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MODIFICATIONS TO CHEVRON N-LAYER COMPUTER PROGRAM

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16. Abstract

ABSTRACT

This report documents changes made to the Chevron N-layer computer program to:

1. Include superposition principles.
2. Calculate strain energy density (or work) at specified locations within the pavement structure.
3. Analyze pavement response at specified radii from one circularly loaded area to permit comparison of analyses by the program as originally written with results incorporating superposition principles.
4. Evaluate pavement response to any combination of loads on circular areas defined by XY coordinates on the surface. Loads and contact pressures are permitted to be different from one loaded area to another, but must be constant for any one loaded area.
5. Simulate dynamic loads as the difference between the root mean squares of the maximum and minimum dynamic loads. This analysis is appropriate for constant vibratory testers such as Road Raters and Dynaflects. Moduli of asphaltic concrete must be adjusted for frequency effects.

17. Key Words			18. Distribution Statement
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INTRODUCTION

The Chevron N-Layer computer program (1) was received in 1968 from the Chevron Research Company, Chevron Oil Corporation. Appendix A contains the background documents as written and distributed by Chevron Research Company. That version permitted analyses of one circular load applied to the surface, and output was requested at specified radii from the center of the loaded area. The program was modified to permit analyses of multiple loads located at coordinates in the X-Y plane, and output for specified locations within the X-Y-Z coordinate system may be requested. Superposition principles were applied to each component of strain and stress to resolve them into a value compatible with the X-Y-Z coordinate system. Appropriate equations are shown in Figure 1. The Kentucky modified Chevron N-layer program is listed in Appendix B. Appendix A also contains the letter from Chevron granting permission to publish the listed program as modified herein.

The original Chevron N-layer program contained a subroutine called COE5 that limited the number of layers to be analyzed to five. Shortly thereafter, another subroutine was written by Chevron called CO15 that expanded the capability to analyze a pavement having 15 layers. Some users noted that slightly different answers for locations near the surface of the pavement were obtained, depending upon whether subroutine COE5 or CO15 was being used. Several years later, these two subroutines were replaced with a subroutine called COFE that resolved the problem, and this subroutine is used in the listing given in Appendix B. The number of layers is still limited to 15.

For those having an original Chevron N-layer program, one additional change will be noted. In Appendix B, Subroutine CALCIN has been split into two sections. The part listed herein as Subroutine CALCIN is the first half of the original subroutine. The output portion of the original subroutine was separated to form Subroutine PRN7.

STRAIN ENERGY

The "work" done by a force when its point of application is displaced is the product of that force (parallel to the direction of movement) and the displacement. When work is done on some systems, the internal geometry is altered in such a way that there is a potential to "give back" work when the force is removed, and the system returns to its original configuration. This stored energy is defined as strain energy. Strain energy per unit volume at a given point in the body is the strain energy density at that point.

Strain energy density is a function of the Young's modulus of elasticity and Poisson's ratio of the material and the nine strain (or stress) components; however, it is independent of the coordinate system. Stress and strain components, referenced to a local cylindrical coordinate system, for each load are calculated by the Chevron program (1). The classical equation for strain energy density derived by Sokolnikoff (2) is as follows:

$$W = (1/2)\lambda \gamma \epsilon_{ii} + G \epsilon_{ij} \epsilon_{ij}$$

$$= (1/2)\lambda \gamma^2 + G(\epsilon^2_{11} + \epsilon^2_{22} + \epsilon^2_{33} +$$

$$2\epsilon^2_{12} + 2\epsilon^2_{23} + 2\epsilon^2_{13}), \qquad (1)$$

in which W = strain energy density, or energy of deformation per unit volume;

ϵ_{ij} = i, jth component of the strain tensor;

G = $E/(2(1 + \mu))$, the "modulus of rigidity" or the "shear modulus";

E = Young's modulus;

μ = Poisson's ratio;

$\lambda = E\mu/((1 + \mu)(1 - 2\mu))$; and

$\gamma = \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$.

Strain energy density may be calculated using stress components by the equation

$$W = -\mu\theta^2/2E + ((1 + \mu)/2E)(\sigma^2_{11} + \sigma^2_{22} + \sigma^2_{33}) +$$

$$(4(1 + \mu)/E)(\sigma^2_{12} + \sigma^2_{23} + \sigma^2_{31}), \qquad (2)$$

in which $\theta = \sigma_{11} + \sigma_{22} + \sigma_{33}$ and

σ_{ij} = i, jth component of the stress tensor.

Equation 2 differs from Sokolnikoff's (2) Equation 26.17 by a factor of "4" in the third term. Shear stress is usually written as $2(1 + \mu)\, \epsilon ij/E$. Sokolnikoff (2, p 25) states, "The factor 1/2 was inserted in the formulas (7.5) in order that the set of quantities may transform according to the Tensor Laws". It has been verified that the Chevron N-Layer Program comuptes shear stresses in a manner that includes the "4" as shown in Equation 2.

Inspection of Equation 1 shows that the terms E & $(1 + \mu)$ are contained by means of the terms λ and G. Also, it is noted that the strain components are squared. Having calculated strain energy density, "work strain" (3) may be obtained from

$$\epsilon_w = (2\ W/E)^{0.5} \qquad (3)$$

in which ϵ_w = work strain. The associated "work stress" is given by $E\epsilon_w$.

INTERPRETATIONS OF WORK STRAIN

Admittedly, work strain is not a true strain because Poisson's ratio has not been eliminated prior to taking the square root; however, it is of the same order of magnitude as any of the strain components. Calculating the work strain is a minor effort since all terms of the equations are either required input to, or calculated output of, the Chevron N-layer (1, 4) program. Work strain is also the composite, or net effect, of all strain components and thus is an indicator of the total strain behavior. Figure 1 illustrates there is a direct correlation between a strain component and work strain.

USES FOR WORK STRAIN

Some thickness design systems for flexible pavements are based partially upon tensile strain criteria at the bottom of the asphaltic concrete layer. Kentucky's system (5-10) is based in part upon the tangential strain component. The tangential component is generally the largest in magnitude, but the radial component often is nearly as large. Only the tangential component has been utilized because laboratory test data yield one component of tensile strain. The net effect of all components of strain (work strain) can be correlated with any component of strain. Thus, design systems based upon one component of strain may be converted to a design system that utilizes the net effect of all component strains. The load-damage factor relationships presented in this paper are based on work strain. All comments concerning component strains also apply to component stresses.

The concept of work strain has been used as the basis for the development of thickness design systems for pavements consisting of portland cement concrete or pozzolanic base courses under asphaltic concrete (11-13).

MODIFICATIONS TO ORIGINAL CHEVRON N-LAYER PROGRAM

GENERAL COMMENTS

The first data card, or line of input data, contains two variable names -- OPTN and OUTPUT. OPTN determines which type of analyses is to be performed. OUTPUT specifies a hard copy form and/or storage of output in the computer's memory for OPTN = 2 or 3 only. For OPTN = 1, OUTPUT has no meaning. A value of 1 for OUTPUT produces a hardcopy and stores the output in the computer memory. A blank or 0 produces only a hardcopy. A value of 2 stores the output in computer memory only.

OPTN = 1

A value of "1" causes the program to run according to the original Chevron N-Layer program. This version restricts the input data to one circular load and output locations are specified as radial distances from center of the load.

OPTN = 2

A value of "2" permits the entry of as many as 99 loads at locations in the form of X-Y coordinates, and up to 99 desired output locations may be specified as X-Y coordinates and up to 99 depths (Z) for each X-Y coordinate. As many as 15 layers of materials with their respective input data for thickness, modulus, and Poisson's ratio may be investigated in each computer run.

OPTN = 3

Kentucky research has shown that dynamic deflection testing may be simulated utilizing the Chevron N-layer computer program with static loads as input. Two adjustments must be made to make the simulation match measured values. First, the modulus of the asphaltic concrete layers is frequency dependent and must be adjusted to an equivalent value appropriate to the testing frequency. The second adjustment involves the oscillating load. While the actual loading is applied as a sine wave, the sensors measure either velocity waves or accelerations and the electronic circuits integrate, or double integrate, the signals respectively to obtain the deflection readings shown on the meters. Thus, the deflections are equivalent to static deflections. Therefore, the dynamic loading applied as a true sine wave may be considered as a static loading by converting the sine wave to an equivalent square wave by

$$MXSL = MEAN + 0.707 * (MXPL - MEAN) \quad (4)$$

and

$$MNSL = MEAN - 0.707 * (MEAN - MNPL), \quad (5)$$

in which MXSL = maximum static load,

MNSL = minimum static load,

MXPL = maximum dynamic load,

MNPL = minimum dynamic load, and

MEAN = dead load about which the dynamic loading oscillates.

Correlation testing of in-place pavements has shown that the measured deflection as displayed on the meters is identical to the calculated values for each sensor location by

$$DEFL = DEFMX - DEFMN \quad (6)$$

in which DEFL = deflection displayed on the meter for that sensor,

DEFMX = calculated deflection corresponding to the adjusted load MXSL, and

DEFMN = calculated deflection corresponding to adjusted load MNSL.

Thus, for OPTN = 3, the calculated deflections, stresses, and strains are obtained by storing the calculated values for the equivalent maximum square-waved loads, then subtracting the calculated values for the minimum square-waved loads and printing the resulting values. This procedure has been used most successfully for both the Road Rater (14-16) and for limited data obtained with the Dynaflect. It has not been attempted for a falling weight tester.

TYPICAL PROBLEMS INVESTIGATED

WHEEL LOADS

The original Chevron N-layer program permitted behavioral analyses of a pavement due to the application of one load. Such analyses corresponded to the use of OPTN = 1. These analyses did provide a major breakthrough at that time. However, it became evident very quickly that analyses were needed for multiple wheel loads and axles.

AXLELOAD ANALYSES

Actual tire loadings and configurations required modifications to be made to the program. The first modifications involved those found in OPTN = 2. Other modifications were made and reported (3). However, the version contained in Appendix B covers all those options. Except for the original version and the simulation of dynamic loadings, the other options primarily were written to choose which variables were to be output. One exception to the above was an attempt to analyze axleloads for each group of axles on a given vehicle. While this type of analysis is possible, analyses may be made by inputting all groups of axles as separate problems using OPTN = 2 and summing the results manually.

SIMULATION OF DYNAMIC DEFLECTIONS

As stated earlier, OPTN = 3 should be used to simulate dynamic deflection tests. Under this option, the stresses, strains, and deflections are calculated for the equivalent maximum and minimum dynamic loads when expressed as the square wave of the applied sinusoidal loading. These deflections have correlated with the measured values as shown on the meters of either the Road Rater or the Dynaflect testers.

EXAMPLES

The input and output data for an example problem for each method of analyses is presented in Appendix C.

RECOMMENDATIONS

The modified Chevron N-Layer Computer Program is recommended for analyzing pavements using a single wheel load, multiple wheelloads on one axle, or multiple axles (up to a total of 99 wheelload locations), and for simulation of dynamic testers such as the Dynaflect and various models of Road Raters.

REFERENCES

1. J. Michelow, "Analysis of Stresses and Displacements in an N-Layered Elastic System under a Load Uniformly Distributed on a Circular Area," Chevron Research Corporation, Richmond, California, 1963; also reproduced in Appendix A.

2. I. A. Sokolnikoff, .us Mathematical Theory of Elasticity, McGraw-Hill Book Company, 1956.

3. R. C. Deen, H. F. Southgate, and J. G. Mayes, "The Effect of Truck Design on Pavement Performance," Proceedings, The Association of Asphalt Paving Technologists, Louisville, Kentucky, Volume 49, 1980.

4. R. C. Deen, H. F. Southgate, and J. H. Havens, "Structural Analysis of Bituminous Concrete Pavements," Division of Research, Kentucky Department of Highways, Research Report 305, Lexington, Kentucky, May 1971.

5. J. H. Havens, R. C. Deen, and H. F. Southgate, "Design Guide for Bituminous Concrete Pavement Structures," University of Kentucky Transportation Research Program, Research Report UKTRP-81-17, Lexington, Kentucky, August 1981.

6. H. F. Southgate, R. C. Deen, and J. H. Havens, "Development of Thickness Design System for Bituminous Concrete Pavements," University of Kentucky Transportation Research Program, Research Report UKTRP-81-20, Lexington, Kentucky, November 1981.

7. H. F. Southgate and R. C. Deen, "Variable Serviceability Concept for Pavement Design Confirmed by AASHO Road Test Fatigue Data," University of Kentucky Transportation Research Program, Research Report UKTRP-84-12, Lexington, Kentucky, April 1984.

8. H. F. Southgate and R. C. Deen, "Effects of Load Distributions and Axle and Tire Configurations on Pavement Fatigue," Second National Conference on Weigh-In-Motion, Atlanta, Georgia, May 20-24, 1985.

9. H. F. Southgate, R. C. Deen, J. H. Havens, and W. B. Drake, "Kentucky Research: A Flexible Pavement Design and Management System," Proceedings, Fourth International Conference on the Structural Design of Asphalt Pavements, University of Michigan, Ann Arbor, Michigan, August 1977.

10. H. F. Southgate and R. C. Deen, "Distribution of Strain Components and Work within Flexible Pavement Structures," University of Kentucky Transportation Research Program, Research Report UKTRP-86-21, Lexington, Kentucky, September 1986.

11. G. W. Sharpe, H. F. Southgate, and R. C. Deen, "Development of Pavement Thickness Designs Using Pozzolanic Base Materials," University of Kentucky Transportation Research Program, Research Report UKTRP-83-25, Lexington, Kentucky, October 1983.

12. H. F. Southgate, J. H. Havens, R. C. Deen, and D. C. Newberry, Jr., "Development of a Thickness Design System for Portland Cement Concrete Pavements," University of Kentucky Transportation Research Program, Research Report UKTRP-83-5, Lexington, Kentucky, February 1983.

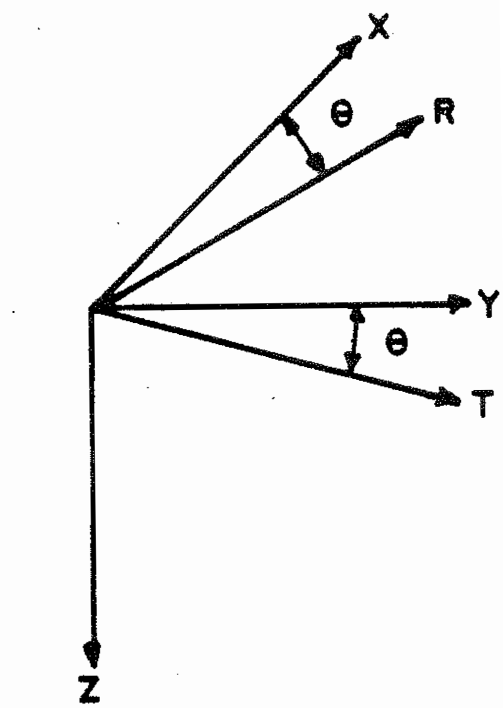
13. H. F. Southgate and R. C. Deen, "Thickness Design Curves for Portland Cement Concrete Pavements," University of Kentucky Transportation Research Program, Research Report UKTRP-84-3, Lexington, Kentucky, February 1984.

14. G. W. Sharpe, H. F. Southgate, and R. C. Deen, "Pavement Evaluation Using Road Rater Deflections," Record 700, Transportation Research Board, Washington, D. C., 1979.

15. G. W. Sharpe, H. F. Southgate, and R. C. Deen, "Dynamic Pavement Deflections," Transportation Engineering Journal, American Society of Civil Engineers, Vol 107, No. TE2, March 1981.

16. H. F. Southgate, G. W. Sharpe, R. C. Deen, and J. H. Havens, "Structural Capacity of In-Place Asphaltic Concrete Pavements from Dynamic Deflections," Proceedings, Fifth International Conference on the Structural Design of Asphalt Pavements, University of Michigan, Ann Arbor, Michigan, August 1982.

SUPERPOSITION EQUATIONS



$$\sigma_x = \sigma_R \cos^2\theta - 2\tau_{RT} \cos\theta \sin\theta + \sigma_T \sin^2\theta$$

$$\tau_{xy} = \sigma_R \cos\theta \sin\theta + \sigma_{RT}(\cos^2\theta - \sin^2\theta) - \sigma_T \sin\theta \cos\theta$$

$$\tau_{xz} = \tau_{RZ} \cos\theta - \tau_{TZ} \sin\theta$$

$$\sigma_y = \sigma_R \sin^2\theta + 2\tau_{RT} \sin\theta \cos\theta + \sigma_T \cos^2\theta$$

$$\tau_{yz} = \tau_{RZ} \sin\theta + \tau_{TZ} \cos\theta$$

$$\sigma_z = \sigma_z$$

$$\tau_{yx} = \tau_{xy}$$
$$\tau_{zx} = \tau_{xz}$$
$$\tau_{zy} = \tau_{yz}$$

Figure 1. Basic Equations by Superposition Principles.

APPENDIX A

ORIGINAL CHEVRON DOCUMENTATION

Chevron Research Company
A Standard Oil Company of California Subsidiary
576 Standard Avenue, Richmond, California
Mail Address: P. O. Box 1627, Richmond, CA 94802

April 18, 1978

Mr. Herbert F. Southgate
Chief Research Engineer
Kentucky Department of
Transportation
Division of Research
533 South Limestone
Lexington, Kentucky 40508

Dear Mr. Southgate:

You requested in your letter of April 4, 1978, our permission to publish modifications to our layered elastic computer program used for pavement design. We are delighted to grant you this permission.

Our current policy is to supply upon request the layered elastic program to universities and public agencies at no charge. With this in mind, you should not feel restricted in the use of the program. We understand you will give appropriate reference and acknowledgement in your publications to Chevron Research Company for providing the original programs. We greatly appreciate the recognition you have given us in the past.

Your proposed modifications to the program sound interesting. Your interim report on a rational overlay design method is particularly timely. I would appreciate a copy of the report, when available, and a copy of the modified elastic layered program.

You and your associates at the Kentucky Department of Transportation have done an excellent job of pioneering the field of rational pavement design. We are delighted that the Chevron program provided some assistance in accomplishing that task. We look forward to your effort on a rational overlay design method.

Sincerely,

Larry Santucci

L. E. Santucci

LES:cic

CALIFORNIA RESEARCH CORPORATION
RICHMOND, CALIFORNIA

NUMERICAL COMPUTATION OF STRESSES AND STRAINS IN A MULTIPLE-LAYERED ASPHALT PAVEMENT SYSTEM

September 24, 1963

By

H. Warren and W. L. Dieckmann

The program numerically carries out the solutions given by Mr. J. Michelow in his analysis of multilayered asphalt pavement systems. The present program computes vertical, tangential, radial, and shear stress together with vertical and radial strain at any point in the system for a given load on a circular area of the surface.

At interfaces between two layers, the stresses are computed only for the lower layer. The corresponding radial and tangential stresses for the upper layer are proportional to those given for the lower, with ratios the same as the ratio of the moduli of elasticity of the two layers.

Input

The order and format of the data cards are given in the attached input sheet. Each system to be analyzed is represented by six cards in the data deck. Any number of systems may be processed on the same run by placing sets of cards one after the other.

The stress and strain solutions may be obtained in either absolute terms (e.g., psi and inches) or relative to a unit pressure. In the latter case, the input total load and tire pressure are used only to define the radius of the loaded area. The type of solution desired (absolute or relative) is indicated on the second input card in Column 36 by a 0 (or blank) for absolute or a 1 for relative output.

Output

The input heading supplied by the user is printed out at the top of each page. The results and a list of the parameters are printed out in columns below the heading.

The Organization of the Computation

The final answers are expressed as integrals of the form

$$\int_0^\infty T(m)dm \quad . \tag{1}$$

For purposes of numerical evaluation (1) is replaced by

$$\sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} T(m)dm \quad . \tag{2}$$

Each of the integrals in (2) is evaluated using four-point Legendre-Gauss quadrature [1]. To maximize the accuracy of

Encl. - Input Sheet
Sample Output

these evaluations, the set of x_i's are chosen from the zeros of $T(m)$, such that in the interior of any interval (x_{i-1}, x_i) there is at most one zero of $T(m)$.

Two criteria determine the value of n - that is, when the integration will stop. One is the size of the integrand; the other is the number of intervals already integrated over. Specifically, the integration is terminated at x_n if either condition a or b is satisfied.

a. The integrals over (x_{n-2}, x_{n-1}) and (x_{n-1}, x_n) both have magnitude less than 10^{-4}.

b. n Reaches its maximum assigned value of 46.

Because $T(m)$ is a product of two Bessel functions times a term which decays as $K\, m^{-p} e^{-zm}$, condition (a) is an approximation to

$$\int_{x_n}^{\infty} T(m)\,dm \leq 10^{-4} . \tag{3}$$

p is an integer greater than one and K is only a function of r and z; where r is the radial coordinate and z is the vertical coordinate of a point in the elastic system. Equation (3) will be satisfied for m greater than one if

$$K\, e^{-zm_n} < 10^{-4}$$

which is equivalent to

$$zm_n > 4 \log 10 - \log K .$$

If $M = \max(a, r)$ where a is the load radius, then m_{46} is approximately equal to $71/M$. It follows that when the integration stops, condition (a) will hold if

$$\frac{z}{M} > \frac{4 \log 10 - \log K}{71} . \tag{4}$$

Conversely, condition (a) will not be satisfied if z is too small, or either a or r is too large. In concrete terms, if the point under consideration is (i) too near or on the surface of the system, or (ii) too far from the center of the load area, or (iii) if the load area is too large, then the convergence of Equation (2) to (1) is slow, and the computations of the present program are more subject to error.

It should be noted, however, that (ii) and (iii) are problem cases only because (1) is being evaluated numerically. The zeros of $T(m)$ are approximately inversely proportional to a and r; hence increasing them shortens the intervals (x_{i-1}, x_i).

The remedy for this problem is to take a great number of intervals; but to limit computing time some number u must be set as the maximum number of intervals (46 is used for u in this program).

The main work of the program lies in evaluating T(m). The parameters of T are the coordinates of the point (r,z) and all the physical constants describing the system. As shown in Michelow's report, a large part of the work of the evaluation of T(m) goes into evaluating the functions $A_j(m)$, $B_j(m)$, $C_j(m)$, and $D_j(m)$, which do not include the parameter z but depend on r. Hence, these evaluations are made once for each r, and the computation then proceeds to finding the solutions for the different z's. A further saving would be made by groupin all points (r,z) such that $r \leq a$, since in this event M = a, and the same subdivision $\langle x_i \rangle$ applies to all these points (although the cutoff point, x_n, may differ). This refinement has not been incorporated into the present program.

The evaluation of $A_j(m)$, $B_j(m)$, $C_j(m)$, and $D_j(m)$ is carried out by one of two subroutines, depending on the size of m. For large m, A_j and B_j decrease exponentially and may be assumed to be zero, in which case the computation of C_j and D_j can be proceed independently. It can be shown that C_j and also D_j asymptotically approach multiples of m^{-3} or m^{-2}, depending on the parameters of the system. Using asymptotic approximations to these variables should be considered if it is desirable to continue the integration in slowly convergent cases of type (i).

Comparison with Published Results

No known published results exist for the upper one third of the top layer. In other regions the results obtained with this program compare almost identically with those published by Jones [2] and by Mehta [3]. We have gotten good results in the difficult upper region but have experienced slow convergence as the z coordinate (depth below surface) goes to zero. The extreme case, at z = 0, where the theoretical results are known, is considered below.

Error Estimation in Slowly Convergent Cases

Whenever the integration of (1) is terminated by condition b (n = 46), the program writes the comment "SLOW CONVERGENCE" for that printed line of output. In computing the deflections, damping factors other than the exponential make it unlikely that slow convergence of type (i) will produce any error over 5%. Further, the vertical and shear stresses for z = 0 - that is, at the surface - are known from the boundary conditions, namely:

$$\text{vertical stress} = \begin{cases} -1, & r < a \\ -0.5, & r = a \\ 0, & r > o \end{cases}$$

$$\text{shear stress} = 0 \text{ for all } r$$

Comparing the computed values with these boundary conditions gives a good idea of the quality of convergence, which may well give results good to 1% even though the "SLOW CONVERGENCE" message is written out.

The results from the analysis of the following two systems which converged slowly illustrate this conclusion:

System 1

System 1 consists of two layers with a load radius of 1.5. The layers have the following characteristics:

Layer	Thickness	Modulus of Elasticity	P's Ratio
1	3.0	1000	0.25
2	∞	2000	0.25

System 2

System 2 consists of four layers with a load radius of 14.82. The layers have the following characteristics:

Layer	Thickness	Modulus of Elasticity	P's Ratio
1	2.0	20,000	0.75
2	3.0	15,000	0.50
3	4.0	12,000	0.50
4	∞	10,000	0.45

On the surface (z = 0), at radial distance (r), vertical stress was computed to be

r	System 1	System 2
0	-1.000000	-1.000000
0.5	-1.005083	-0.998351
1.0	-0.997157	-1.001935
1.45	-1.056223	-1.000057
1.50	-0.499998	-0.999376
1.55	0.048758	-0.998707
2.00	-0.008926	-0.998332
12.00	-0.003021	-0.986899

The shear stresses were all computed to be less than 10^{-6}.

Thus, we see that the computed values for System 1 are close to the theoretical values of the vertical stress which is -1 for values of r less than 1.5, -0.5 at r equal 1.50, and zero for values of r greater than 1.5. None of the computed shear stresses exceeded 10^{-6} which is close to the theoretical value of zero for all radii.

For System 2, the computed values are very close to the theoretical value of -1. In this system, r does not exceed the load radius.

Survey of the Component Routines of the Program

Main Routine

Handles input and output (except for writing the final answers) and calls the subroutines, which do all the arithmetic operations.

PART

Calculates the partition $\{x_i\}$ and the four points in each interval (x_{i-1}, x_i) at which $T(m)$ is to be evaluated for integration.

COEE and COHIGH

Computes $A_i(m)$, $B_i(m)$, $C_i(m)$, and $D_i(m)$. COHIGH works in the range where A_i and B_i are effectively zero.

BESSEL

Computes either $J_0(m)$ or $J_1(m)$ to within an absolute tolerance of less than 10^{-6}. This routine could be used in any program requiring evaluation of zero and first-order Bessel functions.

CALCIN

Completes the evaluation of T(m) and performs the integration of (2). It writes out the final results.

HIGHM

As now compiled, just prints out the message "SLOW CONVERGENCE." There is, however, a FORTRAN deck for an alternate version of HIGHM which would effectively raise the value of u on a point-by-point basis - that is, continue the integration indefinitely, or until some criterion such as an error bound or time limit was met. This alternate version of HIGHM would be used with the subroutines RANGE2-346 and CDHIGH 338, which are already compiled and fulfill the functions of CALCIN and COHIGH, respectively.

References

1. Hildebrand, F. B., "Introduction to Numerical Analysis," McGraw-Hill (1956).

2. Jones, A., "Tables of Stresses in Three-Layer Elastic Systems," Highway Research Board, Bulletin 342, 176-214 (1962).

3. Mehta, M. R., and Veletsos, A. S., "Stresses and Displacements in Layered Systems," Technical Report, Office of Naval Research, University of Illinois (1959).

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ANALYSIS OF STRESSES AND DISPLACEMENTS IN AN n-LAYERED ELASTIC SYSTEM UNDER A LOAD UNIFORMILY DISTRIBUTED ON A CIRCULAR AREA

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By

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Table of Contents

I. Introduction

Asphalt pavements may be idealized as a semiinfinite solid with three or four layers of substances whose elastic characteristics are different from one layer to the other.

In view of this, a computer program for the IBM 7090 is being prepared at the request of the Asphalt Development Group. This program will give the distribution of stresses and deformations of such an n-layered system when subject to a load uniformily distributed on a circular area on the free surface of the semiinfinite solid. The interfaces between layers are considered to be "rough."

Classical papers in this field were consulted [1;2;3]. None of these papers was considered satisfactory for several reasons; hence, we conducted our own analysis of the problem. This analysis constitutes the content of this report and possesses the following desirable characteristics:

1. The system may have any number of layers.

2. The elastic coefficients in each layer can assume any value (where the Poisson modulus ν is different from one).

3. The mathematical analysis is straightforward and self-contained.

4. The constants of integration are given explicitly and can be found by solving a system of two linear equations with two unknowns.

5. The results are given in a form convenient for a digital computer to handle.

The major contribution of this paper is that of Items 3, 4, and 5. Mehta and Veletsos[2] had also considered systems satisfying 1 and 2.

II. Description of the System

A. Geometry of the System

Consider an orthogonal system of coordinates in three dimensional space, where the x and y axes are in a horizontal plane, and the z axis, therefore vertical, is pointing downward.

Consider the semiinfinite solid consisting of all points (x, y, z) where $z \geq 0$. This solid will be divided in n "layers" L_i, $i = 1, 2, \ldots, n$ by the collection of numbers

$$h_0 = 0 < h_1 < h_2 < \ldots < h_{n-1} . \qquad (1)$$

The layer L_i will consist of all points (x, y, z) such that

$$h_{i-1} \leq z < h_i \ , \qquad (2)$$

for i = 1, 2, ... , n-1. The layer L_n will consist of all points (x, y, z) such that

$$h_{n-1} \leq z \ . \qquad (3)$$

The plane $z = h_i$ (i = 0, 1, 2, ... , n-1) will be called the i^{th}-interface.

B. Elastic Characteristics of the System

The semiinfinite solid will be assumed to be perfectly elastic, homogeneous, etc.

E_i will denote the modulus of Young and ν_i, the modulus of Poisson for layer L_i.

C. Description of the Load

We will consider a load f(x, y) normal to the 0^{th}-interface given by the following equation.

$$f(x, y) = \begin{cases} 1 & \text{whenever } x^2 + y^2 < a^2 \\ 1/2 & \text{whenever } x^2 + y^2 = a^2 \\ 0 & \text{whenever } x^2 + y^2 > a^2 \ . \end{cases} \qquad (4)$$

Later on we will see why we define f(x, y) to be 1/2, when $x^2 + y^2 = a^2$, and not one.

We will also consider a load f(x, y) normal to the 0^{th}-interface given by the equation

$$f(x, y) = p(m) \ J_0 \ [m \ (x^2 + y^2)^{1/2}] \ , \qquad (5)$$

where p(m) is an arbitrary function of the parameter m, and $J_0(t)$ denotes the Bessel function of the first kind of order zero.

The load described by equation (4) will be called Load A, and the one described by equation (5) will be called Load B.

D. Description of the Boundary Conditions

For each layer L_i the boundary conditions are the conditions that have to be satisfied at the i^{th}-interface and at the $(i-1)^{th}$-interface.

We will consider the case when the i^{th}-interface is "rough", that is, the components of stress σ_z and τ_{zr} as well as the components of the displacement u and w are continuous at $z = h_i$ ($i = 1, 2, \ldots, n-1$). (See IIIA below.)

At the 0^{th}-interface, sometimes called the free surface, σ_z has to be equal to the load and τ_{zr} has to be zero.

E. Description of the Problem

To determine the components of stress and displacement at any point (x, y, z) in a system with a geometry given by Section IIA, with elastic characteristics as the ones given in Section IIB, with boundary conditions as described in Section IID, when subject to one of the loads defined in Section IIC.

We will consider then two different problems that we will call A and B, depending on which of the loads of Section IIC is acting on the system.

As we will see later on (Section VI), the solution of Problem B leads to the solution of Problem A by means of superposition. In the following sections we will consider it advantageous to adopt a cylindrical system of coordinates: Each point is defined by the coordinates (r, θ, z) where

$$r = (x^2 + y^2)^{1/2} ,$$
$$\cos\theta = x/r, \sin\theta = y/r, \text{ and} \qquad (6)$$
$$z = z .$$

The reason for this is that the z axis is an axis of symmetry of the geometry of the system as well as of the load.

III. Basic Equations

A. For a cylindrical system of coordinates, the components of stress are:

$$\sigma_z, \sigma_r, \sigma_\theta, \tau_{zr}, \tau_{r\theta}, \tau_{z\theta} ;$$

and the components of displacement are:

u, the radial displacement,
v, the tangential displacement, and
w, the vertical displacement.

Because of the symmetry of the system under consideration

$$\tau_{r\theta} = \tau_{z\theta} = v = 0 . \qquad (7)$$

We have, then, to determine six functions: four components of stress, σ_z, σ_r, σ_θ, τ_{rz}, and two components of displacement, u, w.

B. The four components of stress have to satisfy two sets of equations called the equilibrium conditions and the compatibility conditions, respectively.

The equilibrium conditions are:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \text{ and} \qquad (8)$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} = 0. \qquad (9)$$

The compatibility conditions are:

$$\nabla^2 \sigma_r - \frac{2}{r^2}(\sigma_r - \sigma_\theta) + \frac{1}{1+\nu}\frac{\partial^2 Q}{\partial r^2} = 0, \qquad (10)$$

$$\nabla^2 \sigma_\theta + \frac{2}{r^2}(\sigma_r - \sigma_\theta) + \frac{1}{1+\nu}\frac{1}{r}\frac{\partial Q}{\partial r} = 0, \qquad (11)$$

$$\nabla^2 \sigma_z + \frac{1}{1+\nu}\frac{\partial^2 Q}{\partial z^2} = 0, \text{ and} \qquad (12)$$

$$\nabla^2 \tau_{rz} - \frac{1}{r^2}\tau_{rz} + \frac{1}{1+\nu}\frac{\partial^2 Q}{\partial r \partial z} = 0. \qquad (13)$$

Here

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}, \qquad (14)$$

$$Q = \sigma_r + \sigma_\theta + \sigma_z, \text{ and} \qquad (15)$$

ν is Poisson's modulus.

C. We can simplify the problem by expressing the four components of stress and the two components of displacement in the following way:

$$\sigma_r = \frac{\partial}{\partial z}\left[\nu \nabla^2 \phi - \frac{\partial^2 \phi}{\partial r^2}\right], \qquad (16)$$

$$\sigma_\theta = \frac{\partial}{\partial z}\left[\nu \nabla^2 \phi - \frac{1}{r}\frac{\partial \phi}{\partial r}\right], \qquad (17)$$

$$\sigma_z = \frac{\partial}{\partial z}\left[(2-\nu)\nabla^2 \phi - \frac{\partial^2 \phi}{\partial z^2}\right], \qquad (18)$$

$$\tau_{rz} = \frac{\partial}{\partial r}\left[(1-\nu)\nabla^2 \phi - \frac{\partial^2 \phi}{\partial z^2}\right], \qquad (19)$$

$$u = -\frac{1+\nu}{E}\frac{\partial^2 \phi}{\partial r \partial z}, \text{ and} \tag{20}$$

$$w = \frac{1+\nu}{E}\left[2(1-\nu)\nabla^2\phi - \frac{\partial^2 \phi}{\partial z^2}\right]; \tag{21}$$

where $\phi(r, z)$ is a so-called Airy's function.

By expressing σ_r, σ_θ, σ_z, and τ_{rz} in this way, we automatically satisfy equation (8). Moreover, equations (9) through (13) are also satisfied whenever ϕ is a solution of

$$\nabla^4\phi = 0. \tag{22}$$

The problem has been reduced to finding one function ϕ that satisfies equation (22) and the boundary conditions of the problem under consideration.

D. It is easy to check that equation (22) is satisfied by the function

$$\phi(r, z, m) = J_o(mr)[(A + Bz)e^{mz} + (C + Dz)e^{-mz}], \tag{23}$$

where $J_n(t)$ is the Bessel function of the first kind of order n, and m is a parameter. A, B, C, and D are constants of integration; hence they do not depend on r and z, but they may be functions of the parameter m.

IV. Boundary Conditions

A. It follows from the considerations of the previous section that in the layer L_1, the solution of the problem will be given by the Airy's function:

$$\phi_1(r, z, m) = J_o(mr)[(A_1 + B_1 z)e^{mz} + (C_1 + D_1 z)e^{-mz}]; \tag{24}$$

where the constants A_1, B_1, C_1, and D_1 should be determined in such a way that the boundary conditions are satisfied.

The condition of continuity at the interfaces is not difficult to fulfill. [See equation (31).]

B. Equation (24), together with equations (16) through (21), give:

$$\sigma_z^i = m^2 J_o(mr) \{(1-2\nu_i)[B_i e^{mz}+D_i e^{-mz}]-m[(A_i+B_i z)e^{mz} -(C_i+D_i z)e^{-mz}]\}, \qquad (25)$$

$$\tau_{rz}^i = m^2 J_1(mr) \{2\nu_i[B_i e^{mz}-D_i e^{-mz}]+m[(A_i+B_i z)e^{mz} +(C_i+D_i z)e^{-mz}]\}, \qquad (26)$$

$$u^i = \frac{1+\nu_i}{E_i} mJ_1(mr) \{B_i e^{mz}+D_i e^{-mz}+m[(A_i+B_i z)e^{mz} -(C_i+D_i z)e^{-mz}]\}, \qquad (27)$$

$$w^i = \frac{1+\nu_i}{E_i} mJ_o(mr) \{2(1-2\nu_i)[B_i e^{mz}-D_i e^{-mz}]-m[(A_i+B_i z)e^{mz} +(C_i+D_i z)e^{-mz}]\}, \qquad (28)$$

$$\sigma_r^i = m^2 J_o(mr) \{(1+2\nu_i)[B_i e^{mz}+D_i e^{-mz}]+m[(A_i+B_i z)e^{mz} -(C_i+D_i z)e^{-mz}]\}-m^2\frac{J_1(mr)}{mr} \{B_i e^{mz}+D_i e^{-mz} + m[(A_i+B_i z)e^{mz} -(C_i+D_i z)e^{-mz}]\}, \text{ and} \qquad (29)$$

$$\sigma_\theta^i = 2\nu_i m^2 J_o(mr)[B_i e^{mz} + D_i e^{-mz}]+m^2\frac{J_1(mr)}{mr} \{B_i e^{mz}+D_i e^{-mz} +m[(A_i+B_i z)e^{mz}-(C_i+D_i z)e^{-mz}]\}. \qquad (30)$$

C. Consider the matrix

$$S_i(z) = \left\| \begin{matrix} \sigma_z^i \\ \tau_{rz} \\ u^i \\ w^i \end{matrix} \right\| .$$

Continuity conditions at the i^{th}-interface can be expressed by

$$S_i(h_i - 0) = S_{i+1}(h_i) , \qquad (31)$$

$i = 1, 2, \ldots , n-1$. Here $S_i(h_i - 0)$ stands for $\lim_{\epsilon \to 0} S_i(h-|\epsilon|)$.

From equation (25) we see that (in any Layer L_i) σ_z^i is proportional to $J_o(mr)$. Therefore, the Airy's function given by equation (24) allows only a load normal to the 0^{th}-interface that is proportional to $J_o(mr)$. This is the case in Problem B.

V. Solution of Problem B

A. To solve Problem B, we need only to determine the constants A_i, B_i, C_i, and D_i $(i = 1, 2, \ldots, n)$ that fulfill the conditions given by equation (31) and that are compatible with Load B. See last paragraph of Section IID.

We have 4n constants to determine; equation (31) gives us 4n-4 equations. The compatibility of the stresses with Load B gives us two more equations. The fact that at infinity the stresses should be zero allows us to conclude that A_n and B_n should be zero; this gives us the last two equations needed.

B. From equations (25) through (28), we see that the matrix $S_i(z)$ can be expressed in the following form:

$$S_i(z) = K(\nu_i,E_i)M(z,\nu_i)D(z) \begin{Vmatrix} A_i \\ B_i \\ C_i \\ D_i \end{Vmatrix} , \tag{32}$$

where

$$K(\nu,E) = \begin{Vmatrix} -m^2 J_0(mr) & & & \\ & m^2 J_1(mr) & & \\ & & \frac{1+\nu}{E} mJ_1(mr) & \\ & & & -\frac{1+\nu}{E} mJ_0(mr) \end{Vmatrix}, \tag{33}$$

$$M(z,\nu) = \begin{Vmatrix} 1 & mz + 2\nu - 1 & -1 & -mz + 2\nu - 1 \\ 1 & mz + 2\nu & 1 & mz - 2\nu \\ 1 & mz + 1 & -1 & -mz + 1 \\ 1 & mz + 4\nu - 2 & 1 & mz - 4\nu + 2 \end{Vmatrix}, \tag{34}$$

and

$$D(z) = \begin{Vmatrix} me^{mz} & & & \\ & e^{mz} & & \\ & & me^{-mz} & \\ & & & e^{-mz} \end{Vmatrix} . \tag{35}$$

C. Therefore, equation (31) can be written as:

$$K(\nu_i,E_i)M(h_i,\nu_i)D(h_i) \begin{Vmatrix} A_i \\ B_i \\ C_i \\ D_i \end{Vmatrix} = K(\nu_{i+1},E_{i+1})M(h_i,\nu_{i+1})D(h_i) \begin{Vmatrix} A_{i+1} \\ B_{i+1} \\ C_{i+1} \\ D_{i+1} \end{Vmatrix} \tag{36}$$

Hence, we can express A_i, B_i, C_i, and D_i as functions of A_{i+1}, B_{i+1}, C_{i+1}, and D_{i+1} by

$$\begin{Vmatrix} A_i \\ B_i \\ C_i \\ D_i \end{Vmatrix} = D^{-1}(h_i)M^{-1}(h_i,\nu_i)K^{-1}(\nu_i,E_i)K(\nu_{i+1},E_{i+1})$$

$$M(h_i,\nu_{i+1})D(h_i)\begin{Vmatrix} A_{i+1} \\ B_{i+1} \\ C_{i+1} \\ D_{i+1} \end{Vmatrix} . \tag{37}$$

Note that

$$K^{-1}(\nu_i,E_i)K(\nu_{i+1},E_{i+1}) = \begin{Vmatrix} 1 & & & \\ & 1 & & \\ & & L_i & \\ & & & L_i \end{Vmatrix} , \tag{38}$$

where

$$L_i = \frac{E_i}{E_{i+1}} \; \frac{1 + \nu_{i+1}}{1 + \nu_i} . \tag{39}$$

It is easy to check that

$$M^{-1}(z,\nu_i) = \frac{1}{4(\nu_i-1)} \begin{Vmatrix} -mz-1 & mz+4\nu_i-2 & mz+2\nu_i-1 & -mz-2\nu_i \\ 1 & -1 & -1 & 1 \\ -mz+1 & -mz+4\nu_i-2 & mz-2\nu_i+1 & mz-2\nu_i \\ 1 & 1 & -1 & -1 \end{Vmatrix} \tag{40}$$

Let us define the matrix X_i by

$$X_i = 4M^{-1}(h_i,\nu_i)K^{-1}(\nu_i,E_i)K(\nu_{i+1},E_{i+1})M(h_i,\nu_{i+1})(\nu_i-1) , \tag{41}$$

thus

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$$X_i = \left\| \begin{array}{c|c|c|c}
(4\nu_i-3)-L_i & \begin{array}{l}-q(-h_i,\nu_i,\nu_{i+1}) \\ +L_i q(-h_{i+1},\nu_{i+1},\nu_i)\end{array} & (2mh_i+4\nu_i-1)(1-L_i) & \begin{array}{l}p(h_i,\nu_i,\nu_{i+1}) \\ -L_i p(-h_i,\nu_{i+1},\nu_i)\end{array} \\ \hline
0 & L_i(4\nu_{i+1}-3)-1 & 2(L_i-1) & (2mh_i-4\nu_{i+1}+1)(L_i-1) \\ \hline
(2mh_i-4\nu_i+1)(L_i-1) & \begin{array}{l}-p(-h_i,\nu_i,\nu_{i+1}) \\ +L_i p(h_i,\nu_{i+1},\nu_i)\end{array} & (4\nu_i-3)-L_i & \begin{array}{l}q(h_i,\nu_i,\nu_{i+1}) \\ -L_i q(h_i,\nu_{i+1},\nu_i)\end{array} \\ \hline
2(1-L_i) & (2mh_i+4\nu_{i+1}-1)(1-L_i) & 0 & L_i(4\nu_{i+1}-3)-1
\end{array} \right\|$$

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where

$$p(h, \nu, \mu) = m^2(2h^2) + 4mh(\nu-\mu) + (1-8\nu\mu + 2\mu) \tag{43}$$

and

$$q(h, \nu, \mu) = 2mh(2\nu-1) - (1+8\nu\mu-6\mu) \ . \tag{44}$$

Finally, we can write equation (37) as

$$\begin{Vmatrix} A_i \\ B_i \\ C_i \\ D_i \end{Vmatrix} = \frac{D^{-1}(h_i)\ X_i\ D(h_i)}{4(\nu_i-1)} \begin{Vmatrix} A_{i+1} \\ B_{i+1} \\ C_{i+1} \\ D_{i+1} \end{Vmatrix} . \tag{45}$$

We can, therefore, by repeated use of equation (45), express the set of constants A_i, B_i, C_i, and D_i as functions of C_n and D_n for all $i = 1, 2, \ldots, n-1$. We thus obtain

$$\begin{Vmatrix} A_i \\ B_i \\ C_i \\ D_i \end{Vmatrix} = \prod_{j=1}^{n-1} \frac{D^{-1}(h_j)\ X_j\ D(h_j)}{4(\nu_j-1)} \begin{Vmatrix} 0 \\ 0 \\ C_n \\ D_n \end{Vmatrix} . \tag{46}$$

D. From equation (26), using the condition $(\tau^1_{rz})_{z=0} = 0$, we obtain

$$mA_1 + 2\nu_1 B_1 + mC_1 - 2\nu_1 D_1 = 0 \ . \tag{47}$$

From equation (25), imposing the condition that

$$(\sigma^1_z)_{z=0} = -p(m)J_o(mr) \tag{48}$$

we obtain

$$mA_1 + (2\nu_1-1)B_1 - mC_1 + (2\nu_1-1)D_1 = p(m)/m^2 = K(m) \ . \tag{49}$$

Therefore, we can write the matrix equation

$$\begin{Vmatrix} 0 \\ K(m) \end{Vmatrix} = \begin{Vmatrix} m & 2\nu_1 & m & -2\nu_1 \\ m & 2\nu_1-1 & -m & 2\nu_1-1 \end{Vmatrix} \begin{Vmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{Vmatrix} . \tag{50}$$

E. The constants C_n and D_n can be determined by combining equations (50) and (46). We thus obtain

$$\begin{Vmatrix} 0 \\ K(m) \end{Vmatrix} = \begin{Vmatrix} m & 2\nu_1 & m & -2\nu_1 \\ m & 2\nu_1-1 & -m & 2\nu_1-1 \end{Vmatrix} \prod_{j=1}^{n-1} \frac{D^{-1}(h_j)X_jD(h_j)}{4(\nu_j-1)} \begin{Vmatrix} 0 \\ 0 \\ C_n \\ D_n \end{Vmatrix} \tag{51}$$

This gives a system of two linear equations in two unknowns whose solution is immediate. (We do not expect the determinant of the system to be zero.)

F. In summary, Problem B is solved by equations (25) through (30), where C_n and D_n are determined from equation (51) and A_i, B_i, C_i, and D_i are given by equation (46), (i = 1, 2, ... , n-1).

VI. Solution of Problem A

A. From equation (51) we see that C_n and D_n are proportional to p(m); therefore, equation (46) tells us that A_i, B_i, C_i, and D_i are proportional to p(m) for all i = 1, 2,... , n-1. We can conclude from equations (25) through (30) that σ_z^i, τ_{rz}^i, σ_θ^i, σ_r^i, u^i, and w^i are proportional to p(m) for i = 1, 2,... , n.

Therefore, we can put $K(m) = 1/m^2$ in equation (49) and write the factor p(m) in front of equations (25) through (30).

B. Because equations (16) through (21), as well as equation (22), are linear in ϕ, whenever the Airy's function $\phi(r, z, m_j)$ solves Problem B, then the function

$$\phi(r, z) = \sum_{j=1}^{n} \Delta_j \phi(r, z, m_j) \tag{52}$$

solves the problem of finding stresses and displacement when the system is subject to a load

$$\sum_{j=1}^{n} \Delta_j p(m_j) J_o(m_j r) \ . \tag{53}$$

Therefore, the function

$$\phi(r, z) = \int_0^\infty \phi(r, z, m)dm \ , \tag{54}$$

will solve the problem of finding stresses and displacements when the system is subject to a load.

$$\int_0^\infty p(m)J_o(mr)dm \ . \tag{55}$$

C. The inversion formula for the Hankel transform of order zero is:

$$\tfrac{1}{2}[g(r-0)+g(r+0)] = \int_0^\infty m[\int_0^\infty rg(r)J_o(mr)dr]J_o(rm)dm \ , \tag{56}$$

provided g(r) satisfies a certain set of conditions that we will call Condition C [6, p. 52].

In particular we see that if we chose

$$p(m) = m\int_0^\infty rf(r)J_0(mr)dr \ , \quad (57)$$

where $f(r)$ is the function defined by equation (4), then equation (55) gives Load A. This is permissible because the function $f(r)$ satisfies Condition C.

We see now why equation (4) takes that rather peculiar form at $r = a$. Integrating equation (57) we get.

$$p(m) = aJ_1(ma) \ . \quad (58)$$

D. In summary, the chain of reasoning is the following:

1. We chose an Airy's function $\phi(r, z, m)$ as the one given by equation (23).

2. We chose a Load B as the one given in equation (5) where $p(m)$ is defined by equation (58).

3. By the last paragraph of Section VIA, we see that the stresses and displacements are given by equations (25) through (30) with a factor $aJ_1(ma)$ in front of each equation.

4. The constants A_i, B_i, C_i, D_i, $(i = 1, 2, \ldots, n-1)$ C_n and D_n are given as functions of the parameter m; and $A_n = B_n = 0$, by equations (51) and (46), where $K(m) = 1/m^2$.

5. Then we chose an Airy's function given by

$$\phi(r, z) = \int_0^\infty \phi(r, z, m)dm \ ,$$

where $\phi(r, z, m)$ is the function chosen in Step 1.

6. Sections VIB and VIC tell us that the Airy's function $\phi(r, z)$ solves Problem A.

7. The stresses and displacements defined by the function $\phi(r, z)$ are:

$$\overline{\sigma}_z^i = a \int_0^\infty J_1(ma)\sigma_z^i \, dm \ , \quad (59)$$

$$\overline{\sigma}_r^i = a \int_0^\infty J_1(ma)\sigma_r^i \, dm \ , \quad (60)$$

$$\overline{\sigma}_\theta^i = a \int_0^\infty J_1(ma)\sigma_\theta^i \, dm \ , \quad (61)$$

$$\overline{\tau}_{rz}^i = a \int_0^\infty J_1(ma)\tau_{rz}^i dm \ , \quad (62)$$

$$\overline{u}^i = a \int_0^\infty J_1(ma)u^i \, dm \ , \quad (63)$$

and

$$\overline{w}^i = a \int_0^\infty J_1(ma) w^i dm, \quad (i = 1, 2, \ldots, n) \, . \tag{64}$$

σ_z^i, σ_r^i, σ_θ^i, τ_{rz}^i, u^i, and w^i are given by equations (25) through (30). The constants being determined as in Step 4. This finally solves Problem A.

VII. Bibliography

1. A. Jones, "Tables of Stresses in Three-Layered Elastic Systems," Highway Research Board, Bull. 342 (1962).

2. M. R. Mehta and A. S. Veletsos, "Stresses and Displacements in Layered Systems," Civil Engineering Studies, Structural Research Series, No. 178. University of Illinois (1959).

3. D. M. Burmister, "The General Theory of Stresses and Displacements in Layered Systems," J. Appl. Phys. Vol. 16 (1945).

4. S. Timoshenko, "Theory of Elasticity," McGraw-Hill, New York (1934).

5. A. E. H. Love, "A Treatise on the Mathematical Theory of Elasticity," Dover, New York (1944).

6. I. N. Sneddon, "Fourier Transforms," McGraw-Hill, New York (1951).

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Chevron

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A. L. Grossberg
Vice-President

May 9, 1980

Chevron N-Layer Computer
Program - Improved Accuracy

Mr. Herbert F. Southgate
Chief Research Engineer
Kentucky Department of Transportation
533 South Limestone
Lexington, KY 40508

Dear Mr. Southgate:

Attached is a report by Mr. L. J. Painter of our staff describing a new subroutine COFE for the Chevron N-Layer Elastic System computer program. This new routine, which completely replaces the two earlier routines, COE5 and CO15, makes accurate calculations of all stress, strain, and deflection parameters for any number of layers, up to the maximum program dimension of 15 layers.

Additionally, the new subroutine will provide computer run time savings of as much as 45%.

If you have previously received a FORTRAN source deck of the program from us, you will be sent a source deck for the new subroutine.

Very truly yours,

A. L. Grossberg

Encl. - Report, above subject
and date

CHEVRON RESEARCH COMPANY
RICHMOND, CALIFORNIA

CHEVRON N-LAYER COMPUTER PROGRAM - IMPROVED ACCURACY

MAY 11, 1980

Author - L. J. Painter

Summary

This report describes a new subroutine COFE to replace the two subroutines COE5 and CO15. The new routine eliminates the serious numerical errors in CO15 (used for more than 5 layers) which have hampered the use of the program in research on asphalt pavement systems. The results using the new subroutine are every bit as accurate as those obtained from the older subroutine COE5 used for 5 or fewer layers, which is excellent. A listing of the new subroutine is attached.

To use the new routine, it is only necessary to remove the original routines COE5 and CO15, replacing them by the single new routine COFE. COFE has two entry points, COE5 and CO15, so that no change is necessary in the calling (MAIN) program.

A further benefit of the new routine is that typical execution times will be reduced as much as 45%.

Background

The original version of the Chevron n-layer program as developed in 1963 was limited to 5 or fewer layers. An ingenious method of scaling the calculations was used to avoid overflows (generation of numbers larger than can be handled by the computer). This required, however, separate sections of computer code in subroutine COEE for each possible layer. [Subroutine CØEE computed the coefficient matrices A(I,K), B(I,K), C(I,K), and D(I,K) for the K-th layer at the I-th level of the variable of integration AZ(I).]

In 1967 when the program was expanded to handle up to 15 layers, it was found impractical to continue this programming scheme. Hence, a separate and much simpler subroutine CO15 was written to handle the general n-layer case. Unfortunately, numerical difficulties arose because numbers in excess of $e^{174.67}$ were generated in the calculation of the intermediate product matrices PM, leading to overflow. The power of e as used in parts of the calculations is 2*AZ(I)*H(N); AZ

Encl. - Table I
Program Listing

could be as high as $\pi \cdot 23$/radius of the loaded area and H(N) is the depth of the lowest interface between layers (in some research problems, as much as 300 inches). With a load radius of about 10 inches, this could give exponents of $2*\pi*23*300/10 = 4335$, far in excess of the (IBM) computer capacity when exponentiated.

Several reviews of the computer algorithm in CO15 were unsuccessful in solving the problem, so a simple check was made on the magnitude of the offending value and if it approached the critical level, further calculation was halted and the remaining A, B, C, and D values were set equal to zero; hopefully they would have been small anyway. This truncation of the series to be integrated gave rise to numerical errors which were at least annoying for pavement structures of moderate maximum depth. But for deep structures, such as in studies of the effect of subsoil elastic properties changing with depth, the errors could be disastrous.

The original 5-layer routine (renamed COE5) was kept to preserve accuracy for systems of 5 or fewer layers, with CO15 being used only for more than 5 layers.

Solution of Problem

The calculation of the product matrices PM has been reorganized so as to include the scaling by $e^{-AZ[H(N)\pm H(K)]}$ from the very start. This led to a simplification of the exponential multipliers required. After initialization of PM(N,M,J), M = 1,4 and J = 3,4, we obtain:

for M = 1 or 2,

$$PM(K,M,J) = T1*e^{0} + T3*e^{-2P*H(K)};$$

for M = 3 or 4,

$$PM(K,M,J) = T1*e^{2P*H(K)} + T3*e^{0},$$

with T1 = X(K,M,1)*PM(K+1, 1,J)
 + X(K,M,2)*PM(K+1, 2,J),

T3 = X(K,M,3)*PM(K+1, 3,J)
 + X(K,M,4)*PM(K+1, 4,J),

and P = AZ(I).

Detailed study of the course of the calculations has shown that by the time e^{2P*H_K} becomes unmanageable, T1 has been driven to zero.

Judicious checking of the magnitude of the exponent makes it possible to keep the calculations under control at all times.

As a result, it is possible to substitute the new routine COFE for the two older routines COE5 and CO15. Use of entry points COE5 and CO15 in COFE obviates the need for modifications to the MAIN (calling) program.

Timing

Computer runs on a 5-layer problem using the new COFE and the old COE5 routines show a 45% reduction in CPU time (1.15 seconds versus 2.08 seconds). No meaningful comparisons can be made with CO15 because that routine was effectively short-circuited as soon as potential overflow occurred.

Verification of Accuracy

A check on the accuracy has been made between a 4-layer solution (known to be accurate) and a 6-layer problem artificially constructed by splitting one layer into three with identical elastic properties.

Basic problem parameters were: 30,000 lb load, 105 psi tire pressure.

Layer, i	Ei	ni	Thickness, In.
1	150,000	0.35	3
2	17,800	0.40	35
3	6,000	0.40	300
4	555,555	0.40	Semi-infinite

The 6-layer problem split layer 3 above into 3 layers of 100-inches thickness each. The calculated values of several stresses and strains are compared in Table I. In general, the new COFE routine agrees with the 4-layer solution to about 5 significant digits, compared to discrepancies by a factor of almost 2 for the old program at the surface.

References

1. Analysis of Stresses and Displacements in an N-Layered Elastic System under a Load Uniformly Distributed on a Circular Area, by J. Michelow, September 24, 1963.

2. Numerical Computation of Stresses and Strains in a Multiple-Layered Asphalt Pavement System, by H. Warren and W. L. Dieckman, September 24, 1963.

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TABLE I

COMPARISONS OF NEW AND OLD CALCULATIONS
IN LAYERED ELASTIC SYSTEMS

Vertical Stress Values
6-Layer

R	Z	COE5	COFE	CO15
0	0	-105.00	-105.00	-63.29
0	3	-96.72	-96.72	-57.19
0	38	-5.63	-5.63	-5.19
19	0	0.47	0.47	-7.33
19	3	-0.82	-0.82	-8.27
19	38	-3.71	-3.71	-3.73

Tangential Strain

R	Z	COE5	COFE	CO15
0	0	-968.85	-968.83	-491.14
0	3	794.50	794.47	331.02
0	38	408.44	408.44	402.34
19	0	-201.35	-201.35	-252.48
19	3	67.69	67.68	120.22
19	38	314.83	314.84	314.99

Vertical Deflection

R	Z	COE5	COFE	CO15
0	0	0.1013	0.1013	0.0846
0	3	0.1003	0.1003	0.0841
0	38	0.0425	0.0425	0.0424
19	0	0.0408	0.0408	0.0443
19	3	0.0410	0.0410	0.0444
19	38	0.0335	0.0335	0.0335

```
      SUBROUTINE COFE(KIN)
C        USE FOR ALL PROBLEMS, UP TO MAX DIMENSION OF 15 LAYERS
C     REPROGRAMMED 1 MAY 1980 BY L J PAINTER - EXCELLENT ACCURACY
C       NOTE DOUBLE ENTRIES FOR COE5 & CO15 PREVIOUSLY USED
C       ******SUBROUTINE COFE - N-LAYER ELASTIC SYSTEM ******
C
      COMMON   /RMCOY/RR(99),  ZZ(99),   E(15),    V(15),    HH(14),
     1               H(14),    AZ(396),  A(396,15),B(396,15),C(396,15),
     2               D(396,15),AJ(396),  RJ1(396), RJ0(396), TITLE(20),
     3               TEST(11), BZ(100),  X(15,4,4),SC(14),   FN(4),
     4              PH(14,4,4),R,        Z,        AR,       NS,
     5               N,        L,        ITN,      RSZ,      RSR,
     6               RCM,      RNU,      SF,       CSZ,      CST,
     7               CSR,      CTR,      CCM,      CNU,      PSI,
     8               NLINE,    NOUTP,    NTEST,    I,        ITN4,
     9               K,        LC,       JT,       TZZ,      PR,
     A               PA,       P,        ZP,       TIP,      TIM,
     B               T1,       T2,       T3,       T4,       T5,
     C               T6,       T2P,      TCM,      WA,       BJ1,
     D               BJ0,      ZF,       SZ1,      SZ2,      SG1,
     E               SG2,      PH,       PH2,      VK2,      VKP2,
     F               VK4,      VKP4,     VKK8,     RDT,      RDS
      REAL*4 Q(2,2)
      ENTRY  COE5(KIN)
      ENTRY  CO15(KIN)
         LC = KIN
C5-MX           SET UP MATRIX X =DI*MI*KI*K*M*D
C     COMPUTE THE MATRICES X(K)
    1 DO 10 K=1,N
      T1=E(K)*(1.0+V(K+1))/(E(K+1)*(1.0+V(K)))
      T1M=T1-1.0
      PH=PHH(K)
      PH2=PH*2.0
      VK2=2.0*V(K)
      VKP2=2.0*V(K+1)
      VK4=2.0*VK2
      VKP4=2.0*VKP2
      VKK8=8.0*V(K)*V(K+1)
C
      X(K,1,1)=VK4-3.0-T1
      X(K,2,1)=0.0
      X(K,3,1)=T1M*(PH2-VK4+1.0)
      X(K,4,1)=-2.0*T1M*P
C
      T3=PH2*(VK2-1.0)
      T4=VKK8+1.0-3.0*VKP2
      T5=PH2*(VKP2-1.0)
      T6=VKK8+1.0-3.0*VK2
C
      X(K,1,2)=(T3+T4-T1*(T5+T6))/P
      X(K,2,2)=T1M*(VKP4-3.0)-1.0
      X(K,4,2)=T1M*(1.0-PH2-VKP4)
C
      X(K,3,4)=(T3-T4-T1*(T5-T6))/P
C
```

```
      T3=PH2*PM-VKM8+1.0
      T4=PH2*(VK2-VKP2)
C
      X(K,1,4)=(T3+T4+VKP2-T1*(T3+T4+VK2))/P
      X(K,3,2)=    (-T3+T4-VKP2+T1*(T3-T4+VK2))/P
C
      X(K,1,3)=T1M*(1.0-PH2-VK4)
      X(K,2,3)=2.0*T1M*P
      X(K,3,3)=VK4-3.0-T1
      X(K,4,3)=0.0
C
      X(K,2,4)=T1M*(PH2-VKP4+1.0)
      X(K,4,4)=T1*(VKP4-3.0)-1.0
C     K = K
   10 CONTINUE
C     COMPUTE THE PRODUCT MATRICES FM
      SC(N)=4.0*(V(N)-1.0)
      IF (N-2) 13,11,11
   11 DO 12 K1=2,N
      M=N3-K1
      SC(M)=SC(M+1)*4.0*(V(M)-1.0)
   12 CONTINUE
   13 CONTINUE
C
      Q(1,1)=1.
      Q(2,2)=1.
      Q(1,2)=0.
      CQ     =P*2.*H(N)
      IF (CQ-172.) 15,15,16
   15 CONTINUE
      Q(1,2)=EXP(-CQ)
C     Q(2,1) IS NOT NEEDED FOR INITIALIZING THE FM MATRIX
   16 CONTINUE
C      20 LOOP INITIALIZES FM(N,,)
      DO 20 M=1,4
      LL=(M+1)/2
      DO 20 J=3,4
      FM(N,M,J)= X(N,M,J) *      Q(LL,2)
   20 CONTINUE
      DO 26 K1=2,N
      K=N3-K1
      KK=K+1
      CQ     =P*2.*H(K)
      IF (CQ-172.) 22,22,23
   22 CONTINUE
      Q(2,1)=EXP(CQ)
      Q(1,2)=1./Q(2,1)
      GO TO 24
   23 CONTINUE
      Q(1,2)=0.
      Q(2,1)=1.E20
   24 CONTINUE
      DO 25 M=1,4
      LL=(M+1)/2
      DO 25 J=3,4
```

```
      FM(K,M,J)=( X(K,M,1) * FM(KK,1,J)                                 COF01110
     2          +X(K,M,2) * FM(KK,2,J) ) * Q(LL,1)                      COF01120
     3         +( X(K,M,3) * FM(KK,3,J)                                 COF01130
     4          +X(K,M,4) * FM(KK,4,J) ) * Q(LL,2)                      COF01140
   25 CONTINUE                                                          COF01150
   26 CONTINUE                                                          COF01160
C     SOLVE FOR C(NS) AND D(NS)                                         COF01170
                                                                        COF01180
      T3=2.0*V(1)                                                       COF01190
      T4 =T3-1.0                                                        COF01200
      FM(1)= P*(FM(1,1,3)+FM(1,3,3)) + T3*(FM(1,2,3)-FM(1,4,3))         COF01210
      FM(2)= P*(FM(1,1,3)-FM(1,3,3)) + T4*(FM(1,2,3)+FM(1,4,3))         COF01220
      FM(3)= P*(FM(1,1,4)+FM(1,3,4)) + T3*(FM(1,2,4)-FM(1,4,4))         COF01230
      FM(4)= P*(FM(1,1,4)-FM(1,3,4)) + T4*(FM(1,2,4)+FM(1,4,4))         COF01240
      DFAC=SC(1)/((FM(1)*FM(4)-FM(3)*FM(2))*P*P)                        COF01250
         A(LC,NS) = 0.0                                                 COF01260
         B(LC,NS) = 0.0                                                 COF01270
         C(LC,NS) = -FM(3)*DFAC                                         COF01280
         D(LC,NS) =  FM(1)*DFAC                                         COF01290
C     BACKSOLVE FOR THE OTHER A,B,C,D                                   COF01300
      DO 91 K1=1,N                                                      COF01310
      A(LC,K1)=(FM(K1,1,3)*C(LC,NS)+FM(K1,1,4)*D(LC,NS))/SC(K1)         COF01320
      B(LC,K1)=(FM(K1,2,3)*C(LC,NS)+FM(K1,2,4)*D(LC,NS))/SC(K1)         COF01330
      C(LC,K1)=(FM(K1,3,3)*C(LC,NS)+FM(K1,3,4)*D(LC,NS))/SC(K1)         COF01340
   91 D(LC,K1)=(FM(K1,4,3)*C(LC,NS)+FM(K1,4,4)*D(LC,NS))/SC(K1)         COF01350
  100 CONTINUE                                                          COF01360
      RETURN                                                            COF01370
      END                                                               COF01380
**************************************************************************************
```

APPENDIX B

KENTUCKY VERSION OF
CHEVRON N-LAYER COMPUTER PROGRAM

```
C$JOB           ,LIST,T=6000
C                 MODIFIED CHEVRON N-LAYERED PROGRAM
C
C                      AS OF 3/17/87
C
C                 MODIFIED BY H. F. SOUTHGATE
C
C             THIS VERSION USES COEF IN PLACE OF COE5 AND CO15
C
C             THIS VERSION HAS 3 OPTIONS ONLY.
C
C
C
C
C191MN  ****** MAIN ROUTINE - N-LAYER ELASTIC SYSTEM ******
      COMMON/CALMAN/RJ1(396),RJ0(396),AJ(396),TZZ,L,Z
      COMMON/CALPRN/CSZ,CST,CSR,CTR,COM,RDS,RDT,RDZ,SST,JT
      COMMON/CAMAPA/AZ(396),AR,ITN,ITN4,NTEST,TEST(99)
      COMMON/CAPP7M/BZ(100),NLINE,R
      COMMON/COMAIN/H(15),P
      COMMON/DON/        NPAGE
      COMMON/GARY/IM(15),IT(15),IH(15)
      COMMON/HERB1/WORK(99),STENDN(99),ASTEN(99),WKSTRS(99),ZZZ(99)
      COMMON/HERB2/    THETA(99)
      COMMON/HERB3/ IZI,CSXX(99),CSXY(99),CSXZ(99),CSYY(99),CSYZ(99),
     1 CSZZ(99),CSOM(99),CCOM(99)
      COMMON/HERB4/RRCOM(99),TR(99)
      COMMON/HERB5/CRXX(99),CRYY(99),CRZZ(99),CRXY(99),CRXZ(99),CRYZ(99)
      COMMON/JESSE1/RSTB(7,99),RSTS(7,99),RLB(7,99),RLS(7,99),DFB(7,99),
     1 DFS(7,99)
      COMMON/JESSE2/REPB(2),REPS(2),STWB(2),STWS(2)
      COMMON/JESSE3/AJX,AJY
      COMMON/RDA/NS,IZ,E(15),V(15),HH(15),ZZ(99),N,TITLE(20),LLLL(99)
      COMMON/RD0/OPTN
      COMMON/RD1/PSI,WGT,RR(99),IR
      COMMON/RD2/IB2,IB3,IA,IRA,AJXX(99),AJYY(99)
      COMMON/RD3/RX(99),RY(99),BPSI(99),BWGT(99),JRUN,KST
      COMMON/RD4/IEQB,IEQS,JZLB,JZLS,JAI,BINCLD,KAI
      COMMON/JESSE5/JA
      INTEGER OPTN,OUTPUT
      DIMENSION IE(15)
      DATA   ASTER, PERD/4H****, 4H..../
C     COMMON STATEMENTS 'RDA', 'RD0', 'RD1', 'RD2', 'RD3', AND 'RD4'
C        ARE USED IN SUBROUTINES 'READ1', 'READ2', 'READ3', 'READ4', AND
C        'READ5'.  SUBROUTINE 'READ5' ALSO REQUIRES COMMON STATEMENTS
C        'HERB2', 'JESSE2', AND 'JESSE3' FOR ADDITIONAL ARGUMENTS.
C     COMMON STATEMENTS 'DON', 'GARY', 'HERB1', 'HERB2', 'HERB3',
C        'HERB4', 'HERB5', 'JESSE1', 'JESSE2', 'JESSE3', 'RDA', 'RD0',
C        'RD1', RD2', RD3', AND 'RD4' CONTAIN ARGUMENTS USED IN 'WRITE'
C        STATEMENTS.
C     COMMON STATEMENT 'COMAIN' CONTAINS ARGUMENTS USED IN MAIN AND
C        SUBROUTINES COE5 AND CO15.
C     COMMON STATEMENT 'CALCO' CONTAINS ARGUMENTS USED IN SUBROUTINES
C        CALCIN, COE5, AND CO15.
C     COMMON STATEMENT 'CALMAN' CONTAINS ARGUMENTS USED IN MAIN AND
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C          SUBROUTINE CALCIN.
C       COMMON STATEMENT 'CALPRN' CONTAINS ARGUMENTS USED IN SUBROUTINES
C          CALCIN AND PRN7.
C       COMMON STATEMENT 'CAPP7M' CONTAINS ARGUMENTS USED IN MAIN AND
C          SUBROUTINES CALCIN, PART, AND PRN7.
C       COMMON STATEMENT 'CAMAPA' CONTAINS ARGUMENTS USED IN MAIN AND
C          SUBROUTINES CALCIN AND PART.
C       SUBROUTINE 'CALCIN' CALCULATES STRESSES 'CSZ' ETC. AND STRAINS
C          'CRZ' ETC. AT DEPTH 'IZT' AND DESIRED ANSWER LOCATION 'AJX,AJY'
C          DUE TO INDIVIDUAL LOAD 'BWGT' LOCATED AT IT'S 'RX,RY'.
C        'COMMON/RDA/...' CONTAINS INPUT VARIABLES USED IN ALL OPTIONS.
C        'COMMON/RD1/...' CONTAINS INPUT VARIABLES USED WHEN OPTN = 1.
C        'COMMON/RD2/...' CONTAINS INPUT VARIABLES USED WHEN OPTN=2-7.
C        'COMMON/RD3/...' CONTAINS INPUT VARIABLES USED WHEN OPTN=7 OR 8.
C       AJX=X COORDINATE OF POSITION OF DESIRED ANSWERS--INPUT ARGUMENT
C          AJXX.
C       AJY=Y COORDINATE OF POSITION OF DESIRED ANSWERS--INPUT ARGUMENT
C         AJYY.
C       AR=CALCULATED RADIUS OF LOADED AREA.
C       ASTEN=SQUARE ROOT OF THE QUANTITY STENDN/(1/2 MODULUS) OF THAT
C          LAYER AND CALLED 'WORK STRAIN'.
C       BPSI = APPLIED PRESSURE UNDER LOAD 'BWGT' WHEN OPTN = 2.
C       BWGT(JA)= GIVES THE OPTION OF DIFFERENT TIRE LOADS WITHIN THE
C          LOAD GROUP FOR OPTN = 2.
C       CRZZ(IZT)=SUPERPOSITIONED STRAIN(ZZ) AT DEPTH IZT
C       ETC. FOR OTHER COMPONENTS
C       CSZZ(IZT)=SUPERPOSITIONED STRESS(ZZ) AT DEPTH IZT
C       ETC. FOR OTHER COMPONENTS
C       E(I)=YOUNG'S MODULUS OF ELASTICITY FOR ITH LAYER.
C       HH(I)=THICKNESS OF ITH LAYER
C       IA= NO. OF POSITIONS FOR WHICH THE SUPERPOSITIONED STRESSES AND
C          STRAINS ARE DESIRED.
C       IR=NUMBER OF HORIZONTAL RADII FOR WHICH ANALYSES ARE DESIRED.
C       IRA= NUMBER OF LOADS (BWGT) WITHIN LOAD GROUP FOR OPTN = 2 & 3.
C       'RX' AND 'RY' = 'X' & 'Y' COORDINATES OF INDIVIDUAL LOADS WITHIN A
C          LOAD GROUP AND USED WHEN OPTN>1.
C       IZ=NUMBER OF VERTICAL POINTS FOR WHICH ANALYSES ARE DESIRED.
C       IZI=COUNTER USED TO DETERMINE THE LAYER NO. AT THE INTERFACE
C          BETWEEN LAYERS.
C       IZT=COUNTER USED TO DETERMINE I'TH LAYER.
C       JA = COUNTER OF POINTS FOR WHICH ANSWERS ARE DESIRED, 1 TO IA.
C       MAXSTR= MAXIMUM TENSILE STRAIN
C       N = NUMBER OF LAYERS ABOVE THE BOTTOM LAYER -- SUBGRADE.
C       NS=NUMBER OF LAYERS
C       OPTN= CODE NUMBER TO TELL CHEVRON WHICH TYPE OF ANALYSES IS WANTED
C       OPTN=1 IS ORIGINAL CHEVRON N-LAYER PROGRAM WITHOUT MODIFICATIONS
C       OPTN=2 IS OUTPUT USING SUPER-POSITIONING PRINCIPLES.
C       OPTN=3 IS ANALYSES OF SIMULATED DYNAMIC LOADING (ROAD RATER, ETC.)
C       OUTPUT = 0 ; OUTPUT IS TO BE PRINTED ONLY.
C       OUTPUT = 1 ; OUTPUT IS PRINTED AND ALSO SENT TO DEVICE 8 (SPECIFIED
C          BY JCL, IE. TAPE, OS DISK, OR TERMINAL)
C       OUTPUT = 2 ; OUTPUT IS SENT TO DEVICE 8 ONLY
C       PSI= APPLIED UNIT PRESSURE UNDER THE LOAD.
C       R=HORIZONTAL RADIUS FROM CENTER OF APPLIED LOAD
C       RR(I)=RADIAL DISTANCE FOR ITH RADIUS.
```

```
C       STENDN= THE STRAIN ENERGY DENSITY CAUSED BY ALL THE LOADS WITHIN
C          THE "LOAD GROUP" AND AT THE PARTICULAR DEPTH "Z".
C       THETA=ANGLE IN R-V PLANE MEASURED FROM R AXIS;POSITIVE IS
C          CLOCKWISE FROM R AXIS.
C       TR(I)= CALCULATED RADIUS FROM CENTER OF LOAD TO DESIRED ANSWER PT.
C       V(I)=POISSON'S RATIO FOR 2TH LAYER.
C       WGT=APPLIED LOAD, POUNDS
C        WKSTRS = 'WORK STRAIN' MULTIPLIED BY MODULUS OF LAYER AND CALLED
C          'WORK STRESS'.
C       Z=VERTICAL DEPTH FROM SURFACE AT WHICH ANALYSIS IS DESIRED.
C          TYPICAL POINTS ARE SURFACE, INTERFACES OF LAYERS, AND DOWN INTO
C          SUBGRADE.
C       ZZ(I)=VERTICAL DEPTHS AT WHICH ANALYSES ARE DESIRED.
C
C           ** COMPUTE ZEROS OF J1(X) AND J0(X). SET UP GAUSS CONSTANTS **
        BZ(1)=0.0
        BZ(2)=1.0
        BZ(3)=2.4048
        BZ(4)=3.8317
        BZ(5)=5.5201
        BZ(6)=7.0156
        DO 1395 I=7,100
        BZ(I)=0.
 1395   CONTINUE
        ITN=46
        ITN4=184
        K = ITN+1
        DO 2 I=7,K,2
              T = I/2
              TD = 4.0*T - 1.0
    2         BZ(I) = 3.1415927*(T - 0.25+0.050661/TD
     1             -0.053041/TD**3 + 0.262051/TD**5)
        DO 3 I=8,ITN,2
              T = (I-2)/2
              TD = 4.0*T + 1.0
    3   BZ(I) = 3.1415927*(T + 0.25-0.151982/TD
     1             + 0.015399/TD**3-0.245270/TD**5)
C ***     END ROUTINE  SETTING GAUSS CONSTANTS. ****
C         READ  IN THE VALUE FOR OPTION AND THE NUMBER OF LOAD GROUPS.
        READ(11,315) OPTN,OUTPUT
  315   FORMAT(2I5)
        KAI=1
        JA=1
        NPAGE=1
   10   CONTINUE
        IF(OPTN.EQ.1) CALL READ1(IEND)
        IF(OPTN.GE.2) CALL READ2(IEND)
        IF(IEND.EQ.1) GO TO 9999
        MS=NS
        IF(OPTN.EQ.1) GO TO 2001
C         SUBROUTINES INIT1 & INIT2 INITIALIZE COMPUTATIONAL VARIBLES TO 0.
        CALL INIT1
   12   CONTINUE
 2007   CALL INIT2
 2008   CONTINUE
```

```
C     CALCULATE RADIUS AND ANGLE FROM X-AXIS
      AJX=AJXX(JA)
      AJY=AJYY(JA)
      IR=IRA
C     CALCULATE RADIAL DISTANCE FROM LOCATION OF DESIRED ANSWER TO EACH
C        LOAD IN THE LOAD GROUP.
      DO 2003 I=1,IR
      RR(I)=SQRT((RX(I)-AJX)**2 +(RY(I)-AJY)**2 )
      IF(RR(I).EQ.0.) GO TO 625
      COSTH=(RX(I)-AJX)/RR(I)
      GO TO 626
  625 COSTH=0.
  626 THETA(I)=ARCOS(COSTH)
 2003 CONTINUE
      TR(JA)=RR(1)
C  *** END CALCULATION OF RADIUS AND ANGLE FROM X-AXIS.  ****
 2001 CONTINUE
      DO 372 I=1,NS
      IE(I)=E(I)
      IH(I)= IE(I)-IE(I)/1000*1000+1000
      IM(I)=IE(I)/1000000
      IT(I)=IE(I)/1000-IM(I)*1000+1000
  372 CONTINUE
      IF(OPTN.GE.2) GO TO 374
      NPAGE = 1
      IF(OPTN.EQ.1) CALL PRN1
C      SUBROUTINE PRN1 PRINTS TITLE AND COLUMN HEADINGS FOR OPTN=1, I.E.
C        THE ORIGINAL CHEVRON N-LAYER PROGRAM.  CONTROL OF OUTPUT OCCURS
C        IN SUBROUTINES CALCIN AND PRN7.
C      SUBROUTINE PRN2 PRINTS HEADINGS AND PROBLEM INPUT DATA OPTN=2.
C  ** ADJUST LAYER DEPTHS **
  374 H(1)=HH(1)
      IRT=0
      IF(N.LE.1) GO TO 100
      DO 25 I=2,N
   25 H(I)=H(I-1)+HH(I)
C  ** START ON A NEW R **
  100 IRT=IRT+1
      IF(OPTN.GE.2) GO TO 50
      IF(OPTN.EQ.1) WRITE(6,128)
  128 FORMAT(/)
C  *** IF IRT-IR>1., THE CONTROL OF THE PROGRAM WILL TAKE ONE OF TWO
C        MAJOR ROUTES.  IF OPTN = 1, THE PROGRAM IS ROUTED TO READ
C        A NEW PROBLEM.  IF OPTN >= 2, THE STRESSES AND STRAINS OBTAINED
C        IN THE SUBROUTINE "SUPER" ( THE ACCUMULATED VALUES BY SUPER-
C        POSITION PRINCIPLES) ARE PRINTED.
      IF (IRT-IR) 105,105,1010
   50 IF(IRT.GT.IRA) GO TO 1010
  105 R=RR(IRT)
      IF(OPTN.GE.2) WGT=BWGT(IRT)
      IF(OPTN.GE.2) PSI=BPSI(IRT)
      AR = SQRT (WGT/(3.14159*PSI))
      DO 31 I =1,IZ
      DO 31 J=1,N
      TZ = H(J) - ZZ(I)
```

```
      ATZ = ABS(TZ)
C   ** NEXT STATEMENT IS CHECKING TO SEE IF "ZZ" IS AT AN INTERFACE. ***
      IF(ATZ - .0001) 32,32,31
   32 ZZ(I) = -H(J)
   31 CONTINUE
C  *** NLINE IS A COUNTER FOR PRINTOUT CONTROL. ****
      NLINE=NLINE+1
C  ** CALCULATE THE PARTITION **
      CALL PART
C  ** CALCULATE THE COEFFICIENTS **
      DO 125 I=1,ITN4
      P=AZ(I)
  107 CONTINUE
      CALL COFE(I)
  109   IF (R) 115,115,110
  110   PR = P*R
C  ***SUBROUTINE "BESSEL" CALCULATES THE "BESSEL FUNCTIONS". ***
      CALL BESSEL (0,PR,Y)
          RJ0(I) = Y
      CALL BESSEL (1,PR,Y)
          RJ1(I) = Y
  115 PA=P*AR
      CALL BESSEL (1,PA,Y)
      AJ(I)=Y
  125 CONTINUE
  195 IZT=0
      IZI=0
C  ** START ON A NEW Z **
  200 IZT=IZT+1
      IZI=IZI+1
C  *** IF IZT-IZ IS GREATER THAN 0., THE LAST "Z" HAS BEEN INVESTIGATED
C         AND THE PROGRAM INCREMENTS TO THE NEXT DESIRED "RADIUS". ****
      IF(IZT-IZ) 205,205,100
  205 Z=ABS (ZZ(IZT))
      IF(OPTN.GE.2) GO TO 207
      IF ( NLINE - 54 ) 207,206,206
  206 NPAGE = NPAGE + 1
      NLINE = 8
C       SUBROUTINE PRN3 PRINTS TITLE FOR PROBLEMS ON SUCCEEDING PAGES OF
C         SAME PROBLEM.
      CALL PRN3
      CALL PRN6
  207 CONTINUE
C  ** FIND THE LAYER CONTAINING Z **
      TZZ = 0.0
      DO 210 J1=1,N
      J=NS-J1
      TZ = Z-H(J)
      IF(TZ+.001) 210,215,215
  210 CONTINUE
      L = 1
      GO TO 34
  215 L=J+1
      IF (ZZ(IZT)) 33,34,34
   33 L = J
```

```
      TZZ = 1.0
   34 CONTINUE
C ****SUBROUTINE "CALCIN" CALCULATES THE DEFLECTIONS AND THE NINE
C          COMPONENTS OF STRESS AND STRAIN.  IF OPTN = 1, THE OUTPUT
C          STATEMENTS AND CONTROL REMAIN IN "CALCIN".  FOR OPTN = 2,
C          THE CONTROL OF THE OUTPUT IS RETURNED TO "MAIN". ****
C     LLLL(IZI) = TEMPORARY STORAGE LOCATION.
      LLLL(IZI)=L
      CALL CALCIN
      IF(OPTN.EQ.1) CALL PRN7
      ZZZ(IZI)= ZZ(IZT)
      IF (TZZ-.5) 36,36,35
   35 ZZ(IZT) = -ZZ(IZT)
      ZZZ(IZI)=-ZZ(IZT)
      IZT = IZT-1
   36 CONTINUE
      IF(OPTN.EQ.1) GO TO 200
      RDS=RDS*1.0E-06
      RDT=RDT*1.0E-06
      RDZ=RDZ*1.0E-06
      SST=SST*1.0E-06
      COM=COM
      IF(OPTN.LT.3) GO TO 37
      IF(IRT.EQ.1.OR.IRT.EQ.3) GO TO 37
   40 RDS=-RDS
      RDT=-RDT
      RDZ=-RDZ
      SST=-SST
      COM=-COM
      CSR=-CSR
      CTR=-CTR
      CSZ=-CSZ
      CST=-CST
   37 CSOM(IZI)=CSOM(IZI)+COM
C  *** THE PROGRAM NOW CALLS SUBROUTINE "SUPER" TO RESOLVE AND ACCUMU-
C          LATE THE STRESS COMPONENTS BY SUPERPOSITION PRINCIPLES.
      CALL SUPER(CSR,CTR,CST,CSZ,THETA(IRT),CSXX(IZI),CSXY(IZI),
     1 CSXZ(IZI),CSYY(IZI),CSYZ(IZI),CSZZ(IZI))
C  *** THE PROGRAM NOW CALLS SUBROUTINE "SUPER" TO RESOLVE AND ACCUMU-
C          LATE THE STRAIN COMPONENTS BY SUPERPOSITION PRINCIPLES.
      CALL SUPER(RDS,SST,RDT,RDZ,THETA(IRT),CRXX(IZI),CRXY(IZI),
     1 CRXZ(IZI),CRYY(IZI),CRYZ(IZI),CRZZ(IZI))
C       NOW INVESTIGATE THE NEXT SPECIFIED DEPTH 'ZZ(I)'.
      GO TO 200
 1010 CONTINUE
      IZI=IZI-1
C     IF OPTN = 1, ANALYSIS IS BY ORIGINAL CHEVRON N-LAYER PROGRAM.
C          PROCEED TO READ DATA FOR NEXT PROBLEM.
      IF(OPTN.EQ.1) GO TO 10
C   START PRINTOUT OF SUPERPOSITIONED STRESSES, STRAINS, AND DEFLECTIONS
  599 CONTINUE
      IF(JA.EQ.1.AND.NPAGE.EQ.1.AND.OUTPUT.LT.2) CALL PRN2
      JLINE=8+2*IZI
      IF(JA.EQ.1) MLINE=8+NS+IRA
      MLINE=MLINE+JLINE
```

```
      IF(MLINE.LE.61) GO TO 598
  597 NPAGE=NPAGE+1
      MLINE=8+NS+IRA
      CALL PRN2
C      SUBROUTINES PRN4 & PRN5 PRINT COLUMN HEADINGS ONLY AND DIFFER
C         ACCORDING TO FORMAT OF OUTPUT DATA.
  598 CONTINUE
C     SUBROUTINE 'STRNEN' USES THE LAYER MODULI AND POISSON'S RATIOS
C        TOGETHER WITH THE COMPONENT STRAINS RESOLVED THROUGH SUPERPOSI-
C        TION PRINCIPLES IN SUBROUTINE 'SUPER' TO CALCULATE STRAIN
C        ENERGY DENSITY (ARGUMENT STENDN), 'WORK STRAIN' (ARGUMENT
C        ASTEN), AND 'WORK STRESS' (ARGUMENT WKSTRS).
      IF(OUTPUT.EQ.0) GO TO 600
C *** WRITE STATEMENTS USING "UNIT=7" ARE STORING THE OUTPUT IN COMPUTER
C *** MEMORY.  WRITE STATEMENTS USING "UNIT=6" ARE WRITING THE OUTPUT
C *** TO A PRINTER TO OBTAIN A HARD COPY.
      WRITE(8,7772)
 7772 FORMAT(' 0')
      WRITE(8,7774) (I,E(I),V(I),HH(I),I=1,N)
 7774 FORMAT(' 1',I3,F8.0,F5.3,F6.2)
      WRITE(8,7773) NS,E(NS),V(NS)
 7773 FORMAT(' 1',I3,F8.0,F5.3,'999.99')
      WRITE(8,7775) (I,RX(I),RY(I),BWGT(I),BPSI(I),I=1,IRA)
 7775 FORMAT(' 2',I3,2F6.2,F8.2,F6.2)
      WRITE(8,7776)
 7776 FORMAT(' 2','  0')
  600 CALL STRNEN
      IF(OUTPUT.LT.2) CALL PRN4
      DO 601 J=1,IZI
      IF(OUTPUT.LT.2) WRITE(6,586) AJX,AJY,ZZZ(J),CSXX(J),
     1            CSXY(J),CSXZ(J),CSYY(J),CSYZ(J),CSZZ(J),CSOM(J)
  586 FORMAT(1X,F6.2,2X,F6.2,2X,F6.2,E14.6,E15.6,F5.1,E15.6,F5.1,2E15.6)
      IF(OUTPUT.EQ.0) GO TO 601
      WRITE(8,7770) AJX,AJY,ZZZ(J),CSXX(J),
     1            CSXY(J),CSXZ(J),CSYY(J),CSYZ(J),CSZZ(J),CSOM(J)
C7770 FORMAT(' 3',3F6.2,6F8.2,E12.6)
 7770 FORMAT(' 3',F6.2,F6.2,F6.2,E14.6,E15.6,F5.1,E15.6,F5.1,2E15.6)
      WRITE(8,7771) AJX,AJY,ZZZ(J),CRXX(J),
     1         CRXY(J),CRXZ(J),CRYY(J),CRYZ(J),CRZZ(J),STENDN(J)
C7771 FORMAT(' 4',3F6.2,6F8.2,E12.6)
 7771 FORMAT(' 4',F6.2,F6.2,F6.2,E14.6,E15.6,F5.1,E15.6,F5.1,3E15.6)
  601 CONTINUE
      IF(OUTPUT.EQ.2) GO TO 610
      CALL PRN5
      DO 602 J=1,IZI
      WRITE(6,587) AJX,AJY,ZZZ(J),CRXX(J),
     1         CRXY(J),CRXZ(J),CRYY(J),CRYZ(J),CRZZ(J),STENDN(J),ASTEN(J)
  587 FORMAT(1X,F6.2,2X,F6.2,2X,F6.2,E14.6,E15.6,F5.1,E15.6,F5.1,3E15.6)
  602 CONTINUE
  610 JA=JA+1
      IF(JA.LE.IA) GO TO 12
  615 JA=1
C **** NOW THE PROGRAM CHECKS TO SEE IF THERE IS ANOTHER PROBLEM. ***
      NPAGE=1
      GO TO 10
```

```
 9999 CONTINUE
      STOP
      END
C
C     SUBROUTINES ARE ASSEMBLED IN ALPHABETICAL ORDER.
C
      SUBROUTINE BESSEL(NI,XI,Y)
C       ******SUBROUTINE BESSEL - N-LAYER ELASTIC SYSTEM ******
C
      REAL*8 PZ(6)/1.0D0,-1.125D-4,2.8710938D-7,-2.3449658D-9,
     A3.9806841D-11,-1.1536133D-12/,  QZ(6)/-5.0D-3,4.6875D-6,
     B-2.3255859D-8, 2.8307087D-10, -6.3912096D-12,  2.3124704D-12/,
     CP1(6)/ 1.0D0,  1.875D-4,      -3.6914063D-7,   2.7713232D-9,
     D-4.5114421D-11,1.2750463D-12/, Q1(6)/1.5D-2,  -6.5625D-6,
     E 2.8423828D-8,-3.2662024D-10,  7.1431166D-12, -2.5327056D-13/,
     F PI/3.1415927/,D(20)
C
    9 N = NI
      X = XI
      IF (X-7.0) 10,10,160
   10 X2=X/2.0
      FAC=-X2*X2
      IF (N) 11,11,14
   11 C=1.0
      Y=C
      DO 13 I=1,34
      T=I
      C=FAC*C/(T*T)
      TEST=ABS (C) - 10.0**(-8)
      IF (TEST) 17,17,12
   12 Y=Y+C
   13 CONTINUE
   14 C=X2
      Y=C
      DO 16 I=1,34
      T=I
      C=FAC*C/(T*(T+1.0))
      TEST=ABS (C) - 10.0**(-8)
      IF (TEST) 17,17,15
   15 Y=Y+C
   16 CONTINUE
   17 RETURN
  160 IF (N) 161,161,164
  161 DO 162 I=1,6
      D(I) = PZ(I)
      D(I+10) = QZ(I)
  162 CONTINUE
      GO TO 163
  164 DO 165 I=1,6
      D(I) = P1(I)
      D(I+10) = Q1(I)
  165 CONTINUE
  163 CONTINUE
      T1 = 25.0/X
      T2=T1*T1
```

```
      P = D(6)*T2+D(5)
      DO 170 I=1,4
      J = 5-I
      P = P*T2+D(J)
  170 CONTINUE
      Q = D(16)*T2+D(15)
      DO 171 I=1,4
      J = 5-I
      Q = Q*T2+D(J+10)
  171 CONTINUE
      Q = Q*T1
      T4 =DSQRT (X*PI)
      T6 = SIN (X)
      T7 = COS (X)
      IF (N) 180,180,185
  180 T5 = ((P-Q)*T6 + (P+Q)*T7)/T4
      GO TO 99
  185 T5 = ((P+Q)*T6 - (P-Q)*T7)/T4
   99 Y = T5
      RETURN
      END
      SUBROUTINE CALCIN
C     ******SUBROUTINE CALCIN - N-LAYER ELASTIC SYSTEM ******
C
      COMMON/CALCO/A(396,15),B(396,15),C(396,15),D(396,15)
      COMMON/CALMAN/RJ1(396),RJ0(396),AJ(396),TZZ,L,Z
      COMMON/CALPRN/CSZ,CST,CSR,CTR,COM,RDS,RDT,RDZ,SST,JT
      COMMON/CAMAPA/AZ(396),AR,ITN,ITN4,NTEST,TEST(99)
      COMMON/CAPP7M/BZ(100),NLINE,R
      COMMON/RDA/NS,IZ,E(15),V(15),HH(15),ZZ(99),N,TITLE(20),LLLL(99)
      COMMON/RD1/PSI,WGT,RR(99),IR
      REAL*8 W(4)/0.34785485,2*0.65214515,0.34785485/
      REAL MARY,MAN,JUNIOR,MAXSTR
    2 VL=2.0*V(L)
      EL=(1.0+V(L))/E(L)
      VL1=1.0-VL
      CSZ=0.0
      CST=0.0
      CSR=0.0
      CTR=0.0
      COM=0.0
      NTS1 = NTEST + 1
      ITS = 1
      JT = 0
      ARP = AR*PSI
   10 DO 40 I=1,ITN
C     INITIALIZE THE SUB-INTEGRALS
      RSZ=0.0
      RST=0.0
      RSR=0.0
      RTR=0.0
      ROM=0.0
C     COMPUTE THE SUB-INTEGRALS
      K = 4*(I-1)
      DO 30 J=1,4
```

```
      J1 = K + J
      P=AZ(J1)
      EP=EXP (P*Z)
      T1=B(J1,L)*EP
      T2=D(J1,L)/EP
      T1P=T1+T2
      T1M=T1-T2
      T1=(A(J1,L)+B(J1,L)*Z)*EP
      T2=(C(J1,L)+D(J1,L)*Z)/EP
      T2P=P*(T1+T2)
      T2M=P*(T1-T2)
      WA=AJ(J1)*W(J)
      IF (R) 20,20,15
   15 BJ1=RJ1(J1)*P
      BJ0=RJ0(J1)*P
      RSZ=RSZ+WA*P*BJ0*(VL1*T1P-T2M)
      ROM=ROM+WA*EL*BJ0*(2.0*VL1*T1M-T2P)
      RTR=RTR+WA*P*BJ1*(VL*T1M+T2P)
      RSR=RSR+WA*(P*BJ0*((1.0+VL)*T1P+T2M)-BJ1*(T1P+T2M)/R)
      RST=RST+WA*(VL*P*BJ0*T1P+BJ1*(T1P+T2M)/R)
      GO TO 30
C     SPECIAL ROUTINE FOR R = ZERO
   20 PP=P*P
      RSZ=RSZ+WA*PP*(VL1*T1P-T2M)
      ROM=ROM+WA*EL*P*(2.0*VL1*T1M-T2P)
      RST=RST+WA*PP*((VL+0.5)*T1P+0.5*T2M)
      RSR=RST
   30 CONTINUE
      SF = (AZ(K+4) - AZ(K+1))/1.7222726
      CSZ=CSZ+RSZ*SF
      CST=CST+RST*SF
      CSR=CSR+RSR*SF
      CTR=CTR+RTR*SF
      COM=COM+ROM*SF
      RSZ = 2.0*RSZ*AR*SF
      TESTH = ABS (RSZ)-10.0**(-4)
      IF (ITS-NTS1 ) 31,32,32
   31 CONTINUE
      TEST(ITS) = TESTH
      ITS = ITS+1
      GO TO 40
   32 CONTINUE
      TEST(NTS1) = TESTH
      DO 33 J = 1,NTEST
      IF (TESTH-TEST(J)) 35,36,36
   35 CONTINUE
      TESTH = TEST(J)
   36 CONTINUE
      TEST(J) = TEST(J+1)
   33 CONTINUE
      IF (TESTH) 50,50,40
   40 CONTINUE
      JT = 1
   50 CSZ=CSZ*ARP
      CST=CST*ARP
```

```
      CTR=CTR*ARP
      CSR=CSR*ARP
      COM=COM*ARP
      IF (TZZ-.5) 72,72,71
   71 Z=Z
   72 CONTINUE
      RDZ=(1000000./E(L))*(CSZ-V(L)*(CSR+CST))
      RDT=(1000000./E(L))*(CST-V(L)*(CSZ+CSR))
      RDS=(1000000./E(L))*(CSR-V(L)*(CSZ+CST))
      SST=(2000000./E(L))*(1.0+V(L))*CTR
   99 CONTINUE
      RETURN
      END
C
C
C***********************************************************************
C     THIS SUBROUTINE COPIED TO PROGRAM BY DHC, 5/08/84
C***********************************************************************
C
C
      SUBROUTINE COFE(KIN)
      COMMON/CALCO/A(396,15),B(396,15),C(396,15),D(396,15)
      COMMON/COMAIN/H(15),P
      COMMON/RDA/NS,IZ,E(15),V(15),HH(15),ZZ(99),N,TITLE(20),LLLL(99)
      COMMON /SPSCOM/ X(15,4,4),SC(14),PM(14,4,4),FM(4)
C     SUBROUTINE COFE(KIN,PSI)
C     COMMON /SPSCOM/ E(15),V(15),H(15),AZ(396),A(396,15),B(396,15),C(39
C    16,15),D(396,15),AJ(396),BZ(100),X(15,4,4),SC(14),FM(4),PM(14,4,4),
C    2Z,AR,NS,N,L,ITN,RSZ,SF,CSZ,I,ITN4,LC,PA,P,EP,T1P,T1,T2,T3,T4
C    3,T5,T6,T2M,WA,ZF,SZ1,SZ2,SG1,SG2,PH,PH2,VK2,VKP2,VK4,VKP4,VKK8,
C    4HH(15),SIGMA3(15),W(15),MATYP(15),ANSDPT(15)
      REAL*4 Q(2,2)
      LC=KIN
    1 DO 10 K=1,N
      T1=E(K)*(1.0+V(K+1))/(E(K+1)*(1.0+V(K)))
      T1M=T1-1.0
      PH=P*H(K)
      PH2=PH*2.0
      VK2=2.0*V(K)
      VKP2=2.0*V(K+1)
      VK4=2.0*VK2
      VKP4=2.0*VKP2
      VKK8=8.0*V(K)*V(K+1)
C
      X(K,1,1)=VK4-3.0-T1
      X(K,2,1)=0.0
      X(K,3,1)=T1M*(PH2-VK4+1.0)
      X(K,4,1)=-2.0*T1M*P
C
      T3=PH2*(VK2-1.0)
      T4=VKK8+1.0-3.0*VKP2
      T5=PH2*(VKP2-1.0)
      T6=VKK8+1.0-3.0*VK2
C
      X(K,1,2)=(T3+T4-T1*(T5+T6))/P
```

```
      X(K,2,2)=T1*(VKP4-3.0)-1.0
      X(K,4,2)=T1M*(1.0-PH2-VKP4)
C
      X(K,3,4)=(T3-T4-T1*(T5-T6))/P
C
      T3=PH2*PH-VKK8+1.0
      T4=PH2*(VK2-VKP2)
C
      X(K,1,4)=(T3+T4+VKP2-T1*(T3+T4+VK2))/P
      X(K,3,2)=   (-T3+T4-VKP2+T1*(T3-T4+VK2))/P
C
      X(K,1,3)=T1M*(1.0-PH2-VK4)
      X(K,2,3)=2.0*T1M*P
      X(K,3,3)=VK4-3.0-T1
      X(K,4,3)=0.0
C
      X(K,2,4)=T1M*(PH2-VKP4+1.0)
      X(K,4,4)=T1*(VKP4-3.0)-1.0
C     K = K
   10 CONTINUE
C     COMPUTE THE PRODUCT MATRICES PM
      SC(N)=4.0*(V(N)-1.0)
      IF (N-2) 13,11,11
   11 DO 12 K1=2,N
      M=NS-K1
      SC(M)=SC(M+1)*4.0*(V(M)-1.0)
   12 CONTINUE
   13 CONTINUE
C
      Q(1,1)=1.
      Q(2,2)=1.
      Q(1,2)=0.
      QQ    =P*2.*H(N)
      IF (QQ-172.) 15,15,16
   15 CONTINUE
      Q(1,2)=EXP(-QQ)
C     Q(2,1) IS NOT NEEDED FOR INITIALIZING THE PM MATRIX
   16 CONTINUE
C      20 LOOP INITIALIZES PM(N,,)
      DO 20 M=1,4
      LL=(M+1)/2
      DO 20 J=3,4
      PM(N,M,J)= X(N,M,J) *     Q(LL,2)
   20 CONTINUE
      DO 26 K1=2,N
      K=NS-K1
      KK=K+1
      QQ    =P*2.*H(K)
      IF (QQ-172.) 22,22,23
   22 CONTINUE
      Q(2,1)=EXP(QQ)
      Q(1,2)=1./Q(2,1)
      GO TO 24
   23 CONTINUE
      Q(1,2)=0.
```

```
      Q(2,1)=1.E20
   24 CONTINUE
      DO 25 M=1,4
      LL=(M+1)/2
      DO 25 J=3,4
      PM(K,M,J)=( X(K,M,1) * PM(KK,1,J)
     2          +X(K,M,2) * PM(KK,2,J) ) * Q(LL,1)
     3         +( X(K,M,3) * PM(KK,3,J)
     4          +X(K,M,4) * PM(KK,4,J) ) * Q(LL,2)
   25 CONTINUE
   26 CONTINUE
C     SOLVE FOR C(NS) AND D(NS)
      T3=2.0*V(1)
      T4 =T3-1.0
      FM(1)= P*(PM(1,1,3)+PM(1,3,3)) + T3*(PM(1,2,3)-PM(1,4,3))
      FM(2)= P*(PM(1,1,3)-PM(1,3,3)) + T4*(PM(1,2,3)+PM(1,4,3))
      FM(3)= P*(PM(1,1,4)+PM(1,3,4)) + T3*(PM(1,2,4)-PM(1,4,4))
      FM(4)= P*(PM(1,1,4)-PM(1,3,4)) + T4*(PM(1,2,4)+PM(1,4,4))
      DFAC=SC(1)/((FM(1)*FM(4)-FM(3)*FM(2))*P*P)
          A(LC,NS) = 0.0
          B(LC,NS) = 0.0
          C(LC,NS) = -FM(3)*DFAC
          D(LC,NS) =  FM(1)*DFAC
C     BACKSOLVE FOR THE OTHER A,B,C,D
      DO 91 K1=1,N
      A(LC,K1)=(PM(K1,1,3)*C(LC,NS)+PM(K1,1,4)*D(LC,NS))/SC(K1)
      B(LC,K1)=(PM(K1,2,3)*C(LC,NS)+PM(K1,2,4)*D(LC,NS))/SC(K1)
      C(LC,K1)=(PM(K1,3,3)*C(LC,NS)+PM(K1,3,4)*D(LC,NS))/SC(K1)
   91 D(LC,K1)=(PM(K1,4,3)*C(LC,NS)+PM(K1,4,4)*D(LC,NS))/SC(K1)
  100 CONTINUE
      RETURN
      END
C
C
C***************************************************************************
C
C
      SUBROUTINE INIT1
      COMMON/HERB4/RRCOM(99),TR(99)
      COMMON/JESSE1/RSTB(7,99),RSTS(7,99),RLB(7,99),RLS(7,99),DFB(7,99),
     1 DFS(7,99)
      COMMON/JESSE2/REPB(2),REPS(2),STWB(2),STWS(2)
      COMMON/RD2/IB2,IB3,IA,IRA,AJXX(99),AJYY(99)
C**  INITIALIZE THE ACCUMULATING VARIABLES FOR STRAIN ENERGY DENSITY
C         CALCULATIONS.  ********
      DO 414 I=1,7
      DO 414 J=1,99
      DFB(I,J)=0.
      DFS(I,J)=0.
      RSTB(I,J)=0.
      RSTS(I,J)=0.
      RLB(I,J)=0.
      RLS(I,J)=0.
  414 CONTINUE
      RETURN
```

```
      END
      SUBROUTINE INIT2
      COMMON/HERB1/WORK(99),STENDN(99),ASTEN(99),WKSTRS(99),ZZZ(99)
      COMMON/HERB2/     THETA(99)
      COMMON/HERB3/ IZI,CSXX(99),CSXY(99),CSXZ(99),CSYY(99),CSYZ(99),
     1 CSZZ(99),CSOM(99),CCOM(99)
      COMMON/HERB5/CRXX(99),CRYY(99),CRZZ(99),CRXY(99),CRXZ(99),CRYZ(99)
      COMMON/RD2/IB2,IB3,IA,IRA,AJXX(99),AJYY(99)
      DO 11 I=1,99
      CSXX(I)=0.
      CSYY(I)=0.
      CSZZ(I)=0.
      CSXY(I)=0.
      CSXZ(I)=0.
      CSYZ(I)=0.
      CSOM(I)=0.
      CRXX(I)=0.
      CRYY(I)=0.
      CRZZ(I)=0.
      CRXY(I)=0.
      CRXZ(I)=0.
      CRYZ(I)=0.
      ASTEN(I)=0.
      STENDN(I)=0.
   11 CONTINUE
      RETURN
      END
      SUBROUTINE PART
C       ******SUBROUTINE PART - N-LAYER ELASTIC SYSTEM ******
C
      COMMON/CAMAPA/AZ(396),AR,ITN,ITN4,NTEST,TEST(99)
      COMMON/CAPP7M/BZ(100),NLINE,R
      REAL*8  G1/0.86113631/,G2/0.33998104/
    4 ZF = AR
      NTEST = 2
      IF (R) 8,8,9
    9 CONTINUE
      NTEST = AR/R + .0001
      IF (NTEST) 6,6,5
    6 CONTINUE
      NTEST = R/AR + .0001
      ZF = R
    5 CONTINUE
      NTEST = NTEST + 1
      IF (NTEST-10) 8,8,7
    7 CONTINUE
      NTEST = 10
    8 CONTINUE
C           ** COMPUTE POINTS FOR LEGENDRE-GAUSS INTEGRATION **
   15 K = 1
       ZF = 2.0*ZF
       SZ2 = 0.0
      DO 28 I=1,ITN
        SZ1 = SZ2
        SZ2 = BZ(I+1)/ZF
```

```
          SF = SZ2 - SZ1
          PP = SZ2 + SZ1
      SG1=SF*G1
      SG2=SF*G2
      AZ(K)=PP-SG1
      AZ(K+1)=PP-SG2
      AZ(K+2)=PP+SG2
      AZ(K+3)=PP+SG1
         K = K + 4
   28 CONTINUE
   40 RETURN
      END
      SUBROUTINE PRN1
      COMMON/CALMAN/RJ1(396),RJ0(396),AJ(396),TZZ,L,Z
      COMMON/CAMAPA/AZ(396),AR,ITN,ITN4,NTEST,TEST(99)
      COMMON/CAPP7M/BZ(100),NLINE,R
      COMMON/GARY/IM(15),IT(15),IH(15)
      COMMON/RDA/NS,IZ,E(15),V(15),HH(15),ZZ(99),N,TITLE(20),LLLL(99)
      COMMON/RD1/PSI,WGT,RR(99),IR
      DATA ASTER/4H****/
C  *** IF OPTN= 1, THIS NEXT SECTION PRINTS TITLE AND HEADINGS. ****
C *** THESE HEADINGS AND VALUES CORRESPOND TO THE ORIGINAL N-LAYER PROGR
      NLINE = 17+NS
      CALL PRN3
      WRITE(6, 351) WGT,PSI,AR,(I,IM(I),IT(I),IH(I),V(I),HH(I),I=1,N)
  351 FORMAT(1H0, 40X, 26HTHE PROBLEM PARAMETERS ARE/
     1     1H0, 20X, 12HTOTAL LOAD.., 8X, F10.2, 5H LBS/
     2     1H0, 20X, 15HTIRE PRESSURE.., 5X, F10.2, 5H  PSI/
     3     1H0, 20X, 13HLOAD RADIUS.., 7X, F10.2, 5H  IN./ 1H /
     4     (1H , 20X, 5HLAYER, I3, 14H  HAS MODULUS ,3I4,
     6      18H   POISSONS RATIO , F5.3, 17H   AND THICKNESS , F6.2,
     7      4H IN.,T48,1H.,T52,1H.))
      WRITE(6,354) NS,IM(NS),IT(NS),IH(NS),V(NS)
  354 FORMAT (1H , 20X, 5HLAYER, I3, 14H  HAS MODULUS ,3I4,
     1      18H   POISSONS RATIO , F5.3, 24H   AND IS SEMI-INFINITE. ,
     2T48,1H.,T52,1H.)
      WRITE(6, 352)
  352 FORMAT(1H0,4X,8HLOCATION,2X,1H*,14X,15HS-T-R-E-S-S-E-S,14X,12H*DEF
     1LECTION*,21X,13HS-T-R-A-I-N-S,21X,6H*ANGLE/15X,1H*,19X,3HPSI,21X,
     21H*,2X,6HINCHES,2X,1H*,20X,16HMICROINCHES/INCH,19X,1H*,1X,3HDEG/
     315X,1H*,43X,1H*,10X,1H*,55X,1H*/5X,1HR,6X,1HZ,2X,1H*,2X,8HVERTICAL
     4,3X,10HTANGENTIAL,2X,6HRADIAL,5X,5HSHEAR,2X,1H*,1X,8HVERTICAL,
     41X,1H*,
     52X,8HVERTICAL,3X,10HTANGENTIAL,2X,6HRADIAL,2X,8HSHEAR IN,2X,
     511HMAX.PRIN.IN,1X,1H*,1X,4HWITH/
     615X,1H*,43X,1H*,10X,1H*,32X,10HMICRO RAD.,1X,
     719HTENSILE DIR.*R AXIS/1H )
      RETURN
      END
      SUBROUTINE PRN2
      COMMON/CAPP7M/BZ(100),NLINE,R
      COMMON/GARY/IM(15),IT(15),IH(15)
      COMMON/HERB2/    THETA(99)
      COMMON/RDA/NS,IZ,E(15),V(15),HH(15),ZZ(99),N,TITLE(20),LLLL(99)
      COMMON/RD2/IB2,IB3,IA,IRA,AJXX(99),AJYY(99)
```

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      COMMON/RD3/RX(99),RY(99),BPSI(99),BWGT(99),JRUN,KST
      DATA ASTER/4H****/
C     *****************************************************
      NLINE = 13+NS
      CALL PRN3
      WRITE(6,370)
  370 FORMAT(' ',120('*'))
      WRITE(6,371)
  371 FORMAT(' ',41('*'),'THE ANSWERS BELOW ARE BY SUPERPOSITION',
     & 41('*'))
      WRITE(6,370)
      WRITE(6, 361)            (I,IM(I),IT(I),IH(I),V(I),HH(I),I=1,N)
  361 FORMAT(1H0, 40X, 26HTHE PROBLEM PARAMETERS ARE/
     4      (1H , 20X, 5HLAYER, I3, 14H  HAS MODULUS ,3I4,
     6       18H   POISSONS RATIO , F5.3, 17H   AND THICKNESS , F6.2,
     7       4H IN,,T48,1H,,T52,1H,))
      WRITE(6,354) NS,IM(NS),IT(NS),IH(NS),V(NS)
  354 FORMAT (1H , 20X, 5HLAYER, I3, 14H  HAS MODULUS ,3I4,
     1       18H   POISSONS RATIO , F5.3, 24H   AND IS SEMI-INFINITE. ,
     2T48,1H,,T52,1H,)
      WRITE(6, 580)
  580 FORMAT('0',  39X,'COORDINATES OF THE LOAD POINTS AND LOAD VALUES A
     &RE')
  373 CONTINUE
      DO 603 I=1,IRA
      WRITE(6,582) I,RX(I),I,RY(I),I,BWGT(I),BPSI(I)
  582 FORMAT(  31X,'X(',I2,')=',F6.2,' Y(',I2,')=',F6.2,' P(',I2,')=',
     & F8.2,' TIRE PRESSURE=',F6.2,' PSI')
  603 CONTINUE
C   ******  END PRINTING HEADINGS FOR OPTN = 2  *****
C     *****************************************************
  374 CONTINUE
      RETURN
      END
      SUBROUTINE PRN3
      COMMON/DON/            NPAGE
      COMMON/RDA/NS,IZ,E(15),V(15),HH(15),ZZ(99),N,TITLE(20),LLLL(99)
      DATA ASTER/4H****/
      WRITE(6, 350) (ASTER,I=1,5),(TITLE(I),I=1,20),(ASTER,I=1,5),NPAGE
  350 FORMAT (1H1,     5A4,1X,20A4,1X,5A4,6H  PAGE,I3)
      RETURN
      END
      SUBROUTINE PRN4
      WRITE(6,552)
  552 FORMAT(1X,/,2X,'COORDINATES     DEPTH',29X,'S-T-R-E-S-S-E-S',/,4X
     1,'X',7X,'Y',7X,'Z',8X,'XX',13X,'XY',8X,'XZ',8X,'YY',8X'YZ',8X,
     2'ZZ',10X,'DEFLECTION')
      RETURN
      END
      SUBROUTINE PRN5
      WRITE(6,553)
  553 FORMAT(1X,/,2X,'COORDINATES     DEPTH',30X,'S-T-R-A-I-N-S',
     &29X,'STRAIN ENERGY',5X,'WORK',/,4X,'X',7X,'Y',7X,'Z',8X,'XX',13X,
     &'XY',8X,'XZ',8X,'YY',8X,'YZ',8X,'ZZ',11X,'DENSITY',8X,'STRAIN')
      RETURN
```

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      END
      SUBROUTINE PRN6
      WRITE(6,352)
  352 FORMAT(1H0,4X,8HLOCATION,2X,1H*,14X,15HS-T-R-E-S-S-E-S,14X,12H*DEF
     1LECTION*,21X,13HS-T-R-A-I-N-S,21X,6H*ANGLE/15X,1H*,19X,3HPSI,21X,
     21H*,2X,6HINCHES,2X,1H*,20X,16HMICROINCHES/INCH,19X,1H*,1X,3HDEG/
     315X,1H*,43X,1H*,10X,1H*,55X,1H*/5X,1HR,6X,1HZ,2X,1H*,2X,8HVERTICAL
     4,3X,10HTANGENTIAL,2X,6HRADIAL,5X,5HSHEAR,2X,1H*,1X,8HVERTICAL,
     41X,1H*,
     52X,8HVERTICAL,3X,10HTANGENTIAL,2X,6HRADIAL,2X,8HSHEAR IN,2X,
     511HMAX.PRIN.IN,1X,1H*,1X,4HWITH/
     615X,1H*.43X,1H*,10X,1H*,32X,10HMICRO RAD.,1X,
     719HTENSILE DIR.*R AXIS/1H )
      RETURN
      END
      SUBROUTINE PRN7
C
C       THIS SUBROUTINE IS USED TO PRINT OUTPUT WHEN OPTN=1 ONLY.  IT
C         IS ACTUALLY THE LAST HALF OF SUBROUTINE CALCIN IN THE ORIGINAL
C         CHEVRON N-LAYER PROGRAM.
      COMMON/CALMAN/RJ1(396),RJ0(396),AJ(396),TZZ,L,Z
      COMMON/CALPRN/CSZ,CST,CSR,CTR,COM,RDS,RDT,RDZ,SST,JT
      COMMON/CAPP7M/BZ(100),NLINE,R
      COMMON/RDA/NS,IZ,E(15),V(15),HH(15),ZZ(99),N,TITLE(20),LLLL(99)
      REAL MARY,MAN,JUNIOR,MAXSTR
C     CALCULATE MAXIMUM PRINCIPAL STRAIN IN TENSILE DIRECTION
C     AND ITS ANGLE OR ANGLES WITH SPATIAL AXES T,R, AND V.
C     IN PRINTOUT: A NUMERICAL ANGLE IS DIRECTION OF THIS STRAIN
C     WITH R-AXIS IN THE R-V PLANE AND MINUS IS COUNTERCLOCKWISE.
C     IN PRINTOUT: A COMBINATION DIRECTION DEFINES THE PLANE OR
C     PLANES IN WHICH THIS STRAIN IS CONSTANT.
      BSC = ABS(CTR) - 0.0009
      IF(BSC.GT.0.0) GO TO 500
C     WHEN SHEAR STRESS IS ZERO,T,R,& V ARE PRINCIPAL AXES.
      TMPMX1 = (1000000./E(L))*(CSR-V(L)*(CSZ+CST))
      TMPMX2 = (1000000./E(L))*(CSZ-V(L)*(CSR+CST))
      TMPMX3 = (1000000./E(L))*(CST-V(L)*(CSR+CSZ))
      MARY = (CST-CSR)
      THOMP = ABS(MARY) - 0.0009
      SUTTON = (CSR-CSZ)
      JUNIOR = ABS(SUTTON) - 0.0009
      SAM = (CST-CSZ)
      SAMPAT = ABS(SAM) - 0.0009
      IF((JUNIOR.LT.0.0).AND.(THOMP.LT.0.0))  GO TO 530
      GO TO 531
  530 MAXSTR = TMPMX1
      GO TO 501
  531 CONTINUE
      IF((JUNIOR.LT.0.0).AND.(MARY.LE.-0.0009))  GO TO 534
      GO TO  535
  534 MAXSTR = TMPMX1
      GO TO 503
  535 CONTINUE
      IF((SAMPAT.LT.0.0).AND.(MARY.GE.0.0009))  GO TO 532
      GO TO 533
```

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  532 MAXSTR = TMPMX2
      GO TO 505
  533 CONTINUE
      IF((THOMP.LT.0.0).AND.(SUTTON.GE.0.0009)) GO TO 536
      GO TO 537
  536 MAXSTR = TMPMX3
      GO TO 507
  537 CONTINUE
      IF((MARY.GE.0.0009).AND.(SAM.GE.0.0009)) GO TO 538
      GO TO 539
  538 MAXSTR = TMPMX3
      GO TO 509
  539 CONTINUE
      IF((MARY.LE.-0.0009).AND.(SUTTON.GE.0.0009))  GO TO 5400
      GO TO 5401
 5400 MAXSTR = TMPMX1
      GO TO 511
 5401 CONTINUE
      MAXSTR=TMPMX2
      GO TO 513
  501 WRITE(6,502) R,Z,CSZ,CST,CSR,CTR,COM,RDZ,RDT,RDS,SST,MAXSTR
  502 FORMAT(1H ,1X,F6.2,1X,F6.2,1H*,F10.4,1X,F10.4,1X,F10.4,1X,F10.4,
     11H*,F10.6,1H*,F10.2,1X,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
     21X,7H*  TRV )
      GO TO 557
  503 WRITE(6,504)R,Z,CSZ,CST,CSR,CTR,COM,RDZ,RDT,RDS,SST,MAXSTR
  504 FORMAT(1H ,1X,F6.2,1X,F6.2,1H*,F10.4,1X,F10.4,1X,F10.4,1X,F10.4,
     11H*,F10.6,1H*,F10.2,1X,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
     21X,6H*  RV )
      GO TO 557
  505 WRITE(6,506)R,Z,CSZ,CST,CSR,CTR,COM,RDZ,RDT,RDS,SST,MAXSTR
  506 FORMAT(1H ,1X,F6.2,1X,F6.2,1H*,F10.4,1X,F10.4,1X,F10.4,1X,F10.4,
     11H*,F10.6,1H*,F10.2,1X,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
     21X,6H*  TV )
      GO TO 557
  507 WRITE(6,508)R,Z,CSZ,CST,CSR,CTR,COM,RDZ,RDT,RDS,SST,MAXSTR
  508 FORMAT(1H ,1X,F6.2,1X,F6.2,1H*,F10.4,1X,F10.4,1X,F10.4,1X,F10.4,
     11H*,F10.6,1H*,F10.2,1X,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
     21X,6H*  TR )
      GO TO 557
  509 WRITE(6,510)R,Z,CSZ,CST,CSR,CTR,COM,RDZ,RDT,RDS,SST,MAXSTR
  510 FORMAT(1H ,1X,F6.2,1X,F6.2,1H*,F10.4,1X,F10.4,1X,F10.4,1X,F10.4,
     11H*,F10.6,1H*,F10.2,1X,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
     21X,7H* T DIR)
      GO TO 557
  511 WRITE(6,512)R,Z,CSZ,CST,CSR,CTR,COM,RDZ,RDT,RDS,SST,MAXSTR
  512 FORMAT(1H ,1X,F6.2,1X,F6.2,1H*,F10.4,1X,F10.4,1X,F10.4,1X,F10.4,
     11H*,F10.6,1H*,F10.2,1X,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
     21X,7H* R DIR)
      GO TO 557
  513 WRITE(6,514)R,Z,CSZ,CST,CSR,CTR,COM,RDZ,RDT,RDS,SST,MAXSTR
  514 FORMAT(1H ,1X,F6.2,1X,F6.2,1H*,F10.4,1X,F10.4,1X,F10.4,1X,F10.4,
     11H*,F10.6,1H*,F10.2,1X,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
     21X,7H* V DIR)
      GO TO 557
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C       WHEN SHEAR STRESS IS NOT ZERO, PRINCIPAL STRAIN MAY BE AN ANGLE
C       IN R-V PLANE OR IN T-DIRECTION OR MAY BE IN A COMBINATION
C       (OF THESE) PLANE.
  500 TEMP = 0.5*(SQRT((CSR-CSZ)*(CSR-CSZ)+4.*CTR*CTR))
      CS1TEN = 0.5*(CSZ+CSR) + TEMP
      CS2TEN = CS1TEN - 2.*TEMP
      CS3TEN = CST
      GEORGE = CS1TEN-CS3TEN
      TUT = ABS(GEORGE) - 0.0009
      IF(GEORGE.LE.-0.0009) GO TO 554
      MAN = 1000.
      COFF = CSR-CSZ
      IF(COFF.LE.-0.0009) MAN = 0.
      SAM = ABS(COFF) -0.0009
      IF(SAM.GT.0.) GO TO 520
C       WHEN STRESSES IN R AND V DIRECTIONS ARE EQUAL, THETA IS PLUS OR
C       MINUS 45.0 DEGREES WITH R-AXIS.
      IF(CTR.GE.0.0009) THETA = 45.
      IF(CTR.LE.-0.0009) THETA = -45.
      GO TO 540
C       IN PRINTOUT:THETA CAN VARY FROM PLUS OR MINUS ZERO TO NINETY
C       DEGREES (WITH R- AXIS).
  520 THETA = 0.5*ARSIN(CTR/TEMP)
      THETA = (180./3.1415927)*THETA
      IF ((MAN.EQ.0.0).AND.(CTR.GE.0.0009)) THETA = 90. - THETA
      IF ((MAN.EQ.0.0).AND.(CTR.LE.-0.0009)) THETA = -(90.-ABS(THETA))
  540 CS1 = CS1TEN
      IF((CS3TEN-CS2TEN).GE.0.0009) GO TO 545
      CS2 = CS2TEN
      CS3 = CS3TEN
      GO TO 550
  545 CS2 = CS3TEN
      CS3 = CS2TEN
  550 MAXSTR = (1000000./E(L))*(CS1-V(L)*(CS2+CS3))
      IF(TUT.LT.0.0) GO TO 551
C       WHEN MAXIMUM STRESS IN R-V PLANE IS GREATER THAN TANGENTIAL
C       STRESS, MAXIMUM TENSILE STRAIN IS UNIDIRECTIONAL IN R-V PLANE.
      WRITE(6,553)R,Z,CSZ,CST,CSR,CTR,COM,RDZ,RDT,RDS,SST,MAXSTR,THETA
  553 FORMAT(1H ,1X,F6.2,1X,F6.2,1H*,F10.4,1X,F10.4,1X,F10.4,1X,F10.4,
     11H*,F10.6,1H*,F10.2,1X,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
     21X,1H*,2X,F5.1)
      GO TO 557
C       WHEN TANGENTIAL STRESS EQUALS MAXIMUM TENSILE STRESS IN R-V
C       PLANE,MAXIMUM TENSILE STRAIN IS IN PLANE DEFINED BY T-AXIS
C       AND ANGLE IN R-V PLANE.
  551 WRITE(6,552)R,Z,CSZ,CST,CSR,CTR,COM,RDZ,RDT,RDS,SST,MAXSTR,THETA
  552 FORMAT(1H ,1X,F6.2,1X,F6.2,1H*,F10.4,1X,F10.4,1X,F10.4,1X,F10.4,
     11H*,F10.6,1H*,F10.2,1X,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
     21X,3H* T,F5.1)
      GO TO 557
C       IF TANGENTIAL STRESS IS MAJOR PRINCIPAL TENSILE STRESS,MAXIMUM
C       TENSILE STRAIN IS IN TANGENTIAL DIRECTION, ONLY.
  554 CS1 = CS3TEN
      CS2 = CS1TEN
      CS3 = CS2TEN
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```
      MAXSTR = (1000000./E(L))*(CS1-V(L)*(CS2+CS3))
      WRITE(6,555)R,Z,CSZ,CST,CSR,CTR,COM,RDZ,RDT,RDS,SST,MAXSTR
  555 FORMAT(1H ,1X,F6.2,1X,F6.2,1H*,F10.4,1X,F10.4,1X,F10.4,1X,F10.4,
     11H*,F10.6,1H*,F10.2,1X,F10.2,1X,F10.2,1X,F10.2,1X,F7.2,
     21X,7H* T DIR)
  557 NLINE=NLINE+1
      IF(OPTN.EQ.2) GO TO 99
      IF (JT) 99,99,60
   60 WRITE(6,316)
  316 FORMAT(1H+,131X,1HS)
   99 RETURN
      END
      SUBROUTINE READ1(IEND)
      COMMON/CAMAPA/AZ(396),AR,ITN,ITN4,NTEST,TEST(99)
C       SUBROUTINE READ1 IS USED ONLY WHEN OPTN = 1.
      COMMON/RDA/NS,IZ,E(15),V(15),HH(15),ZZ(99),N,TITLE(20),LLLL(99)
      COMMON/RD1/PSI,WGT,RR(99),IR
      IEND=0
      READ(11,310,END=9999) (TITLE(I),I=1,20)
  310 FORMAT (20A4)
      READ(11,314)NS,IZ,IR,PSI,WGT
  314 FORMAT(3I5,2F10.3)
      AR=SQRT(WGT/(3.14159*PSI))
      READ(11,302)(E(I),V(I),I=1,NS)
  302 FORMAT(5(F7.0,F6.5))
      READ(11,313)(RR(I),I=1,IR)
  313 FORMAT(10F6.3)
      N=NS-1
      READ(11,313)(HH(I),I=1,N)
      READ(11,313)(ZZ(I),I=1,IZ)
      GO TO 9998
 9999 IEND=1
 9998 CONTINUE
      RETURN
      END
      SUBROUTINE READ2(IEND)
C       SUBROUTINE READ2 IS USED ONLY WHEN OPTN = 2
      COMMON/RDA/NS,IZ,E(15),V(15),HH(15),ZZ(99),N,TITLE(20),LLLL(99)
      COMMON/RD2/IB2,IB3,IA,IRA,AJXX(99),AJYY(99)
      COMMON/RD3/RX(99),RY(99),BPSI(99),BWGT(99),JRUN,KST
      COMMON/JESSE5/JA
      IEND=0
      READ(11,310,END=9999) (TITLE(I),I=1,20)
  310 FORMAT (20A4)
      READ(11,311)NS,IZ,IA,IRA
  311 FORMAT(4I5)
      READ(11,302)(E(I),V(I),I=1,NS)
  302 FORMAT(5(F7.0,F6.5))
      N=NS-1
      READ(11,313)(HH(I),I=1,N)
  313 FORMAT(10F6.3)
      READ(11,313)(ZZ(I),I=1,IZ)
C  ****READ IN THE LOCATIONS OF THE DESIRED ANSWER POINTS AS X,Y COORDI-
C       NATES
      READ(11,317)(AJXX(I),AJYY(I),I=1,IA)
```

```
  317 FORMAT(16F5.2)
C  **** READ IN THE X,Y COORDINATE LOCATIONS AND RESPECTIVE LOADS.
C      WHEN OPTN=2, READ MAGNITUDE OF LOAD FOR EACH LOAD LOCATION.
C      WHEN OPTN=3, ANALYSES SUBTRACTS CALCULATED ANSWERS FOR MINIMUM
C         LOADS FROM ANSWERS FOR MAXIMUM LOADS.
C      WHEN OPTN=3, LOADS MUST BE READ IN AS THE MAXIMUM, THEN MINIMUM
C         FOR FIRST LOADED AREA FOLLOWED BY THE MAXIMUM THEN MINIMUM FOR
C         OTHER LOADED AREA.
C      BWGT(1)=MAXIMUM LOAD ON LOADED AREA(1).
C      BWGT(2)=MINIMUM LOAD ON LOADED AREA(1).
C      BWGT(3)=MAXIMUM LOAD ON LOADED AREA(2).
C      BWGT(4)=MINIMUM LOAD ON LOADED AREA(2).
       IF(JA.GT.1) GO TO 2008
  403 READ(11,316)(RX(I),RY(I),BPSI(I),BWGT(I),I=1,IRA)
  316 FORMAT(4(2F5.2,F4.1,F6.1))
 2008 CONTINUE
      GO TO 9998
 9999 IEND=1
 9998 CONTINUE
      RETURN
      END
      SUBROUTINE STRNEN
      COMMON/HERB1/WORK(99),STENDN(99),ASTEN(99),WKSTRS(99),ZZZ(99)
      COMMON/HERB3/ IZI,CSXX(99),CSXY(99),CSXZ(99),CSYY(99),CSYZ(99),
     1 CSZZ(99),CSOM(99),CCOM(99)
      COMMON/HERB5/CRXX(99),CRYY(99),CRZZ(99),CRXY(99),CRXZ(99),CRYZ(99)
      COMMON/RDA/NS,IZ,E(15),V(15),HH(15),ZZ(99),N,TITLE(20),LLLL(99)
      COMMON/RD2/IB2,IB3,IA,IRA,AJXX(99),AJYY(99)
      DO 602 IZT=1,IZI
C    ** START THE CALCULATION OF THE STRAIN ENERGY DENSITY. *****
      L=LLLL(IZT)
      ALAMB=E(L)*V(L)/((1.+V(L))*(1.-2.*V(L)))
      AMU=E(L)/(2.*(1.+V(L)))
      PHI=CRXX(IZT)+CRYY(IZT)+CRZZ(IZT)
      STENDN(IZT)=(0.5*ALAMB*PHI**2 +AMU*((CRXX(IZT))**2 +(CRYY(IZT))**2
     1+(CRZZ(IZT))**2 +2.*(CRXY(IZT))**2 +2.*(CRYZ(IZT))**2 +2.*(CRXZ(IZ
     2T))**2))
      ASTEN(IZT)=SQRT(STENDN(IZT)*2./E(L))
  609 CONTINUE
  602 CONTINUE
      RETURN
      END
      SUBROUTINE SUPER(RR,RT,TT,ZZ,THETA,XXT,XYT,XZT,YYT,YZT,ZZT)
      CO=COS(THETA)
      SI=SIN(THETA)
      SICO=SI*CO
      COSQ=CO*CO
      SISQ=SI*SI
      RZ=0
      TZ=0
      XX=COSQ*RR-2*SICO*RT+SISQ*TT
      XY=SICO*RR+(COSQ-SISQ)*RT-SICO*TT
      XZ=CO*RZ-SI*TZ
      YY=SISQ*RR+2*SICO*RT+COSQ*TT
      YZ=SI*RZ+CO*TZ
```

```
      ZZ=ZZ
      XXT=XXT+XX
      XYT=XYT+XY
      XZT=XZT+XZ
      YYT=YYT+YY
      YZT=YZT+YZ
      ZZT=ZZT+ZZ
      CONTINUE
      RETURN
      END
```

APPENDIX C

EXAMPLE PROBLEMS

FILE: CHOPTN1 SCALE A1 University of Kentucky Computing Center

```
|...+....1....+....2....+....3....+....4....+....5....+....6....+....7....+....8
    1
OPTN = 1    (OLD CHEVRON)
    3    3    1      265      4500
 550000 400    11350 400     3000 450
  65
  7    16
  0     7     23
OPTN = 1    (OLD CHEVRON)
    2    2    1      265      4500
 200000 400    12000 450
  65
 1825
  0    1825
|...+....1....+....2....+....3....+....4....+....5....+....6....+....7....+....8
```

******************** OPTN = 1   (OLD CHEVRON)   ******************** PAGE 1

THE PROBLEM PARAMETERS ARE

TOTAL LOAD.. 4500.00 LBS

TIRE PRESSURE.. 26.50 PSI

LOAD RADIUS.. 7.35 IN.

LAYER 1 HAS MODULUS 0,550,000 POISSONS RATIO 0.400 AND THICKNESS 7.00 IN,
LAYER 2 HAS MODULUS 0,011,850 POISSONS RATIO 0.400 AND THICKNESS 16.00 IN,
LAYER 3 HAS MODULUS 0,003,000 POISSONS RATIO 0.450 AND IS SEMI-INFINITE.

LOCATION		*	S-T-R-E-S-S-E-S PSI				*DEFLECTION* INCHES *	S-T-R-A-I-N-S MICROINCHES/INCH					*ANGLE * DEG
R	Z	*	VERTICAL	TANGENTIAL	RADIAL	SHEAR	* VERTICAL *	VERTICAL	TANGENTIAL	RADIAL	SHEAR IN MICRO RAD.	MAX.PRIN.IN TENSILE DIR.	* WITH *RAXIS
6.50	0.0	*	-26.8080	-85.2049	-79.1467	-0.0000*	0.016684*	70.79	-77.86	-62.44	-0.00	70.79	* V DIR
6.50	7.00	*	-2.5858	65.8185	56.4206	-0.8012*	0.016585*	-93.60	80.52	56.60	-4.08	80.52	* T DIR
6.50	7.00	*	-2.5858	-0.2686	-0.4711	-0.8013*	0.016585*	-193.24	80.52	56.60	-189.33	88.41	* -18.6
6.50	23.00	*	-0.7782	1.4931	1.4297	-0.1009*	0.014046*	-164.33	104.01	96.52	-23.84	104.01	* T DIR
6.50	23.00	*	-0.7782	-0.0820	-0.0975	-0.1009*	0.014046*	-232.45	104.01	96.52	-97.51	103.59	* T -8.3

******************** OPTN = 1   (OLD CHEVRON)   ******************** PAGE 1

THE PROBLEM PARAMETERS ARE

TOTAL LOAD.. 4500.00 LBS

TIRE PRESSURE.. 26.50 PSI

LOAD RADIUS.. 7.35 IN.

LAYER 1 HAS MODULUS 0,200,000 POISSONS RATIO 0.400 AND THICKNESS 18.25 IN,
LAYER 2 HAS MODULUS 0,012,000 POISSONS RATIO 0.450 AND IS SEMI-INFINITE.

LOCATION		*	S-T-R-E-S-S-E-S PSI				*DEFLECTION* INCHES *	S-T-R-A-I-N-S MICROINCHES/INCH					*ANGLE * DEG
R	Z	*	VERTICAL	TANGENTIAL	RADIAL	SHEAR	* VERTICAL *	VERTICAL	TANGENTIAL	RADIAL	SHEAR IN MICRO RAD.	MAX.PRIN.IN TENSILE DIR.	* WITH *R AXI
6.50	0.0	*	-26.8080	-31.5199	-31.3013	0.0000*	0.005000*	-8.40	-41.38	-39.85	0.00	-8.40	* V DIR
6.50	18.25	*	-1.2497	11.4819	10.5039	-0.2303*	0.004354*	-50.22	38.90	32.05	-3.22	38.90	* T DIR
6.50	18.25	*	-1.2497	-0.2201	-0.2768	-0.2303*	0.004354*	-85.51	38.90	32.05	-55.67	38.90	* T DIR

FILE: CHOPTN2 SCALE A1 University of Kentucky Computing Center

```
|...+....1....+....2....+....3....+....4....+....5....+....6....+....7....+....8
    2    1
OPTN = 2
    3    3    2    4
 480000 400    30000 400       4500 450
  8     16
  0     8     24
 2000 2927 2000 3150
 2000 2000 80  4500  2000 3350 80  4500  2000 9430 80  4500  200010780 80  4500
OPTN = 2    (TANDEM)
    4    4    2    8
 480000 400    30000 400     24500 400      4500 450
  8     6     16
  0     8     14    30
 2000 2927 2000 3150
 2000 2000 80  4500  2000 3350 80  4500  2000 9430 80  4500  200010780 80  4500
 7000 2000 80  4500  7000 3350 80  4500  7000 9430 80  4500  700010780 80  4500
|...+....1....+....2....+....3....+....4....+....5....+....6....+....7....+....8
```

******************** OPTN = 2 ********************

****

THE ANSWERS BELOW ARE BY SUPERPOSITION

****

THE PROBLEM PARAMETERS ARE

LAYER 1 HAS MODULUS 0,480,000 POISSONS RATIO 0.400 AND THICKNESS 8.00 IN,
LAYER 2 HAS MODULUS 0,030,000 POISSONS RATIO 0.400 AND THICKNESS 16.00 IN,
LAYER 3 HAS MODULUS 0,004,500 POISSONS RATIO 0.450 AND IS SEMI-INFINITE.

COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE

X(1)= 20.00 Y(1)= 20.00 P(1)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(2)= 20.00 Y(2)= 33.50 P(2)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(3)= 20.00 Y(3)= 94.30 P(3)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(4)= 20.00 Y(4)=107.80 P(4)= 4500.00 TIRE PRESSURE= 80.00 PSI

COORDINATES		DEPTH	S-T-R-E-S-S-E-S						
X	Y	Z	XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
20.00	29.27	0.0	-0.120267E 03	-0.290459E-03	0.0	-0.933130E 02	0.0	-0.407353E 02	0.294233E-01
20.00	29.27	-8.00	0.833339E 02	0.362830E 01	0.0	0.580410E 02	0.0	-0.766529E 01	0.292195E-01
20.00	29.27	8.00	0.417582E 00	0.362834E 01	0.0	-0.116322E 01	0.0	-0.766527E 01	0.292195E-01
20.00	29.27	-24.00	0.585323E 01	0.329623E 00	0.0	0.460769E 01	0.0	-0.162761E 01	0.262580E-01
20.00	29.27	24.00	-0.208028E 00	0.329625E 00	0.0	-0.388416E 00	0.0	-0.162760E 01	0.262580E-01

COORDINATES		DEPTH	S-T-R-A-I-N-S						STRAIN ENERGY	WORK
X	Y	Z	XX	XY	XZ	YY	YZ	ZZ	DENSITY	STRAIN
20.00	29.27	0.0	-0.138849E-03	-0.171903E-08	0.0	-0.602336E-04	0.0	0.931176E-04	0.926312E-02	0.196460E-03
20.00	29.27	-8.00	0.131632E-03	0.211651E-04	0.0	0.578617E-04	0.0	-0.133782E-03	0.783021E-02	0.180626E-03
20.00	29.27	8.00	0.131633E-03	0.338645E-03	0.0	0.578616E-04	0.0	-0.245567E-03	0.339243E-02	0.475565E-03
20.00	29.27	-24.00	0.155373E-03	0.307648E-04	0.0	0.972479E-04	0.0	-0.193732E-03	0.856702E-03	0.238984E-03
20.00	29.27	24.00	0.155373E-03	0.212425E-03	0.0	0.972482E-04	0.0	-0.302044E-03	0.350797E-03	0.394854E-03

COORDINATES		DEPTH	S-T-R-E-S-S-E-S						
X	Y	Z	XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
20.00	31.50	0.0	-0.149705E 03	0.135765E-03	0.0	-0.131929E 03	0.0	-0.793907E 02	0.298439E-01
20.00	31.50	-8.00	0.871106E 02	0.281239E 01	0.0	0.672242E 02	0.0	-0.803096E 01	0.293667E-01
20.00	31.50	8.00	0.425102E 00	0.281244E 01	0.0	-0.817795E 00	0.0	-0.803090E 01	0.293668E-01
20.00	31.50	-24.00	0.580480E 01	0.320587E 00	0.0	0.447658E 01	0.0	-0.162247E 01	0.264120E-01
20.00	31.50	24.00	-0.213086E 00	0.320591E 00	0.0	-0.405447E 00	0.0	-0.162246E 01	0.264120E-01

COORDINATES		DEPTH	S-T-R-A-I-N-S						STRAIN ENERGY	WORK
X	Y	Z	XX	XY	XZ	YY	YZ	ZZ	DENSITY	STRAIN
20.00	31.50	0.0	-0.135786E-03	0.775684E-09	0.0	-0.839382E-04	0.0	0.692972E-04	0.129500E-01	0.232290E-03
20.00	31.50	-8.00	0.132153E-03	0.164056E-04	0.0	0.741508E-04	0.0	-0.145343E-03	0.892420E-02	0.192832E-03
20.00	31.50	8.00	0.132153E-03	0.262494E-03	0.0	0.741507E-04	0.0	-0.262460E-03	0.252816E-02	0.410541E-03
20.00	31.50	-24.00	0.155438E-03	0.299214E-04	0.0	0.934549E-04	0.0	-0.191167E-03	0.834587E-03	0.235879E-03
20.00	31.50	24.00	0.155438E-03	0.206603E-03	0.0	0.934553E-04	0.0	-0.298693E-03	0.339272E-03	0.388314E-03

```
********************* OPTN = 2    (TANDEM)                                                 ********************
*****************************************************************************************************************
*******************************************THE ANSWERS BELOW ARE BY SUPERPOSITION*******************************************
*****************************************************************************************************************
```

THE PROBLEM PARAMETERS ARE

LAYER 1 HAS MODULUS 0,480,000 POISSONS RATIO 0.400 AND THICKNESS 8.00 IN,
LAYER 2 HAS MODULUS 0,030,000 POISSONS RATIO 0.400 AND THICKNESS 6.00 IN,
LAYER 3 HAS MODULUS 0,024,500 POISSONS RATIO 0.400 AND THICKNESS 16.00 IN,
LAYER 4 HAS MODULUS 0,004,500 POISSONS RATIO 0.450 AND IS SEMI-INFINITE.

COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE

X(1)= 20.00 Y(1)= 20.00 P(1)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(2)= 20.00 Y(2)= 33.50 P(2)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(3)= 20.00 Y(3)= 94.30 P(3)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(4)= 20.00 Y(4)=107.80 P(4)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(5)= 70.00 Y(5)= 20.00 P(5)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(6)= 70.00 Y(6)= 33.50 P(6)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(7)= 70.00 Y(7)= 94.30 P(7)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(8)= 70.00 Y(8)=107.80 P(8)= 4500.00 TIRE PRESSURE= 80.00 PSI

COORDINATES X	Y	DEPTH Z	S-T-R-E-S-S-E-S XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
20.00	29.27	0.0	-0.115413E 03	0.265308E 01	0.0	-0.955747E 02	0.0	-0.404865E 02	0.444388E-01
20.00	29.27	-8.00	0.771379E 02	0.139842E 01	0.0	0.567631E 02	0.0	-0.798010E 01	0.442516E-01
20.00	29.27	8.00	0.109602E 00	0.290853E 01	0.0	-0.171584E 01	0.0	-0.798006E 01	0.442517E-01
20.00	29.27	-14.00	0.178759E 01	0.112623E 01	0.0	0.765196E 00	0.0	-0.489560E 01	0.429506E-01
20.00	29.27	14.00	0.914400E 00	0.116384E 01	0.0	-0.263284E-01	0.0	-0.489560E 01	0.429507E-01
20.00	29.27	-30.00	0.396793E 01	-0.288301E 00	0.0	0.355357E 01	0.0	-0.171936E 01	0.400697E-01
20.00	29.27	30.00	-0.279942E 00	0.364713E-01	0.0	-0.567113E 00	0.0	-0.171935E 01	0.400696E-01

COORDINATES X	Y	DEPTH Z	S-T-R-A-I-N-S XX	XY	XZ	YY	YZ	ZZ	STRAIN ENERGY DENSITY	WORK STRAIN
20.00	29.27	0.0	-0.127059E-03	0.773787E-05	0.0	-0.691978E-04	0.0	0.914759E-04	0.880766E-02	0.191569E-03
20.00	29.27	-8.00	0.120910E-03	0.128553E-04	0.0	0.597664E-04	0.0	-0.128209E-03	0.693684E-02	0.170010E-03
20.00	29.27	8.00	0.146671E-03	0.276160E-03	0.0	0.340055E-04	0.0	-0.244584E-03	0.260558E-02	0.416780E-03
20.00	29.27	-14.00	0.128120E-03	0.114689E-03	0.0	0.534845E-04	0.0	-0.197224E-03	0.910364E-03	0.246355E-03
20.00	29.27	14.00	0.134164E-03	0.142584E-03	0.0	0.474404E-04	0.0	-0.214319E-03	0.953614E-03	0.279009E-03
20.00	29.27	-30.00	0.139432E-03	-0.103897E-04	0.0	0.100911E-03	0.0	-0.192978E-03	0.626213E-03	0.226096E-03
20.00	29.27	30.00	0.208286E-03	0.460626E-04	0.0	0.320559E-04	0.0	-0.297374E-03	0.235431E-03	0.323474E-03

```
******************** OPTN = 2    (TANDEM)                                                                              ********************
*******************************************************************************************************************************************
****************************************THE ANSWERS BELOW ARE BY SUPERPOSITION*************************************************************
*******************************************************************************************************************************************
```

THE PROBLEM PARAMETERS ARE

LAYER 1 HAS MODULUS 0,480,000 POISSONS RATIO 0.400 AND THICKNESS 8.00 IN,
LAYER 2 HAS MODULUS 0,030,000 POISSONS RATIO 0.400 AND THICKNESS 6.00 IN,
LAYER 3 HAS MODULUS 0,024,500 POISSONS RATIO 0.400 AND THICKNESS 16.00 IN,
LAYER 4 HAS MODULUS 0,004,500 POISSONS RATIO 0.450 AND IS SEMI-INFINITE.

COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE

X(1)= 20.00 Y(1)= 20.00 P(1)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(2)= 20.00 Y(2)= 33.50 P(2)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(3)= 20.00 Y(3)= 94.30 P(3)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(4)= 20.00 Y(4)=107.80 P(4)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(5)= 70.00 Y(5)= 20.00 P(5)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(6)= 70.00 Y(6)= 33.50 P(6)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(7)= 70.00 Y(7)= 94.30 P(7)= 4500.00 TIRE PRESSURE= 80.00 PSI
X(8)= 70.00 Y(8)=107.80 P(8)= 4500.00 TIRE PRESSURE= 80.00 PSI

COORDINATES		DEPTH	S-T-R-E-S-S-E-S						
X	Y	Z	XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
20.00	31.50	0.0	-0.144916E 03	0.269884E 01	0.0	-0.134258E 03	0.0	-0.791189E 02	0.449040E-01
20.00	31.50	-8.00	0.809289E 02	0.564432E 00	0.0	0.659088E 02	0.0	-0.833767E 01	0.444442E-01
20.00	31.50	8.00	0.128675E 00	0.209859E 01	0.0	-0.137333E 01	0.0	-0.833762E 01	0.444443E-01
20.00	31.50	-14.00	0.179167E 01	0.896993E 00	0.0	0.749988E 00	0.0	-0.480360E 01	0.431289E-01
20.00	31.50	14.00	0.930068E 00	0.935206E 00	0.0	-0.285906E-01	0.0	-0.480360E 01	0.431289E-01
20.00	31.50	-30.00	0.397600E 01	-0.295529E 00	0.0	0.350451E 01	0.0	-0.172382E 01	0.402765E-01
20.00	31.50	30.00	-0.279808E 00	0.352985E-01	0.0	-0.581564E 00	0.0	-0.172382E 01	0.402765E-01

COORDINATES		DEPTH	S-T-R-A-I-N-S						STRAIN ENERGY	WORK
X	Y	Z	XX	XY	XZ	YY	YZ	ZZ	DENSITY	STRAIN
20.00	31.50	0.0	-0.124094E-03	0.787175E-05	0.0	-0.930082E-04	0.0	0.678135E-04	0.125737E-01	0.228889E-03
20.00	31.50	-8.00	0.121502E-03	0.806519E-05	0.0	0.759412E-04	0.0	-0.139735E-03	0.803076E-02	0.182925E-03
20.00	31.50	8.00	0.147787E-03	0.200641E-03	0.0	0.496563E-04	0.0	-0.261324E-03	0.194220E-02	0.359833E-03
20.00	31.50	-14.00	0.127510E-03	0.934462E-04	0.0	0.514195E-04	0.0	-0.194009E-03	0.797797E-03	0.230622E-03
20.00	31.50	14.00	0.133678E-03	0.116608E-03	0.0	0.452510E-04	0.0	-0.210783E-03	0.818746E-03	0.258527E-03
20.00	31.50	-30.00	0.140789E-03	-0.107952E-04	0.0	0.986949E-04	0.0	-0.192491E-03	0.623567E-03	0.225618E-03
20.00	31.50	30.00	0.211080E-03	0.457273E-04	0.0	0.284046E-04	0.0	-0.296933E-03	0.236737E-03	0.324371E-03

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FILE: CHOPTN3  SCALE    A1  University of Kentucky Computing Center

|...+....1....+....2....+....3....+....4....+....5....+....6....+....7....+....8

    3    1
OPTN = 3     (ROAD RATER)
    3    3    4    4
1200000 400     11850 400      3000 450
  7     16
  0     7     23
 1525 1000 1525 2200 1525 3400 1525 4600
 1000 1000 335  9410 1000 1000 265  7290 2050 1000 335  9410 2050 1000 265  7290
OPTN = 3     (ROAD RATER)
    4    4    4    4
2000000 400    111850 400     27500 450      7500 450
  7     6     16
  0     7     13    29
 1525 1000 1525 2200 1525 3400 1525 4600
 1000 1000 335  9410 1000 1000 265  7290 2050 1000 335  9410 2050 1000 265  7290

|...+....1....+....2....+....3....+....4....+....5....+....6....+....7....+....8
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******************** OPTN = 3     (ROAD RATER)                                                               ********************
*******************************************************************************************************************************
*******************************************THE ANSWERS BELOW ARE BY SUPERPOSITION***********************************************
*******************************************************************************************************************************

                                THE PROBLEM PARAMETERS ARE
           LAYER  1  HAS MODULUS    1,200,000   POISSONS RATIO 0.400   AND THICKNESS   7.00 IN,
           LAYER  2  HAS MODULUS    0,011,850   POISSONS RATIO 0.400   AND THICKNESS  16.00 IN,
           LAYER  3  HAS MODULUS    0,003,000   POISSONS RATIO 0.450   AND IS SEMI-INFINITE.

                         COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE
                  X( 1)= 10.00 Y( 1)= 10.00 P( 1)=  941.00 TIRE PRESSURE= 33.50 PSI
                  X( 2)= 10.00 Y( 2)= 10.00 P( 2)=  729.00 TIRE PRESSURE= 26.50 PSI
                  X( 3)= 20.50 Y( 3)= 10.00 P( 3)=  941.00 TIRE PRESSURE= 33.50 PSI
                  X( 4)= 20.50 Y( 4)= 10.00 P( 4)=  729.00 TIRE PRESSURE= 26.50 PSI
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COORDINATES X	Y	DEPTH Z	S-T-R-E-S-S-E-S XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
15.25	10.00	0.0	-0.729191E 01	0.857135E-05	0.0	-0.903232E 01	0.0	-0.143685E-01	0.139009E-02
15.25	10.00	-7.00	0.800193E 01	-0.502803E-01	0.0	0.981052E 01	0.0	-0.172346E 00	0.138950E-02
15.25	10.00	7.00	-0.347374E-01	-0.502837E-01	0.0	-0.168786E-01	0.0	-0.172338E 00	0.138951E-02
15.25	10.00	-23.00	0.874503E-01	-0.491461E-02	0.0	0.899600E-01	0.0	-0.558273E-01	0.122256E-02
15.25	10.00	23.00	-0.112061E-01	-0.491464E-02	0.0	-0.105927E-01	0.0	-0.558273E-01	0.122256E-02

COORDINATES X	Y	DEPTH Z	S-T-R-A-I-N-S XX	XY	XZ	YY	YZ	ZZ	STRAIN ENERGY DENSITY	WORK STRAIN
15.25	10.00	0.0	-0.306105E-05	0.196691E-10	0.0	-0.509151E-05	0.0	0.542944E-05	0.341155E-04	0.754050E-05
15.25	10.00	-7.00	0.345556E-05	-0.117320E-06	0.0	0.556557E-05	0.0	-0.608110E-05	0.416619E-04	0.833286E-05
15.25	10.00	7.00	0.345561E-05	-0.118814E-04	0.0	0.556554E-05	0.0	-0.128009E-04	0.219093E-05	0.192296E-04
15.25	10.00	-23.00	0.622759E-05	-0.116126E-05	0.0	0.652411E-05	0.0	-0.106997E-04	0.875837E-06	0.121581E-04
15.25	10.00	23.00	0.622758E-05	-0.475081E-05	0.0	0.652411E-05	0.0	-0.153393E-04	0.405424E-06	0.164403E-04

COORDINATES X	Y	DEPTH Z	S-T-R-E-S-S-E-S XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
15.25	22.00	0.0	-0.509708E 01	-0.381470E-05	0.0	-0.380148E 01	0.0	0.294432E-01	0.124039E-02
15.25	22.00	-7.00	0.484048E 01	0.333700E-01	0.0	0.335420E 01	0.0	-0.974073E-01	0.124101E-02
15.25	22.00	7.00	-0.164985E-01	0.333738E-01	0.0	-0.311753E-01	0.0	-0.974097E-01	0.124101E-02
15.25	22.00	-23.00	0.846033E-01	0.703996E-02	0.0	0.764930E-01	0.0	-0.499151E-01	0.111606E-02
15.25	22.00	23.00	-0.841232E-02	0.703999E-02	0.0	-0.103947E-01	0.0	-0.499151E-01	0.111607E-02

COORDINATES X	Y	DEPTH Z	S-T-R-A-I-N-S XX	XY	XZ	YY	YZ	ZZ	STRAIN ENERGY DENSITY	WORK STRAIN
15.25	22.00	0.0	-0.299022E-05	-0.363798E-11	0.0	-0.147869E-05	0.0	0.299072E-05	0.104753E-04	0.417838E-05
15.25	22.00	-7.00	0.294814E-05	0.778673E-07	0.0	0.121414E-05	0.0	-0.281273E-05	0.931361E-05	0.393988E-05
15.25	22.00	7.00	0.294813E-05	0.788578E-05	0.0	0.121419E-05	0.0	-0.661098E-05	0.805096E-06	0.116568E-04
15.25	22.00	-23.00	0.624235E-05	0.166345E-05	0.0	0.528419E-05	0.0	-0.965010E-05	0.730426E-06	0.111031E-04
15.25	22.00	23.00	0.624234E-05	0.680529E-05	0.0	0.528418E-05	0.0	-0.138172E-04	0.386941E-06	0.160612E-04

********************* OPTN = 3 (ROAD RATER) *********************

****

THE ANSWERS BELOW ARE BY SUPERPOSITION

****

THE PROBLEM PARAMETERS ARE

LAYER 1 HAS MODULUS 1,200,000 POISSONS RATIO 0.400 AND THICKNESS 7.00 IN.
LAYER 2 HAS MODULUS 0,011,850 POISSONS RATIO 0.400 AND THICKNESS 16.00 IN.
LAYER 3 HAS MODULUS 0,003,000 POISSONS RATIO 0.450 AND IS SEMI-INFINITE.

COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE

X(1)= 10.00 Y(1)= 10.00 P(1)= 941.00 TIRE PRESSURE= 33.50 PSI
X(2)= 10.00 Y(2)= 10.00 P(2)= 729.00 TIRE PRESSURE= 26.50 PSI
X(3)= 20.50 Y(3)= 10.00 P(3)= 941.00 TIRE PRESSURE= 33.50 PSI
X(4)= 20.50 Y(4)= 10.00 P(4)= 729.00 TIRE PRESSURE= 26.50 PSI

COORDINATES X	Y	DEPTH Z	S-T-R-E-S-S-E-S XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
15.25	34.00	0.0	-0.274444E 01	-0.143051E-05	0.0	-0.117191E 01	0.0	-0.440079E-01	0.108245E-02
15.25	34.00	-7.00	0.246831E 01	0.368044E-01	0.0	0.854230E 00	0.0	-0.483718E-01	0.108290E-02
15.25	34.00	7.00	-0.755604E-02	0.368095E-01	0.0	-0.234949E-01	0.0	-0.483733E-01	0.108290E-02
15.25	34.00	-23.00	0.588683E-01	0.114495E-01	0.0	0.368380E-01	0.0	-0.367032E-01	0.100918E-02
15.25	34.00	23.00	-0.736357E-02	0.114496E-01	0.0	-0.127485E-01	0.0	-0.367032E-01	0.100918E-02

COORDINATES X	Y	DEPTH Z	S-T-R-A-I-N-S XX	XY	XZ	YY	YZ	ZZ	STRAIN ENERGY DENSITY	WORK STRAIN
15.25	34.00	0.0	-0.188173E-05	-0.187583E-11	0.0	-0.471120E-07	0.0	0.126878E-05	0.258183E-05	0.207438E-05
15.25	34.00	-7.00	0.178831E-05	0.858761E-07	0.0	-0.947886E-07	0.0	-0.114782E-05	0.220064E-05	0.191513E-05
15.25	34.00	7.00	0.178828E-05	0.869758E-05	0.0	-0.947739E-07	0.0	-0.303401E-05	0.708045E-06	0.109317E-04
15.25	34.00	-23.00	0.496323E-05	0.270537E-05	0.0	0.236050E-05	0.0	-0.632790E-05	0.367643E-06	0.787715E-05
15.25	34.00	23.00	0.496321E-05	0.110679E-04	0.0	0.236049E-05	0.0	-0.921759E-05	0.389284E-06	0.161097E-04

COORDINATES X	Y	DEPTH Z	S-T-R-E-S-S-E-S XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
15.25	46.00	0.0	-0.148220E 01	-0.214577E-05	0.0	-0.107921E 00	0.0	0.300466E-01	0.923956E-03
15.25	46.00	-7.00	0.132406E 01	0.285433E-01	0.0	-0.187398E-01	0.0	-0.257961E-01	0.924301E-03
15.25	46.00	7.00	-0.395171E-02	0.285472E-01	0.0	-0.172118E-01	0.0	-0.257948E-01	0.924301E-03
15.25	46.00	-23.00	0.383940E-01	0.110592E-01	0.0	0.101315E-01	0.0	-0.254841E-01	0.881836E-03
15.25	46.00	23.00	-0.600359E-02	0.110592E-01	0.0	-0.129118E-01	0.0	-0.254842E-01	0.881841E-03

COORDINATES X	Y	DEPTH Z	S-T-R-A-I-N-S XX	XY	XZ	YY	YZ	ZZ	STRAIN ENERGY DENSITY	WORK STRAIN
15.25	46.00	0.0	-0.120921E-05	-0.483169E-11	0.0	0.394115E-06	0.0	0.555079E-06	0.883215E-06	0.121327E-05
15.25	46.00	-7.00	0.111822E-05	0.666007E-07	0.0	-0.448365E-06	0.0	-0.456598E-06	0.754187E-06	0.112115E-05
15.25	46.00	7.00	0.111822E-05	0.674531E-05	0.0	-0.448372E-06	0.0	-0.146239E-05	0.405628E-06	0.827408E-05
15.25	46.00	-23.00	0.375823E-05	0.261313E-05	0.0	0.419206E-06	0.0	-0.378855E-05	0.180342E-06	0.551702E-05
15.25	46.00	23.00	0.375816E-05	0.106905E-04	0.0	0.419237E-06	0.0	-0.565741E-05	0.294554E-06	0.140132E-04

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********************* OPTN = 3     (ROAD RATER)                                                  *********************
**********************************************************************************************************************
*****************************************THE ANSWERS BELOW ARE BY SUPERPOSITION***************************************
**********************************************************************************************************************
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THE PROBLEM PARAMETERS ARE

LAYER 1 HAS MODULUS 2,000,000 POISSONS RATIO 0.400 AND THICKNESS 7.00 IN.
LAYER 2 HAS MODULUS 0,111,850 POISSONS RATIO 0.400 AND THICKNESS 6.00 IN.
LAYER 3 HAS MODULUS 0,027,500 POISSONS RATIO 0.450 AND THICKNESS 16.00 IN.
LAYER 4 HAS MODULUS 0,007,500 POISSONS RATIO 0.450 AND IS SEMI-INFINITE.

COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE

X(1)= 10.00 Y(1)= 10.00 P(1)= 941.00 TIRE PRESSURE= 33.50 PSI
X(2)= 10.00 Y(2)= 10.00 P(2)= 729.00 TIRE PRESSURE= 26.50 PSI
X(3)= 20.50 Y(3)= 10.00 P(3)= 941.00 TIRE PRESSURE= 33.50 PSI
X(4)= 20.50 Y(4)= 10.00 P(4)= 729.00 TIRE PRESSURE= 26.50 PSI

COORDINATES		DEPTH	S-T-R-E-S-S-E-S						
X	Y	Z	XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
15.25	10.00	0.0	-0.532060E 01	-0.126658E-04	0.0	-0.694563E 01	0.0	-0.143704E-01	0.545423E-03
15.25	10.00	-7.00	0.536591E 01	-0.231150E 00	0.0	0.697501E 01	0.0	-0.354200E 00	0.545572E-03
15.25	10.00	7.00	0.771638E-01	-0.231152E 00	0.0	0.167153E 00	0.0	-0.354199E 00	0.545573E-03
15.25	10.00	-13.00	0.413847E 00	-0.395753E-01	0.0	0.451414E 00	0.0	-0.149176E 00	0.521777E-03
15.25	10.00	13.00	0.162013E-01	-0.395755E-01	0.0	0.251194E-01	0.0	-0.149176E 00	0.521778E-03
15.25	10.00	-29.00	0.821806E-01	-0.407468E-02	0.0	0.838363E-01	0.0	-0.475746E-01	0.449110E-03
15.25	10.00	29.00	-0.589589E-02	-0.407473E-02	0.0	-0.544437E-02	0.0	-0.475744E-01	0.449111E-03

COORDINATES		DEPTH	S-T-R-A-I-N-S						STRAIN ENERGY	WORK
X	Y	Z	XX	XY	XZ	YY	YZ	ZZ	DENSITY	STRAIN
15.25	10.00	0.0	-0.126830E-05	-0.179174E-10	0.0	-0.240583E-05	0.0	0.244606E-05	0.117115E-04	0.342220E-05
15.25	10.00	-7.00	0.135879E-05	-0.323609E-06	0.0	0.248516E-05	0.0	-0.264528E-05	0.129307E-04	0.359592E-05
15.25	10.00	7.00	0.135880E-05	-0.578652E-05	0.0	0.248517E-05	0.0	-0.404044E-05	0.365080E-05	0.807962E-05
15.25	10.00	-13.00	0.261915E-05	-0.990709E-06	0.0	0.308936E-05	0.0	-0.442808E-05	0.164795E-05	0.542837E-05
15.25	10.00	13.00	0.261915E-05	-0.417341E-05	0.0	0.308938E-05	0.0	-0.610073E-05	0.845387E-06	0.784109E-05
15.25	10.00	-29.00	0.239501E-05	-0.429692E-06	0.0	0.248231E-05	0.0	-0.444663E-05	0.311740E-06	0.476151E-05
15.25	10.00	29.00	0.239500E-05	-0.157556E-05	0.0	0.248230E-05	0.0	-0.566283E-05	0.133725E-06	0.597160E-05

COORDINATES		DEPTH	S-T-R-E-S-S-E-S						
X	Y	Z	XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
15.25	22.00	0.0	-0.356742E 01	0.476837E-05	0.0	-0.249925E 01	0.0	0.294378E-01	0.474326E-03
15.25	22.00	-7.00	0.290104E 01	0.116350E 00	0.0	0.171735E 01	0.0	-0.122363E 00	0.475160E-03
15.25	22.00	7.00	0.852298E-01	0.116353E 00	0.0	0.190307E-01	0.0	-0.122362E 00	0.475160E-03
15.25	22.00	-13.00	0.297449E 00	0.380129E-01	0.0	0.228191E 00	0.0	-0.936099E-01	0.462658E-03
15.25	22.00	13.00	0.188618E-01	0.380129E-01	0.0	0.242144E-02	0.0	-0.936096E-01	0.462656E-03
15.25	22.00	-29.00	0.821649E-01	0.617017E-02	0.0	0.764221E-01	0.0	-0.422792E-01	0.403834E-03
15.25	22.00	29.00	-0.274921E-02	0.617026E-02	0.0	-0.431548E-02	0.0	-0.422791E-01	0.403835E-03

COORDINATES		DEPTH	S-T-R-A-I-N-S						STRAIN ENERGY	WORK
X	Y	Z	XX	XY	XZ	YY	YZ	ZZ	DENSITY	STRAIN
15.25	22.00	0.0	-0.128975E-05	0.437694E-11	0.0	-0.542027E-06	0.0	0.122805E-05	0.299593E-05	0.173088E-05
15.25	22.00	-7.00	0.113152E-05	0.162892E-06	0.0	0.302938E-06	0.0	-0.984859E-06	0.199958E-05	0.141406E-05
15.25	22.00	7.00	0.113153E-05	0.291273E-05	0.0	0.302937E-06	0.0	-0.146683E-05	0.818654E-06	0.382602E-05
15.25	22.00	-13.00	0.217806E-05	0.951595E-06	0.0	0.131118E-05	0.0	-0.271673E-05	0.673033E-06	0.346909E-05
15.25	22.00	13.00	0.217805E-05	0.400863E-05	0.0	0.131119E-05	0.0	-0.375225E-05	0.502510E-06	0.604534E-05
15.25	22.00	-29.00	0.242912E-05	0.650680E-06	0.0	0.212631E-05	0.0	-0.413249E-05	0.276432E-06	0.448376E-05
15.25	22.00	29.00	0.242911E-05	0.238583E-05	0.0	0.212630E-05	0.0	-0.521334E-05	0.131723E-06	0.592673E-05

******************** OPTN = 3 (ROAD RATER) ********************

****

***THE ANSWERS BELOW ARE BY SUPERPOSITION**

****

THE PROBLEM PARAMETERS ARE

LAYER 1 HAS MODULUS 2,000,000 POISSONS RATIO 0.400 AND THICKNESS 7.00 IN.
LAYER 2 HAS MODULUS 0,111,850 POISSONS RATIO 0.400 AND THICKNESS 6.00 IN.
LAYER 3 HAS MODULUS 0,027,500 POISSONS RATIO 0.450 AND THICKNESS 16.00 IN.
LAYER 4 HAS MODULUS 0,007,500 POISSONS RATIO 0.450 AND IS SEMI-INFINITE.

COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE

X(1)= 10.00 Y(1)= 10.00 P(1)= 941.00 TIRE PRESSURE= 33.50 PSI
X(2)= 10.00 Y(2)= 10.00 P(2)= 729.00 TIRE PRESSURE= 26.50 PSI
X(3)= 20.50 Y(3)= 10.00 P(3)= 941.00 TIRE PRESSURE= 33.50 PSI
X(4)= 20.50 Y(4)= 10.00 P(4)= 729.00 TIRE PRESSURE= 26.50 PSI

S-T-R-E-S-S-E-S

COORDINATES X	Y	DEPTH Z	XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
15.25	34.00	0.0	-0.183914E 01	-0.143051E-05	0.0	-0.666670E 00	0.0	-0.440081E-01	0.411368E-03
15.25	34.00	-7.00	0.133920E 01	0.878593E-01	0.0	0.234329E 00	0.0	-0.457913E-01	0.411876E-03
15.25	34.00	7.00	0.460747E-01	0.878636E-01	0.0	-0.157149E-01	0.0	-0.457909E-01	0.411876E-03
15.25	34.00	-13.00	0.165103E 00	0.435366E-01	0.0	0.602903E-01	0.0	-0.461258E-01	0.406593E-03
15.25	34.00	13.00	0.131762E-01	0.435367E-01	0.0	-0.117048E-01	0.0	-0.461257E-01	0.406593E-03
15.25	34.00	-29.00	0.592006E-01	0.107113E-01	0.0	0.419036E-01	0.0	-0.322688E-01	0.371024E-03
15.25	34.00	29.00	-0.305570E-02	0.107114E-01	0.0	-0.777307E-02	0.0	-0.322689E-01	0.371025E-03

S-T-R-A-I-N-S

COORDINATES X	Y	DEPTH Z	XX	XY	XZ	YY	YZ	ZZ	STRAIN ENERGY DENSITY	WORK STRAIN
15.25	34.00	0.0	-0.777434E-06	-0.193268E-11	0.0	0.432951E-07	0.0	0.479155E-06	0.689930E-06	0.830620E-06
15.25	34.00	-7.00	0.631893E-06	0.123003E-06	0.0	-0.141516E-06	0.0	-0.337601E-06	0.435879E-06	0.660211E-06
15.25	34.00	7.00	0.631890E-06	0.219954E-05	0.0	-0.141514E-06	0.0	-0.517965E-06	0.414046E-06	0.272096E-05
15.25	34.00	-13.00	0.142546E-05	0.108987E-05	0.0	0.113539E-06	0.0	-0.121844E-05	0.244096E-06	0.208919E-05
15.25	34.00	13.00	0.142545E-05	0.459114E-05	0.0	0.113540E-06	0.0	-0.170137E-05	0.447731E-06	0.570633E-05
15.25	34.00	-29.00	0.199509E-05	0.112956E-05	0.0	0.108306E-05	0.0	-0.282785E-05	0.151571E-06	0.332014E-05
15.25	34.00	29.00	0.199509E-05	0.414172E-05	0.0	0.108306E-05	0.0	-0.365279E-05	0.140405E-06	0.611892E-05

S-T-R-E-S-S-E-S

COORDINATES X	Y	DEPTH Z	XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
15.25	46.00	0.0	-0.964500E 00	-0.357628E-06	0.0	-0.373195E-02	0.0	0.300480E-01	0.351245E-03
15.25	46.00	-7.00	0.678181E 00	0.534807E-01	0.0	-0.148317E 00	0.0	-0.203486E-01	0.351570E-03
15.25	46.00	7.00	0.251206E-01	0.534824E-01	0.0	-0.211011E-01	0.0	-0.203479E-01	0.351570E-03
15.25	46.00	-13.00	0.935783E-01	0.333237E-01	0.0	-0.209437E-02	0.0	-0.239948E-01	0.349317E-03
15.25	46.00	13.00	0.828268E-02	0.333240E-01	0.0	-0.144287E-01	0.0	-0.239949E-01	0.349316E-03
15.25	46.00	-29.00	0.394136E-01	0.108064E-01	0.0	0.156445E-01	0.0	-0.230474E-01	0.328922E-03
15.25	46.00	29.00	-0.296496E-02	0.108064E-01	0.0	-0.944743E-02	0.0	-0.230474E-01	0.328924E-03

S-T-R-A-I-N-S

COORDINATES X	Y	DEPTH Z	XX	XY	XZ	YY	YZ	ZZ	STRAIN ENERGY DENSITY	WORK STRAIN
15.25	46.00	0.0	-0.487508E-06	-0.454747E-12	0.0	0.185025E-06	0.0	0.208670E-06	0.237888E-06	0.487738E-06
15.25	46.00	-7.00	0.372824E-06	0.748729E-07	0.0	-0.205725E-06	0.0	-0.116147E-06	0.150867E-06	0.388416E-06
15.25	46.00	7.00	0.372823E-06	0.133885E-05	0.0	-0.205724E-06	0.0	-0.196295E-06	0.152060E-06	0.164894E-05
15.25	46.00	-13.00	0.929942E-06	0.834210E-06	0.0	-0.267568E-06	0.0	-0.541690E-06	0.105888E-06	0.137601E-05
15.25	46.00	13.00	0.929940E-06	0.351418E-05	0.0	-0.267573E-06	0.0	-0.771970E-06	0.249256E-06	0.425767E-05
15.25	46.00	-29.00	0.155436E-05	0.113958E-05	0.0	0.301078E-06	0.0	-0.173903E-05	0.776558E-07	0.237649E-05
15.25	46.00	29.00	0.155435E-05	0.417845E-05	0.0	0.301085E-06	0.0	-0.232824E-05	0.113411E-06	0.549836E-05

******************** OPTN = 3 (ROAD RATER) ********************
****
THE ANSWERS BELOW ARE BY SUPERPOSITION
****

THE PROBLEM PARAMETERS ARE

LAYER 1 HAS MODULUS 2,000,000 POISSONS RATIO 0.400 AND THICKNESS 7.00 IN.
LAYER 2 HAS MODULUS 0,111,850 POISSONS RATIO 0.400 AND THICKNESS 6.00 IN.
LAYER 3 HAS MODULUS 0,027,500 POISSONS RATIO 0.450 AND THICKNESS 16.00 IN.
LAYER 4 HAS MODULUS 0,007,500 POISSONS RATIO 0.450 AND IS SEMI-INFINITE.

COORDINATES OF THE LOAD POINTS AND LOAD VALUES ARE

X(1)= 10.00 Y(1)= 10.00 P(1)= 941.00 TIRE PRESSURE= 33.50 PSI
X(2)= 10.00 Y(2)= 10.00 P(2)= 729.00 TIRE PRESSURE= 26.50 PSI
X(3)= 20.50 Y(3)= 10.00 P(3)= 941.00 TIRE PRESSURE= 33.50 PSI
X(4)= 20.50 Y(4)= 10.00 P(4)= 729.00 TIRE PRESSURE= 26.50 PSI

COORDINATES X	Y	DEPTH Z	S-T-R-E-S-S-E-S XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
15.25	34.00	0.0	-0.183914E 01	-0.143051E-05	0.0	-0.666670E 00	0.0	-0.440081E-01	0.411368E-03
15.25	34.00	-7.00	0.133920E 01	0.878593E-01	0.0	0.234329E 00	0.0	-0.457913E-01	0.411876E-03
15.25	34.00	7.00	0.460747E-01	0.878636E-01	0.0	-0.157149E-01	0.0	-0.457909E-01	0.411876E-03
15.25	34.00	-13.00	0.165103E 00	0.435366E-01	0.0	0.602903E-01	0.0	-0.461258E-01	0.406593E-03
15.25	34.00	13.00	0.131762E-01	0.435367E-01	0.0	-0.117048E-01	0.0	-0.461257E-01	0.406593E-03
15.25	34.00	-29.00	0.592006E-01	0.107113E-01	0.0	0.419036E-01	0.0	-0.322688E-01	0.371024E-03
15.25	34.00	29.00	-0.305570E-02	0.107114E-01	0.0	-0.777307E-02	0.0	-0.322689E-01	0.371025E-03

COORDINATES X	Y	DEPTH Z	S-T-R-A-I-N-S XX	XY	XZ	YY	YZ	ZZ	STRAIN ENERGY DENSITY	WORK STRAIN
15.25	34.00	0.0	-0.777434E-06	-0.193268E-11	0.0	0.432951E-07	0.0	0.479155E-06	0.689930E-06	0.830620E-06
15.25	34.00	-7.00	0.631893E-06	0.123003E-06	0.0	-0.141516E-06	0.0	-0.337601E-06	0.435879E-06	0.660211E-06
15.25	34.00	7.00	0.631890E-06	0.219954E-05	0.0	-0.141514E-06	0.0	-0.517965E-06	0.414046E-06	0.272096E-05
15.25	34.00	-13.00	0.142546E-05	0.108987E-05	0.0	0.113539E-06	0.0	-0.121844E-05	0.244096E-06	0.208919E-05
15.25	34.00	13.00	0.142545E-05	0.459114E-05	0.0	0.113540E-06	0.0	-0.170137E-05	0.447731E-06	0.570633E-05
15.25	34.00	-29.00	0.199509E-05	0.112956E-05	0.0	0.108306E-05	0.0	-0.282785E-05	0.151571E-06	0.332014E-05
15.25	34.00	29.00	0.199509E-05	0.414172E-05	0.0	0.108306E-05	0.0	-0.365279E-05	0.140405E-06	0.611892E-05

COORDINATES X	Y	DEPTH Z	S-T-R-E-S-S-E-S XX	XY	XZ	YY	YZ	ZZ	DEFLECTION
15.25	46.00	0.0	-0.964500E 00	-0.357628E-06	0.0	-0.373195E-02	0.0	0.300480E-01	0.351245E-03
15.25	46.00	-7.00	0.678181E 00	0.534807E-01	0.0	-0.148317E 00	0.0	-0.203486E-01	0.351570E-03
15.25	46.00	7.00	0.251206E-01	0.534824E-01	0.0	-0.211011E-01	0.0	-0.203479E-01	0.351570E-03
15.25	46.00	-13.00	0.935783E-01	0.333237E-01	0.0	-0.209437E-02	0.0	-0.239948E-01	0.349317E-03
15.25	46.00	13.00	0.828268E-02	0.333240E-01	0.0	-0.144287E-01	0.0	-0.239949E-01	0.349316E-03
15.25	46.00	-29.00	0.394136E-01	0.108064E-01	0.0	0.156445E-01	0.0	-0.230474E-01	0.328922E-03
15.25	46.00	29.00	-0.296496E-02	0.108064E-01	0.0	-0.944743E-02	0.0	-0.230474E-01	0.328924E-03

COORDINATES X	Y	DEPTH Z	S-T-R-A-I-N-S XX	XY	XZ	YY	YZ	ZZ	STRAIN ENERGY DENSITY	WORK STRAIN
15.25	46.00	0.0	-0.487508E-06	-0.454747E-12	0.0	0.185025E-06	0.0	0.208670E-06	0.237888E-06	0.487738E-06
15.25	46.00	-7.00	0.372824E-06	0.748729E-07	0.0	-0.205725E-06	0.0	-0.116147E-06	0.150867E-06	0.388416E-06
15.25	46.00	7.00	0.372823E-06	0.133885E-05	0.0	-0.205724E-06	0.0	-0.196295E-06	0.152060E-06	0.164894E-05
15.25	46.00	-13.00	0.929942E-06	0.834210E-06	0.0	-0.267568E-06	0.0	-0.541690E-06	0.105888E-06	0.137601E-05
15.25	46.00	13.00	0.929940E-06	0.351418E-05	0.0	-0.267573E-06	0.0	-0.771970E-06	0.249256E-06	0.425767E-05
15.25	46.00	-29.00	0.155436E-05	0.113958E-05	0.0	0.301078E-06	0.0	-0.173903E-05	0.776558E-07	0.237649E-05
15.25	46.00	29.00	0.155435E-05	0.417845E-05	0.0	0.301085E-06	0.0	-0.232824E-05	0.113411E-06	0.549936E-05