Research Report
KTC-93-29

Computer Program for Analysis of Embankments with Tensile Elements

by

MIKHAIL E. SLEPAK
Research Engineer

and

TOMMY C. HOPKINS
Head of Geotechnology

Kentucky Transportation Center
College of Engineering
University of Kentucky

in cooperation with
Transportation Cabinet
Commonwealth of Kentucky

and

Federal Highway Administration
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DON C. KELLY, P.E.
SECRETARY OF TRANSPORTATION

JERRY D. ANGLIN
DEPUTY SECRETARY
AND
COMMISSIONER OF HIGHWAYS

COMMONWEALTH OF KENTUCKY
TRANSPORTATION CABINET
FRANKFORT, KENTUCKY 40622

BRERETON C. JONES
GOVERNOR

July 21, 1995

Mr. Paul E. Toussant
Division Administrator
Federal Highway Administration
330 West Broadway, Frankfort, Kentucky 40602-0536

SUBJECT: Implementation Statement: Research Study KYHPR-88-122, "Computer Program for Analysis of Embankments Reinforced With Tensile Elements"

Dear Mr. Toussant:

Tensile reinforcing elements have increasingly become a very important construction material on highway projects over the past several years. Tensile reinforcing elements, such as geofabrics, geogrids, or metal strips are used to construct and reinforce such highway structures as embankments on soft foundations, steep embankment slopes, retaining structures, and other earthen structures. In certain design situations, especially where right-of-way problems, or space limitations, may be present, tensile reinforced slopes and walls offer good economical alternatives when compared to more conventional approaches. Reinforcing embankments that are to be located on soft, compressive foundations may prevent the development of embankment tension cracks and prevent failure during construction. Mechanical reinforcement reduces tensile strains. Consequently, in the coming years, we anticipate that the use of reinforced slopes and walls will increase in Kentucky.

In anticipating an increase in the use of reinforced slopes and walls, our geotechnical staff felt that a comprehensive and generalized mathematical model and computer program were needed to reduce the time required for designing these types of structures and to ease the use of this approach. Several mathematical models and approaches for designing reinforced slopes and walls currently exist, as shown in this report. Many of these methods may be applied only to certain design situations. The method proposed by one manufacturer -- the Tensar® Corporation -- represents a good approach, but it is limited to analyzing mainly the internal stability of the reinforced earthen structure. If the reinforced slope or wall is located on a firm foundation, then the approach appears to yield a reasonable and safe design. If the reinforced portion of the embankment is located on a soft, compressible foundation then a question arises concerning the external, or global, stability of the reinforced earthen structure. Although the internal stability may be adequate here, the failure surface may pass behind and below the portion of the embankment containing reinforcement. During early stages of this research study, members of the Study Advisory Committee and the authors of this report were also concerned with the similarity of solutions obtained from different mathematical stability models modified to account for tensile element forces. The "so-called" Bishop model equations -- a widely used limit equilibrium model -- have been modified in two different manners to account for tensile forces. Since this model can only be used to analyze circular shear surfaces, what mathematical model should be used when the potential failure surface is non-circular? In the design of reinforced slopes and walls, the designer must determine and specify the number, spacing, and lengths of tensile elements.

The authors developed two computer programs. As a means of determining the number, spacing, and lengths of tensile elements, the proposed method and algorithms published by the Tensar® Corporation were programmed for the personal computer (PC® IBM) so that manual calculations would not be necessary. Solutions obtained from the PC program and manual calculations for several examples were compared to check the accuracy of the PC computer program. A reinforced slope or wall may be designed in a matter of seconds using the PC program. The design cross section (including the number, spacing, and lengths of tensile elements) may be viewed on the computer screen. The design cross section and data may also be printed. The PC computer program was used successfully to check the design (performed manually by Cabinet engineers) of a reinforced slope constructed on Kentucky (Relocated) Route 34. The PC computer program was transmitted to the geotechnical staff of the Cabinet during the early portion of the study. Training sessions were held with the geotechnical staff of the Cabinet to instruct them on the use and limitations of the PC computer program. A complete discussion of this PC program, which considers only internal stability, was not included in this report. This computer program provides a good approach for obtaining a preliminary design of a reinforced slope or wall.

The second computer program -- which is the main focus of this report -- is a generalized approach to designing and analyzing both the internal and external stabilities of a slope or wall reinforced with tensile elements. The computer program was developed so several limit equilibrium mathematical models may be used to calculate the factors of safety of a reinforced slope or wall. The user may readily compare solutions from different stability models and make an assessment of the safety of a design. These models may be used to calculate the factors of safety of the unreinforced slope or wall so that a quick assessment of the increased stability provided by reinforcement may be determined. The necessary data for making an economical assessment are obtained. Limit equilibrium models included in the generalized computer program include Bishop's (two versions model), Morgenstern-Price's model, the original model developed by Raulin, and a model developed by Hopkins. The authors have developed, or extended, Raulin's perturbation model approach. The extended model is a very generalized approach. The generalized algorithms were programmed for the mainframe computer (IBM® 3090) at the University of Kentucky. This program was made available to the Cabinet's engineers.

Several important findings were obtained during the study. These findings emerged because of the literature search and a comprehensive analysis of different generic examples and case histories using the newly developed generalized computer program. This study showed that two versions of the Bishop model are possible in the case of reinforced slopes. One version, which appears in many publications and computer programs, was found incorrect from a mathematical viewpoint. The authors formulate in the report the correct approach for accounting for tensile element forces in Bishop's model. Both approaches were programmed for the computer so that solutions could be compared. The incorrect formulation was found to yield lower factors of safety than the correct formulation. The exception to this finding was when the factor of safety approaches one. All of the statically consistent, limit equilibrium methods yield essentially the same results when only analyzing the internal stability of reinforced slopes and walls. Bishop's "incorrect" approach, which is frequently used by others, yields very conservative results and should not be used. The approach proposed by the Tensar® Corporation yields essentially the same results as those obtained from the more rigorous methods when the slope angle is less than 45 degrees for internal design. For steeper slopes, the results obtained from the Tensar® method are conservative.

When analyzing the global, or external, stability of reinforced walls and slopes located on soft foundations, the situation is much more complex than the situation involving only internal analysis. Most of the limit equilibrium models tend to violate physical admissibility criteria and may yield different factors of safety. The least violation of admissibility criteria occurred when using the perturbation method proposed by the authors. The designer should exercise extreme caution in attempting to use either of the Bishop models for checking the external stability. Incorrect solutions may be obtained.

Members of the Study Advisory Committee felt that location of the generalized computer program on the mainframe at the University of Kentucky or on the Frankfort mainframe would limit their accessibility to the program and pose an inconvenience in using the program. The committee members recommended that a "PC Version" of the generalized approach be developed. Development of a PC version of the generalized computer program was much beyond the scope of the current study. The major objective of this research study was the development of the model algorithms and the generalized computer program. Committee members recommended that this research study be finalized and that a new study should be initiated to develop a PC version of the generalized program. A new study was initiated. The PC version will be "menu-driven" so that data input and output will be relatively easy. The PC version will contain graphics so that the user can view input and output geometry. The PC program will also contain "help" guidance and error detection statements.

The two computer programs developed during this research study will be combined into one PC computer program. If the user is designing a reinforced slope or wall, then the geometry of the slope to be reinforced is input and the internal design option is first executed. This option provides the user with the number, spacing, and lengths of geofabrics. Also, this option provides the x - and y - coordinates of each layer of geofabric for global analyses. These coordinates are stored internally. If the user chooses to examine the external (as well as the internal) stability, then the user would select the generalized option of the computer program. In this option, additional coordinates of foundation layers, shear strengths, and other conditions, such as pore pressures, would be provided by the user. After providing this information, the global stability would be determined. The user has the option of changing the x - and y - coordinates of the geofabric layers. The user also will have the option of determining the stability of the unreinforced slope so that economies of design may be compared.

Sincerely,

J.M. Yowell, P.E.
State Highway Engineer

Enclosure

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16. Abstract
A generalized computer program for analyzing stability of earth structures reinforced with geosynthetics (geogrids, geotextiles etc.) was developed in this research study. The newly proposed computer program was built in such a way that a reinforced slope stability problem may be analyzed using a variety of limit equilibrium methods including Hopkins', Morgenstern and Price's, Bishop's methods, Raulin's perturbation method, as well as new perturbation methods proposed by the authors of this study. It was shown in the research study that two versions of the Bishop's method were possible in the case of reinforced analyses: the traditionally used version referred to as "incorrect" Bishop's method, and a version that strictly follows the philosophy of the original Bishop's method referred to as "correct" Bishop's method. A comprehensive analysis of different generic examples and case histories was undertaken in the research study using different limit equilibrium methods. It was shown that in an internal stability analysis of reinforced earth structures all statically consistent methods and Bishop's "correct" method provide almost the same values for safety factors. Bishop's "incorrect" method was found to significantly underestimate the factor of safety. It was also shown that the commonly used Tensar® design method tends to be conservative especially for steep slopes. In the case of overall stability of reinforced embankments on soft foundations, most methods led to some violation of admissibility criteria. The least violation of admissibility criteria was observed in a perturbation method proposed by the authors of this research study. Consequently, this method generally yields safety factors that are close to the "right" answers.

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APPROXIMATE CONVERSIONS TO SI UNITS					APPROXIMATE CONVERSIONS FROM SI UNITS				
Symbol	When You Know	Multiply By	To Find	Symbol	Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH					**LENGTH**		
in.	inches	25.40000	millimetres	mm	mm	millimetres	0.03937	inches	in.
ft	feet	0.30480	metres	m	m	metres	3.28084	feet	ft
yd	yards	0.91440	metres	m	m	metres	1.09361	yards	yd
mi	miles	1.60934	kilometres	km	km	kilometres	0.62137	miles	mi
		AREA					**AREA**		
in.^{2}	square inches	645.16000	millimetres squared	mm^2	mm^2	millimetres squared	0.00155	square inches	in.
ft^2	square feet	0.09290	metres squared	m	m^2	metres squared	10.76392	square feet	ft
yd^2	square yards	0.83613	metres squared	m	m^2	metres squared	1.19599	square yards	yd
ac	acres	0.40469	hectares	ha	ha	hectares	2.47103	acres	ac
mi^2	square miles	2.58999	kilometres squared	km^2	km^2	kilometres squared	0.38610	square miles	mi
		VOLUME					**VOLUME**		
fl oz	fluid ounces	29.57353	millilitres	ml	ml	millilitres	0.03381	fluid ounces	fl oz
gal.	gallons	3.78541	litres	l	l	litres	0.26417	gallons	gal.
ft^3	cubic feet	0.02832	metres cubed	m	m^3	metres cubed	35.31448	cubic feet	ft
yd^3	cubic yards	0.76455	metres cubed	m	m^3	metres cubed	1.30795	cubic yards	yd
		MASS					**MASS**		
oz	ounces	28.34952	grams	g	g	grams	0.03527	ounces	oz
lb	pounds	0.45359	kilograms	kg	kg	kilograms	2.20462	pounds	lb
T	short tons (2000 lb)	0.90718	megagrams	Mg	Mg	megagrams	1.10231	short tons (2000 lb)	T
		FORCE AND PRESSURE					**FORCE**		
lbf	pound-force	4.44822	newtons	N	N	newtons	0.22481	pound-force	lbf
psi	pound-force per square inch	6.89476	kilopascal	kPa	kPa	kilopascal	0.14504	pound-force per square inch	psi
		ILLUMINATION					**ILLUMINATION**		
fc	foot-candles	10.76426	lux	lx	lx	lux	0.09290	foot-candles	fc
fl	foot-Lamberts	3.42583	candela/m	cd/m^2	cd/m^2	candela/m^2	0.29190	foot-Lamberts ^{2}	fl

* SI is the symbol for the International System of Units. Appropriate

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EXECUTIVE SUMMARY

The purposes of this study were to develop a generalized and comprehensive computer program for analyzing the stability of earth structures reinforced with geosynthetics (geogrids, geotextiles etc.). The HOPK-I computer program developed at the University of Kentucky Transportation Center by Hopkins was used as a basis for this study. HOPK-I is a generalized slope stability computer program capable of solving a variety of problems for unreinforced earth structures.

A comprehensive analysis of mathematical formulations of a stability problem was undertaken in this research study. It was shown that there are two possible equivalent formulations of a stability problem, using either differential equations of equilibrium for interslice forces, or overall equations of equilibrium for the whole soil mass bounded by the ground surface and the potential failure surface. In both cases, some criteria should be satisfied to provide a physically admissible solution of a problem.

The newly proposed computer program was built in such a way that a reinforced slope stability problem can be analyzed using a variety of limit equilibrium methods including Hopkins' method (originally proposed for HOPK-I computer program), Morgenstern and Price's method, Bishop's method, the original Raulin's perturbation method, as well as new perturbation methods proposed in this study. In Raulin's perturbation method, a simple function for describing the distribution of normal stresses along a potential failure surface was proposed with two unknown parameters determined from equilibrium equations. Although Raulin's method is a statically consistent limit equilibrium method, this method in some cases may violate admissibility criteria and lead to unreasonable values of factors of safety. To improve this situation, two more versions of the perturbation method were proposed by the authors. In these new versions, combinations of normal components of reinforced forces and unreinforced normal stress distribution obtained from Hopkins' method were used as the basic functions in the perturbation equations to obtain a better approximation for the normal stress distribution. In all methods, reinforcement forces were treated as known, horizontal external forces applied at intersections of geosynthetic sheets and the potential shear surface. The magnitude of the reinforcement force is computed in the program as a minimum of a long-term tensile strength and pullout resistances of a geosynthetic sheet in active and passive zones. It was also shown in the research study that two versions of the Bishop's method were possible in the case of reinforced analyses: the traditionally used version referred to as "incorrect" Bishop's method, and a version that strictly follows the philosophy of the original Bishop's method referred to as "correct" Bishop's method.

A comprehensive analysis of different generic examples and case histories was undertaken in the research study using different limit equilibrium methods. It was shown that Hopkins' method, originally proposed for unreinforced slope stability analyses, usually provides a rapid convergence of the factor of safety, but, in some cases involving a high density of reinforcement sheets, convergence problems may arise. Morgenstern and Price's method usually gives reasonable answers, but it is too sensitive to initial approximations. Perturbation methods always provide rapid convergence.

In an internal stability analysis of reinforced earth structures, all statically consistent methods (Hopkins' method, Morgenstern and Price's method and any of the perturbation methods) provide physically admissible solutions and almost the same values of the safety factors. In circular analyses, Bishop's "correct" method yields essentially the same factors of safety as other statically consistent methods. In other words for internal stability analyses using circular failure surfaces, there is no need to use more sophisticated methods than Bishop's "correct" method. Bishop's "incorrect" method was found to significantly underestimate the factor of safety. It was also shown that the two versions of the Bishop's method yield the same results only in two extreme cases: when either there is no reinforcement (unreinforced case) or the factor of safety is equal to one.

The results of the computer analyses of internal stability of reinforced slopes were compared to the results obtained from the Tensar® design method, which is commonly used by practitioners. The comparisons showed that for slopes with angles less than 45° the Tensar® design method yields results that are very close to the ones obtained by more rigorous methods. For steep slopes with angles greater than 45°, the Tensar® method tends to yield conservative results. As the slope of the reinforced structure increases, or becomes steeper, results obtained from this method become more conservative.

The situation is somewhat different in the case of overall stability of reinforced embankments on soft foundations. Generally, in this case, different methods may lead to different factors of safety. To decide on which method yields the "right" answer, admissibility criteria should be further analyzed. It was shown in the research study that most methods led to some violation of admissibility criteria. The least violation of admissibility criteria was observed in a perturbation method proposed in this research study. Consequently this method is supposed to yield safety factors that are close to the "right" answers. It is worth mentioning that either version of Bishop's method (in circular analyses) did not yield results close to the "right" answers in cases involving reinforced embankments on soft foundations. Consequently, caution should be exercised in applying this commonly used method.

Based on the results obtained in this research study the following conclusions and recommendations can be made.

1. All limit equilibrium methods are approximate. None of the equations of solid mechanics is explicitly satisfied everywhere inside or outside of the failure surface. A solution obtained using limit equilibrium methods is not necessarily an upper or a lower bound. However, usage of the method quite often gives acceptable results. It is commonly agreed that statically consistent limit equilibrium methods that satisfy admissibility criteria yield factors of safety that differ less than 5%.

2. Hopkins' method, Morgenstern and Price's method, and Perturbation methods are all statically consistent methods. However, these methods are based on different assumptions and, generally, yield different factors of safety. In all the cases analyzed in this research study, the newly proposed perturbation method converged rapidly and exhibited the least violation of the admissibility criteria. Therefore, this method is recommended for practical use in the stability analyses of reinforced earth structures.

3. Bishop's method is not a statically consistent method. Two options of this method are available in reinforced earth stability analyses. Bishop's "correct" method strictly follows the philosophy of the original Bishop's method (for unreinforced analyses) and can be successfully used in circular analyses of internal stability of reinforced earth structures. However, this method should not be used for analyzing stability of reinforced embankments on soft foundations, but rather the perturbation method mentioned above should be used. Bishop's "incorrect" method does not satisfy the equilibrium equations of the original Bishop's method and usually underestimates the factors of safety. This method should not be used in practice.

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1. LIMIT EQUILIBRIUM METHODS FOR STABILITY ANALYSIS OF EARTH STRUCTURES. BRIEF REVIEW.

1.1. Limit equilibrium method. General definitions.

According to mathematical theory of plasticity, a mass of soil is considered to be in elasto-plastic equilibrium if the following conditions are satisfied within and at the boundary of a mass under consideration:

- stress equilibrium equations,
- stress-strain elasto-plastic constitutive relationships,
- compatibility equations relating strains and displacements, and
- appropriate boundary conditions.

Usual formulation of stress-strain elasto-plastic constitutive relationships involves a yield surface (f) and a flow rule. For bearing capacity and stability problems, elasto-plastic material is usually considered perfectly plastic. For a perfectly plastic material, f depends only on the stress tensor (σ_{ij}). Plastic flow can occur only when the yield condition is satisfied.

$$f(\sigma_{ij})=0 \tag{1.1}$$

Stress states for which $f(\sigma_{ij})>o$ are excluded, and $f(\sigma_{ij})<o$ corresponds to elastic behavior. In soil mechanics, yield function $f(\sigma_{ij})$ is usually assumed in the form:

$$f(\sigma_{ij}) = \tau - c' - \sigma' \tan \phi' \tag{1.2}$$

first suggested by Coulomb (1773).

In Equation 1.2, it is assumed that plastic flow occurs when, on any plane, the shear stress (τ) reaches an amount that depends linearly upon cohesion (c'), and the effective normal stress

$$\sigma' = \sigma - u \tag{1.3}$$

where σ is a total normal stress and u is a pore pressure.

The angle (ϕ') in Equation 1.2 is known as the angle of internal friction of a soil. Bearing capacity and stability problems can be formulated as the problems of finding collapse conditions. These collapse conditions correspond to the stage when the yielding of soil governed by the Coulomb's criterion:

$$\tau = c' + \sigma' \tan \phi' \tag{1.4}$$

has spread to such an extent that the remaining elastic soil plays a relatively insignificant role in sustaining the load. This condition has been termed (Chen, 1975) uncontained or unrestricted plastic flow. This collapse condition can be used as a realistic basis for design.

Limit equilibrium methods have traditionally been used to obtain approximate solutions for the stability problems in soil mechanics. These methods can probably best be described as (Chen, 1975) approximate methods to the construction of a slip-line field and generally entail an assumed failure surface of various simple shapes. With this assumption, each of the stability problems is reduced to one of finding the most dangerous position for the failure or slip surface of the shape chosen. According to limit equilibrium methods, a mass of soil (Fig. 1.1) is considered to be in a state of limit equilibrium if Coulomb's failure condition 1.4 is satisfied along a potential slip surface, y(x) and if equilibrium equations are satisfied for the mass bounded by the slope surface Y(x), and the slip surface y(x). It is worth mentioning here (Chen, 1975) that none of the equations of solid mechanics mentioned is explicitly satisfied everywhere inside or outside the failure surface. The method gives no consideration to soil kinematics and does not satisfy Coulomb's failure criterion 1.4 everywhere inside or outside the failure surface. A solution obtained using the limit equilibrium method is not necessarily an upper or a lower bound. However, useage of the method quite often gives acceptable results.

In general, a soil mass of given properties and geometry that is acted upon by a given set of loads, is not in a state of limiting equilibrium as previously defined. In order to quantify the margin of safety relative to a state of imminent failure, one may replace the soil's real strength parameters c' and ϕ' by artificial ones c_f and ϕ_f, for which a state of limiting equilibrium may be realized. There are many possible ways by which c' and ϕ' may be related to c_f and ϕ_f, and still realize a state of limiting equilibrium. It is customary however (Baker and Garber, 1978), to adjust the real strength parameters by a single factor F in the following manner:

$$\begin{aligned} c_f &= c'/F; \\ \tan\phi_f &= \tan\phi'/F. \end{aligned} \tag{1.5}$$

1.2. Two equivalent formulations of a stability problem.

It was mentioned that none of the equations of solid mechanics is explicitly satisfied everywhere inside or outside the failure surface. As an alternative to equilibrium equations of solid mechanics, limit equilibrium method considers either equations of equilibrium for vertical slices shown in Figures 1.1 and 1.2, or overall equilibrium

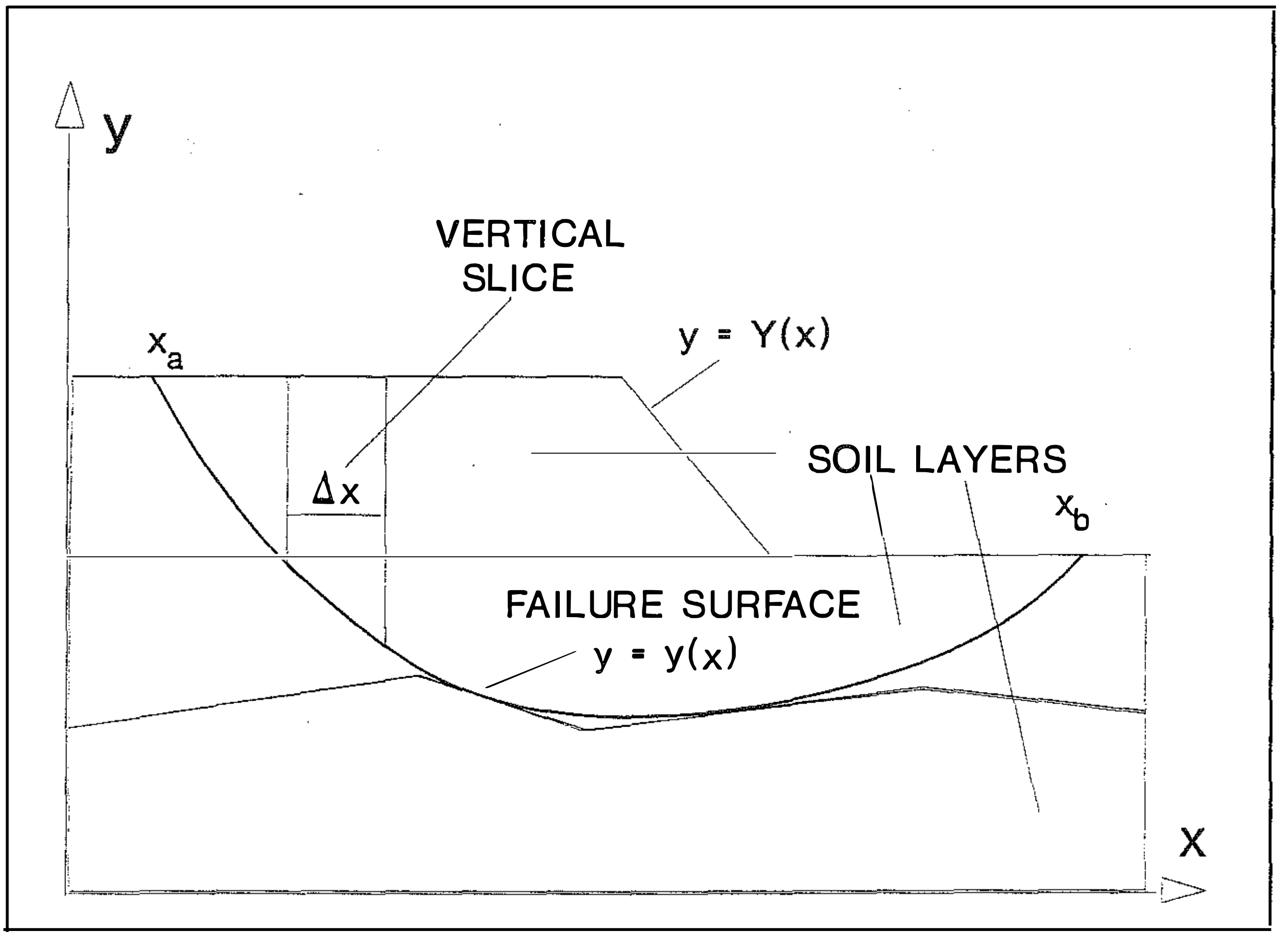

Figure 1.1. Failure Surface in Limit Equilibrium Analysis

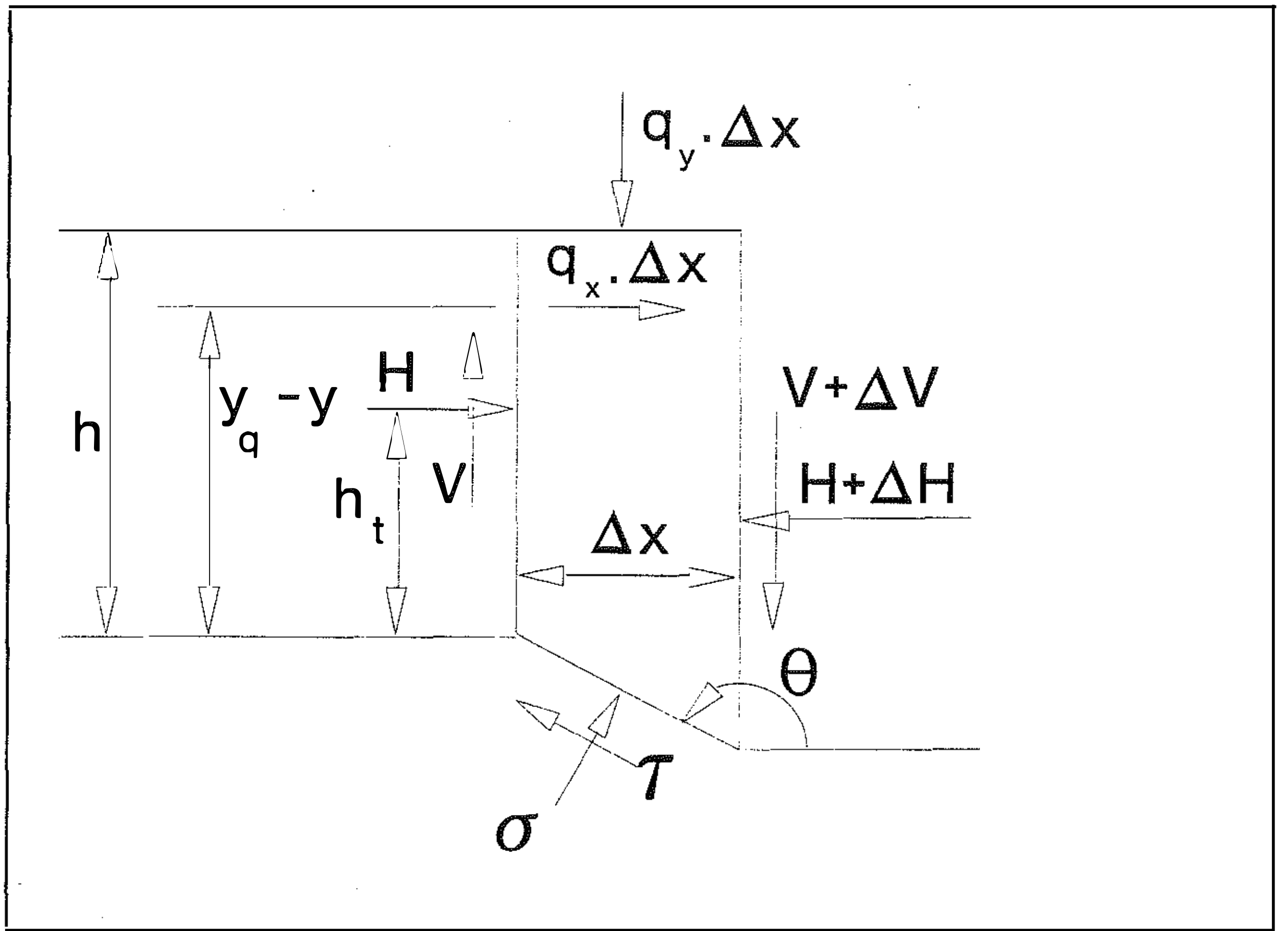

Figure 1.2. Forces Acting on a Vertical Slice.

equations for a soil mass bounded by the slope surface and the shear surface. Therefore, two equivalent formulations of a stability problem are possible.

1.2.1. Limit equilibrium method. Formulation 1.

Taking into consideration Equations 1.4 and 1.5 the slice equilibrium equations for the soil mass bounded by the slope surface Y(x) and any particular slip surface y(x) could be written in a form (Janbu, 1954, 1957; Morgenstern and Price, 1965; Hopkins, 1991):

$$\frac{dH}{dx} = -(\tau + \sigma \tan\theta) + q_x \qquad (1.6)$$

$$\frac{dV}{dx} = \sigma - \tau\tan\theta - (q_y + \gamma_{av}\, h) \qquad (1.7)$$

$$\frac{d(Hh_t)}{dx} = V - H\tan\theta + q_x\,(y_Q - y) \qquad (1.8)$$

where

$$\tau = \frac{c' + (\sigma - u)\tan\phi'}{F}, \qquad (1.9)$$

q_x and q_y = distributed horizontal and vertical external forces, psf,

H and V = horizontal and vertical interslice forces, lbs/ft,

$h_t = y_t - y$, ft,

y_t = y - coordinates of the thrust line, ft,

y_Q = y - coordinates of points of application of q_x, ft,

θ = angle between the tangent to the failure surface and horizontal,

γ_{av} = is the distribution of the average total unit weight of the soil above y(x), pcf.

This quantity is related to the conventional unit weight (γ), by the relation

$$\gamma_{av} = \frac{\int_{y(x)}^{Y(x)} \gamma\, dy}{h} \qquad (1.10)$$

$$h = Y(x) - y(x) \tag{1.11}$$

where h = height of slices, ft.

All the variables in Equations 1.6 through 1.11 are shown in Figures 1.1 and 1.2. Appropriate boundary conditions should be added to the system of Equations 1.6 to 1.9, namely:

$$\begin{aligned} H(x_a) &= H_a \\ H(x_b) &= H_b \\ V(x_a) &= V_a \\ V(x_b) &= V_b \end{aligned} \tag{1.12}$$

As one can see, the system 1.6 to 1.9 with boundary conditions 1.12, is statically indeterminate because there are only four equations available to determine five unknown functions H(x), V(x), $\sigma(x)$, $\tau(x)$ and $h_t(x)$. Therefore, this system may have an infinite number of solutions. In order to narrow the range of possible solutions, some physical admissibility criteria are usually considered (Chen and Morgenstern, 1983). Let us introduce two additional functions related to the set of original unknown functions:

the thrust ratio

$$\eta = h_t/h \tag{1.13}$$

and the average factors of safety on vertical sides of slices:

$$F_v = \frac{c_{av}\, h + (H - u_{av}\, h)\tan\phi_{av}}{V} \tag{1.14}$$

where c_{av}, ϕ_{av}, u_{av} are the average weighted values of c', ϕ' and u.

It is required usually that:

$$o \le \eta \le 1 \tag{1.15}$$

$$F_{ve} = F_v \,/\, F \ge 1 \tag{1.16}$$

where F is defined by Equation 1.9.

It is worth mentioning here that taking account for admissibility criteria 1.15 and 1.16 does not necessarily lead to a rigorous solution of a problem because this solution is still obtained within the framework of approximations of a limit equilibrium method (see above). At the same time, any rigorous solution of a problem formulated as a problem of plasticity theory will satisfy both criteria 1.15 and 1.16.

Admissibility criteria 1.15 and 1.16 narrow a range of possible solutions of the system 1.6 to 1.9, and 1.12, but this system still remains statically indeterminate. In order to provide statical determinacy to the system under consideration, some additional assumptions are usually made. These assumptions will be considered next. Here we will only emphasize that any additional assumption that leads to a statical determinacy of the system 1.6 to 1.9, and 1.12, automatically defines a function $\sigma(x)$ of a normal stress distribution along the slip line y(x). Therefore, the factor F in Equation 1.9 that is completely defined by the functions $\sigma(x)$ and y(x) represents a functional of two functions:

$$F = F\{\, y(x),\ \sigma(x)\,\} \tag{1.17}$$

This functional is termed (Baker and Garber, 1978) the safety functional, to be distinguished from the factor of safety F_s, which is the minimum value of F:

$$F_s = \min F\{\, y(x),\ \sigma(x)\,\} \tag{1.18}$$

Consequently, a stability problem can be formulated in the following way.

Formulation 1.

Among all functions y(x), H(x), V(x), $h_t(x)$, $\sigma(x)$ and $\tau(x)$ which satisfy equilibrium Equations 1.6 to 1.9 and boundary conditions 1.12, find the functions which provide the minimum value F_s of the safety functional 1.17 and meet admissibility conditions 1.15, and 1.16.

1.2.2. Limit equilibrium method. Formulation 2.

There is another possibility to formulate a stability problem using the limit equilibrium method. Let us consider overall equilibrium equations for a soil mass bounded by the slope surface Y(x) and a slip surface y(x) with Coulomb's failure criterion 1.9 satisfied along the slip surface (Baker and Garber, 1978):

$$-\int_{x_a}^{x_b} (c' - u\tan\phi')\,dx - \int_{x_a}^{x_b} \sigma(\tan\phi' + F\tan\theta)\,dx + F\int_{x_a}^{x_b} q_x\,dx = 0 \tag{1.19}$$

$$\int_{x_a}^{x_b} (c' - u\tan\phi')\tan\theta\,dx - \int_{x_a}^{x_b} \sigma(F - \tan\phi'\tan\theta)\,dx + F\int_{x_a}^{x_b} (q_y + \gamma_{av}\,h)\,dx = 0 \tag{1.20}$$

$$\int_{x_a}^{x_b} (c' - u\tan\phi')(y - x\tan\theta)\,dx +$$
$$\int_{x_a}^{x_b} [\sigma y(\tan\phi' + F\tan\theta) + \sigma x\,(F - \tan\phi'\tan\theta)]\,dx - \qquad (1.21)$$
$$F\int_{x_a}^{x_b} y_Q\, q_x\,dx - F\int_{x_a}^{x_b} x\,(\gamma_{av}\,h + q_y)\,dx = 0$$

In these equations, variables have the same meaning as before. It is easy to see from these equations that the quantity F depends on the functions y(x) and σ(x). Therefore, F can be again considered as a safety functional 1.17 of two functions y(x) and σ(x). Its minimum value will provide the value of the factor of safety F_s. So far, there was no attention paid to interslice forces and the Equations 1.6 to 1.12. At the same time it is easy to see from these equations that any assumption made with respect to selecting a function σ(x) will lead to a certain set of functions H, V, h_t and the admissibility criteria of equations 1.15, 1.16 may not be satisfied. In other words, admissibility criteria 1.15, 1.16 put some restrictions to selecting a function of normal stress distribution σ(x). Therefore, a stability problem can be formulated in the following way.

Formulation 2.

Among all functions y(x) and σ(x) which satisfy Equations of overall equilibrium 1.19-1.21, find the functions which provide the minimum value F_s of the safety functional 1.17 and physically admissible interslice characteristics H, V, and h_t (Equations 1.15, 1.16).

It is easy to see that formulation 1 and formulation 2 of a stability problem are equivalent to each other. In the following sections, different limit equilibrium methods are discussed.

1.3. Limit equilibrium methods based on formulation 1.

Several methods have been developed for the stability of slopes in which the failure surface may have any arbitrary shape and the differential equations of equlibrium 1.6 through 1.9 are satisfied. In each method, some additional assumptions are made to render a statical determinacy to the system under consideration.

In Janbu's method (1954, 1957), the thrust line h_t is assumed. This assumption makes the problem statically determinate, because for four unknown functions H, V, σ, τ, four Equations 1.6 to 1.9 are available. Using the overall horizontal equilibrium equation as a stability criterion, an iterative procedure was developed for the factor of safety determination. It was shown by Morgenstern and Price (1965) that convergence problems may arise using Janbu's method, especially in the cases with high cohesion. To overcome these difficulties, Hopkins (1986, 1991) used a special numerical technique to obtain derivatives of interslice forces involved in the Janbu's iterative procedure. In contrast to original Janbu's method, Hopkins' method provides rapid convergence for a wide range of practical problems (Hopkins, 1986, 1991).

Another assumption to make the problem statically determinate was made by Morgenstern and Price (1965). They assumed a linear relationship between interslice forces:

$$V = \lambda \; f(x) \; H \tag{1.22}$$

where λ - is an unknown parameter;
$f(x)$ - is an assumed but known function.

By subsituting Equation 1.22 into the original system of equations 1.6 through 1.9 the system finally results in a system of two equations with respect to λ and F:

$$\begin{aligned} H_{x_b}(\lambda, F) &= H_b \\ M_{x_b}(\lambda, F) &= 0 \end{aligned} \tag{1.23}$$

where

$$\begin{aligned} H_{x_b} &= H(x_b) \\ M_{x_b} &= M(x_b) \\ M(x) &= H(x)\,h_t\,(x) \end{aligned} \tag{1.24}$$

To obtain a solution of the system 1.23, initial approximations for λ and F are assumed. Successive approximations are obtained using the Newton-Raphson technique.

A similar approach was proposed by Hardin (1984). He used the same assumption 1.22 as in the Morgenstern and Price's method but another equation to obtain λ and F. Instead of Equations 1.23, he used overall equilibrium equations.

Spencer (1967, 1973) proposed a method that assumes the inter-slice forces to be parallel. He found the results to be fairly accurate and gave some attention to obtaining an acceptable position for the line of thrust in terms of effective stresses.

The most detailed discussion of the admissibility criteria (1.15, 1.16) is by Chen and Morgenstern (1983). They extended the original generalized method of slices (Morgenstern and Price, 1965) and developed a special numerical procedure for formally exploring the bounds of the factor of safety within the limits of physical admissibility. It was shown that, consistent with earlier studies, the variation in the factor of safety when subjected to conditions of physical admissibility is small for all practical purposes. This analysis confirms the view that variations in the factor of safety between several methods in common use are of little practical significance.

1.4. Limit equilibrium methods based on formulation 2.

In this class of methods, overall equilibrium equations are considered as main equations. Differential equations of equilibrium for interslice forces should be considered to check admissibility criteria.

Taylor's method (Taylor, 1937) satisfies all equilibrium requirements, but makes arbitrary assumptions with respect to both the kinematical function y(x) and the stress function $\sigma(x)$. Taylor's method is based on the assumption that kinematical function represents a circular arc, while the stress function is distributed as sin x.

Another group of methods is represented by logarithmic-spiral methods (Rendulic, 1935; Taylor, 1937; Fröhlich, 1953; Wright, 1969; Huang and Avery, 1976). Log-spiral methods are based on the properties of log-spiral functions, that the resultant of the elementary normal and frictional forces passes through the pole of the spiral. Consequently, in this case the moment equation about the pole is independent of σ and may be utilized for the determination of the factor of safety regardless of the normal stress distribution; i.e., the problem of sliding along a logarithmic spiral is statically determinate. Furthermore, the stress function $\sigma(x)$ can always be selected to satisfy admissibility criteria (1.15, 1.16.) That makes log-spiral methods not only statically consistent but also physically admissible. Therefore, these methods could be good independent checks for other methods. It is necessary to note that log-spiral methods are only applicable in the case of homogeneous soils with $c' = \text{const}$, $\tan \phi' = \text{const}$.

The first attempt to formulate the slope stability problem as a variational problem in terms of Formulation 2 (see above) was made by Kopacsy (1955). A reappraisal of Kopacsy's analysis by Baker and Garber (1977a) shows that this analysis contains a number of serious errors and misconceptions. An improved variational formulation of the slope stability problem was presented by Baker and Garber (1977b). This formulation applies to the case of homogeneous and isotropic soil, without pore water pressure or external loads. Baker and Garber (1978) later extended their approach to the general case of non-homogeneous, non-isotropic soil with arbitrary distribution of pore water pressure and external loads. They proved that the minimal factor of safety had to occur on slip surfaces with a special geometrical property. The geometrical property ensures that the resultant of the infinitesimal normal and frictional forces either pass through a common point or are parallel to a common direction. It is shown that as a result of this geometrical property the minimal factor of safety is independent of the normal stress distribution along the critical slip surface. In the homogeneous and isotropic case, the analysis shows that the critical slip surface may be either a log-spiral (rotational failure mode) or a straight line (translational failure mode). In a layered profile, the critical slip surface may consist of a series of log-spirals that have a common pole or a series of straight lines. In some cases, the boundary between layers may be part of the critical slip surface. Baker and Garber (1978) suggested a simple computational scheme for the determination of the factor of safety and the critical slip surface. This computational scheme is only slightly more laborious than generally used simplified Bishop's method. It was also proved by Baker and Garber (1978) that for the homogeneous and isotropic case, without pore water pressure or external loads, the solutions provided by Rendulic (log-spiral), and Culmann (straight-line) are not only convenient, but correct. The two methods are related to the two possible modes of failure (rotational and translational).

It is necessary to note that these variational formulations of the problem are cirticized by some researchers (De Jong, 1980, 1981; Luceno and Castillo, 1981). According to De Jong (1980), the variational approach leads to a weak extremum (or no extremum at all) and therefore it produces unsafe predictions for slope stability problems. At the same time, it was shown by Castillo and Luceno (1983), Leshchinsky et al. (1985) that the procedure based on variational formulation of the problem yields a result that is equivalent to an upper-bound solution in the strict framework of limit analyses of plasticity. Subsequently, although the variationally obtained y(x) signifies a stationary result by virtue of satisfying Euler's equation, it may actually yield a local minimum or inflection value of F_s in the context of limit analysis (Leshchinsky, 1990).

At the same time, encouraging results were obtained by Leshchinsky (1990). Based on the results by Baker and Garber (1978), he developed an approximate procedure for evaluating safety factors for any kind of arbitrarily selected slip surfaces. For each arbitrarily selected y(x), Leshchinsky (1990) used Euler's equation for normal stress distribution $\sigma(x)$ obtained by Baker and Garber (1978). The specified y(x) that yields the minimum F_s is considered the critical surface. Using this approach, however, it is explicitly being assumed that among all possible stress distributions, the variationally determined $\sigma(x)$ leads to F_s, which is a genuine minimum for all specified y(x). Leshchinsky considered three example problems. For all three examples, he found the line of thrust to be reasonable. This result indirectly supports the original variational approach by Baker and Garber (1978).

Another group of methods which is based on overall equilibrium equations is known as a group of perturbation methods (Raulin et al, 1974). In these methods, any arbitrarily selected slip line can be analyzed. As a first step, an approximation for the normal stress distribution is considered.

$$\sigma_o(x) = (\gamma_{av}\, h + q_y)\cos^2\theta + q_x \sin\theta \cos\theta \qquad (1.25)$$

Equation 1.25 is based on the assumption that the normal stress distribution is not affected by interslice forces. The real normal stress distribution is assumed in the following form:

$$\sigma(x) = \omega(x)\ \sigma_o(x) \qquad (1.26)$$

where $\omega(x)$ is a perturbation coefficient. Raulin et al (1974) consider three possible expressions for $\omega(x)$:

$$\omega(x) = \lambda + \mu \tan\theta \tag{1.27}$$

$$\omega(x) = \lambda + \mu \tan^2\theta \tag{1.28}$$

$$\omega(x) = \lambda \tan\theta + \mu \tan^2\theta \tag{1.29}$$

By substituting any of expressions 1.27 to 1.29 into Equation 1.26 and then into equilibrium equations, one can obtain three algebraic equations with respect to three unknowns λ, μ, and F.

It was shown by Raulin et al (1974) that all three perturbation methods based on Equations 1.27, 1.28 or 1.29 provide practically the same values of factors of safety.

1.5. Simplified limit equilibrium methods.

All the methods considered above were statically consistent; i.e., three equilibrium equations (either overall or differential) were always satisfied. There are also some methods which satisfy only some of the equilibrium equations and ignore the others. These methods will be referred to as simplified methods.

1.5.1. Ordinary method of slices.

Ordinary method of slices, also known as Fellenius' (1927, 1936) method, satisfies only one condition of equilibrium, which is overall moment equilibrium around the center of a circular slip surface (the method is only applicable to circular slip surfaces). The method assumes that the resultant of all side forces on any slice acts parallel to the base of a slice. Ordinary method of slices does not satisfy either horizontal or vertical force equilibrium for the mass above the slip surface, but it provides a simple procedure for the determination of the safety factor.

1.5.2. Bishop's method

Bishop's (1955) modified method provides a more rigorous approach by including the inter-slice forces in the equations of equilibrium of a typical slice. This method satisfies the overall moment equilibrium equation around the center of the circle (the method is also applicable only to circular slip surfaces) and vertical equilibrium equation for each slice. The method does not satisfy horizontal force equilibrium or individual slice moment equilibrium equations. The solution of the problem is obtained by iteration.

1.5.3. Force equilibrium methods.

Another group consists of methods which use only force equilibrium conditions. These include: the method described by Lowe and Karafiath (1960); the method developed by Seed and Sultan (1967); various methods (commonly known as "wedge" or "sliding block" methods) in which the slip surface is assumed to consist oftwo or three plane segments (Chowdhury, 1978); and any other methods which satisfy only force (not moment) equilibrium. In all such methods, the analysis can be accomplished by trial-and-error graphical procedures wherein a value F is assumed and a trial force polygon is drawn for each slice or wedge; if the last slice is in equilibrium, the assumed value of F is correct. The analysis may also be accomplished by a numerical equivalent of this graphical procedure.

1.6. Comparison of different limit equilibrium methods.

According to Duncan and Wright (1980), methods which satisfy all conditions of equilibrium (log spiral, Janbu's, Spencer's, and Morgenstern and Price's methods) all give essentially the same value of F_s. Studies of non-homogeneous slopes and dams, and non-circular slip surfaces, show a slightly wider disparity in the values of F_s calculated by these methods. These studies indicate that for any practical slope stability problem, any method which satisfies all conditions of equilibrium will give a value of F_s which differs by no more than ±5% from what may be considered the "correct" answer. Thus, although there is no mathematical proof that the values of F_s calculated by Janbu's, Morgenstern and Price's, and Spencer's methods are rigorously correct, from a practical point of view there is no doubt that they may be considered to be correct for all pratical purposes.

Bishop's method, which does not satisfy all condition of equilibrium, gives virtually the same value of F_s as methods which satisfy all conditions of equilibrium. Thus, for analyses of circular slip surfaces, no more elaborate method need be used (Morgenstern and Price, 1965; Duncan and Wright, 1980).

Ordinary method of slices which is also applicable only to circluar slip surfaces gives values of F_s which are lower than those calculated by more accurate methods. For u=o conditions (in practical terms these would be total stress analyses), the inaccuracy is no more than a few percent. For effective stress analyses with high pore pressures, the inaccuracy may be as much as 50%. Thus, while ordinary method of slices may be applied to total stress analyses, it should not be used for effective stress analyses with high pore pressure (Duncan and Wright, 1980).

For $\phi' = 0$ conditions, any method which satisfies moment equilibrium around the center of a circular slip surface will give the correct value of F_s, regardless of what other equilibrium conditions it does or does not satisfy (Duncan and Wright, 1980). Thus, the ordinary method of slices, Bishop's method, Janbu's method, Morgenstern and Price's method and Spencer's method all give the same value of F_s for circular slip surfaces and $\phi' = 0$ conditions.

The factor of safety calculated by force equilibrium procedures is significantly affected by the assumed side force inclination (Duncan and Wright, 1980). Of all the possible assumed inclinations for these forces, the one suggested by Lowe and Karafiath (1960) appears to be the most generally applicable. They proposed that the side force inclination at each interslice boundary should be assumed to be the average of: (a) the inclination of the ground surface at the top of the interslice boundary; and (b) the inclination of the slip surface at the bottom of the interslice boundary.

1.7. Use of limit equilibrium methods in the stability analysis of reinforced earth structures.

1.7.1. Reinforced earth structures. General concepts.

The concept of reinforcing an earth fill by incorporating geosynthetics which possess a much higher tensile strength than soil, and the capability to bond with soil through friction has begun to gain popularity in recent years. Polymer based soil reinforcing materials are finding increased use in permanent, critical applications such as reinforced soil retaining walls, steep fills, and earth dams. These uses are in contrast to earlier soil reinforcement applications such as unpaved roads, temporary retaining walls, and low-height embankments where the reinforcement function was often temporary or where the consequences of failure were not severe.

In the walls reinforced with geosynthetics, the sheets of geosynthetic are used to wrap layers of compacted soil producing a stable composite structure. Advantages of reinforced walls over conventional concrete walls include (Leshchinsky and Perry, 1987):

- in many cases, the reinforced wall cost-effectiveness compares favorably with conventional walls;
- the construction of reinforced walls is simple and rapid; and
- the reinforced wall is flexible, thus it can undergo significant deformation or sustain significant dynamic impacts.

Geogrids and other geosynthetic reinforcement materials have expanded the practical options for design of soil slopes. These materials permit construction of slopes at angles steeper than the angle of repose of the soil fill, thereby reducing land requirements for slope construction and often eliminating the need for retaining walls. Steep reinforced soil slopes frequently provide economic advantages over traditional design alternatives.

The main performance criterion for a reinforced soil structure is adequate stability against sliding of the soils comprising the structure. The common practice concerning reinforced soil structures analysis consists in utilizing traditional limiting equilibrium methods originally developed for unreinforced soil stability analysis. Reinforcing forces are treated as known external forces.

1.7.2. Two definitions of the safety factor.

There are two commonly used approaches for incorporating the factor of safety into limiting equilibrium design equations (Bonaparte et al., 1987). The factor of safety can be defined as the ratio of the resistance (forces or moments) of the soil structure and foundation to the applied loading effects. This approach is usually used in design methods that treat the unstable soil zone as a rigid body (Schneider and Holtz, 1986; Verduin and Holtz, 1989; Jewell, 1982; Ingold, 1982; Murray, 1982). For example in the case of a circular analysis, this definition will lead to a following expression for the safety factor (Langston and Williams, 1989):

$$F=\frac{M_{res}+M_g}{M_{dr}} \tag{1.30}$$

where M_{res} and M_{dr} are resisting and driving moments correspondingly (the same values as in unreinforced soil stability analysis) and M_g is the geosynthetic resisting moment.

Alternatively, the factor of safety can be applied to the soil shear strength to produce factored shear strength parameters. This is the approach when the soil is considered to behave as a continuum (Schmertmann et al., 1987; Leshchinsky and Reinschmidt, 1985; Giroud and Beech, 1989; Gourc et al., 1989; Leshchinsky and Perry, 1987). In this case, the safety factor is defined using the same expression 1.9 as in unreinforced stability analysis.

These two approaches give answers that are close to each other for cases in which the soil weight and soil shear strength are the major destabilizing and stabilizing forces, respectively (Bonaparte et al., 1987). For reinforced soil structures, the reinforcement forces can be large, and the two cited methods for calculating the factor of safety give different answers (Bonaparte et al., 1987). In fact, since the soil and reinforcement often exhibit markedly different stress-strain behavior, no meaningful overall factor of safety can be defined for the reinforced soil structure. That is why Bonaparte et al. (1987) recommended to apply the factor of safety to the soil shear strength, i.e. to use the second definition of the factor of safety based on Equation 1.9.

1.7.3. Treatment of reinforcement forces in stability analysis.

Internal failure may result from reinforcement rupture, reinforcement pullout, or a combination of both. Reinforcement rupture can occur when the tensile force required to maintain equilibrium at any elevation within the slope exceeds the available tensile strength of the reinforcement. Reinforcement pullout can occur when the frictional forces in active or passive zone (Fig. 1.3) are less than the tensile forces required to maintain equilibrium. The direction of the reinforcement resistance (T) is characterized by the angle β ($o \leq \beta \leq \theta$). In most applications, reinforcement forces are assumed horizontal ($\beta = o$) and their magnitude is assumed equal to long-term tensile strength of a geosynthetic sheet under consideration.

However, since geosynthetics have no significant lateral stiffness and T is activated by soil differential movement, it is obvious that T is not horizontal ($\beta > o$). Leshchinsky and Reinschmidt (1985), Leshchinsky and Perry (1987) assume that when failure of the composite structure occurs, the membrane at the slip surface will be inclined so as to contribute the most resistance, i.e., be most effective. A more rigorous approach is based on the principle of strain compatibility. Although limiting equilibrium methods don't allow taking strains into account, in the last few years, the importance of strains in the design of reinforced structures has been recognized (McGown et al., 1984; Bonaparte et al. 1987; Beech, 1987; Wallace and Fluet, 1987; Delmas et al., 1986; Gourc et al., 1989). One of the primary reasons for this is that various types of strain are involved (soil and reinforcement). Since soil and reinforcement have different deformation characteristics, their strength can not usually be mobilized at the same strains. Consequently, both magnitude and direction of the geosynthetic resistance will depend on soil strains or displacements along the potential slip surface. In other words, strains of soil and reinforcement must be compatible. The most rigorous limit equilibrium procedure accounting for strain compatibility is developed by Delmas et al. (1986) and Gourc et al (1986, 1989). In this method, the relationship between the displacement along the potential slip surface and the inclination of the geosynthetic sheets at the points of intersection with the slip surface is established accounting for both rupture and pullout resistance of geosynthetics. This allows calculation of the magnitude and the direction of the geosynthetic resistance at any displacement Δ of a sliding soil mass. In the proposed procedure the factor of safety F_s is referenced to the soil strength parameters only (Equation 1.9). The required value F_s of the safety factor is chosen (e.g. $F_{sr} = 1.5$). Different slip surfaces having initial values of $F_s = F_{so} < F_{sr}$ for $\Delta = 0$ are considered. Then, the displacement Δ is increased by small increments. By increasing Δ, the reinforcement resistance is mobilized and shear soil resistance necessary to maintain equilibrium is decreased that results in an increase of the factor of safety. Calculations stop when $F_s = F_{sr}$. The slip surface having a maximum value of Δ (or maximum values of mobilized reinforcement resistance) is considered critical.

Another possibility to estimate the reinforcement forces consists of using finite element solutions of the plasticity theory. This possibility is discussed by Rowe (1984), Rowe and Soderman (1985, 1987), Rowe and Mylleville (1989). Rowe and Soderman

$$T = \min(T_r\ ,\ T_a\ ,\ T_p\ )$$

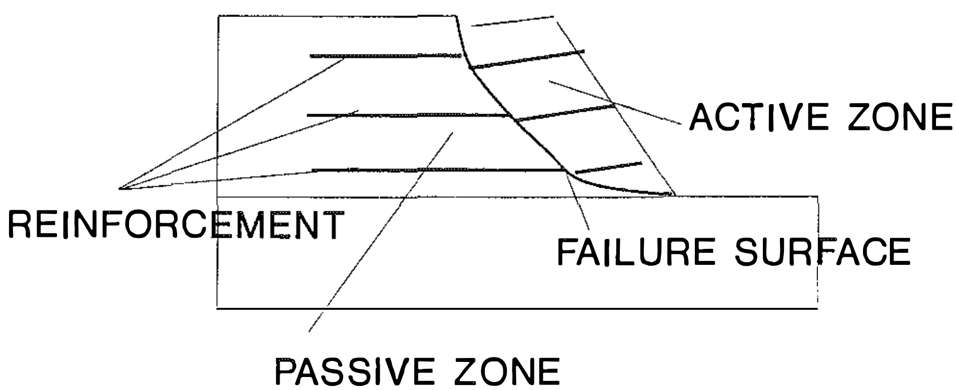

Figure 1.3. Treatment of reinforcement Forces in Stability Analysis

(1985) developed a method for estimating the short-term stability of reinforced embankments constructed on a uniform purely cohesive foundation. This approach maintains the simplicity of simple limit equilibrium techniques while incorporating the effects of soil-geosynthetic interaction in terms of an allowable compatible strain for the geosynthetic. This allowable compatible strain may be deduced from a design chart and depends on the foundation stiffness, the embankment geometry, the deposit depth, the unit weight of the fill, and the critical height of an unreinforced embankment. As it was shown further (Rowe and Soderman, 1987; Rowe and Mylleville, 1989), this approach worked well since the reinforced collapse mechanism was similar to the unreinforced collapse mechanism. However, this is not the case when embankments are constructed on a foundation where there is a significant strength increase with depth.

1.7.4. Methods commonly used in reinforced earth stability analysis.

Most methods commonly used in reinforced earth stability analysis are based on either force equilibrium methods (Tensar technical note, 1986a, b; Steward et al., 1977; Bonaparte et al, 1987; Murray, 1982; Schneider and Holtz, 1986; Jewell et al., 1986; Schmertmann et al., 1987) or circular slip methods (Ingold, 1982; Jewell, 1982; Verduin and Holtz, 1989; Berg et al, 1989).

Some authors used statically consistent methods. For example, investigations carried out by Leshchinsky (1984, 1985), Leshchinsky and Reinschmidt (1985), Leshchinsky and Perry (1987), Leshchinsky and Boedeker (1989) are based on a variational approach proposed by Baker and Garber (1978). Gourc et al. (1989) used a perturbation method.

A comparative analysis of different methods for reinforced soil stability analysis was done by Langston and Williams (1989) and Wright and Duncan (1991). It was shown that methods that satisfy all conditions of equilibrium result in essentially the same value of factor of safety regardless of the assumptions they may involve. Bishop's method, although it does not satisfy all conditions of equilibrium, results in values of safety factors that are essentially the same as values calculated using methods that do satisfy all conditions of equilibrium. As in the case of unreinforced analysis, force equilibrium methods provide safety factors which are strongly affected by the assumed inclination of side forces.

1.8. The major objective of the current research study.

Different methods are available for evaluating the stability of reinforced earth structures. All the methods built within the framework of limit equilibrium methods are approximate. The major objective of the current research study consists in finding a method which provides reasonable accuracy and at the same time can be easily used by practitioners. In the following sections, different methods will be considered in detail. The merits based on a comprehensive comparative analysis and the shortcomings of each method will be outlined. Recommendations with respect to the selection of the method will be done for certain classes of practical problems.

2. DEVELOPMENT OF MULTIPURPOSE LIMIT EQUILIBRIUM COMPUTER PROGRAM FOR REINFORCED EARTH STABILITY ANALYSIS.

2.1. General characteristics of the program.

The computer program developed by Hopkins (1986, 1991) has been used as a basis for the proposed multipurpose limit equilibrium computer program. Calculations are based on assumptions that the safety factor is referred to the soil shear strength only (Equation 1.9) and the reinforcement forces are treated as known external forces. As mentioned (see section 1.7.3.), these forces should be derived from a principle of strain compatibility. As a first step in this study, a simple assumption with respect to reinforcement forces is made and based on this assumption the most reliable limit equilibrium method is selected. Once the basic limit equilibrium method is set up to handle the reinforced earth stability analysis, any other extensions to this method can be made including more rigorous calculations of reinforcement forces based on a principle of strain compatibility. However, a complete implementation of this principle is beyond the scope of this study and may be considered as a proposal for future research.

In this report, reinforcement forces T are considered horizontal and equal to (see Figure 1.3):

$$T = \min(T_r, T_a, T_p) \tag{2.1}$$

where

T_r = is a long-term tensile strength of a fabric;

T_a = is a pullout resistance of a fabric in an active zone;

T_p = is a pullout resistance of a fabric in a passive zone;

$$T_a = k_1 \sum_{1_a} (c_i' + \sigma_i' \tan\phi_i')\Delta x_i + k_2 \sum_{1_a} (C_j' + \sigma_j' \tan\psi_j')\Delta x_j \tag{2.2}$$

$$T_p = k_1 \sum_{l_p} (c'_i + \sigma'_i \tan\phi'_i)\ \Delta x_i + \qquad (2.3)$$
$$+ k_2 \sum_{l_p} (C'_j + \sigma'_j \tan\psi'_j)\ \Delta x_j$$

l_a and l_p = are lengths of a fabric in an active and passive zone correspondingly;

c'_i and ϕ'_i = are the soil effective strength parameters above the fabric;

C'_j and ψ'_j = are the soil effective strength parameters below the fabric;

k_1 and k_2 = are the soil-fabric interaction coefficients for soils above and below the fabric correspondingly;

$\sigma'_i = \sigma_i - u_i$ - is an effective normal stress;
u_i = is a pore pressure;

$$\sigma_i = \gamma_{av,i}\ h_i + q_y^i \qquad (2.4)$$

Though assumptions 2.1 to 2.4 simplify the problem, very often they may lead to reasonable answers. Besides, as it was mentioned, this research is basically methodological and it is an authors' belief that from the methodological point of view these assumptions should work very well.

The computer program is set up in such a way that it can use different limit equilibrium methods according to the user's choice. These methods are discussed in the following sections. All of these methods assume two dimensional failure mechanism. Detailed information about the program is included in the user's manual (section 4) and in the appendices.

2.2. Limit equilibrium methods used in the computer program.

2.2.1. Basic equations.

Differential and integral equations in section 1.2 are rewritten here to take into account reinforcement forces which are treated as known external concentrated forces. The differential equations of equilibrium are as follows:

$$\frac{dH}{dx} = -(\tau + \sigma\tan\theta) + q_x - t(x) \qquad (2.5)$$

$$\frac{dv}{dx} = \sigma - \tau\tan\theta - (q_y + \gamma_{av}\ h) \qquad (2.6)$$

$$\frac{d(Hh_t)}{dx} = V - H\tan\theta + q_x\,(y_Q - y) \tag{2.7}$$

$$\tau = \frac{c' + (\sigma - u)\tan\phi'}{F} \tag{2.8}$$

Integral equations of equilibrium are as follows:

$$-\int_{x_a}^{x_b}(c' - u\tan\phi')\,dx - \int_{x_a}^{x_b}\sigma(\tan\phi' + F\tan\theta)\,dx + F\int_{x_a}^{x_b}q_x\,dx - F\sum T_i = o \tag{2.9}$$

$$\int_{x_a}^{x_b}(c' - u\tan\phi')\tan\theta\,dx - \int_{x_a}^{x_b}\sigma(F - \tan\phi'\tan\theta)\,dx + F\int_{x_a}^{x_b}(q_y + \gamma_{av}\,h)\,dx = o \tag{2.10}$$

$$\int_{x_a}^{x_b}(c' - u\tan\phi')(y - x\tan\theta)\,dx + \int_{x_a}^{x_b}[\sigma y(\tan\phi' + F\tan\theta) + \sigma x(F - \tan\phi'\tan\theta)]\,dx - F\int_{x_a}^{x_b}y_Q q_x\,dx - F\int_{x_a}^{x_b}x(\gamma_{av}\,h + q_y)\,dx + F\sum T_i y_r^i = o \tag{2.11}$$

In equations 2.5 to 2.11 T_i are reinforcement forces that are treated here as known external forces applied at points of intersection between the potential failure surface and geosynthetic sheets;

$$t(x) = \sum T_i\,\delta(x - x_r^i); \tag{2.12}$$

x_r^i and y_r^i are x- and y-coordinates of points of intersection between an assumed failure surface and geosynthetic sheets;

$\delta(x)$ - Delta-function:

$$\delta(x) = \begin{cases} 0, & if\ x \neq 0; \\ \infty, & if\ x = o; \end{cases} \tag{2.13}$$

$$\int_{-\epsilon}^{\epsilon} \delta(x)\, dx = 1 \qquad (2.14)$$

for any $\epsilon > o$.

The rest of the variables have the same meaning as before (see Equations 1.6 - 1.21). Admissibility criterion 1.15 doesn't change for reinforced cases

$$o \le \eta \le 1 \qquad (2.15)$$

The second admissisbility criterion described by Equations 1.14 and 1.16, gets very complicated in reinforced case because of the presence of the fabric layers. That is why this criterion will be ignored in the future discussion. Most articles ignore both criteria even in unreinforced cases. To the best of the authors' knowledge none of them consider any of the criteria in reinforced cases. Therefore, even considering only one criterion (1.15) in the reinforced earth stability analysis is a noncontroversial advantage of the current research study.

Differential Equations 2.5 through 2.8 can be rewritten after eliminating σ and τ:

$$-\frac{dH}{dx} = \frac{1}{F} \frac{c' + (q_y + \gamma_{av} h + \frac{dv}{dx} - u) \tan\phi'}{1 - \frac{\tan\theta \tan\phi'}{F}} \sec^2\theta - q_x + t(x) + (q_y + \gamma_{av} h + \frac{dv}{dx}) \tan\theta; \qquad (2.16)$$

$$\frac{d(Hh_t)}{dx} = V - H \tan\theta + q_x (y_Q - y). \qquad (2.17)$$

Equations 2.16 and 2.17 are used in Janbu's method, Hopkins' method and Morgenstern and Price's method.

2.2.2. Hopkins' method.

Hopkins' (1986, 1991) method involves the same assumption as the original Janbu's (1954, 1957) method. It assumes the thrust ratio η, and consequently, the values of h_t. Then Equations 2.16, 2.17 become a system of two equations having two unknown functions H and V. As in the original Janbu's method, Hopkins uses an overall horizontal equilibrium equation as a criterion for the determination of the factor of safety:

$$F = \frac{\int_{x_a}^{x_b} A dx}{H_a - H_b + \int_{x_a}^{x_b} B dx}$$

$$A = \frac{(c' + (q_y + \gamma_{av} h + \frac{dV}{dx} - u) \tan\phi')(1 + \tan^2\theta) dx}{1 - \frac{\tan\theta \tan\phi'}{F}} \tag{2.18}$$

$$B = q_x - t(x) - (q_y + \gamma_{av} h + \frac{dv}{dx}) \tan\theta$$

In this equation, the factor of safety, F, appears in both parts of the equation; therefore, iterations are needed to solve it. As a first approximation both Janbu and Hopkins, start with an assumption that

$$\frac{dv}{dx} = 0 \ (x_a \le x \le x_b) \tag{2.19}$$

With this assumption, Equation 2.18 leads to a simple iteration procedure and allows obtainment of an initial approximation for F:

$$F_o = F|_{\frac{dv}{dx} = o} \tag{2.20}$$

This approximation is used to obtain a more rigorous solution through the following steps:

-determine dH/dx and H from the Equation 2.16 based on $F=F_0$ and dV/dx=o;
-determine V and dV/dx from the Equation 2.17;
-substitute dV/dx into the Equation 2.18 to obtain the next approximation of F;
-repeat steps 1 through 3 until the difference between subsequent approximations of F becomes appropriatary small.

The steps 1 through 4 are involved in both original Janbu's and Hopkins' procedures. Hopkins uses a special smoothing technique to obtain dH/dx and dV/dx in steps 1 and 2. As it was shown by Hopkins (1986, 1991) with this smoothing technique, his method converged rapidly for a big variety of practical problems for unreinforced earth stability analysis. It will be shown in the following sections that this method is applicable for reinforced earth stability analysis but it has convergence problems in some particular cases.

2.2.3. Morgenstern and Price's method.

Similar to the original Morgenstern and Price's (1965) method, its version for reinforced case uses the basic Equations 2.16 and 2.17 and the assumption of 1.22, which makes the problem statically determinate.

$$V = \lambda\ f(x) H \tag{2.21}$$

With this assumption, Equation 2.16 takes the form:

$$\frac{dH}{dx} = \{(c' - u\tan\phi')\sec^2\theta + t(x)(F - \tan\theta\ \tan\phi') + (q_y + \gamma_{av} h)(\tan\phi' + F\tan\theta) + \lambda(\tan\phi' + F\tan\theta) f'(x) H\} / \{\tan\theta\ \tan\phi' - F - \lambda(\tan\phi' + F\tan\theta) f(x)\} \tag{2.22}$$

As in the original Morgenstern and Price's method, this equation can be solved with respect to H with an initial condition

$$H(x_a) = H_a \tag{2.23}$$

By substituting H(x) into Equation 2.17, the value of M(x) = H(x) h_t(x) can be easily determined. Finally, Equations 1.23 are used as criteria to obtain the values of λ and F (see section 1.3).

Three options can be considered in the program based on different definitions of f(x):

$$f(x) = 1 \tag{2.24}$$

$$f(x) = \sin\frac{\pi(x - x_a)}{x_b - x_a} \text{ - half sine wave} \tag{2.25}$$

$$f(x) = \left[\sin\frac{\pi(x - x_a)}{x_b - x_a}\right]^2 \text{ - full sine wave} \tag{2.26}$$

2.2.4. Bishop's method.

Bishop's (1955) method is not a statically consistent method and it was classified in section 1.5 as a simplified method. However, it was proven in the previous sections that this method gives reasonable answers. That is why it is also considered in the program.

The original Bishop's method (for unreinforced analysis) consists of finding the safety factor F by subsequent approximations based on the following equation:

$$F = \frac{M_{res}}{M_{dr}} \tag{2.27}$$

where M_{res} is a resisting moment with respect to the center of a trial circle;

$$M_{res} = \sum \frac{(c' + (q_y + \gamma_{av}\, h - u)\tan\phi')\ \Delta x}{\cos\theta\left(1 - \dfrac{\tan\theta\ \tan\phi'}{F}\right)} \tag{2.28}$$

and M_{dr} is a driving moment;

$$M_{dr} = -\sum \sin\theta\ (\gamma_{av}\, h + q_y)\ \Delta x \tag{2.29}$$

Ingold (1982) first proposed an extension of the original Bishop's method to take account for reinforcing fabrics. He proposed the following equation:

$$F = \frac{M_{res} + \sum T\cos\theta}{M_{dr}} \tag{2.30}$$

Where the term Σ T cosθ is an additional resisting moment provided by the fabric.

An approach based on Equation 2.30 does not strictly follow the logic involved in Bishop's method. Bishop's method satisfies the overall moment equilibrium condition and the vertical equilibrium condition for each slice. An approach proposed by Ingold (1982) satisfies the overall moment equilibrium condition but it doesn't satisfy the vertical equilibrium condition.

To overcome this misconception, one has to consider all the derivations involved in Bishop's method treating reinforcement forces as known external forces. It is easy to show that this approach will lead to an Equation different from 2.30:

$$F = \frac{M_{res}}{M_{dr} - \sum T\cos\theta} \tag{2.31}$$

where M_{res} and M_{dr} are described by the same Equations 2.28 and 2.29. In future discussions, the method based on Equation 2.30 will be referred to as Bishop's incorrect method while the second method based on Equation 2.31 will be referred to as Bishop's correct method. Most authors use Bishop's incorrect method following the original work by Ingold (1982), but in a recent publication (Wright and Duncan, 1991) the necessity of using Bishop's correct method is strongly emphasized.

It will be shown that the difference in answers between Bishop's correct and incorrect methods may be very large. It is easy to see comparing Equations 2.30 and 2.31 that both methods yield the same results only in two special cases: when F = 1 or when T = 0 (unreinforced case).

2.2.5. Perturbation methods.

In contrast to slices methods, perturbation methods use overall equilibrium equations in an integral form (see section 1.4). In the original perturbation method proposed by Raulin et al (1974), basic approximation for the normal stress distribution is chosen in the form 1.25. A modification of this approximation for reinforced analysis was made by Gourc et al (1989):

$$\sigma_o(x) = (\gamma_{av}\, h + q_y)\cos^2\theta + q_x \sin\theta\cos\theta - t(x)\sin\theta\cos\theta, \qquad (2.32)$$

where t(x) is described by Equation 2.12.

As it was shown by Raulin et al (1974), all three modifications 1.27 to 1.29 of the perturbation method with the same basic function 1.25 yield essentially the same results. Therefore, only one version described by Equation 1.27 will be considered in future discussions. Instead, two other versions will be introduced. These versions consider normal stress distribution obtained by Hopkins' (1986, 1991) method for the unreinforced case. It was mentioned in section 2.2.2 that Hopkins' method provided reasonable answers in unreinforced earth stability analysis and therefore the normal stress distribution obtained by this method may be a good initial approximation for perturbation methods. In other words, the following three perturbation methods will be used in the further discussion.

Method 1. (Gourc et al, 1989).

$$\sigma = (\lambda + \mu \tan\theta)\,\sigma_o \qquad (2.33)$$

where σ_0 is described by Equation 2.32.

Method 2.

$$\sigma = \lambda\,\sigma^* - \mu\, t(x)\sin\theta\cos\theta \tag{2.34}$$

Where $\sigma^*(x)$ - is the normal stress distribution in a corresponding unreinforced problem obtained according to Hopkins' method.

Method 3.

$$\sigma = (\lambda + \mu\tan\theta)(\sigma^* - t(x)\sin\theta\cos\theta) \tag{2.35}$$

where σ^* is defined in the same way as in method 2.

On substituting any of expressions 2.33, 2.34 or 2.35 into the system of equilibrium Equations 2.9 to 2.11, one arrives at the following system of nonlinear algebraic equations with respect to λ, μ, and F:

$$\begin{aligned} a_{11}\lambda + a_{12}\mu + a_{13}\lambda F + a_{14}\mu F + a_{15}F - b_1 &= 0 \\ a_{21}\lambda + a_{22}\mu + a_{23}\lambda F + a_{24}\mu F + a_{25}F - b_2 &= 0 \\ a_{31}\lambda + a_{32}\mu + a_{33}\lambda F + a_{34}\mu F + a_{35}F - b_3 &= 0 \end{aligned} \tag{2.36}$$

Expressions for a_{ij} and b_i for each of the above mentioned methods are given in Appendix 2. The system 2.36 is solved by the Newton-Raphson method.

In the following sections applications of the proposed limit equilibrium computer program to reinforced earth stability analyses will be considered. The program will be referred to as HOPSLEP. For each of the following examples, detailed input information is included in Appendix 4.

3. APPLICATION OF THE PROPOSED LIMIT EQUILIBRIUM COMPUTER PROGRAM TO REINFORCED EARTH STABILITY ANALYSES.

3.1. Internal stability of reinforced slopes and retaining walls.

3.1.1. Tensar example. Comparison with Wright and Duncan (1991) results.

The example slope considered in this section is a 1:1 (45 degree) slope, 38 ft high, as shown in Figure 3.1. The soil is cohesionless and has an angle of internal friction of 32° and a total unit weight of 120 pcf. The slope contains 17 layers of reinforcement, varying from 23.9 to 29.2 ft in length and spaced vertically as shown in Figure 3.1. This example was taken from Tensar Technical Note (1986a). The original example presented by Tensar had a surcharge of 240 psf, equivalent to about 2 ft of an additional slope height, which was not used in the current analyses. Each layer of reinforcement has an axial force of 1,000 lb. Although the force would actually decrease to zero near the embedded ends of the reinforcement, the force was assumed to be constant along the entire length of the reinforcement for the current stability computations.

The computed minimum factors of safety for this example are summarized in Table 3.1. For comparison, results obtained by Wright and Duncan (1991) for the same example are also shown in the Table. For all methods except log spiral, only circular search analyses were performed.

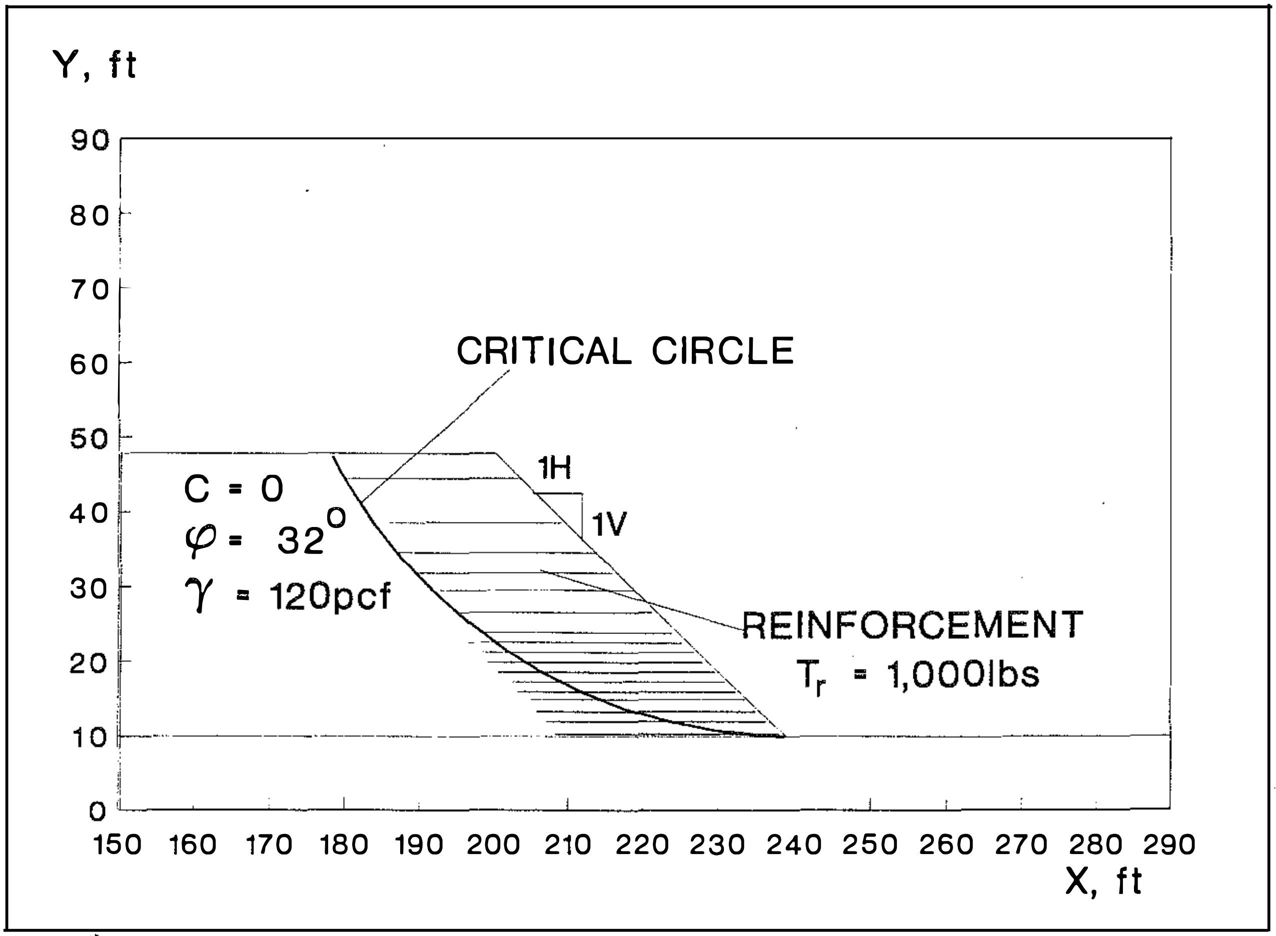

Figure 3.1. Tensar Manual Example.

Table 3.1. Comparison of factors of safety computed for Tensar (1986a) example.

Authors	Method of analysis	Factor of safety
Wright	Spencer's	1.44
and	Bishop's	1.45
	Force equilibrium	1.30
Duncan	Log. spiral	1.46
Hopkins	Hopkins'	1.445
and	Bishop's correct	1.452
	Bishop's incorrect	1.361
Slepak	Perturbation 1	1.440
	Perturbation 2	1.450
	Perturbation 3	1.449

One can see from the Table 3.1, that both results obtained by Wright and Duncan and by using HOPSLEP computer program are close to each other. All the methods which satisfy either all equilibrium equations or moment equilibrium equation (Spencer's, Bishop's correct, log. spiral, Hopkins', perturbation 1, 2, and 3) produce factors of safety that agree within approximately 2 percent. Bishop's incorrect method and force equilibrium method yield lower answers.

3.1.2. Generic examples. Comparison with Tensar method.

The Tensar (1986a) method provides a simplified method for reinforced slope design. This method is based on earth pressure theory and can be classified as a force equilibrium method. Two options of the Tensar method are available. In the first option, the design procedure strictly follows force equilibrium equations and consequently reinforcement spacings and reinforcement lengths are continuously changing within the slope height. This option will be referred to as the Tensar rigorous method. In the second option, the slope height is divided into zones and within each zone both reinforcement lengths and spacings are assumed constant. This option will be referred to as the Tensar simplified method.

Four generic examples are considered in this section. The design characteristics are summarizied in Figure 3.2. Each of four examples corresponds to one of the values of a slope angle β: 30°, 45°, 60°, 80°. In all examples, the soil is assumed cohesionless with an angle of internal friction of 36° and a total unit weight of 125 pcf. H = 39.37 ft is assumed as a slope height for all four examples. The slope is supposed to be reinforced with fabric layers which have the design tensile strength of R = 2,020 lb/ft. The uniform surcharge q = 819 lb/ft is acting on the top of the slope. The soil-fabric interaction coefficients (Equations 2.2 and 2.3) were assumed: $k_1 = k_2 = 0.9$. In all examples, slopes are designed in order to provide the required factor of safety of F_r= 1.5. Each slope was designed according to both rigorous and simplified Tensar methods. The results are shown in the Table 3.2. One can see from the Table 3.2 that simplified and rigorous Tensar methods provide slightly different results. As expected, the simplified method tends to be more

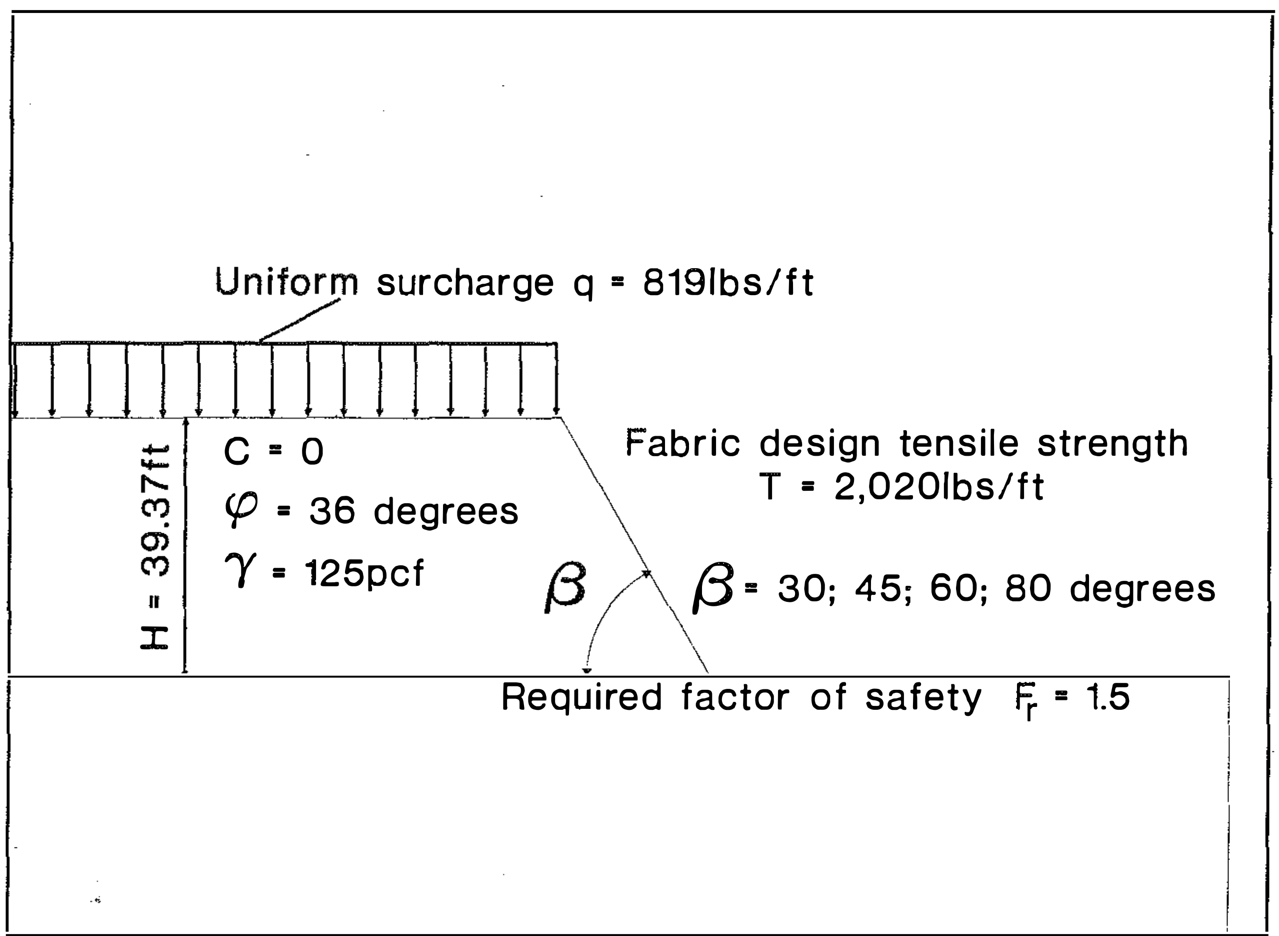

Figure 3.2. Generic Examples. Design Characteristics.

Table 3.2. Number of fabric layers in generic examples.

β,°	Design method*)	No. of fabric layers
30	1	2
	2	2
45	1	11
	2	10
60	1	18
	2	16
80	1	30
	2	25

*) 1 - simplified Tensar method
2 - rigorous Tensar method

conservative than the rigorous method.

As a first step for each of the slopes designed according to the simplified Tensar method, four circles passing through the toe of the slopes were selected and each circle was subjected to the analysis by different methods. The circles are shown in Figures 3.3 - 3.6, coordinates of their centers are given in the Table 3.3 and the computed factors of safety

Table 3.3. Coordinates of centers of the selected circles.

β,°	Circle No.	x_0, ft.	y_0, ft.
30	1	391.0	350.0
	2	377.6	358.7
	3	303.5	236.0
	4	314.0	317.0
45	1	312.5	157.3
	2	283.0	143.8
	3	269.4	135.8
	4	255.5	127.5
60	1	307.2	109.7
	2	293.3	130.5
	3	281.3	139.6
	4	272.0	150.7
80	1	270.8	73.8
	2	263.1	91.7
	3	255.2	103.8
	4	248.0	118.6

are shown in Tables 3.4 through 3.7. In all cases considered in Tables 3.4 through 3.7, Morgenstern and Price's function f(x) was assumed in the form of Equation 2.25 (half sine wave).

It can be seen from Tables 3.4 to 3.7 that all statically consistent methods (Hopkins' method, Morgenstern and Price's method, Perturbation methods) and Bishop's correct

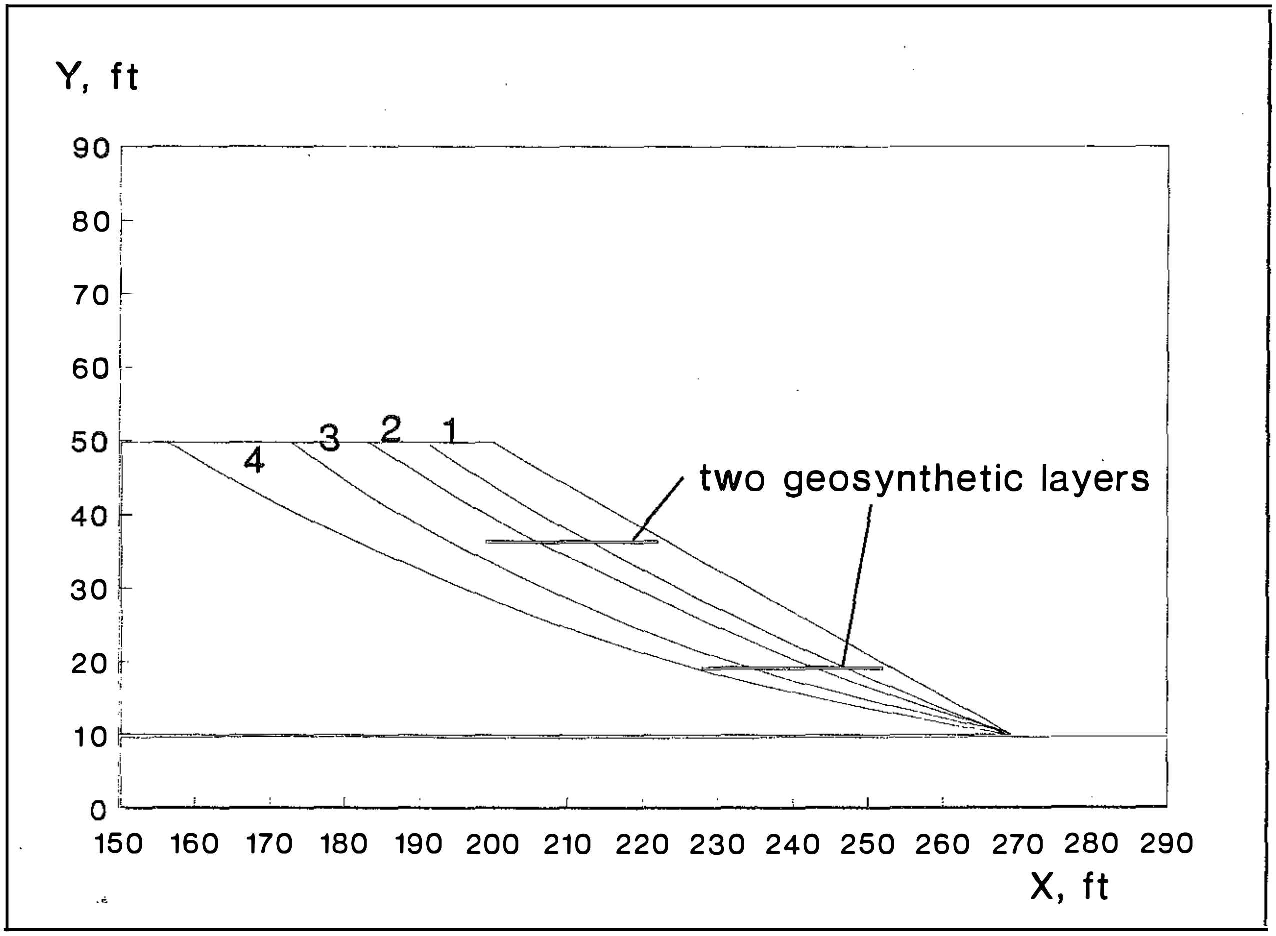

Figure 3.3. 30 degrees slope Simplified Tensar Design Method.

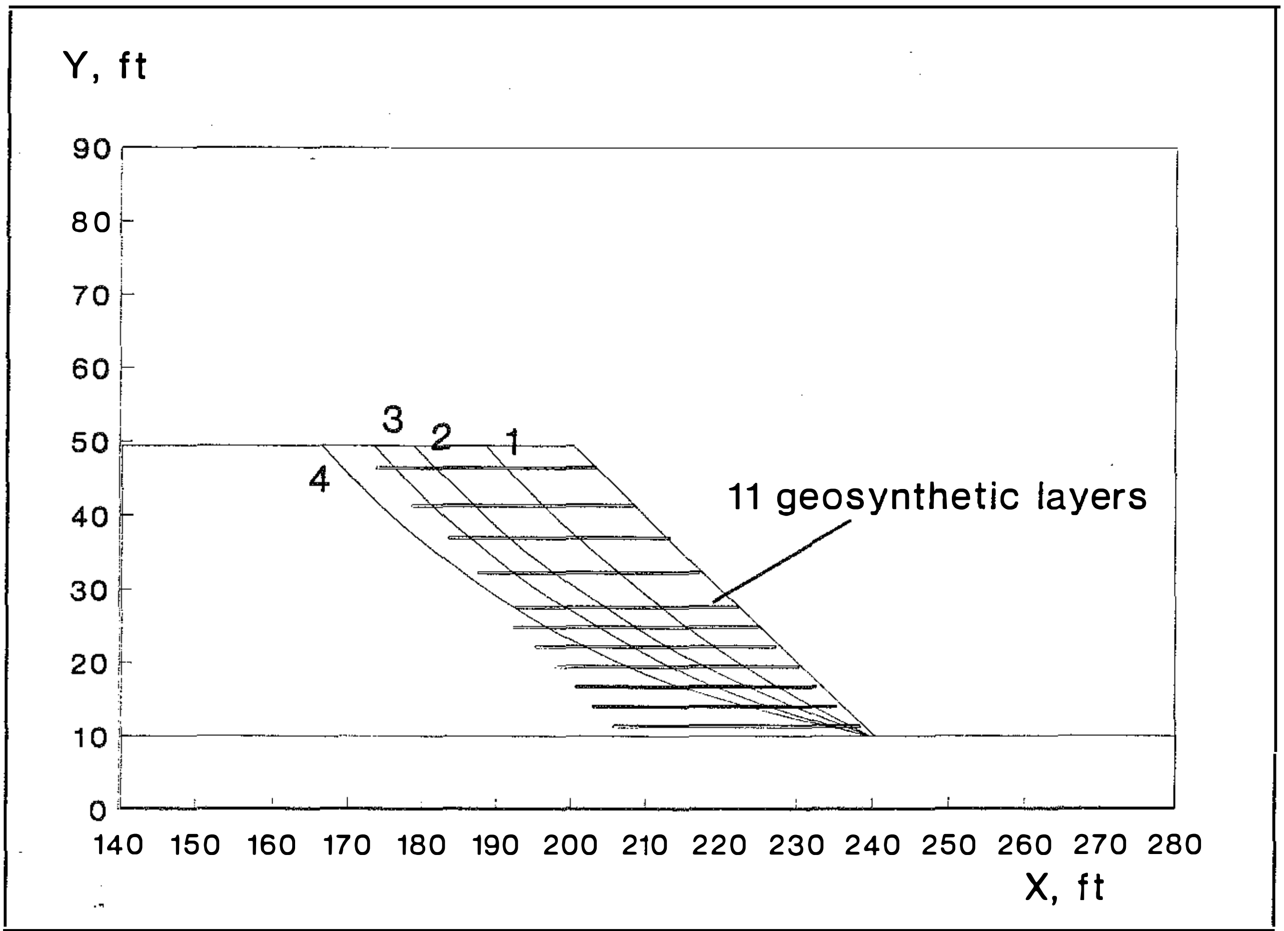

Figure 3.4. 45 degrees slope. Simplified Tensar Design Method.

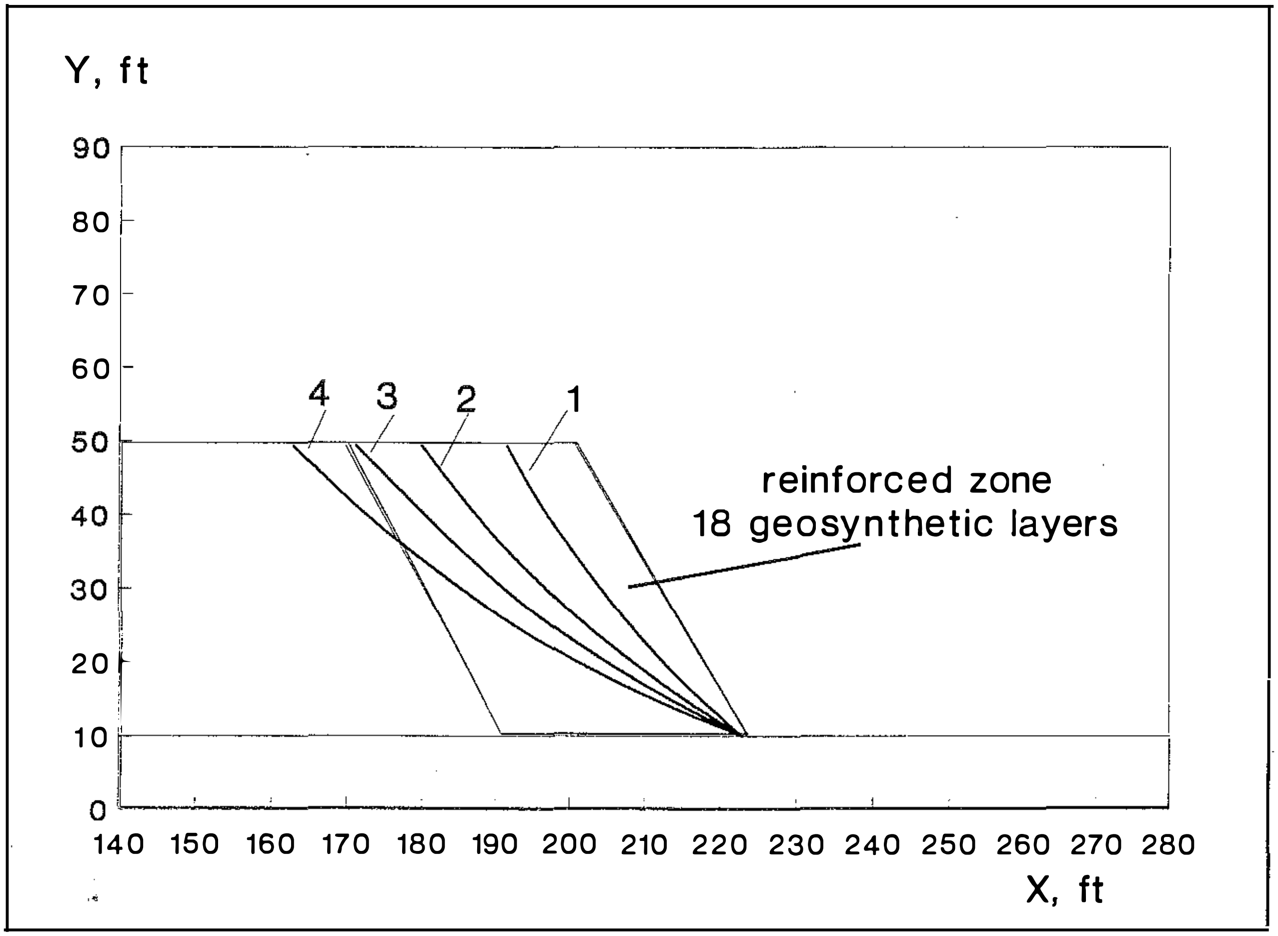

Figure 3.5. 60 degrees slope. Simplified Tensar Design Method.

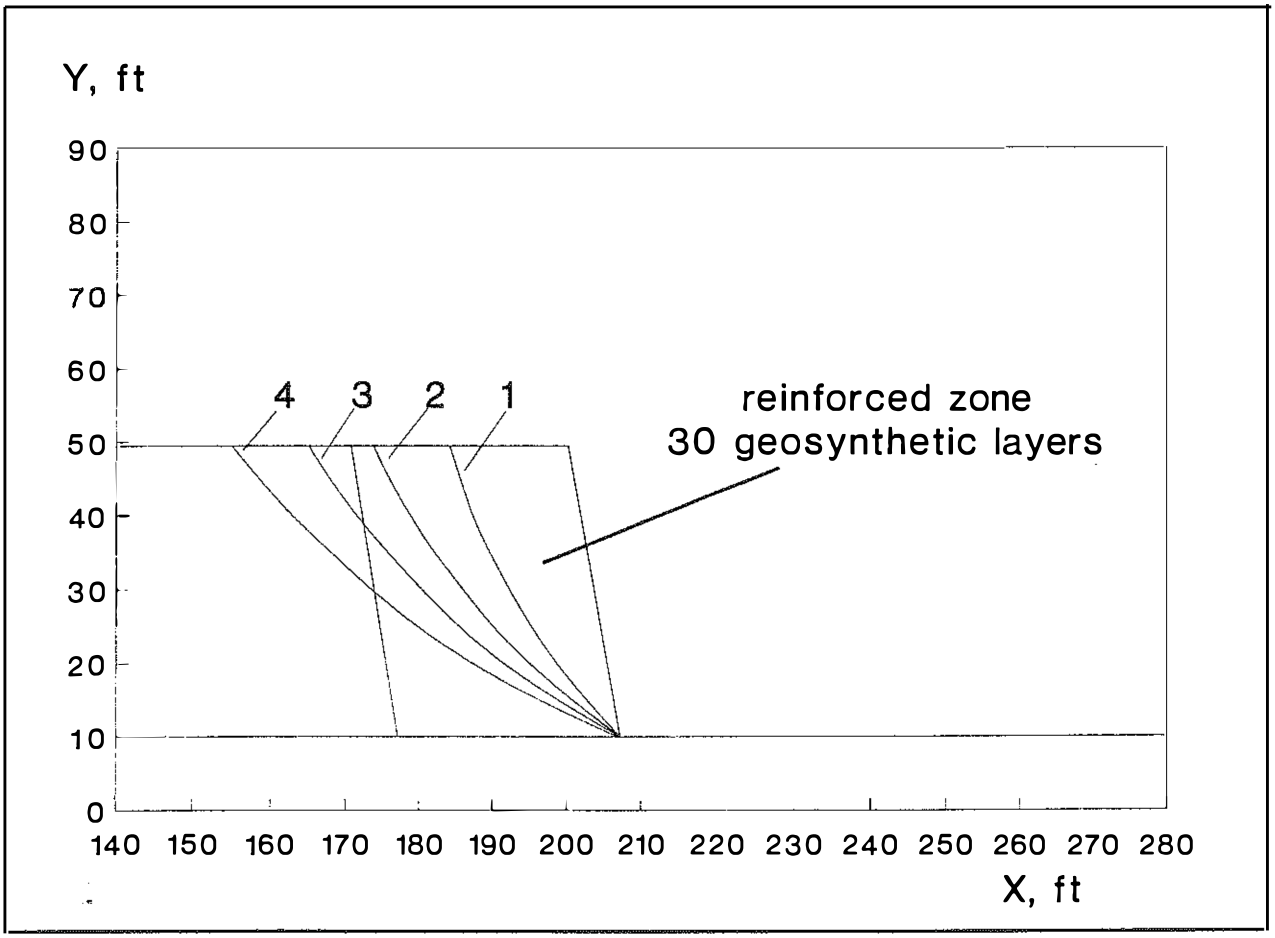

Figure 3.6. 80 degrees slope. Simplified Tensar Design Method.

method yield essentially the same results. The difference between the answers is

Table 3.4. Factors of safety for selected circles for 30° slope (simplified Tensar design).

Circle No.	Method of analysis						
	Hopkins	Bishop's		Perturbation			Morgenstern & Price *)
		correct	incorrect	1	2	3	
1	1.622	1.622	1.516	1.621	1.621	1.621	1.619
2	1.677	1.677	1.593	1.677	1.677	1.677	1.676
3	1.710	1.710	1.681	1.710	1.710	1.710	1.706
4	1.927	1.927	1.927	1.927	1.927	1.928	1.924

*) f(x) is given by Equation 2.25.

Table 3.5. Factors of safety for selected circles for 45° slope (simplified Tensar design).

Circle No.	Method of analysis						
	Hopkins	Bishop's		Perturbation			Morgenstern & Price *)
		correct	incorrect	1	2	3	
1	2.050	2.061	1.503	2.049	2.049	2.049	2.043
2	1.768	1.773	1.486	1.768	1.769	1.769	1.763
3	1.789	1.793	1.536	1.789	1.791	1.791	1.784
4	1.662	1.665	1.540	1.660	1.663	1.663	1.648

*) f(x) is given by Equation 2.25.

Table 3.6. Factors of safety for selected circles for 60° slope (simplified Tensar design).

Circle No.	Method of analysis						
	Hopkins	Bishop		Perturbation			Morgenstern & Price *)
		correct	incorrect	1	2	3	
1	**)	2.891	1.535	2.799	2.799	2.799	2.766
2	1.955	1.972	1.447	1.954	1.954	1.954	1.942
3	1.956	1.964	1.507	1.954	1.955	1.955	1.945
4	1.924	1.927	1.567	1.920	1.921	1.921	1.909

*) f(x) is given by Equation 2.25.
**) no convergence

Table 3.7. Factors of safety for selected circles for 80° slope (simplified Tensar design).

Circle No.	Method of analysis						
	Hopkins	Bishop's		Perturbation			Morgenstern & Price *)
		correct	incorrect	1	2	3	
1	**)	2.760	1.476	2.653	2.655	2.656	2.615
2	**)	2.448	1.521	2.407	2.409	2.410	2.378
3	2.342	2.367	1.579	2.336	2.340	2.339	2.307
4	2.159	2.190	1.115	2.171	2.174	2.174	2.146

*) f(x) is given by Equation 2.25.
**) no convergence

practically insignificant. Bishop's incorrect method yields lower values. Depending on the number of reinforcement sheets, the difference in factors of safety provided by Bishop's correct and incorrect methods varies from 0% to 47% (0% corresponds to the case when there is no intersections between a selected failure surface and reinforcement sheets). It should also be mentioned that Morgenstern and Price's method is very sensitive to initial approximations of λ and F. Depending on how close these approximations are to the "true" answer, this method may or may not converge. Hopkins' method usually provides a rapid convergence, but in some cases (see Tables 3.6 and 3.7) convergence was not obtained.

In all cases analyzed in Tables 3.4 to 3.7, statically consistent methods observed reasonable thrust ratios. In Figures 3.7 through 3.10, thrust ratio curves are shown for circles #2 for each of the selected slope angles. Similar curves were also observed for the rest of the circles. Figures 3.7 to 3.10 show that the largest part of the curves satisfies the admissibility criterion (1.15). There is some violation of this criterion at the ends of a trial mass which obviously doesn't affect the factor of safety (see Tables 3.4 to 3.7). For each of the selected slopes, circular search analyses were performed using Hopkins' method, Bishop's correct and incorrect methods and Perturbation 1 method. Both cases were considered when fabric layout was designed based on Tensar simplified and rigorous methods. Minimum factors of safety are shown in Table 3.8 and in Figures 3.11 and 3.12.

It can be seen from Table 3.8 and Figures 3.11 and 3.12, that minimum factors of safety provided by Hopkins' method, Perturbation method and Bishop's correct method are practically the same. Bishop's incorrect method underestimates the factor of safety (especially for steep slopes with $\beta > 45°$). The following conclusion about the Tensar design method can be made. For slopes with angles $\beta \leq 45°$, actual values of the minimum factors of safety reasonably agree with $F_r = 1.5$ for which the slopes were designed. For steeper slopes with $\beta > 45°$, actual values of factors of safety are much higher than F_r. In other words, it may be concluded that both Tensar methods work reasonably well for mild slopes ($\beta \leq 45°$). For steeper slopes ($\beta > 45°$), these methods should be considered as too conservative. The simplified Tensar method appears to be more conservative than the rigorous Tensar method.

Table 3.8. Minimum factors of safety based on circular search analyses.

Slope Angle, β.°	Design method *)	No. of fabric sheets	Method of analysis			
			Hopkins	Perturbation 1	Bishop's	
					correct	incorrect
30	1	2	1.47	1.46	1.47	1.46
	2	2	1.40	1.39	1.40	1.40
45	1	11	1.57	1.57	1.58	1.43
	2	10	1.51	1.50	1.51	1.39
60	1	18	1.75	1.73	1.75	1.44
	2	16	1.68	1.66	1.68	1.38
80	1	30	2.14	2.15	2.16	1.46
	2	25	1.68	1.68	1.68	1.32

*) 1 - simplified Tensar method
2 - rigorous Tensar method

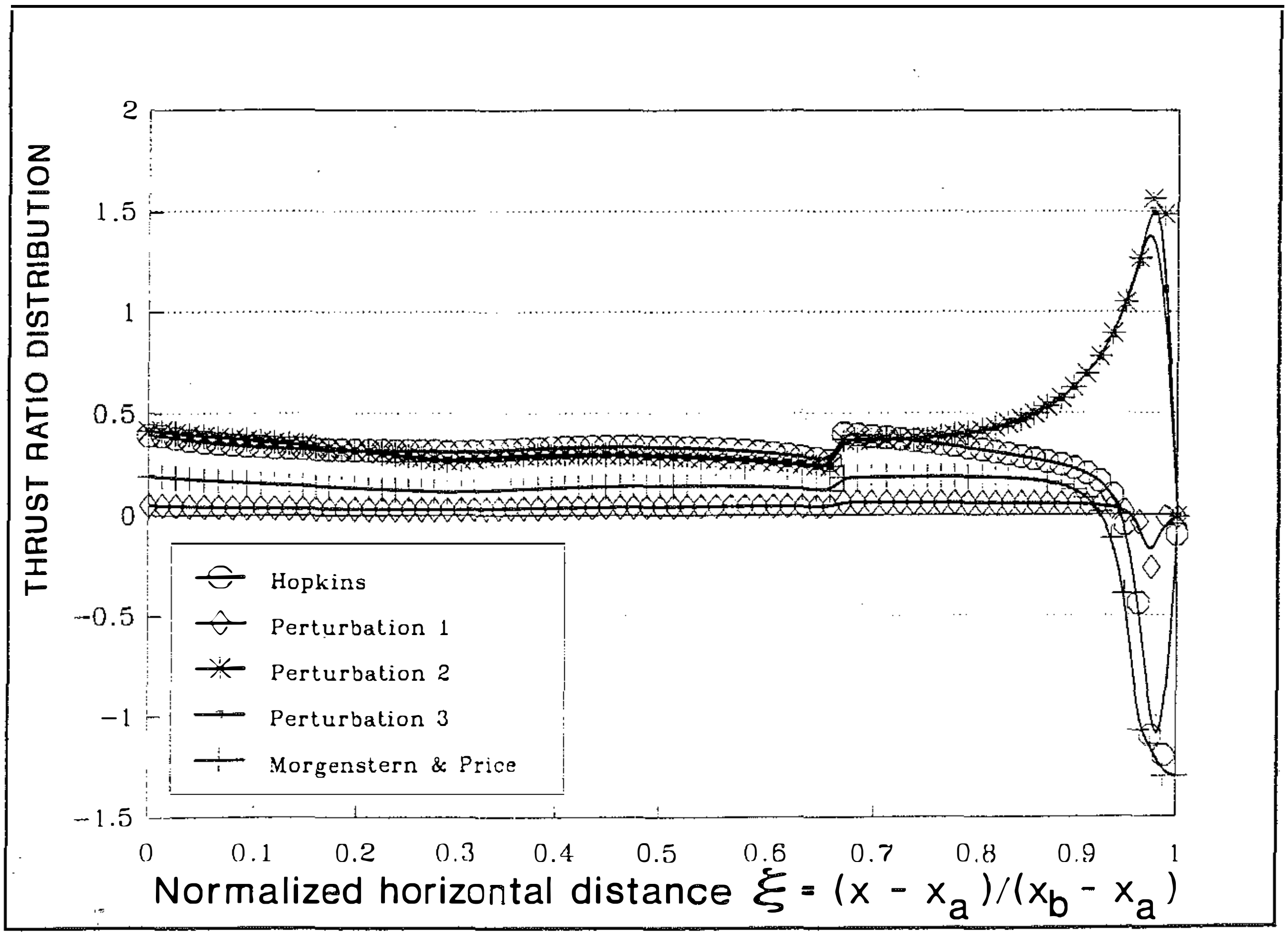

Figure 3.7. Thrust ratio in 30 degrees example slope problem. Circle #2. Tensar Simplified Design Method.

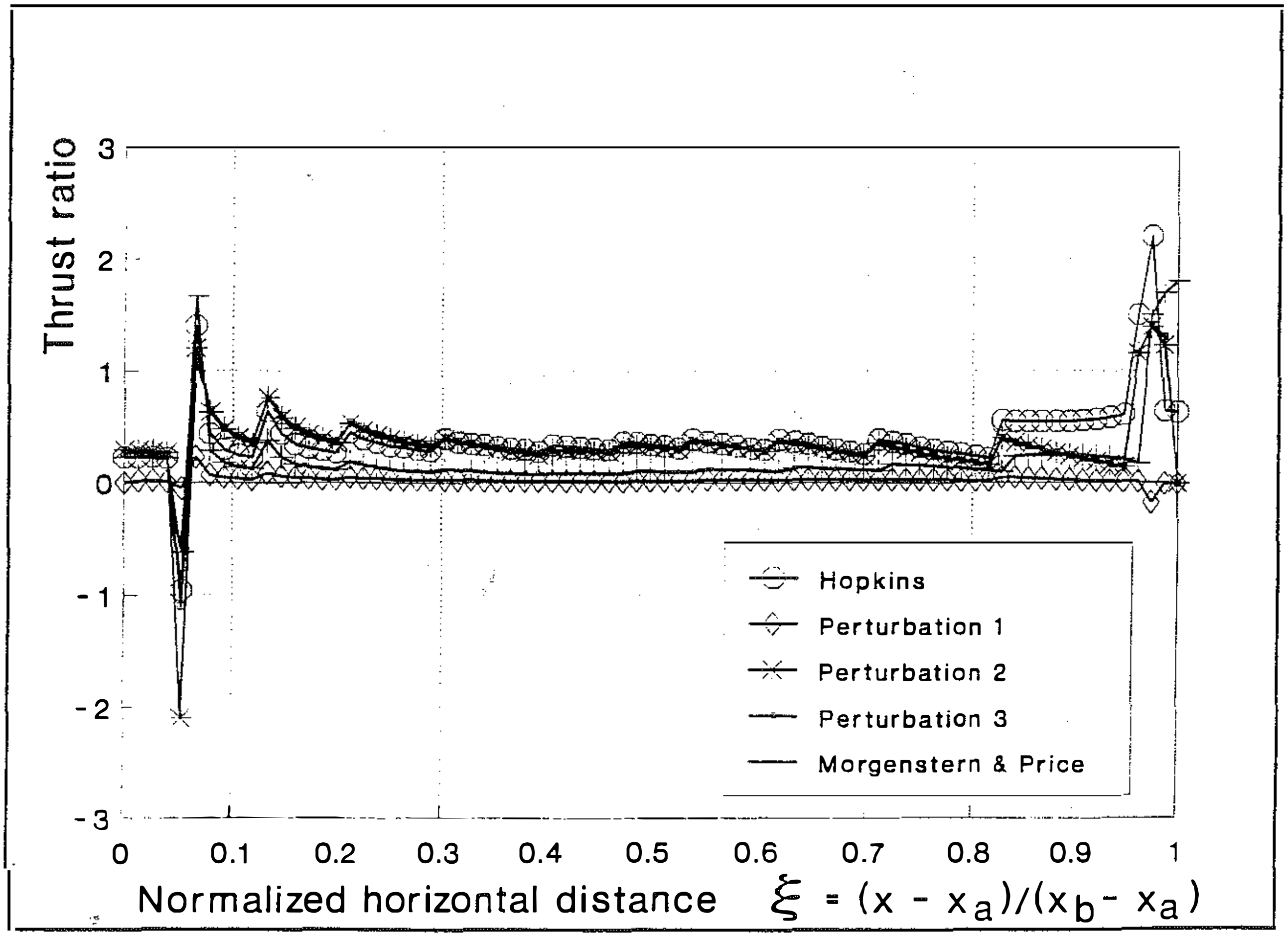

Figure 3.8. ***Thrust ratio in 45 degrees example slope problem. Circle #2. Tensar Simplified Design Method.***

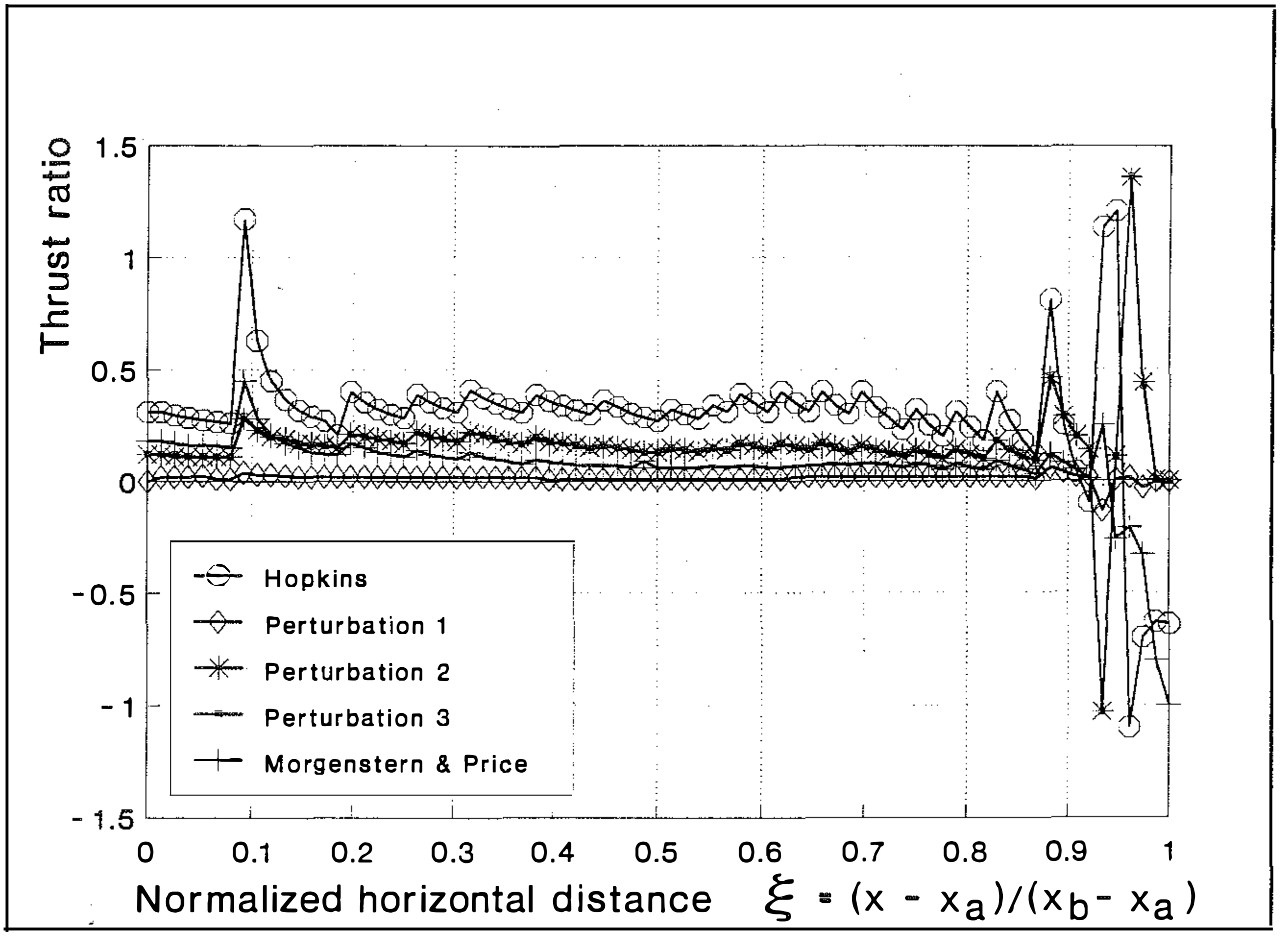

Figure 3.9. ***Thrust ratio in 60 degrees example slope problem. Circle #2. Tensar Simplified Design Method.***

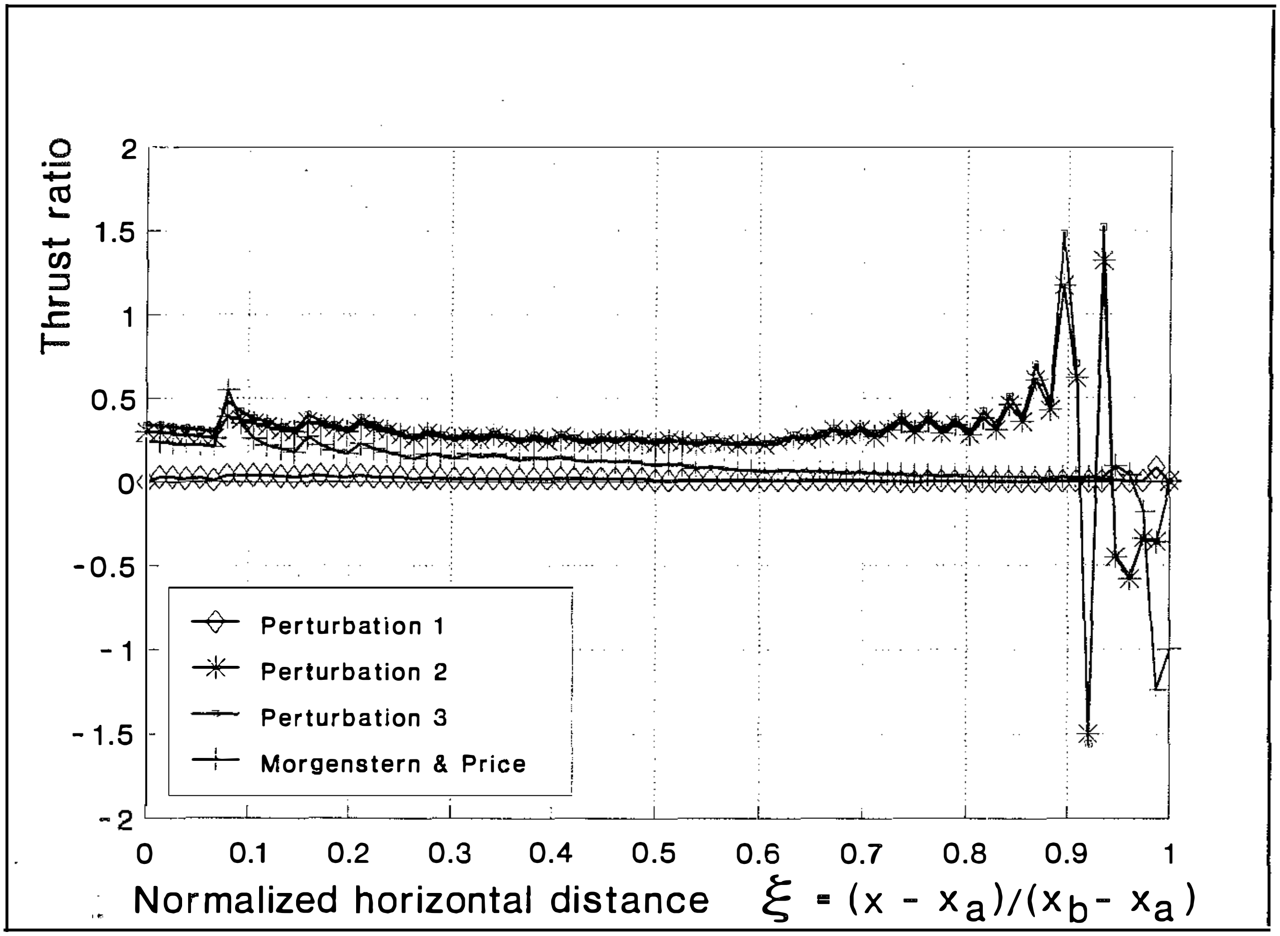

Figure 3.10. ***Thrust ratio in 80 degrees example slope problem. Circle #2. Tensar Simplified Design Method.***

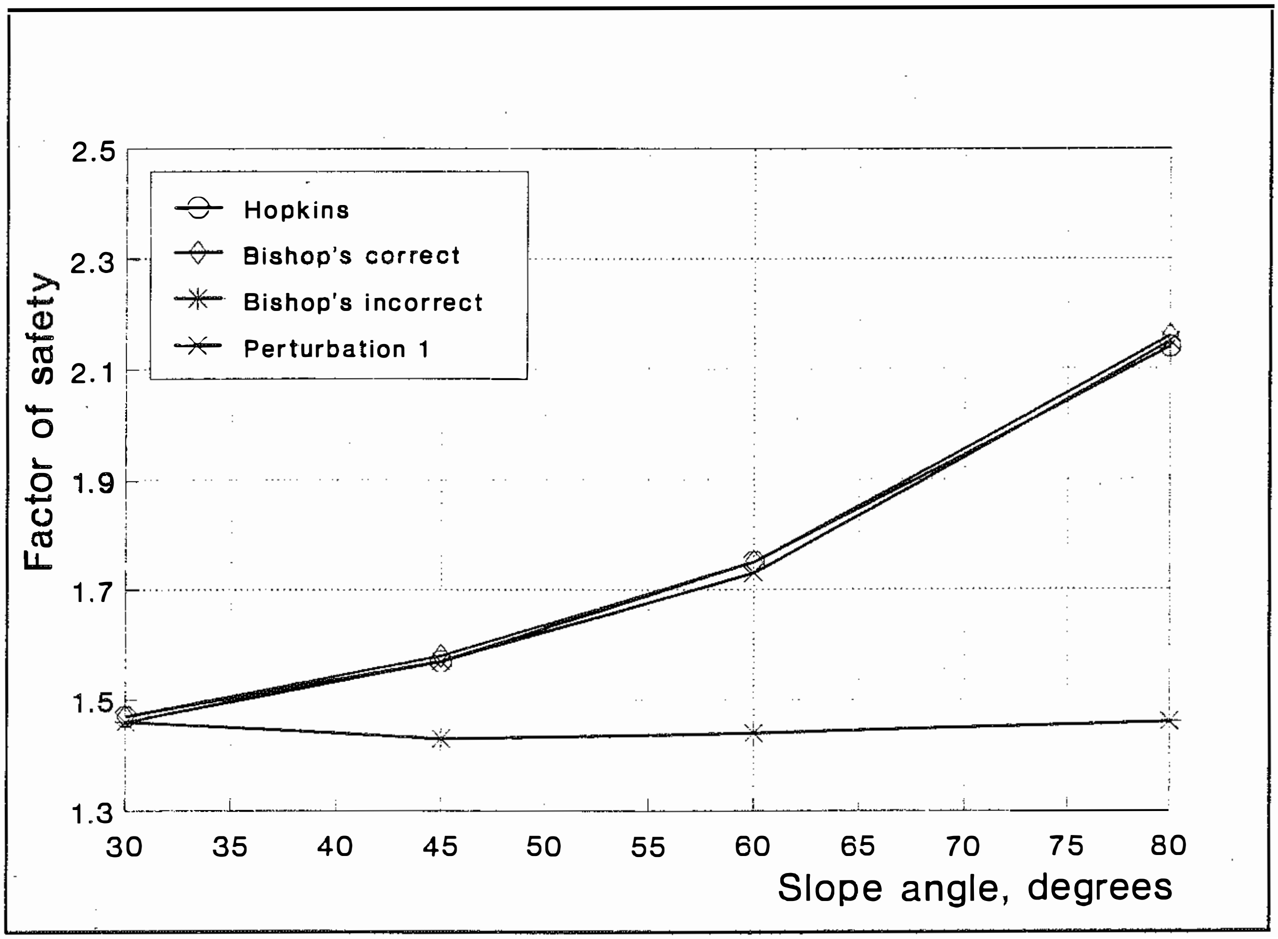

Figure 3.11. ***Minimum factors of safety. Tensar Simplified Design Method Circular Search Analysis.***

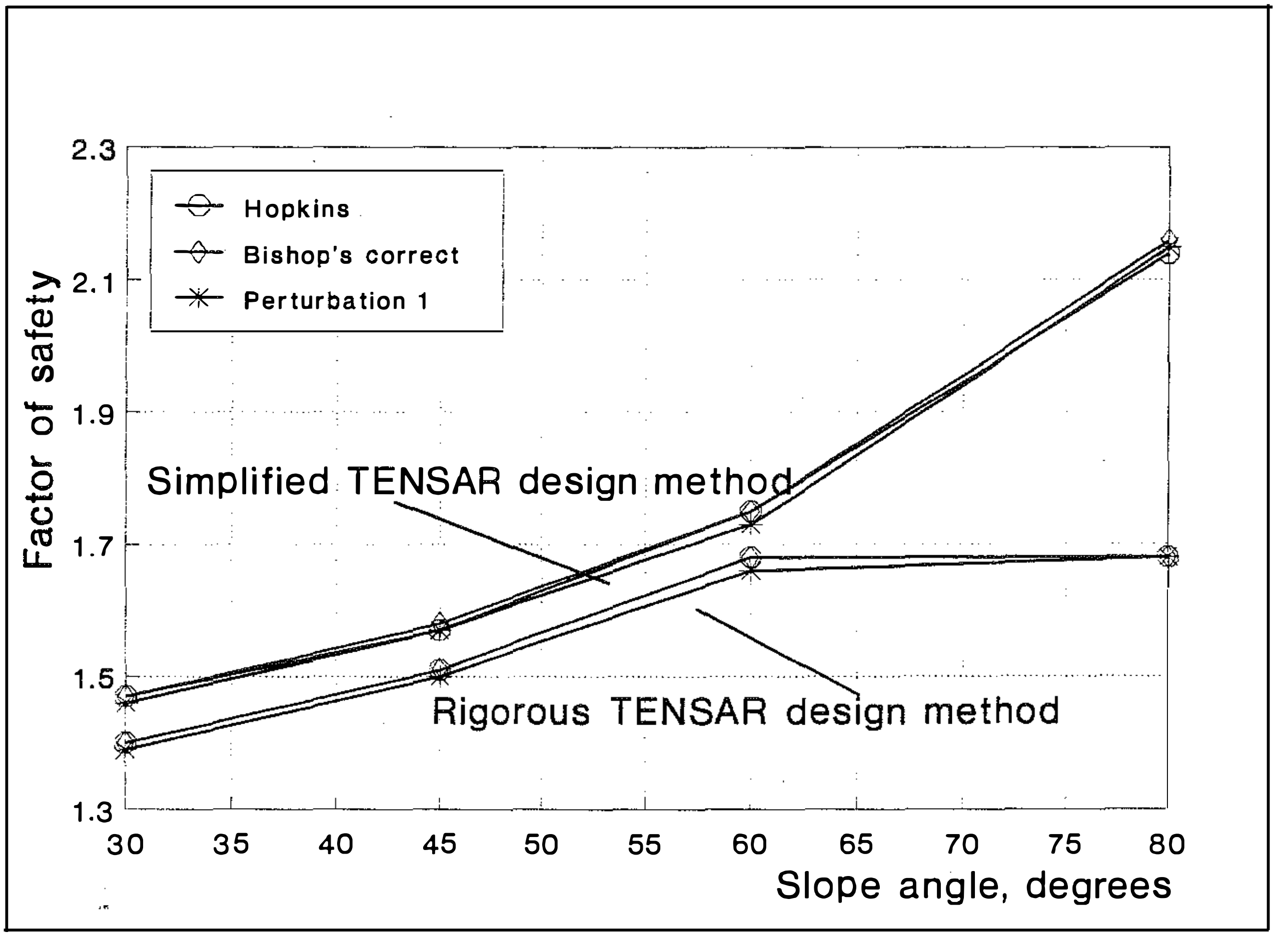

Figure 3.12. Minimum factors of safety. Circular Search Analysis.

3.1.3. Load test of a large-scale geotextile-reinforced retaining wall. Billiard and Wu (1991) example.

Billiard and Wu (1991) performed a controlled load test to investigate the performance of a geotextile-reinforced retaining wall until a failure state was reached. The test wall geometry is illustrated in Figure 3.13. This test wall was erected in the laboratory using a typical sequential construction technique. The test wall was loaded by applying incremental vertical surcharge loads on the top surface until excessive deformation of the facing had occurred. To provide insight into the behavior of the retaining wall under load,the wall was instrumented to measure the strain of the geotextile, deflection of the top surface and vertical wall face. The wall was constructed using a low weight spun bonded nonwoven polypropylene geotextile with a wide width tensile strength of 420 lbs/ft at 60% elongation. The soil was a gravelly sand cohesionless soil having a ϕ angle of 39°. Placement unit weight of the sand was estimated to be approximately 95 pcf. The test was terminated at a surcharge load of q_u = 2,855 psf as a result of excessive lateral face deformation. However, because of the highly deformable reinforcing elements even the ultimate load did not produce a "classic" Rankine failure plane.

For each of the incremented surcharge loads q = 850; 1,380; 2,660 and 2,855 psf, the test wall was analyzed using HOPSLEP computer program by different methods. The actual geotextile layout was modeled as shown on the Figure 3.13. Only circular search analyses were performed. The results are given in Table 3.9 and in Figure 3.14.

The results show that Hopkins' method, perturbation method and Bishop's correct method provide almost the same answers. Bishop's incorrect method underestimates the factor of safety when $F > 1$. When F is close to 1, all the methods, including Bishop's incorrect method, yield the same answers (see also section 2.2.4). The results also indicate that at a surcharge load of q = 2,660 psf the factor of safety $F \approx 1.0$. This is consistent with the observations made by Billiard and Wu (1991). They mentioned that at the average surcharge pressure of 2,660 psf the sound of sand "flowing" inside the test wall was detected. It was suspected that a failure condition might have been reached at that surcharge. Both experimentally determined and computed critical failure surfaces are plotted in the Figure 3.13.

Table 3.9. Factors of safety for different surcharge loads. Billiard and Wu example.

Surcharge q, psf	Method of analysis			
	Hopkins	Perturbation 1	Bishop's	
			correct	incorrect
850	1.685	1.706	1.699	1.471
1380	1.469	1.470	1.475	1.262
2660	1.005	0.986	0.993	0.996
2855	0.973	0.940	0.948	0.969

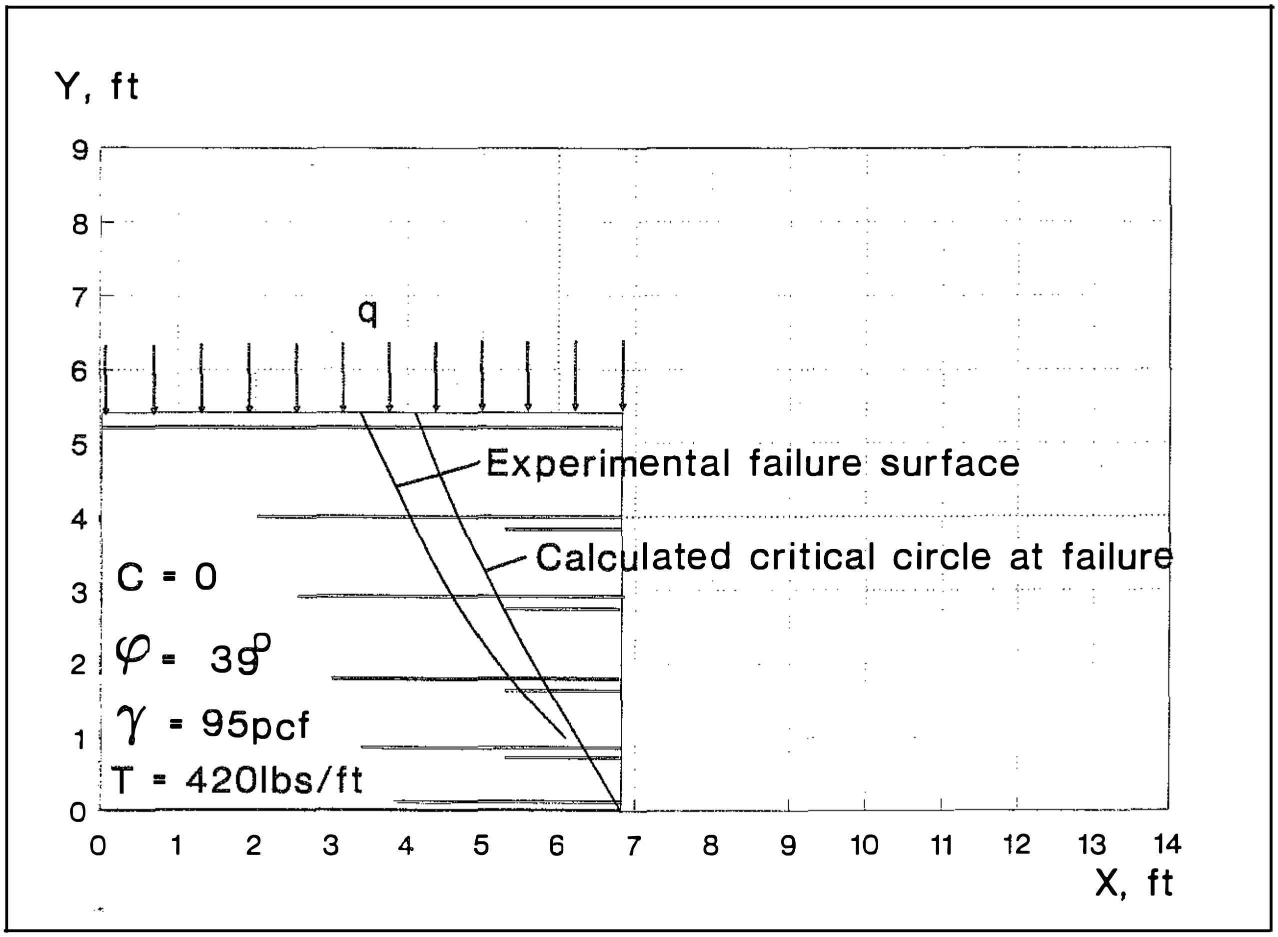

Figure 3.13. Experimental and Calculated Failure Surfaces in Billiard and Wu Example.

3.1.4. RMC load test of a large-scale model geogrid-reinforced wall.

Another case history concerning the internal stability of a reinforced retaining wall is one of the large-scale model geogrid-reinforced walls constructed as part of a long-term research project at the Royal Military College of Canada (RMC). The test wall used for this example is Test 3 presented in a paper by Bathurst et al (1988). The geometry of this wall is shown in Figure 3.15. The wall is 9.8 ft high with four layers of geogrid reinforcement, each approximately 9.8 ft long and at depths 1.6, 4.1, 6.6 and 9.0 ft. The wall was constructed by slightly pretensioning the reinforcements and then carefully compacting granular backfill with a reported internal friction angle of 53°. The reinforcement consisted of a high-density polyethylene geogrid. This material possessed an ultimate wide width tensile strength of 840 lbs/ft at 14% strain in the direction it was oriented. The model was surcharged in increments to approximately 250; 625; 1,045; 1,250; 1,465 and 2,090 psf. The 1,045-psf surcharge was sustained for 162 hours, during which time failure occurred.

Circular search analysis for q = 1,045 psf using the HOPSLEP computer program was performed. As it was mentioned in the previous sections, all statically consistent methods work reasonably well in the case of internal stability of reinforced slopes and retaining walls. That's why in this example only the perturbation method was used. The search analysis provides the minimum factor of safety F = 1.523 which does not indicate a failure. At the same time, it should be emphasized that the mode of failure reported was creep. It means that the long-term strength should be used as a design strength of the fabric. Analyzing this example, Claybourn and Wu (1991) came to the conclusion that the effective strength of 500 lbs/ft should be used to represent the average of the maximum mobilized stresses in the four reinforcement layers. Circular search analysis using R = 500 lbs/ft as a fabric strength provides the factor of safety F = 1.064 which definitely indicates failure.

This example shows that the stability of reinforced walls can be predicted by limit equilibrium methods reasonably well provided the design fabric strength is close to the tensile stresses actually mobilized in the fabric.

3.2. Stability of reinforced embankments on soft foundations.

In the previous sections, it was shown that Hopkins' method, Morgenstern and Price's method, Bishop's correct method and perturbation methods yield reasonable answers in the analyses of internal stability of reinforced slopes and retaining walls. In the following sections, stability analyses of reinforced embankments on soft foundations are considered.

3.2.1. Wright and Duncan's (1991) example.

This example consists of a 10-ft. high cohesionless fill resting on a 10-ft layer of saturated (ϕ = o) clay, as shown in Figure 3.16. Much stronger soils are assumed to exist below the clay. The fill has an angle of internal friction (ϕ) of 35 degrees and a total unit weight of 105 pcf. The clay has a uniform undrained shear strength of 200 psf. One layer of

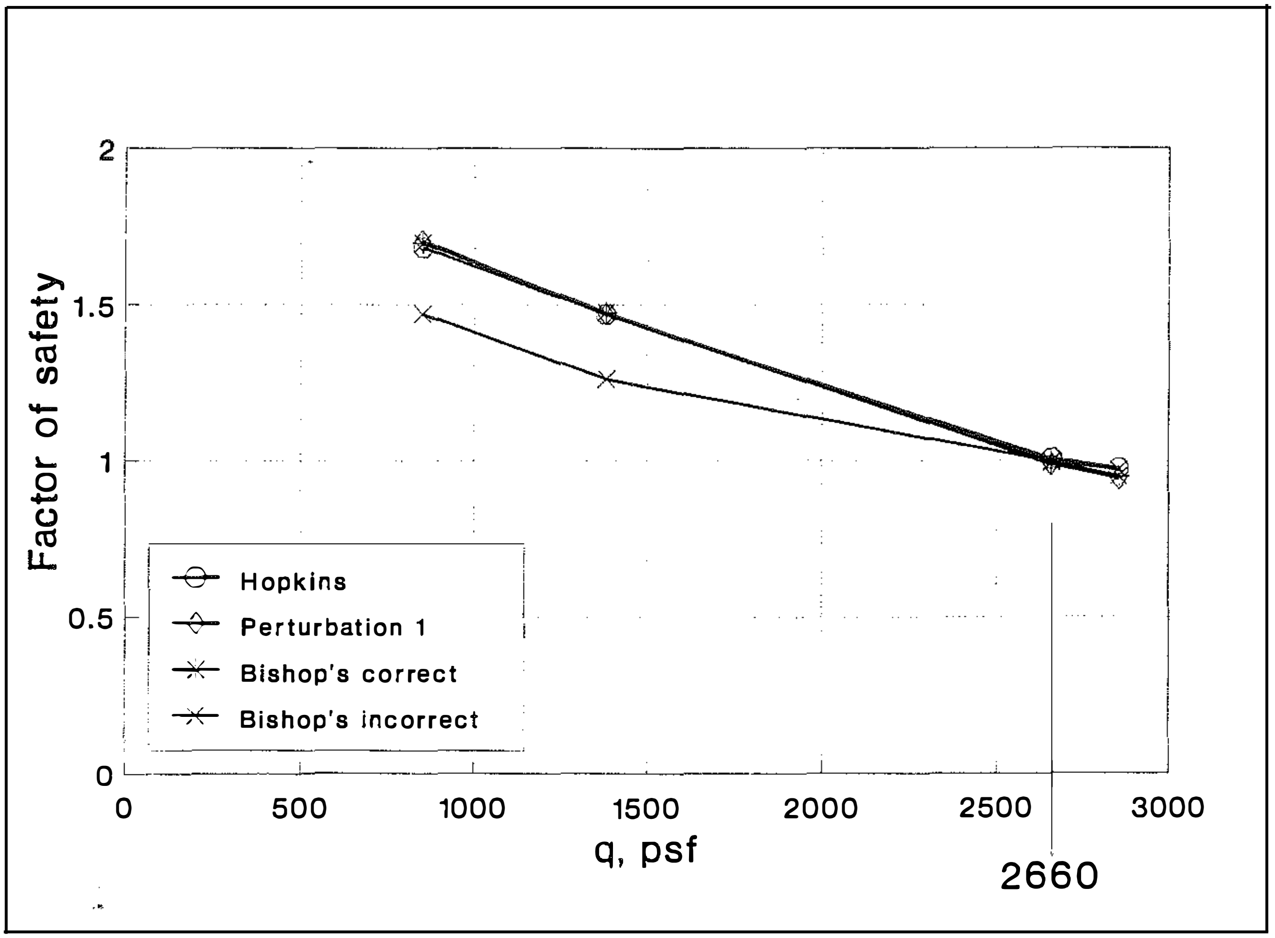

Figure 3.14. Factors of safety for different surcharge loads. Billiard and Wu example.

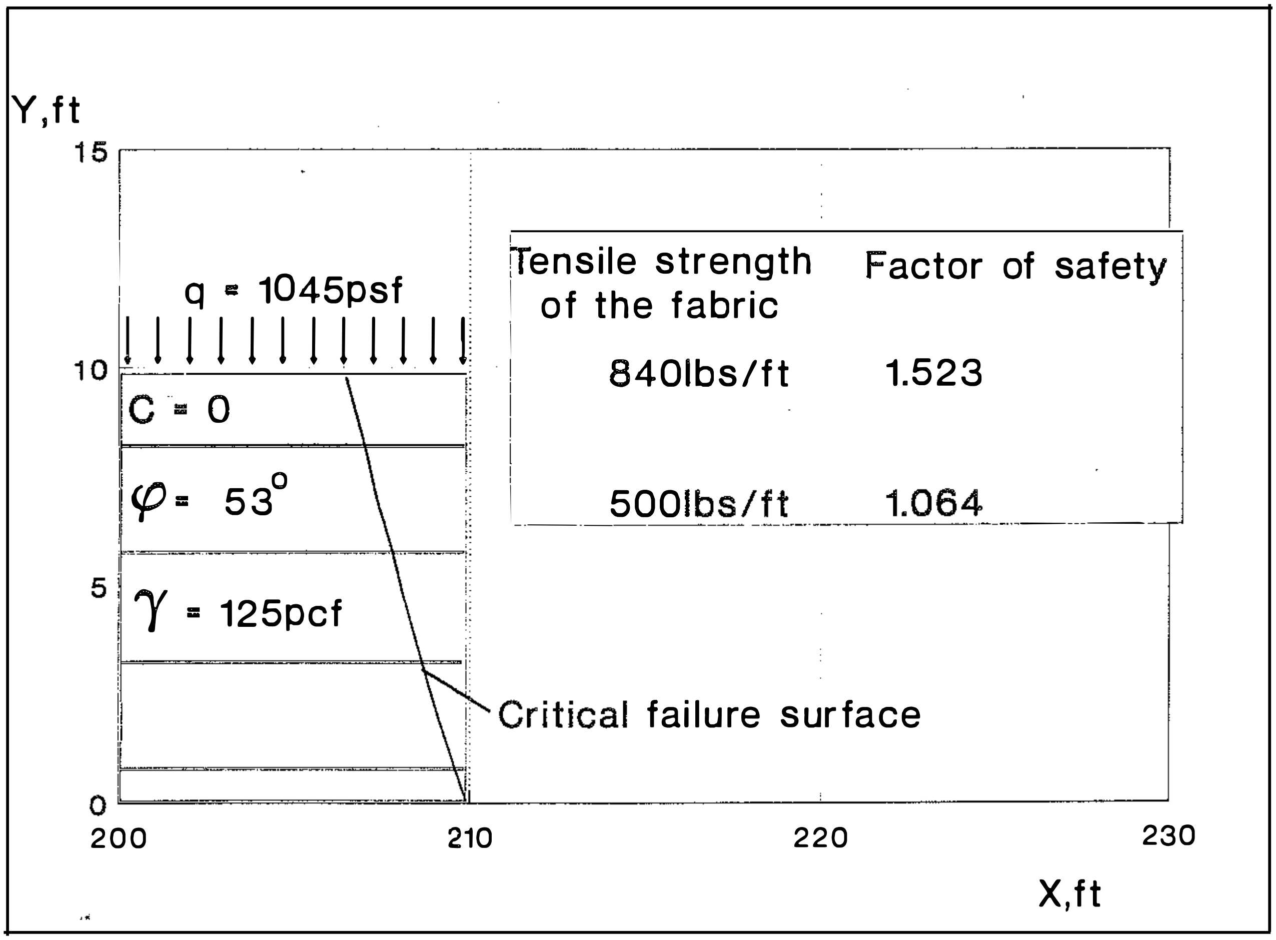

Figure 3.15. RMC example.

reinforcement is placed at the base of the fill on the surface of the clay. The reinforcement carries a constant force of 3,000 lbs/ft. This example is the Wright and Duncan's (1991) example 2.

Circular search analyses were performed with this example using the HOPSLEP computer program by different methods listed in Table 3.10. The minimum safety factors are also shown in the Table. For comparison, the values obtained by Wright and Duncan (1991) are also presented in the Table.

Table 3.10. Minimum factors of safety and admissibility violation values in Wright and Duncan's (1991) example.

Authors	Method	Factor of safety	ϵ
Wright and Duncan	Spencer	1.37	--
	Bishop	1.36	--
	Force equilibrium	1.35	--
Hopkins and Slepak	Hopkins	1.383	0.118
	Bishop's correct	1.355	--
	Bishop's incorrect	1.288	--
	Perturbation 1	1.232	0.171
	Perturbation 2	1.365	0
	Perturbation 3	1.360	0

One can see from the Table 3.10 that even statically consistent methods in HOPSLEP analysis yield different factors of safety. More detailed analyses using the admissibility criterion 1.15 are needed. The results of the analyses are shown in the Table 3.10 and in Figure 3.17. For all of the cases, the following value is considered.

$$\varepsilon = \frac{\int_{x_a}^{x_b} \eta^*(x)\, dx}{x_b - x_a} \tag{3.1}$$

where

$$\eta^*(x) = \begin{cases} 0, if\ 0 \leq \eta \leq 1 \\ 1,\ if\ either\ \eta < o\ or\ \eta > 1 \end{cases} \tag{3.2}$$

It is obvious from Equations 3.1 and 3.2 that for an admissible thrust ratio we have $\epsilon = 0$; for a thrust ratio that violates the admissibility criterion 1.15 at all points x ($x_a \leq x \leq x_b$) we have $\epsilon = 1$. For all intermediate cases, $0 \leq \epsilon \leq 1$. Therefore, the value of ϵ may be considered as a measure of the admissibility criterion violation. The value of ϵ was computed for all of the methods analyzed and it is shown in Table 3.10. As one can see from the Table, Hopkins' method and Perturbation 1 method violate the admissibility criterion in 11.8% and in 17.1% points, respectively. Both Perturbation 2 and Perturbation 3 methods provide admissible thrust ratios and therefore the correct

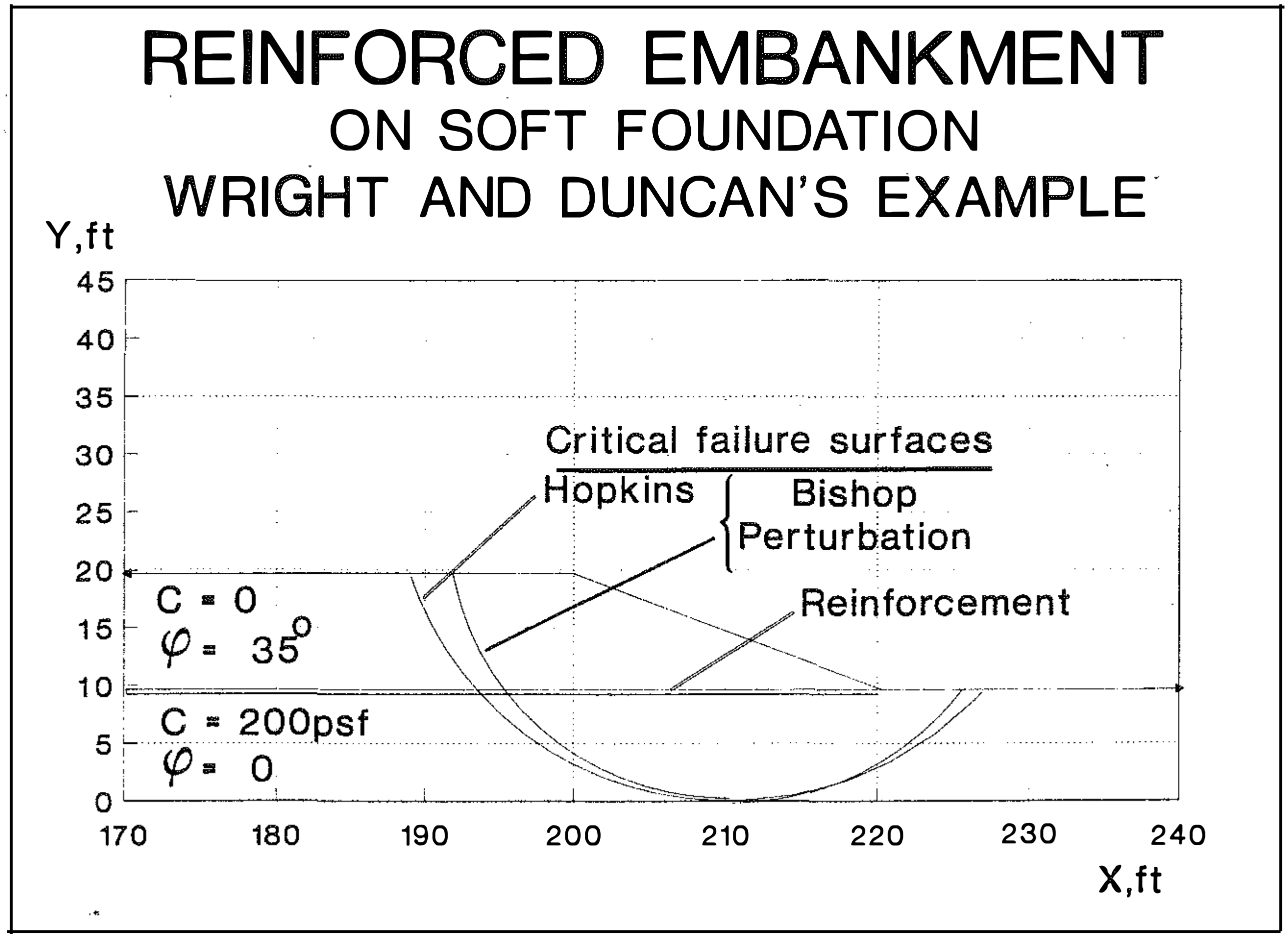

Figure 3.16. Reinforced embankment on soft foundation. Wright and Duncan's example.

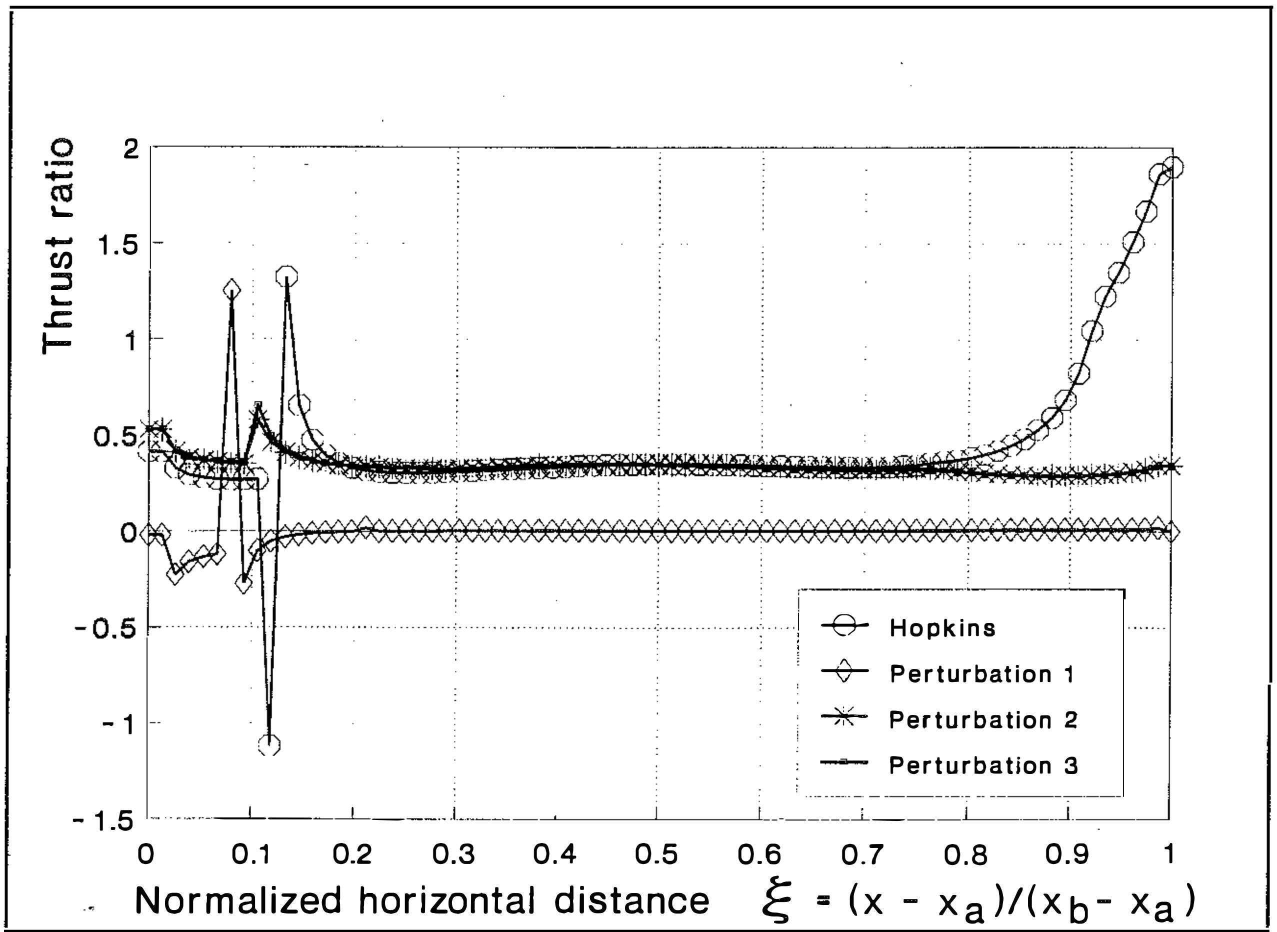

Figure 3.17. Thrust ratio in Wright and Duncan's example.

factor of safety ranges between 1.360 and 1.365 which is close to the factors of safety obtained by Wright and Duncan (1991). It should be also noted that in this example Bishop's correct method yields results very close to the correct answer. Bishop's incorrect method and Perturbation 1 method significantly underestimate the factor of safety.

3.2.2. Rowe et al (1984) example of an embankment on a very poor foundation.

Rowe et al (1984) described the design, instrumentation, and field performance of two instrumented sections of a geotextile-reinforced embankment on peat. One of them, the section at station A, is reanalyzed here using HOPSLEP computer program. The geometry of the embankment is illustrated in Figure 3.18. It was designed assuming negligible geometry changes under "undrained" conditions and also assuming that the embankment was constructed to the maximum design height in one stage. Reinforcing geotextile was placed near the peat-fill interface (Figure 3.19) to provide the necessary stability of the embankment-foundation structure. The geotextile was primarily selected on the basis of a circular stability analysis. The geotextile tensile strength of T = 2,750 lbs/ft was considered sufficient to provide the required factor of safety of 1.3. The field observations revealed, however, that the embankment had undergone significant settlements. Therefore, the initial assumption of negligible geometry changes is not valid. It is apparent from the field data that the use of a single layer of even a very strong geotextile did not prevent large deformations. Nevertheless, the example shown in the Figures 3.18 and 3.19, may be used to compare different limit equilibrium methods using the HOPSLEP computer program.

As a first step, unreinforced circular search analyses were performed by different methods. The results are shown in the Table 3.11. The critical circles are shown in the Figure 3.18. As seen in Table 3.11, unreinforced case factors of safety vary from 1.07 to 1.12 which is close to F = 1.15 given by Rowe et al (1984) for station A.

The results of reinforced circular search analyses are also shown in the Table 3.11. In this case, minimum factors of safety vary from 1.17 to 1.29. Different methods indicate different critical circles (shown in the Figure 3.19) which is also unsatisfactory. One can see four different circles in Table 3.11. More detailed analysis is needed in order to find a real critical circle and a real minimum factor of safety. Let us consider circles marked in Table 3.11, as the most critical circles and analyze all these circles using different statically consistent methods with a special emphasis on the thrust ratio curves. The results of the analyses are shown in the Table 3.12. For all of the cases as in the previous example, the value of ϵ (see Equations 3.1 and 3.2) is considered. This value is shown in Table 3.13.

As seen in the Table, all methods provide acceptable values of ϵ ($\epsilon \leq 0.039$) for circle #3 and the corresponding factors of safety (see Table 3.12) only slightly differ from each other ($1.278 \leq F \leq 1.292$). That happened because circle #3 (see Figure 3.19) barely touches the reinforcement sheet, therefore, this is almost an unreinforced case where all methods work reasonably well.

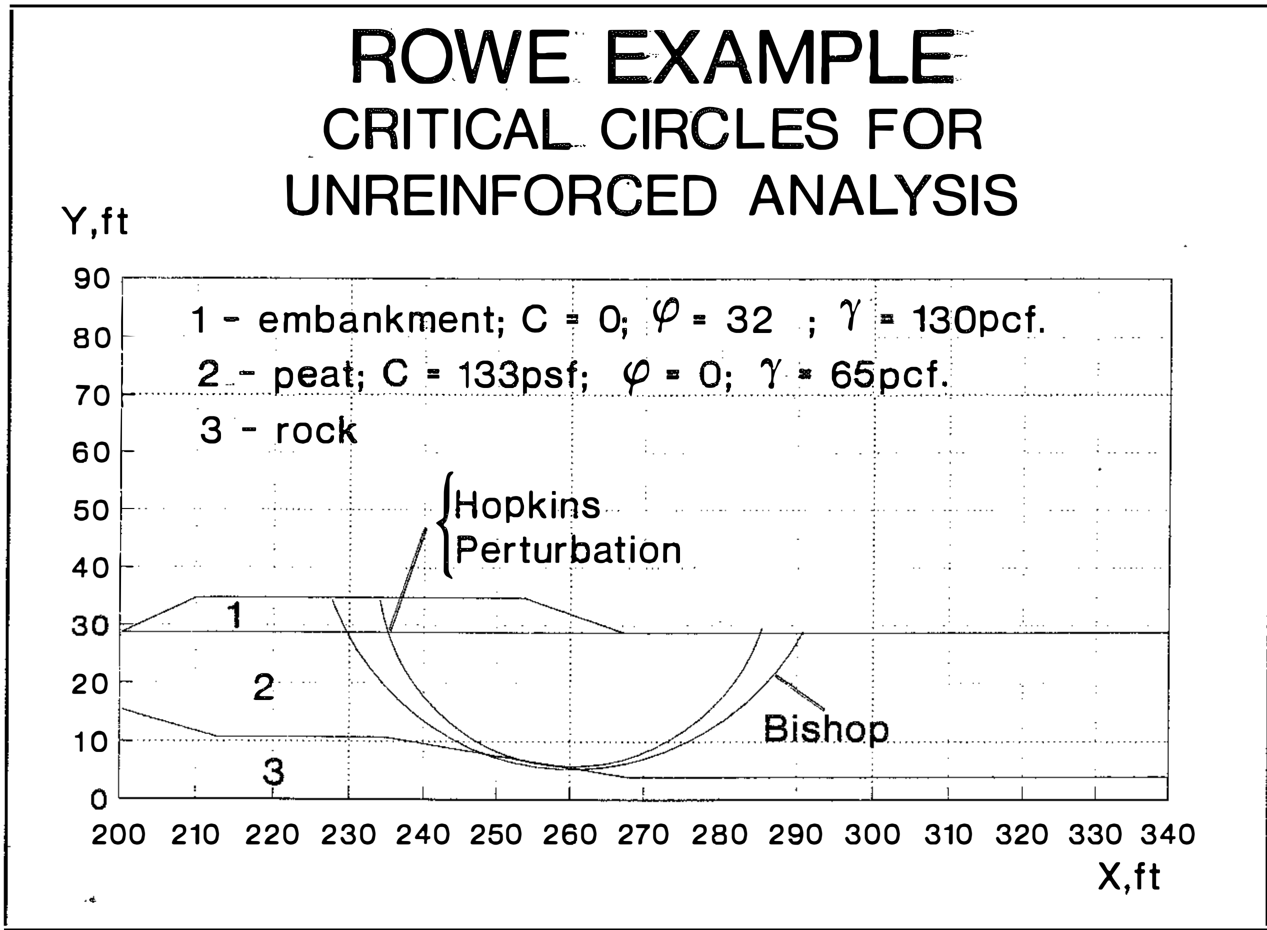

Figure 3.18. Rowe example. Critical circles for unreinforced analysis.

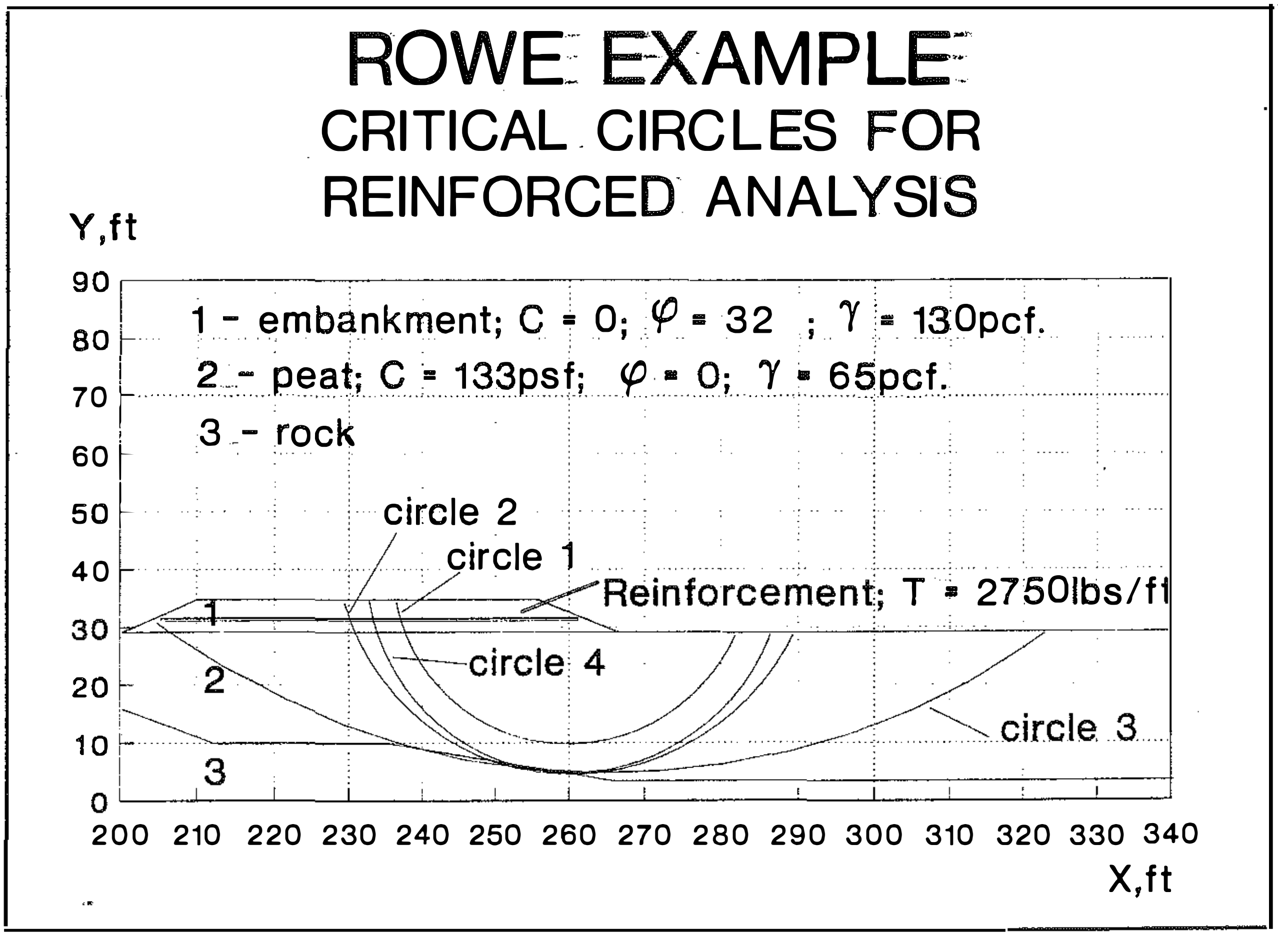

Figure 3.19. Rowe example. Critical circles for reinforced analysis.

Table 3.11. Minimum factors of safety and critical circles parameters in Rowe et al (1984) example. Station A.

Method of analysis		Unreinforced				Reinforced				
		F	X_o ft	Y_o ft	R_o ft	F	X_o ft	Y_o ft	R_o ft	Circle no.
Hopkins	76 slices					1.171	260	40	26.662	1
	598 slices	1.067	260	40	26.662	1.272	260	50	42.882	2
Perturbation	1	1.073	260	40	21.662	1.283	260	80	72.189	3
	2	-	-	-	-	1.278	260	80	72.189	3
	3	-	-	-	-	1.292	260	80	72.189	3
Bishop	correct					1.207	260	40	33.113	4
	incorrect	1.116	260	50	40.881	1.195	260	40	33.113	4

X_o, Y_o - X and Y coordinates of centers of critical circles.
R_o - radii of centers of critical circles.

Table 3.12. Factors of safety for the most critical circles in reinforced case. Rowe et al (1984) example. Station A.

Circle No.	Method of analysis				
	Hopkins	Morgenstern & Price	Perturbation		
			1	2	3
1	1.339	1.296	1.446	1.420	1.478
2	1.272	1.256	1.373	1.326	1.381
3	1.280	1.282	1.283	1.278	1.292
4	1.277	1.234	1.341	1.305	1.363

Table 3.13. Values of ϵ (Equations 3.1 and 3.2) for the most critical circles in reinforced case. Rowe et al (1984) example. Station A.

Circle No.	Method of Analysis				
	Hopkins	Morgenstern & Price	Perturbation		
			1	2	3
1	0.079	0.079	0.092	0.013	0.039
2	0.084	0.053	0.079	0.026	0.039
3	0.027	0.013	0.000	0.039	0.026
4	0.069	0.066	0.132	0.039	0.013

For circles No. 1, 2, and 4, the minimum values of ϵ are provided by perturbation methods No. 2 and 3. In other words, these perturbation methods tend to be the most physically admissible methods and the search analysis should be based on one of these methods. The Table 3.11 shows that Perturbation 2 and Perturbation 3 methods provide the minimum values of the factor of safety of 1.278 and 1.292, respectively, which are very close to each other. It is interesting to note that both values are close to the design value of the factor of safety of 1.3 even though the design method was not rigorous and it was originally assumed that reinforcing force acts tangentially to a slip circle (Rowe et al, 1984). It should be also emphasized that both versions of the Bishop's method provide lower values of the factor of safety (see Table 3.11).

3.2.3. Non-circular analysis. Hadj-Hamou et al (1990) example.

This example deals with the stability analysis of a hurricane protection levee constructed in Louisiana. The test section is 350 ft long, 10 ft high, 10 ft wide at the crown, and 136 ft wide at the base, including the two stabilizing berms. The levee is constructed with a central core of hauled semicompacted clay fill placed on a working pad of hauled sand fill. The stabilizing berms are constructed of hauled uncompacted clay fill placed from the sand pad. The reinforcement consists of two layers of high-density polyethylene Tensar SR 2 geogrids. A cross section of the test section, the location of the reinforcement and design parameters of the three fills used to build the test embankment, and the foundation soils are shown in Figure 3.20 and 3.21.

The stability of the levee was analyzed by Hadj-Hamou et al (1990) using the wedge method, which can be classified as a force equilibrium method. The most critical slip surfaces and corresponding factors of safety for both reinforced and unreinforced analyses are shown in Figure 3.22 and in Table 3.14.

Table 3.14. Factors of safety of the test section according to Hadj-Hamou et al (1990).

Failure Surface	Factor of Safety	
	Unreinforced	Reinforced
A-1	1.05	1.31
A-2	1.16	1.54
B-1	1.09	1.26
B-2	1.10	1.29
C-1	1.24	1.36

All the surfaces shown in Figure 3.22 were also analyzed using the HOPSLEP computer program by different statically consistent methods. The results are shown in Tables 3.15 and 3.16.

In Tables 3.15 and 3.16, ϵ is a measure of the admissibility violation according to the Equations 3.1 and 3.2. The analysis of the results given in Tables 3.15 and 3.16 shows that for an unreinforced case for the surfaces A-1; A-2; B-1; B-2 Perturbation method no. 3 can be considered the most physically admissible (it is worth mentioning here that Perturbation method no. 2 is not applicable for unreinforced analysis). For the surface C-1, the least admissibility violation corresponds to Morgenstern and Price's method, but the factors of safety for Morgenstern and Price's method and for Perturbation method no. 3 only slightly differ from each other for this surface. Therefore even in this case, Perturbation method no. 3 provides a reasonable

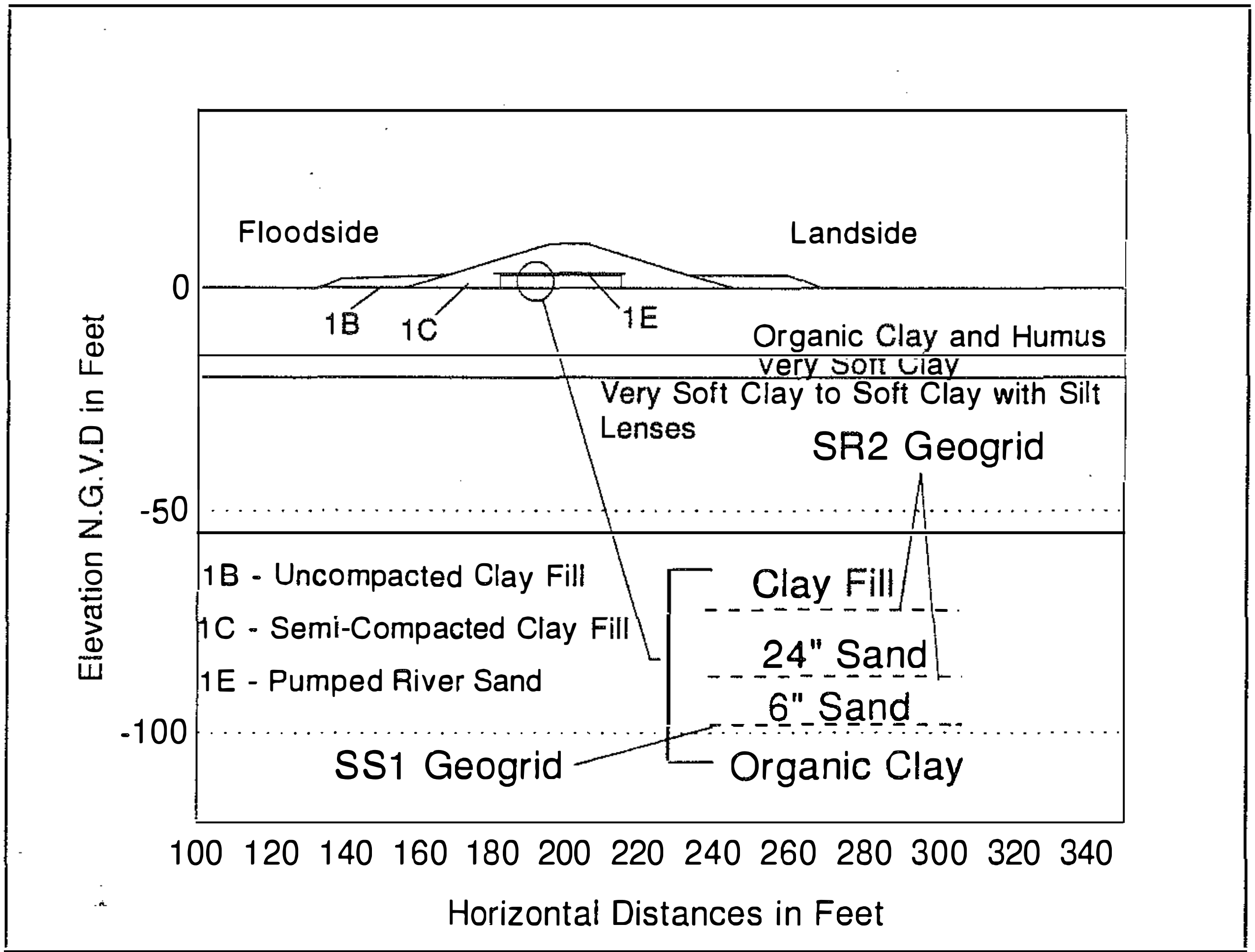

Figure 3.20. Cross section of test section in Hadj-Hamoe et al (1990) example.

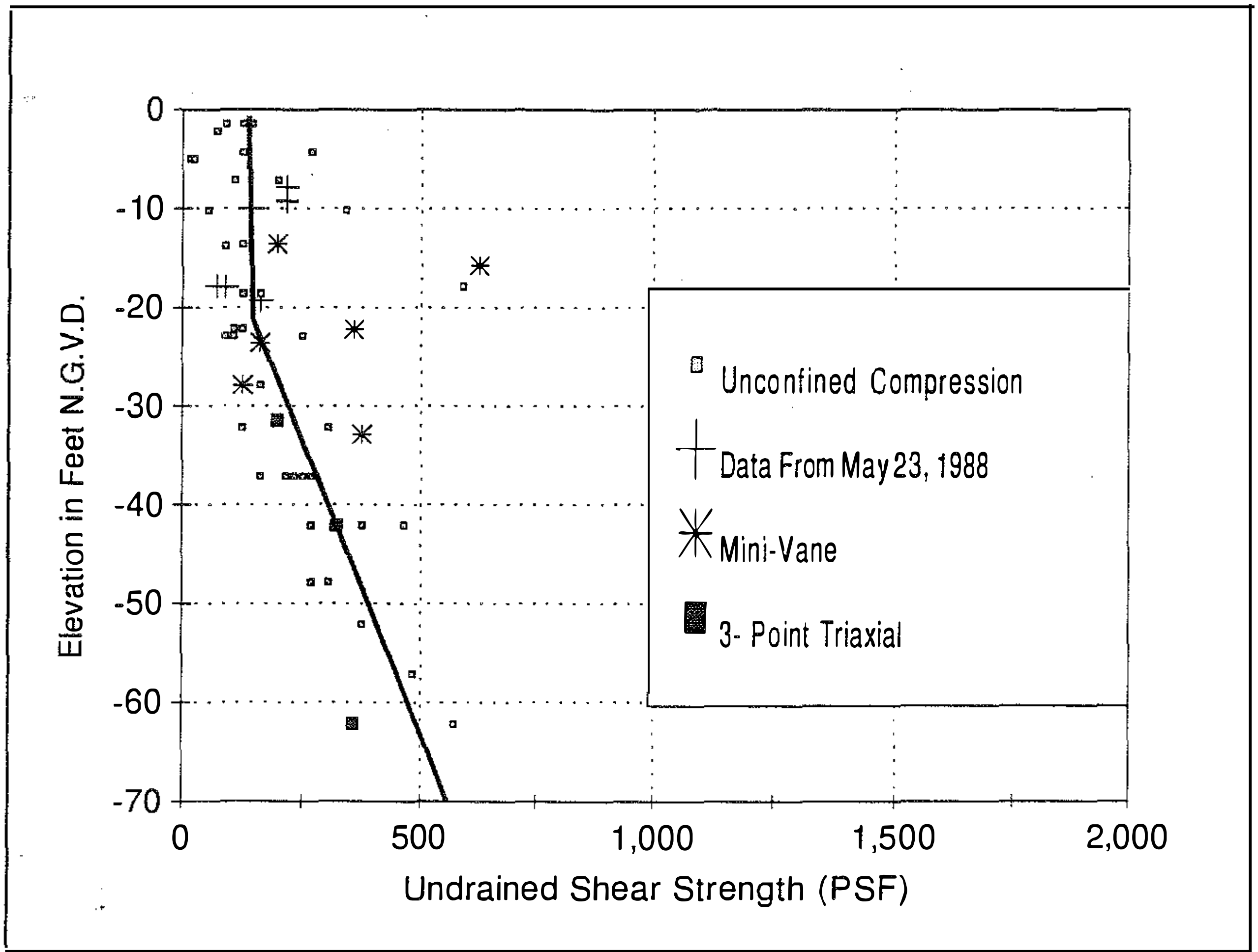

Figure 3.21. Shear strength profile in Hadj-Hamoe et al (1990) example.

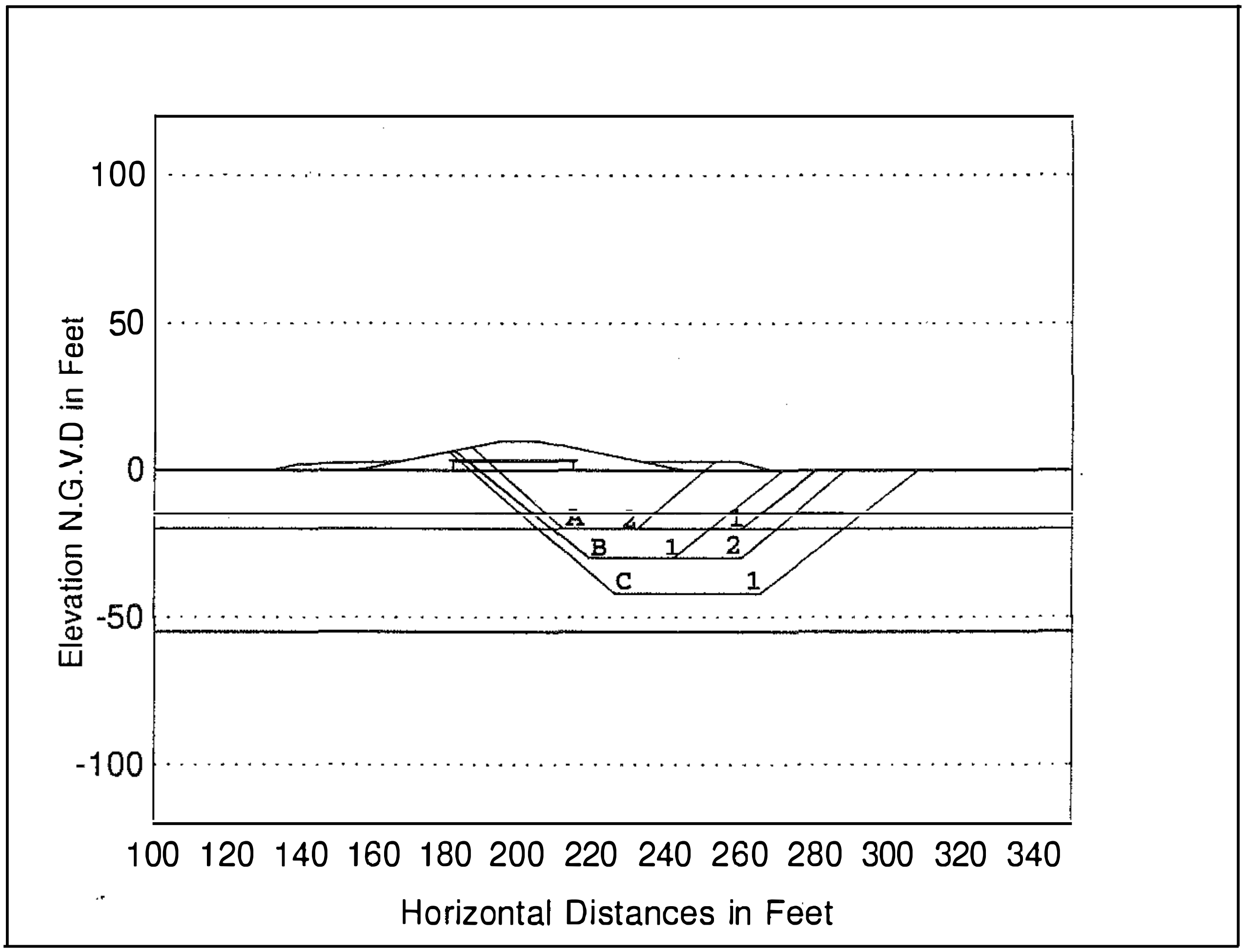

Figure 3.22. Critical failure surfaces in Hadj-Hamoe et al (1990).

answer. For the reinforced case, both Perturbation method no. 2 and Perturbation method no. 3 appear to be the most admissible for the surfaces A-1; A-2; B-1; B-2. At the same time for the most critical surfaces (surface A-1 and B-1), Perturbation method no. 3 shows less values of the admissibility violations and therefore, this method should be considered most admissible. For the surface C-1, the least admissibility violation is provided by Morgenstern and Price's method. The difference between the factors of safety obtained by Perturbation method no. 3 and

Table 3.15. Results of unreinforced stability analysis of the test section by HOPSLEP computer program.

Failure Surface	Method of Analysis							
	Hopkins		Morgensterin & Price *)		Perturbation 1		Perturbation 3	
	F	ε	F	ε	F	ε	F	ε
A-1	0.979	0.132	1.105	0.053	1.253	0.474	1.046	0.026
A-2	1.475	0.158	1.159	0.079	1.404	0.092	1.680	0.039
B-1	1.084	0.145	1.201	0.053	1.121	0.224	1.228	0.026
B-2	1.477	0.158	1.383	0.053	1.323	0.066	1.707	0.026
C-1	1.630	0.171	1.796	0.026	1.894	0.079	1.808	0.158

*) f(x) is given by Equation 2.25

Table 3.16. Results of reinforced stability analysis of the test section by HOPSLEP computer program.

Failure Surface	Method of Analysis									
	Hopkins		Morgenstern & Price *)		Perturbation 1		Perturbation 2		Perturbation 3	
	F	ε	F	ε	F	ε	F	ε	F	ε
A-1	1.136	0.105	1.291	0.079	1.702	0.513	1.198	0.053	1.322	0
A-2	1.578	0.237	1.179	0.118	2.380	0.132	2.450	0.039	2.698	0.053
B-1	1.265	0.118	1.298	0.092	1.395	0.224	1.308	0.092	1.503	0.013
B-2	1.875	0.118	1.426	0.092	1.916	0.118	2.122	0.013	2.416	0.066
C-1	1.814	0.145	1.847	0.039	2.257	0.118	1.886	0.171	2.059	0.132

*) f(x) is given by Equation 2.25

Morgenstern and Price's method is about 11%. But even in this case, the factor of safety provided by Perturbation method no. 3 can be accepted because the surface C-1 is far from critical and both values of the factor of safety (1.847 - for Morgenstern and Price's method and 2.059 - for Perturbation method no. 3) are much bigger than the minimum value of 1.322. It is worth mentioning here that the minimum factor of safety obtained by Hadj-Hamou et al (1990) was F = 1.26 which is only slightly conservative. In the corresponding unreinforced analysis, the Hadj-Hamou's et al (1990) factor of safety (F = 1.05) and the one obtained by Perturbation method no. 3 (F = 1.046) agree within less than 0.5%.

In addition to the noncircular analysis described, the circular search was also performed for this example. The results are shown in the Table 3.17 for the unreinforced case and in the Table 3.18 for the reinforced case. In both tables, characteristics of critical circles are also shown. It is interesting to note that in the unreinforced analysis all methods including Bishop's method provide almost the same minimum values of factors of safety varying between 1.15 and 1.17. All methods indicate the same critical circle (see Table 3.17).

Table 3.17. Results of circular search analysis in unreinforced case by HOPSLEP computer program. Hadj-Hamou et al (1990) example.

Method of Analysis	F_{min}	Critical Circle Parameters		
		X - Coordinates of the centers (ft.)	Y - Coordinates of the centers (ft.)	Radii (ft.)
Hopkins	1.168	240	45	64.643
Perturbation 1	1.146	240	45	64.643
Perturbation 3	1.164	240	45	64.643
Bishop	1.163	240	45	64.643

Table 3.18. Results of circular search analysis in reinforced case by HOPSLEP computer program. Hadj-Hamou et al (1990) example.

Method of Analysis	F_{min}	Critical Circle Parameters			ϵ
		X - Coordiantes of the centers (ft.)	Y - Coordinates of the centers (ft.)	Radii (ft.)	
Hopkins	1.285	235	40	67.694	0.105
Perturbation 1	1.357	240	45	74.643	0.066
Perturbation 2	1.310	235	20	49.480	0.053
Perturbation 3	1.361	240	45	74.643	0.039
Bishop's correct	1.310	240	45	74.643	--

Reinforced case limit equilibrium methods, listed in Table 3.18, give different minimum factors of safety ranging between 1.29 and 1.36. Besides, critical circles obtained using different methods don't coinside (see Table 3.18). Therefore, in this case the admissibility analysis is highly desirable. In Table 3.18, admissibility violation values (ϵ) are computed for each of the methods, except for the Bishop's method. Again, as in noncircular analysis Perturbation method no. 3 can be considered the most admissible. In other words, the value of F = 1.361 provided by Perturbation method no. 3, should be considered as the real minimum factor of safety in reinforced circular search analysis.

It is interesting to note that in both the reinforced and unreinforced cases circular search analysis provides a slightly higher value for the factor of safety than the one obtained by noncircular wedge analysis. This fact emphasizes the importance of noncircular analysis.

4. MULTIPURPOSE LIMIT EQUILIBRIUM COMPUTER PROGRAM. GENERAL GUIDE TO DATA ENTRY.

A general entry guide for the HOPSLEP limit equilibrium computer program is discussed below. All data must be submitted in the sequence described.

Utility data and number of problems

Different computers and computer installations have different input and output codes, or UTILITY codes. The particular codes used by each installation must be supplied when submitting a problem. The user may submit at any one time any number of problems. Information for utility codes and number of problems is contained on Record Number 1:

IN IOUT NOP,

where IN = a code designating which input device will be used at a given computer installation,

IOUT = a code designating which output device will be used at a given computer installation, and

NOP = the number of problems per submission.

Regardless of the number of problems submitted at a given time, data for IN, IOUT, and NOP need be entered only once.

Problem identification data

Data for problem identification are contained on Record Number 2 as follows:

MONTH KDAY KYEAR IH(I),

where

MONTH = the month of the year,
KDAY = the day of the week,
KYEAR = the year (use only the last two digits), and
IH1(I) = description of the problem. This information may be a project

number, a route number, or an assigned problem number.

Problem control data

These data entered on Record Number 3 include the unit weight of water (UNITWW), the number of slices (NSLICE), the maximum number of iterations (MAXIT) allowed for each problem, limit equilibrium method number (NMETH), routing gimmick code (NPULL) that controls the way of calculating the reinforcement forces, the number of slices for calculating pullout resistances of geosynthetic sheets (NSLICER), and routing gimmick code (NVERS) that controls the version of a limit equilibrium method.

UNITWW NSLICE MAXIT NMETH NPULL NSLICER NVERS

The units of all data entered into the program must be consistent. The numerical value and units specified for the unit weight of water control the units of all other input data. If the user does not specify a value for the unit weight of water (UNITWW), the computer program assumes a value of 62.4 pounds per cubic foot. Therefore, in this case, length must be in feet and force and weight must be in pounds to be consistent with the units of the unit weight of water. For example, all coordinate data must be in feet and such values as the cohesion of a soil would have the units of pounds per square foot. If units other than those assumed by the computer program are used, then the user must specify a numerical value of the unit weight of water consistent with the intended units. For instance, if the unit weight of the pore fluid is specified as 1.0, then the consistent set of metric units would be metric tons and meters, or grams and centimeters (not kilograms and centimeters). If a value of the unit weight of the pore fluid other than fresh water is desired, then the desired unit weight of the pore fluid should be specified.

The maximum number of slices (NSLICE) that may be specified by the user is 598. If the user fails to specify a number for NSLICE, then the computer program assumes a value of 76. When the user specifies the number of slices, the specified number MUST be an EVEN integer. The computer program divides each trial mass into the number of slices specified by the user. It is recommended that no fewer slices than 76 be used.

If a value for the maximum number of iterations (MAXIT) allowed is not specified, the program uses a value of 15. Based on experience, the computer solution usually converges in less than 15 iterations. Generally, in most problems convergence is obtained in less than about 10 iterations. If convergence is not achieved, the program alerts the user with a message, "Convergence was not obtained". When this occurs, the user may specify a value for MAXIT larger than 15. However, if more than 15 iterations are required, then the user should exercise caution in accepting the results obtained from the program. A listing of the interslice side forces should be reviewed to determine if they are reasonable.

Several limit equilibrium methods can be considered. These methods and the corresponding values of the parameter NMETH are listed in Table 4.1.

Table 4.1. Limit equilibrium methods allowed in HOPSLEP computer program.

Method	NMETH
Hopkins'	1
Bishop's correct	2
Bishop's incorrect	3
Morgenstern and Price	5
Perturbation	7

There are two options to calculate reinforcement forces in the program. These options are controlled by the parameter NPULL. When NPULL = o reinforcement forces are calculated according to Equation 2.1. When NPULL = 1, then reinforcement forces are still calculated based on Equation 2.1, but reinforcement sheets are considered fixed at the right ends, i.e. $t_a = T_r$ in Equation 2.1.

The number of slices NSLICER for calculating pullout resistances T_a and T_p (see Equations 2.2 and 2.3) is usually less than 10. NVERS is a routing gimmick code that controls a version of a certain limit equilibrium method.

Gimmick Routing Data

The computer program contains three routing options that allow the user to control certain operations of the program. Data for these options are contained on Record Number 4 as follows:

GSEARC GIMTH GIMPO

The routing optional code, GSEARC, specifies the mode of solution with regard to the shear surface. Two options are available. If the user chooses to analyze only one shear surface, then GSEARC is assigned a value of zero. In this case, the user must supply x- and y-coordinates of the shear surface. If the user chooses to perform a search analysis to locate the critical circle having a minimum safety factor, GSEARC is assigned a value of 1. In this case, the program generates the x- and y-coordinates of the shear surface. When GSEARC equals 1, the user must supply x- and y-coordinates of the search grid as described below in the section entitled "Search Grid -- Predetermined Trial Centers." The coordinates are entered on Record Number 15.

The user may generate coordinates for one circle. In this case, GSEARC is set equal to 1 and search coordinates of the grid are entered in a manner described in the section entitled "Search Grid -- Predetermined Trial Centers." The search coordinates are entered on Record Number 15. The radius of the circle is entered on Record Number 16.

When GIMTH is assigned a value of 0, the x- and y-coordinates of the thrust line must be supplied as discussed in the section entitled "Coordinate System and Method of Describing Geometry." Thrust-line coordinates are entered on Record Number 9. To avoid supplying these coordinates, the location of the thrust line may be defined by one parameter, ETA (Record Number 9): ETA = $(y_t - y)/(Y - y)$ (see Figures 1.1 and 1.2). In this case, GIMTH is set equal to 1; the x- and y-coordinates of the thrust line are generated.

A third option is available for defining the location of the thrust line. When the effective stress parameter c′ is greater than 0, the line of thrust may be located slightly below the bottom third points of the slices in a tensile zone (active condition), while in a compressive zone (passive condition), the thrust line is located slightly above the bottom third points of the slices. This option is executed by assigning appropriate values on Record Number 9 to the computer parameters ETAC and ETAT, respectively. Thrust-line coordinates are then generated by the program.

The amount of data printed from the computer program is controlled by the routing parameter, GIMPO. Three options are available when only one shear surface is analyzed, that is, when GSEARC is assigned a value of 0. A limited amount of data is printed when GIMPO is set equal to 0. These data include all coded data and the factor of safety pertaining to each iteration. If GIMPO is assigned a value of 1, then all coded data, the factor of safety pertaining to each iteration, a table of side-force calculations corresponding to the last iteration, and a table of geometric data are printed. When GIMPO is set equal to 2, detailed information is printed. The printed information includes all coded data, the factor of safety corresponding to each iteration, a table of side-force calculations pertaining to each iteration, and a table of geometric data.

When GSEARC is set equal to 1, the format of printed data is fixed. In this case, GIMPO has no meaning. Printed data include a table summarizing the minimum factors of safety at each search grid point, geometric data, search grid coordinates and corresponding minimum factors of safety, and the minimum factor of safety obtained from the search analysis.

Properties of Soil Layers

Properties of each layer of soil are contained on Record Number 5 as follows:

CO(M) PHI(NL) WT(NL) RU(M) YEL(M,N)

where

CO(NL) or CO(M,NL)=	effective or total stress parameter, cohesion, (c′), of each soil layer; also, the undrained shear strength parameter (S_u);
PHI(NL)=	effective or total stress parameter, angle of internal friction (φ′), of each soil layer;
WT(NL)=	total unit weight of each soil layer (γ);
RU(M)=	pore pressure ratio (r_u: the ratio of the pore pressure at a given point in the soil layer to the total overburden pressure at the given point) of each soil layer (options associated with this parameter are described below); and
YEL(M,N)=	elevation of the saturated undrained shear strength (S_u), at a given point in a soil layer.

Shear strength of each soil is defined in terms of the Mohr-Coulomb-Terzaghi strength criterion,

$$\tau = c' + (\sigma - u) \tan \phi',$$

where c′ = cohesion of the soil,

φ′ = angle of internal friction of the soil,

σ = total normal stress acting on the shear surface, and

u = pore water pressure acting on the shear surface.

When the strength of a particular layer is expressed in terms of total stress parameters, the values of φ and c are total stress shear strength parameters. Pore pressures within the particular layers are specified as zero. The values, c′, φ′, γ, and u, used for each slice are values applied at the midpoint of the base of each slice. If an effective stress analysis is specified, values of φ′ and c′ are effective stress strength parametes and an appropriate method of representing the pore pressures is selected as described below in the section entitled "PORE-WATER PRESSURE and r_u DATA." Two different methods of entering soils data are available. The method selected will depend on the particular problem to be solved by the user. Details of the two methods are described as follows.

Method 1: Soil Properties of Each Layer -- normal case

Typically, in many problems, the user may wish to describe the soils properties c′, ϕ', γ_t, and r_u of each soil layer. Corresponding computer variable names are CO(L,M), PHI(L), WT(L), and RU(L), respectively. For each soil layer, a set of values for ϕ', c′, γ_t, and r_u is entered.

When the properties of each soil layer are described using ϕ', c′, γ_t, values of the parameter YEL(L,M) are entered as zero. At the end of the soil properties data set, a value of 999 is inserted. This number is used in the computer program to terminate reading of soil properties records.

Method 2: Variation of Saturated Undrained Shear Strength with Elevation

Variations of the saturated undrained shear strength (S_u), with depth (or elevation) are specified in terms of the variable CO(L,M) and corresponding elevations, YEL(L,M). When the data are entered in this manner, the computer program interpolates a value of undrained shear strength from the S_u -elevation curve for the ith slice (or for each slice). Where the unit weight of soil is constant within a given depth range, values of CO(L,M) and YEL(L,M) are entered for that depth range. If the entire foundation stratum has a constant unit weight, then the profile values CO(L,M) and YEL(L,M) are entered. In this case, the stratum need not be divided into several layers. However, the stratum may always be subdivided into layers. When entering the elevations of corresponding values of S_u for a given soil layer, the values of S_u -elevation at the top of a soil layer (elevation of the top boundary line of the layer) and values of S_u -elevation at the bottom of a soil layer (elevation at the bottom boundary line of the layer) must be entered. Otherwise, a portion of the soil strength of the layer would be undefined. For the case where more than one soil layer is involved, values of S_u -elevation common to adjacent soil layers must be entered twice.

When soil boundary interfaces are not horizontal, the profile of values of S_u -elevation must be entered for the full distance (deepest part) of the soil stratum or strata. However, it is better to subdivide the foundation into layers using horizontal lines. When CO-YEL data are entered, the computer program automatically enters the Method 2 mode. No optional gimmick is needed to direct the program to enter the Method 2 mode. (Default is Method 2 Mode.)

Pore-Water Pressure and r_u Data

Pore pressures must be described for all soils in which the effective stress parameters ϕ' and c′ are used to obtain the effective normal stresses acting at the bases of slices. Pore-pressure problems, according to Bishop and Bjerrum (1969), may be divided into two main classes:

CLASS 1. Pore pressure is a dependent variable controlled by the magnitude of the stresses acting in the soil or tending to lead to instability. Problems of this type may involve the rapid construction in or excavation of low-permeability soils.

CLASS 2. Pore pressure is an independent variable and does not depend on the magnitude of the total stresses acting in the soil. In this case, pore pressures are controlled by the groundwater level or by the flow pattern of the groundwater.

There are four methods, or options, in the computer program that handle the two classes of problems. These methods are described in the following.

Option 1.

Pore pressure in a given soil layer may be defined by a pore pressure ratio (r_u.) This dimensionless parameter is the ratio of the pore pressure (u) to the vertical stress (σ_v) of soil above the element considered

$$r_u = u/\sigma_v = u/\gamma_t h_s \tag{4.1}$$

where γ_t = total unit weight of soil above the element and

h_s = height of soil above the element.

The r_u ratio was first used by Daehn and Hilt (1951) as a means of expressing the results of the stability analysis of four earth dams. Bishop (1955) showed that, for a constant value of r_u, the relationship between the factor of safety and r_u is almost linear. Later work by Bishop and Morgenstern (1960) showed that, for both classes of problems, the pore-pressure ratio is a very convenient means of expressing the influence of pore pressure on stability.

For Class 1 problems, the r_u ratio is obtained either from field measurements of pore pressures or estimates from triaxial tests and consolidation theory as described by Bishop and Bjerrum (1969) and Bishop and Henkel (1957). In the latter case, an estimate of the stress distribution within the soil must be made. In Class 2 problems, the r_u value is obtained from a flow net; it is expressed as an average value. Details of this technique have been given by Bishop and Morgenstern (1960).

In the computer program, each soil type or layer may have only one value of r_u. When Method 1 is used to define the pore pressure in a given layer, the value of r_u must be some real number less than 1.0. This value is entered on Record Number 5 as previously described above in the section entitled "Properties of Soil Layers."

Option 2.

Pore pressure in a given soil layer may be defined by piezometric lines. This method is convenient to use when piezometers are used to obtain pore pressures. However, both Class 1 and 2 problems may be solved using this method. Pore pressures are obtained as described in Method 1. Each soil layer may have only one piezometric line. Each peizometric level is approximated by straight line segments and x- and y-coordintes.

When the trial shear surface passes through a soil layer where the pore pressures are defined by piezometric coordinates, the pore pressure for each slice of the unstable mass is calculated by multiplying the vertical distance between the shear surface of the slice and the appropriate piezometer line by the unit weight of water. The pore pressure (u_i) at the base of slice i is equal to the vertical distance (h_w)times the unit weight of water (γ_w), or

$$u_i = \gamma_w h_w. \tag{4.2}$$

Whenever Method 2 is used to define pore pressure in a layer, the value of r_u for that layer is set equal to 1.5 on the soils properties data Record Number 5 and the appropriate x-and y-piezometric coordinates are entered on Record Number 10 as described under "COORDINATE SYSTEM".

Both Options 1 and 2 may be intermixed. For example, pore pressures in one soil layer may be defined by a r_u -value while pore pressures in another layer may be defined using x- and y-piezometric coordinates.

Option 3.

Pore pressures may be defined using an infinitely sloping groundwater table or phreatic surface. In this option, the groundwater level is approximated by x- and y-coordinates defining straight-line segments. Pore pressure (u_i), at the base of each slice is computed

from

$$u_i = h_p \gamma_w = h_i \gamma_w \cos^2 j \tag{4.3}$$

where

- h_p= pressure head;
- γ_w= unit weight of water;
- j = gradient, or angle between a horizontal line and the ground-water line, and
- h_i= vertical distance between the surface of the ground-water table and the shear surface.

Pore pressures for the flow net may be computed using Option 3. This method applies to Class 2 problems where pore pressures are independent of the magnitude of the total stresses acting in the soil.

To execute Option 3, the first value of r_u of the first soil layer (see section on "SOIL PROPERTIES") is set equal to a real number, 2.5. All values of r_u for subsequent soil layers are left blank or set equal to 0. The groundwater table is approximated using x- and y-coordinates defining straight-line segments. The coordinates are entered in a manner described under the section entitled "COORDINATE SYSTEM". When Option 3 is used, Options 1 and 2 cannot be used.

Option 4.

Pore pressures may be defined by assuming or specifying a groundwater table. This method primarily applies to Class 2 problems. The groundwater table is described by x- and y-coordinates defining straight-line segments. The coordinates are entered on Record Number 11 as described in the section entitled "COORDINATE SYSTEM". Pore pressures (u_i,) acting at a point on the base of each slice are computed from the equation

$$u_i = h_i \gamma_w \qquad (4.4)$$

where h_i = vertical head of water between base of slice and groundwater table at the center of the slice. To execute Option 4, the first value of r_u of the first soil layer is set equal to a real value, 3.5.

Coordinate System and Method of Describing Slope Geometry

The method used in the computer program to describe the geometry of a slope and the arrangement of the soil types comprising the slope is illustrated below. Only two-dimensional problems can be solved by the computer program. All geometry of the slope is defined by x- and y-coordinates and line segments. The slope should face to the right. The x-coordinate direction must be horizontal and increase positively from left to right. The y-coordinate direction is vertical and must increase positively from bottom to top. The origin of the coordinate system is located to the left and below the slope.

The entire cross section is approximated by straight line segments. This applies to the groundline surface, layer boundary interfaces, water table surface or piezometric lines, shear surface and thrust lines. The line segments are defined by x- and y-coordinates. The uppermost line segments in the cross section are identified in the computer program as the groundline surface. Groundline coordinates are entered on Record Number 6. All boundary-line coordinates are entered on Record Number 7. X- and y-coordinates of the shear surface (if GSERC=0) are entered on Record Number 8. The thrust line is defined by x- and y-coordinates if GIMTH=0. Alternately, if GIMTH=1 the thrust line coordinates may be generated by specifying a ratio, ETA, defined previously. Thrust line coordinates or the value of ETA are entered on Record Number 9. Water table or piezometric lines are defined by x- and y-coordinates entered on Records Number 10 or 11, respectively.

A maximum of twenty-five soil layers and layer boundaries may be specified. Line segments of different boundary layers, including the groundline surface, may have the same coordinate points. A maximum of 25 sets of x- and y-coordinates may be used to define a boundary layer, groundline surface, water table, or peizometric surface. Each layer boundary, ground-water table, or piezometer line should extend from the left-most groundline coordinate point to the right-most groundline coordinate point. Coordinate points should be defined at sufficient distances to the left and right of the slope so that a trial shear surface intersects the ground surface or slope. Vertical slopes may be analyzed. Overhanging slopes are not permitted.

End Boundary Loads

End boundary loads are entered on Record Number 12. An example of an end boundary load would be the hydrostatic force exerted by a body of water resting against the slope or the hydrostatic thrust (EA) exerted by a water-filled tension crack located at the top of the slope. Whenever a body of water rests against the upstream or downstream slopes, the end boundary forces are automatically calculated by the program. However, the body of water resting against the slope must be treated as a soil layer having no shear strength. In this case the unit weight of the water layer is assumed to be 62.4 pounds per cubic feet. EA or EB need not be entered. However, for the case involving a water-filled tension crack, the end boundary force due to the hydrostatic thrust, EA, must be calculated and entered on Record Number 12.

External Vertical Distributed Loads

External vertical ditributed loads are approximated as shown in Figure 4.15 and are entered on Record Number 13. These loads are assumed constant between two consequent x-coordinates x_i, x_{i+1} and equal q_i .

Earthquake Forces

Earthquake loading is simulated using a psuedo-statical method. The seismic force is assumed to act horizontally on each slice in the direction of the failure (away from the slope). The force acting on each slice is computed from

$$F_i = aW_i/g = \psi W_i \tag{4.5}$$

where F_i = horizontal seismic force acting on slice i,

W_i = weight of slice i,
g = acceleration of gravity,
a = horizontal earthquake acceleration, and
Ψ = seismic coefficient in the region in which the earth structure is located.

Seismic loading is executed in the program by inserting values for SEMC and HQ on Record Number 14 SEMC is the seismic coefficient. HQ is the ratio of the distance S_q to the height of the slice Z_q

$$H_Q = S_q / Z_q \tag{4.6}$$

where

S_q= the distance between the elevation of the point of application, Y_q, of the seismic force and the elevation of the shear surface, Y_f ($S_q = Y_q - Y_f$)

Z_q= the distance between the elevation of the groundline surface, Y_s, and the elevation of the shear surface ($Z_q = Y_s - Y_f$).

By specifying a value of HQ, the user may locate the earthquake forces at any position of the slices. However, the earthquake forces are generally located at the midpoints of the slices; a value of 0.5 is usually inserted for HQ.

Search Grid-Predetermined Trial Centers

If GSERC = 1 (Record Number 4), the user must specify a grid for centers of trial shear surfaces. The grid is rectangular in shape. The rectangular grid is described by two points. The x- and y-coordinates, XSTART and YSTART, of the upper left-hand corner of the grid, and the x- and y-coordinates, XFIN and YFIN, of the lower right-hand corner must be specified. To establish the number of trial centers of the grid in the horizontal direction, the user must specify the width, XDEL, of each increment. The value selected for the increment, XDEL, must be such that

$$(XFIN - XSTART)/XDEL = m(\text{integer value}). \tag{4.7}$$

The number of trial centers of the grid in the vertical direction is established by specifying the width, YDEL, of each increment. The value selected for YDEL must be such that

$$(YSTART - YFIN)/YDEL = n(\text{integer value}). \tag{4.8}$$

The grid of centers specified above may be reduced to a single point by setting

XSTART = XFIN and YSTART = YFIN.

Generation of trial centers starts from the top, left-hand corner (XSTART, YSTART) of the grid and proceeds to the right in the x-direction until

$$XSTART + (m - 1)*XDEL = XFIN, \tag{4.9}$$

The integer m is defined by Equation 4.7. After generating the top row of the trial centers and solving for the factors of safety, the program selects the trial centers for the next lower row. Each time a row of trial centers is computed, the program starts at XSTART and moves downward an amount YDEL. This operation procedes until

$$YSTART - (n - 1) * YDEL = YFIN, \tag{4.10}$$

where n is defined by Equation 4.8.

When Equation 4.10 is satisfied, all trial centers (and safety factors) have been solved. For each trial center, the computer program stores the minimum factor of safety and later prints these factors of safety in the form of a grid of minimum factors of safety. However, factors of safety for all trial circles as well as other geometric data are printed. Additionally, the program selects and prints the minimum factor of safety of all trial circles obtained from the search-grid operation.

The radius of each trial shear surface is generated by specifying the length of the radius increment, RDEL:

$$RO = RMIN + RDEL \tag{4.11}$$

where RO = radius at a trial center,
RMIN = the initial radius, and
RDEL = the radius increment.

If XDEL, YDEL, and RDEL remain blank, then the default values for each of these increments is 5.0.

Information for the search grid is contained on Record Number 15 as follows:

XSTART YSTART YFIN XFIN XDEL YDEL RDEL.

Initial Radius Coordinates and Analysis of Individual Shear Surfaces

To establish an initial radius, RMIN, at a given trial center, (when GSERC = 1) the user must specify a point on the cross section by the coordinates XE and YE. The initial radius, RMIN, at each trial center is computed as the distance from the center of the circle and the specified point. Additional radii are generated from Equation 4.11. When circular shear surfaces are generated, the starting coordinate point XE, YE must never be placed above the groundline. By placing the starting radius coordinate point below the groundline, the user can control the depth of shear surfaces and avoid the analysis of shallow shear surfaces.

The radius RMIN at a trial center is incremented until a trial circle intersects the bottom layer boundary. When this occurs, the computer program determines a circle (and corresponding radius) that is tangent to the bottom layer boundary. After computing the factor of safety for the tangent circle, the computer program proceeds to the next trial center of the grid. The minimum factor of safety at each trial center is determined and stored.

If the factor of safety of a single circle is required, then only values of XSTART, YSTART, and RO1, the value of the known radius, need be supplied. Values of XFIN, YFIN, XDEL, YDEL, RDEL, XE, and YE are not needed. Information for the starting radius point (XE, YE) and the radius (RO1) for a single circle is contained on Record Number 16 as follows:

XE YE RO1 DVS

When performing a search analysis on a slope containing a small vertical line segment(s), a circle may intersect the vertical slope and yield a small factor of safety. To avoid this situation, a vertical distance, DVS, may be specified. Usually DVS is set equal to the height of the vertical line segment. The value of DVS is entered on Record Number 16. In the computer program, the slice having the maximum height, ZMAX, is determined and compared to DVS. If ZMAX is less than DVS, then the factor of safety is not computed.

There is a special opportunity in the computer program for analyzing retaining walls. In this case, in performing a search analysis, the user need not supply the x-coordinate of an initial point but rather XE is input as zero. The computer program calculates XE as an x-coordinate of a point of intersection of a vertical wall face and a horizontal line y = YE.

Tension Crack Analysis

In cases where embankments are constructed on soft clay foundations, stresses in the upper reaches of the potential failure mass may be tensile. A problem arises in the design of embankments on soft foundations because it is uncertain as to what portion of the shear strength of the embankment is mobilized and may be relied on for stability. Uncertainties arise in the stability analysis because of differences in the stress-strain behaviors of the embankment and soft foundation soils. Embankments normally will be constructed of compacted soils that will be stiff and overconsolidated. The peak strength of the compacted embankment soils occurs at a relatively small failure strain while the peak strength of the compacted embankment soils occurs at a relatively small strain. At this stage, only a small portion of the shear strength of the foundation soils may be mobilized. This situation leads to the development of tensile stresses in the upper zone of the embankment. Since soils cannot sustain tensile stresses, at least for a prolonged period of time, a tension crack may develop in the embankment. If the embankment cracks, a smaller portion of the shear strength of the embankment may contribute to overall stability. Therefore, the use of the peak strength of the embankment in the stability analysis may be under conservative if the embankment is prone to crack. Assuming no shear strength for the embankment soils may be over conservative since overturning moments are too large. For this later case, the safety factor is too low.

As shown elsewhere (Lambe and Whitman, 1969; Chowdhury, 1978), the depth of the tension crack may be expressed as (in terms of effective stress)

$$Z = (2c'/\gamma_t (1 - r_u))((1 + \sin \phi')/(1 - \sin \phi'))^{1/2} \tag{4.12}$$

where

- Z = depth of tension crack;
- c' = effective stress parameter, cohesion;
- γ_t = total unit weight;
- r_u = pore-pressure ratio; and
- ϕ' = effective stress parameter, angle of internal friction.

Equation 4.12 allows the maximum depth of the tension crack to be computed at failure ($F = 1.0$) in terms of effective stress or total stress. However, for cases other than failure, that is, when the factor of safety is greater than one, the tension crack depth may be computed using the mobilized shear strength parameters: $\tan \phi_f = \tan \phi'/F$ and $c_f = c'/F$. Substituting the mobilized strength parameters into Equation 4.12, the depth of the tension crack may be expressed as

$$Z = (2c_f/\gamma_t (1 - r_u))((1 + \sin \phi_f)/(1 - \sin \phi_f))^{1/2} \tag{4.13}$$

According to Equation 4.13, the depth of the tension crack is a function of F, γ_t, c_m, r_u, and ϕ_m. However, the factor of safety and, therefore, the mobilized depth of tension crack is unknown (except at failure when $F = 1.0$). To solve this problem when the factor of safety is greater than one and for a given shear surface, that is, to obtain the depth of the crack compatible with the factor of safety, iteration may be performed using Equation 4.13. The iteration is performed on Z and F and c_m, γ_t, r_u, and ϕ_m are constant.

To start the iteration, an initial factor of safety must be assumed. A reasonable initial estimate of the factor of safety (F_o), may be obtained by solving the problem assuming no tension crack. Substituting F_o into Equation 4.13 and solving yields the first estimate of the depth of tension crack (Z_o.) Using Z_o, a new value of the safety factor (F), is computed. The iteration is continued until

$$|Z_n - Z_{n-1}| < \varepsilon, \tag{4.14}$$

where ϵ = a selected numerical error.

The numerical value of ϵ in the computer program is 0.001. A detailed treatment of the tension crack problem, as used in the computer program is given elsewhere (Hopkins, 1984) and is beyond the scope of this report.

Two options are available for performing stability problems involving potential tension cracks. The user may specify the iteration scheme described above and allow the computer program to obtain a compatible value of tension crack and factor of safety. If the depth of tension crack is known, or estimated, then the user can input the tension crack depth. To invoke the first option, a value of 1.0 is input for DTC. The second option is executed when the fixed value of the tension crack is inserted for DTCF X. If DTC is specified, the value of DTCFIX is left blank or set equal to 0. When either option is used, the user must specify the layer of soil in which the tension crack will develop. The layer number where the tension crack occurs is specified by NSOLY. Pore pressures in the soil layer designated for the tension crack are handled by inserting a value of pore-pressure ratio for RURT. Data for the tension crack calculations are entered on Record Number 17.

Reinforcement Geometry and Strength Properties

Reinforcement geometry and strength properties are entered on Record Number 18 as follows:

GSL X2 YR GSR1 GSR2 RSTREN,

where

GSL - reinforcement sheet length;
X2 - coordinate of the reinforcement;
YR - reinforcement sheet elevation;
GSR1 and GSR2 - soil-fabric interaction coefficients for soils above and below the reinforcement sheet correspondingly (see coefficients k_1 and k_2 in Equations 2.2 and 2.3);
RSTREN - tensile strength of the reinforcement sheet.

Initial Approximations for Morgenstern and Price's Method

nitial approximations for Morgenstern and Price's method are entered on Record Number 19 as follows:

FOLD LAMOLD F LAM

where

FOLD and F - two consequent approximations of the safety factor: FOLD = F_o, F = F_1;
LAMOLD and LAM = two consequent approximations of λ (see Equation 1.22): LAMOLD = λ_o, LAM = λ_1.

Record Number 19 is only important when Morgenstern and Price's method is used. In all other cases this line may be left blank.

5. CONCLUSIONS

Different limit equilibrium methods are considered in this report and adopted to take account of the reinforcement. Among these methods there are traditionally used Bishop's method, Morgenstern and Price's method, and also relatively rarely used Perturbation method and Hopkins' method, which is essentially the modification of the Janbu's method. Besides, two new methods were proposed in this report. These methods can be considered as modifications of Hopkins' and original Perturbation methods. Based on the comparative analysis of the above mentioned methods the following conclusions can be made.

1. Two modifications of the original Bishop's method are currently being used in practice. These modifications are referred to as Bishop's correct and Bishop's incorrect methods. It was shown that in most cases these methods lead to essentially different factors of safety. In the unreinforced case, both modifications are reduced to the original Bishop's method. In the case of the reinforced earth stability analysis, factors of safety obtained by two modifications, coinside only when the factor of safety F = 1. This case, however, is not practically important because usually reinforced earth structures are designed to provide the required factor of safety $F_r \geq 1.3$. Therefore, Bishop's incorrect method should not be used in the stability analysis of reinforced earth structures.

2. Bishop's correct method and all statically consistent methods such as Hopkin's method, Morgenstern and Price's method, original Perturbation method and new Perturbation methods, as proposed by the authors, provide reasonable factors of safety for internal stability analysis of reinforced slopes and retaining walls. However, Hopkins' method discovers convergence problems in some cases and Morgenstern and Price's method appears to be very sensitive to the initial approximations of λ and F parameters as well as the function f(x) (see Equation 2.21). Therefore, only Bishop's correct method and any of the perturbation methods are recommended for practical use.

3. The Tensar method for designing reinforced slopes and retaining walls usually provides acceptable results, though this method is conservative. It tends to overestimate the number of reinforcement layers to provide the required factor of safety. It was shown that the steeper the slope the bigger the difference in the number of layers according to the Tensar method and rigorous limit equilibrium methods.

4. In the case of a reinforced embankment on a soft foundation, different limit equilibrium methods may lead to essentially different values of factors of safety, especially in the case of noncircular slip surfaces. The selection of the method was done in this research study based on the comprehensive investigation of the admissibility criterion (1.15.) It was shown that in both circular and noncircular analyses only Perturbation method Number 3 proposed in this report appears to be statically consistent and physically admissible. Therefore, the potential user should be careful using other methods including Bishop's correct method.

5. Perturbation method Number 3 can be recommended for practical use as a universal limit equilibrium method. This method proposed by the authors appears to be statically consistent and physically admissible. It works reasonably in the case of internal stability of reinforced

slopes and retaining walls as well as in the case of a reinforced embankment on a soft foundation involving deep failure surfaces. This method does not have any restrictions with respect to the kind of failure surfaces and in all cases a rapid convergence was observed by the authors.

6. A multipurpose limit equilibrium computer program has been developed which allows the user to analyze a broad variety of the stability problems of reinforced earth structures. The analysis can be done using any of the limit equilibrium methods considered in this report.

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APPENDIX 1

Coding sheets for the HOPSLEP computer program.

HOPSLEP

1. Input Output Data

IN	IOUT	NOP

COL 5 10 15

2. Problem Identification

MO	DA	YR	PROBLEM IDENTIFICATION

COL 6 80

3. Problem Control

UNITWW	NSLICE	MAXIT	NMETH	NPULL	NSLICER	NVERS

COL 10 15 20 25 30 35 40

4. Gimmick Routine Data

GSEARC	GIMTH	GIMPO

COL. 5 10 15

5. Properties Of Soil Layers

CO(M)	PHI(NL)	WT(NL)	RU(M)	YEL

COL 10 20 30 40 50 60

6. Groundline Coordinates

X( )	Y( )

COL 10 20

7. Soil Layer Boundary Coordinates

XLS( )	YLS( )

10 20

8. Shear Surface Coordinates

XF( )	YF( )

COL 10 20

9. Thrust Line Coordinates

XTH( )	YTH( )

10 20

or

If GIMTH Equal to 1

ETA	ETAT	ETAC

COL 10 20 30

10. Piezometric Coordinates

XP( )	YP( )

COL 10 20

11. Groundwater Level Coordinates

XWT( )	YWT( )

10 20

12. End Boundary Loads

EA	EB	TA	TB

COL 10 20 30 40

13. External Vertical Distributed Loads

X( )	q( )

COL 10 20

14. Earthquake Forces

SMEC	HQ

COL 10 20

15. Grid Search Coordinates and Increments

XSTART	YSTART	XFIN	YFIN	XDEL	YDEL	RDEL

COL 10 20 30 40 50 60 70

16. Starting Coordinates of Radius

XE	YE	ROI	DVS

COL 10 20 30 40

17. Tension Crack Parameters

DTC	DTCFIX	NSOLY	RURT

COL 10 20 25 30

18. Reinforcement Geometry and Strength Properties

GSL	X2	YR	GSR1	GSR2	RSTREN

COL 10 20 30 40 50 60

19. Initial Approximations for Morgenstern and Price's Method

FOLD	LAMOLD	F	LAM

COL 10 20 30 40

APPENDIX 2

Coding Formats for the HOPSLEP Computer Program

Method of Coding Data

A complete guide describing the method of coding data for a slope stability problem is detailed below. All input data must be coded in a rigid format. Consequently, it is essential that careful attention be given to the coding instructions and that all input data be properly placed in the allocated column spaces.

Comments on the Data Coding Instructions

Although prior knowledge of computer programming is not required, the following basic points must be understood to properly apply the data coding instructions.

1. There are two basic types of data fields. The first is referred to as alphanumeric. Both numbers and letters may be entered in an alphanumeric field. In the computer program, this type of field is used only once (see Line-type 2 below). The second type of data field is referred to as numeric. Only numbers are permitted. With the exception of Line 2 below, all data will be entered in numeric fields.

2. Numeric data are specified as integer or real. An integer number is a whole number. A real number is a decimal point. Whenever a variable is specified as an integer number, the numerical value of that variable is "Justified Right" in the field; that is, a number is inserted on a data line so as to leave no blank spaces to the right of the number in the allotted spaces. For example, if the integer number 7654 is to be punched in Column 1 through 10 (justified right), the digit 7 must be placed in Column 7 to allow the last digit (4), to be located in the last allocated column, 10.

3. Each capitalized term appearing in the coding instructions below, under each line type, refers to the input variable exactly as it is found in the computer program. Additionally, each of the input variables is identified as real or integer.

4. In certain cases, the column spaces allotted to an input variable may remain blank. Where default values are listed in the instructions for an input variable and the user chooses to use the default value, the allotted column spaces for that variable may remain blank. For example, if only one problem is to be performed, the column spaces for the input variable NOP (Number of Problems) may remain blank; the program assigns the default value of 1 to the variable NOP. In cases where a variable has a value of zero, the allotted column spaces may remain blank.

5. Frequent reference is made in the coding instructions to gimmick routing variables. Values assigned to these variables merely instruct the computer program either to use certain source statements while ignoring certain others or to proceed to subsequent source statements after a particular operation has been performed. Two types of gimmick variables are used in the program. The first is an optional variable that provides the user some means of controlling certain operations in the program. For example, the variable listed as GIMPO controls the

amount of printed output. This variable may be assigned the integer values of 0 (or blank), 1, or 2. For instance, if GIMPO is set equal to 0, or left blank, all cross-section data and calculations yielding the factor of safety are printed out. Other similar gimmick routing variables appearing in the instructions are GSEARC, GIMTH, RU(1). Options associated with these variables are explained in the data coding instructions listed below.

The second type of input (gimmick) routing variable used in the program is referred to as a terminal routing gimmick. The function of this gimmick is to instruct the program that the end of a data set has been reached and to proceed to the next operation in the program or to read a subsequent data set. For example, assume the following x-y coordinate data sets are to be read:

```
         X           Y
      35.000      36.000
      70.000      10.000
     100.000      10.000
999
```

To terminate the reading of this data set, the x-coordinate is assigned an integer value of 999; this value is placed in the first three columns at the end of each data set as shown in the example above.

Data Coding Instructions

The following instructions illustrate the manner in which all problems are to be coded. This listing includes the type of line data, the field width of each variable, the column spaces allotted to each variable, whether the variable is an integer, real or alphanumeric, the default value of the variable and a definition of the input variations and general comments concerning the variable. Additionally, the value or range of values that may be assigned to each variable is listed under each variable name. The instructions are as follows:

COLUMNS	DATA TYPE	VARIABLE NAME	REMARKS

1. Input Output Data (3I5):

COLUMNS	DATA TYPE	VARIABLE NAME	REMARKS
1-5	Integer	IN	No Default Value 0-99999 Symbolic input number. This number designates which one of the various input devices in the computing system is to be utilized. The number may differ between computers and computing installations.
6-10	Integer	IOUT	No Default Value 0-99999 Symbolic output number. This number designates which one of the various output devices in the computing system is to be utilized. The number may differ between computers and computing installations.

11-15	Integer NOP		Default Value = 1 0-99999 The number of problems to be analyzed for each submission.

2. Problem Identification Data (A6, A74):

1-2	Alphanumeric	MONTH 0-12	No Default Value The month recorded as a number.
3-4	Alphanumeric	KDAY 0-31	No Default Value The day recorded as a number.
5-6	Alphanumeric	KYEAR 0-9	No Default Value The Year. Only the last two digits of the year are placed in these spaces.
7-80	Alphanumeric	IH(I)	No Default Value Alphanumeric information. These spaces are used to describe the problem. This information might be the project number, route designation, station number. The information may be arranged in any desired order in the allotted column spaces.

If all information on this record is omitted, a blank record must be inserted at this position in the data to insure the proper execution of the computer program.

3. Problem Control (F10.4, 6I5):

1-10	Real	UNITWW	Default Value = 0.0624 Unit weight of water in force per length cubed. All subsequent units of variables must conform to the units designated for the unit weight of water. If the default value is used, then leave these spaces blank.
11-15	Integer	NSLICE	Default Value = 76 Number of slices. A maximum of 598 slices may be specified. Normally, 76 slices are sufficient to obtain an accurate value of the safety factor. If the default value is used, then leave these spaces blank. The number of slices must be an even integer value.
16-20	Integer	MAXIT	Default Value = 15 1-00000 Number of iterations. In most problems, 15 iterations are sufficient to obtain convergence of the safety factor. If the default value is used, leave these spaces blank.
21-25	Integer	NMETH	No Default Value Routing gimmick code that specifies the limit equilibrium method.
		= 1	Hopkins' method

		= 2	Bishop's correct method
		= 3	Bishop's incorrect method
		= 5	Morgenstern and Price's method
		= 7	Perturbation method
26-30	Integer	NPULL	Default Value = 0 Routing gimmick code that controls the method of handling reinforcement forces.
		= 0 (or blank)	Reinforcement force is calculated based on Equation 2.1.
		= 1	Reinforcement force is calculated based on Equation 2.1, but reinforcement sheets are considered fixed at the right ends, i.e. $T_a = T_r$ in Equation 2.1.
31-35	Integer	NSLICER	No Default Value Number of slices for calculating pullout resistances T_a and T_p (see Equations 2.2 and 2.3).
36-40	Integer	NVERS	Default Value = 0 Routing gimmick code that controls a version of a certain limit equilibrium method. Non zero value of this gimmick code should be used only for Morgenstern and Price's method (NMETH = 5) and for Perturbation method (NMETH = 7). For all other methods this gimmick code does not have any effect and may be left blank.
		= 0 (or blank)	For Morgenstern and Price's method the function f(x) (see Equation 1.22) is constant: f(x) = 1; for Perturbation method the program uses Perturbation method #1 (Equation 2.33).
		= 1	For Morgenstern and Price's method f(x) is a half size function; for Perturbation method the program uses Perturbation method #2 (Equation 2.34).
		= 2	For Morgenstern and Price's method f(x) is a full size wave; for Perturbation method the program uses Perturbation Method #3 (Equation 2.35).

4. Gimmick Route Data (3I5):

1-5	Integer	GSEARC	Default Value = 0 Routing gimmick code that specifies the mode of solution. Two options are available:

			= 0 (or blank)	Option 1: Only one shear surface is analyzed.
			= 1	Option 2: A search analysis is performed. Coordinates of the search grid and x-, y-, and R(radius) - increments must be specified on Line 15 below.
6-10	Integer	GIMTH		Default Value = 0 Routing gimmick code that specifies the method of handling the thrust line. Two options are available:
			= 0 (or blank)	Option 1: x- and y-coordinates of the thrust must be supplied by the user in a format described below on Line 9.
			= 1	Option 2: A value of ETA, the ratio of the height of the interslice force above the shear surface to the total height of the side of a slice, must be input as described below on Line 9. Three ratio values (ETA, ETAT, and ETAC) can be specified as described below on Line 9.
11-15	Integer	GIMPO		Default Value = 0 Routing gimmick code that controls the amount of printed output. Three options are available:
			= 0 (or blank)	Option 1: All information is printed out. This includes the input data, a table of cross-sectional data, and a table of calculations corresponding to each iteration. This option should not be used when GSEARC = 1 (Line 2 above).
			= 1	Option 2: The same information printed out by Option 1 is given by Option 2, except the table of cross-sectional data is omitted. This option should not be used when GSEARC = 1 (Line 2 above).
			= 2	Option 3: A limited amount of information is printed out. This includes input data, a safety factor corresponding to each iteration, and a table of calculations corresponding to the last iteration.

If each gimmick routing code remain blank, then a blank record must be inserted at this position in the data to insure the proper execution of the program.

5. Properties of Soil Layers (5F10.3)

1-10	Real	CO(M)	No Default Value Cohesion (c′), of a layer in terms of effective stress or the undrained shear strength (SU), in force per length squared.
11-20	Real	PHI(NL)	No Default Value Angle of shearing resistance (ϕ'), in terms of effective stress or total stress in degrees.

21-30	Real	WT(NL)	No Default Value Total unit weight of a soil layer in force per length cubed.
31-40	Real	RU(M)	No Default Value Pore-pressure ratio. The pore pressure ratio also is used as an optional gimmick routing code. Therefore, the value entered for each soil layer dictates the method by which pore pressures are computed. Four options are available for computing pore pressures:
		< 1.0	Option 1: Pore pressures in a given soil layer are defined using the pore-pressure ratio (RU). To use this option, the value of RU for each layer must be some real number less than 1.0.
		= 1.5	Option 2: Pore pressures in a given layer are defined by a piezometric line. Whenever the value of RU is set equal to 1.5, x- and y-coordinates on the piezometric line for each soil layer where this option is to be used must be supplied in a format described under Line-type 10.
		= 2.5	Option 3: Pore pressures are defined by an infinitely sloping groundwater level and a flow net appropriate for this case. The pore pressure is calculated by multiplying the vertical head of water by the unit weight of water and the square of the cosine of the angle measured between a horizontal line and the groundwater level. The coded value of RU may be some real number between 2.0 and 2.9. However, the value of 2.5 is recommended. To use this option, only the value of RU corresponding to the first soil layer needs to be coded; the x- and y-coordinates of the groundwater level must be supplied in a format described below under Line-type 11.
		> 3.0	Option 4: Pore pressures are defined by assuming the groundwater level within a slope is a piezometric line. Pore pressures are computed by multiplying the vertical head of water by the unit weight of water. The coded value of RU may be some real number greater than 3.0 (use 3.5). This option may be used by setting the first RU value, which corresponds to the first soil layer, equal to some real number greater than 3.0: x- and y-coordinates must be supplied in a format described below under Line-type 12. Subsequent RU values, if any, remain blank. Options 1 and 2 may be intermixed. Options 1 or 2 may not be intermixed with Options 3 or 4. If only piezometric or water-table coordinates (Options 3 or 4) are used in a given problem, then only one value of RU is input on the first layer line (Columns 31-40, Record Number 5.1). subsequent RU values are not required.

41-50	Real	YEL	The corresponding y-coordinate (elevation) of each value (input in Columns 1 through 10 above) of undrained shear strength, SU. These values are input only for the problems involving variable undrained shear strength with depth or elevation. The entire soil unit is treated as one soil layer. In cases where the undrained shear strength is constant for a given layer, the y-coordinates need not be input. Where the undrained shear strength varies with depth, use of the y-coordinates and corresponding undrained shear values eliminates the need to subdivide a soil unit into many layers.

Record Number 5 as described above is repeated for each soil layer. A maximum of 25 soil layers may be described.

1-3	Integer	X( ) = 999	No Default Value Place the value of 999 in these columns and place at the end of the above data set. Reading of these data is ended when this value is encountered.

6. Groundline Coordinates (2F10.3):

1-10	Real	X( )	No Default Value x-coordinate of the groundline.
11-20	Real	Y( )	No Default Value y-coordinate of the groundline.

Record Number 6 is repeated for each set of x- and y-coordinates. The groundline coordinates are placed in sequence of increasing x values. A maximum of 25 sets of x- and y-coordinates may be used to describe the groundline. The groundline should extend beyond each end of the shear surface. Where water masses (lakes or pools) rest against the upstream or downstream slopes, the coordinates of the lake or pool should be input as groundline coordinates. Bodies of water are assumed to be soil layers having no shear strength but having unit weights.

1-3	Integer	X( ) = 999	No Default Value Place the value 999 in these columns and place at the end of the above data set.

7. Soil Layer Boundaries Coordinates (2F10.3):

1-10	Real	XLS( )	No Default Value x-coordinate of a layer boundary.
11-20	Real	YLS( )	No Default Value y-coordinate of a layer boundary.

Repeat Record Number 7 for each set of x- and y-coordinates of each layer boundary; repeat these cards for subsequent layers. The layer coordinate records are placed in a sequence of increasing x-coordinates. A maximum of 25 sets of x- and y-coordinates may be used to describe each layer boundary. Each layer boundary should extend beyond each end of the shear surface. The sequence of coding groundline and layer line data is from the top of the profile to the bottom of the profile. A maximum of 25 layer boundaries may be used.

1-3	Integer	XLS( )	No Default Value Place the value 999 in these columns and place after each layer line data set.

8. Shear Surface Coordinate Card (2F10.3):

1-10	Real	XF( )	No Default Value x-coordinate of shear surface.
11-20	Real	YF( )	No Default Value y-coordinate of shear surface.

Repeat Record Number 8 for each set of x- and y-coordinates of the shear surface. A maximum of 25 sets of x- and y-coordinates may be used to describe the shear surface. IF GSEARC = 1, do not input these coordinates.

1-3	Integer	XF( ) = 999	No Default Value Place the value 999 in these columns and place this card at the end of the above data set.

9. Thrust Line Coordinate Card (2F10.3):

If the optional routing gimmick code, GIMTH, on Record Number 4 is set equal to 0, x- and y-coordinates of the thrust line must be supplied. These coordinates are coded as follows:

1-10	Real	XTH( )	No Default Value x-coordinate of thrust line.
11-20	Real	YTH( )	No Default Value y-coordinate of thrust line.

Repeat Record Number 9 for each set of x- and y-coordinates of the thrust line. A maximum of 25 sets of x-and y-coordinates may be used to describe the thrust line.

1-3	Integer	XTH( ) = 999	No Default Value Place the value 999 in these columns and place this card at the end of the above data set.

If GIMTH is set equal to 1 on Record Number 4, the above lines are omitted and a value of ETA must be supplied. For this case, Record Number 9 is coded as follows:

1-10	Real	ETA	No Default Value The ratio of the distance between the shear surface and the point of action of the interslice forces to the height of the side of a slice. Generally, ETA should be

assigned any value lying in the range 0.25 to 0.65. Normally, a value of 0.33 is used.

The user has an additional option for defining the thrust line. If c′ = 0, then the ETA above should be set equal to about 0.33. If c′ > 0, then the line of thrust in a compression zone (passive condition) may be located, perhaps slightly above the third-point of the slice while in a tensile zone (active condition) the thrust line is located slightly below the third-point of the slices. To exercise this option, ETA remains blank and the following values are input:

11-20	Real	ETAT	No Default Value The ratio of the distance between the shear surface and the point of action of the interslice forces to the height of the side of the slice. The value of ETAT in the tension zone (active condition) is usually slightly smaller than 0.33.
21-30	Real	ETAC	No Default Value The ratio of the distance between the shear surface and the point of action of the interslice forces to the height of the side of a slice. The values of ETAC in the compression zone (active condition) is usually slightly larger than 0.33.

If x-and y-coordinates are used to define the thrust line, then ETA, ETAT, and ETAC are not input. If ETA is used to define the thrust line, then ETAT and ETAC are not input. If ETAT and ETAC (both values must be entered) are used, then ETA is not input.

10. Piezometric Coordinates (2F10.3):

If the pore-pressure ratio (RU), as input on Record Number 5 above (Columns 31-40), is set equal to a value greater than 2.0, piezometric coordinates are omitted. The user should go to Record Number 11 and enter groundwater coordinates. If the pore-pressure ratio (RU), of a given soil layer is less than 1.0 (the actual value of the pore pressure ratio is entered on Record Number 5 above, Columns 31-40 for this case), then the pore pressures in the given layer are calculated using the actual value of RU. However, if the pore-pressure ratio is set equal to a value greater than 1.0 and less than 1.9 (normally, a value of 1.5 is used), then x- and y-piezometric coordinates for the given soil layer must be supplied as described below:

1-10	Real	XP( )	No Default Value x- coordinate of the piezometric line.
11-20	Real	YP( )	No Default Value y-coordinate of the piezometric line.

Repeat Record Number 10 for each set of x- and y-coordinates and for each soil layer where the RU-value is set equal to 1.5. A maximum of 25 sets of x- and y-coordinates may be used to describe each piezometric level corresponding to each layer.

1-3	Integer	XP( ) = 999	No Default Value Place the value 999 in these columns and place this card after each piezometric data set.

In the computer program it is permissible to specify that the pore pressures in certain designated soil layers be computed using the actual pore- pressure ratios while in other layers the pore pressures may be computed using piezometric coordinates. The two options may be intermixed.

11. Groundwater Level coordinates (2F10.3):

If the first RU-value of the first soil layer on Record Number 5 above, columns 31-40, is set equal to a real number between 2.0 and 2.9 (use 2.5) or to a real number greater than 3.0 (use 3.5), x- and y-coordinates of the groundwater level must be supplied as described as follows:

1-10	Real	XWT( )	No Default Value x-coordinate of groundwater level.
11-20	Real	YWT( )	No Default Value y-coordinate of groundwater level.

When RU-value is set equal to 2.5, pore pressures are computed assuming an infinitely sloping water table.

Repeat Record Number 11 above for each set of x- and y-coordinates of the groundwater level. A maximum of 25 sets of x- and y-coordinates may be used to describe the groundwater level.

1-3	Integer	XWT( ) = 999	No Default Value Place the value 999 in these columns and place this card after the groundwater level data set.

When RU is set equal to a value greater than 2.0, piezometric coordinates are omitted.

12. End Boundary Loads (4F10.0):

1-10	Real	EA	Default Value = 0.0 Value of the horizontal force acting on the boundary of the uphill side of the potentially unstable soil mass.
11-20	Real	EB	Default Value = 0.0 Value of the horizontal force acting on the boundary of the downhill side of the potentially unstable soil mass.
21-30	Real	TA	Default Value = 0.0 Value of the vertical force acting on the boundary of the uphill side of the potentially unstable soil mass.
31-40	Real	TB	Default Value = 0.0 Value of the vertical force acting on the boundary of

the downhill side of the potentially unstable soil mass.

If all values on Record Number 12 remain blank, then a blank record must be inserted at this position in the data to insure the proper execution of the program.

13. External Vertical Distributed Loads (2F10.0):

1-10	Real	x( )	No Default Value x-coordinates of vertical distributed loads.
11-20	Real	q( )	No Default Value Magnitude of vertical distributed loads.

Repeat Record Number 13 for each vertical distributed load. A maximum of 25 vertical distributed loads is allowed. If there are no vertical distributed loads, then a blank record must be inserted at this position.

1-3	Integer	x( ) = 999	No Default Value Place the value 999 in these columns and place this card after the vertical distributed loads.

14. Earthquake Forces (2F10.3):

1-10	Real	SEMC	No Default Value Seismic coefficient. Values of seismic coefficient can be obtained elsewhere.
11-20	Real	HQ	No Default Value Ratio of the distance between the shear surface and the point of action of the earthquake force on the side of the slice to the height of the side of the slice. This parameter allows the user to select the point of action of the earthquake force on the side of the slice. Normally, a value of 0.5 is assumed. Forces acting a mid-height of each slice are oftentimes assumed.

A pseudo-static method (a traditional approach) is used to solve for the earthquake forces. The user should be aware of the shortcomings of this approach.

15. Predetermined Trial Centers (7F10.3):

If GSEARC is set equal to 1, Record Number 4 (Columns 1-5), then x- and y-coordinates and x- and y-increments of the search grid must be specified. Additionally, the radius increment must be specified. These coordinates and increments are input as shown below:

1-10	Real	XSTART	No Default Value x-coordinate of the left upper corner of search grid.
11-20	Real	YSTART	No Default Value y-coordinate of the left upper corner of search grid.
21-30	Real	XFIN	No Default Value

			x-coordinate of the right bottom corner of search grid.
31-40	Real	YFIN	No Default Value y-coordinate of the right bottom corner of search grid.
41-50	Real	XDEL	Default Value = 5 x-increment of search grid. The x-increment should be selected such that, when the difference (XFIN - XSTART) is divided by the x-increment, the result is an integer number.
51-60	Real	YDEL	Default Value = 5 y-increment of the search grid. The y-increment should be selected such that, when the difference (YSTART - YFIN) is divided by the y-increment, the result is an integer number.
61-70	Real	RDEL	Default Value = 5 Radius increment.

16. Starting Coordinates of Radius (4F10.3):

To start the search routine (if GSEARC = 1), the starting coordinates of the radius must be specified. The x- and y-starting coordinates can be located on the groundline or any location below the groundline. The input is as follows:

1-10	Real	XE	No Default Value x-coordinate of radius starting coordinate.
11-20	Real	YE	No Default Value y-coordinate of radius starting coordinate.

There are certain situations where the user may want to solve for the safety factor of one shear surface. The user may want to specify the radius length. In this case, the x-and y-coordinates of the center of the circle are entered on Record Number 15 (x- and y-increments and the radius increment and the XE and YE coordinates above need not be entered) and the radius length is input as shown below:

21-30	Real	ROI	No Default Value Radius length for a given circle.
31-40	Real	DVS	No Default Value A vertical distance that controls shallow failures. For a given circle, the slice having the maximum height, ZMAX, is computed and compared to the value of DVS. IF ZMAX is less than DVS, then the factor of safety is not computed.

In the case of a retaining wall with a vertical face, the user may not need to input both XE and YE, but rather XE may be left blank.

17. Tension Crack Parameters (2F10.1, I5, F10.2):

Two options are available for performing stability problems that may involve tension cracks. A typical problem where tension cracks may develop consists of a compacted embankment on a soft foundation. The embankment consists of stiff compacted soil and is brittle. Failure strain may be small. The foundation consists of soft soil that may strain considerably before failure. If sufficient settlement occurs in the soft foundation, the embankment soil will fail much sooner than the foundation material. Consequently, a tension crack will develop in the stiff embankment soils. The first option available in the computer program consists of specifying a variable crack. In this case, the computer program computes the depth of crack for each trial shear surface based on the assumption that the horizontal effective stress is zero at the bottom of the crack and according to the Equation (4.13). To invoke this first option, a value of DTC is input as follows:

1-10	Real	DTC	Default Value = Variable Tension Crack Iteration is performed until a compatible tension crack depth (DTC), and factor of safety (F), are determined for each trial shear surface. Generally, insert a value of 1.0 to invoke this procedure. DTCFIX below is set equal to 0.0. A layer number must be input.

If the depth of tension crack is known or estimated, then the tension crack depth can be entered (Option 2) as follows:

11-20	Real	DTCFIX	Default Value = Fixed Tension Crack Depth All trial shear surfaces have the same depth of tension crack. DTC above is set equal to 0.0.

When either option is used, the layer of soil in which the tension crack will develop must be specified as shown below:

21-25	Integer	NSOLY	Default Value = 1 This parameter designates the soil layer in which the tension crack will develop. Soil layers are counted (numbered) downward from the groundline.

If the groundwater table is located in the soil layer where the tension crack is specified, then an estimate of the pore-pressure ratio in the soil can be made.

The pore-pressure ratio is input as follows:

26-35	Real	RURT	No Default Value Pore-pressure ratio in the soil layer where the tension crack is specified.

If a tension crack is not specified, then a blank must be inserted at this position in the data.

18. Reinforcement Geometry and Strength Properties (6F10.3):

1-10	Real	GSL( )	No Default Value Length of geosynthetic sheet.
11-20	Real	X2( )	No Default Value x-coordinate of the right end of geosynthetic sheet.
21-30	Real	YR( )	No Default Value Elevation of the geosynthetic sheet.
31-40	Real	GSR1( )	No Default Value Soil-fabric interacion coefficient for soil above geosynthetic sheet.
41-50	Real	GSR2( )	No Default Value Soil-fabric interacion coefficient for soil below geosynthetic sheet.
51-60	Real	RSTREN( )	No Default Value Tensile strength of geosynthetic sheets.

Repeat Record Number 18 for each geosynthetic sheet. A maximum of 100 geosynthetic sheets may be used.

1-3	Real	GSL( )	No Default Value Place the value 999 in these columns and place this card at the end of the above data set.

19. Initial Approximations for Morgenstern and Price's Method.

1-10	Real	FOLD	No Default Value Initial approximation of the safety factor.
11-20	Real	LAMOLD	No Default Value Initial approximation of λ.
21-30	Real	F	No Default Value First approximation (F_1) of the safety factor.
31-40	Real	LAM	No Default Value First approximation (λ_1) of λ.

Data on Record Number 19 are only important if Morgenstern and Price's method is used. For any other method, these data need not be entered and the corresponding line may remain blank.

APPENDIX 3

Derivations Concerning Perturbation Methods

As it was mentioned in the section 2.2.5, all Perturbation methods are described by the same system of nonlinear algebraic equations.

$$a_{11}\,\lambda + a_{12}\,\mu + a_{13}\,\lambda F + a_{14}\,\mu F + a_{15}\,F - b_1 = 0$$

$$a_{21}\,\lambda + a_{22}\,\mu + a_{23}\,\lambda F + a_{24}\,\mu F + a_{25}\,F - b_2 = 0$$

$$a_{31}\,\lambda + a_{32}\,\mu + a_{33}\,\lambda F + a_{34}\,\mu F + a_{35}\,F - b_3 = 0$$

For Perturbation methods No. 1 and No. 3 values of a_{ij}, b_i are as followes:

$$a_{11} = \Sigma\, T_i \sin\theta \cos\theta \tan\phi' - \int_{x_a}^{x_b} a_2 \tan\phi'\, dx$$

$$a_{12} = \Sigma\, T_i \sin^2\theta \tan\phi' - \int_{x_a}^{x_b} a_2 \tan\theta \tan\phi'\, dx$$

$$a_{13} = \Sigma\, T_i \sin^2\theta - \int_{x_a}^{x_b} a_2 \tan\theta\, dx$$

$$a_{14} = \Sigma\, T_i \sin^2\theta \tan\theta - \int_{x_a}^{x_b} a_2 \tan^2\theta\, dx$$

$$a_{15} = \int_{x_a}^{x_b} q_x\, dx - \Sigma\, T_i$$

$$b_1 = \int_{x_a}^{x_b} (c' - u \tan\phi')\, dx;$$

$$a_{21} = -\,a_{12};$$

$$a_{22} = \int_{x_a}^{x_b} a_2 \tan^2\theta \tan\phi' \, dx - \Sigma\, T_i \sin^2\theta \tan\theta \tan\phi';$$

$$a_{23} = \Sigma\, T_i \sin\theta \cos\theta - \int_{x_a}^{x_b} a_2 \, dx;$$

$$a_{24} = a_{13};$$

$$a_{25} = \int_{x_a}^{x_b} (\gamma_{av}\, h + q_y) \, dx;$$

$$b_2 = - \int_{x_a}^{x_b} (c' - u \tan\phi') \tan\theta \, dx;$$

$$a_{31} = \int_{x_a}^{x_b} a_2 (y - x \tan\theta) \tan\phi' \, dx -$$

$$\Sigma\, T_i \sin\theta \cos\theta \,(y - x \tan\theta) \tan\phi'$$

$$a_{32} = \int_{x_a}^{x_b} a_2 \tan\theta \tan\phi' (y - x \tan\theta) \, dx -$$

$$\Sigma\, T_i \sin^2\theta \tan\phi' (y - x \tan\theta)$$

$$a_{33} = \int_{x_a}^{x_b} a_2 (y \tan\theta + x) \, dx - \Sigma\, T_i \sin\theta \cos\theta \,(y \tan\theta + x);$$

$$a_{34} = \int_{x_a}^{x_b} a_2 \tan\theta\,(y\tan\theta + x)\,dx - \Sigma\, T_i \sin^2\theta\,(y\tan\theta + x);$$

$$a_{35} = -\int_{x_a}^{x_b} y_q\, q_x\, dx + \Sigma\, T_i\, y_i - \int_{x_a}^{x_b} x\,(\gamma_{av} h + q_y)\, dx;$$

$$b_3 = -\int_{x_a}^{x_b} (c' - u\tan\phi')\,(y - x\tan\theta)\, dx.$$

where for Perturbation Method No. 1

$$a_2 = (\gamma_{av} h + q_y)\cos^2\theta + q_x \sin\theta\cos\theta$$

and for Perturbation Method No. 3

$$a_2 = \sigma^*(x)$$

where $\sigma^*(x)$ is a normal stress distribution in a corresponding unreinforced case obtained according to Hopkins' method.

For Perturbation Method No. 2

$$a_{11} = -\int_{x_a}^{x_b} \sigma^* \tan\phi'\, dx$$

$$a_{12} = \Sigma\, T_i \sin\theta\cos\theta\tan\phi';$$

$$a_{13} = -\int_{x_a}^{x_b} \sigma^* \tan\theta\, dx$$

$$a_{14} = \Sigma\, T_i \sin\theta\cos\theta\tan\theta;$$

$$a_{15} = \int_{x_a}^{x_b} q_x \, dx - \Sigma \, T_i$$

$$b_1 = \int_{x_a}^{x_b} (c' - u \tan \phi') \, dx$$

$$a_{21} = \int_{x_a}^{x_b} \sigma^* \tan \phi' \tan \theta \, dx$$

$$a_{22} = - \Sigma \, T_i \sin \theta \cos \theta \tan \phi' \tan \theta$$

$$a_{23} = - \int_{x_a}^{x_b} \sigma^* \, dx$$

$$a_{24} = \Sigma \, T_i \sin \theta \cos \theta$$

$$a_{25} = \int_{x_a}^{x_b} (\gamma_{av} \, h + q_y) \, dx$$

$$b_2 = - \int_{x_a}^{x_b} (c' - u \tan \phi') \tan \theta \, dx$$

$$a_{31} = \int_{x_a}^{x_b} \sigma^* \tan\phi' \, (y - x \tan \theta) \, dx$$

$$a_{32} = - \Sigma \, T_i \sin \theta \cos \theta \tan\phi' \, (y - x \tan \theta)$$

$$a_{33} = \int_{x_a}^{x_b} \sigma^* \ (y \tan \theta + x) \ dx$$

$$a_{34} = - \Sigma \ T_i \sin \theta \cos \theta \ (y \tan \theta + x)$$

$$a_{35} = - \int_{x_a}^{x_b} y_q \ q_x \ dx + \Sigma \ T_i \ y_i - \int_{x_a}^{x_b} x \ (\gamma_{av} \ h + q_y) \ dx$$

$$b_3 = - \int_{x_a}^{x_b} (c' - u \tan \phi') \ (y - x \tan \theta) \ dx .$$

where $\sigma^*(x)$ is the same as in Perturbation Method No. 3.

APPENDIX 4

Input Data for the Examples

1. Tensar Manual Example (Figure 3.1).

```
    5     6     1
021192    TENSAR MANUAL EXAMPLE, WRIGHT AND DUNCAN (1991)
    0.0000    76    15    1     0     5     0
    1     1     2
    0.0000    32.0000    0.1200    0.0000    0.0000
999             0.          0.          0.          0.
     0.000    48.000
   200.000    48.000
   238.000    10.000
   240.000     9.500
   600.000     9.500
999             0.
     0.000     9.500
   600.000     9.500
999             0.
      0.33      0.00      0.00
   0.    0.    0.    0.
         0.          0.
999             0.
      0.00      0.00
   240.000   140.000   320.000    60.000    10.000    10.000    50.00
   238.000    10.000    00.000     5.000
      0.00      0.00    0        0.00
    29.200   237.330    10.670     0.900     0.900     1.00
    29.200   236.000    12.000     0.900     0.900     1.00
    29.200   234.670    13.330     0.900     0.900     1.00
    29.200   233.340    14.660     0.900     0.900     1.00
    29.200   232.000    16.000     0.900     0.900     1.00
    29.200   230.680    17.330     0.900     0.900     1.00
    29.200   229.340    18.670     0.900     0.900     1.00
    29.200   228.010    20.000     0.900     0.900     1.00
    29.200   226.680    21.330     0.900     0.900     1.00
    29.200   225.350    22.670     0.900     0.900     1.00
    26.500   224.020    24.000     0.900     0.900     1.00
    26.500   221.350    26.670     0.900     0.900     1.00
    26.500   218.680    29.330     0.900     0.900     1.00
    26.500   216.000    32.000     0.900     0.900     1.00
    26.500   213.340    34.670     0.900     0.900     1.00
    23.900   209.340    38.670     0.900     0.900     1.00
    23.900   203.340    44.670     0.900     0.900     1.00
999             0.          0.
2.0       -0.5       2.1       -0.4
```

2. Generic Example. Tensar Simplified Design Method. β = 30°.

```
    5     6     1
021392    TENSAR EXAMPLE BETA = 30 DEGREES
```

```
      0.0000    76    15     5    1    10    1
      1     1     2
      0.0000    36.0000    0.1250    0.0000    0.0000
999                 0.         0.        0.         0.
  -1500.000      49.370
    200.000      49.370
    268.230      10.000
    269.100       9.500
   1600.000       9.500
999                 0.
  -1500.000       9.500
   1600.000       9.500
999                 0.
       0.33       0.00       0.00
    0.    0.    0.    0.
     -1500.         0.
       140.         0.
       140.      0.819
       200.      0.819
       200.         0.
     268.23         0.
     269.10         0.
      1600.         0.
999                 0.
       0.00       0.00
    303.500    236.000    303.500    236.000    2.000    2.000    0.00
    268.230     10.000    228.000      5.0
       0.00       0.00    0         0.00
     23.000    222.000     36.000    0.9    0.9    2.02
     24.000    252.000     18.670    0.9    0.9    2.02
999                 0.         0.
      1.7     -0.5         1.6      -0.4
```

3. Generic Example. Tensar Simplified Design Method. β = 45°.

```
      5     6     1
021192    TENSAR EXAMPLE BETA = 45 DEGREES
      0..0000    76    15     7    1     5    2
      1     1     2
      0.0000    36.0000    0.1250    0.0000    0.0000
999                 0.         0.        0.         0.
      0.000      49.370
    200.000      49.370
    239.370      10.000
    240.000       9.500
    600.000       9.500
999                 0.
      0.000       9.500
    600.000       9.500
999                 0.
```

```
       0.33       0.00       0.00
   0.    0.    0.    0.
         0.          0.
       140.          0.
       140.        0.819
       200.        0.819
       200.          0.
     239.37          0.
     240.00          0.
       600.          0.
999                  0.
       0.00       0.00
    312.500    157.300   312.500   157.300     2.000     2.000     50.00
    239.370     10.000   164.200     5.000
       0.00       0.00   0       0.00
     29.600    203.000    46.000     0.900     0.900    2.02
     29.600    208.000    41.330     0.900     0.900    2.02
     29.600    213.000    36.670     0.900     0.900    2.02
     29.600    217.000    32.000     0.900     0.900    2.02
     29.600    222.000    27.330     0.900     0.900    2.02
     32.500    224.500    24.670     0.900     0.900    2.02
     32.000    227.000    22.000     0.900     0.900    2.02
     32.500    230.000    19.330     0.900     0.900    2.02
     32.300    232.500    16.670     0.900     0.900    2.02
     32.200    235.000    14.000     0.900     0.900    2.02
     32.800    238.000    11.330     0.900     0.900    2.02
999                  0.        0.
2.0       -0.5        2.1       -0.4
```

4. Generic Example. Tensar Simplified Design Method. $\beta = 60°$.

```
    5     6     1
022592   TENSAR GENERIC EXAMPLE,BETA = 60 DEGREES
    0.0000    76    15     5     1     3     1
    1     1     2
    0.0000    36.0000     0.1250     0.0000     0.0000
999                0.         0.         0.         0.
     0.000     49.370
   200.000     49.370
   222.730     10.000
   223.000      9.500
   600.000      9.500
999                0.
     0.000      9.500
   600.000      9.500
999                0.
      0.33       0.00       0.00
0.33
   0.    0.    0.    0.
         0.          0.
```

```
      140.         0.
      140.      0.819
      200.      0.819
      200.         0.
    222.73         0.
    223.00         0.
      600.         0.
999                0.
      0.00       0.00
   272.000    150.700    272.000    150.700       2.000       2.000       0.00
   222.730     10.000    148.200      5.000
      0.00       0.00    0         0.00
    30.700      203.7      44.67       0.90        0.90        2.02
    31.000      206.0      38.67       0.90        0.90        2.02
    31.000      207.7      36.00       0.90        0.90        2.02
    31.000      209.0      33.33       0.90        0.90        2.02
    31.000      210.7      30.67       0.90        0.90        2.02
    31.000      212.1      28.00       0.90        0.90        2.02
    31.000      213.6      25.33       0.90        0.90        2.02
    31.500      214.7      24.00       0.90        0.90        2.02
    31.500      215.5      22.67       0.90        0.90        2.02
    31.500      216.0      21.33       0.90        0.90        2.02
    31.500      217.0      20.00       0.90        0.90        2.02
    31.500      217.8      18.67       0.90        0.90        2.02
    31.500      218.5      17.33       0.90        0.90        2.02
    31.500      219.5      16.00       0.90        0.90        2.02
    31.500      220.0      14.67       0.90        0.90        2.02
    31.500      220.9      13.33       0.90        0.90        2.02
    31.500      221.5      12.00       0.90        0.90        2.02
    31.500      222.2      10.67       0.90        0.90        2.02
999                0.         0.
2.0       1.0       2.1       0.9
```

5. Generic Example. Tensar Simplified Design Method. $\beta = 80°$.

```
    5    6    1
022692   TENSAR EXAMPLE BETA = 80 DEGREES
    0.0000    76    15     5    1    1    1
    1    1    2
    0.0000   36.0000    0.1250    0.0000    0.0000
999                0.        0.        0.        0.
     0.000    49.370
   200.000    49.370
   206.940    10.000
   207.500     9.500
   600.000     9.500
999                0.
     0.000     9.500
   600.000     9.500
999                0.
```

```
      0.33      0.00      0.00
   0.   0.   0.   0.
        0.        0.
      140.        0.
      140.      0.819
      200.      0.819
      200.        0.
    206.94        0.
    207.50        0.
      600.        0.
999               0.
      0.00      0.00
   270.800    73.800   270.800    73.800    20.000    20.000    0.00
   206.940    10.000    90.100     5.000
      0.00      0.00   0        0.00
    29.640     200.8     45.00     0.900     0.900     2.020
    29.640     201.6     40.30     0.900     0.900     2.020
    29.640     202.1     37.00     0.900     0.900     2.020
    29.640     202.7     34.30     0.900     0.900     2.020
    29.640     203.0     32.30     0.900     0.900     2.020
    29.640     203.2     31.00     0.900     0.900     2.020
    29.640     203.5     29.70     0.900     0.900     2.020
    29.640     203.7     28.30     0.900     0.900     2.020
    29.640     203.9     27.00     0.900     0.900     2.020
    29.640     204.1     25.70     0.900     0.900     2.020
    29.640     204.4     24.30     0.900     0.900     2.020
    29.640     204.6     23.00     0.900     0.900     2.020
    29.640     204.8     21.70     0.900     0.900     2.020
    29.640     205.0     21.00     0.900     0.900     2.020
    29.640     205.1     20.30     0.900     0.900     2.020
    29.640     205.2     19.70     0.900     0.900     2.020
    29.640     205.3     19.00     0.900     0.900     2.020
    29.640     205.4     18.30     0.900     0.900     2.020
    29.640     205.6     17.70     0.900     0.900     2.020
    29.640     205.6     17.00     0.900     0.900     2.020
    29.640     205.8     16.30     0.900     0.900     2.020
    29.640     205.9     15.70     0.900     0.900     2.020
    29.640     206.0     15.00     0.900     0.900     2.020
    29.640     206.1     14.30     0.900     0.900     2.020
    29.640     206.2     13.70     0.900     0.900     2.020
    29.640     206.4     13.00     0.900     0.900     2.020
    29.640     206.5     12.30     0.900     0.900     2.020
    29.640     206.6     11.70     0.900     0.900     2.020
    29.640     206.7     11.00     0.900     0.900     2.020
    29.640     206.8     10.30     0.900     0.900     2.020
999               0.        0.
    2.2       -0.5      2.10     -0.55
```

6. Generic Example. Tensar Rigorous Design Method. $\beta = 30^\circ$.

```
      5     6     1
021392   TENSAR EXAMPLE BETA = 30 DEGREES RIGOROUS DESIGN
    0.0000   76   15    1    1   10
    1    1    2
    0.0000   36.0000    0.1250    0.0000    0.0000
999             0.         0.         0.         0.
 -1500.000    49.370
   200.000    49.370
   268.230    10.000
   269.100     9.500
  1600.000     9.500
999             0.
 -1500.000     9.500
  1600.000     9.500
999             0.
      0.33      0.00       0.00
   0.   0.   0.   0.
    -1500.         0.
      140.         0.
      140.     0.819
      200.     0.819
      200.         0.
    268.23         0.
    269.10         0.
     1600.         0.
999             0.
      0.00      0.00
   270.000   105.000   280.000    95.000     2.000     2.000    50.00
   268.230    10.000    00.000     5.0
      0.00      0.00    0       0.00
    15.000   223.000    36.010     0.900     0.900     2.020
    21.000   254.000    17.650     0.900     0.900     2.020
999             0.         0.
```

7. Generic Example. Tensar Rigorous Design Method. β = 45°.

```
    5     6     1
021192   TENSAR EXAMPLE BETA = 45 DEGREES RIGOROUS DESIGN
    0.0000   76   15    1    1    5
    1    1    2
    0.0000   36.0000    0.1250    0.0000    0.0000
999             0.         0.         0.         0.
  -500.000    49.370
   200.000    49.370
   239.370    10.000
   240.000     9.500
  1600.000     9.500
999             0.
  -500.000     9.500
  1600.000     9.500
```

```
999                0.
      0.33        0.00        0.00
   0.    0.    0.    0.
    -500.          0.
     140.          0.
     140.        0.819
     200.        0.819
     200.          0.
   239.37          0.
   240.00          0.
    1600.          0.
999                0.
      0.00        0.00
   240.000    100.000    258.000    80.000     2.000     2.000    50.00
   239.370     10.000     00.000     5.000
      0.00        0.00     0       0.00
    26.000    204.500     45.430     0.9      0.9       2.02
    27.000    211.000     38.820     0.9      0.9       2.02
    28.200    216.200     33.730     0.9      0.9       2.02
    29.000    220.000     29.460     0.9      0.9       2.02
    29.500    223.500     25.720     0.9      0.9       2.02
    30.300    227.000     22.350     0.9      0.9       2.02
    31.000    230.000     19.270     0.9      0.9       2.02
    31.500    233.000     16.440     0.9      0.9       2.02
    32.400    235.700     13.760     0.9      0.9       2.02
    32.800    237.800     11.230     0.9      0.9       2.02
999                0.         0.
1.0        0.0        0.9        -0.3
```

8. Generic Example. Tensar Rigorous Design Method. β = 60°.

```
    5     6     1
022592   TENSAR EXAMPLE BETA = 60 DEGREES RIGORODS DESIGN
    0.0000    76    15     1     1     3
    1     1     2
    0.0000    36.0000    0.1250    0.0000    0.0000
999                0.         0.         0.         0.
  -100.000     49.370
   200.000     49.370
   222.730     10.000
   223.000     09.500
  1600.000     09.500
999                0.
  -100.000      9.500
  1600.000      9.500
999                0.
      0.33        0.00        0.00
   0.    0.    0.    0.
    -100.          0.
     140.          0.
```

```
      140.      0.819
      200.      0.819
      200.         0.
    222.73         0.
    222.00         0.
     1600.         0.
999                0.
      0.00       0.00
   223.000     75.000   233.000    65.000      2.000      2.000     50.00
   222.730     10.000    00.000     5.000
      0.00       0.00    0       0.00
    30.500      203.0     43.85     0.900      0.900      2.020
    30.500      205.5     39.77     0.900      0.900      2.020
    30.500      207.0     36.39     0.900      0.900      2.020
    31.000      209.0     33.45     0.900      0.900      2.020
    31.000      210.5     30.82     0.900      0.900      2.020
    31.000      212.0     28.42     0.900      0.900      2.020
    31.000      213.0     26.20     0.900      0.900      2.020
    31.000      214.0     24.13     0.900      0.900      2.020
    31.000      215.0     22.18     0.900      0.900      2.020
    31.000      216.0     20.33     0.900      0.900      2.020
    31.300      217.5     18.56     0.900      0.900      2.020
    31.300      218.3     16.87     0.900      0.900      2.020
    31.300      219.3     15.25     0.900      0.900      2.020
    31.400      220.2     13.69     0.900      0.900      2.020
    31.500      221.0     12.18     0.900      0.900      2.020
    31.500      222.0     10.72     0.900      0.900      2.020
999                0.        0.
2.0        1.0        2.1       0.9
```

9. Generic Example. Tensar Rigorous Design Method. β = 80°.

```
    5    6    1
022692   TENSAR EXAMPLE BETA = 80 DEGREES RIGOROUS DESIGN
    0.0000    76    15    1    1    1    0
    1    1    2
    0.0000   36.0000    0.1250    0.0000    0.0000
999                0.        0.        0.        0.
     0.000    49.370
   200.000    49.370
   206.940    10.000
   207.500     9.500
   600.000     9.500
999                0.
     0.000     9.500
   600.000     9.500
999                0.
      0.33      0.00      0.00
   0.    0.    0.    0.
         0.         0.
```

```
       140.         0.
       140.     0.819
       200.     0.819
       200.         0.
     206.94         0.
     207.50         0.
     600.00         0.
999                 0.
       0.00      0.00
   210.000    140.000    290.000     60.000     20.000     20.000      0.00
   206.940     10.000     00.000      5.000
      0.00       0.00    0       0.00
    29.640      200.3      47.28      0.900      0.900      2.020
    29.640      201.0      43.70      0.900      0.900      2.020
    29.640      201.6      40.80      0.900      0.900      2.020
    29.640      202.0      38.31      0.900      0.900      2.020
    29.640      202.3      36.10      0.900      0.900      2.020
    29.640      202.8      34.09      0.900      0.900      2.020
    29.640      203.0      32.24      0.900      0.900      2.020
    29.640      203.4      30.52      0.900      0.900      2.020
    29.640      203.6      28.90      0.900      0.900      2.020
    29.640      203.9      27.37      0.900      0.900      2.020
    29.640      204.1      25.91      0.900      0.900      2.020
    29.640      204.4      24.52      0.900      0.900      2.020
    29.640      204.7      23.18      0.900      0.900      2.020
    29.640      204.9      21.89      0.900      0.900      2.020
    29.640      205.0      20.65      0.900      0.900      2.020
    29.640      205.2      19.45      0.900      0.900      2.020
    29.640      205.5      18.29      0.900      0.900      2.020
    29.640      205.8      17.16      0.900      0.900      2.020
    29.640      205.9      16.06      0.900      0.900      2.020
    29.640      206.0      14.99      0.900      0.900      2.020
    29.640      206.1      13.95      0.900      0.900      2.020
    29.640      206.3      12.93      0.900      0.900      2.020
    29.640      206.6      11.93      0.900      0.900      2.020
    29.640      206.8      10.96      0.900      0.900      2.020
    29.640      206.9      10.48      0.900      0.900      2.020
999                0.         0.
```

10. Billiard and Wu (1991) Example.

```
    5     6     1
042892   WU EXAMPLE (GEOS.'91,P.537-548)) CORRECT
   0.0000    76   15     3     0     1
   1    1     2
   0.0000   39.0000    0.0950    0.0000    0.0000
  10.0000    0.0000    0.0950    0.0000    0.0000
999               0.        0.        0.        0.
 -500.000     5.200
  106.800     5.200
```

```
    106.800     -0.200
    600.000     -0.200
999               0.
   -500.000      5.200
    100.000      5.200
    100.000     -0.200
    600.000     -0.200
999               0.
   -500.000     -0.200
    600.000     -0.200
999               0.
       0.33       0.00       0.00
    0.   0.    0.    0.
      -500.       0.
       100.       0.
       100.     2.855
      106.8     2.855
      106.8       0.
      600.0       0.
999               0.
       0.00       0.00
    112.600      7.000   112.600      7.000      2.000      2.000     50.00
      0.000     -0.100     0.000      0.200
       0.00       0.00     0        0.00
      6.800     106.80      5.19        0.9        0.9       0.42
      4.800     106.80      4.00        0.9        0.9       0.42
      1.500     106.80      3.83        0.9        0.9       0.42
      4.300     106.80      2.90        0.9        0.9       0.42
      1.500     106.80      2.73        0.9        0.9       0.42
      3.800     106.80      1.80        0.9        0.9       0.42
      1.500     106.80      1.63        0.9        0.9       0.42
      3.400     106.80      0.85        0.9        0.9       0.42
      1.500     106.80      0.70        0.9        0.9       0.42
      3.000     106.80      0.00        0.9        0.9       0.42
999               0.          0.
1.0      -0.9       1.1      -0.8
```

11. RMC Example.

```
    5    6    1
051292   CLAYBOURN & WU EXAMPLE (NO.2),GEOSYNTHETIC'91,PP.549-559
    0.0000   76   15    7    0     1
    1    1    2
    0.0000   53.0000    0.1250    0.0000    0.0000
999               0.          0.        0.        0.
  -500.000      9.800
   209.800      9.800
   209.800     -0.100
   600.000     -0.100
999               0.
```

```
   -500.000    -0.100
    600.000    -0.100
999                0.
      0.33       0.00       0.00
   0.   0.   0.   0.
     -500.0      1.045
      209.8      1.045
      209.8        0.
      600.0        0.
999                0.
      0.00       0.00
   315.000    43.000   315.000    37.000     2.000     2.000    10.00
   000.000     0.000     0.000     5.000
      0.00       0.00   0       0.00
      0.90       0.50   0
     9.800      209.8      8.20    0.9000    0.9000    0.5
     9.800      209.8      5.70    0.9000    0.9000    0.5
     9.800      209.8      3.20    0.9000    0.9000    0.5
     9.800      209.8      0.80    0.9000    0.9000    0.5
999                0.         0.
1.0      -0.9       1.1      -0.8
```

12. Wright and Duncan's (1991) Example (Figure 3.16).

```
    5    6    1
021192   WRIGHT AND DUNCAN, TRR 1330, EXAMPLE 2
    0.0000   76   15    1    0    5    1
    1    1    2
    0.0000   35.0000    0.1050    0.0000    0.0000
    0.2000   00.0000    0.1000    0.0000    0.0000
999                0.        0.        0.        0.
     0.000    10.000
   200.000    10.000
   220.000     0.000
   600.000     0.000
999                0.
     0.000     0.000
   600.000     0.000
999                0.
     0.000   -10.000
   600.000   -10.000
999
      0.33       0.00       0.00
   0.   0.   0.   0.
        0.         0.
999                0.
      0.00       0.00
   211.000    14.000   211.000    14.000     1.000     1.000     5.00
   220.000    00.000    23.990     5.000
      0.00       0.00   0       0.00
```

```
    100.000    220.000     0.000      0.900      0.900     3.00
999               0.         0.
2.0      -0.5        2.1      -0.4
```

13. Rowe et al (1984) Exmaple.

```
    5    6    1
052992     ROWE  EXAMPLE,CAN  G.J.,21,1984,289-304,STATION  A,DESIGNED
SECT.
    0.0000   76   15    2    1    1    0
    1    1    2
    0.0000   32.0000    0.1300    0.0000    0.0000
    0.1330   00.0000    0.0650    0.0000    0.0000
999             0.         0.        0.        0.
     0.000    30.000
   200.000    30.000
   211.000    35.000
   255.000    35.000
   266.000    30.000
   900.000    30.000
999             0.
     0.000    30.000
   900.000    30.000
999             0.
     0.000    16.000
   200.000    16.000
   213.000    12.000
   233.000    12.000
   265.000     5.000
   900.000     5.000
999             0.
      0.33      0.00      0.00
   0.   0.   0.   0.
        0.        0.
999             0.
      0.00      0.00
   260.000    40.000   260.000    40.000    10.000    10.000    5.00
   266.000    30.000    33.113     5.000
      0.00      0.00    0     0.00
    56.000     261.0    32.00      1.0       1.0    2.750
999
    1.0        0.6      1.10       0.59
```

14. Hadj-Hamoe et al (1990) Example.

```
    5    6    1
101292   HADJ-HAMOE EXAMPLE, TRR, NO.1277, 1990, 80-89
    0.0000   76   15    1    1    1    0
    0    1    2
    0.2000   00.0000    0.1000    0.0000    0.0000
```

```
      0.4000    00.0000     0.1050     0.0000     0.0000
      0.0000    30.0000     0.1200     0.0000     0.0000
      0.1500    00.0000     0.0740     0.0000     0.0000
      0.1500    00.0000     0.0950     0.0000     0.0000
      0.2000    00.0000     0.0980     0.0000     0.0000
      0.2750    00.0000     0.0980     0.0000     0.0000
      0.4000    00.0000     0.0980     0.0000     0.0000
999                  0.         0.         0.         0.
       0.000      0.000
     132.000      0.000
     138.000      2.000
     167.000      3.000
     195.000     10.000
     205.000     10.000
     233.000      3.000
     259.000      3.000
     268.000      0.000
     600.000      0.000
999                  0.
       0.000      0.000
     155.000      0.000
     167.000      3.000
     195.000     10.000
     205.000     10.000
     233.000      3.000
     245.000      0.000
     600.000      0.000
999                  0.
       0.000      0.000
     168.000      0.000
     181.600      3.000
     214.400      3.000
     234.000      0.000
     600.000      0.000
999
       0.000      0.000
     600.000      0.000
999
       0.000    -15.000
     600.000    -15.000
999
       0.000    -20.000
     600.000    -20.000
999
       0.000    -30.000
     600.000    -30.000
999
       0.000    -40.000
     600.000    -40.000
999
```

```
     0.000    -55.000
   600.000    -55.000
999
   187.000      7.990
   212.000    -20.000        SURFACE A-1
   260.000    -20.000
   280.000      0.000
999
      0.33       0.00       0.00
   0.     0.    0.     0.
          0.           0.
999                    0.
      0.00       0.00
   215.000    20.000   245.000     10.000     0.000     0.000    110.00
   200.000    00.000     0.000      5.000
      0.00      0.00    0        0.00
    32.800     214.4      3.00        1.0       1.0     2.350
    49.200     224.2      1.50        1.0       1.0     2.350
999                0.            0.
    1.25       0.20      1.30        0.30
```

Coordinates of other shear surfaces:

```
   187.000      7.990
   212.000    -20.000             SURFACE A-2
   232.000    -20.000
   253.000      3.000
999

   182.000      7.000
   219.000    -30.000             SURFACE B-1
   260.000    -30.000
   288.000      0.000
999

   182.000      6.750
   219.000    -30.000             SURFACE B-2
   242.000    -30.000
   271.000      0.000
999

   180.500      6.380
   226.000    -42.000             SURFACE C-1
   265.000    -42.000
   308.000      0.000
999
```