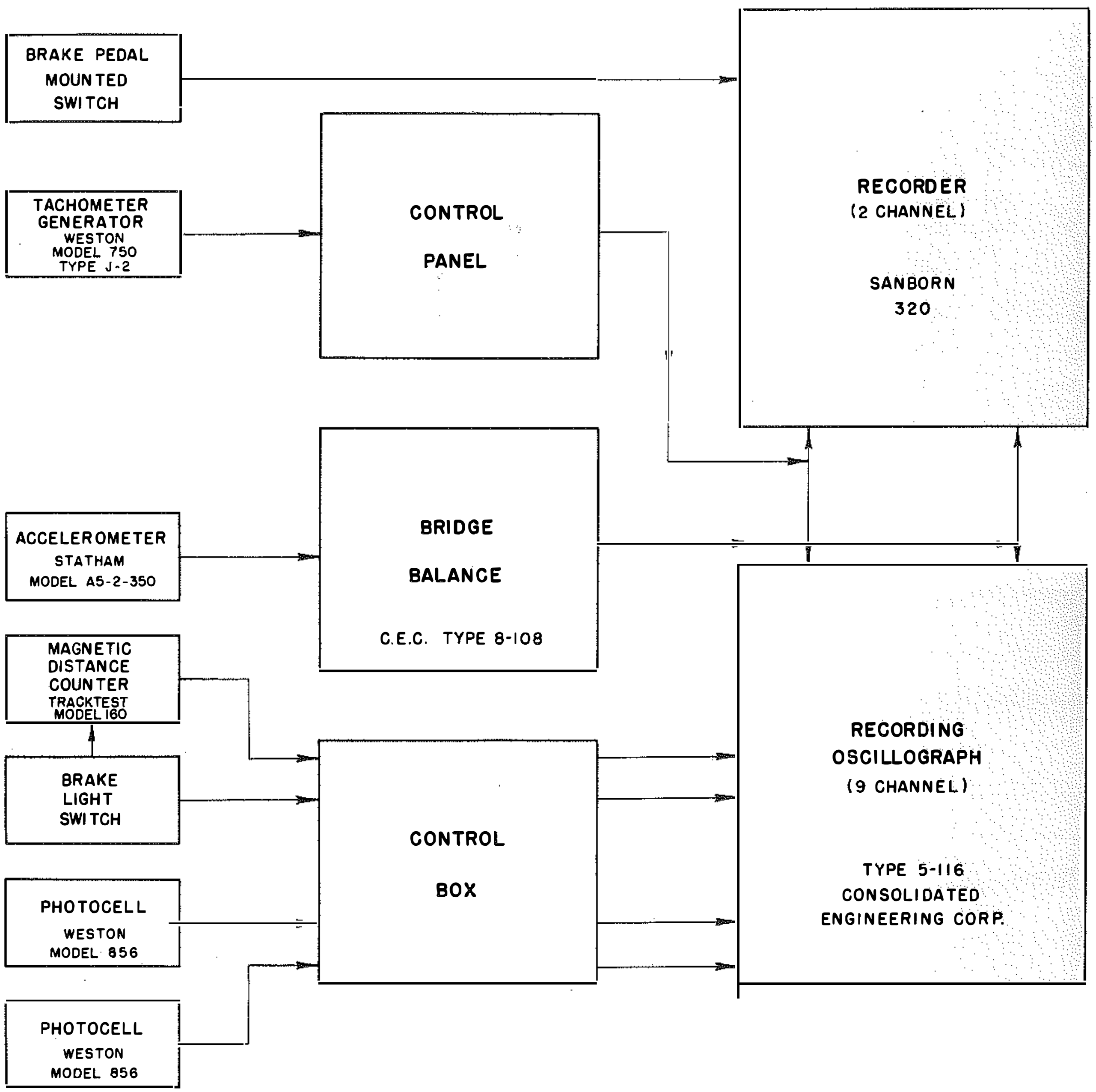

Figure 2. Block Diagram of Skid-Resistance Measuring System.

of the cam was unknown for any given test; thus a maximum error of one count could result. The operation of the micro-switch was recorded on a 9-channel, Consolidated Engineering, recording oscillograph. The resultant rectangular wave, representing one count per cycle, could then be counted on the oscillographic chart.

The "observed" stopping-distance, from the approximate point of wheel-lock to where the vehicle stopped, was measured with a metallic tape. Figure 4 is a photograph of the test vehicle and shows the measurement of "observed" stopping-distance in progress.

Velocity: A tachometer generator, mounted on the axle of the fifth wheel, was used in conjunction with a Weston, Model 910, Speedmeter to indicate test velocity. The output of the tachometer generator was recorded by both the Sanborn and the C. E. recorders.

Deceleration: A Statham, ±2G, resistive-type, horizontally sensitive accelerometer was used to detect deceleration. A C. E. Wheatstone bridge balance was used to balance and to calibrate the accelerometer. The bridge voltage was recorded by both the Sanborn and the C. E. recorders.

Brake Application: A push-button switch, mounted in the brake pedal (Figure 5), activated an event marker in the Sanborn recorder to indicate the instant of brake application.

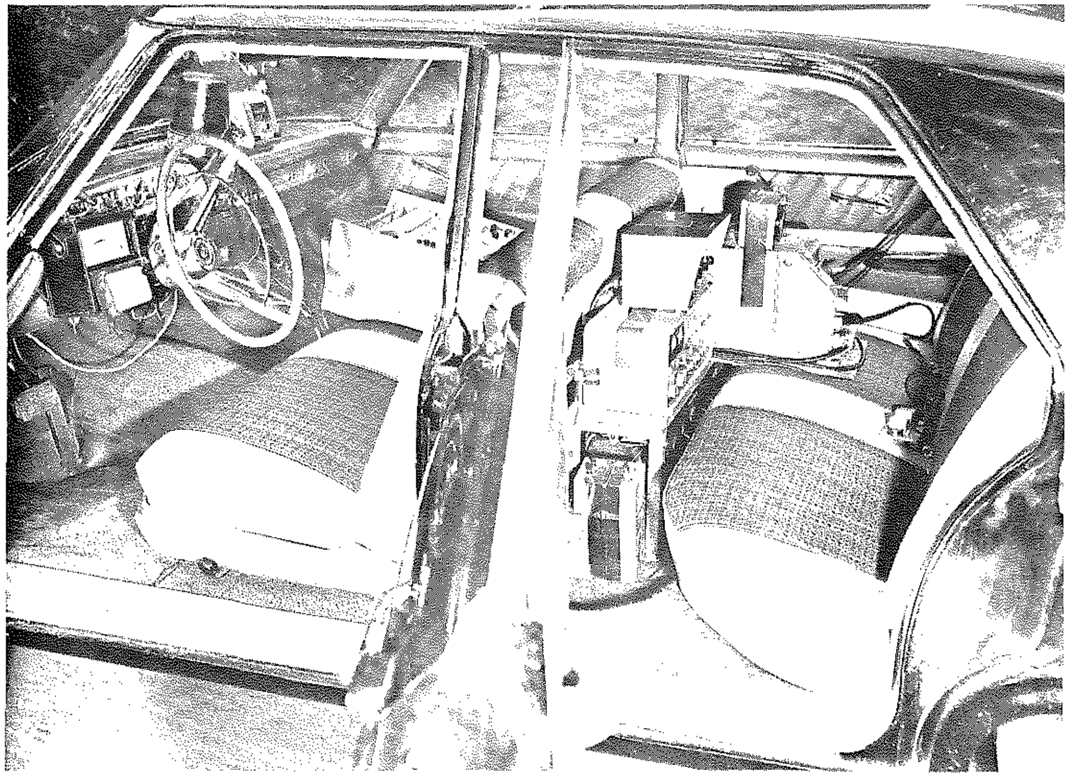

Figure 3. Accomodation of Equipment in the Test Vehicle.
Front: Weston 901 Speedometer, Magnetic Counter and Sanborn Recorder.
Back: C. E. Recording Oscillograph and Balance, Control Panel, Frequency Meter, D. C. Power Supply and Accelerometer.

Figure 4. Test Vehicle During Measurement of "Observed" Stopping Distance.

Brake Lights: Voltage on the pressure-activated brake-light switch in the brake master cylinder was recorded by the C. E. recorder.

Wheel Rotation: The rear wheel rotation was monitored to determine when the tires were fully skidding. Weston photocells, mounted in black, water-proof tubes, 6 in. in length, were clamped on the fender of the vehicle (Figure 6) and aimed at the tires.

PROCEDURES

a) Skid Test: All skid measurements were made using ASTM, E-17, Standard Tires inflated to 24 p.s.i. The front suspension of the vehicle was partially neutralized at the test site by placing 5-in. wood blocks, padded with 3/4 in. of rubber at both ends, near the coil springs.

The fifth-wheel speedometer was accurately calibrated on a two-mile section of Interstate highway. The magnetic distance counter was also found to be reliable for the purpose of speed calibration. The velocity calibration of the Sanborn and C. E. recorders was then based on the accurately calibrated Weston, Model 910, Speedmeter.

Two operators were required in the vehicle during the test. The driver's responsibilities were to monitor test speed and to operate the Sanborn recorder; the other operator,

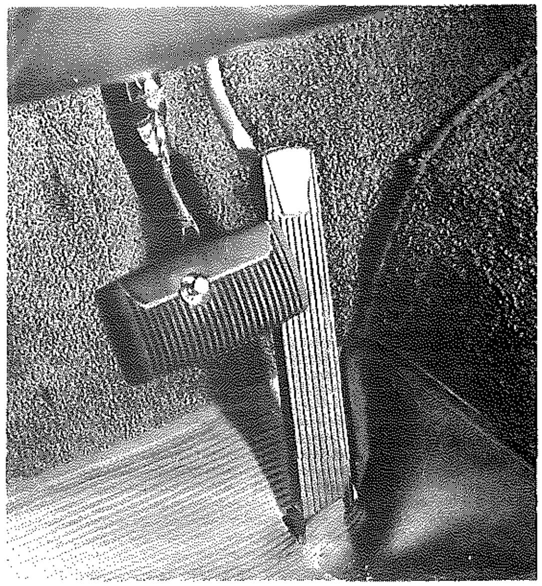

Figure 5. Push-Botton Switch Mounted in Brake Pedal.

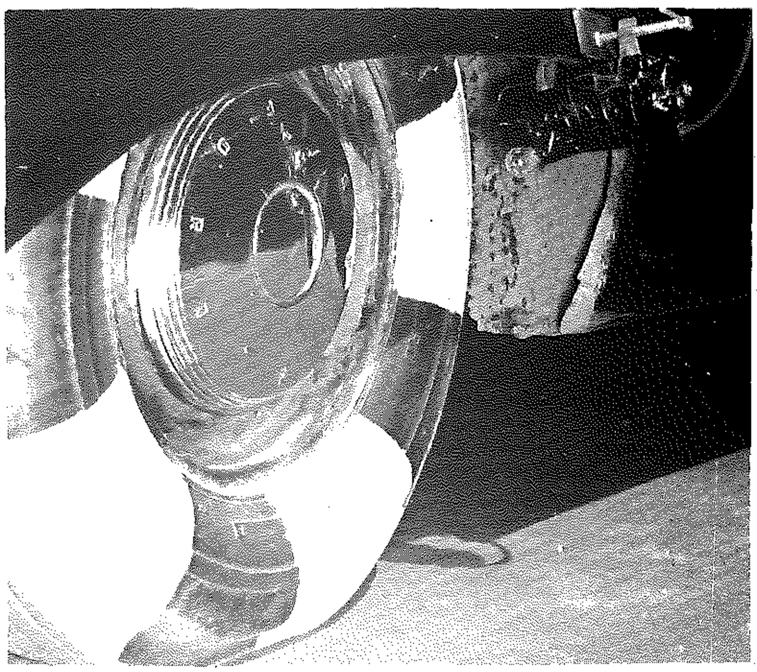

Figure 6. Photocell Aimed At the Test Tire.

seated in the back seat, calibrated and checked all the equipment, operated the C. E. recorder, and monitored the power supply frequency; he also held a platform-mounted accelerometer, slightly angled to compensate for vehicle tilt, during the period of testing.

After traffic control had been established, a 1-1/2-ton, GMC, water truck, equipped with spray bar and water pump, wetted the pavement in the test lane. Two or three applications of water were required before testing, and one re-wetting was required for every two or three repeat tests. The beginning of each successive test was advanced approximately 10 ft. to minimize skid overlaps. The wetted pavement could be described as well-saturated--i.e., surface cavities filled with water until runoff resulted.

Sufficient starting distance preceded the test section to permit the vehicle to attain the desired speed. The vehicle was accelerated to above test speed, the recorders turned on and the transmission placed in neutral. The last two maneuvers, executed a few seconds before brake application, insured a steady A. C. power supply throughout the test and permitted the recorders to attain the desired chart speed. At the appropriate velocity, the vehicle brakes were applied quickly and firmly to facilitate rapid wheel lock. During the skid, the vehicle was guided so as to remain in the wheel tracks.

Immediately after the completion of a skid, the recorders were turned off; and the power supply frequency, magnetic counter and "observed" stopping distances, velocity at brake application, etc., were recorded.

b) Coefficient Determination: The Sanborn and the C. E. recorder charts, illustrated in Figures 7 and 8 respectively, were carefully analyzed to obtain the various recorded parameters. Both charts display velocity and deceleration, but only the Sanborn chart was used to obtain the readings of recorded velocity. The deceleration curves were used to transpose a particular instant during the skid from one chart onto the other. The Sanborn chart was used to measure time, velocity, and deceleration, and to determine brake-pedal application. The C. E. chart provided a record of distance skidded, the instant the brake-light was energized, and the time of wheel-lock.

The following procedure was used in arriving at the individual coefficients using the data derived from chart analysis and "on-the-site" measurements:

f_{O_o} - coefficient, computed from equation 5, obtained from measurement of "observed" stopping distance and meter-indicated velocity at the instant of brake application.

f_{O_w} - coefficient (eq. 5) obtained from the measurement of "observed" stopping distance and the actual (Sanborn chart) velocity at wheel-lock.

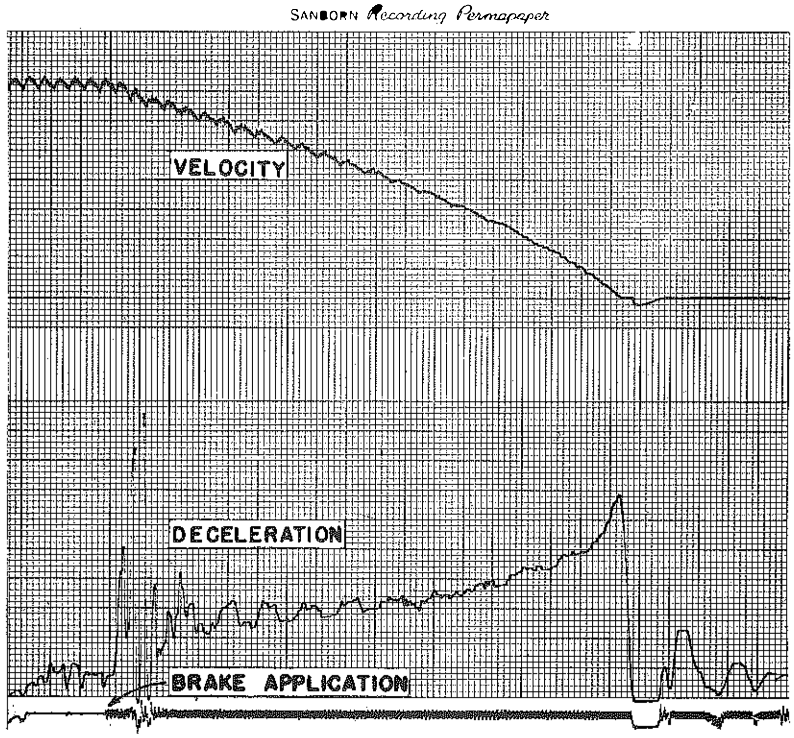

Figure 7. Example of Sanborn Recording Taken on Site 1, at Reduced Chart Speed.

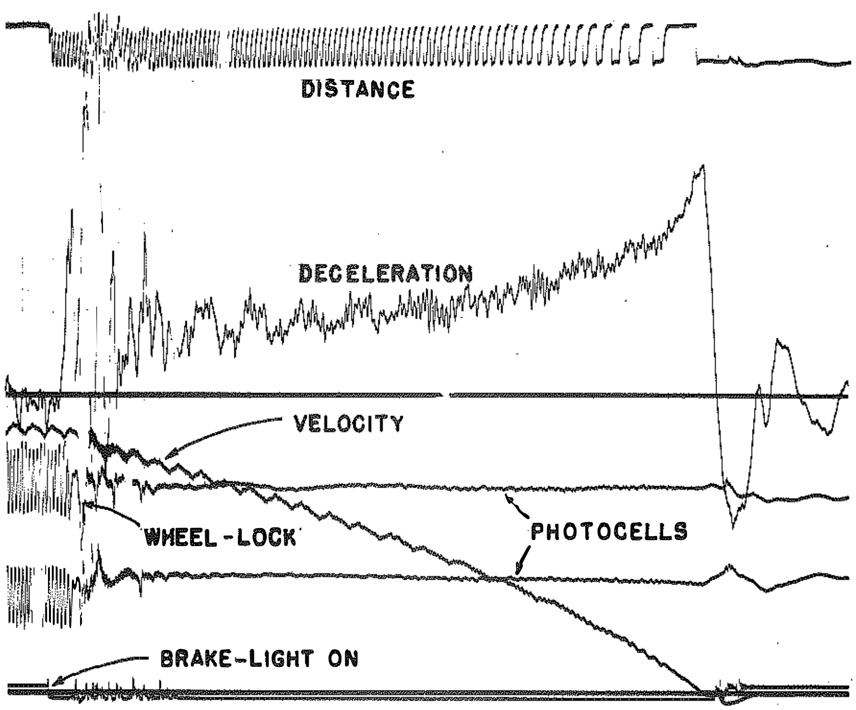

Figure 8. Example of C.E. Recording Taken on Site 1, at Reduced Chart Speed.

f_{M_o} - coefficient (eq. 5) obtained from measurement of magnetic-counter-indicated stopping distance and meter-indicated velocity at the instant of brake application.

f_{M_1} - coefficient (eq. 5) obtained from measurement of magnetic-counter-indicated stopping distance and actual velocity at the instant brake-light was energized.

f_{M_w} - coefficient (eq. 5) obtained from measurement of skid distance by counting impulses of the imput to magnetic counter on the C. E. chart, in the velocity increment between wheel-lock and 0 m.p.h.

$f_{M(30-0)}$ - coefficient (eq. 5) obtained from measurement of skid distance by counting impulses of the imput to magnetic counter in the velocity increment between 30 m.p.h. and 0. m.p.h.

$f_{M(20-0)}$ - coefficient (eq. 5) obtained from measurement of skid distance by counting impulses of the imput to magnetic counter in the velocity increment between 20 m.p.h. and 0 m.p.h.

$f_{M(10-0)}$ - coefficient (eq. 5) obtained from measurement of skid distance by counting impulses of the imput to magnetic counter in the velocity increment between 10 m.p.h.and 0 m.p.h.

$f_{M(V_w-30)}$ - coefficient (eq. 5) obtained from measurement of skid distance by counting impulses of the imput to magnetic counter in the velocity increment between wheel-lock and 30 m.p.h.

$f_{M(30-20)}$ - coefficient (eq. 5) obtained from measurement of skid distance by counting impulses of the imput to magnetic counter in the velocity increment between 30 m.p.h. and 20 m.p.h.

$f_{M(20-10)}$ - coefficient (eq. 5) obtained from measurement of skid distance by counting impulses of the imput to magnetic counter in the velocity increment between 20 m.p.h. and 10 m.p.h.

$f_M(V_w-10)$ - coefficient (eq. 5) obtained from measurement of skid distance by counting impulses of the imput to magnetic counter in the velocity increment between wheel-lock and 10 m.p.h.

f_{V1} - coefficient (eq. 9) obtained from measurement of elapsed time in the veloctiy increment between brake-light energization and 0 m.p.h.

f_{Vw} - coefficient (eq. 9) obtained from measurement of elapsed time in the velocity increment between wheel-lock and 0 m.p.h.

$f_{V(30-0)}$ - coefficient (eq. 9) obtained from measurement of elapsed time in the velocity increment between 30 m.p.h. and 0 m.p.h.

$f_{V(20-0)}$ - coefficient (eq. 9) obtained from measurement of elapsed time in the velocity increment between 20 m.p.h. and 0 m.p.h.

$f_V(10-0)$ - coefficient (eq. 9) obtained from measurement of elapsed time in the velocity increment between 10 m.p.h. and 0 m.p.h.

$f_{V(V_w-30)}$ - coefficient (eq. 9) obtained from measurement of elapsed time in the velocity increment between wheel-lock and 30 m.p.h.

$f_{V(30-20)}$ - coefficient (eq. 9) obtained from measurement of elapsed time in the velocity increment between 30 m.p.h. and 20 m.p.h.

$f_{V(20-10)}$ - coefficient (eq. 9) obtained from measurement of elapsed time in the velocity increment between 20 m.p.h. and 10 m.p.h.

$f_V(V_w-10)$ - coefficient (eq. 9) obtained from measurement of elapsed time in the velocity increment between wheel-lock and 10 m.p.h.

f_{Dw} - average coefficient obtained from measurement of area under the deceleration curve on the Sanborn recording between wheel-lock and 0 m.p.h. divided by the corresponding chart length (eq. 12).

$f_{D(15)}$ - coefficient obtained from measurement of deceleration at 15 m.p.h. by interpolating the deceleration trace between 12 and 18 m.p.h. (eq. 11).

$f_{D(25)}$ - coefficient obtained from measurement of deceleration at 25 m.p.h. by interpolating the deceleration trace between 22 and 28 m.p.h. (eq. 11).

$f_{D(35)}$ - coefficient obtained from measurement of deceleration at 35 m.p.h. by interpolating the deceleration trace between 32 and 38 m.p.h. (eq. 11).

Reference to Figure 9 may be helpful in visualizing what measurements were used in the computation of some of the skid-resistance values.

TEST RESULTS AND DISCUSSION

The various skid-resistance values determined in the correlation study are presented in Table I, Appendix A, according to the parameters used in their computation. All subsequent tables may also be found in Appendix A.

The most obvious observation that can be made about the three groups of skid-resistance data--other than the fact that skid resistance varies with velocity--is that the corresponding coefficients were generally quite different. The coefficients determined from the velocity and time measurements were larger than the coefficients computed from the velocity and distance measurements for the same interval of skid. This is not surprising because the equations used do not properly describe the relationship between the measured parameters. The only exceptions were the coefficients determined over small increments of velocity above 10 m.p.h. using the velocity, distance and time measurements. The closeness of these coefficients must be attributed to the nearly linear skid-characteristics of the pavements at those velocities and to the accuracy of the velocity, distance and time measurements.

The error due to the nonlinearity of the coefficient-velocity relationship in the velocity increment measurements was determined for one-hundred skid tests of low-coefficient surfaces. Velocity and time were accurately measured in the velocity increments of 30 to 20 m.p.h. and 27.5 to 22.5 m.p.h. The approximate coefficient at 25 m.p.h. was found to be about 1.8% less than f_V(30-20). This difference reflects the "incremental" error which has been discussed elsewhere in the report. The "incremental" error, which also applied to coefficients f_0 and f_M, increases as the velocity increment widens and is dependent of course, on the extent of the non-linearity of the coefficient-velocity relationship. Particularly susceptible to this error are the coefficients f_V, as evidenced in the tabulation of percent differences between corresponding coefficients f_M, in Table II.

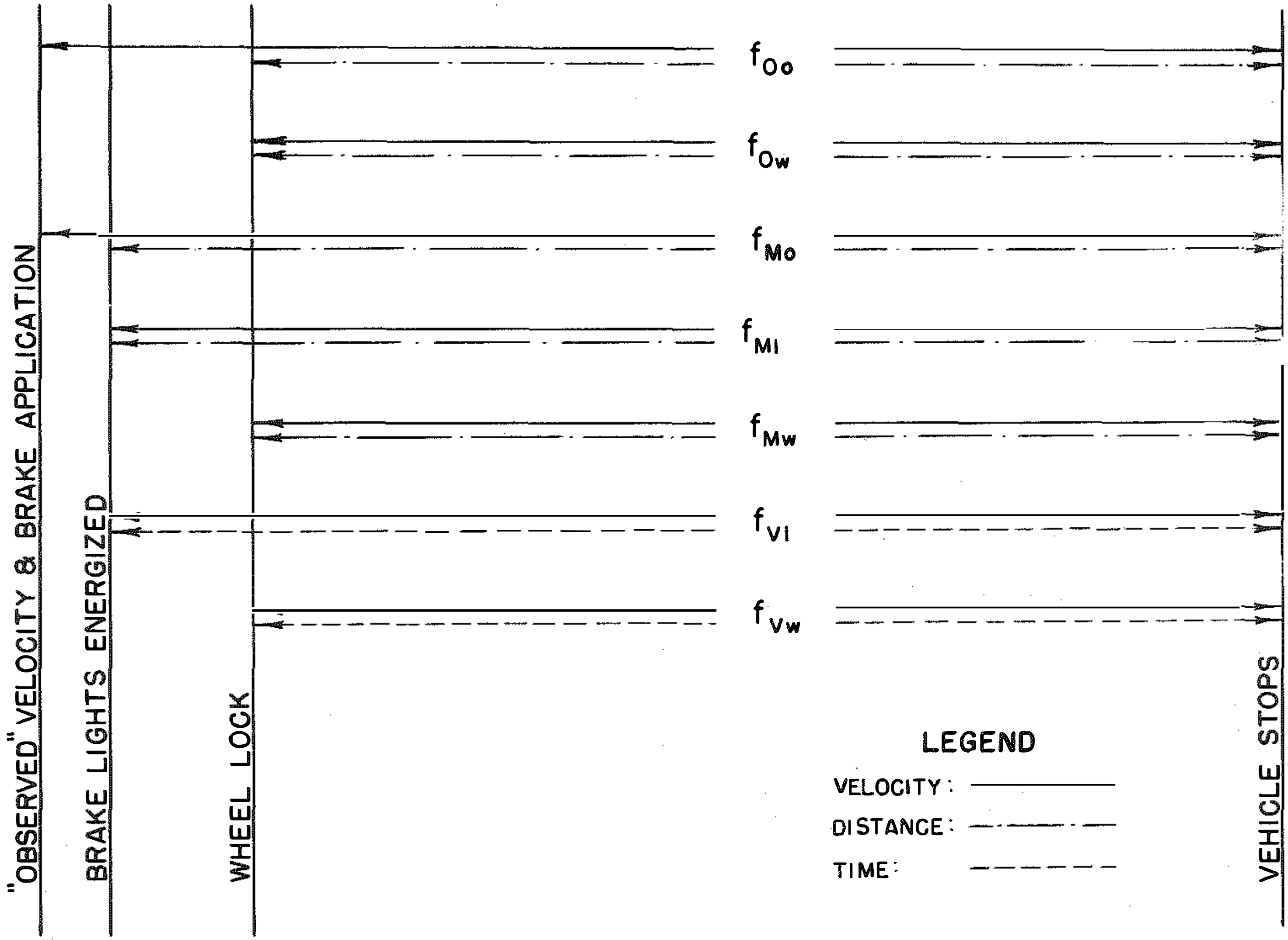

Figure 9. Illustration of Measurements Made for Determination of Several Skid-Resistance Values.

A number of different measurements were made to determine skid-resistance values using the stopping-distance equation. The coefficient f_{0o}, determined from the measurement of the "observed" stopping distance and "observed" velocity at brake application, was slightly higher than f_{0w} which was based on the actual velocity at wheel-lock. This variation was caused by the difference between the observed velocity and velocity at wheel-lock. The accuracy of the observed velocity, which was later checked on the velocity recording, was found to be biased. This was probably caused by the driver viewing the meter movement at an angle while carrying out other tasks during the test. The coefficients f_{0w} and f_{Mw} should be identical since the same velocity measurement was used in their calculation; however, a difference of 3.1% resulted; and this was attributed to error in the measurement of "observed" stopping distance, which was usually less than the actual skid distance. Because of this error in the distance measurement and inasmuch as the velocity at brake application was used, f_{0o} was much larger than f_{Mw}; but, of course, f_{Mw} represented the correct measurement of velocity and distance. Coefficients f_{Mo} and f_{Ml} were determined from the measurement of distance with the magnetic counter utilizing the "observed" velocity at brake application and the recorded velocity at the moment of brake-light energization, respectively. The difference between these coefficients was quite small because the velocities were quite similar.

The repeatability of a particular skid-resistance measurement was judged largely on the basis of the standard deviation of the tests made in Round 3. The pavement and the magnitude of the skid resistance, as well as instrumentation errors, influence the standard deviation; therefore, careful examination of the data in Table IV is warranted. The influence of the pavement is evident on Site 3 where the standard deviation shows a significant deviation from the trend of the other sites. This pavement was extrememly pitted, and the non-homogeneous surface was the apparent cause of the deviation. The magnitude of the skid resistance affected the standard deviation; that is: the standard deviation increased as skid-resistance increased. The standard deviation was used to determine the number of tests required to achieve a desired degree of accuracy. The number of required tests for a few selected coefficients is presented in Table III. The complete mathematical procedure used in the statistical analysis of the data is presented in Appendix B.

Further examination of the standard deviations reveals that the most repeatable test results were obtained when the largest velocity increment was chosen for the computation.

There are three main reasons for this: first, the larger the velocity increment, the more accurate are the velocity, distance and time measurements; second, influences due to variability of skid resistance below 10 m.p.h. are minimized; third, errors due to premature or delayed front-wheel locking and errors in establishing the instant of rear-wheel locking are reduced. These latter errors were noted in the coefficients f_V and f_M for the velocity increment of V_W-30 m.p.h.

The primary cause of poor repeatability of the coefficients in the 10-0 m.p.h. increment was the inability to steer the skidding vehicle in wheel-tracks near the end of the skid. As the vehicle skids out of the wheel-tracks, it encounters higher skid resistance; and the degree of "skid-out" varied from test to test.

Velocity-time and velocity-distance coefficients in the 30-20 m.p.h. increment exhibited smaller deviations than the coefficients in the other 10 m.p.h. increments. This could be attributed to the higher velocity and therefore to longer skid intervals which permitted more accurate distance and time measurements; of course, these measurements were not affected by differential wheel-lock or by "skid-out".

The measurement of deceleration yielded much lower coefficients than those obtained from the measurement of other parameters for similar velocities. The conclusion is that the measurement of deceleration was in error because of improper correction for vehicle tilt. Even more significant was the fact that the coefficients f_D had very poor repeatability, indicating that the method of holding the plate-mounted accelerometer was not satisfactory.

Data from the five test sites were used to correlate selected coefficients as shown in Table V. The results of these analyses were arbitrarily divided into three classifications on the basis of the standard error of estimate, E_S, and the correlation coefficient, R. The regression equations were linear, permitting simple conversion from one coefficient to another. This enabled a prediction of skid distance or duration of the skid without actually making the measurement. For example, by knowing $f_V(30\text{-}20)$, the coefficients f_{Vw} and f_{Mw} can be calculated from the appropriate regression equations. Then by using equations 5 and 9 and the estimated velocity at wheel-lock, the approximate skid distance and time in skid, from the moment of wheel-lock, can be determined. The actual velocity at wheel-lock can be closely estimated by subtracting one m.p.h. from the "observed" velocity. This method is applicable to the determination of the skid distance and skid duration at the velocity of 40 m.p.h. only.

Selection of a standard test was subjected to several criteria; accuracy, repeatability, rapid availability of test results, simplicity of measurement and minimum instrumentation. Several measurements fulfill most of these requirements. Coefficient f_{MO} in particular offers a number of advantages; the magnetic distance counter provides a simple and quick measurement of skid distance with little equipment, the velocity is obtained visually, and the test results are highly repeatable. Coefficient f_{OO} provides highly repeatable test results and requires little or no equipment other than the test vehicle. However, the measurement of "observed" skid distance is cumbersome and requires additional personnel. Neither coefficient measures skid distance from the moment of wheel-lock, and the velocity at brake application does not correspond to the beginning of either measurement of distance.

The measurement of time in the velocity increment of 30 to 20 m.p.h., yielding coefficient f_V(30-20), was selected as the standard for the following reasons:

1. Time can be measured accurately to $\pm$ 1%.

2. Coefficient of friction in this velocity increment is nearly linear.

3. Good repeatability, requiring five tests for a 5% error or less.

4. Requires only one channel for recording purposes.

5. Relative ease of chart interpretation.

Safety considerations prohibited the selection of a test which would require initiation of the skid above a speed of approximately 35 m.p.h.

Coefficient f_V(30-20) cannot be directly equated to a steady-state sliding coefficient of friction at 25 m.p.h.; this is due to the influence of air resistance and "incremental" error. The net effect of these influences is an increase in the magnitude of the coefficient. On the basis of this study, the increase, in terms of coefficient of friction, appeared to be about the same regardless of the skid resistance of the pavement--approximately 0.01.

AFFECTING FACTORS

SPEED

The coefficient of friction between a tire and a road surface decreases as the velocity of the vehicle increases. No theory for this phenomena has met universal approval, but several have been offered. For some time, the reigning theory has been that higher speeds allow less time for penetration of the water film that covers the pavement. This is similar to hydroplaning in the sense that hydrodynamic lift is provided by the water film. It differs from hydroplaning because the tire is still deformed by the asperities and does not ride above them. Obertop presented a theory to show that the decrease in skid resistance is caused by the development of steam resulting from a transformation of energy (5). The kinetic energy of the moving vehicle is irreversibly converted into other types of energy, including heat created at the tire-pavement contact area. This heat raises the temperature of the water at this contact area to a point where the pressure exerted by the tire creates steam. When this occurs, adhesion becomes zero at the point of contact. As the speed of the vehicle increases, the amount of steam generated increases, and the average coefficient drops. Obertop further suggested a mathematical equation to define the coefficient at any speed after calculating the coefficient at any two speeds. A comparison of values obtained from this formula and observed values showed a maximum difference of 0.03 (5). Total hydroplaning results in almost complete loss of braking traction and cornering capability. Before total hydroplaning can occur, the depth of water must exceed the tread depth of the tire plus an amount necessary to submerge the asperities of the pavement. The latter depends on the texture of the pavement surface. For E-17 tires on a typical bituminous pavement, the minimum, necessary depth is approximately 0.5 in. If this water condition exists, a formula using only the tire inflation pressure in p.s.i. as a parameter can be used to obtain the velocity in m.p.h., V_p, at which total hydroplaning will occur ($V_p = 10.35\sqrt{p}$) (2). The normal operating tire pressure of 24 p.s.i. requires a minimum velocity of 51 m.p.h. to produce total hydroplaning.

While hydroplaning definitely does cause a decrease in the coefficient of friction with increased velocity when the depth of water exceeds about 0.15 in., evidence exists that on wet pavements where water depth is small, significant hydroplaning does not occur. The NASA Langley Research Center encountered no hydroplaning in wet runway tests (2). They also found that a 4-groove, rib tread, passenger car tire, traveling 30 m.p.h. on a level, textured concrete runway surface covered with 0.04 in. of water, developed 88 percent of its dry-pavement cornering force. This loss of 12 percent is probably caused by loss of contact due to the presence of the lubricating film of water rather than by partial hydroplaning. When the velocity is greater than 50 m.p.h., partial hydroplaning is small, but possibly not negligible. The degree of hydroplaning depends on the condition of the tire, the tire inflation pressure, and the depth of the water.

Total hydroplaning can never result in complete loss of traction because of viscous friction. When a lubricating liquid causes loss of contact between two surfaces, the friction depends on the viscosity of the liquid and hence on the temperature (7). When this friction is present, the force required to move the skidding tire at a constant rate, F_V, is given by:

$$*F_V = \mu \frac{v}{h} A_C$$

where μ is the viscosity of the fluid, v is the velocity at which the tire is moving, h is the thickness of the water film and A_C is the contact area of the tire. This formula indicates an increasing force with increasing velocity--causing a higher "drag" on the vehicle. This drag would decelerate the vehicle

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*Reference No. 7

asymptotically if it were the only drag or resistance encountered.

The variation of coefficient of friction with velocity is illustrated in Figure 10 for a number of different types of pavements, including the five correlation study test sites.

AIR RESISTANCE

Any object moving through a fluid encounters a resistance. When an automobile moves through air, this resistance in pounds, R_a, is given by the equation:

$$*R_a = \frac{C_d v^2 A \gamma}{2g} \qquad (1a)$$

which is the general equation for drag. In this equation C_d is a dimensionless drag coefficient, v is the velocity of the automobile in ft./sec., A is the projected frontal area of the vehicle in ft.^{2} and γ is the unit weight of air in lb./ft.^{3} To make the equation dimensionally correct when V is in m.p.h., it is necessary to use a conversion factor of 1.47^2 to change m.p.h. to ft./sec. The result is then:

$$R_a = \frac{2.16\ C_d\ A\ V_a^2\ \gamma}{2g} \qquad (2a)$$

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*King, A. W., Wisler, C.O., and Woodburn, J. G.; Hydraulics, John Wiley and Sons, Inc., New York, 1958, pp. 304-305.

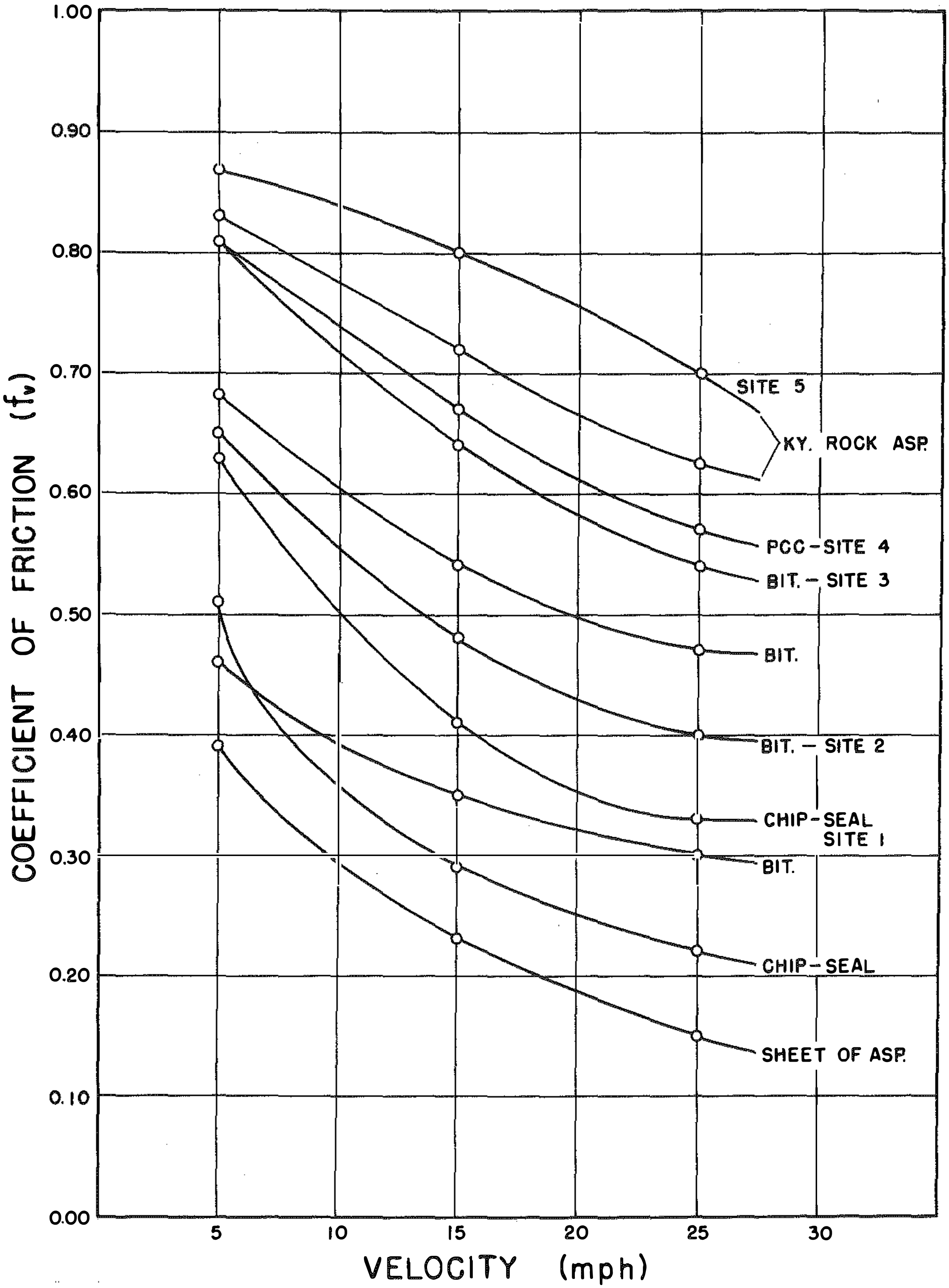

Figure 10. A Plot of Coefficient of Friction (f_V) vs Velocity for a Number of Pavements.

where V_a is the velocity of the vehicle in m.p.h., with respect to the air mass. The unit weight of air, γ, varies with temperature T, and atmospheric pressure, p, according to the equation:

$$** \ \gamma = \frac{p}{53.3T} \qquad (3a)$$

Under normal atmospheric pressure and an ambient temperature of 85°F,

$$=0.0732 \text{ lbs./ft.}^3$$

The terms C_d and A are constant for any given vehicle. For the test vehicle:

$$*** \ C_d = 0.51 \text{ and}$$
$$*** \ A = 24.1 \text{ sq. ft.}$$

Substituting these values, and $g = 32.2$ ft./sec.^{2}, into equation (2a), R_a can be plotted as a function of V_a, as shown in Figure 11. The equation is:

$$R_a = 0.030176 \ V_a^2 \qquad (4a)$$

Since air resistance is a force retarding a skidding vehicle, it can be considered as a portion of the total friction force--thereby affecting the computed coefficient of friction.

** Ibid., p. 11

*** Information supplied by Ford Motor Company, Dearborn, Michigan.

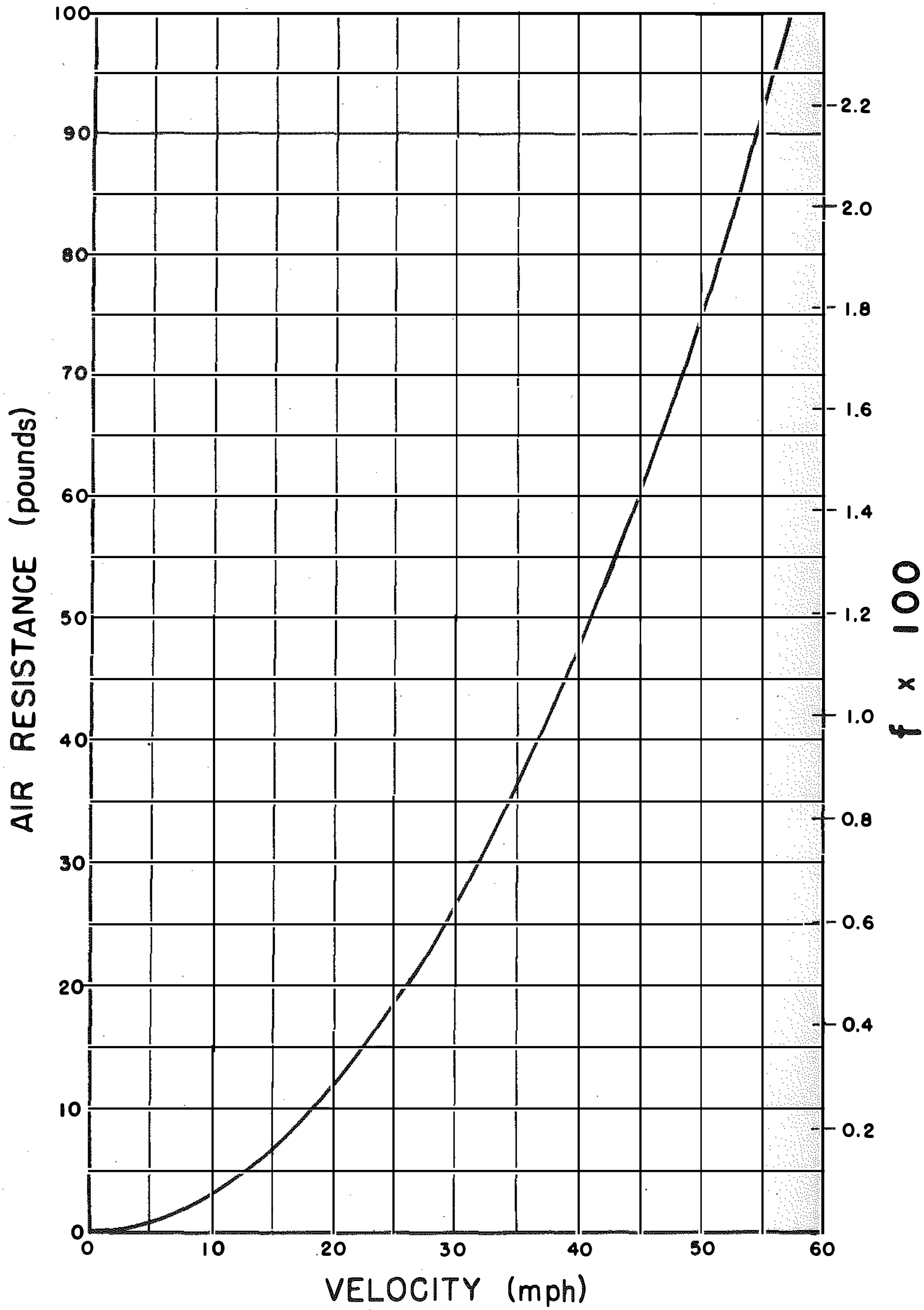

Figure 11. A Plot of Air Resistance and Coefficient of Friction vs Velocity.

The influence can be determined by substituting R_a for the friction in Coulomb's equation F = fN, or:

$$R_a = fN \qquad (5a)$$

Using the value of R_a from equation (4a) and solving for f, equation (5a) becomes:

$$f = \frac{0.030176 \, V_a^2}{N} \qquad (6a)$$

and by substituting the weight of the vehicle, 4,200 pounds, for the normal force, N,

$$f = 7.18 \times 10^{-6} V_a^2 \qquad (7a)$$

From the graph in Figure 12, f can be indicated opposite R_a as a function of V_a.

The velocity V_a is the sum of the vehicle velocity, V_v, and the wind velocity V_r or $V_a = V_r + V_v$. If wind velocity, V_r, is assumed to be zero, then $V_a = V_v$, or the vehicle speed. At 30 m.p.h. and 20 m.p.h. the air resistance is equivalent to f = 0.0065 and f = 0.0029, respectively. The calculated coefficient $f_v(30-20)$, therefore, is higher than the actual coefficient. This error is largest for low coefficient surfaces, approximately 2% for a coefficient of friction of 0.25.

VEHICLE DYNAMICS

The behavior of the vehicle body on it's suspension system is referred to in this report as the vehicle dynamics.

When brakes are rapidly applied to a moving vehicle, the body of the vehicle surges forward and oscillates in a pitching manner as illustrated by the deceleration recording in Figures 7 and 8. The motion of the body near the wheels is both vertical and horizontal and, to a large extent, is dampened out during the skid. The body tilt, however, continues to change with the change in skid resistance as the vehicle decelerates.

The vehicular dynamic behavior could affect a friction measurement as a result of weight transfer from rear to front or as a result of energy stored in the suspension system. To determine the extent of this influence, skid tests were conducted with the suspension system partially neutralized and with the suspension system acting freely. Rubber-cushioned wood blocks were constructed for this purpose and inserted near the suspension system components. A summary of the test results follows:

BLOCKS	V_1	V_w	S_M	f_{Vw}	$f_V(10-0)$
None	44.5	43.0	117.5	0.64	0.80
Front	45.2	43.3	119.2	0.64	0.80
Front and Rear	44.5	42.7	118.3	0.64	0.80

Note: All values are averages of ten tests conducted on Site 4.

In this investigation, the test results were not altered to a measurable degree by partially neutralizing the suspension system. From these results, it is surmised that the vehicle dynamics have insignificant effect on the skid distance or on the skid resistance at any given velocity during the skid.

It should be pointed out however, that the velocity increment measurements at velocities near the instant of wheel-lock may be quite susceptible to error; this is due to the behavior of the vehicle body and the fact that the fifth wheel is attached to the body of the car rather than the rear axle. The velocity at wheel-lock could be in error simply because the body of the car oscillates and momentarily increases or decreases the velocity of the fifth wheel. It becomes imperative then to provide the test automobile with a very stiff suspension so as to shorten the period of dynamic activity. While it is recognized that the vehicle continues to undergo a change in displacement with respect to the wheels and the chassis during skid, the resultant error in the measured velocity is assumed to be negligible.

TIRE INFLATION PRESSURE

The infuence of tire inflation pressure was observed in tests on all five test sites. Beginning at 32 p.s.i., the tire pressure was decreased in 4-p.s.i. increments to 20 p.s.i.

Five tests were run at each tire pressure. The results of these tests showed a general decrease in the coefficient of friction with increase in tire pressure. The coefficient f_V(30-20) has slightly less than a 5% decrease between 20 p.s.i. and 32 p.s.i.; therefore, a variation of 2 p.s.i. was considered insignificant.

THE BRITISH PORTABLE TESTER

The British Portable Tester (BPT), shown in Figure 12, was employed by the Department of Highways in skid testing for three years, including 1964. The validity of the test results yielded by the tester for certain types of pavements has been in doubt for some time. Consequently, a comparison was made between values obtained with this tester and $f_V(30\text{-}20)$. The surfaces tested were generally classified as bituminous, concrete, and bituminous sealcoats. The results are presented graphically in Figure 13.

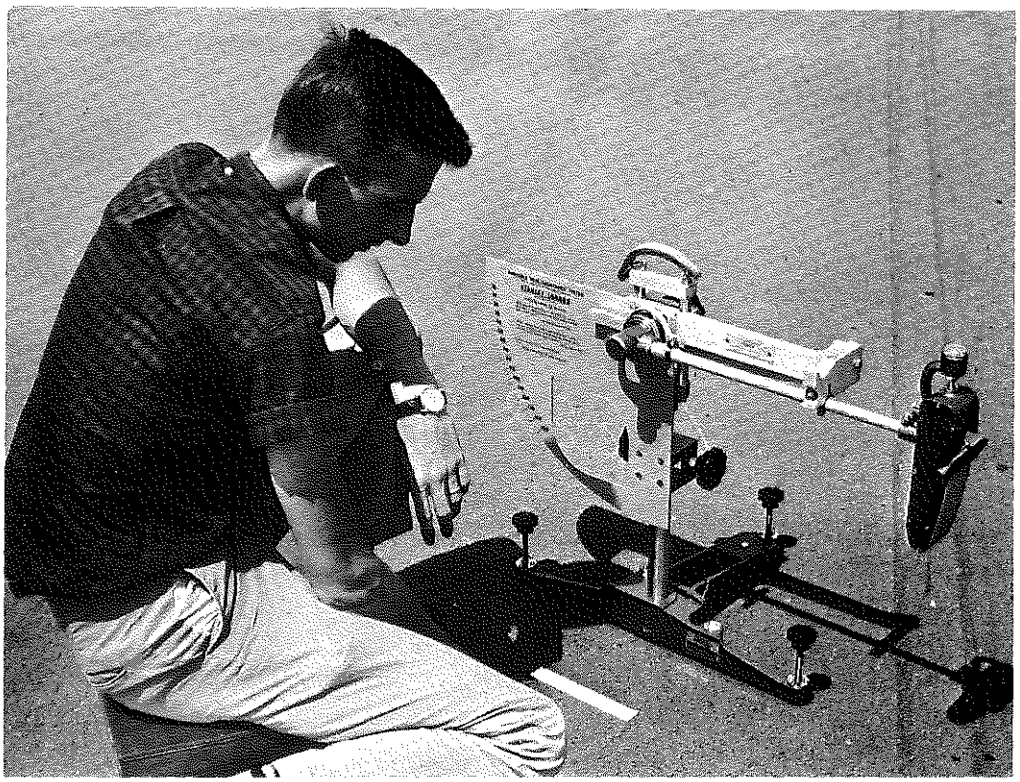

Figure 12. The British Portable Tester

Regression equations were found for data of several combinations of pavements, but only the equation for bituminous pavements was plotted in Figure 13. The correlation was as follows:

PAVEMENTS	EQUATION	R	E_s
Combined	*Y = 0.162 + 0.777X	0.877	0.045
Concrete and Bituminous	Y = 0.066 + 0.957X	0.928	0.036
Bituminous	Y = 0.052 + 0.985X	0.944	0.031

*Y = BPT Reading

Obviously, a correlation exists between the two sets of values; but, on the basis of the criterion established for the correlation of the other coefficients in Table V, the correlation is poor. The influence of sealed and concrete pavements is easily discernible (sealed pavements are included in the combined group in the above table). On sealed surfaces, the BPT indicates much higher skid resistance, and a separate correlation would be warranted. Unfortunately, the number of these surfaces tested was insufficient to perform such a correlation.

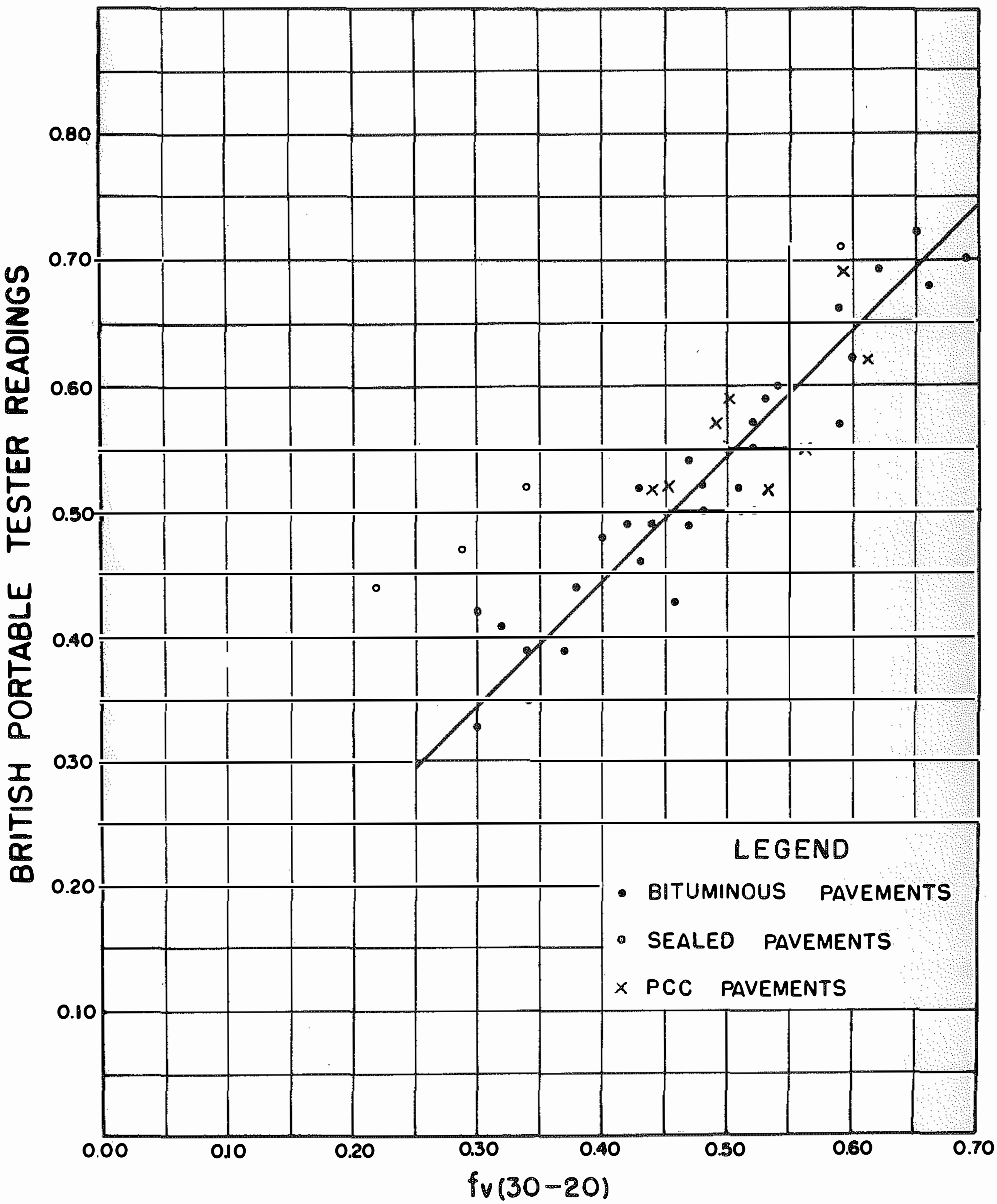

Figure 13. A Plot of British Portable Tester Readings vs Coefficient $f_V(30\text{-}20)$.

CONCLUSIONS

1. The measurement of time and velocity of a skidding automobile fulfills the requirement of an interim standard method of testing. The coefficient $f_V(30\text{-}20)$ was found to be a good indicator of pavement-tire friction but does not fully describe the frictional characteristics of the surface.

2. The use of an automobile as a regular pavement-slipperiness testing device is extremely unsatisfactory. The test interfers with traffic flow and is time consuming, hazardous to testing personnel and expensive, since it requires the services of four technicians. The average cost of testing was $25.00 per site per lane.

3. Air resistance has little effect on the retardation of a skidding vehicle because of the low test velocities. The influence of vehicle dynamics was found to be negligible. Other test influences, such as speed and tire pressure, were determined and have been previously discussed. Much remains to be learned concerning the variation of skid resistance with seasons, temperature, and pavement washing.

4. The British Portable Tester results correlate poorly with coefficient $f_V(30\text{-}20)$, particularly when concrete and

bituminous sealcoat surfaces are included. From the standpoint of safety, traffic interference, test personnel, and time required to perform the test, the instrument offers little advantage over testing with an automobile. It does provide repeatable results and is quite usefull in laboratory testing and in testing small areas and otherwise inaccessible locations.

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APPENDIX A

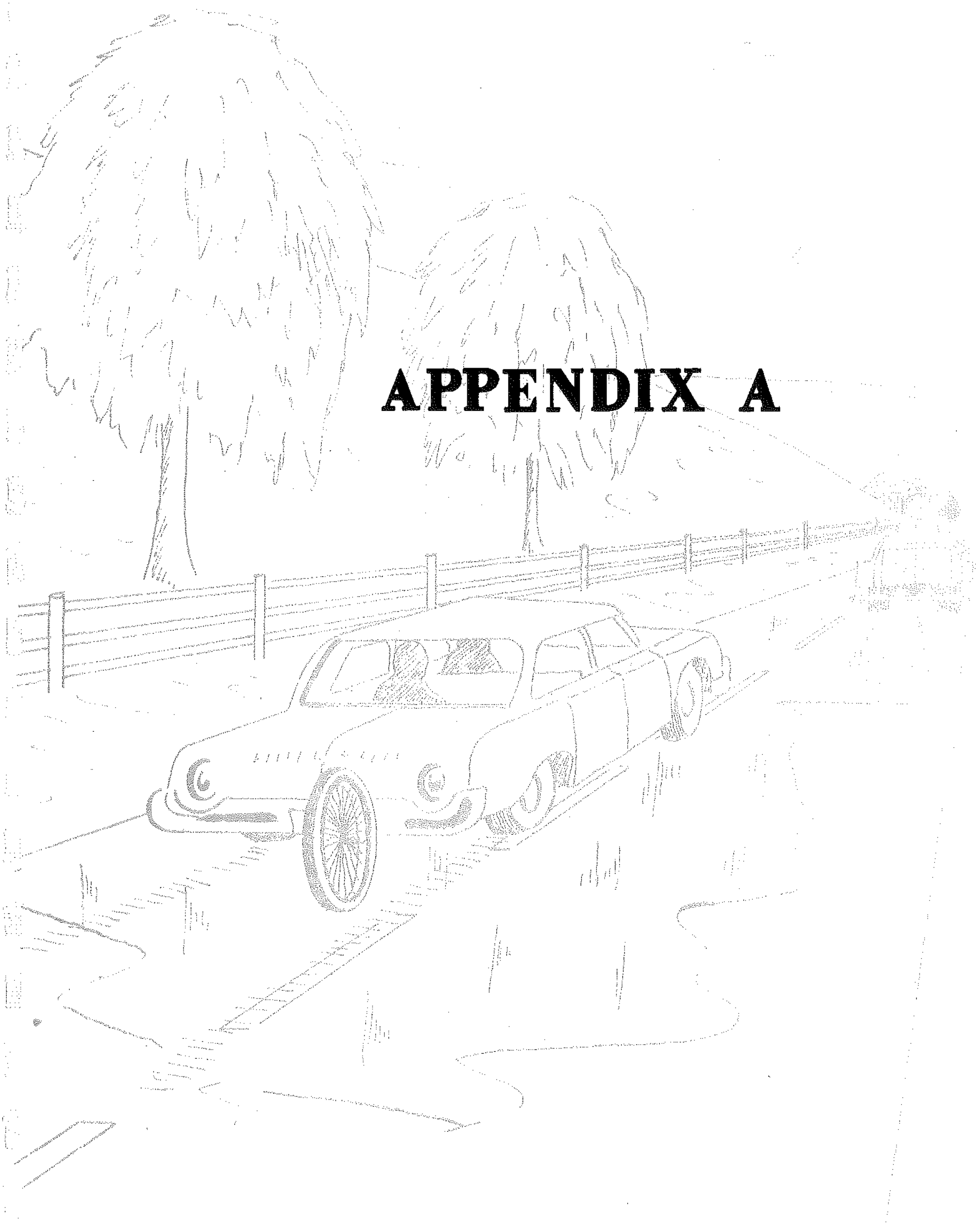


TABLE I

CORRELATION STUDY TEST DATA

Velocity (m.p.h.)	Site 1a	Site 1b	Site 2	Site 3	Site 4	Site 5
V_o	34.9		40.0	40.1	40.0	40.1
V_b	35.4		40.4	40.5	40.1	40.4
V_1	35.3		40.3	40.5	39.9	40.1
V_w	34.3		39.4	39.6	38.5	38.9
Coefficients			**Velocity and Distance**			
f_{0o}	0.38	0.49	0.43	0.60	0.65	0.79
f_{0w}	0.37	0.46	0.41	0.58	0.61	0.74
f_{Mo}	0.34	0.43	0.39	0.52	0.56	0.68
f_{M1}	0.35	0.43	0.40	0.52	0.55	0.68
f_{Mw}	0.36	0.46	0.41	0.55	0.58	0.72
$f_M(30-0)$	0.37	0.52	0.44	0.60	0.61	0.74
$f_M(20-0)$	0.45	0.60	0.50	0.66	0.68	0.79
$f_M(10-0)$	0.59	0.69	0.61	0.76	0.77	0.83
$f_M(V_w-30)$	0.35	0.39	0.37	0.50	0.54	0.71
$f_M(30-20)$	0.33	0.46	0.40	0.53	0.57	0.69
$f_M(20-10)$	0.41	0.58	0.47	0.62	0.66	0.79
$f_M(V_w-10)$	0.35	0.45	0.41	0.54	0.57	0.72
			Velocity and Time			
f_{V1}	0.40	0.49	0.44	0.48	0.62	0.73
f_{Vw}	0.41	0.52	0.44	0.60	0.64	0.76
$f_V(30-0)$	0.42	0.57	0.47	0.64	0.67	0.78
$f_V(20-0)$	0.50	0.66	0.54	0.72	0.73	0.83
$f_V(10-0)$	0.63	0.75	0.62	0.81	0.81	0.87
$f_V(V_w-30)$	0.34	0.41	0.36	0.50	0.55	0.71
$f_V(30-20)$	0.33	0.47	0.40	0.54	0.57	0.70
$f_V(20-10)$	0.41	0.59	0.48	0.64	0.67	0.80
$f_V(V_w-10)$	0.36	0.47	0.41	0.55	0.59	0.73
			Deceleration			
f_{Dw}	0.36	0.41	0.38	0.56	0.60	0.66
$f_D(15)$	0.38	0.46	0.41	0.60	0.61	0.68
$f_D(25)$	0.28	0.36	0.34	0.45	0.51	0.61
$f_D(35)$		0.30	0.31	0.42	0.49	0.54

TABLE II

PERCENT DIFFERENCE BETWEEN COEFFICIENTS

COEFFICIENTS	SITE 1	SITE 2	SITE 3	SITE 4	SITE 5	AVG.	PRIMARY SOURCES OF DIFFERENCE
$f_{0o} - f_{0w}$	2.7	4.8	3.4	6.3	6.5	4.7	Unlike velocities
$f_{0o} - f_{Mo}$	11.1	9.8	14.3	14.9	15.0	13.0	Unlike distances
$f_{0o} - f_{Mw}$	5.4	4.8	8.7	11.4	9.3	7.9	Unlike velocities and error in distance
$f_{0w} - f_{Mw}$	2.7	0.0	5.3	5.0	2.7	3.1	Error in distance
$f_{Ml} - f_{Mo}$	2.9	2.5	0.0	-1.8	0.0	0.7	Unlike velocities
$f_{Mw} - f_{Ml}$	2.8	2.5	5.6	5.3	5.7	4.5	Unlike velocites and distance
$f_{Vw} - f_{0w}$	10.3	7.1	3.4	4.8	2.7	5.7	"Incremental" error
$f_{Vw} - f_{0o}$	7.6	2.3	0.0	-1.5	-3.9	0.9	Unlike velocities and "incremental" error
$f_{Vw} - f_{Mw}$	13.0	7.1	8.7	9.8	5.4	8.8	Difference in "incremental" error
$f_{Vl} - f_{Ml}$	13.3	9.5	10.9	12.0	8.3	10.8	"Incremental" error
$f_{Ml} - f_{Mo}$	2.9	2.5	0.0	-1.8	0.0	0.7	Unlike velocities
$f_V(10-0) - f_M(10-0)$	6.6	1.6	6.4	5.1	4.7	4.9	Difference in "incremental" errors
$f_V(20-0) - f_M(20-0)$	10.5	7.7	8.7	7.1	4.9	7.8	Difference in "Incremental" errors
$f_V(30-0) - f_M(30-0)$	12.7	6.6	6.5	9.4	5.3	8.1	Difference in "incremental" errors
$f_V(20-10) - f_M(20-10)$	0.0	2.1	3.2	1.5	1.3	1.6	Difference in "incremental" errors
$f_V(30-20) - f_M(30-20)$	0.0	0.0	1.9	0.0	1.4	0.9	Difference in "incremental" errors
$f_V(Vw-30) - f_M(Vw-30)$	-2.9	-2.7	0.0	1.8	0.0	-0.8	Difference in "incremental" errors
$f_V(Vw-10) - f_M(Vw-10)$	2.8	0.0	1.8	3.5	1.4	1.9	Difference in "incremental" errors
$f_V(20-10) - f_D(15)$	7.6	15.7	6.5	9.4	16.2	11.1	Deceleration error
$f_V(30-20) - f_D(25)$	16.4	16.2	18.2	11.1	13.7	15.1	Deceleration error
$f_V(Vw-30) - f_D(35)$	--	15.0	17.4	11.5	27.2	17.8	Deceleration error
$f_{Vw} - f_{Dw}$	13.0	14.6	6.9	6.5	14.1	11.0	Deceleration and "incremental" errors

TABLE III

NUMBER OF TESTS REQUIRED FOR 5% ERROR OR LESS

COEFFICIENTS	SITE 1	SITE 2	SITE 3	SITE 4	SITE 5	AVG.
f_{Oo}	5	4	5	5	3	4
f_{Ow}	4	3	5	6	4	4
f_{Vw}	3	4	4	3	4	4
f_{Mo}	5	4	6	4	3	4
f_{Mw}	3	4	5	3	3	4
f_{Ml}	3	3	6	4	4	4
$f_V(30-20)$	4	5	7	3	4	5
$f_M(30-20)$	4	5	7	5	5	5
$f_V(10-0)$	5	4	7	5	6	6
f_{Dw}	5	6	5	6	7	6
$f_D(25)$	13	15	18	9	14	14

TABLE IV

STANDARD DEVIATIONS

COEFFICIENTS	SITE 1	SITE 2	SITE 3	SITE 4	SITE 5
f_{Oo}	0.016	0.012	0.025	0.025	0.016
f_{Ow}	0.013	0.011	0.024	0.026	0.023
f_{Mo}	0.015	0.010	0.018	0.015	0.011
f_{M1}	0.008	0.007	0.019	0.018	0.013
f_{Mw}	0.007	0.010	0.025	0.013	0.016
$f_M(30-0)$	0.009	0.010	0.029	0.021	0.018
$f_M(20-0)$	0.014	0.014	0.046	0.022	0.039
$f_M(10-0)$	0.047	0.041	0.075	0.055	0.042
$f_M(V_w-30)$	0.030	0.017	0.040	0.029	0.055
$f_M(30-20)$	0.010	0.016	0.033	0.021	0.023
$f_M(20-10)$	0.015	0.015	0.050	0.020	0.051
$f_M(V_w-10)$	0.012	0.009	0.026	0.012	0.018
f_{V1}	0.010	0.005	0.019	0.013	0.022
f_{Vw}	0.006	0.009	0.020	0.012	0.023
$f_V(30-0)$	0.010	0.011	0.030	0.009	0.022
$f_V(20-0)$	0.012	0.014	0.037	0.016	0.033
$f_V(10-0)$	0.024	0.018	0.044	0.030	0.035
$f_V(V_w-30)$	0.031	0.012	0.034	0.027	0.062
$f_V(30-20)$	0.010	0.015	0.031	0.012	0.024
$f_V(20-10)$	0.010	0.016	0.038	0.017	0.044
$f_V(V_w-10)$	0.009	0.013	0.021	0.012	0.025
f_{Dw}	0.015	0.018	0.020	0.050	0.033
$f_D(15)$	0.036	0.050	0.039	0.030	0.027
$f_D(25)$	0.033	0.036	0.042	0.041	0.060
$f_D(35)$	--	0.057	0.054	0.056	0.083

Note: Data from Rounds I and II.

TABLE V

CORRELATION EQUATIONS

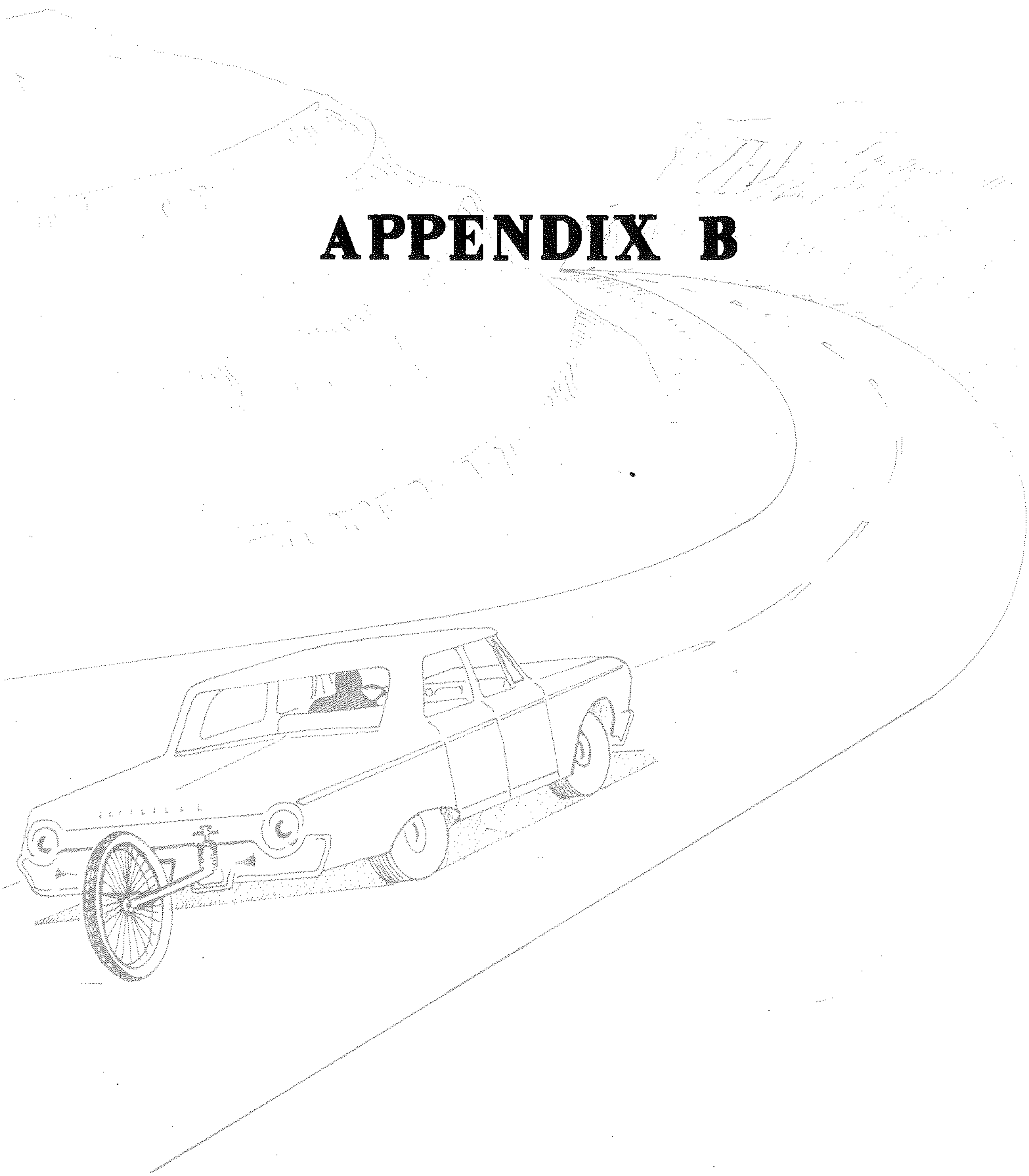
X	Y	EQUATION	R	E_s
		GOOD CORRELATION		
f_{Oo}	f_{Ow}	Y = 0.024 + 0.909X	1.000	0.004
f_{Oo}	f_{Mo}	Y = 0.036 + 0.811X	0.999	0.004
$f_V(30-20)$	$f_V(20-10)$	Y = 0.058 + 1.067X	1.000	0.006
$f_V(30-20)$	$f_M(30-20)$	Y = 0.009 + 0.974X	1.000	0.006
$f_V(30-20)$	$f_V(V_w-10)$	Y =-0.031 + 1.084X	0.998	0.006
f_{Vw}	f_{V1}	Y = 0.020 + 0.932X	0.997	0.006
$f_V(30-20)$	f_{Mw}	Y =-0.023 + 1.058X	0.998	0.007
$f_V(30-20)$	f_{V1}	Y = 0.036 + 1.000X	1.000	0.008
f_{Oo}	f_{Mw}	Y = 0.036 + 0.857X	0.999	0.008
f_{Vw}	f_{Mw}	Y =-0.036 + 0.981X	1.000	0.009
$f_V(30-20)$	f_{Vw}	Y = 0.917 + 1.073X	0.997	0.009
f_{Vw}	$f_V(30-0)$	Y = 0.065 + 0.948X	0.996	0.009
		FAIR CORRELATION		
$f_V(30-20)$	f_{Mo}	Y =-0.024 + 1.006X	0.994	0.011
f_{Vw}	$f_V(20-10)$	Y = 0.071 + 0.955X	0.993	0.012
$f_V(30-20)$	$f_V(30-0)$	Y = 0.082 + 1.012X	0.992	0.012
f_{Vw}	$f_V(Vw-10)$	Y =-0.045 + 1.005X	0.996	0.013
$f_V(30-20)$	f_{Oo}	Y =-0.074 + 1.242X	0.995	0.014
$f_V(30-20)$	$f_V(V_w-30)$	Y =-0.094 + 1.120X	0.995	0.015
f_{Vw}	$f_V(V_w-30)$	Y =-0.151 + 1.111X	0.990	0.017
$f_V(30-20)$	$f_V(20-0)$	Y = 0.184 + 0.944X	0.988	0.017
f_{Vw}	$f_D(25)$	Y =-0.070 + 0.889X	0.971	0.017
$f_V(30-20)$	$f_D(25)$	Y =-0.062 + 0.967X	0.982	0.018

TABLE V

CORRELATION EQUATIONS
(Cont.)

X	Y	EQUATION	R	E_s
		POOR CORRELATION		
f_{Vw}	$f_V(20-0)$	$Y = 0.187 + 0.860X$	0.981	0.027
f_{Vw}	f_{Dw}	$Y = -0.054 + 0.973X$	0.981	0.029
f_{Vw}	$f_V(10-0)$	$Y = 0.336 + 0.735X$	0.938	0.029
$f_V(30-20)$	f_{Dw}	$Y = -0.027 + 1.024X$	0.964	0.032
$f_V(30-20)$	$f_V(10-0)$	Y = 0.364 + 0.[illegible]54X	0.939	0.032

APPENDIX B



STATISTICAL CALCULATIONS

Regression Lines

All regression lines were of the form

$$Y = a + bX \qquad (1)$$

where

$$b = \frac{\eta(\Sigma XY) - (\Sigma X)(\Sigma Y)}{\eta(\Sigma X^2) - (\Sigma X)^2} \qquad (2)$$

$$a = \frac{\Sigma Y - b(\Sigma X)}{\eta} \qquad (3)$$

η = number of observations

X and Y = observed values of data

Coefficient of Correlation

$$R = \frac{\eta(\Sigma XY) - (\Sigma X)(\Sigma Y)}{\sqrt{\eta(\Sigma X^2) - (\Sigma X)^2}\sqrt{\eta(\Sigma Y)^2 - (\Sigma Y)^2}} \qquad (4)$$

Standard Error of Estimate

$$E_s = \sqrt{\frac{\Sigma(Y - Y_1)^2}{\eta}} \qquad (5)$$

Y_1 = calculated values of Y for observed values of X

Standard Deviation

$$\sigma = \sqrt{\frac{\Sigma_1^{\eta}(X - \overline{X})^2}{\eta}} \qquad (6)$$

where

$\overline{X}$ = mean of η number of X's

Required Number of Tests

$$N = \left(t \frac{\sigma}{E}\right)^2 \qquad (7)$$

where

t = student value

σ = standard deviation, from equation (6)

E = maximum allowable error, 5%