Uncertainty Analysis of Carbon Ablation in the VKI Plasmatron

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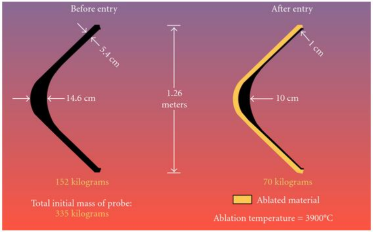
JOINT EXP/NUM WORK IS MANDATORY

Motivations

- **To understand** the operational behavior of the TPS materials
- **To study** the gas/surface interaction physics occurring during reentry
- **To improve** the prediction capacity and **reduce** the design margins

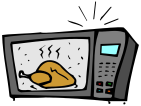


GALILEO MISSION
Destination: Jupiter
Date: 1989–2003

...THE BEST RACE CAR IS THE ONE THAT FALLS APART RIGHT AFTER THE FINISH LINE...

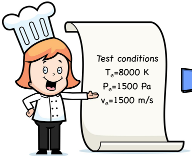
LET'S INTRODUCE OUR PLAYERS

Player 1



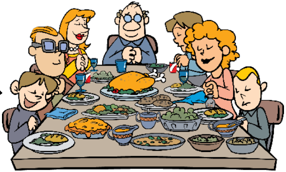
"the oven"

Player 2



"the recipe"

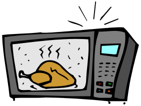
Player 3



"the customer"

Let's introduce our players

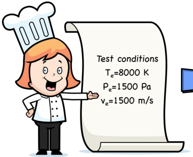
Player 1



"the oven"

PLASMATRON

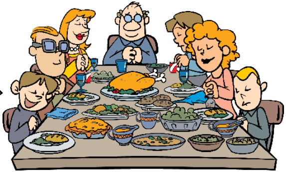
Player 2



"the recipe"

Rebuilding Code (boundary layer)

Player 3



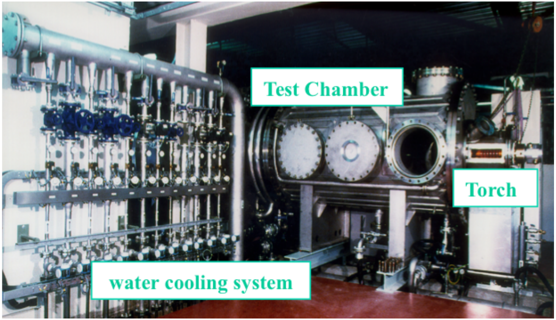
"the customer"

Stagnation-line code (w/ ablative b.c.)

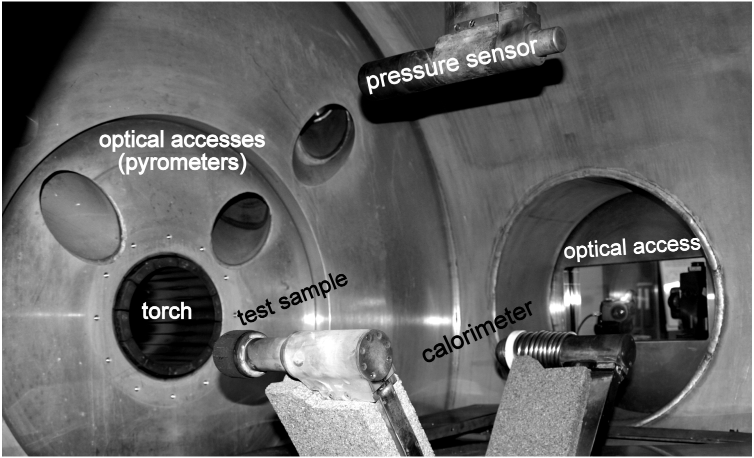
PLAYER #1: PLASMATRON FACILITY

Role: performing reusable/ablative TPS tests

- Gas: Air, N_2, CO_2, Ar
- Power: 1.2 MW – most powerful ICP in the world –
- Heat-flux: up to 16 MW/m^2 (superorbital re-entry)
- Pressure: 10 – 800 mbar



PLAYER #1: PLASMATRON FACILITY



TPS MATERIAL OPERATIONAL TESTING IS **ACHIEVABLE!!**

PLAYER #2: BOUNDARY-LAYER CODE*

Role: rebuilding of enthalpy (calorimeter)

Description

- Solves the reacting boundary layer equations along the stagnation line
- Assumes catalytic surface ($N + N \rightarrow N_2$ and $O + O \rightarrow O_2$)
- Rebuilds the boundary layer edge conditions to match the measured wall heat flux:

$$\dot{q}_{cw} = \dot{q}_{cw}\left(T_{cw},\, \gamma_{ref},\, h_e,\, p_e,\, \delta,\, \frac{\partial u_e}{\partial x},\, v_e \frac{\partial}{\partial y}\left(\frac{\partial u_e}{\partial x}\right)\right)$$

Pros & Cons

👍 Limited computational cost

👎 Ablative boundary condition not yet implemented

* P.F. Barbante, G. Degrez, G.S.R. Sarma, J. Thermophys. Heat Transfer 16 (2002) 490–497

PLAYER #3: STAGNATION-LINE CODE*

Role: rebuilding of the ablation test (test sample)

Description

- Solves a reduced form of the Navier–Stokes equations along the stagnation line
- Applicable to both sub- and supersonic flow over spheres and cylinders
- Chemistry solved via the the Mutation^{++} Library. Up-to-date thermodynamic and transport properties dataset

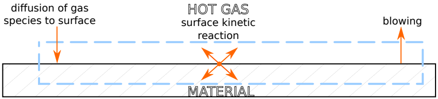
Pros & Cons

- Ablative boundary condition implemented
- Medium computational cost

* A. Munafò, PhD thesis, Ecole Central Paris (2014)

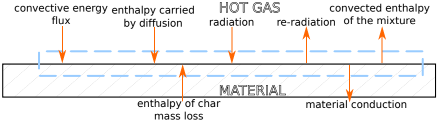
Player #3: Stagnation-Line Code

Surface Mass Balance



$$\rho D_{im} \left. \frac{\partial y_i}{\partial \eta} \right|_w + \dot{m}_{\mathbf{i},\mathbf{c}} = (\rho v)_w \, y_{iw}$$

Surface Energy Balance



$$k \left. \frac{\partial T}{\partial \eta} \right|_w + \sum_{i=1}^{N_C} h_{iw} \, \rho D_{im} \left. \frac{\partial y_i}{\partial \eta} \right|_w + \dot{m}_c h_{cw} + \dot{q}_{rad_{net}} = (\rho v)_w h_w + \dot{q}^{\mathbf{ss}}_{\mathbf{cond}}$$

PLAYER #3: STAGNATION-LINE CODE

THE THERMOCHEMICAL ABLATION MODEL CONSIDERS THE FOLLOWING REACTIONS:

Oxidation

- $C_s + O \rightarrow CO$
- $2\,C_s + O_2 \rightarrow 2\,CO$

Nitridation*

- $C_s + N \rightarrow CN$

Sublimation

- $3\,C_s \rightarrow C_3$

SURFACE SOURCE TERMS ARE GIVEN IN THE FORM:

$$\dot{m}_i = \beta_{0i} \left(m_i n_i \sqrt{\frac{k T_w}{2\pi m_i}} \right)$$

$\Uparrow$

REACTION PROBABILITIES EVALUATED EXPERIMENTALLY

* C. Park, H. K. Ahn., J Thermophys Heat Transfer 13 (1999) 60–67
T. Suzuki, K. Fujita, T. Sakai, J Thermophys Heat Transfer 25 (2010) 589–597
L. Zhang, D. Pejakovic, J. Marschall, D. Fletcher, J Thermophys Heat Transfer 26 (2012) 10–21

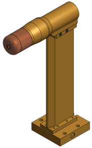
PUT THE PLAYERS TOGETHER

PLASMATRON

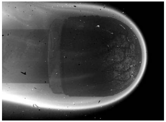
Experimental conditions

Geometry

Measurements

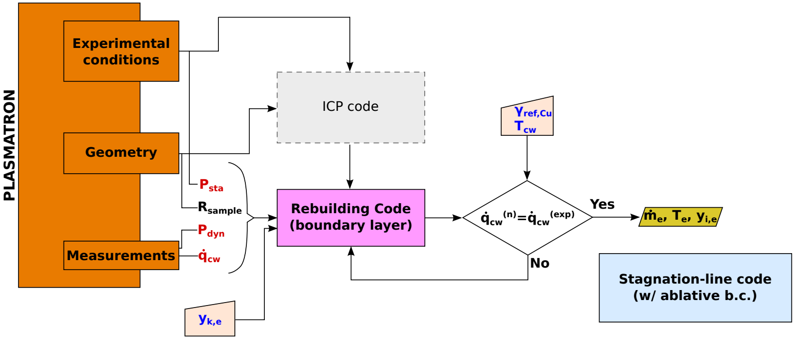


Rebuilding Code (boundary layer)

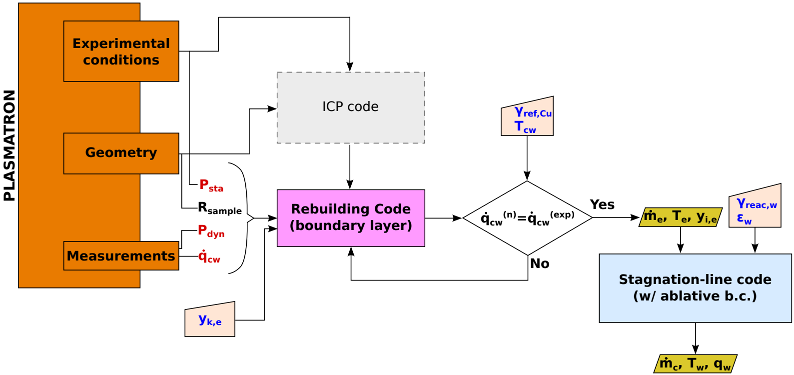


Stagnation-line code (w/ ablative b.c.)

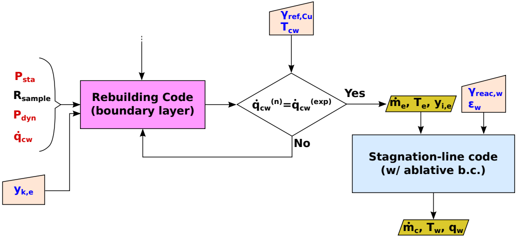
PUT THE PLAYERS TOGETHER



PUT THE PLAYERS TOGETHER



UNCERTAIN INPUTS GENERATE...UNCERTAIN OUTPUTS!!!

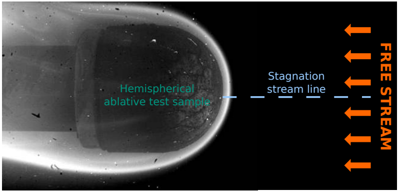


	VARIABLE
STEP 1: BOUNDARY-LAYER CODE	Dynamic Pressure Static Pressure Cold Wall Heat Flux Cold Wall Temperature Catalycity Nitrogen/Oxygen ratio
STEP 2: STAGNATION-LINE CODE	$C_s + O \rightarrow CO$ $2C_s + O_2 \rightarrow 2CO$ $C_s + N \rightarrow CN$ $3C_s \rightarrow C_3$ $N + N \rightarrow N_2$ TPS wall emissivity

UNCERTAINTIES WON'T MAGICALLY DISAPPEAR

Objectives

1. EVALUATE THE ABLATIVE MODEL UNCERTAINTY IMPACT ON THE FINAL QOIS
2. QUANTIFY THE INFLUENCE OF THE FREE-STREAM CONDITION UNCERTAINTIES ON THE FINAL QOIS



POLYNOMIAL CHAOS (PC) EXPANSIONS

1. The QOI u is expanded in a convergent series*

$$u(\xi) \approx u^{\mathrm{PC}}(\xi) = \sum_{\alpha=0}^{\mathrm{P}} u_\alpha \Psi_\alpha(\xi),$$

- $\mathrm{P} = (n_\xi + \mathrm{No})!/n_\xi!\mathrm{No}!$, No: expansion degree
- $\{\Psi_\alpha\}_{\alpha=0,\dots,\mathrm{P}}$ polynomial functions orthogonal w.r.t p_ξ (input PDF)
- correspondence between p_ξ and $\{\Psi_\alpha\}$
- $\{u_\alpha\}_{\alpha=0,\dots,\mathrm{P}}$: deterministic spectral coefficients

2. A non-intrusive spectral method is used to determine $\{u_\alpha\}$

$$u_\alpha = \|\Psi_\alpha\|^{-2} \int u(\xi)\Psi_\alpha(\xi) \approx \|\Psi_\alpha\|^{-2} \sum_{i=1}^{n} u(\mathbf{x}, t, \xi_i)\Psi_\alpha(\xi_i)\omega_i$$

- (ξ_i, ω_i) quadrature formulae points and weigths $\rightarrow$ deterministic code used as a black box

* Wiener 38; Cameron & Martin 47; Ghanem & Spanos 91

SENSITIVITY ANALYSIS

From PC expansions of QOIs

1. MEANS AND VARIANCES ARE OBTAINED

$$E(u^{\mathrm{PC}}) = u_0, \quad Var(u^{\mathrm{PC}}) = \sum_{\alpha=1}^{\mathrm{P}} u_\alpha^2(\mathbf{x}) \left\langle \Psi_i^2 \right\rangle$$

2. SENSITIVITY ANALYSIS BY ANOVA DECOMPOSITION

- Sobol first order indices $\{S_i\}_{i=1,\ldots,n_{\boldsymbol{\xi}}}$

  $\Downarrow$

  Quantifies the contribution to the QOI variance of the i^{th} random parameter

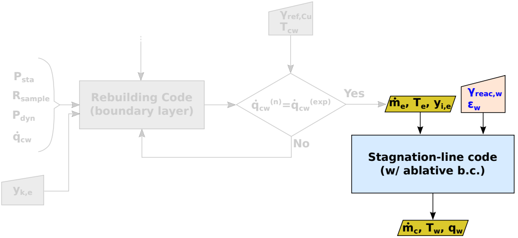
- Sobol total order indices $\{S_{T,i}\}_{i=1,\ldots,n_{\boldsymbol{\xi}}}$

  $\Downarrow$

  Quantifies the contribution to the QOI variance of the i^{th} random parameter including interactions with other parameter $j \in \{1, \ldots, n_{\boldsymbol{\xi}}\}, j \neq i$

* Crestaux, Le Maitre & Martinez 09

LET'S START...FROM THE END



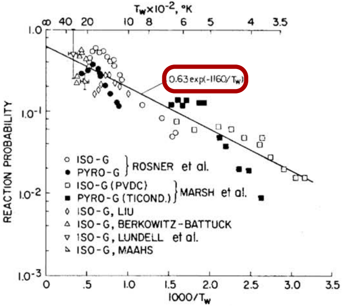
	VARIABLE
STEP 1: BOUNDARY-LAYER CODE	Dynamic Pressure
	Static Pressure
	Cold Wall Heat Flux
	Cold Wall Temperature
	Catalycity
	Nitrogen/Oxygen ratio

	VARIABLE	DISTRIBUTION	RANGE
STEP 2: STAGNATION-LINE CODE	$C_s + O \rightarrow CO$	?	?
	$2C_s + O_2 \rightarrow 2CO$	?	?
	$C_s + N \rightarrow CN$	?	?
	$3C_s \rightarrow C_3$	?	?
	$N + N \rightarrow N_2$	?	?
	TPS wall emissivity	?	?

Reaction probability uncertainties assessment

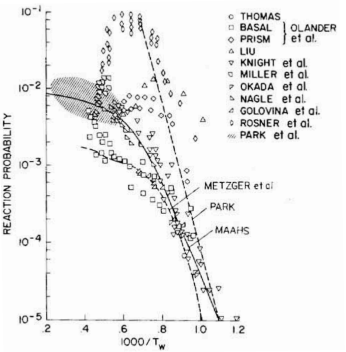
Atomic oxygen

$C_s + O \rightarrow CO$



Molecular oxygen

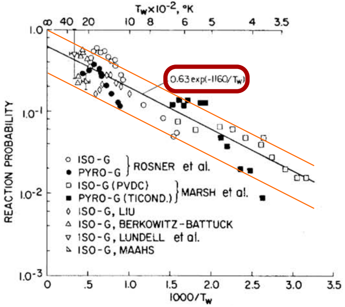
$2\,C_s + O_2 \rightarrow 2\,CO$



Reaction probability uncertainties assessment

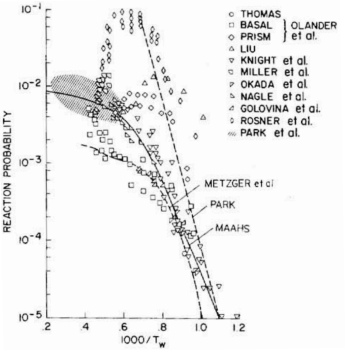
Atomic oxygen

$C_s + O \rightarrow CO$

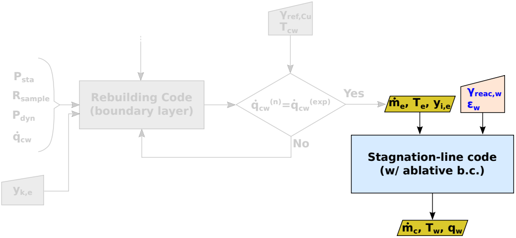


Molecular oxygen

$2\,C_s + O_2 \rightarrow 2\,CO$



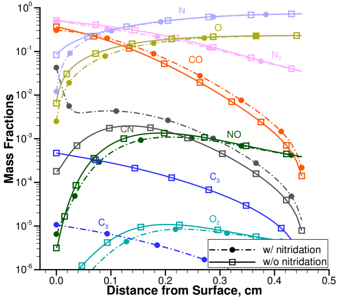
DEFINE THE INPUT UNCERTAINTIES

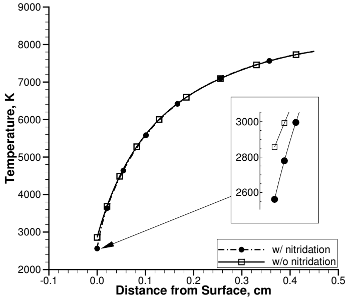


	VARIABLE
STEP 1: BOUNDARY-LAYER CODE	Dynamic Pressure Static Pressure Cold Wall Heat Flux Cold Wall Temperature Catalycity Nitrogen/Oxygen ratio

	VARIABLE	DISTRIBUTION	RANGE
STEP 2: STAGNATION-LINE CODE	$C_s + O \rightarrow CO$	Uniform	0.37–1
	$2C_s + O_2 \rightarrow 2CO$	LogUniform	0.00001–0.1
	$C_s + N \rightarrow CN$	Uniform	0–0.3
	$3C_s \rightarrow C_3$	LogUniform	0.01–1
	$N + N \rightarrow N_2$	Uniform	0–0.5
	TPS wall emissivity	Uniform	0.8–0.95

STAGNATION-LINE CODE NOMINAL OUTPUTS





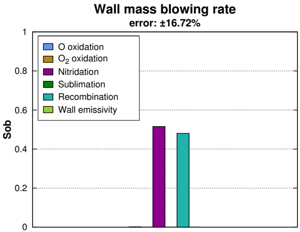
ABLATION QOI

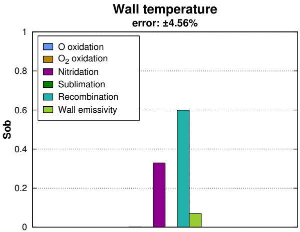
	VARIABLE	MEAN
w/ nitridation	mass blowing rate	0.041 [$kg/m^2 s$]
	temperature	2534 [K]
w/o nitridation	mass blowing rate	0.021 [$kg/m^2 s$]
	temperature	2840 [K]

STAGNATION-LINE CODE W/ NITRIDATION

ABLATION QOI

VARIABLE	MEAN	VARIANCE	$\Delta_{stoch-nom}$
mass blowing rate	0.031 [kg / m^2s]	2.69e-05	–24.4%
temperature	2722 [K]	1.54e+04	+7.4%



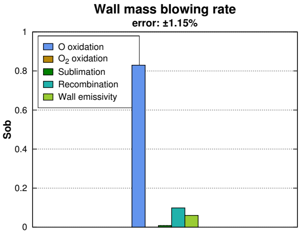


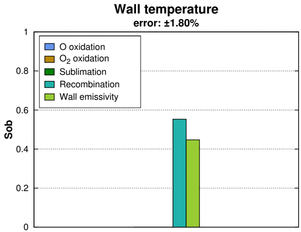
NITRIDATION AND RECOMBINATION ARE STRONGLY RELATED!

Stagnation-line code w/o nitridation

ABLATION QOI

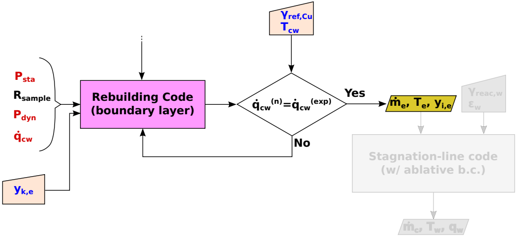
VARIABLE	MEAN	VARIANCE	$\Delta_{stoch-nom}$
mass blowing rate	0.021[kg / m^{2}s]	2.63e-10	–0.6%
temperature	2903 [K]	2.74e+03	2.2%





OXYGEN DIFFUSION LIMITS THE ABLATION RATE!

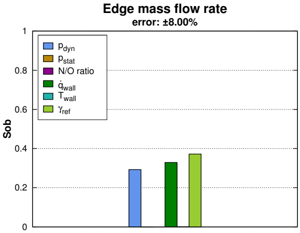
DEFINE THE INPUT UNCERTAINTIES

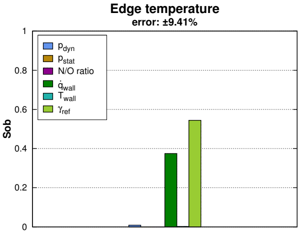


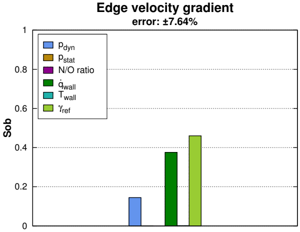
	VARIABLE	DISTRIBUTION	MEAN	ERROR (±)
STEP 1: BOUNDARY-LAYER CODE	Dynamic Pressure	Normal	48 Pa	8.0%
	Static Pressure	Normal	20000 Pa	0.3%
	Cold Wall Heat Flux	Normal	2962 kW / m^2	10.0%
	Cold Wall Temperature	Normal	350 K	10.0%
				RANGE
	Catalycity	Uniform		0.001–1
	Nitrogen/Oxygen ratio	Uniform		(79 / 21) ± 2%

	VARIABLE	DISTRIBUTION	RANGE
STEP 2: STAGNATION-LINE CODE	$C_s + O \to CO$	Uniform	0.37–1
	$2C_s + O_2 \to 2CO$	LogUniform	0.00001–0.1
	$C_s + N \to CN$	Uniform	0–0.3
	$3C_s \to C_3$	LogUniform	0.01–1
	$N + N \to N_2$	Uniform	0–0.5
	TPS wall emissivity	Uniform	0.8–0.95

BOUNDARY-LAYER CODE ANALYSIS





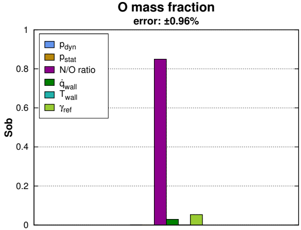


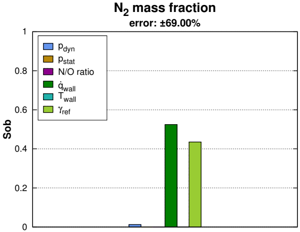
EXPERIMENTAL UNCERTAINTIES ARE AFFECTING THE QOI THE MOST!

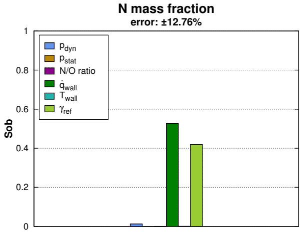
Boundary-layer code analysis

MEAN EDGE MASS FRACTIONS

O_2	1.24e-05
N_2	1.79e-01
NO	1.29e-03
O^+	2.42e-04
N^+	6.95e-04
O	2.32e-01
N	5.87e-01
e^-	3.55e-08

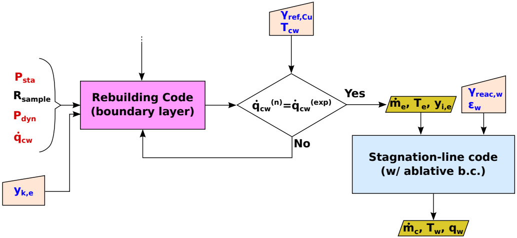


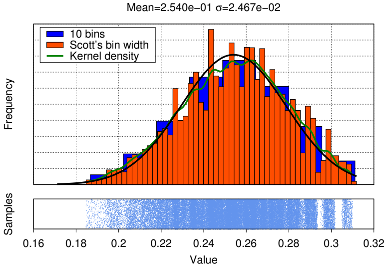


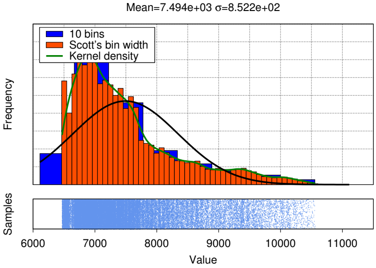


OXYGEN PRACTICALLY UNAFFECTED BY THE UNCERTAINTIES!

Coupled analysis: input uncertainty distributions



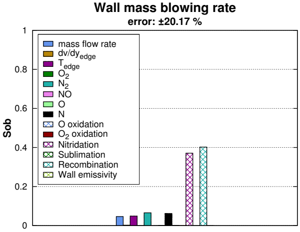


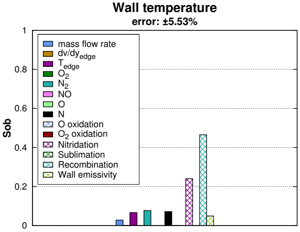


COUPLED ANALYSIS W/ NITRIDATION

ABLATION QOI

VARIABLE	MEAN	VARIANCE	$\Delta_{stoch-nom}$	$\varepsilon_{old}(\pm)$
mass blowing rate	0.029 [kg / m^{2}s]	3.48e-5	–28.4%	16.72%
temperature	2661 [K]	2.17e+4	+5.0%	4.56%



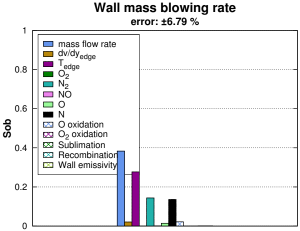


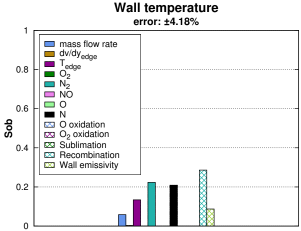
CONSIDERING ALL THE UNCERTAINTIES SLIGHTLY AFFECT THE ERROR!

COUPLED ANALYSIS W/O NITRIDATION

ABLATION QOI

VARIABLE	MEAN	VARIANCE	$\Delta_{stoch-nom}$	$\varepsilon_{old}(\pm)$
mass blowing rate	0.020 [kg / m^{2}s]	1.94e-6	−2.9%	1.15%
temperature	2818 [K]	1.39e+4	+0.8%	1.80%





REBUILDING UNCERTAINTIES AFFECT THE MASS BLOWING RATE!

Concluding Remarks

Conclusions

- Decoupled analiysis
  - Strong impact on the QOIs of a questionable phenomenon such as the surface nitridation when considered
  - Small variations of the QOIs uncertainties when nitridation is neglected: consequence of the analyzed ablation regime
- Coupled analiysis
  - The influence of the nitridation uncertainties remains the bigger
  - Measurement and model uncertainties from the rebuilding procedure cause the error to grow when nitridation is neglected

Perspectives

- Assess more plausible ranges for the most influential parameters
- Analyze different ablation regimes
- Compare the obtained results with the experimental measurements

Uncertainty Analysis of Carbon Ablation in the VKI Plasmatron

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