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A NEW GEOMETRIC MODEL AND METHODOLOGY FOR UNDERSTANDING PARSIMONIOUS SEVENTH-SONORITY PITCH-CLASS SPACE

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A New Geometric Model and Methodology for Understanding Parsimonious Seventh-Sonority Pitch-Class Space

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Dissertation

A dissertation submitted in partial fulfillment of the
requirements of the degree of Doctor of Philosophy in the
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By
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ABSTRACT OF DISSERTATION

A NEW GEOMETRIC MODEL AND
METHODOLOGY FOR UNDERSTANDING PARSIMONIOUS
SEVENTH-SONORITY PITCH-CLASS SPACE

Parsimonious voice leading is a term, first used by Richard Cohn, to describe non-diatonic motion among triads that will preserve as many common tones as possible, while limiting the distance traveled by the voice that does move to a tone or, better yet, a semitone. Some scholars have applied these principles to seventh chords, laying the groundwork for this study, which strives toward a reasonably comprehensive, usable model for musical analysis.

Rather than emphasizing mathematical proofs, as a number of approaches have done, this study relies on two- and three-dimensional geometric visualizations and spatial analogies to describe pitch-class and harmonic relationships. These geometric realizations are based on the organization of the neo-Riemannian *Tonnetz*, but they expand and apply the organizational principles of the *Tonnetz* to seventh sonorities. It allows for the descriptive "mapping" or prescriptive "navigation" of harmonic paths through a defined space.

The viability of the theoretical model is examined in analyses of passages from the repertoire of Frédéric Chopin. These passages exhibit a harmonic syntax that is often difficult to analyze as anything other than "tonally unstable" or "transitional." This study seeks to analyze these passages in terms of what they are, rather than what they are not.

KEYWORDS: Neo-Riemannian Theory, Seventh Chords, Parsimonious, Pitch-Class Space, Tonnetz

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17 October 2012

A New Geometric Model and Methodology for Understanding Parsimonious Seventh-Sonority Pitch-Class Space

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17 October 2012

To:

my mother, who taught me how to make music

my father, who taught me how to listen to it.

ACKNOWLEDGEMENTS

As I complete this labor-intensive project, thinking to myself what a feat I have accomplished, the words "All is vanity," come to mind. It is hubris to think I have expanded the boundary of human knowledge by inventing something brand new. "What has been is what will be, and what has been done is what will be done, and there is nothing new under the sun" (Ecclesiastes 1:9 ESV). In short, everything is a remix.

This dissertation is a remix of seminal ideas others have left behind for me, many of which were breaking ground around the time I was still watching new episodes of *Star Trek: The Next Generation*. Much like Sir Isaac Newton, "If I see farther, it is by standing on the shoulders of giants." As you can see, I cannot even write these acknowledgements without remixing what others have expressed better than I. So let me begin by confessing my debt to those scholars, stretching back more than a hundred years, on whose shoulders I stand.

The more tangible scholars who deserve recognition belong to my Committee. Dr. Lance Brunner and Dr. Gerald Janecek have given both their time and improving counsel. Dr. Michael Baker has sparked a number of suggestions for further research and has been a kind of biological bibliography of sources that relate to my topic. Dr. Karen Bottge has honed my writing to a sharper edge, and although that edge can be painful, she has always been one of my strongest

advocates. And Dr. Kevin Holm-Hudson, whose encyclopedic memory of the structural minutiae of musical repertoire, afforded me a great deal of aid beyond that of reading each chapter. I am grateful to have had him as the helmsman for this project.

To the "Berea Contingent." For their encouragement, experience, goodwill, and generosity that helped to give me structure, focus my writing process, and keep me accountable, I give my esteemed appreciation to Dr. Irene Burgess and Rev. Curt Gardner, my "Alpha Readers." Without them, this would have been a much longer process. To Daniel Roush, who saved me a lot of trouble when I was in a jam, and to Kim Gardner and Lisa Roush, who adopted me into their families and were just as excited for me as I was to have successfully completed this dissertation.

Dr. Vicki Bell gets credit for being the one to spark my interest in music theory, setting off a chain of events that has led to this document.

I am deeply grateful to my family. My parents, to whom I have dedicated this work, have been instrumental in my musical awareness and education from the cradle. My brother had very little to do with this project, but he is my longest friend (both in length of time and length of leg); we both love music and that is something I love sharing with him. My wife, Celia, has encouraged me, kept me fed, and paid the bills while I labored over this document. I'm so thankful she

showed up at the coffee shop that fateful Election Day in 2008. I had a pineapple smoothie.

Finally, I am ever in debt to the One Who "is before all things, and by Him all things consist" (Colossians 1:17 KJV).

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CHAPTER I: SCHOLARLY CONTEXT

INTRODUCTION

Parsimonious voice leading is a term, first used by Richard Cohn, to describe non-diatonic motion among triads that is very smooth.[1] *Parsimonious*, of course, means *stingy*, or perhaps the more complimentarily nuanced *frugal*. That is to say, this sort of harmonic context will preserve as many common tones as possible, while limiting the distance traveled by the voice that does move to a tone or, better yet, a semitone. Richard Cohn has concisely described this kind of harmonic language as "chromatic music that is triadic but not altogether tonally unified."[2]

Strangely enough, it was in the writings of some nineteenth-century theorists, such as Hugo Riemann, that modern music theorists found the germ of inspiration they needed for developing a conceptual framework and analytical approach to parsimonious passages. This led to an outpouring of research under the banner of what is now commonly referred to as neo-Riemannian theory, intended to address these "indeterminate," "coloristic," and "aimless" passages

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[1] Richard Cohn, "Neo-Riemannian Operations, Parsimonious Trichords, and Their *Tonnetz* Representations," *JMT* 41, no. 1 (Spring 1997): 1–2.

[2] Richard Cohn, "Introduction to Neo-Riemannian Theory: A Survey and a Historical Perspective," *JMT* 42, No. 2 (Fall 1998), 167.

that are scattered throughout musical literature of the nineteenth and twentieth centuries.[3]

With neo-Riemannian theory fully entrenched in the study of triadic harmonic structures, even having entered some undergraduate textbooks, several scholars have extended its applications to the realm of seventh chords. The contribution I offer here further advances the application of neo-Riemannian concepts to seventh chords via two- and three-dimensional models of pitch-class space in a way I hope musicians will find both relatively accessible and inclusive enough to apply to musical analysis.[4]

Historical Perspective

The work of several prominent theorists contributed to the work of Riemann and, by extension, neo-Riemannian theory. His harmonic dualism and function theory are more closely aligned with the works of Gottfried Weber (1779–1839), Moritz Hauptmann, and Arthur Joachim von Oettingen. Harmonic monism can be thought of as assuming the de facto preeminence of the major triad, speciously evidenced in the acoustic structure of the overtone series.

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[3] Cohn, "Introduction," 169.

[4] Since the term *seventh chord* often carries the implication of functional harmony, I will in the future refer to these as *seventh sonorities*.

Harmonic dualism, on the other hand, can be thought of as assuming the structural equality of the major triad and its reciprocal manifestation, the minor triad.[5] Although Weber did not necessarily identify himself as a harmonic dualist, his grid of key relationships is one of the first *Tonnetze*, impacting German musical thought in the works of theorists like Oettingen and Arnold Schönberg.[6] Further, Otakar Hostinsky (1847–1910), remembered not so much for his theoretical contributions as for his diagram, was the first to assimilate both major and minor thirds in a single *Tonnetz*, lending equal weight to both.[7]

Hauptmann (1792–1868) forms the first member in a trinity of nineteenth-century harmonic dualists (including Oettingen and Riemann). It was Hauptmann who referred to major and minor triads as *Klänge*, a term later reiterated by Riemann, and then revived by David Lewin in his *Generalized Musical Intervals and Transformations* of 1987.[8] Hauptmann also articulated the

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[5] Henry Klumpenhouwer, "Dualist Tonal Space and Transformation in Nineteenth-Century Musical Thought" in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 459.

[6] David W. Bernstein, "Nineteenth-Century Harmonic Theory: The Austro-German Legacy" in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 784–6.

[7] Brian Hyer, "Tonality" in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 736–7.

[8] Cohn, "Introduction," 170.

notion of the positive *Einheit* (Riemann's *Oberklang*) and negative *Einheit* (Riemann's *Unterklang*), what today we would refer to as inversional symmetry.

Oettingen (1836–1920) not only adopted Weber's practice of making tonal charts, as seen in Oettingen's 1866 treatise *Harmoniesystem in dualer Entwicklung*, but he also adapted the logic of Hauptmann's dualism to reconcile it with acoustical and physiological criticisms leveled against it by the harmonic monist Hermann von Helmholz.[9] In Oettingen, then, we find the synthesis in his paired notions of *Tonictät* (tonicity) and *Phonicität* (phonicity). Under Oettingen's rubric, tonicity refers to the trait of pitches of an interval or chord to be conceived as partials of a common fundamental (the tonic fundamental); in opposition to this is phonicity, which refers to the trait of pitches of an interval or chord to be conceived as fundamentals united by a lowest common partial (the phonic overtone).[10]

In Hugo Riemann (1849–1919) we find the culmination of these various influences, primarily through the conduit of Oettingen. Whereas many of Riemann's pursuits (the undertone series, for one) never yielded positive results, the elegance and logic of his theory continue to appeal to neo-Riemannian

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[9] Klumpenhouwer, 462–3.

[10] Ibid., 464–5.

theorists.[11] In particular, his notion of the minor triad as the mutual and opposite manifestation of the *Klang* explained the dominant and subdominant harmonies as extensions of the "over" fifth and "under" fifth of the tonic *Klang*. All other harmonic structures were variants of these three (*S* for subdominant, *T* for tonic, and *D* for dominant), with some element of them slightly displaced. To Riemann, the submediant chord could be achieved by means of the tonic parallel function or the subdominant leading-tone function. For example, in the key of C major, the tonic parallel function would displace the G of the C major triad (tonic) with the A of the A minor triad (submediant), corresponding to the neo-Riemannian relative function. Likewise, the subdominant leading-tone exchange function would displace the F of the F major triad (subdominant) with the E of the A minor triad (submediant), corresponding to the neo-Riemannian leittonwechsel function.[12] It is these function labels and Riemann's *Tonnetz* that inspired so much scholarly activity since the 1990s under the banner of neo-Riemannian theory.

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[11] Klumpenhouwer, 469.

[12] Bernstein, 796–8.

NEO-RIEMANNIAN THEORY

A great deal of the current study owes to years of neo-Riemannian scholarship that laid a conceptual foundation. But whereas neo-Riemannian theory deals primarily with triads, outgrowths of this theory have attempted to tackle the more complex relationships inherent in seventh-sonority parsimony. I will briefly recapitulate the main motivators behind its initial materialization here, followed by a review of sources more specifically apt for my study.

David Lewin is one of the key figures responsible for the resurgent interest in Riemann. His essay "A Formal Theory of Generalized Tonal Functions" in 1982 and his *Generalized Musical Intervals and Transformations* in 1987 lay the foundation for neo-Riemannian thought, even retaining Riemann's term for triads, *Klänge*.[13] Lewin continued to contribute to the field of transformational and neo-Riemannian theory well into the 1990s.[14]

Brian Hyer is largely responsible for popularizing neo-Riemannian theory among the music theory community with his article "Reimag(in)ing Riemann."[15]

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[13] David Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987), 175–80, cited in Richard Cohn, "Introduction to Neo-Riemannian Theory," 170.

[14] David Lewin, "Cohn Functions," *JMT*, 40, No. 2 (Fall 1996): 181–216; "Some Ideas about Voice-Leading," *JMT*, 42, No. 1 (Spring 1998): 15–72.

[15] Brian Hyer, "Reimag(in)ing Riemann," *JMT* 39, no. 1 (Spring 1995): 101–38.

His modernization of Hugo Riemann's *Tonnetz* is an elegant visualization of pitch/pitch-class space. Although his work lies in the realm of triadic structures in a post-Common-Practice context, his *Tonnetz* and his reimagining of Riemann's harmonic functions as parallel, leittonwechsel, and relative transformations lay the foundation for the scholarship that followed him.[16] This is particularly relevant to my study since his revitalization of the *Tonnetz* departs from Lewin's more mathematical approach in favor of a more accessible visual medium.

Richard Cohn has also been an instrumental and prolific contributor to the field's understanding of triadic parsimony in terms of both mathematical and geometric representation. He is, in fact, credited with coining the term "parsimonious voice leading."[17] Because of his productive work, neo-Riemannian theory has become more mainstream in music theory parlance and pedagogy.[18]

A number of studies have addressed the question of seventh-sonority parsimony, but with constraints that might be seen as problematic.

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[16] After Hyer, I leave the term *leittonwechsel* without italics or capitalization.

[17] Robert C. Cook, "Parsimony and Extravagance," *JMT* 49, no. 1 (Spring 2008): 109.

[18] Cohn, "Introduction"; "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Progressions" *Music Analysis* 15, No. 1 (March 1996): 9–40; "Neo-Riemannian Operations"; "Square Dances with Cubes," *JMT* 42, no. 2 (Fall 1998): 283-96.

1. **Abstraction**. Some studies focus on mathematical relationships and equations. Although abstract ideas present a valuable line of inquiry, many readers may find such ideas difficult to conceptualize.
2. **Exclusion**. Some studies address only a few seventh-sonority types and omit others. This avoids overcomplication, aiding the reader, but it also potentially limits applicability in musical contexts.
3. **Speculation**. To some extent, all music theory is speculative, but those that are particularly so often do not exercise the idea in musical conditions.

In some instances a study may posses more than one of these features, blurring the differences among what I have made into distinct categories. These constraints can potentially inhibit both understanding among a widening readership, especially students, and the practical application of concepts to actual music. I will address each of these constraints in turn.

ABSTRACTION

Clifton Callender writes on the parsimonious fission and fusion of pitches between pitch collections in his article "Voice-Leading Parsimony in the Music of Alexander Scriabin." Although his discussion is not directly related to mine, his

idea of "split" and "fuse" functions has influenced my perceptions and understanding of cross-type transformations.[19] Callender's approach is also very algebraic, but I find the underlying concept he presents to be persuasive and applicable in some of the repertoire I am studying. Addressing the issue of cross-type transformations informed how I went about analyzing triads interrupting a string of seventh sonorities.

In "Half-Diminished Functions and Transformations in Late Romantic Music," Richard Bass examines parsimonious relations among half-diminished sevenths, but he does little in the way of graphic representation of these relationships.[20] Moreover, Bass treats only one seventh-sonority type, even if he does draw the discussion back to musical application.

Steven Scott Baker's dissertation, "Neo-Riemannian Transformations and Prolongational Structures in Wagner's *Parsifal*," includes a treatment of a wide variety of sonority types and favors an algebraic approach.[21] However, this study was less applicable to the current discussion.

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[19] Clifton Callender, "Voice-Leading Parsimony in the Music of Alexander Scriabin," *JMT* 42, no. 2 (Fall 1998): 219–33.

[20] Richard Bass, "Half-Diminished Functions and Transformations in Late Romantic Music," *Music Theory Spectrum* 23, no. 1 (Spring 2001): 41–60.

[21] Steven Scott Baker, "Neo-Riemannian Transformations and Prolongational Structures in Wagner's *Parsifal*," Ph.D. diss., Florida State University, 2003.

EXCLUSION

Edward Gollin and Adrian Childs each take cues directly from Riemann by discussing members of set class (0258), major-minor and half-diminished sevenths, sonorities that are asymmetrical and whose qualities are inversionally related. Gollin, in his article "Some Aspects of Three-Dimensional *Tonnetze*," devises a three-dimensional geometric model that grows naturally from a neo-Riemannian *Tonnetz*. In his model, each seventh sonority resembles a tetrahedron; upright tetrahedrons, like upright triangles in a *Tonnetz*, denote the major-minor quality, whereas inverted tetrahedrons denote the half-diminished quality. Gollin's model retains the structural principle of the *Tonnetz* in which individual pitch classes form the vertices of the model; when these vertices are related to each other in the shape of a tetrahedron, they form a member of set class (0258).[22] Although Gollin creates an elegant adaptation of a neo-Riemannian *Tonnetz*, the model does not incorporate other varieties of seventh sonorities. Childs also deals with major-minor and half-diminished sevenths by representing their interrelationships on a cube, in his study "Moving Beyond Neo-Riemannian Triads: Exploring a Transformational Model for Seventh Chords." Childs's model places complete sonorities at each vertex. Each vertex,

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[22] Edward Gollin, "Some Aspects of Three-Dimensional '*Tonnetze*,'" *JMT* 42, no. 2 (Fall 1998): 195–206.

then, represents four pitch classes, as opposed to Gollin's approach of one pitch per vertex.[23] Both Gollin and Childs fulfill their respective purposes, but they exclude the other three normative seventh sonorities. Furthermore, Childs' model obscures the individual pitch-class members involved during an harmonic transformation.

Like Callender's article, Julian Hook's "Cross-Type Transformations and the Path Consistency Condition" wrestles with the difficulties of connecting triads and sevenths.[24] Hook focuses especially on the omnibus progression and its rocking back and forth between triads and sevenths. Hook primarily addresses what he finds lacking in Lewin's same-type transformations and the strict criteria that govern them. The benefits I have gained from Hook's work lie in his graphic realizations of cross-type transformations, and I can foresee helpful application of his technique in an extended neo-Riemannian context.

SPECULATION

Jack Douthett and Peter Steinbach, in "Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition,"

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[23] Adrian Childs, "Moving Beyond Neo-Riemannian Triads: Exploring a Transformational Model for Seventh Chords," *JMT* 42, no. 2 (Fall 1998): 181–93.

[24] Julian Hook, "Cross-Type Transformations and the Patch Consistency Condition," *Music Theory Spectrum* 29, no. 1 (Spring 2007): 1–39.

illustrate numerous models and relationships among major-minor, minor-minor, half-diminished, and fully-diminished sevenths in various combinations.[25] These authors' approach is, like many in this area of study, rather algebraic, but it also includes a number of graphs describing parsimonious connections among triads and among seventh sonorities. The article includes an extended application of the "power towers" model that shows a number of relationships presented in my models, namely the wormhole (pp. 20, 27).[26] Like Childs, Douthett and Steinbach place entire sonorities at each vertex of their models, rather than pitch classes. Yet, with all the models and equations these authors put forth, none incorporate the major-major seventh, nor do they ever use the models in any sort of analysis.

Richard Cohn deals with tetrachords in a generic sense in "A Tetrahedral Graph of Tetrachordal Voice-Leading Space."[27] Cohn's title initially suggests his article may bear some similarities with the tetrahedral graphs of Edward Gollin. However, there is little to compare between his graphs and Gollin's. Cohn places

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[25] Jack Douthett and Peter Steinbach, "Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition," *JMT* 42, no. 2 (Fall 1998): 241–63.

[26] Ibid., 256.

[27] Richard Cohn, "A Tetrahedral Graph of Tetrachordal Voice-Leading Space," *Music Theory Online* 9, no. 4 (October 2003), http://www.mtosmt.org/index.php (accessed June 1, 2012).

whole tetrachords at each vertex of the model and includes a wide variety of four-note sonorities that are not constrained even to those of tertian structure.

Callender, Quinn, and Tymoczko's article "Generalized Voice-Leading Spaces" is especially abstract, despite its geometric representations of pitch-class space.[28] While this testifies to the applicability of the authors' work to mathematics, it contained no direct musical application, likely because it was published for the science and mathematics community. The authors map parsimonious space among tetrachords but give equal inclusion of non-normative, non-tertian, and even non-chord "sonorities" (such as [0000] or [0066]). This approach is more inclusive than the present study will be.

OTHER

With so much neo-Riemannian scholarship relying on a sort of calculus, Peter Westergaard's clever "Geometries of Sound in Time" provides what I consider to be needed justification for geometric understanding of transformational theory.[29] Westergaard makes a case for the validity, logic, historical underpinnings, and even the shortcomings of geometric embodiments

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[28] Clifton Callener, Ian Quinn and Dmitri Tymoczko, "Generalized Voice-Leading Spaces," *Science* 320 (2008): 346–8.

[29] Peter Westergaard, "Geometries of Sound in Time," *Music Theory Spectrum* 18, no. 1 (Spring 1996): 1–21.

of pitch/pitch-class space.[30] Candace Brower's "A Cognitive Theory of Musical Meaning" is listed here for many of the same reasons as Westergaard's article.[31] Her spatial realizations of pitch relations have enlightened and deepened my own understanding. Her article has influenced this study, particularly in the way that I graphically represent pitch and sonority relationships. Brower's diagrams of cyclical triadic space (see her Figures 21–25)[32] are particularly relevant to my discussion.

Conclusion

In answer to the issue of abstraction, I prefer to translate mathematical relationships into visual ones. Spatial representation of obscure information is ubiquitous in our society as a means to better encode information through some kind of metaphor or analogy we can better grasp. Precedent for spatial representation of music (besides the musical notation itself) is nowhere more evident than in the work of Weber, Oettingen, and Riemann, who all used

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[30] This justification is echoed in Richard Cohn, *Audacious Euphony: Chromaticism and the Triad's Second Nature* (New York: Oxford University Press 2012), 14–5.

[31] Candace Brower, "A Cognitive Theory of Musical Meaning," *JMT* 44, no. 2 (Fall 2000): 323–79.

[32] Ibid., 345–7.

geometry to metaphorically describe musical concepts. Brian Hyer revived the *Tonnetz*, and others have adopted and adapted it or developed their own.

To address the second problem of exclusivity, I include all normative seventh sonority types (major-major, major-minor, minor-minor, half-diminished, and fully diminished). This is still a limitation, but no study I have read includes all five normative types. In trying to balance the extremes of exclusivity and inclusivity, I aim to ease the transition from theory to practice.

The third issue of speculative theory is not an issue in itself, but in my survey of the literature I found that few studies moved from the conceptual model to practical application. After I make a case for my theoretical model, I will 1) demonstrate its descriptive use in the analysis of preexisting music and 2) posit its prescriptive germaneness to composition. As an analytical tool, it can address passages that are typically glossed over as "transitional" or "unstable." These very words ought to spark curiosity as to how such passages work and what they are, rather than be dismissed because of what they are not. As a compositional tool, it can open up new ways for composers to execute those "transitional" passages smoothly, and perhaps even serve as an improvisational template.

CHAPTER II: CONCEPTUAL DICHOTOMIES

In this chapter, I will discuss three dichotomies that influence the ways musicians in general, and music theorists in particular, conceive of harmony and how these dichotomies factor into a discussion of seventh sonorities. While these matters are not the heart of this dissertation, my answers to the questions they raise underpin my theory. I will discuss each dichotomy in turn, with the purpose of clarifying and specifying the conceptual assumptions I will make.

PARSIMONY VS. EXTRAVAGANCE

The first is the relatively recent delineation drawn between what Robert C. Cook calls parsimony and extravagance.[33] Is it necessary to differentiate between maximal smoothness and minimal perturbation? The differentiation Cook makes between these is perhaps silently acknowledged among neo-Riemannian theorists, but is rarely discussed. I include it here to clarify the conditions under which my theory and analyses will and will not apply.

The term *parsimonious* was first used by Richard Cohn to describe musical passages that exhibit (1) maximally smooth, (2) minimally perturbed, motion

[33] Cook, 109–140.

between triads.[34] The former quality refers to the preservation of common tones from one chord to the next; the latter refers to the allowance of a pitch to move only by tone or semitone. The term *parsimonious* has been widely adopted by other theorists and remains the implicit basis of neo-Riemannian theory. Cohn, and others, have intuitively grouped these two properties (maximal smoothness and minimal perturbation) into essentially the same idea, and in many cases these two properties are inextricable.

Robert C. Cook, however, sets out to uncouple them into *parsimonious* motion (retention of common tones) and *extravagant* motion (stepwise motion). He reflects upon the scholarly use of the term *parsimonious,* as it is typically used, to imply both the retention of common tones and the stepwise motion of the remaining voice(s). To demonstrate the distinction between these, Cook cites two passages from César Franck's *Piano Quintet in F minor*, calling one passage extravagant and the other parsimonious. The extravagant passage is composed almost entirely of very smooth voice leading (i.e., by semitone), but all voices move, thus destroying the continuity from chord to chord that common tones afford. Cook's distinction between parsimony and extravagance is crucial. Even though he deals exclusively with triadic structures, I have found that some passages of chromatic seventh-sonority usage can also be extravagant.

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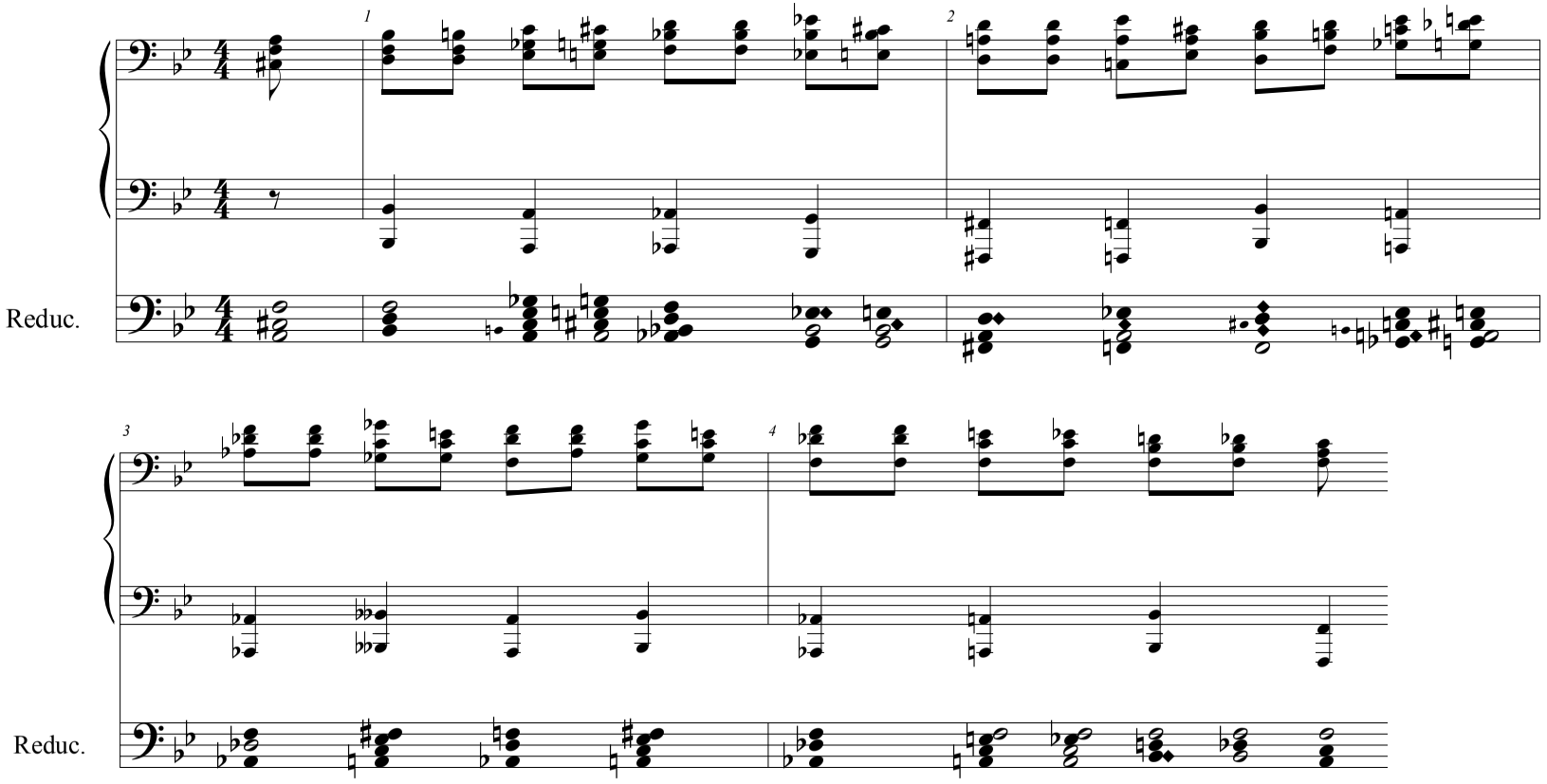
[34] Cohn, "Neo-Riemannian Operations," 1–2.

While I do not place so rigid a set of guidelines as does Cook on what does and does not constitute parsimony or extravagance, his ideas have guided my understanding of what musical passages will and will not apply to my own theory. Musical passages I will explore must be minimally perturbed, and what perturbation there is must be maximally smooth (although I allow whole tone motion as well as semitone motion). This concept is illustrated in Example 2.1 by way of a negative example. I have provided a harmonic reduction below the piano staves to illustrate just how extravagant this passage is.[35] A key is provided below the example. This highly chromatic passage from Brahms features maximal smoothness at the expense of minimal perturbation; very few pitches are held in common with the preceding chord. There are brief instances of relative parsimony, such as those in mm. 4.1–4.2 and 4.3.1–4.3.2, but these cannot be said to characterize the passage. In contrast to Example 2.1, I will examine passages that favor pitch retention and therefore can be characterized as parsimonious.

[35] The harmonic reduction may in some ways resemble a Schenkerian-inspired rhythmic reduction, but it is not intended as such.

Example 2.1: Brahms, "Variation No. 20" from *Variations and Fugue on a Theme by Handel*, mm. 1–4



REDUCTION KEY:
black note heads = pitch has moved by semitone from prior sonority
diamond black note heads = pitch has moved by whole tone from prior sonority
hollow note heads = pitch held in common with prior sonority
cue-sized note heads = pitch smooths the transition of a voice without strictly belonging to either sonority

HORIZONTALITY VS. VERTICALITY

The second dichotomy deals with the longstanding struggle, which has existed in some form since the emergence of polyphony, between the conceptual frameworks dealing with simultaneous versus chronological unfolding of sound. Are seventh sonorities to be considered sonorities in their own right or the mere coincidence of independent musical lines? The answer to this question is one of the foundations of my theory to follow.

The differentiation theorists make between the horizontal (occurring over time) and vertical (simultaneous) parameters of music is in many ways artificial. These parameters are, rather, extremes which in themselves have little or no practical application. The degrees of horizontality or verticality that exist between those extremes are what is more relevant to my interests. Theorists encode some understanding of this tug-of-war in words common to any undergraduate musician–*polyphony*, *homophony*, *prolongation*, *counterpoint*, *linear harmony*, etc. All these words communicate some assumed understanding of music's motivating force as either harmonically driven or linearly driven.

One could certainly make the argument that even music with a strong sense of verticality, something homophonic such as a hymn, is ultimately driven by the tendency of tones within each sonority to resolve linearly to some other tone. However, one has only to sing the alto part from a hymnal to learn that

linearity is not a motivational force at all times. The fugues of J. S. Bach clearly receive their propulsive force from the subject by means of inertia, rhythmic motion, and the gravity of tendency tones that make up the melodic integrity of the individual line. Even so, harmonic considerations are very important in the fugal process, particularly during the exposition. Even the more harmonically free and more horizontally oriented polyphony of the Renaissance still had rules governing simultaneous intervals. In short, these two opposing notions of verticality and horizontality can never be considered apart from one another.

The I^{6}_{4} is an ideal example of this dualistic horizontal/vertical nature of certain harmonies. Some would label it as a suspension figure, V^{6-5}_{4-3}, to reflect its embellished-dominant function. Yet there are instances in which the resolution is not to the dominant, or in which the chord is not approached in such a way that a suspension figure is appropriate. The chord, if I may call it a chord, results most of the time, and has been historically understood to arise, from embellishment of the more structurally important dominant. Yet, due to the ubiquity and consistency of its doubling, voice-leading, and tonicizing force, it is still given a label that implies some kind of vertical identity even by theorists who favor a linear perspective.

How one answers the issues posed by this dichotomy will directly influence how one perceives and analyzes four-pitch, tertian-structure

simultaneities–as a seventh sonority or as a triad with added decoration. In some musical contexts, it is very clear how the would-be seventh ought to be regarded. But in other contexts that delineation is blurred.

The fully diminished seventh sonority is one such example. The typical assessment seems to be that if the sonority performs functionally (i.e., tonicizes something else), then thinking of it as a chord is perfectly legitimate. However, if it behaves as a common-tone diminished sonority, then regarding it as a chord is objectionable. Richard Bass notes the hierarchy placed on fully-diminished seventh sonorities based on their tonicizing power (whether that be diatonic, secondary, or modally inflected).[36] In terms of non-functional manifestations of the fully-diminished sonority, such as the common-tone diminished seventh, it is typically regarded as specious. Laitz calls the common-tone diminished seventh a "contrapuntal chord" (821); Kostka and Payne describe it as having "weak harmonic function" and its "non-essential flavor" (453); Aldwell and Schachter say the sonority is an "apparent…, rather than true, seventh chord" (621) while simultaneously acknowledging its pervasiveness; Clendinning and Marvin

[36] Richard Bass, "Enharmonic Position Finding and the Resolution of Seventh Chords in Chromatic Music," *Music Theory Spectrum* 29, no. 1 (Spring 2007): 73.

describe it as the interaction of chromatic and diatonic non-chord tones that "happen to make a fully-diminished seventh sonority" (583).[37]

Consider the beginning of the development section from the first movement of Mozart's *Symphony No. 40*, shown in Example 2.2. The G♯$^{\circ 4/2}$ in m. 101 is quite obviously linear, judging by the way it is approached, and it even shares two pitches in common with the following measure. It is certainly transitioning the listener from the old key to the new key, but functions in neither (although it could be understood enharmonically as vii$^{\circ 7}$ in F♯). Yet, as if in opposition to a purely horizontal reading, rests on either side of the simultaneity seem to bracket it off, forcing our ear to hear it as a sonority in its own right. The rests may merely interrupt the linear design, but one cannot help but hear it as a sonority given the silence on either side of it.

I have no desire to dissent with the notion that such sonorities are contrapuntal and are the outgrowth of confluent musical lines, but that idea can apply to the genesis of any harmony not just one particular type. Regarding the common-tone fully-diminished seventh, or any seventh sonority, as a quasi-

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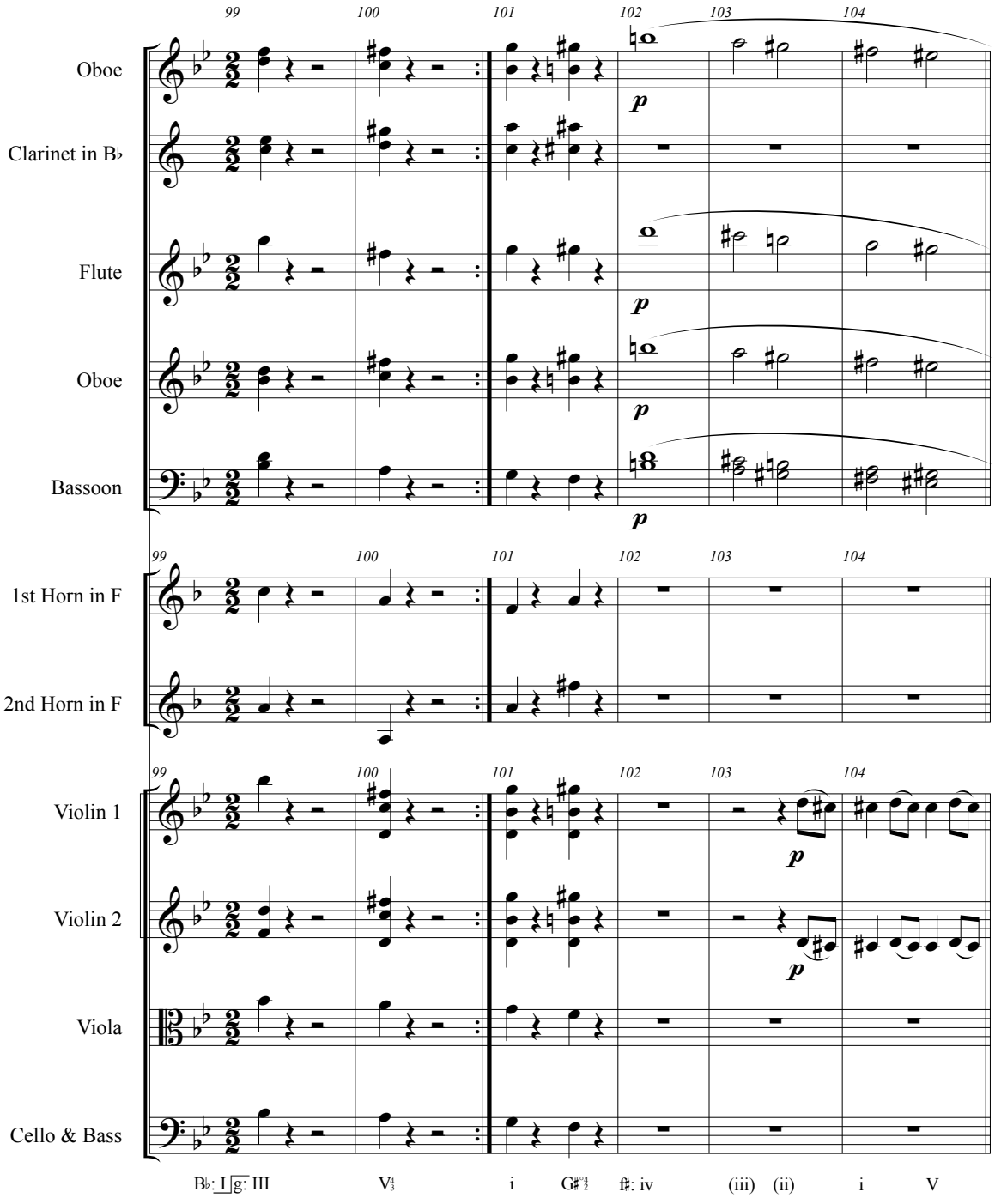
[37] Steven Laitz, *The Complete Musician: An Integrated Approach to Tonal Theory, Analysis, and Listening* 2nd ed. (New York and Oxford : Oxford University Press, 2008); Stefan Kostka and Dorothy Payne, *Tonal Harmony,* 6th ed. (Boston: McGraw Hill, 2004); Edward Aldwell, Carl Schachter and Allen Cadwallader *Harmony and Voice Leading*, 4th ed. (Boston: Schirmer, 2011); Jane Piper Clendinning and Elizabeth West Marvin, *The Musician's Guide to Theory and Analysis* (New York: W. W. Norton, 2005).

Example 2.2: Mozart, *Symphony No. 40 in G minor*, K. 550, I, mm. 99–102



verticality is an over-simplification. Richard Bass would seem to agree. There are circumstances, he asserts, in which imposing a key can lessen one's understanding of these chromaticisms as inherently and stylistically important in and of themselves. He goes on to observe that the musical evidence suggests that composers did not always conceive of non-functional resolutions of seventh sonorities as inevitably weaker than functional resolutions.[38] For these reasons, I have adopted a perspective for this study that allows for a more vertical awareness than might otherwise be called for. The repertoire I will examine requires a certain degree of acknowledgement that the simultaneities heard have a valid vertical identity as well as a horizontal one.

EXTRAPOLATION VS. INTERACTION

The third dichotomy is a contribution of my own that distinguishes two contrasting theoretical conceptions of seventh sonorities, which I have termed extrapolation and interaction. Is it always valuable to understand seventh sonorities as adaptations of a functionally similar triad? Extrapolation is a well-established perspective that understands the seventh purely as an extension of its base triad. Its antithesis, interaction, is a less-traditional perspective that understands seventh sonorities as comprising two triads, an upper and a lower,

[38] Bass, "Enharmonic Position Finding," 74.

both of which can be of equal importance. I address the issue because it provides a conceptual foundation uniquely helpful in my approach to a parsimonious seventh-sonority *Tonnetz*.

In this section I will deal with two contrasting approaches to conceiving of seventh sonorities. To my knowledge, a dichotomy between bottom-up and top-down conceptions of seventh sonorities has never been raised. It is common to think of a seventh as the upper extension of a base triad, most likely due to the lingering affect of older understandings, such as *Stufentheorie* and its underlying presupposition of a key. While this notion is absolutely necessary and beneficial under certain musical conditions, namely functional harmony, it favors a bottom-to-top conceptual framework, which may, in other conditions, be undesirable.

In current music theory pedagogy, the typical use of the term *seventh chord* usually indicates one of five qualities of four-pitch sonorities that commonly occur in Western music as a result of diatonicism. Yet *seventh chord* is sometimes applied more loosely to any four-pitch sonority forming a tertian structure. I will distinguish between these two nuances by referring to the more typical use as *normative* and the looser use as *non-normative*.

EXTRAPOLATION

Extrapolation is the term I will use to denote the typical bottom-to-top conception of harmonic structures. The term is so called because it acknowledges the seventh chord as an upward extension of triadic norms. Extrapolation tends to work in a prescriptive way. It is more speculative and reasons by means of "if-then" relationships. That is, *if* stacking a normative third (M3 or m3) upon another normative third results in a normative triad (Table 2.1), *then* stacking another normative third atop the triad should result in a normative seventh chord. That seventh is *measured* against the root (and named in reference to the quality of the seventh formed), but it is still extrapolated from that root based on the template originating in the distance of the third above the root and the fifth above the third. Ninth chords and larger extended tertian sonorities are further extrapolations along this same line of reasoning, i.e., the continued stacking of thirds (amounting to the same thing as the continued addition of odd-numbered interval quantities above the root). While this process does generate some normative seventh sonorities, it does not produce all of them, and it generates non-normative sonorities. Table 2.2 illustrates this point, even including non-normative thirds ($^{+}3$ and $^{o}3$).[39]

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[39] The augmented triad's semi-normative status leads me to exclude it from Table 2.2.

Table 2.1: Triads formed from stacked thirds

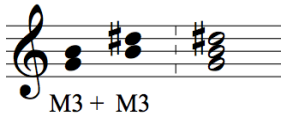
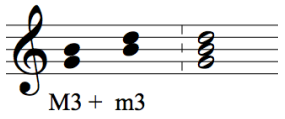
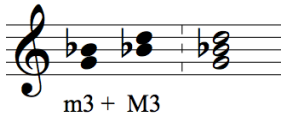
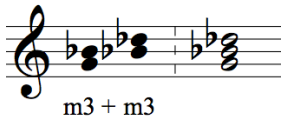
Name	Composed of
augmented	
major	
minor	
diminished	

Table 2.2: Seventh-sonority formation from extrapolated stacking of thirds

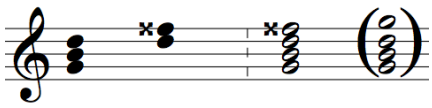
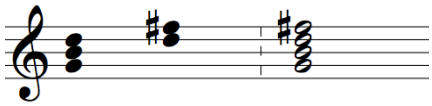
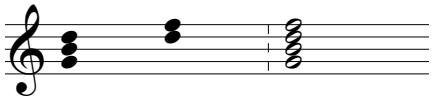
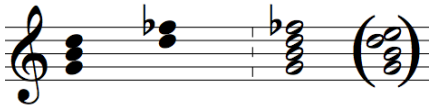
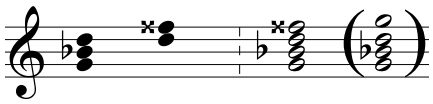
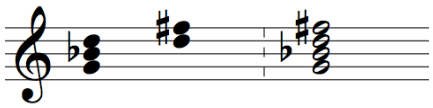
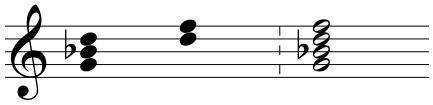
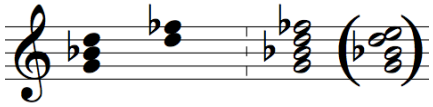
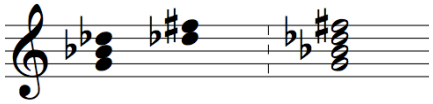
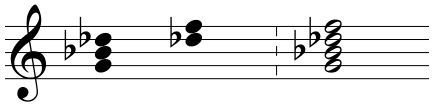
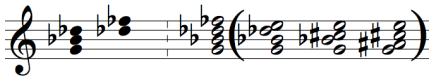
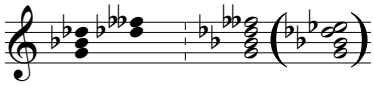
Triad	Added Interval	Seventh Sonority Extrapolation	Example
major	$^{+}3$	***non-normative*** (≈ major triad)	
major	M3	major-major	
major	m3	major-minor	
major	$^{o}3$	***non-normative*** (≈ minor-minor)	

Table 2.2 continued

Triad	Added Interval	Seventh Sonority Extrapolation	Example
minor	+3	***non-normative*** (≈ minor triad)	
minor	M3	***non-normative***	
minor	m3	minor-minor	
minor	º3	***non-normative*** (≈ half-dim.)	
diminished	+3	***non-normative***	
diminished	M3	half-diminished	
diminished	m3	fully-diminished	
diminished	º3	***non-normative*** (≈ major-minor)	

Seventh sonorities in musical contexts often seem to have originated as extensions of triadic counterparts. In other words, when a brief melodic passage might be harmonized by a given triad, a composer can add a diatonic seventh above the root of that triad to achieve a more colorful chord of the same harmonic function. The chord's function within the key does not change but is

enhanced by dissonance, or given a new luster. I will presently discuss examples of these sorts of transformations, such as 8–7 motion in an upper voice over two root-position chords, in the context of Brahms Op. 117, No. 2.

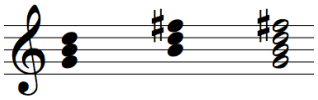
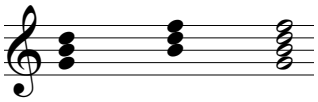
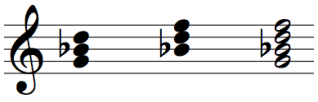
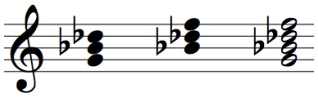
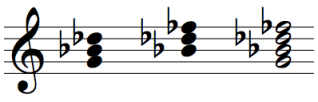
Extrapolating all types of triads with all types of added thirds results in a broad array of tertian combinations that have very narrow, if any, application in musical works. One might argue that good music theory does the opposite: narrow the possible combinations in order to have broader applicability when analyzing music. Normative seventh sonorities emerged from this pool of possibilities as more common because of the limitations of diatonicism. As chromaticism increasingly broke down the barriers of diatonicism, the limitations imposed upon extrapolated seventh sonorities would, one would think, break down as well. But in most of the late-nineteenth-century literature, composers continued to favor normative seventh sonorities, even when diatonicism only loosely applied, if at all. This peculiarity necessitates the need to think differently about seventh sonorities in highly chromatic, parsimonious contexts.

Interaction

Interaction is the term I will use to denote the flexible conception of harmonic structures from either bottom-up or top-down. The term is so called because it acknowledges that seventh sonorities contain two overlapping

normative triads. Interaction is descriptive and is based more on what is, than what might be. Reasoning from this perspective reverses the "if-then" logic statement to become a "so-because" statement. That is, normative seventh sonorities are *so*, *because* they consist of two overlapping normative triads whose roots are a third apart and which share two common tones (illustrated in Table 2.3). These conditions are considerably more stringent than those used in the extrapolative theory, thus allowing for wider applicability.

Table 2.3: Seventh-sonority formation from interacting triads

Lower Triad		Upper Triad	Seventh-Sonority Result	Example
major	+	minor	major-major	
major	+	diminished	major-minor	
minor	+	major	minor-minor	
diminished	+	minor	diminished-minor	
diminished	+	diminished	diminished-diminished	

Interaction can be thought of as more in line with the theoretical legacy of Hugo Riemann (or the "reimag(in)ing" of his theories),[40] which suggests a more fluid perception of the root and an emphasis on common tones. Riemann himself was interested in sonority symmetry, noting the inversional equivalence of major and minor triads. Figure 2.1 illustrates similar inversional symmetries among seventh sonorities in a nod to Riemann's notion of the *Oberklang* and *Unterklang*. Like Riemann's treatment of major and minor triads, other authors (Gollin,[41] Childs,[42] et al.) have commented on the invertible relationship of the major-minor and half-diminished sonorities, and the inversional symmetry of the fully-diminished sonority is well-known. The inversional symmetries of the major-major and minor-minor sevenths, however, have been overlooked, likely due to the relatively less important nature of these sonorities in functional contexts.[43]

Let us examine, as a point of departure, the most pervasive and arguably most essential seventh sonority, the major-minor seventh. This sonority occurs

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[40] Hyer, "Reimag(in)ing Riemann," 101–138.

[41] Gollin, 195–206.

[42] Childs, 181–93.

[43] In other words, the basic interval pattern (BIP) of the major-major sonority, whether read from bottom to top or top to bottom, is M3-m3-M3. The situation is similar in the cases of the minor-minor seventh, whose BIP is m3-M3-m3, and the fully-diminished seventh, whose BIP is m3-m3-m3. This is contrasted with the major-minor seventh's BIP, M3-m3-m3, which when read in reverse is the half-diminished seventh's BIP, m3-m3-M3.

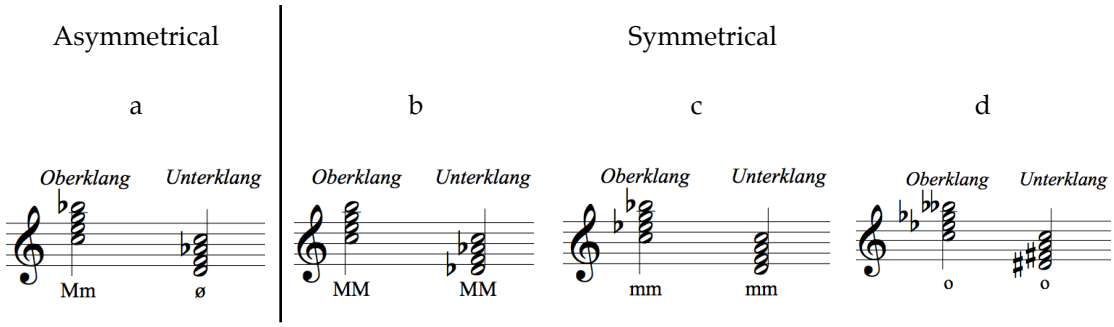


Figure 2.1a–d: Seventh-sonority inversional symmetry

diatonically in only one form, on the dominant (and so is frequently labeled "V^7" or *dominant seventh*), and as such performs the vital function in tonal harmony of drawing the ear toward tonic. The V chord performs the same tonicizing function, as does the vii°, and the roots of these two chords are a major third apart. When these two chords are joined into one sonority, the combined harmonic and functional stability V and the tension of vii° create a stronger gravitational pull toward tonic than either the V or vii° achieves individually.[44]

Extrapolative reasoning explains this relationship in terms of a diatonic seventh atop an already functional V chord; in this regard, it behaves in much the same way a diatonic seventh added to any already functional triad, such as a I^7 or IV^7. But this perception hardly accounts for the aural power and ubiquity of

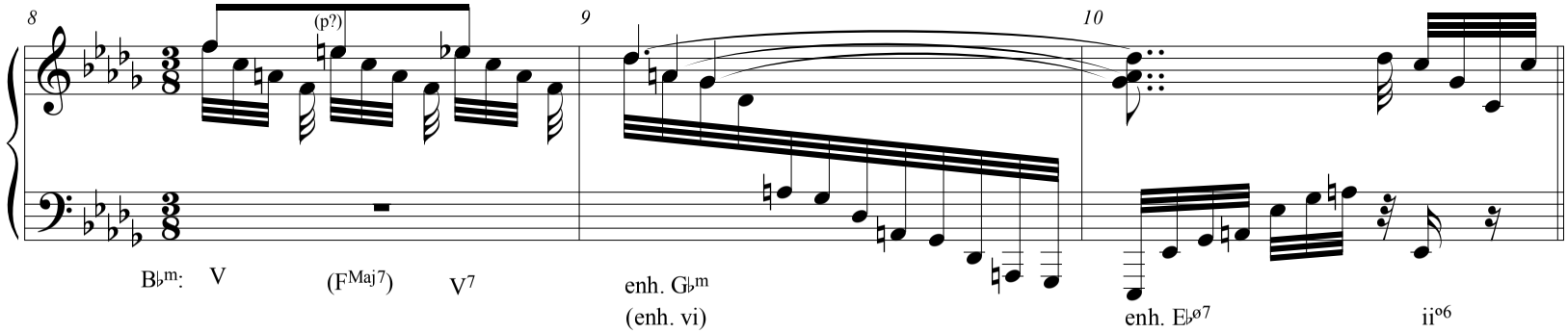
[44] In the context of a key, the V chord is consonant, acts as a harmonic pillar, places emphasis on the relatively stable ◊5, particularly when it is in the bass. The vii° chord provides a dissonant alternative to the V chord, made up entirely of tendency tones (◊4 in addition to ◊2 and ◊7 shared with V).

the V^7 in contrast to that of the I^7 or IV^7. On the other hand, interactive reasoning provides an explanation as to why the V^7 plays such an important role in Western music. Whereas the extrapolative view subsumes the vii^o under the umbrella of "V-ness," the interactive view acknowledges that there is a significant difference between the tonicizing powers of V and V^7, and this is due to the powerful tendency tones belonging to the vii^o portion of V^7.

Brahms's Op. 117, No. 2 provides in microcosm an illustration of the merits of an interactive perspective, and even how it may merge with an extrapolative one. Example 2.3, m. 8 demonstrates the extrapolative labeling of seventh chords (F^{Maj7} in m. 8.2 and F^7 in m. 8.3) as mere variants of their base triad (F in m. 8.1). Typically, the seventh ($E♮$) of the F^{Maj7} would be dismissed as a passing tone, even though it is given just as much durational, articulative, and dynamic weight as the 7th of the F^7. For this reason, I have added a question mark to the passing-tone label. This harks back to Richard Bass's argument that simultaneities can certainly have a valid vertical identity as well as a horizontal one. More to the point, the content of m. 8 is easily understood, and accurately so, as extrapolative; i.e., it is simply the metamorphosis of an upper extension to the same fundamental triad, F major.

In contrast, mm. 9–10 are better explained from an interactive perspective. Measure 9 features a "misspelled" $G♭^m$ which is reinterpreted after passing the

Example 2.3: Brahms, *Three Intermezzi*, Op. 117, No. 2, mm. 8–10



bar line as an enharmonic E♭ø7.[45] Conventional labels, which are inherently extrapolative, mask the reality that G♭m and E♭ø7 have just as close a relation as do F and F7 (in terms of parsimonious voice leading); in each case, the triad is embedded within the seventh sonority. An interactive explication can accommodate such behavior by recognizing the triadic identity of a seventh chord's upper three pitches. In contrast, an extrapolative perspective only recognizes the triadic identity of the lower three pitches of a seventh chord, which implies a difference between F to F7 and G♭m to E♭ø7. In some contexts that difference matters; in others, the difference is less important than the similarity.

Example 2.4 provides a passage by Brahms from later in the same piece which is considerably more "normal." This is a complementary passage to that in Example 2.3 and is similar in its initial rhythm and a sweeping downward arpeggio that transforms a triad into a seventh sonority. However, it differs significantly–it is functional, tertian chords are spelled correctly, and it even features a brief circle-of-fifths sequence. One could quite comfortably understand these three measures in terms of extrapolation (or interaction).

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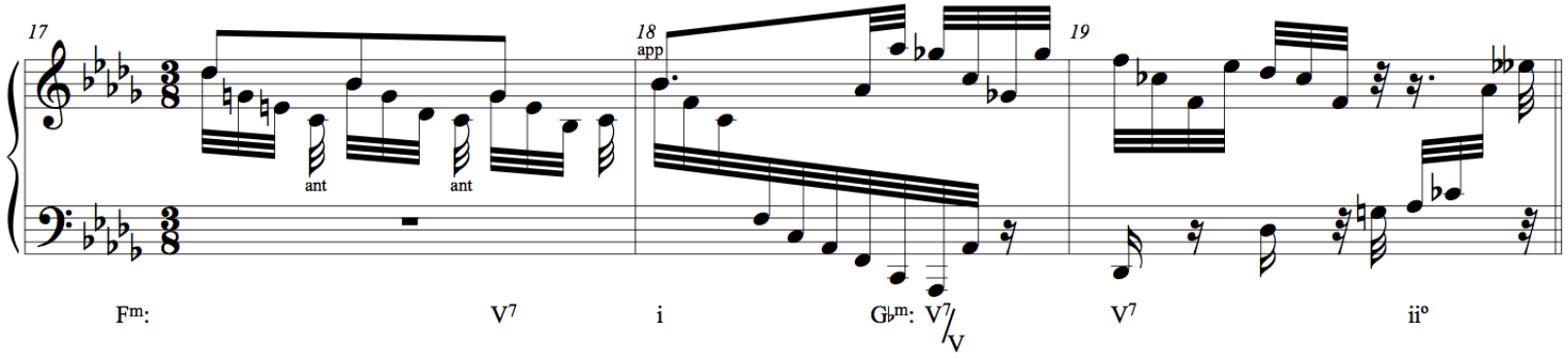
[45] The transition from m. 8.3 to 9.1, from F7 to an enharmonic GΩm, is similar to a purely triadic slide transformation in which A, the third of F7, continues as the third of GΩm, substituting for BΩΩ. Additionally, the incorporation of GΩm into EΩø7 is an example of Julian Hook's *inclusion transformation* (2002), a concept that is closely related to my idea of interaction.

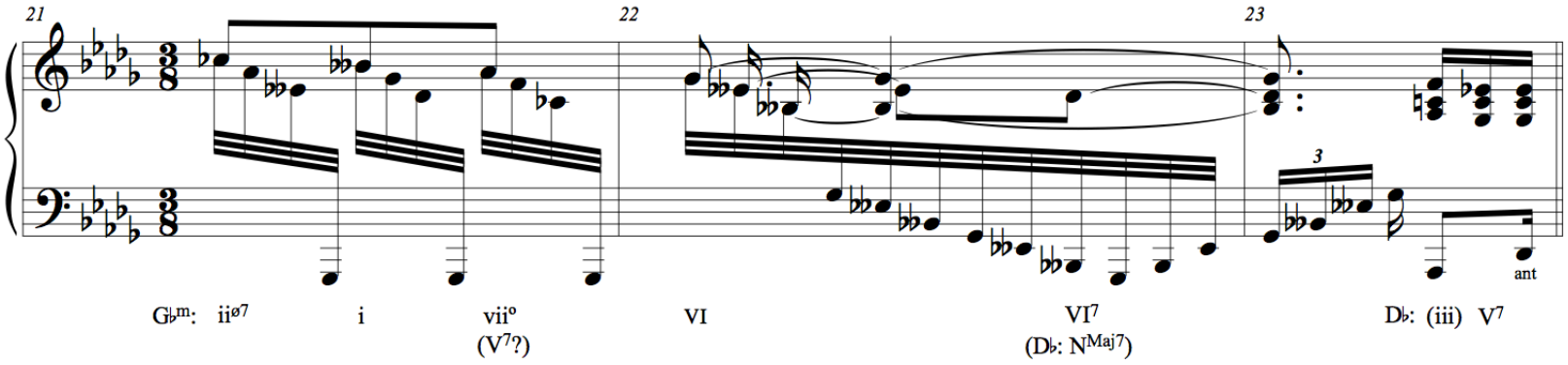
If Example 2.3 can be thought of as a thesis, then Example 2.4 is its antithesis, and Example 2.5 will provide the synthesis. This last complementary passage from Op. 117, No. 2 follows right on the heels of that in Example 2.4. A lingering aural sense of D♭ from m. 21.2 projects itself onto the vii° of m. 21.3, implying an interaction. Measure 22 features a triad transformed into a seventh sonority via a sweeping arpeggio, as have the foregoing examples. But in this iteration, the addition of D♭ in beat three creates a major-major seventh sonority, consistent with an extrapolative perspective, whose function is possible (though unusual and/or awkward) in both the old and new keys. That D♭ works both linearly (as a means of transition first to the E𝄫 of m. 22.1 and then to the C of m. 23.2) and harmonically (as a chord tone implied by the sheer length of its presence in mm. 22.3–23.2). Again, interaction can explain the whole passage, while extrapolation can only account for m. 22.

For the purposes of the theory put forward in the present study, and given the nature of the music to which it applies, I favor the interactive argument over the extrapolative one. In the highly chromatic music of the late-nineteenth century, the principles of functional harmony often gave way to chord connections that were simultaneously more free and more conservative. In that kind of musical climate seventh sonorities could emerge from directions other

Example 2.4: Brahms, *Three Intermezzi*, Op. 117, No. 2, mm. 17–19



Example 2.5: Brahms, *Three Intermezzi*, Op. 117, No. 2, mm. 21–23



than the root. Extrapolation and interaction are not universally applicable thought processes. Different harmonic contexts require different approaches. Just as physics has shown that light can behave as both particles and waves, so music can be considered to feature extrapolative and interactive qualities, or horizontal and vertical qualities.

In physics, classical mechanics refers to the understanding of the physical universe as articulated by Sir Isaac Newton and his followers. Classical mechanics assumed certain constants, such as Euclidian geometry and Galileo's principle of relativity, perfectly addressing the vast majority of contexts anyone encountered. What Newton did not know is that these physical principles were only valid for an object above a certain size and below a certain speed. If one considers the atomic level, or a traveling object exceeding the speed of light, these laws become moot. In some ways, classical mechanics versus quantum field theory is an apt analogy for the extrapolative theory versus the interactive theory. In a majority of the contexts musicians will analyze, an extrapolative perspective, like Newtonian physics, is perfectly apt. But in less ordinary circumstances, interaction better explains observed phenomena.

My discussion of these dichotomies should in no way indicate that I hold some to be irrelevant or outmoded. On the contrary, those I do not adhere to are simply not appropriate to the musical contexts I wish to examine. The musical

examples I will analyze later tend to adhere to a relatively parsimonious harmonic syntax that, while perhaps behaving linearly, certainly has a harmonic identity I wish to recognize. An extrapolative approach only accounts for seventh-sonority construction as an upward extension; this is inadequate to my task. The methodology of interaction solves this problem by neutrally allowing for upward or downward extensions of the triad. The theoretical models I will present in the following chapter, and the musical analyses in the chapter after that, rely to a certain extent on understanding these assumptions.

CHAPTER III: THEORETICAL METHODOLOGY

INTRODUCTION

The theory of parsimonious seventh-chord relationships I propose in this chapter will be both reasonably comprehensive and analytically applicable. Readers may note some similarities between my proposed theory and that published by others, most recently a 2008 article by Callender, Quinn, and Tymoczko.[46] However, even though their article does address parsimony among tetrachords, it is fair to say that their approach is distinctly different, primarily in its more purely mathematical approach and in its equal inclusion of non-tertian "sonorities" (e.g. [0000] or [0066]). They do illustrate parsimony among tetrachords (I use the term broadly here) in three-dimensional space, which I will also do, but their approach takes a step away from actual music and focuses more closely on theoretical possibilities.

What I propose in this chapter is an alternative model of harmonic space that can be used to map actual musical passages. In some ways, my work owes more to Edward Gollin's theory[47] than to those of Cohn,[48] or Callender, Quinn,

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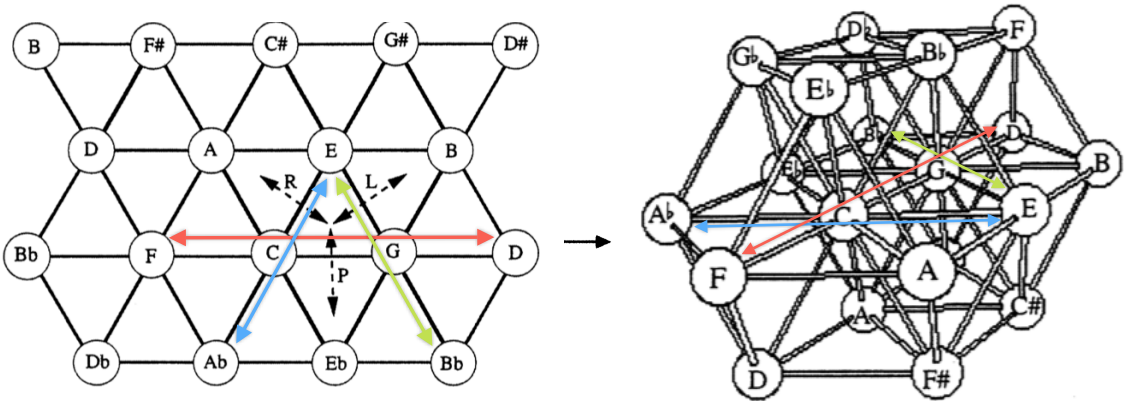
[46] Callender, Quinn, and Tymoczko, 346–348.

[47] Gollin, 195–206.

[48] Cohn, "A Tetrahedral Graph"

and Tymoczko. Gollin generates his three-dimensional, seventh-chord *Tonnetz* quite logically from the preexisting two-dimensional triadic *Tonnetz* of the neo-Riemannian school. In so doing, he creates a very elegant system of major-minor and half-diminished seventh chords, both of which are manifestations of set class (0258). Just as a *Tonnetz* can represent the inversional attribute of major and minor triads with upward and downward pointing triangles, respectively, so Gollin illustrates the analogous inversional quality of set class (0258) with upward and downward tetrahedrons. Figure 3.1 reproduces a neo-Riemannian *Tonnetz* (left) and his molecule-like model (right), a complicated image that requires some decipherment. I have annotated Gollin's figures with colored arrows to help orient the reader when viewing his 3D model. The red arrows I have added run the length of an axis of perfect fifths, the blue arrows run along an axis of major thirds, and the green arrows run along an axis of minor thirds. The 2D *Tonnetz* on the left is then turned about and foreshortened, making the red P5 axis seem to recede into the background. Thus the plane on the left is incorporated into the figure on the right to form the bases of upward- and downward-pointing tetrahedrons. For clarity, I have rendered my own simplified cutaway region of Gollin's model in Figure 3.2 to isolate an upper tetrahedron (a major-minor seventh sonority).



Gollin's "Figure 1" (p. 197)

Gollin's "Figure 2" (p. 198)

Figure 3.1: Annotated versions of Gollin's seventh-chord *Tonnetz*

a

b

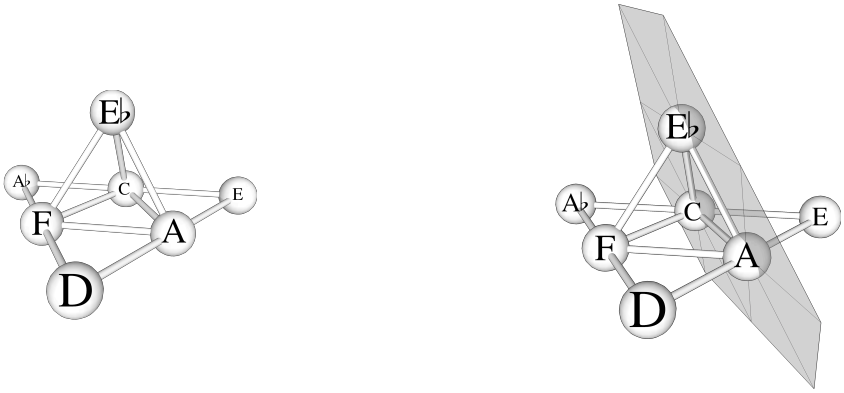


Figure 3.2a–b: Seventh-chord tetrahedron extrapolated from a horizontal 2D *Tonnetz*, after Gollin

Figure 3.2 continued

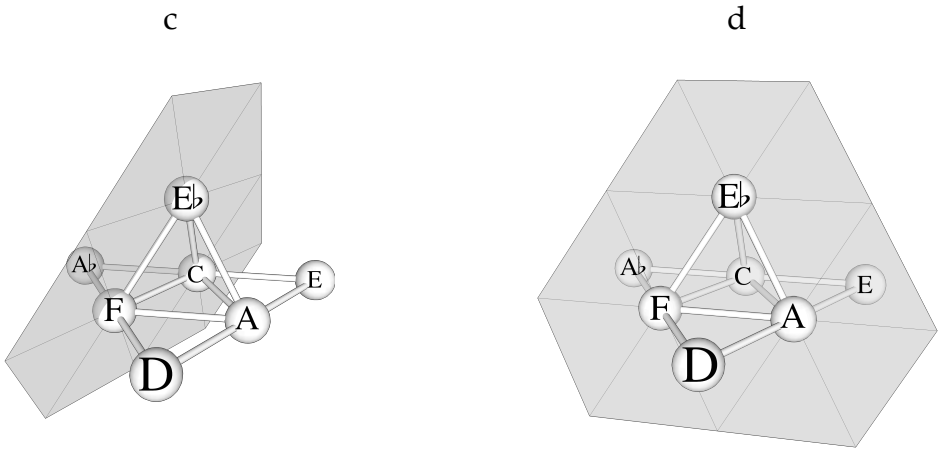


Figure 3.2c–d: Seventh-chord tetrahedron extrapolated from a horizontal 2D *Tonnetz*, after Gollin

As elegant as it is, Gollin's 3D model only accommodates two seventh-chord types, the major-minor and half-diminished. These two types are admittedly prolific in common practice tonal music and serve an important tonicizing role within functional harmony, but what about other types of seventh chords, or contexts that involve non-functional sonorities (such as "linearly generated" sonorities)? The reason that Gollin's model does not incorporate more chord types is because his approach assumes that the seventh is extrapolated above the base triad. At first, Gollin's model may appear interactive, given that the plane highlighted in Figure 3.2b is made up of diminished triads (036). That plane, plus the horizontal plane of consonant triads (037), implies interaction.

However, Gollin's model positions the seventh of the chord equidistant from all other chord members. This placement implies two assumptions: (1) the sonority is really conceived as extrapolation, i.e., F major + E♭; and (2) the geometry gives equal importance to two additional planes of irrelevant trichords, shown in Figures 3.2c (025) and 3.2d (026).

The implications of such a disparity lead me to a geometric model with an internal logic that distinctly favors an interactive perspective. From an interactive approach, if all normative seventh chords can be thought of as the combination of two normative triads, then they can begin to be understood in terms of a typical 2D *Tonnetz*.

SONORITY GEOMETRIES

Figure 3.3 shows that the major-major and minor-minor sonorities already exist within the plane of a 2D *Tonnetz*.[49] (These sonorities are latent within Gollin's model.) The major-major and minor-minor sonorities are simple to accommodate because they result from the interaction of consonant (major or minor) triads. The major-minor, half-diminished, and fully-diminished sonorities all incorporate one or more diminished triads not immanent in the structure of a conventional

[49] Figures 3.3 and following assume enharmonic equivalence, using pitch-class integers rather than letter names.

2D *Tonnetz*. As we have observed, Gollin's model contains a plane of diminished triads (Figure 3.2b), a dissonant *Tonnetz*, by virtue of layering conventional consonant *Tonnetze*.

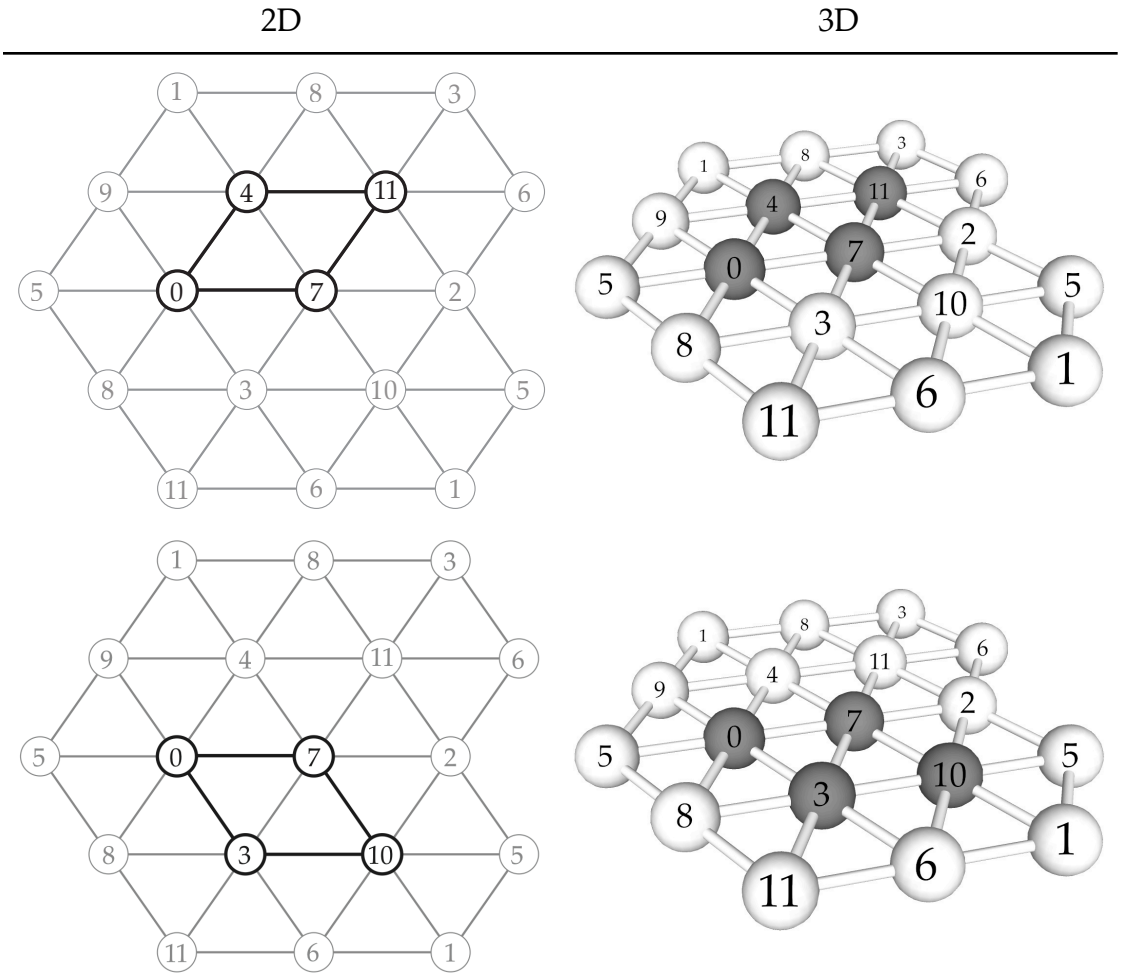


Figure 3.3: Interacting triads within a *Tonnetz* generate major-major and minor-minor chords

In Figure 3.4, I have layered consonant *Tonnetze* in such a way that three different vertical, dissonant *Tonnetze* can connect them. This model preserves the plane containing diminished triads while eliminating the irrelevant ones. Such stratification of *Tonnetze* could hypothetically continue in either direction ad infinitum.

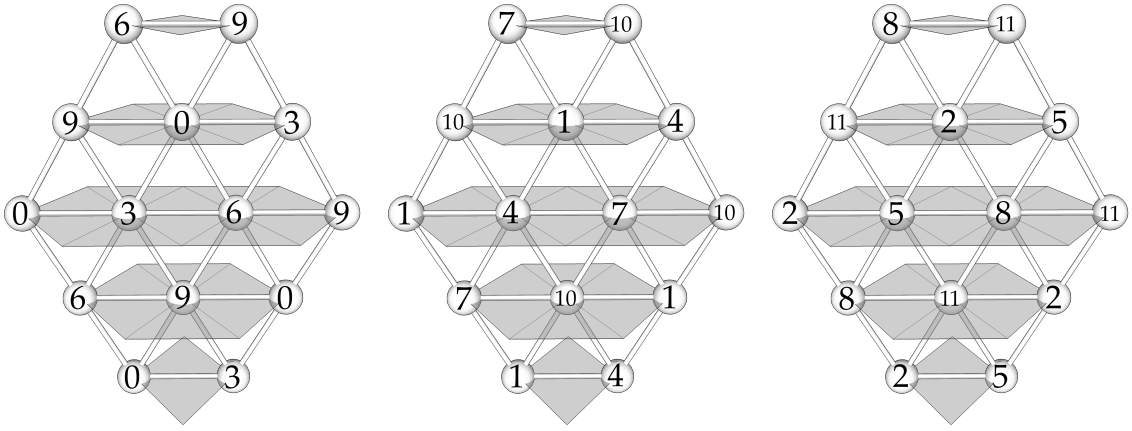


Figure 3.4: Three vertical "dissonant *Tonnetze*" of diminished triads resulting from horizontally layered consonant *Tonnetze*

Each vertically oriented diminished triad shares a common boundary with two horizontal consonant triads (one major and one minor). Combining triads that share such a boundary results in major-minor and half-diminished sonorities. Figure 3.5 illustrates these two chords as they would appear in what I

will refer to as a tiered-*Tonnetz*. The opposing orientation of the major-minor and half-diminished chord geometries is analogous to their inversional relation and to Gollin's depiction of them as upward and downward tetrahedrons.

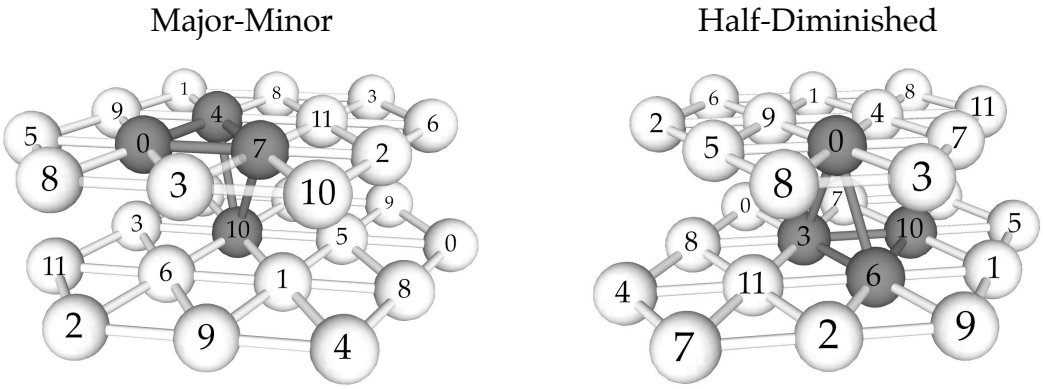


Figure 3.5: Interacting triads within a tiered-*Tonnetz* generate major-minor and half-diminished chords

Only the fully-diminished sonority remains to be explained in terms of a *Tonnetz* (or tiered-*Tonnetz*). Each diminished triad in the vertical plane not only borders two consonant triads, but also borders other diminished triads. Combining these diminished triads results in fully-diminished seventh chords, as shown in Figure 3.6.

It is noteworthy that the geometries of the major-major, minor-minor, and fully diminished sonorities also reflect their respective inversional symmetries by existing in only one plane—the major-major and minor-minor in the horizontal

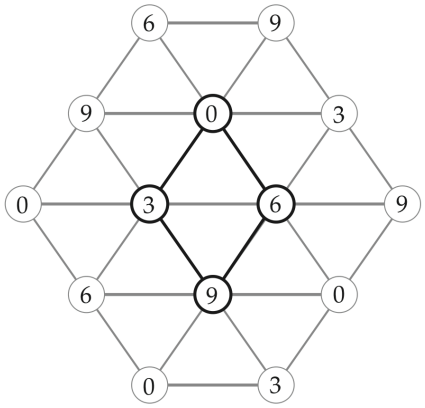


Figure 3.6: Interacting diminished triads within the vertical "dissonant *Tonnetz*" generate fully-diminished sonorities

plane (Figure 3.2), and the fully-diminished in the vertical plane (Figure 3.6). Indeed, we can see from Figure 3.6 that the same fully diminished sonority could easily be reoriented in several directions but always staying within its plane. This property is perhaps a geometric analogue to the multiple enharmonic manifestations of fully-diminished sonorities.

SONORITY NETWORKS IN TWO DIMENSIONS

It is difficult to understand how these chord types, as depicted thus far, can form far-reaching parsimonious networks. If we were interacting with the forgoing model in physical space, we could lay out the major-major, minor-minor, and fully-diminished geometries on the floor and they would all have

certain features in common. Each sonority, would be depicted as a square set on point, with each corner representing one pitch member of the chord. By orienting the root at the top, the third is always immediately clockwise from the root, and the seventh is always opposite the root (just as on the *Tonnetz*). Figure 3.7 summarizes this structure in what I will refer to as a seventh chord *hub*. Note that the root is partitioned off to help orient the viewer.

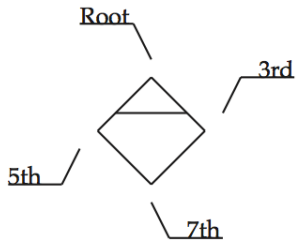


Figure 3.7: Anatomy of a seventh-chord hub

In the figures that follow, I adopt a shorthand labeling system for normative seventh chords, one that utilizes pitch-class integers rather than letter names. Taking a cue from Brian Hyer's use of plus and minus signs to indicate major and minor, I will do the same with the addition of a small circle to denote the diminished quality.[50] Thus the major-major quality is denoted with ++, etc., as summarized in Table 3.1.

[50] Hyer, "Reimag(in)ing Riemann," 107ff.

Table 3.1: Summary and guide to normative seventh chords

Name	Symbol Used in This Study	Set Class
major-major	++	(0158)
major-minor	+–	**(0258)**
minor-minor	– –	(0358)
diminished-minor (half-diminished)	°–	**(0258)**
diminished-diminished (fully-diminished)	°°	(0369)

By "flattening" all the seventh-chord geometries formed from the consonant and dissonant *Tonnetze* (Figures 3.2 and 3.5), all five types become easily comparable and, when combined, can form an array of relationships. In Figure 3.8, 0+– forms the center of an array. Included for each chord is its musical notation equivalent. A hollow note head indicates a pitch in common with its 0+– complement; a filled note head indicates a pitch one-semitone removed from its 0+– complement. A line connecting points (pitches) of two hubs (chords) indicates which chord member moves by semitone and which chord member it becomes. Ascending along a line represents upward motion, and descending along a line represents downward motion. The array in Figure 3.8 illustrates the property that raising the seventh of 0+– results in 0++, but lowering the seventh of 0+– results in 9– –. Similarly, various members of a family of enharmonically equivalent fully-diminished chords (10^{oo}, 1^{oo}, 4^{oo}, and 7^{oo}) are all related to 0+– by

single semi-tone displacement. Their various spellings each claim a different pitch as the root, but the same pitch class, whatever chord member it may be or enharmonic spelling it may have, can descend by semitone to form 0+–.

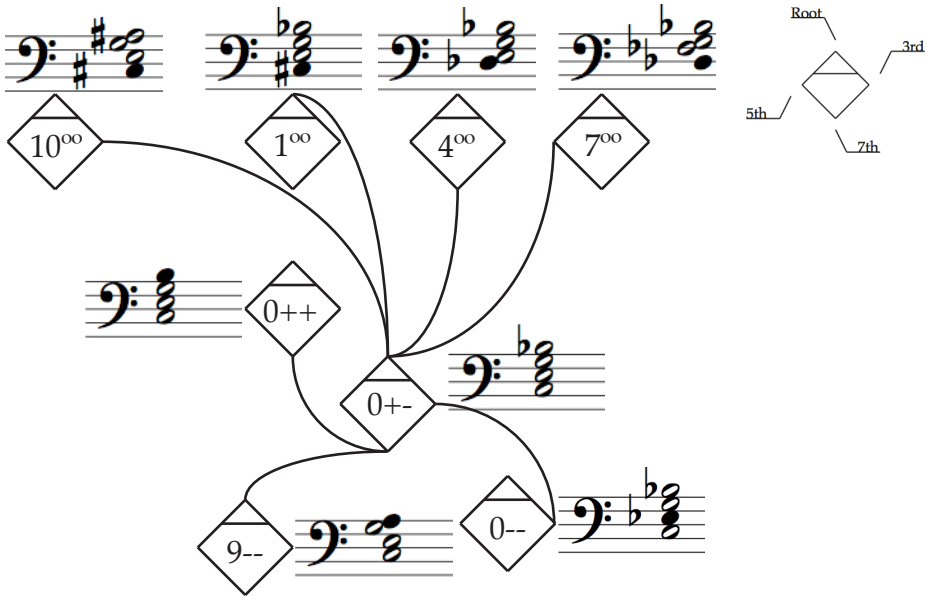


Figure 3.8: Array of parsimonious relationships to 0+–

Such an array is helpful when examining all relationships to a particular sonority, but it cannot accommodate any relationships more than one semitone displacement away from the "nucleus" sonority. Figure 3.9 depicts a strand of seventh chords derived from the array, but continuing the pattern beyond one transformation. This strand is but a fragment of a longer chain of possible semitone transformations. In this chain, the transformational pattern (++, +–, – –, °–) repeats every four chords. We could then parse the entire chain into twelve

families, each family built on a different chromatic pitch of the octave, and each family would contain four members (++, +–, – –, o-). The entire chain would contain forty-eight chords, eventually linking back to its beginning.

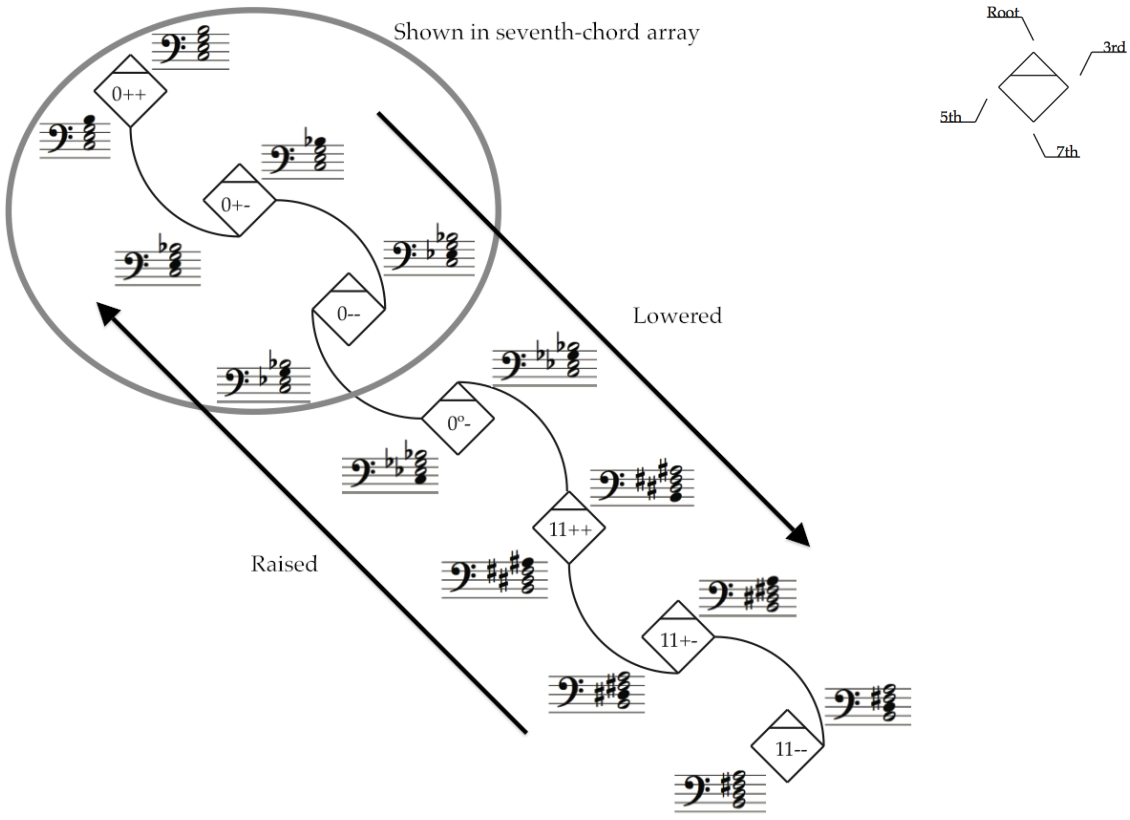


Figure 3.9: Segment of the continuous chain of ++, +–, – –, °– transformations

Even though Figure 3.9 only visualizes a segment of an infinitely looping chain, one can conceive of such a loop as an extension of the mod-12 pitch-class integer loop. I refer to the chain that this segment represents as the Primary Axis (or P axis). A transformation along this axis made by lowering a member by

semitone (e.g. 5– – to 5°–) is labeled -*P*.[51] Likewise, a transformation along this axis made by raising a member by semitone is labeled +*P*. Figure 3.10 shows such an arrangement, based on the segment used in Figure 3.9. The connections that link these three P-axis segments form an intermittent axis that runs roughly perpendicular to the P axis. I refer to these intermittent connections as lying along the auxiliary axis (or A axis), which, when combined with the P axis, form a portion of a lattice of seventh-chord parsimony.

Four strands of the P axis can be aligned along the A axis before sonorities are replicated. In Figure 3.11, the rightmost P-strand seems to end with 6°–. Lowering the root of 6°– by semitone results in the 5++ chord that appears in the leftmost P- strand. From Figure 3.11, then, we can infer that not only can the P axis eventually be wrapped upon itself, but so too can the A axis. Such a cyclical structure would result in a three-dimensional geometric figure resembling a torus. Along this torus the P axes would bend and wrap around the surface like candy cane stripes.

The organization of pitch-class space into the seventh-chord lattice I have shown remains incomplete insofar as it neglects the fully-diminished seventh chord. Although the fully-diminished sonority is best assimilated into the lattice

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[51] This should not be confused with the neo-Riemannian P (parallel) transformation, which represents crossing between major and minor triads with the same root.

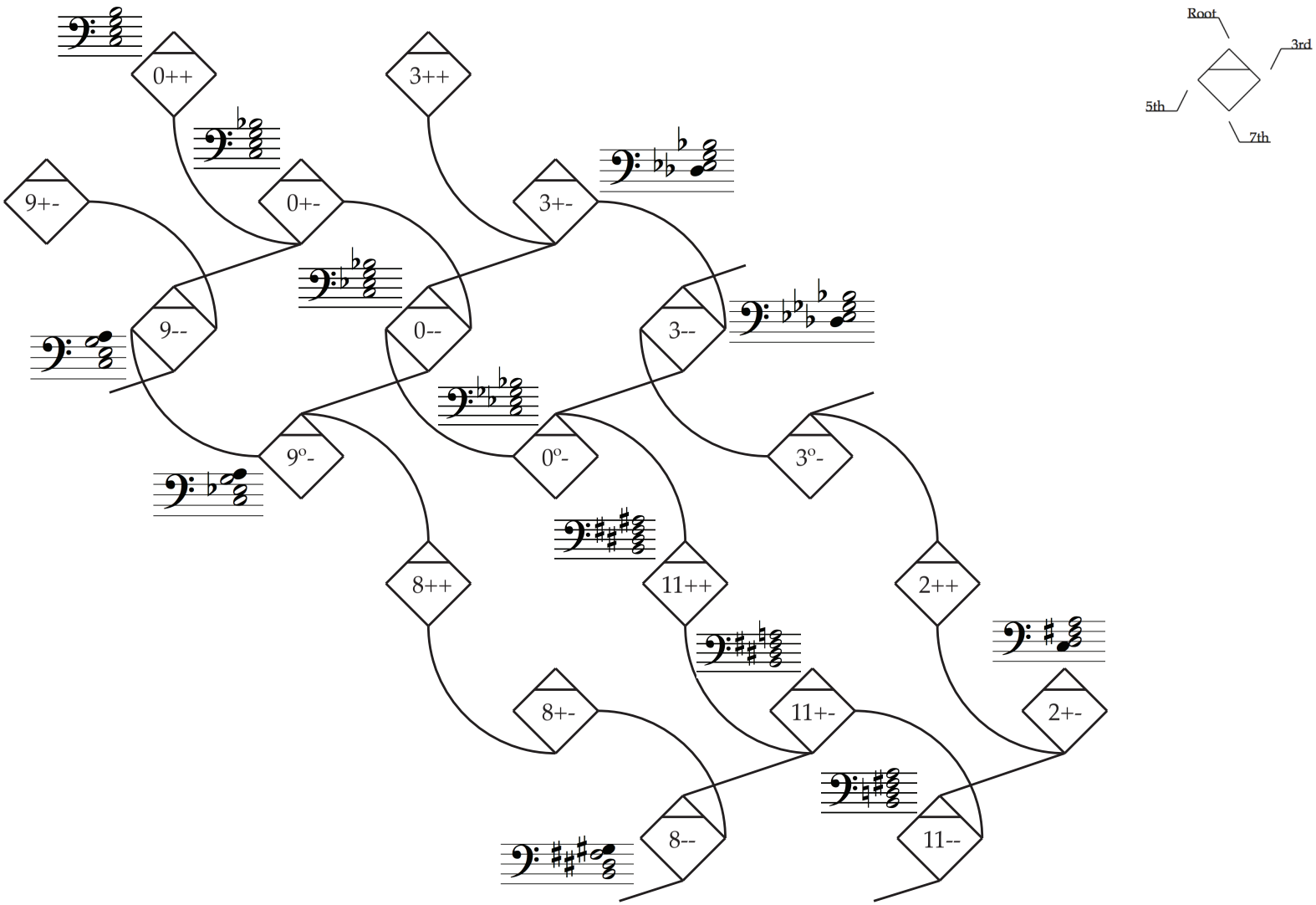

Figure 3.10: Auxiliary axes resulting from aligned P-axis segments

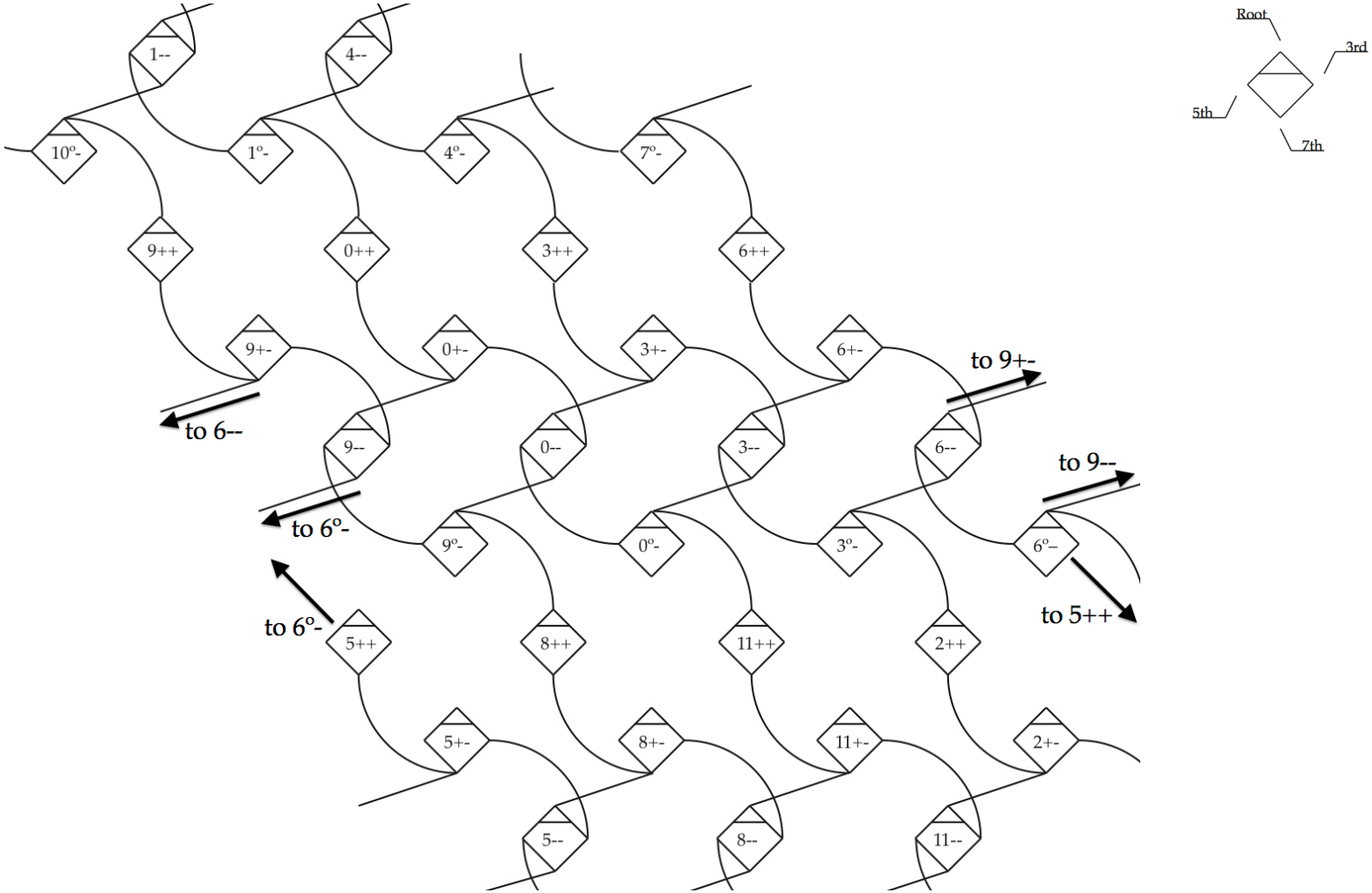

Figure 3.11: Sonority replication after four P-strands

in three dimensions, it can be squeezed into the margins of the two- dimensional lattice, with the understanding that these chords exist in another plane or dimension.[52] Figure 3.12 represents the fully-diminished sonorities in grey as a reminder that they do not inhabit the same dimension as the rest of the lattice. Enharmonically equivalent fully-diminished chords are grouped by proximity, with four chords per group, a phenomenon that meshes well with the logic of the lattice (which replicates sonorities after four strands are aligned). Whereas four P-strands produce nominally different fully-diminished chords, these strands actually generate exactly the same sonority, each claiming a different chord member as the root. Each of these chord spellings can act as the mouth to a harmonic "wormhole" network that can whisk the listener to a distantly related point on the lattice.

Sonority Networks in Three Dimensions

If we were to curve the two-dimensional surface of the lattice of Figure 3.12 such that it wrapped back upon itself along both the P and A axes, the resulting geometry would resemble a skeletal torus. This can be represented two ways. The first, in Figure 3.13, orients the P axis as stretching around the torus

[52] In keeping with my physics metaphor, these "other" dimensions inhabited by fully-diminished seventh chords are perhaps analogous to the theoretical Multiverse, which posits the possibility of alternate universes.

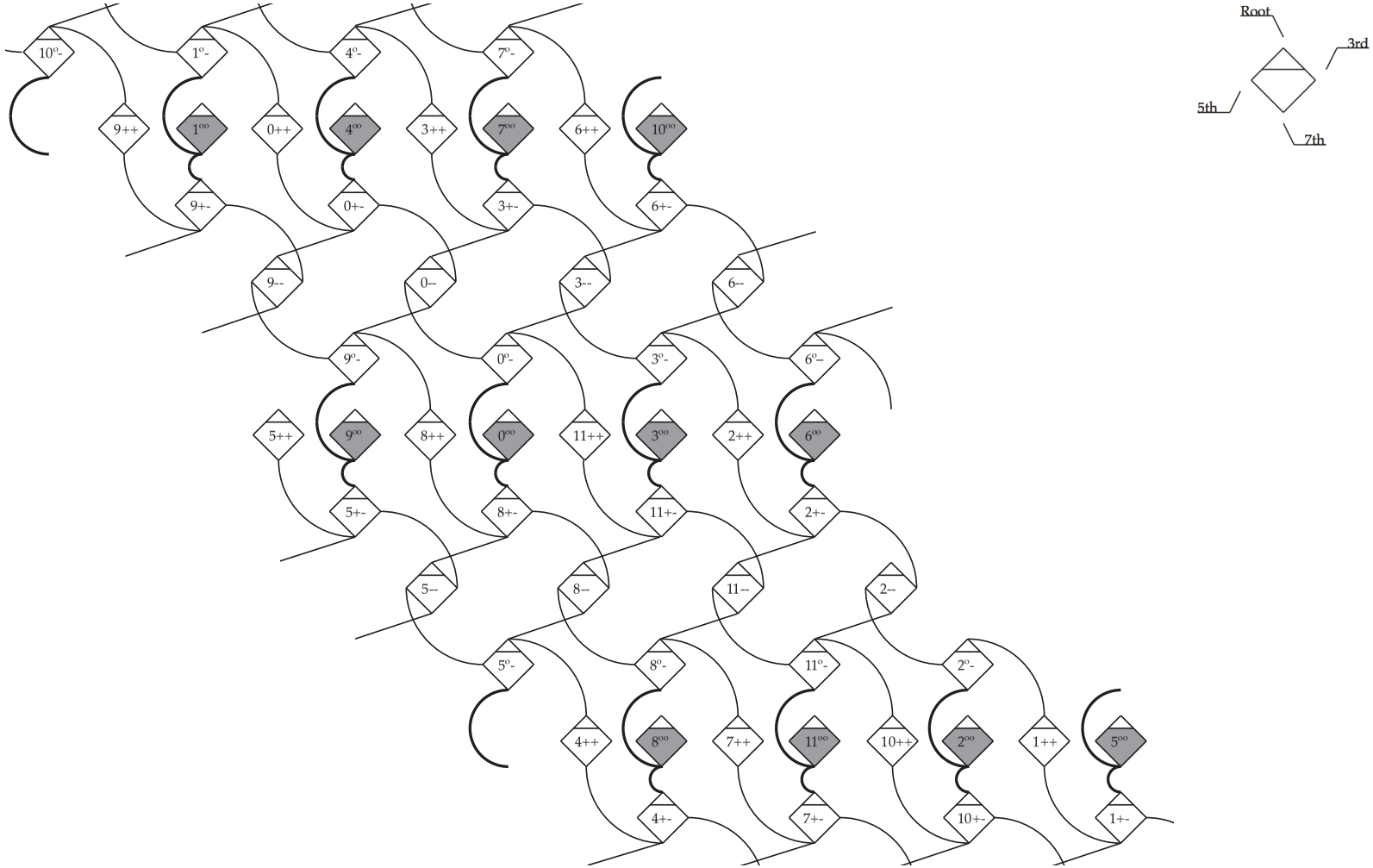

Figure 3.12: Comprehensive lattice region

primarily in the toroidal direction while the A axis is oriented in a quasi-poloidal direction. The second in Figure 3.14 orients the P axis to spiral through the poloidal direction while the A axis stretches around the toroidal direction.[53] Granted, neither axis

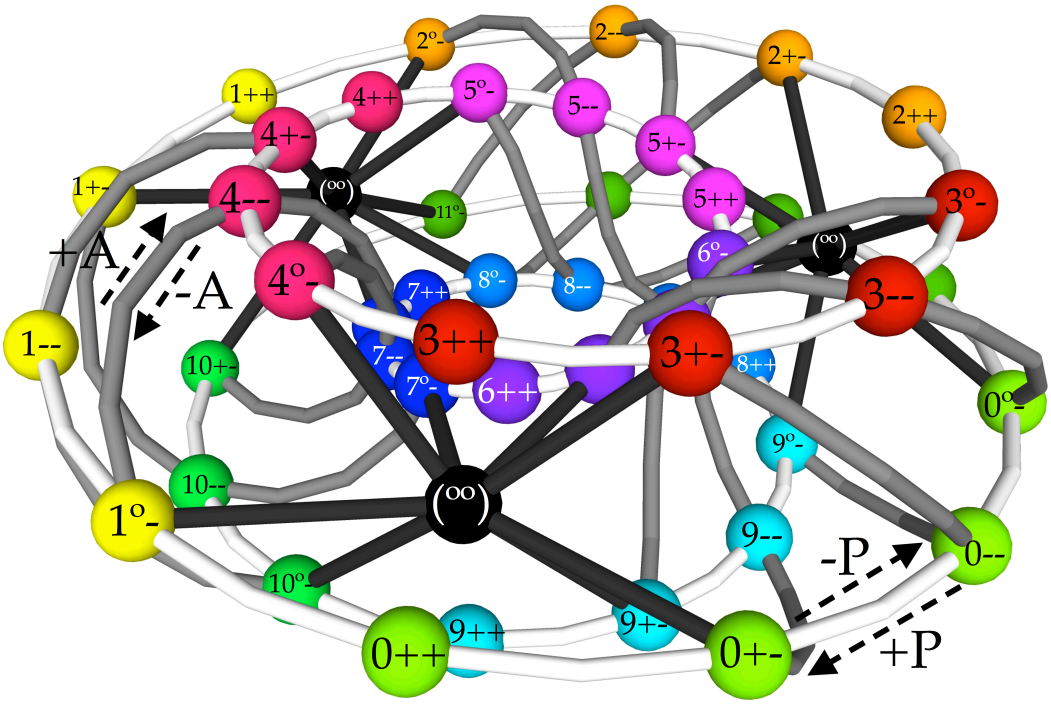

Figure 3.13: Torus with A axis oriented quasi-poloidally

[53] In my prior 3D images, each spherical node represented a single pitch class. However, from this "zoomed-out" perspective, where we are no longer examining networks of individual pitch classes but whole sonorities, the spherical nodes represent chords. Just as Schenkerian sketches can feature the minutiae of the foreground or the substructure of the background, so I have done in an analogous way in these models. The colors added to Figures 3.13, 3.14, and 3.20 bear no significance other than as a means of clarification to organize images containing so many sonorities.

truly adheres to these directions because they spiral around the torus. This trait is analogous to the earth's magnetic and geographic poles' misalignment, resulting in magnetic field lines that are offset from the geographic longitudinal lines.

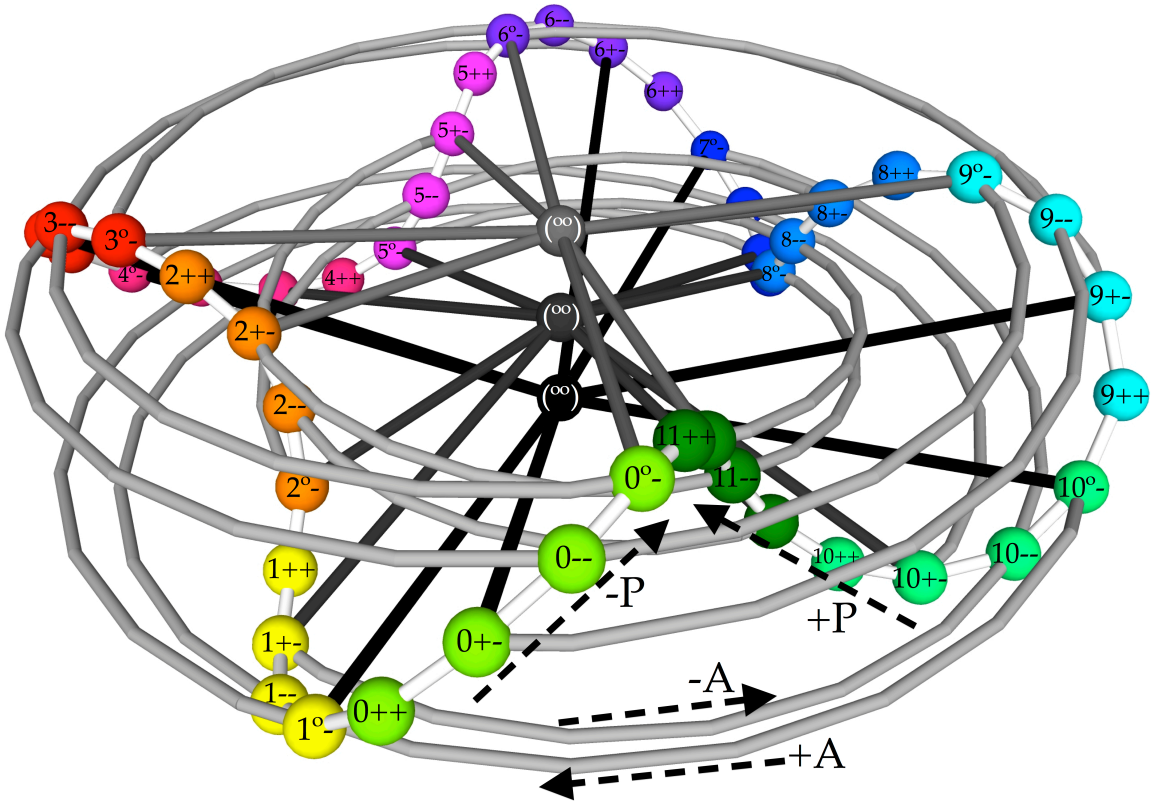

Figure 3.14: Torus with P axis oriented quasi-poloidally

The foregoing models offer a means of representing seventh-sonority movement by semitone, but a discussion of seventh-chord parsimony would be incomplete without also addressing whole-tone motion. Seventh-chord

connection by whole tone is arguably less parsimonious than by semitone,[54] but I include it as a practical consideration. The cycle of chords pictured in Figure 3.15 bears some resemblance to the circle of fifths in that all twelve pitch classes are present as roots (although each occurs twice as two different chord qualities), with tritones polarized. Around the circle, chords alternate between ++ and – – qualities, with roots related by alternating major and minor thirds.

A similar cycle can be constructed of +– and °– chords, a relationship treated at length by Childs[55] and Gollin.[56] Figure 3.16 illustrates this cycle. The ordering of chord roots is identical to that of Figure 3.15. The transformational pattern of the cycle in Figure 3.16 is slightly different, however. For example, 4°– must move its seventh by whole tone to transform into 0+–, an operation like those found in the first whole-tone cycle of Figure 3.15. Conversely, 0+– must have its seventh *and* its third lowered by *semitone* (a divided whole tone) in order to transform into 9°–. Any transformation in Figure 3.16 is the product of two discrete semitone motions achieved alternately by one member or divided between two members. Each cycle bears a closely related internal symmetry and

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[54] The neo-Riemannian R transformation, which involves whole-tone motion, is also less parsimonious than the L or P transformations, which each involve semitone motion.

[55] Childs, 181–93.

[56] Gollin, 195–206.

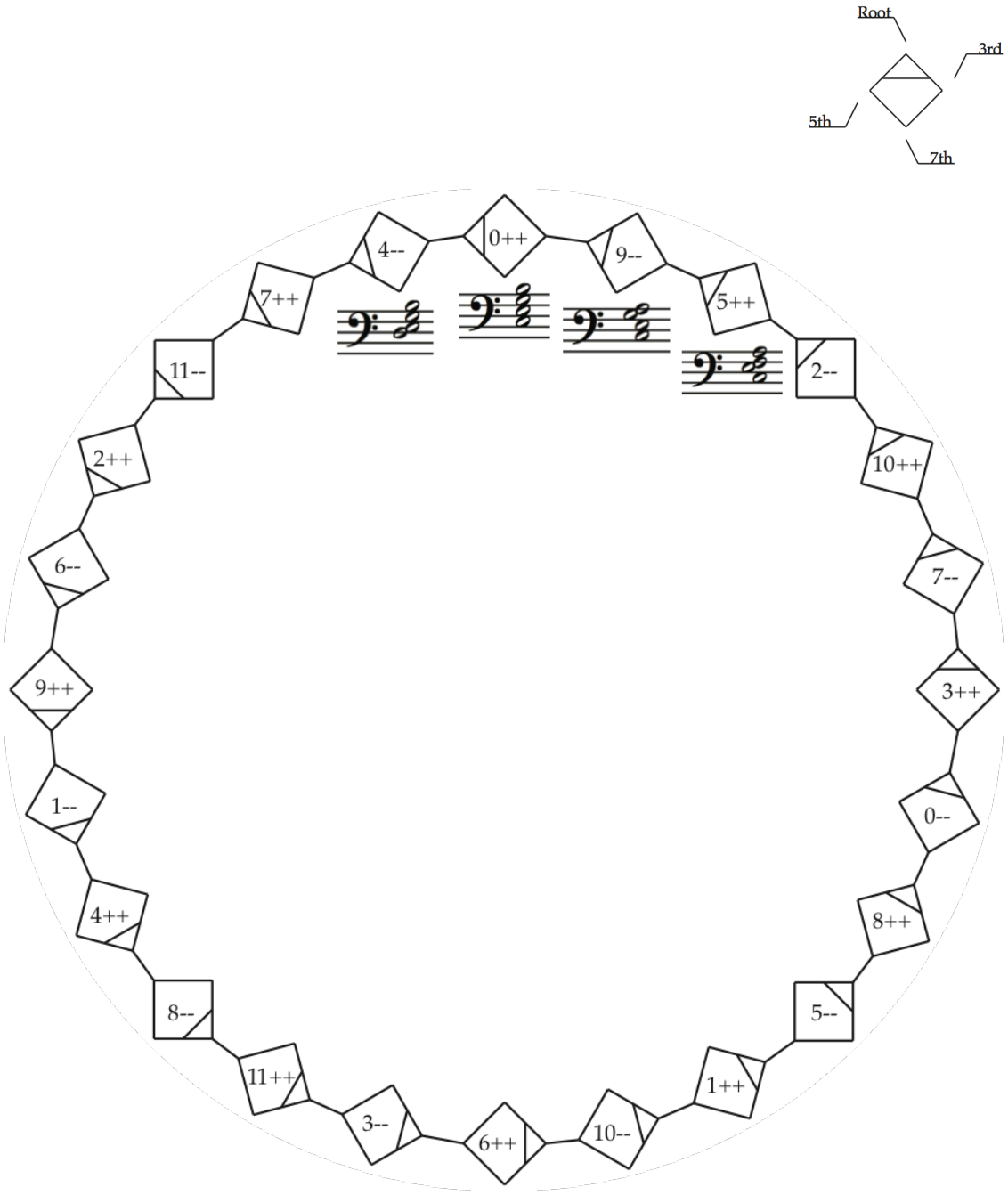

Figure 3.15: Whole-tone cycle of ++ and – – seventh chords

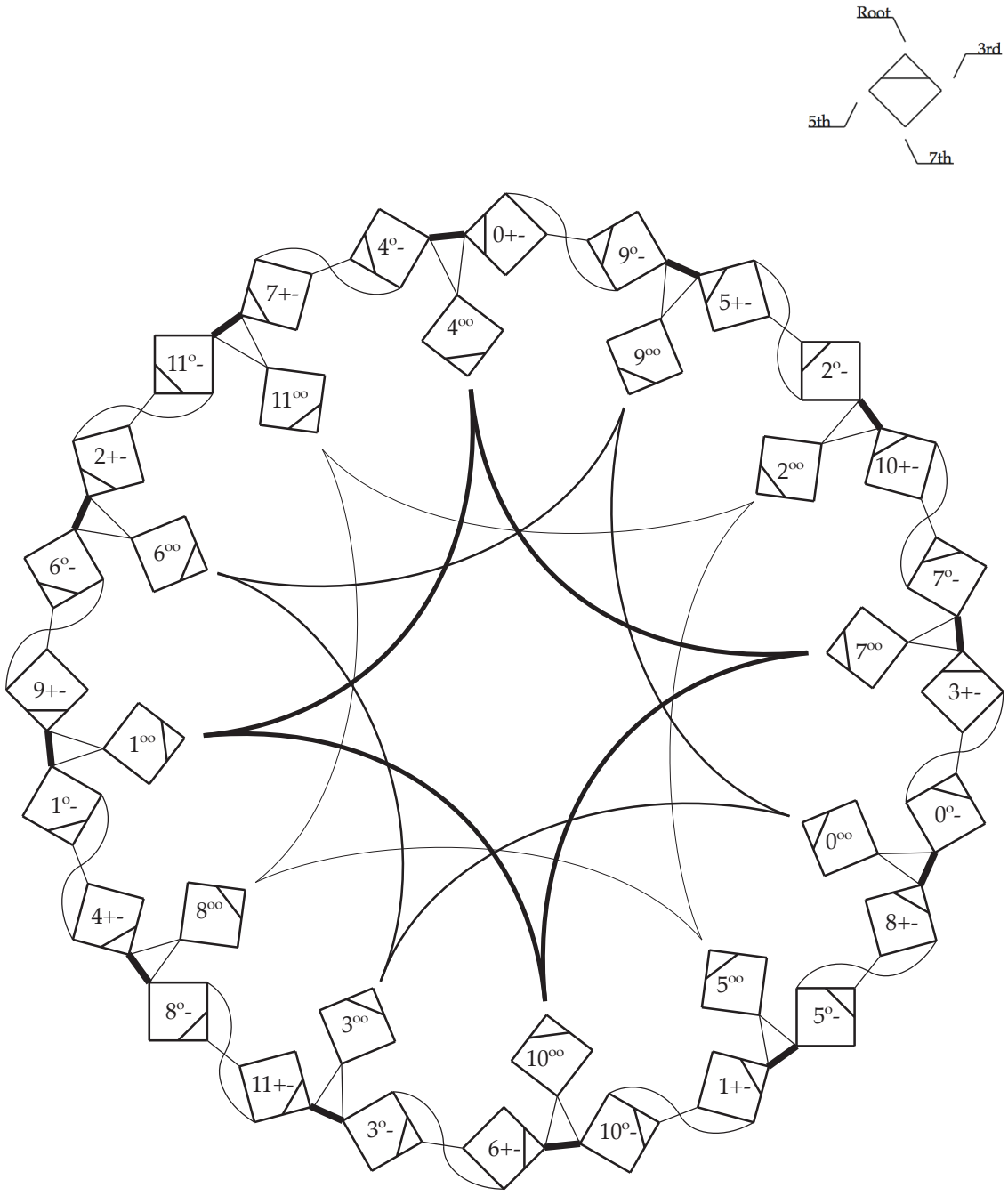

Figure 3.16: Whole-tone cycle of +– and °– seventh chords

regularity of sonority quality, inversion, and root relation, reminiscent of the circle of fifths.

Lines in the midst of Figure 3.16 show the short-cut provided by fully-diminished chords of the same enharmonic family. This is exactly how physicists describe (at least in layman's terms) what a wormhole is. Physicist Richard F. Holman describes them this way:[57]

> Wormholes are solutions to the Einstein field equations for gravity that act as "tunnels," connecting points in space-time in such a way that the trip between the points through the wormhole could take much less time than the trip through normal space.

Figure 3.17 provides visualization of what Holman describes. The curving plane represents normal space, but the wormhole provides a shortcut. In the theoretical models I am building, fully-diminished sonorities often behave as shortcuts to more distant points of the seventh-chord lattice or whole-tone cycles. Looking back at Figures 3.13 and 3.14, we can see that the black connections to the fully-

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[57] Richard F. Holman, "Follow-Up: What exactly is a 'wormhole'? Have wormholes been proven to exist or are they still theoretical?," Scientific American (September 15, 1997), http:// www.scientificamerican.com/article.cfm?id=follow-up-what-exactly-is (accessed October 10, 2011).

diminished spheres shortcut the longer, "surface" lattice distances. We will see this sort of occurrence in the music analyses of the next chapter.[58]

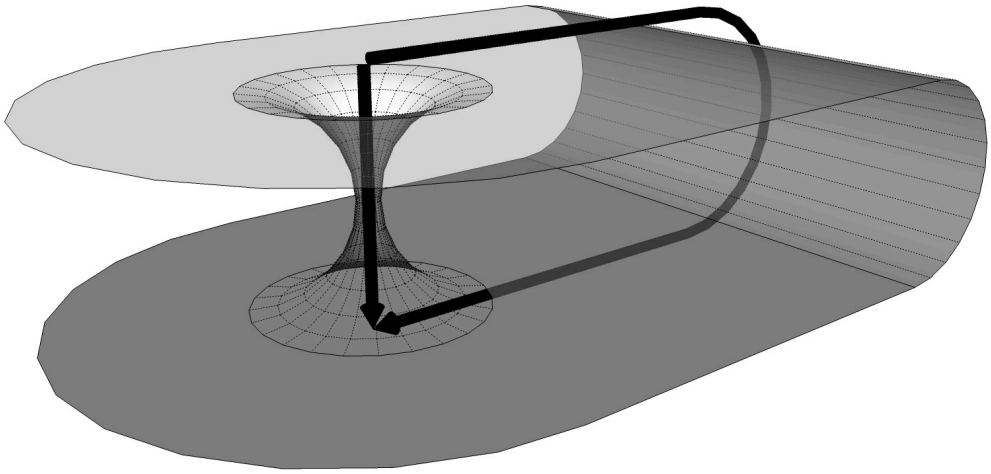


Figure 3.17: A wormhole as a space-time "shortcut"

The chords seen at the top of the first whole-tone cycle (4– –, 0++, 9– –, and 5++) appeared vertically aligned in the same order in the lattice, and these are compared in Figure 3.18. The same principle holds true for the +–/°– whole-tone cycle. The correspondence between the lattice and whole-tone cycles seems to imply a third axis that runs through the lattice. So long as the lattice is held flat, this axis appears to run vertically, as shown in Figure 3.19, with red lines correlating to segments of the rim of the ++/– – whole-tone cycle, and blue lines to segments of the rim of the +–/°– whole-tone cycle. Viewing the lattice in this

[58] This "wormhole property" of the fully-diminished seventh sonority allows for swift enharmonic modulation between distantly related keys as well. See Beethoven's Sonata Op. 13 ("Pathétique"), Mvt. 2, mm. 47–50 for an example.

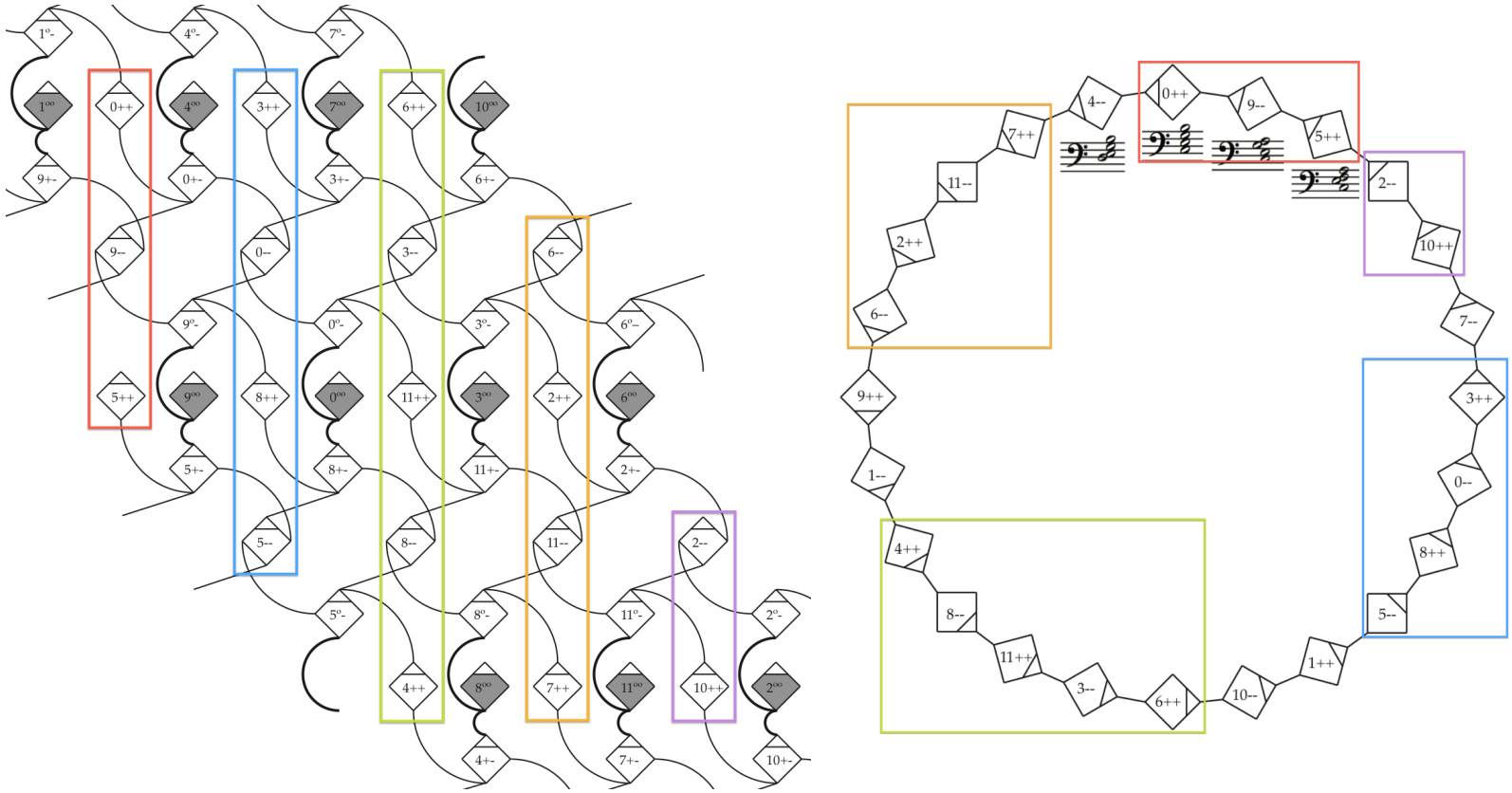
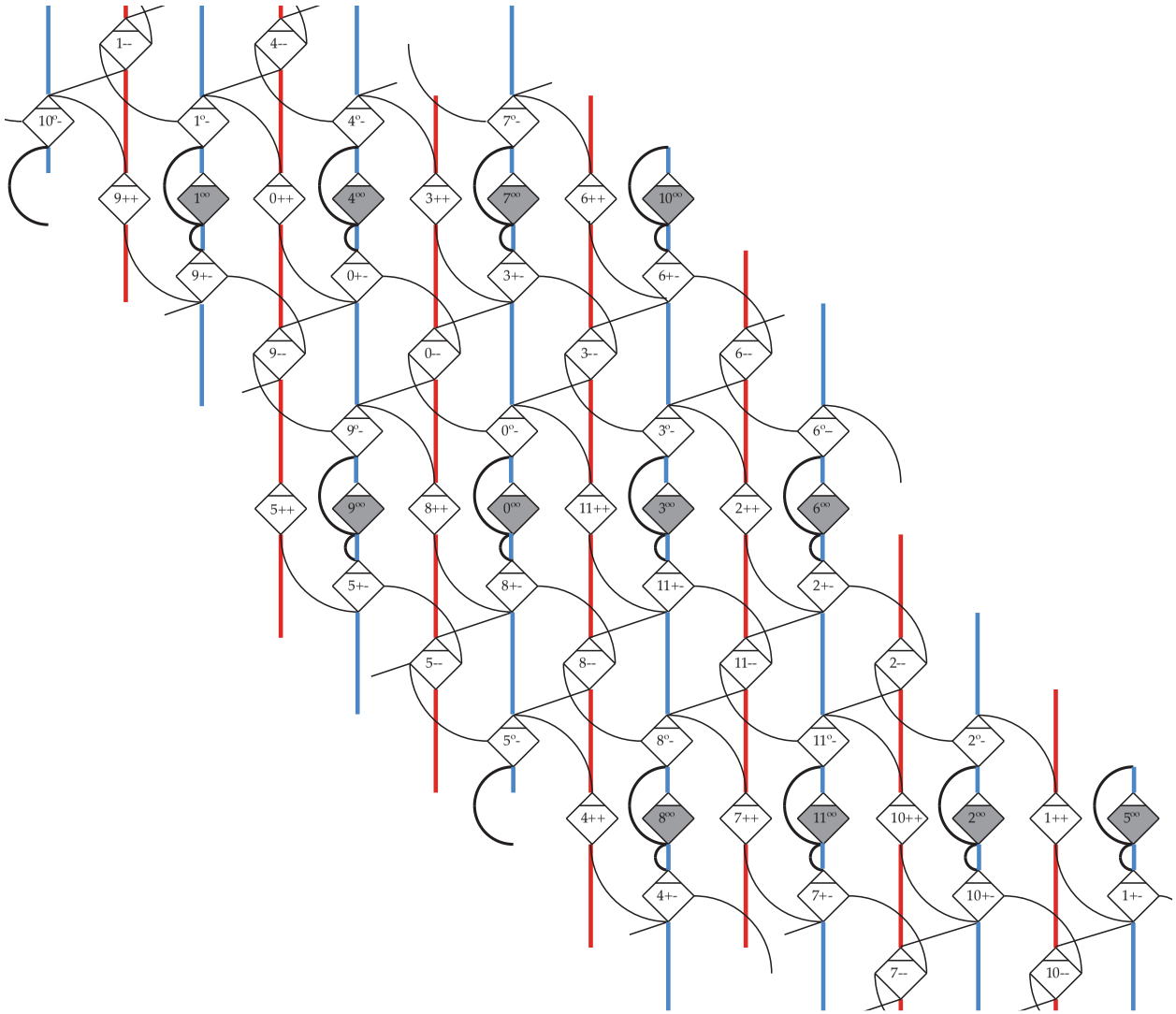


Figure 3.18: Correspondence of the lattice to the ++/– – whole-tone cycle



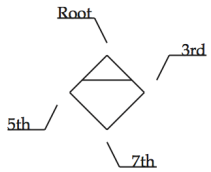


Figure 3.19: Lattice including whole-tone cycle rims

way may be convenient, but it not completely accurate. Like the problems cartographers have long faced when representing a spherical earth on a flat sheet of paper, the lattice is merely a two-dimensional simplification of three-dimensional geometry. Using the whole-tone cycles as cross-sections of a torus (viz. slicing a doughnut), a single lattice can be wrapped around the surface of the torus, uniting the semi-tone and whole-tone transformational models into one "harmonious" model.

Visualizing this in three dimensions in Figure 3.20 is much more helpful than simply describing it. As with the previous toroid models, our view must be "zoomed out" in order to see the big picture, so each seventh chord is represented by a single sphere. In Figure 3.20, only the chords that lie at the intersections between the whole-tone cycles and this one lattice are represented with spheres. This is only to keep the model from becoming cluttered, but chords lie all around each whole tone cycle, even if they are not shown in particular. Only one complete lattice of P and A axes is shown, covering 16.7% of the surface. Five more lattices could fit on the same torus, resulting in six lattice-regions of identical content that differ from each other only in their orientation on the torus. The six lattice-regions of the whole-tone-torus are illustrated in Figure 3.21.

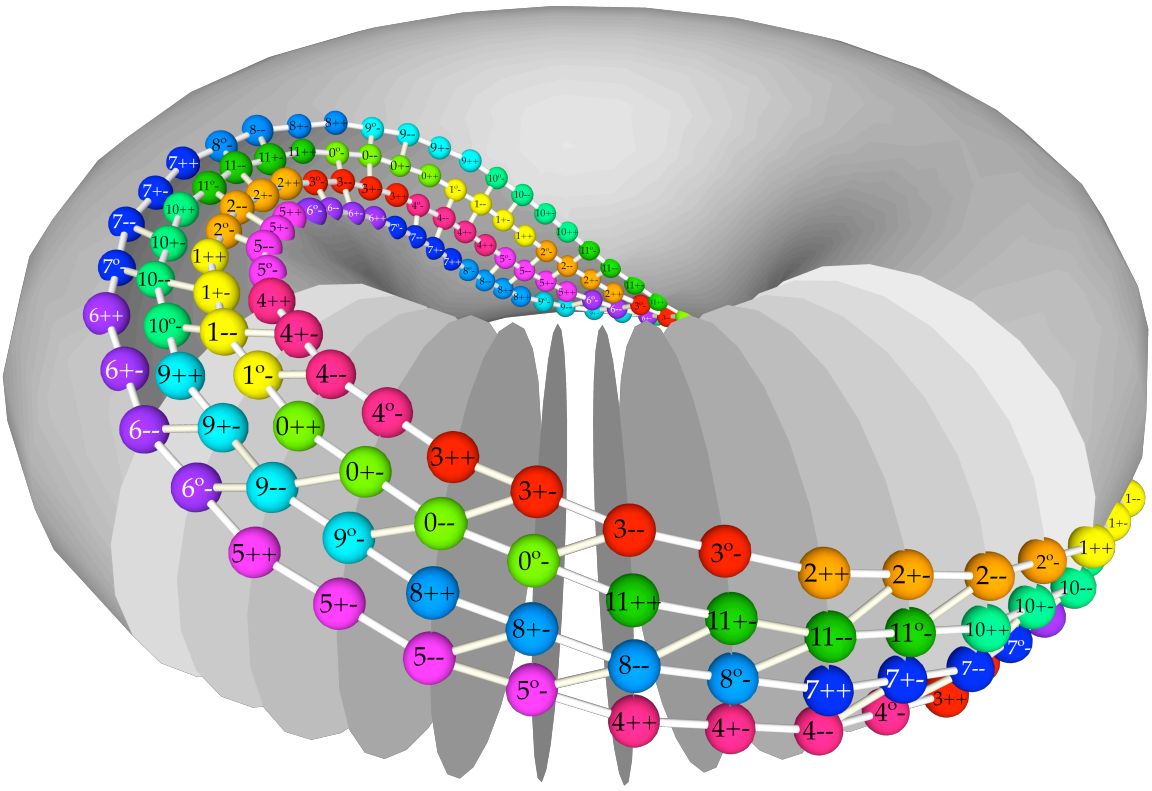


Figure 3.20: Torus accommodating whole-tone cycles

Figure 3.21: Six lattice-regions fitted to one torus

These geometric models provide a number of graphic tools and spatial metaphors for better understanding certain pitch-class space conditions. Each geometric model relates to the others in some way, creating a lattice-family of visual options for understanding admittedly abstract relations. As such, the theory is reasonably comprehensive, in that it accommodates all five normative seventh chords, and in time it may be possible to integrate non-normative seventh chords.

What remains is to see if and how this theory can be applied to music "in the wild," a step sadly lacking in many of the more comprehensive studies. As it happens, this chapter has described the pitch-class landscape sometimes used by certain late-nineteenth-century composers. The theory has mapped that landscape, but the next chapter will retrace the paths that certain composers have chosen to explore it.

CHAPTER IV: ANALYTICAL APPLICATIONS

INTRODUCTION

In this chapter I employ the foregoing theoretical model in the analysis of art music, charting composers' paths through pitch class space in the process. In so doing, passages that have typically been difficult to analyze as anything other than "tonally unstable" or "transitional" can now be analyzed for what they are, rather than what they are not. Understanding the internal structure of these passages via visual maps may answer questions such as: might this help us understand specific composers' harmonic preferences; might this provide a means of comparison among a variety of composers who used a similar harmonic syntax; might this illustrate narrative elements; and might this provide a means of comparison among multiple treatments within a single piece of the same region of harmonic space?

The lattice model, while extensive, is composed of discrete semitone motions (and whole tones if we include the whole-tone cycles). Each of these discrete motions is given a label analogous to the familiar *P*, *L*, and *R* transformations represented by a neo-Riemannian *Tonnetz*. I use +/–*P* and +/–*A* to identify motion along the **P**rimary and **A**uxiliary axes. Transformational letters are coupled with plus and minus signs ("+"/"–") to indicate rising or falling

pitches along these axes.[59] I use *E* to signify **E**nharmonic respelling between two fully-diminished sonorities of the same set class. +/–*V* denotes semitone motion along the **V**ertical axis (i.e., into or out of a fully-diminished seventh chord), and +/–*W*, or "double-*V*," denotes motion by **W**hole tone along that same vertical axis (i.e., a segment of the whole tone cycle). The transformational orthography is summarized in Table 4.1.[60] Examples are illustrated on the lattice in Figure 4.1.

Table 4.1: Transformation symbols

+	=	Ascension along an axis
–	=	Descension along an axis
P	=	Primary Axis
A	=	Auxiliary Axis
V	=	Vertical axis by semitone (into/out of fully diminished 7th [°°])
W	=	Vertical axis by whole tone (some segment of a whole-tone cycle)
E	=	Transit through enharmonic "wormhole"

[59] Thus, *P* refers to the parallel function in neo-Riemannian theory, whereas +*P* refers to upward semitone motion along the P axis and –*P* refers to downward semitone motion along the P axis.

[60] The plus (+) and minus (–) in this context represent movement of one or more pitches. It should not be confused with the plus and minus signs used in conjunction with pitch-class integers, in which plus represents a major triad and minus represents a minor triad. Both uses will appear in the following analyses, but should be easy to differentiate. When plus or minus signs are used with integers they refer to the "majorness" or "minorness" of a given sonority; when they appear with the letters *P*, *A*, *V*, or *W* they refer to upward or downward motion between two sonorities.

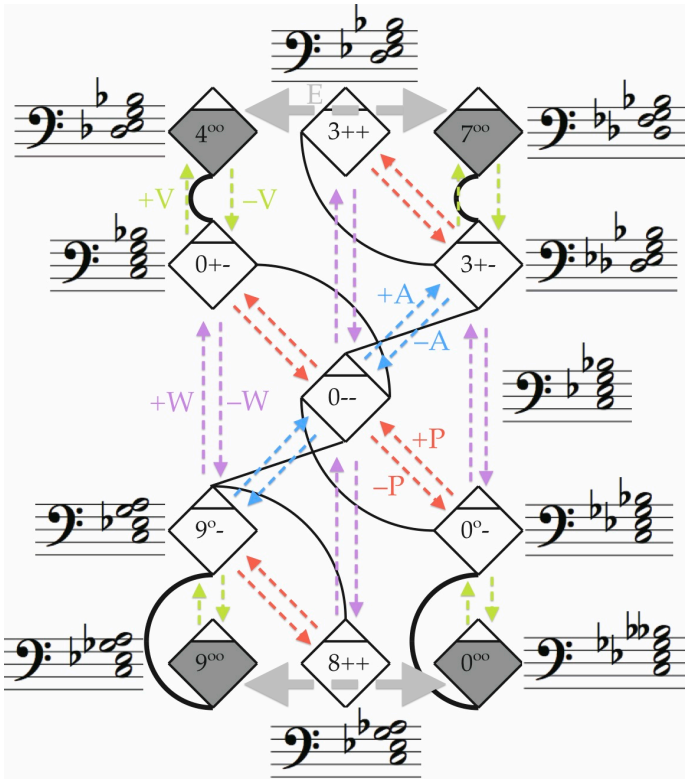


Figure 4.1: Transformational signatures on the lattice model

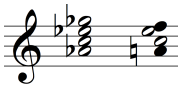
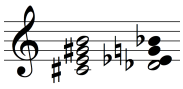
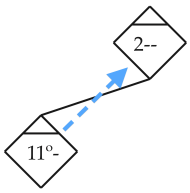
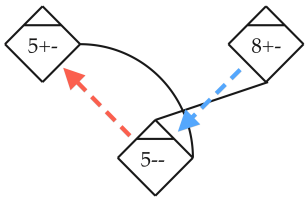
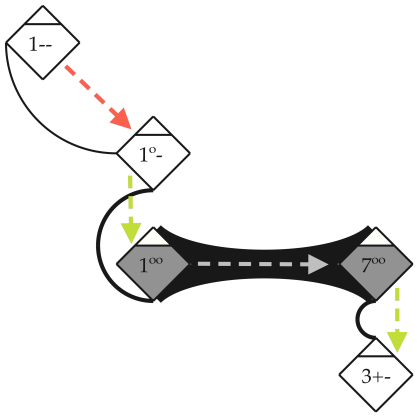
The *P*, *L*, and *R* transformations in neo-Riemannian theory can be applied to any major or minor triad, and are therefore universally applicable operations within that system. However, they do not accommodate diminished triads or any kind of seventh sonority. With a wider range of sonorities, the complexity of their relationships precludes a universally applicable transformation. Thus, the +/–*W* and +/–*P* operations can apply to all but fully-diminished chords. The +/–*A* operation only applies to major-minor (only –*A*), minor-minor (both), and half-diminished (only +*A*) sonorities. Similar to the +/–*A*, the +/–*V* operation applies

exclusively to major-minor (only $+V$), fully-diminished (both), and half-diminished (only $-V$) sonorities.

It would be slow going indeed if composers contented themselves only with moving one chord member by semitone each time, so most chord connections I will examine are conglomerations of the discrete motions shown on the lattice model. One transformation, then, is summed up in a term, not unlike the term of an algebraic equation. Operation E does not in itself consist of a pitch change, only an enharmonic respelling. Therefore E is typically found paired with V, which does involve a pitch change; these together are considered one term. Table 4.2 gives examples of transformational terms.

In addition to transformational labels, I provide a voice leading reduction below the staves of original notation for each musical example that follows. This is intended to aid the reader in recognizing the chord (or what I consider to be the chord) more immediately. It is by no means intended to emulate a Schenkerian reduction. Additionally, I draw slurs between pitches that would be constant, were it not for some intervening motion that is immediately reversed. In some sense then, slurs indicate the protracted influence of the pitches to which they are attached (not the prolongation of a sonority or harmonic function). On occasion, I will use dotted lines to connect a pitch with a chord from which I consider it to be temporally displaced.

Table 4.2: Sample correlation of notation to lattice geometry and term signatures[61]

	One Term	Two Terms	Three Terms
Notation			
Lattice Path			
Transform. Signature	+A	–A+P	–P–VE–V

There are a few stylistic tendencies worth mentioning before examining specific musical excerpts. The first is the tendency of composers to construct a parsimonious passage, and then repeat it, perhaps with some adjustments, in order to extend the length of the passage. The second is the nearly ubiquitous preference to descend, rather than ascend, along any of the axes already described. Therefore, in the transformational terms that accompany each excerpt, minus signs (–) are far more common than plus signs (+). This makes a great deal

[61] In this figure, and in the voice leading reductions in the examples to follow:
1) hollow note heads = pitch class held in common with prior chord.
2) black note heads = pitch has moved by semitone from prior chord.
3) diamond note heads = pitch has moved by whole tone from prior chord.

of sense in terms of the traditional resolution of chord sevenths, which are properly resolved down by step, but downward motion seems to prevail regardless of the changing chord member. Candace Brower has address this idea as an embodied understanding of music in the listener (or perhaps the composer, who listens internally before composing). The listener, through experience of the physical world, interprets the downward pull toward tonic as something akin to gravity.[62] Aside from the leading tone, all tendency tones have a propensity to resolve downward. Schenkerian theory would even say that $\hat{7}$ - $\hat{8}$ motion is incidental in the larger scheme of the *Urlinie*, in which upward motion is exceedingly rare. As a result, ascending motion of a chord member becomes a more significant event by virtue of its relative scarcity.

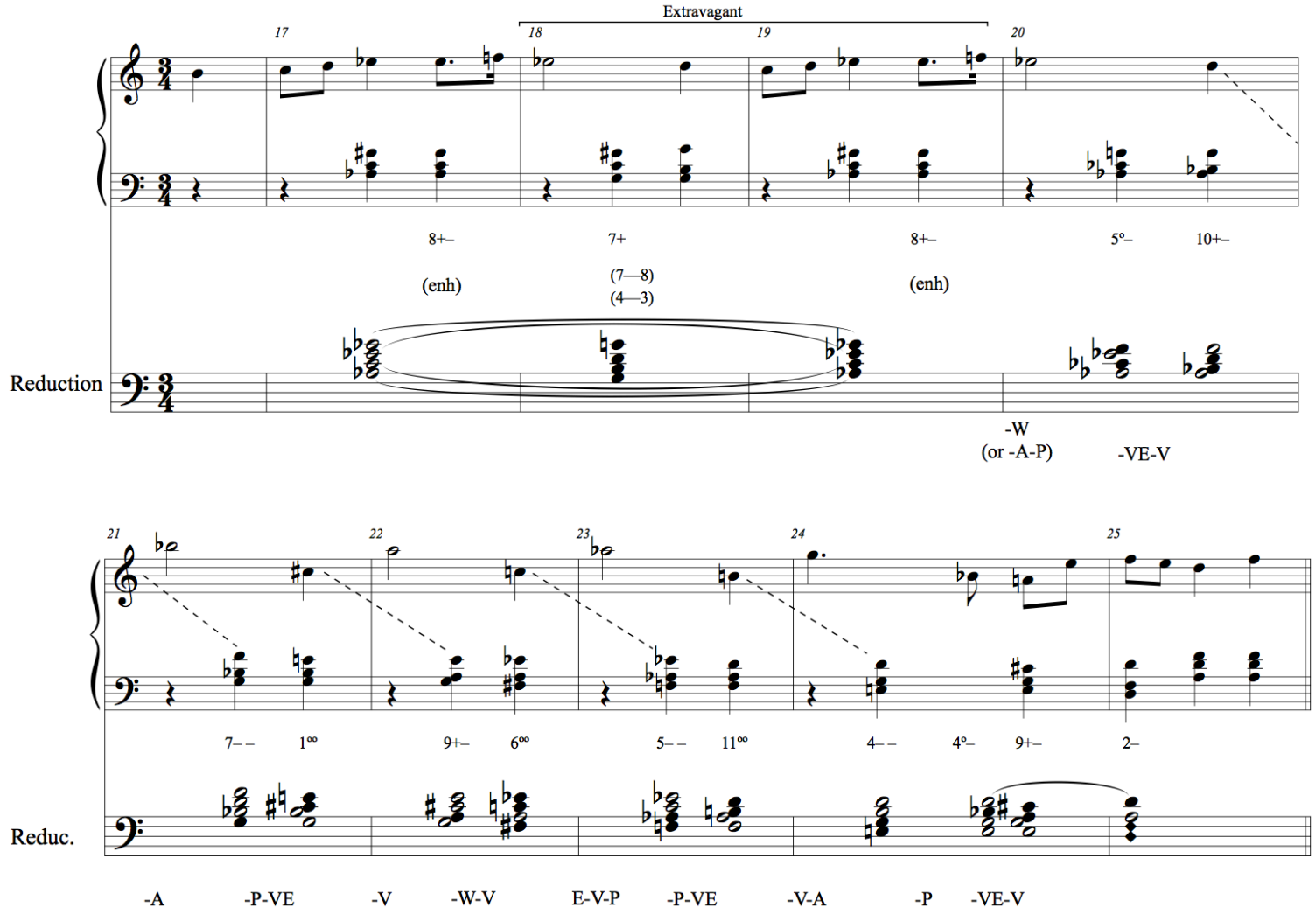
CHOPIN, MAZURKA, OP. 7, NO. 2

The excerpt shown in Example 4.1 is a good introduction to the system I intend to use throughout my analyses. Note the reduction's enharmonic respelling of the Gr^{+6} in mm. 17 and 19 as a major-minor seventh chord. This adjustment is made purely for the sake of conveniently comparing sonorities.

Chopin's use of the Gr^{+6} is curious in that it is at first resolved as one would expect in m. 18, only to revert back to the Gr^{+6} in m. 19. With the listener's

[62] Brower, 323-379.

Example 4.1: Frédéric Chopin, Mazurka, Op. 7, No. 2, mm. 17–25



ears still ringing with a relatively conventional use of the chord, Chopin launches into an unconventional chain of sonorities that, by any traditional reading, must be considered non-functional but can be described in some detail by the parsimonious models previously discussed. Reading the passage this way implies that the "proper resolution" of m. 18 is, in fact, not a resolution, but a feint that is immediately undone. This property is summed up by the slurs connecting the pitches of mm. 17 and 19, such that one might ignore the G major chord of m. 18 altogether, for all the impact it makes on the reaching of a harmonic goal.

Measures 18–19 are marked by a bracket labeled "Extravagant." This refers to Robert Cook's differentiation between parsimony and extravagance.[63] Cook's study was concerned with triadic transformation, but in the passage from Chopin's Op. 7, No. 2 (Example 4.1), we see another example like those given by Cook, with the exception that this involves a triad moving to a seventh chord. The proper, traditional resolution of the Gr^{+6} must, by definition, be extravagant. The passage, then, provides examples of an extravagant and a parsimonious path out of 8+– ($A\flat^{7}$ or Gr^{+6} in C).

The diagonal dotted lines in mm. 20–24 illustrate the temporal displacement between the high register and the supporting harmony. In this

[63] Cook, 109–140.

example, the higher pitch is missing from the chord that is about to arrive. Listeners will likely hear the higher pitch incorporated into the following sonority, even if the pitch has disappeared from the texture.

For clarity, Figure 4.2 reproduces the score of the Mazurka with the aid of a concise "map" of its path through pitch-class space-time. In this map, seventh-chord hubs outlined in red are sonorities that sound within the passage, whereas those outlined in black merely lie along the path. Measure numbers are given along the right margin of the figure. The passage begins at the top of the the map and proceeds downward in agreement with the minus signs of each term of the transformational signatures annotated in the score. This, of course, denotes descending motion of chord members. The map also marks the frequency of fully-diminished "wormholes" that Chopin employs to move among distant positions on the lattice (five wormholes in nine measures).

A less concise map of the same passage in Figure 4.3 shows how distant the connecting points of the wormholes can actually be. This map appears much more scattered and less parsimonious, and the linear path through the space is difficult to discern. Further, from Figure 4.3 we find that beginning around m. 23, previously visited regions of the map are traversed a second time, in many ways complementing the path previously chosen. The passage would then begin at the red hub in the upper right region of the map. Blue hubs are those that are visited

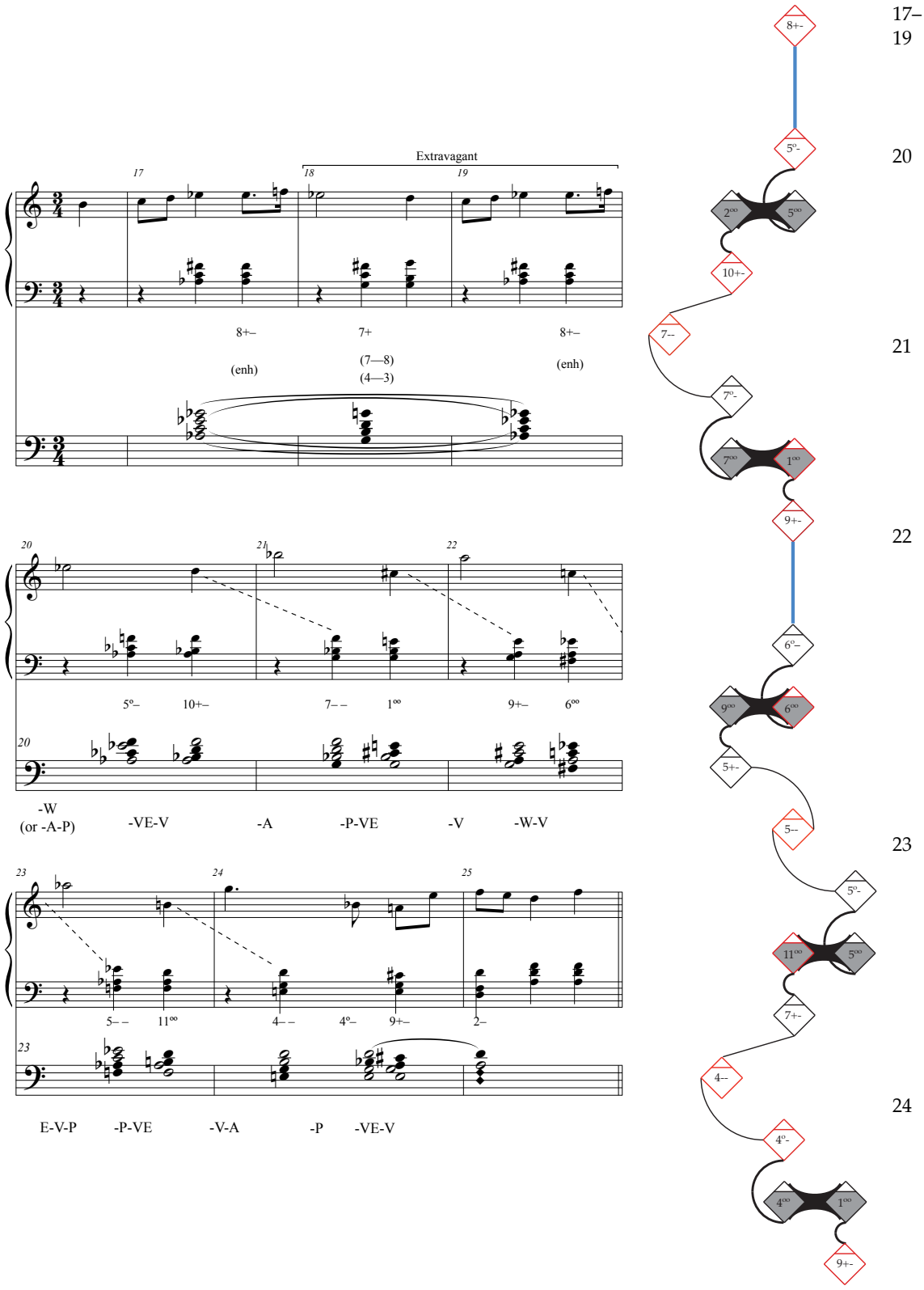


Figure 4.2: Map of Mazurka, Op. 7, No. 2, mm. 17–25

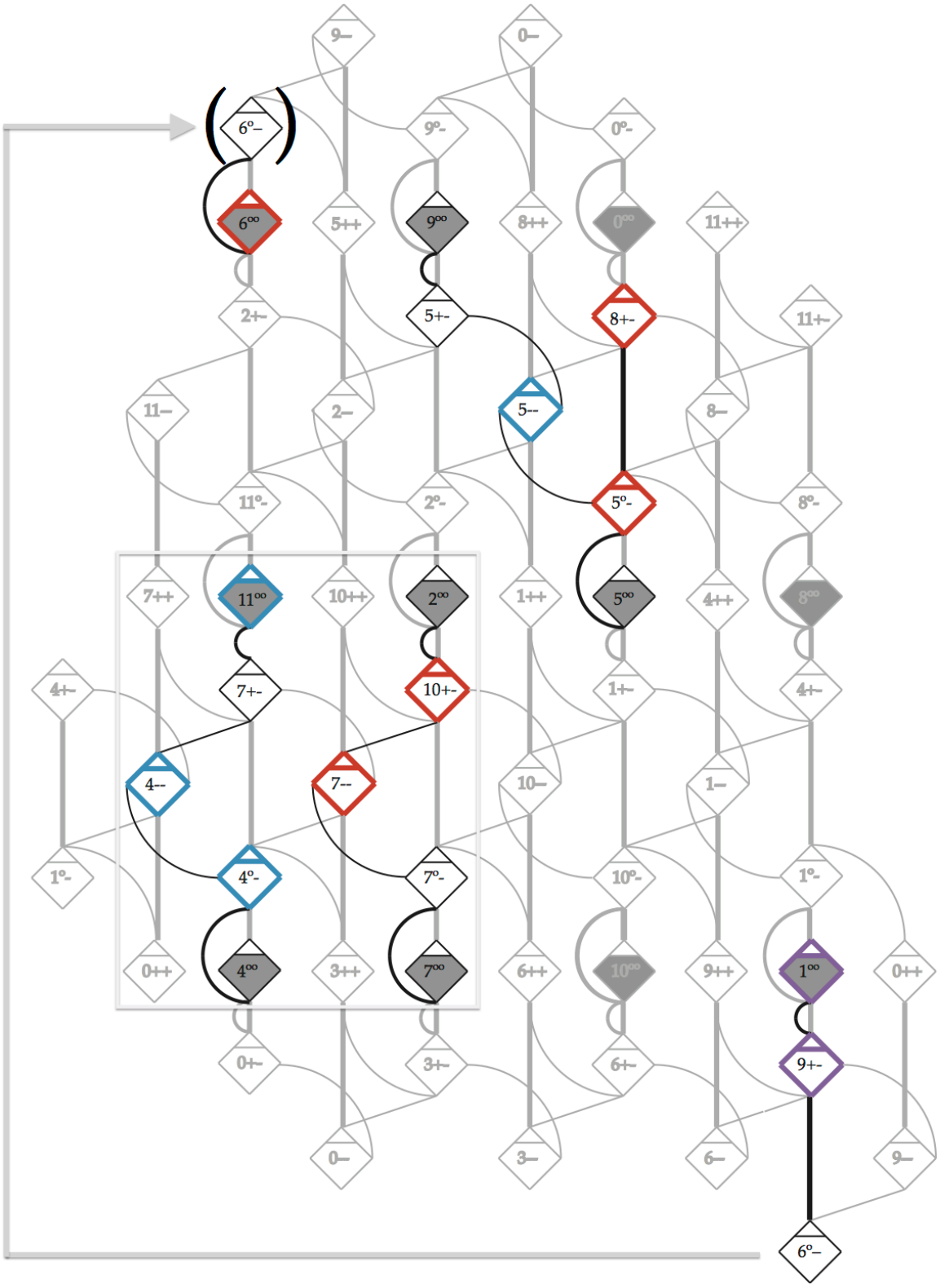


Figure 4.3: Expanded map of Mazurka, Op. 7, No. 2, mm. 17–25

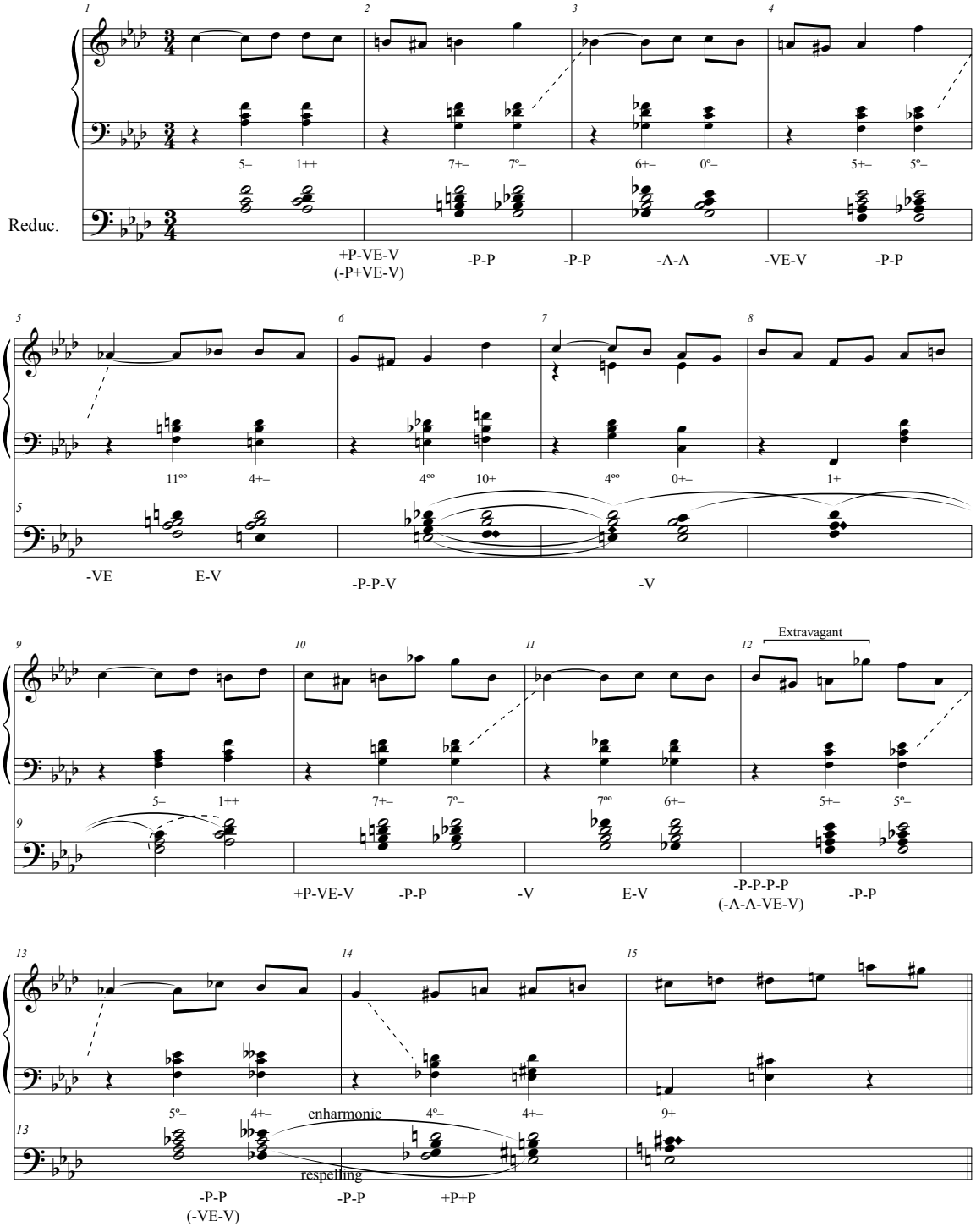
during the second pass through familiar territory; purple hubs are those that are heard in both the first and second passes. Compare, for instance, the region in the box. The paths taken are congruent, and even though the actual sounding chords along these congruent paths differ, they are complementary and end with the same two sonorities (purple hubs).

CHOPIN, MAZURKA, OP. 68, NO. 4

Another of Chopin's Mazurkas, Op. 68, No. 4, can also be analyzed with this system. A score of the first fifteen measures is shown in Example 4.2. We find right away that transformational patterns seem to emerge, in particular the double transformations along the P axis from a +– sonority to a – – sonority. Most transformations in this passage, we will find, retain at least two common tones, due in no small part to the pervasiveness of the –P–P transformation. Once again we see a distinct favoring of downward semitone motion.

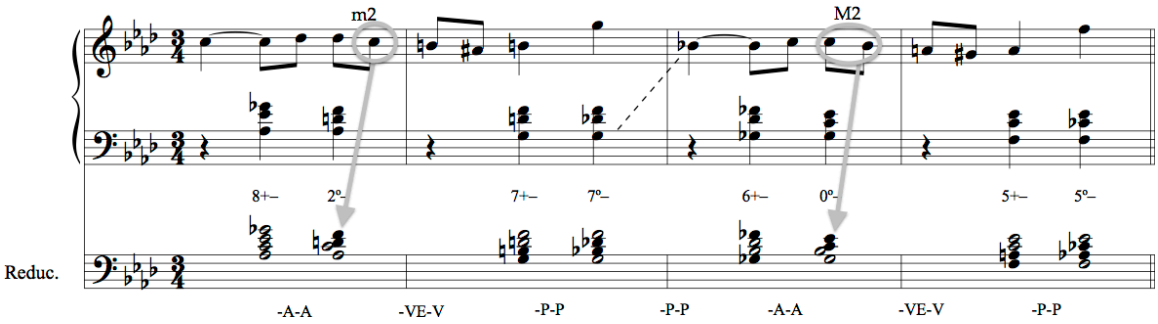
The transformation of 7+– into 7^{o}– in m. 2 is the first occurrence of –P–P that will characterize the passage, and its complement is heard in m. 4 (5+– to 5^{o}–). In between, m. 3 features an –A–A transformation that behaves similarly to the –P–P transformation. The melodic motive of mm. 1–2 is sequenced in mm. 3–4, but the harmony seems to fall short of the symmetry of the melody. If Chopin had desired a harmonic sequence, he might have composed m. 1 as it is

Example 4.2: Chopin, Mazurka, Op. 68, No. 4, mm. 1–15



presented in Example 4.3, which alters the first two chords. My recomposition erases any sense of F minor, the Mazurka's home key, at the outset. Chopin is by no means obligated to establish the key at the start,[64] but even as he does, he wastes no time in sliding out of functional tonality. The –P–P in m. 4 is followed by –VE going into m. 5. The transformation –VE is aurally identical to –V, so it is recognizable when –P–P–V occurs again in mm. 5.3–6.2 as a combined transformation. Measures 7–8 once again bring the ear back to a sense of F minor, only to end deceptively in m. 8.

Example 4.3: Mazurka, Op. 68, No. 4, mm. 1–4 hypothetical recomposition



Measure 9 restarts the process as it began, but with some notable alterations in m. 11 (analogous to m. 3). Initially, these alterations make for more parsimonious transformations (7°– to 7°° to 6+–). However, the smoothness is exchanged for an extravagant transformation back to the original material (6+– to

[64] One such example in which Chopin begins a piece deceptively is his Ballade No. 1 in G minor. Rather than choosing to begin with the tonic or even dominant seventh chord, Chopin arpeggiates an AΩ major triad (in retrospect, the Neapolitan of G minor)!

5+–). In m. 14, we hear the first true upward motion.[65] On one hand this +P+P simply reverses the –P–P immediately prior, but on the other it is significant in that it ends a long pattern of downward motion and signals the end of the section.

Visualizing a map of the harmonic space will again aid us in comparing Chopin's first and second passes through that space; Figure 4.4 does just that. To the left of the dotted line is Chopin's initial pass through PC space; to the right is his variation of the original route. The zones in grey boxes are common to both routes. We can see that in spite of departures from the original path, Chopin does revisit previous sonorities. Further, we note that these deviations from the original path only require different wormholes, not more wormholes, in order to retrace sonorities from the original path.

CHOPIN, PRELUDE, OP. 28, NO. 4

Perhaps the best and most famous instance of parsimonious seventh-chord mutation is in Chopin's E minor Prelude, Op. 28, No. 4. In this rare instance, parsimony pervades the majority of the piece, not merely a passage of it. Example 4.4 shows the whole Prelude, bars.

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[65] Measures 1.3–2.2 and 9.3–10.2 also contain a +P transformation. Since each instance groups the +P with –VE–V, I hear the overall transformation as primarily downward.

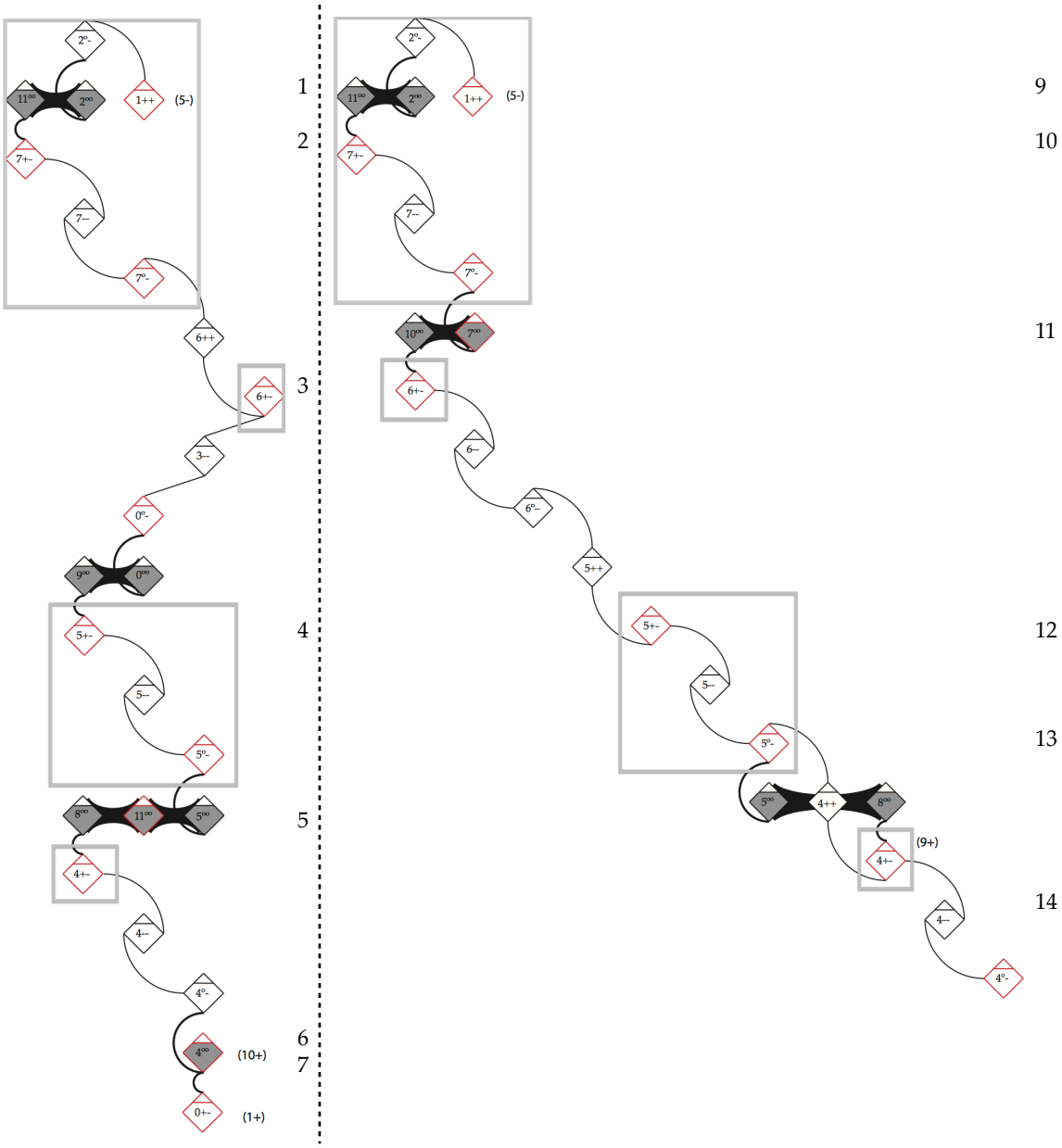
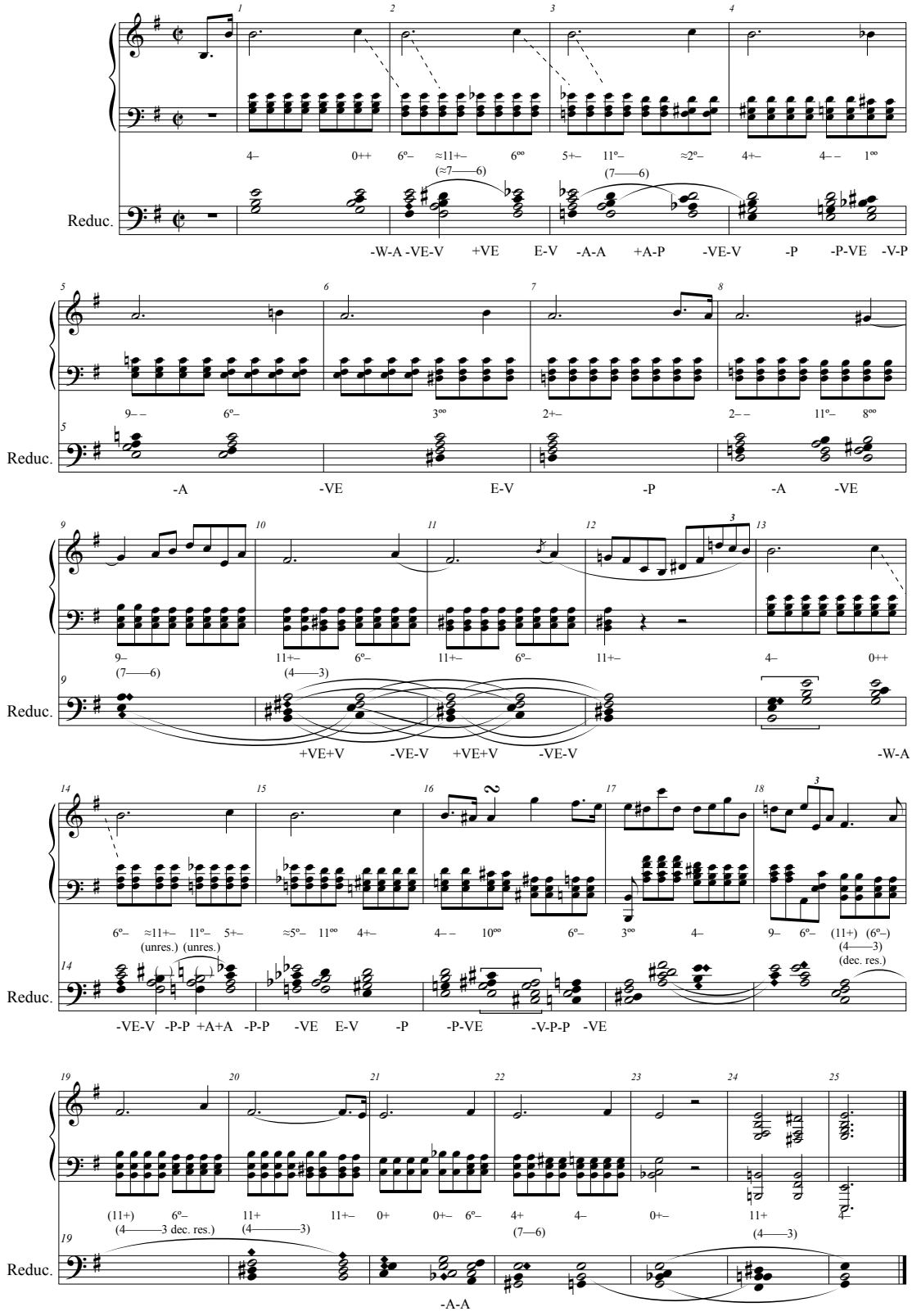


Figure 4.4: Map of Mazurka, Op. 68, No. 4, mm. 1–7, 9–14

Example 4.4: Chopin, Prelude in E minor, Op. 28, No. 4



As in earlier examples, downward motion predominates throughout. Measures 10–12 feature some upward motion, but, much like Example 4.2, this upward motion participates in a series of immediate reversals. The E minor Prelude has more single-term transformations (any transformation that has only one + or – sign) than we have encountered thus far, implying a slower unfolding of the transformational process in this piece than in the others. Chopin restarts the material beginning in m. 13, but accelerates the descent and derails at the height of the piece's tension (mm. 16–17). After the climax has been reached, Chopin primarily uses triads until the conclusion.

Even though I am primarily concerned with Chopin's use of seventh sonorities, his use of triads is significant in this example. Other than the initial 4– to establish the key in m. 1, the first triad he inserts is in m. 9. Up to m. 9, the gradual harmonic transformations in the left hand have been the motivating force of the music. In m. 9 the melody finally has a moment of activity, ascending although the lattice model is not particularly helpful in analyzing the last eight after eight bars of static or descending motion. Again ascension of surface pitches and on the lattice model marks a significant event, even in the melody. The second melodic ascension occurs in m. 12 over silence in the left hand and transitions the listener back to the opening material and register. In both instances, the harmonic activity is reduced in some way as a counterpoint to the

added melodic activity. This is not so much the case during the tension of mm. 16–18, in which the increased activity in both melody and harmony is the very thing that creates the climax. From m. 19 to the end, the effect is something of a denouement; stability is largely restored, and activity decreases, particularly in the right hand.

Figure 4.5 maps the path of the E minor Prelude. To the left of the dotted line is the first pass through harmonic territory covering mm. 1–9. On the right is the second pass, covering mm. 13–17. As in Figure 4.4, the zones in grey boxes are common to both passages through the harmonic territory, even though they are reached by different means or by expanding or contracting the path in between. I have not included the last eight bars of the prelude in the map; with the reassertion of the triad as the primary harmonic structure in these measures, including them in a map of seventh sonorities is not particularly helpful.

CHOPIN, PRELUDE, OP. 28, NO. 6

Chopin's B-minor Prelude holds some unique characteristics in light of the excerpts we have been examining. The passage from this prelude in Example 4.5 shows noticeably more "+" terms in transformations than there were in prior examples (although "–" terms still predominate). The first transformation that moves upward occurs at a strategic point. Measures 1–4 are harmonically static

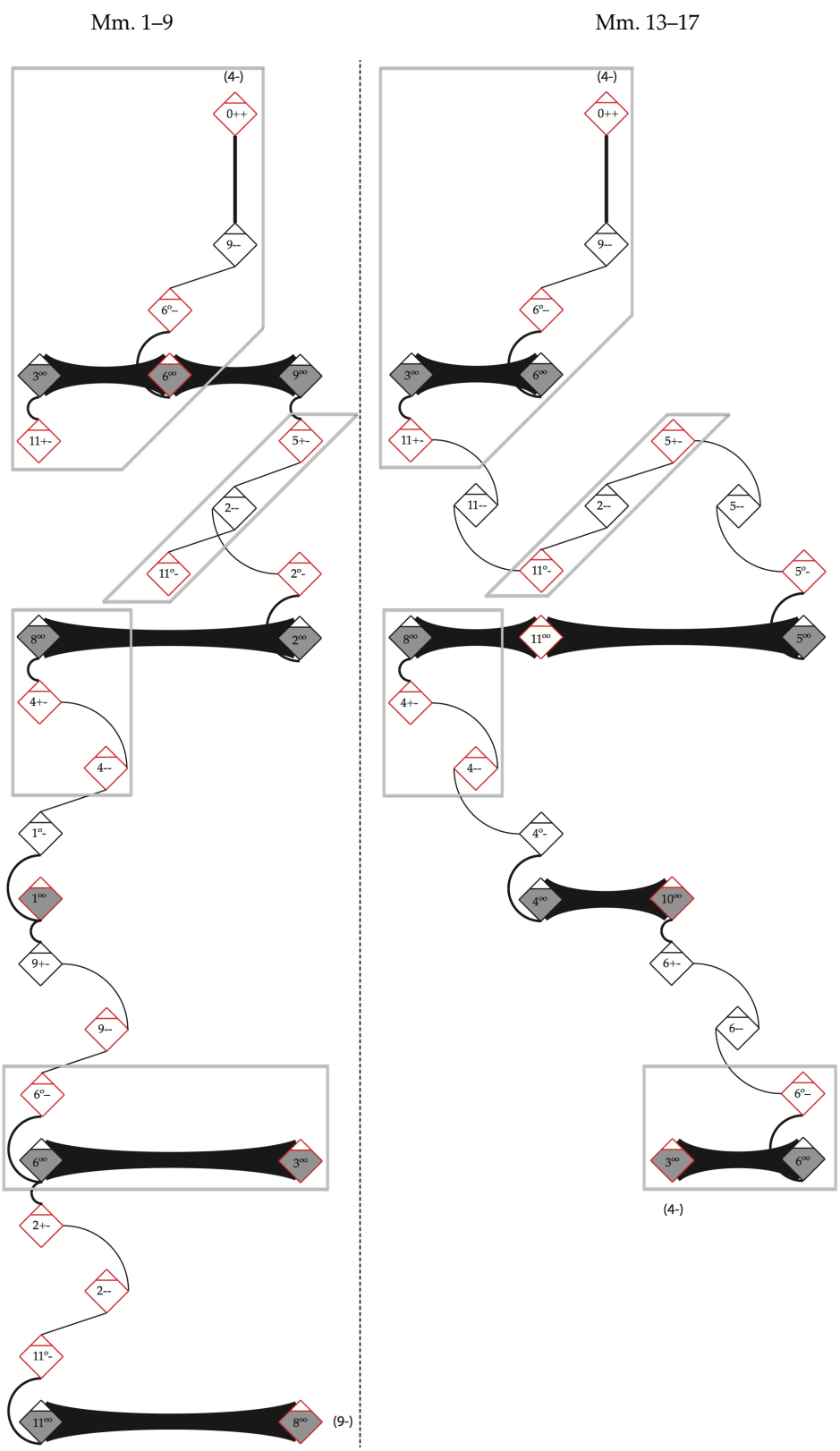
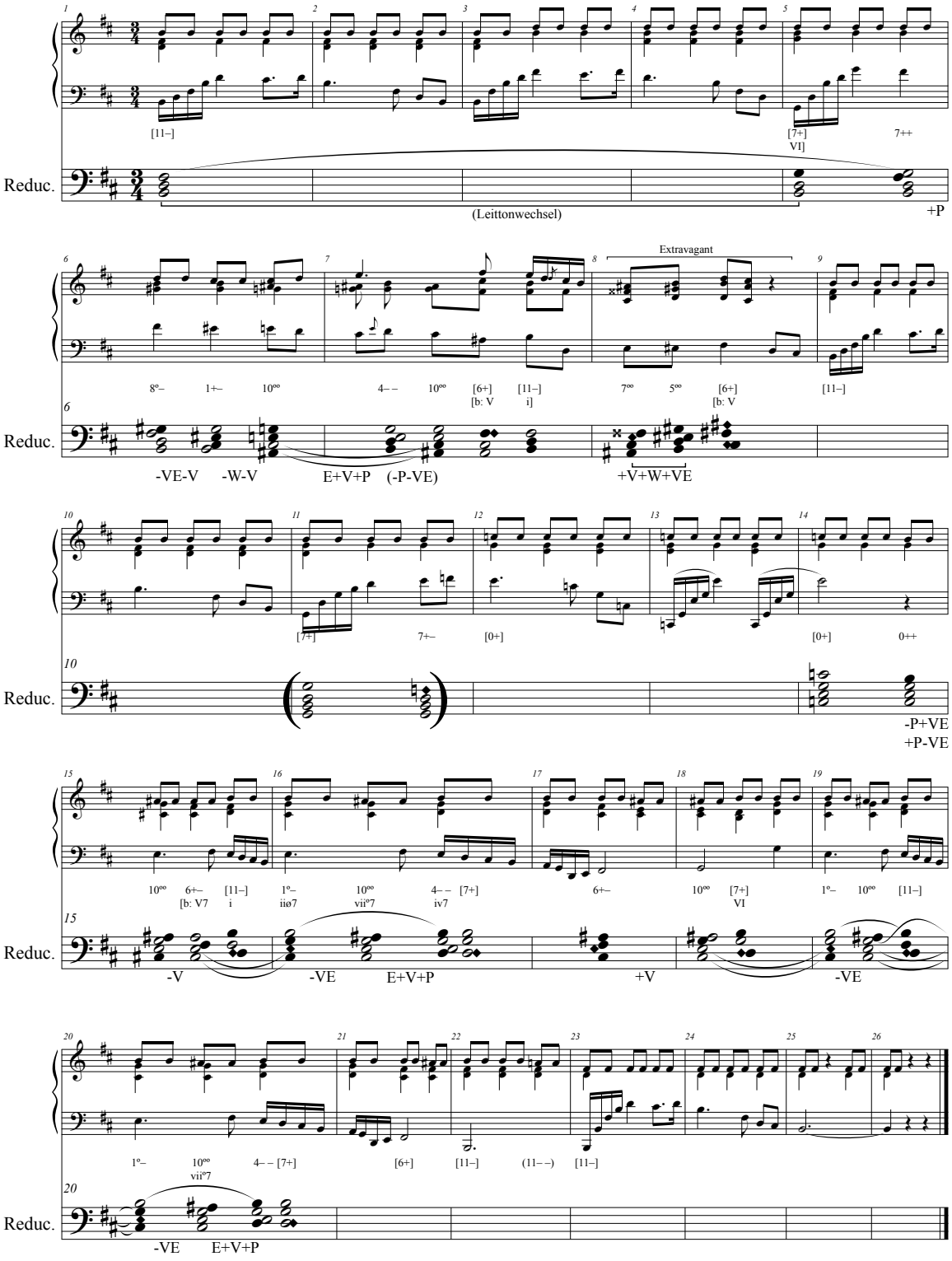


Figure 4.5: Map of Prelude in E minor, Op. 28, No. 4

Example 4.5: Chopin, Prelude in B minor, Op. 28, No. 6



(sounding the tonic chord) but the motive in m. 5 is generated by a similar motive found in mm. 1 and 3. Thus the listener hears this opening idea once (mm. 1–2), then twice (mm. 3–4), but the third time (m. 5), the motive sets up a dramatic harmonic shift into non-diatonicism, beginning in m. 6.

I have included in my analysis the intermittent occurrence of functional harmonies, such as in mm. 7.3, 8.2–9.1, 9–13, and 17–18. While some would perceive the non-functional passages as transitory connections between harmonically stable moments, for the purposes of this discussion, I think of these diatonic moments as a way to break up the harmonic activity of seventh sonorities. These functional moments always involve a triad; since triads are not integrated into the lattice paradigm, they prove to be a convenient way to parse this passage into smaller pieces.

The addition of more upward motion in terms of the lattice and the intermittent incidence of triads within a plausibly functional-harmonic context complicates the mapping of this passage. Rather than traveling linearly on the lattice in primarily one direction, sections curl back on themselves or change direction because of the increased number of "+" terms. I will build this map in steps to avoid confusion and information overload, as well as add arrows to the model to ensure that the directional path is clear.

Figure 4.6a maps Op. 28, No. 6 beginning with mm. 5–7. In it the initial triad of 7+ gives way to 7++. This section curls around (actually reaching near its beginning) before ending with a "V–i" in B minor. Measure 8, as we saw in Example 4.5, was mostly extravagant. I have seen fit to largely ignore m. 8, because the seventh sonorities (7^{oo} and 5^{oo}) are immediately preceded by "V–i" and immediately followed by "V–i," making them irrelevant.

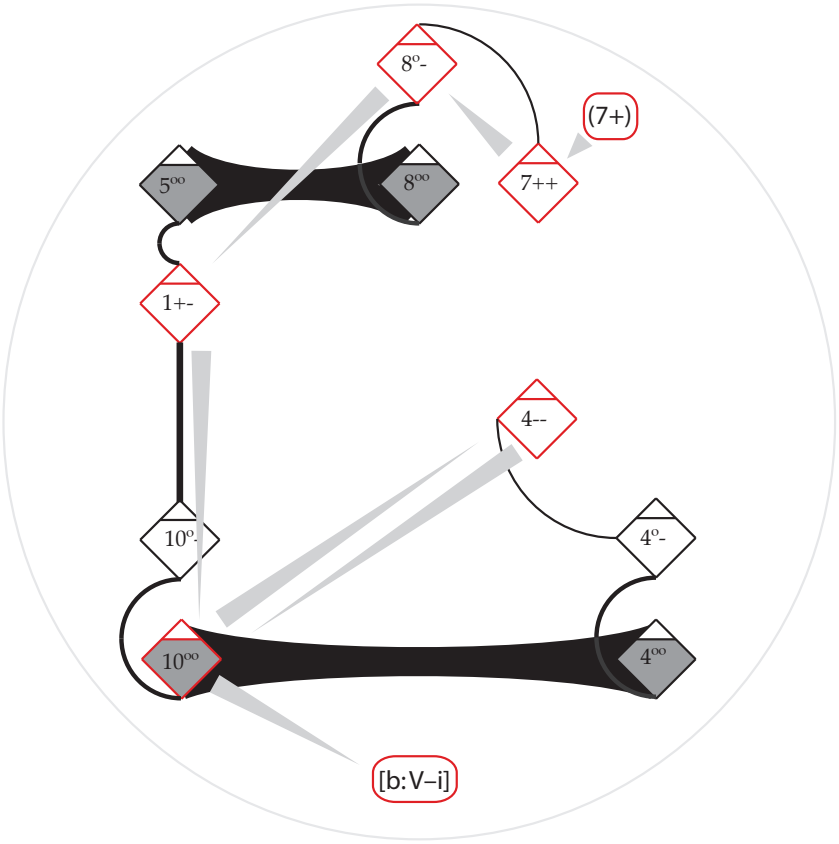


Figure 4.6a: Map of Prelude in B minor, Op. 28, No. 6, mm. 5–7

Measures 9–10 make as if to repeat mm. 1–4. Instead we find the VI chord, which is itself a neo-Riemannian *L* transformation (11– to 7+).[66] This might deceive the listener into assuming that what he is hearing is the analogue to m. 5. Instead, Chopin, rather than touching on 7++ as he did in m. 5, now uses 7+–. By the down-beat of m. 12, the 7+– is proven to behave as V^7 in C major. I map it in Figure 4.6b so as to highlight its similarities to m. 5. Indeed, that 7+– sonority is the "missing link" between the 7++ and 4– – of Figure 4.6a. The newly established key of C major is short-lived. Measure 14 marks a transformation from a major triad to major-major seventh, remarkably similar to that in m. 5. Figure 4.6c maps mm. 11–15. The seemingly excessive geometry surrounding 0++ merely reflects the analysis in Example 4.5, where -P+VE and +P-VE are aurally equidistant paths to 10^{oo}. The section ends with 6+– proceeding to 11– (B minor, the home tonic), which is another instance of Chopin weaving in and out of functional harmony.

The remainder of the passage under consideration (mm. 16–20) can be analyzed as functional harmony, but the seventh chords in m. 16 still map conveniently onto the lattice, as shown in Figure 4.6d. What I find most curious about this passage is that not only do these sections seem to take circuitous

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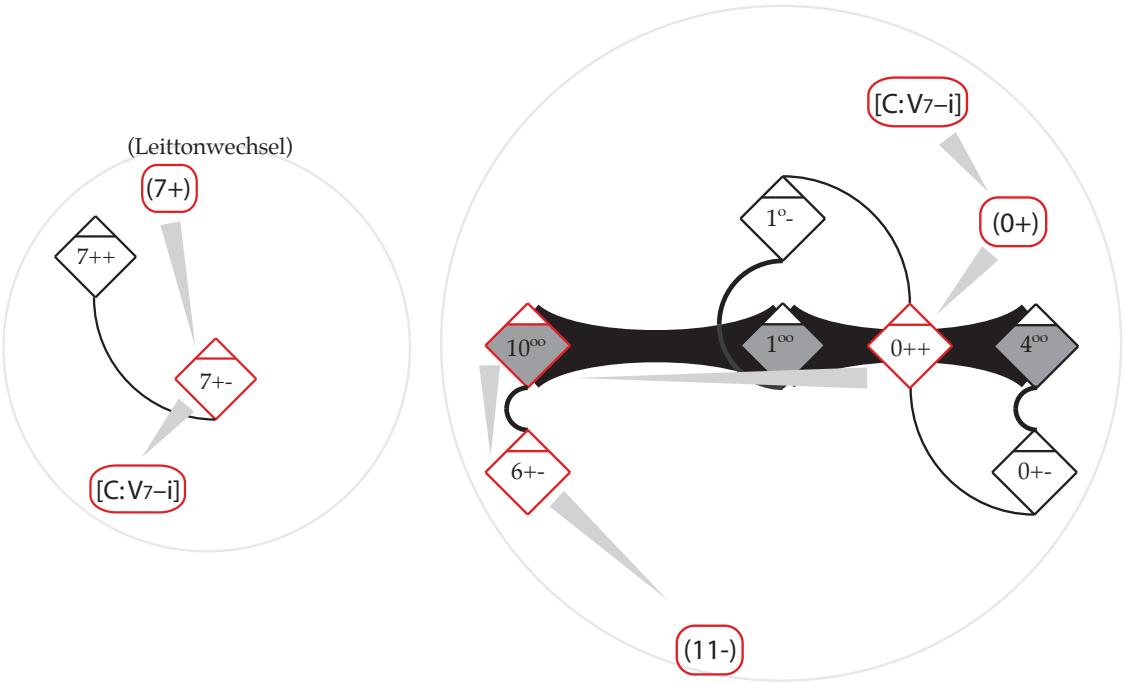
[66] The first time, the VI chord is delayed until m. 5. If the second time were to follow suit, the VI would not arrive until m. 13.



b: mm. 9–11 *c*: mm. 11–15

Figure 4.6b–c: Map of Prelude in B minor, Op. 28, No. 6, mm. 9–15

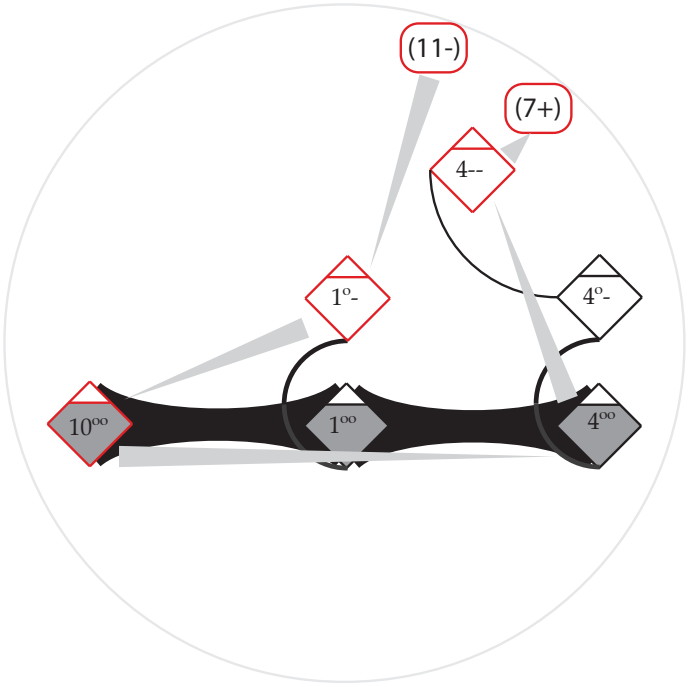


Figure 4.6d: Map of Prelude in B minor, Op. 28, No. 6, mm. 15.3–16

routes through the lattice, but the VI chord (7+) that breaks four measures of static tonic harmony in m. 5 is instrumental in the brief modulation to C major in the middle of the Prelude (m. 11); in fact, the VI chord is preceded by two measures of B minor tonic (mm. 9–10) and followed by two measures of C major tonic (mm. 12–13) for another total of four measures. And the VI chord ends the last seventh-sonority passage in m. 20, just before another relatively static stretch of B minor tonic (beginning in m. 22).[67]

The totality of the passage in question (mm. 5–20) is combined into a large circle made up of over overlapping smaller circles, something like a Venn Diagram, in Figure 4.6e. The triadic/functional "links" fall in the overlapping regions of the circles. I have rotated some of the circles to better accommodate the larger cycle. In some sense, then, this Prelude is harmonically cyclical on two levels, one large-scale and one small.

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[67] For more in-depth studies of this prelude, see: Charles Burkhart, "The Polyphonic Melodic Line of Chopin's B-Minor Prelude," in *Chopin Preludes, Op. 28: An Authoritative Score, Historical Background, Analysis, Views and Comments*, ed. Thomas Higgins, (New York: W. W. Norton, 1973): 80–89; and Howard Cinnamon, "New Observations on Voice Leading, Hemiola, and Their Roles in Tonal and Rhythmic Structures in Chopin's Prelude in B Minor, Op. 28, No. 6," *Intégral* 6 (1992): 66–106.

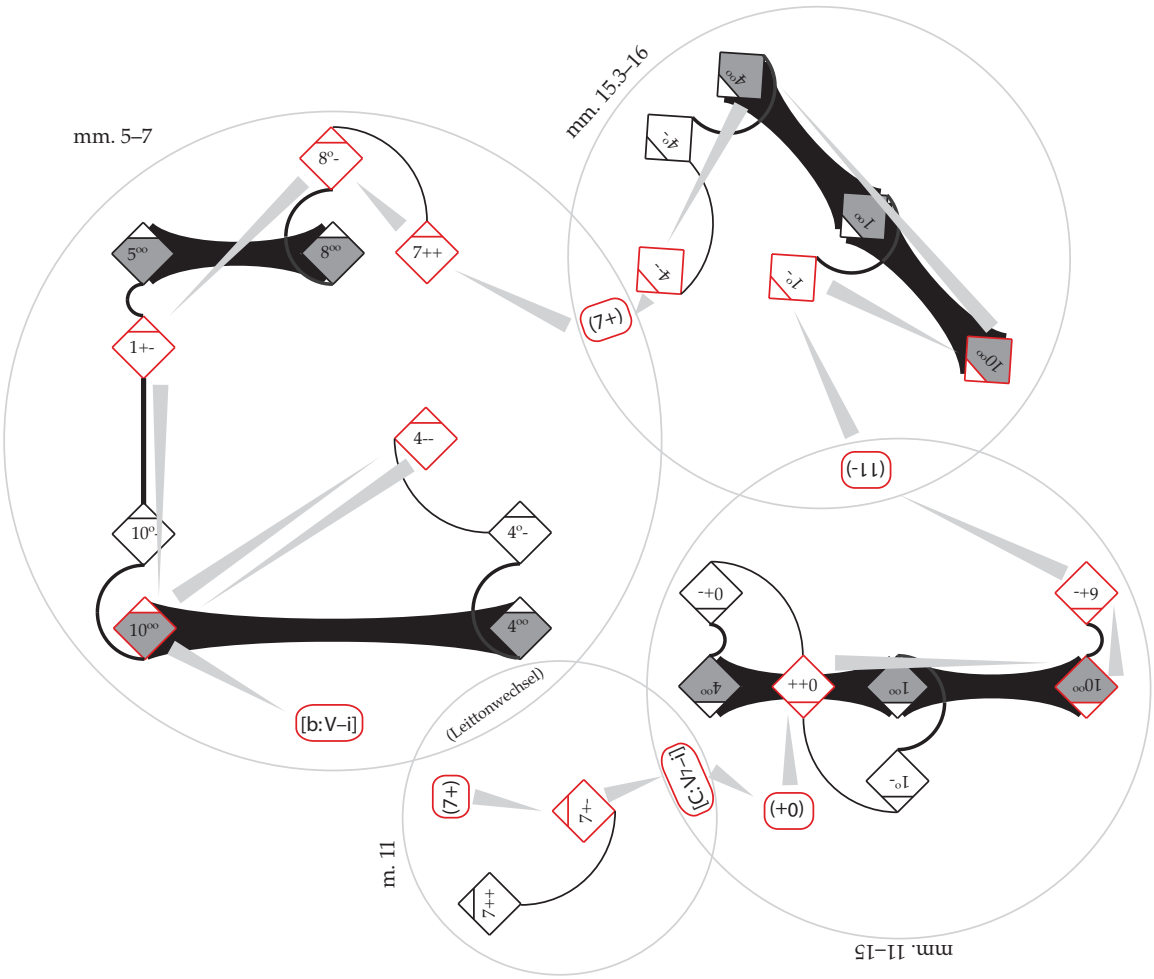


Figure 4.6e: Map of Prelude in B minor, Op. 28, No. 6, mm. 5–16

CHOPIN, NOCTURNE, OP. 27, NO. 2

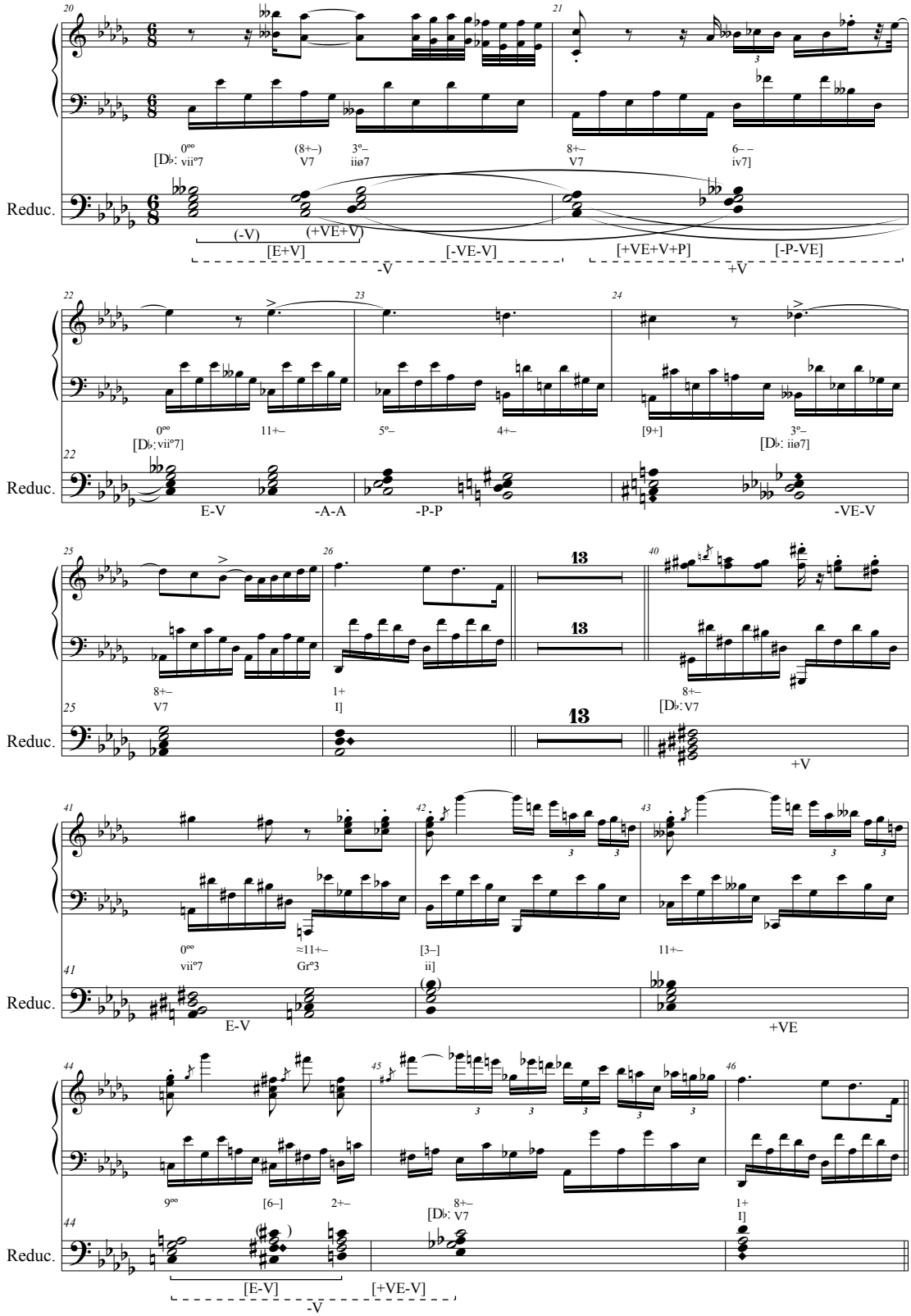
The Nocturne in D♭ major, Op. 27, No. 2, reveals yet another way Chopin revisits of old harmonic territory, this time via fragmentary variations made to each previous iteration. Example 4.6 shows the passages in question. The first section, mm. 20–26 follows the Nocturne's initial A and B sections. The lattice seems unnecessary for analysis here because a functional Roman numeral

analysis is possible. But by m. 22.2, the chain of seventh chords, at first diatonic in D♭, moves outside the key. The key of A major attempts a coup d'état in mm. 23–24.1, but D♭ is immediately reestablished in m. 24.2. The 9+ triad ends the first seventh-sonority chain in m. 24.1; mm. 24.4–25 encapsulate the first fragmentary variation. The fragment ends on D♭ in m. 26, which is the return of the A section (and eventually the return of the B section), covered in the multi-measure rest. Measure 40 interrupts the B´ section with three more variations of fragments of familiar harmonic territory. In each instance, the $0/3/6/9^{oo}$ wormhole network is used to shortcut longer paths.

I use some unfamiliar notation in my analysis of this passage, for example, at mm. 20–21 and 44–45. These brackets indicate various levels (micro or macro) of harmonic transformation. For instance, m. 20 contains two surface-level transformations, *(–V)* and *(+VE+V)*. Since *(–V)* is reversed by one of the *+V* terms, their cumulative effect is given as *[E+V]*. The *[–VE–V]* transformation at mm. 20.2–21.1 (3^{o}– to 8+–) ends up canceling out *[E+V]*, resulting in *–V* from the downbeat of m. 20 to the downbeat of m. 21.

Figure 4.7 maps the initial harmonic passage and each of its fragmentary variations. The vertical dotted line denotes the thirteen-bar break. The differences

Example 4.6: Chopin, Nocturne in D♭ major, Op. 27, No. 2, mm. 20–26, 40–46



between each iteration are easy to compare visually. The first fragment even appears in shape to be a miniature of the first full-bodied passage (mm. 20–23).

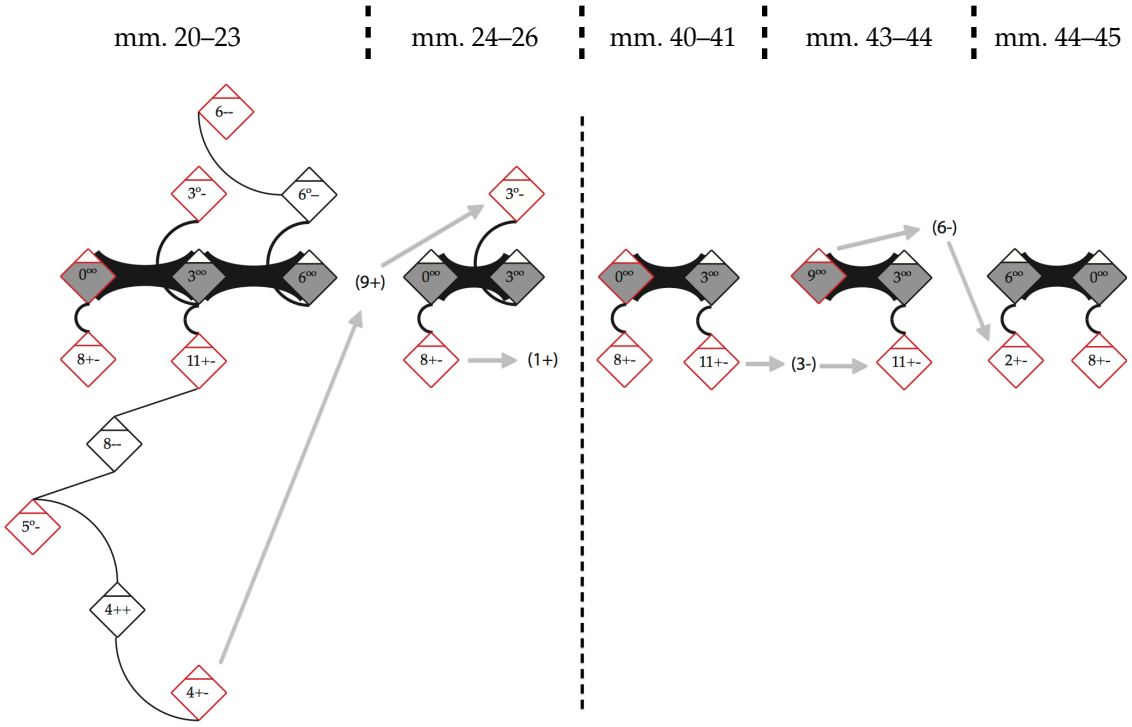


Figure 4.7: Map of Nocturne in D♭ major, Op. 27, No. 2, mm. 20–26 and 40–46

WAGNER, PRELUDE TO ACT I, *TRISTAN UND ISOLDE*

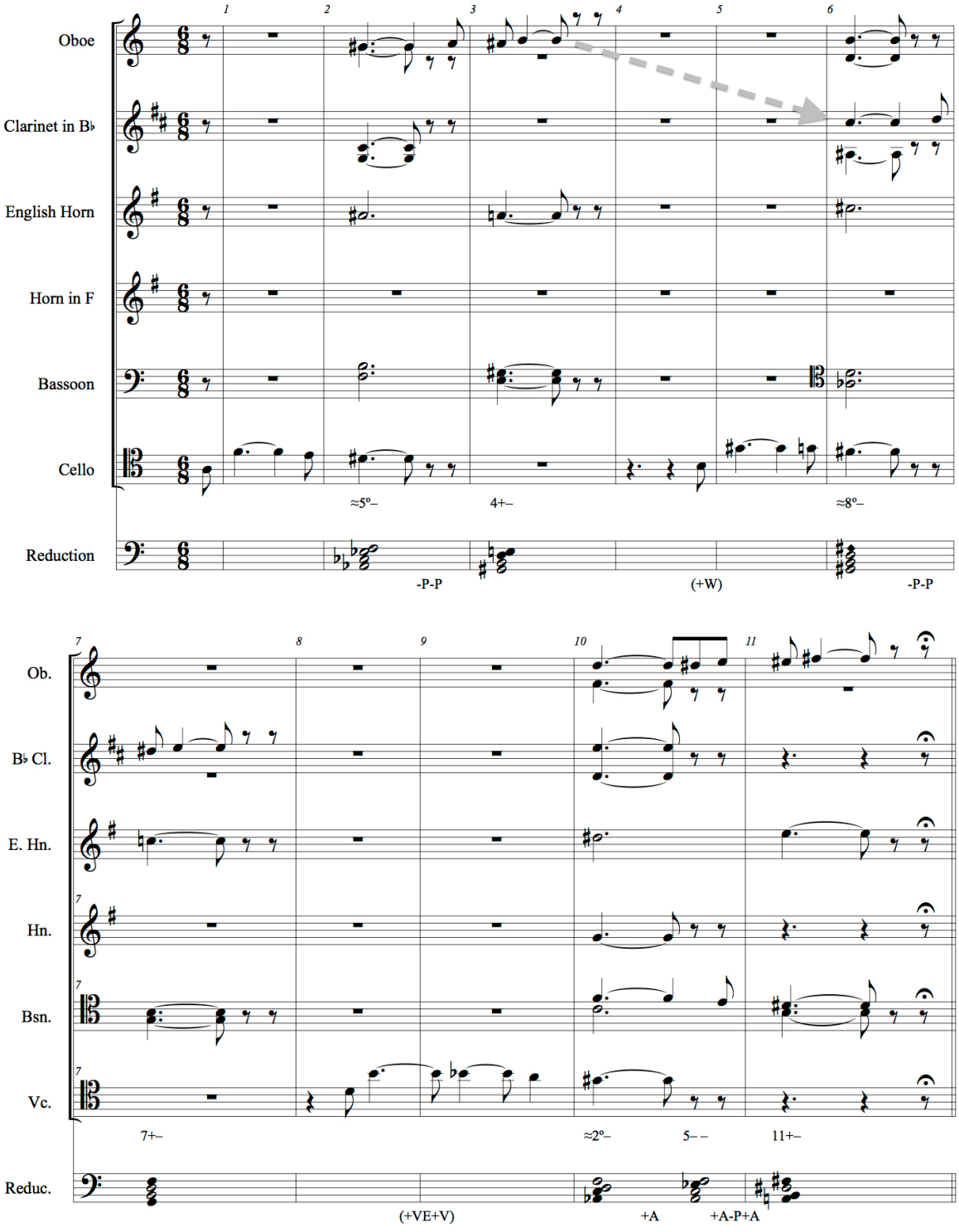
Whereas Chopin's piano music is particularly rich with this kind of harmonic language, he is not alone among late-nineteenth-century composers in its use. His repertoire seems to contain larger and more developed usage, but he is certainly not alone. Another example comes from Wagner's famous Prelude to Act I of *Tristan und Isolde*. Wagner's and Chopin's uses of harmonic language

within the excerpts I have selected to discuss are very different. Whereas Chopin tends to create chains of linear passages that recap the same or overlapping regions of the lattice, Wagner's usage tends toward the brief, in snippets that are adjacent on the lattice but not overlapping. Example 4.7 gives the first eleven bars of the prelude.

For all of Wagner's innovation in harmonic language, this passage follows a very time-honored form-building technique. Wagner states something (mm. 1–3), he states it again (mm. 5–7), but the third time, he varies the statement (mm. 8–11). This owes something to what Arnold Schönberg described as Developing Variation, a compositional principle he believed has been highly influential since the decline of the Baroque fugue. I have analyzed certain motions in parentheses; I am not convinced that the listener will hear these motions, but I include them to show how the next stage of the harmonic sequence is set up.

A cursory hearing of the chromatic climb of the melody in each iteration gives the distinct impression that the primary thrust is ascending. In fact, the ascending line of mm. 6–7 continues upward from where the last ascending line left off in m. 3 (see the grey arrow connecting the oboe to the clarinet). However, analysis of the harmonic transformations in mm. 2–3 and 6–7 reveal descension along the P axis. It is not until the third iteration that the harmonic transformations reflect a generally upward movement.

Example 4.7: Richard Wagner, Prelude to Act I, *Tristan und Isolde*, mm. 1–11.



Given the "Suffering" leitmotif that initiates each statement (mm. 1, 5 and varied in 8–9) by moving chromatically downward, it stands to reason that the harmonic *–P–P* transformations are a continuation of that idea. The harmonic motion itself underscores the rising "Desire" leitmotif. In the third statement, although a *–P* remains a part of one of the transformations, the rising "Desire"-esque motion ultimately triumphs. This reading could imply interesting narrative meaning given the subject matter of the opera.

Figure 4.8 provides a map of the passage just discussed. The transformations that were analyzed in parentheses are shown in grey. It becomes immediately evident that unlike Chopin, Wagner plays with adjacent regions of the lattice, without reproducing anything or exploring alternate paths to the same goal. Granted, this passage is shorter and less dense with sonorities than most of the Chopin examples, but Chopin often did not need to take long passages to retrace his steps, as in the Nocturne in D♭ major, Op. 27, No. 2 (Example 4.6). Also evident is the similarity of the two first statements, with their downward movement, versus the third with its expansion and upward trajectory.

These are only some of the possible applications of my geometric/theoretical models. This chapter is not intended to be exhaustive but to introduce the analytical viability of these models. Certainly, we have seen harmonic trends

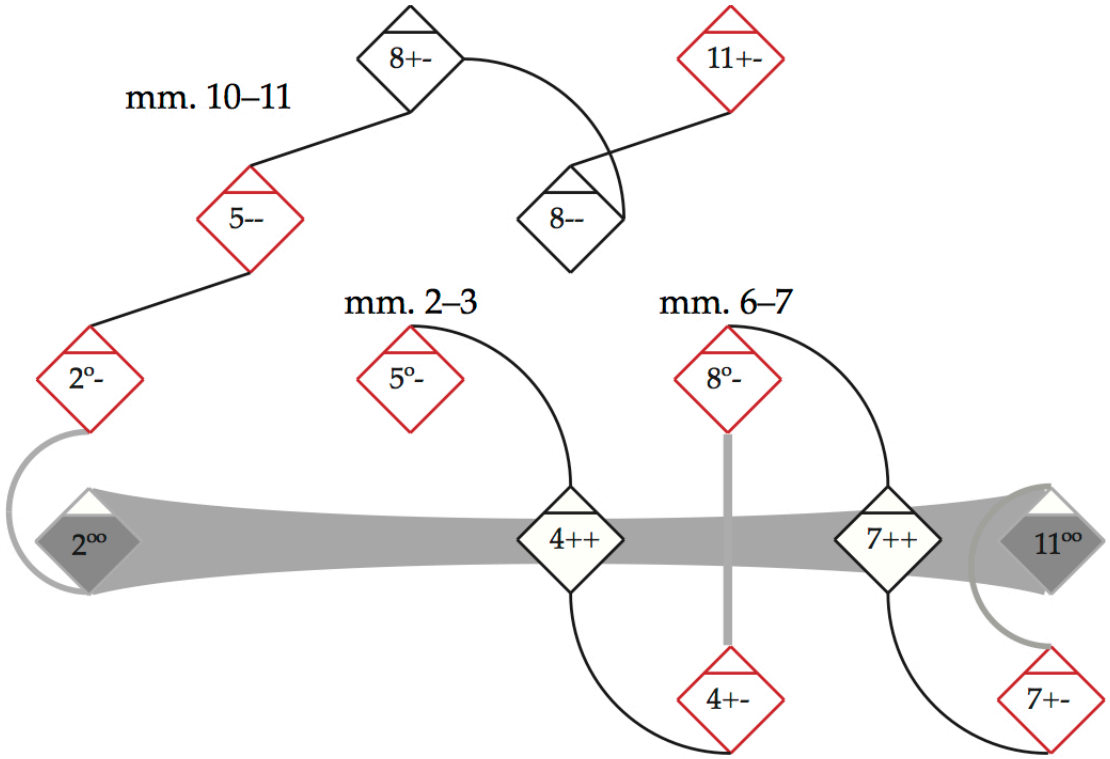


Figure 4.8: Map of Prelude to Act I, *Tristan und Isolde*, mm. 1–11

in Chopin's treatment of seventh sonority parsimony and can infer expectations for further study of his music. Downward motion predominates; upward motion, when it does occur, is often immediately canceled out (Examples 4.2, 4.4, and 4.6). When that ascending motion is not canceled, it signals something (Examples 4.2, 4.5, and 4.7) or provides heightened emotion (Example 4.4). These expectations may prove similar or contrasting among examples from other composers.

Narrative elements seem to emerge, as in the case of Chopin's Prelude in B minor, Op. 28, No. 6 (Example 4.5) and in the passage from Wagner (Example

4.7). This may also apply to the heightened emotional return in Chopin's Prelude in E minor, Op. 28, No. 4 (Example 4.4.).

Wagner's passage exhibited an exquisite instance of parallelism (Figure 4.8), as did a number of Chopin's, but they treated them very differently. In Figure 4.3, we saw how a portion of Chopin's first pass through harmonic space was geometrically congruent with a complementary portion in his second pass. In Figures 4.5 and 4.6, Chopin makes two passes each through the same harmonic territory. Both passes for each figure begin identically, deviate, but still reach a number of the same waypoints along their diverging paths. In Figure 4.7, Chopin creates a series of fragmentary variations, with each variation sharing at least two sonorities in common with its predecessor. Although Chopin's technique is not starkly consistent, his compositional preference treats the same region of the lattice multiple times. Contrast this with Wagner's practice, which would seem to prefer tangential, non-overlapping units.

This discussion by no means depletes the usefulness of this model, considering so much repertoire remains unexamined. In the concluding chapter I will discuss directions for further study and application, and additional research that, in tandem with this model, would open up new analytical possibilities.

CHAPTER V: CONCLUSIONS AND FURTHER RESEARCH

INTRODUCTION

Having demonstrated the viability of a conception of harmonic space in musical analyses, as well as grounding it in precedents set by Riemann and his successors, the neo-Riemannians, I turn my attention now to further possible applications of the family of lattice models. These include:

1. the continued examination of passages from Chopin's repertoire, which this study has by no means exhausted, particularly with regard to deeper harmonic structures and non-proximate transformations;
2. the exploration of other composers' paths through parsimonious harmonic spaces to discern their individual norms, and, I hope, any common stylistic traits among composers;
3. the integration of triadic structures into the lattice in order to better accommodate passages that frequently feature cross-type transformations;
4. the utilization of the lattice as a prescriptive template in composition and improvisation.

I will briefly discuss each of these possibilities in turn.

1. Examination of Non-Contiguous and Deeper-Level Structures

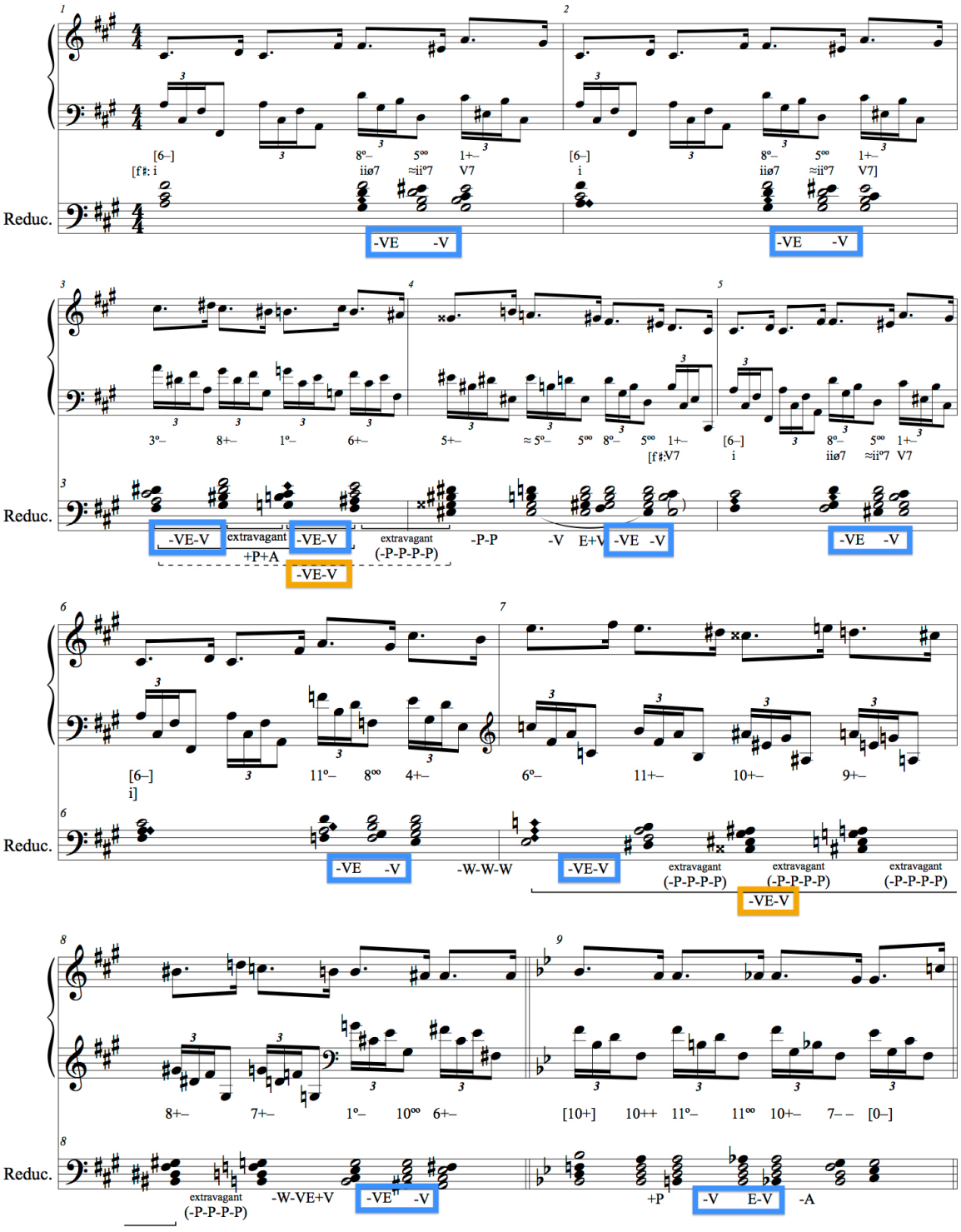
So far, I have only treated passages containing contiguous harmonic transformations of normative seventh sonorities. Other analytical applications could include comparing the congruency of very brief passages interspersed over the course of a piece. This approach is inspired by the Wagner excerpt previously examined, but it can be applied to a greater extent in yet another of Chopin's preludes from Op. 28, the F♯ minor. In Example 5.1 the –VE–V transformation, boxed in blue, can be found repeatedly, but not in a continuous, lengthy passage that could all be analyzed with the lattice model; indeed, there is a fair amount of extravagant motion in this example.[68] Sometimes the –VE–V transformation is lumped into a single transformation (mm. 3.1–3.2, 3.3–3.4, and 7.1–7.2), but more often it is divided into two transformations (mm. 1.3–1.4, 2.3–2.4, 4.3–4.4, 5.3–5.4, 6.3–6.4, 8.3–8.4, and 9.2–9.3). Below the surface, the –VE–V transformation sometimes connects longer, sometimes disconnected, passages; these are spanned by brackets and boxed in orange (mm. 3–4.1 and 7–8.1).

The –VE–V transformation can potentially be subdivided into three varieties based on the nature of the enharmonic reinterpretation of the fully-diminished seventh involved (the *E* of *–VE–V*):

1) those whose roots are related by T_3;

[68] In Example 5.1, the right hand has been simplified for clarity.

Example 5.1: Chopin, Prelude in F♯ minor, Op. 28, No. 8, mm. 1–9



2) those whose roots are related by T_6;

3) those whose roots are related by T_9;

All three types are present in this passage at some point. Nearly all the transformations highlighted in blue belong to the T_9 category. The only one that does not is found in m. 9.2–9.3 (the transformation from 11°– to 10+–); it is from the T_3 category and is the mirror of T_9. The T_3 version may play a significant structural role since it does not occur until after a change of key, precipitating a B section in which the frequent –VE–V transformation all but disappears; and it is at the first point in the music that could actually be considered parsimonious.

Even though the entire passage is not ideally suited to the lattice, it is still possible to examine the individual instances and compare them in Figure 5.1. As in prior mappings, occurring sonorities are highlighted in red. The addition of enclosures in blue and orange serve to encapsulate each instance of the –VE–V transformation, color coded to match those in Example 5.1. The thickness of the borders of the enclosures bears no significance other than to aid the viewer in grouping each set of the –VE–V transformation. Note the mirrored geometry of the 11°– to 10+– transformation (the lone T_3 variety of enharmonic reinterpretation). Orange arrows match up the elements of the deeper-level transformations, which fit the T_6 category. Thus, even though I cannot describe

the music by "charting a course" across the lattice as in my other analyses, I can compare the relationships of the "islands" of transformations.

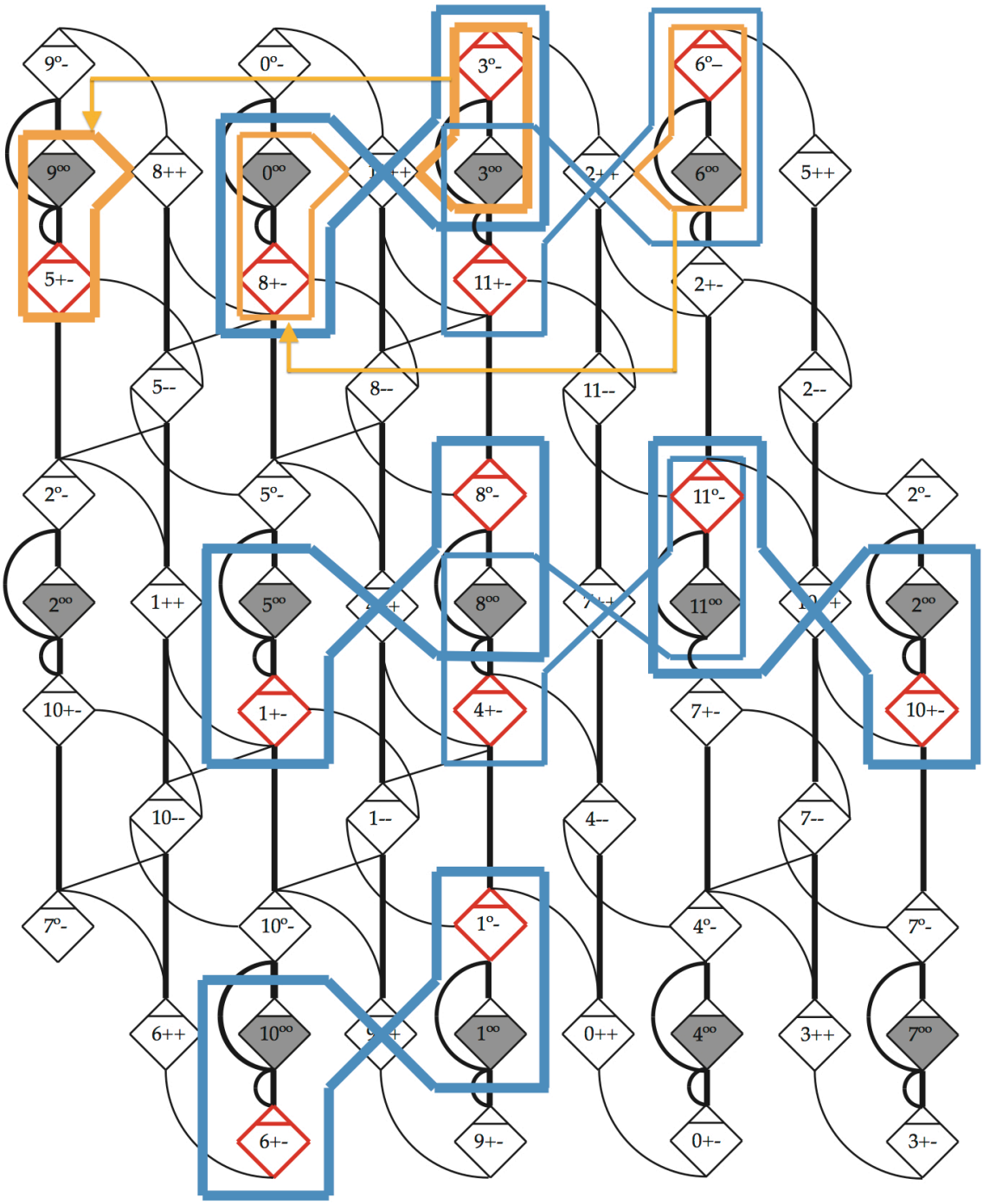


Figure 5.1: Lattice maps from Prelude in F♯ minor, Op. 28, No. 8, mm. 1–9

The amount of overlap among both the blue, surface-level iterations and the orange, deeper-level iterations makes for a cluttered diagram. The nature of the 2D lattice allows Figure 5.1 to illustrate details at the expense of a clear overall picture. Figure 5.2 expresses the lattice as the skeleton of a torus. The torus of Figure 5.2a depicts the surface transformations as constellations ordered around their respective fully-diminished nuclei. I have colored each instance of the –VE–V transformation in differing shades of blue to help the eye track them. Since two transformations begin with 11^{o}– (mm. 6.3–6.4, and 9.2–9.3), the first instance, ending with 4+–, I have colored it in a lighter blue. The second instance, ending with 10+– (the T_3 occurrence) I have colored in purple to draw attention to its difference from the other iterations. Figure 5.2b expresses the same torus, this time mapping constellations of deeper structure in two shades of orange.

2. Exploration of Other Composers' Approaches

Other composers have used similar harmonic syntax even though their usage differs from those I have already analyzed. Passages from Wagner, Grieg, and others have given me reason to believe there may be other approaches to similar parsimonious seventh-sonority space. For instance, Saint-Saëns's beloved "Le Cygne" ("The Swan") movement from *Carnaval des animaux* features a passage that can be understood in terms of functional harmony, but it can also be

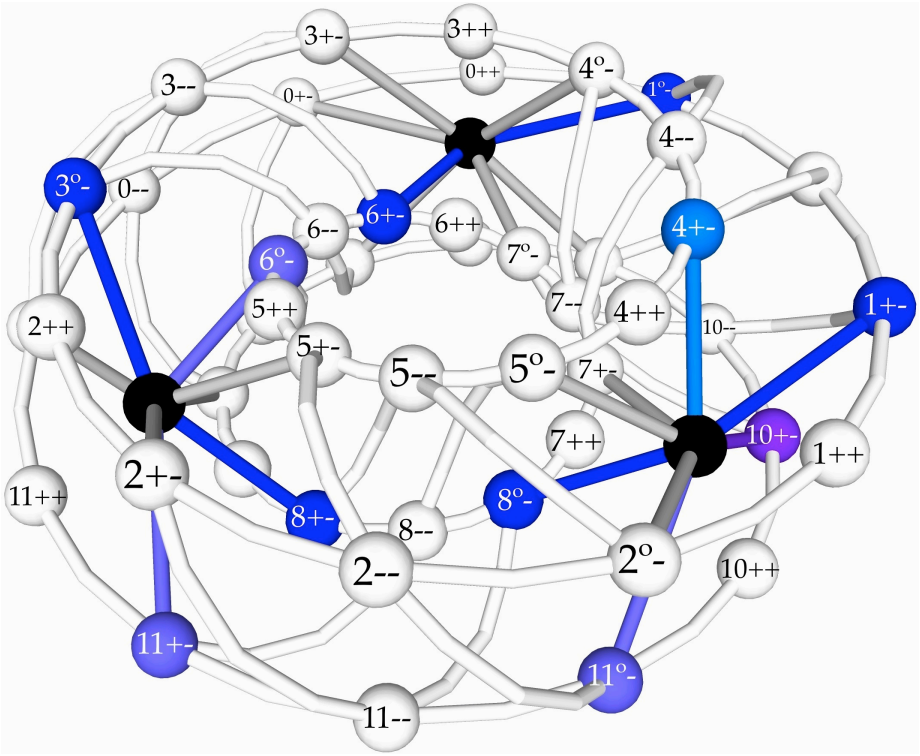


Figure 5.2a: Constellation maps of surface transformations, mm. 1–9

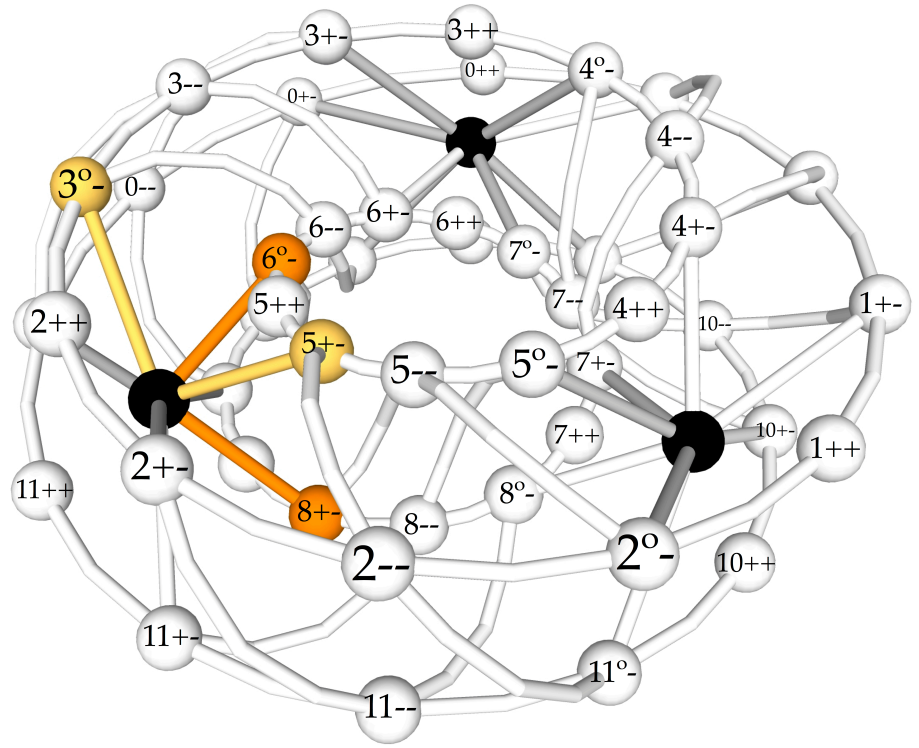


Figure 5.2b: Constellation maps of deeper transformations, mm. 3–4.1 and 7–8.1

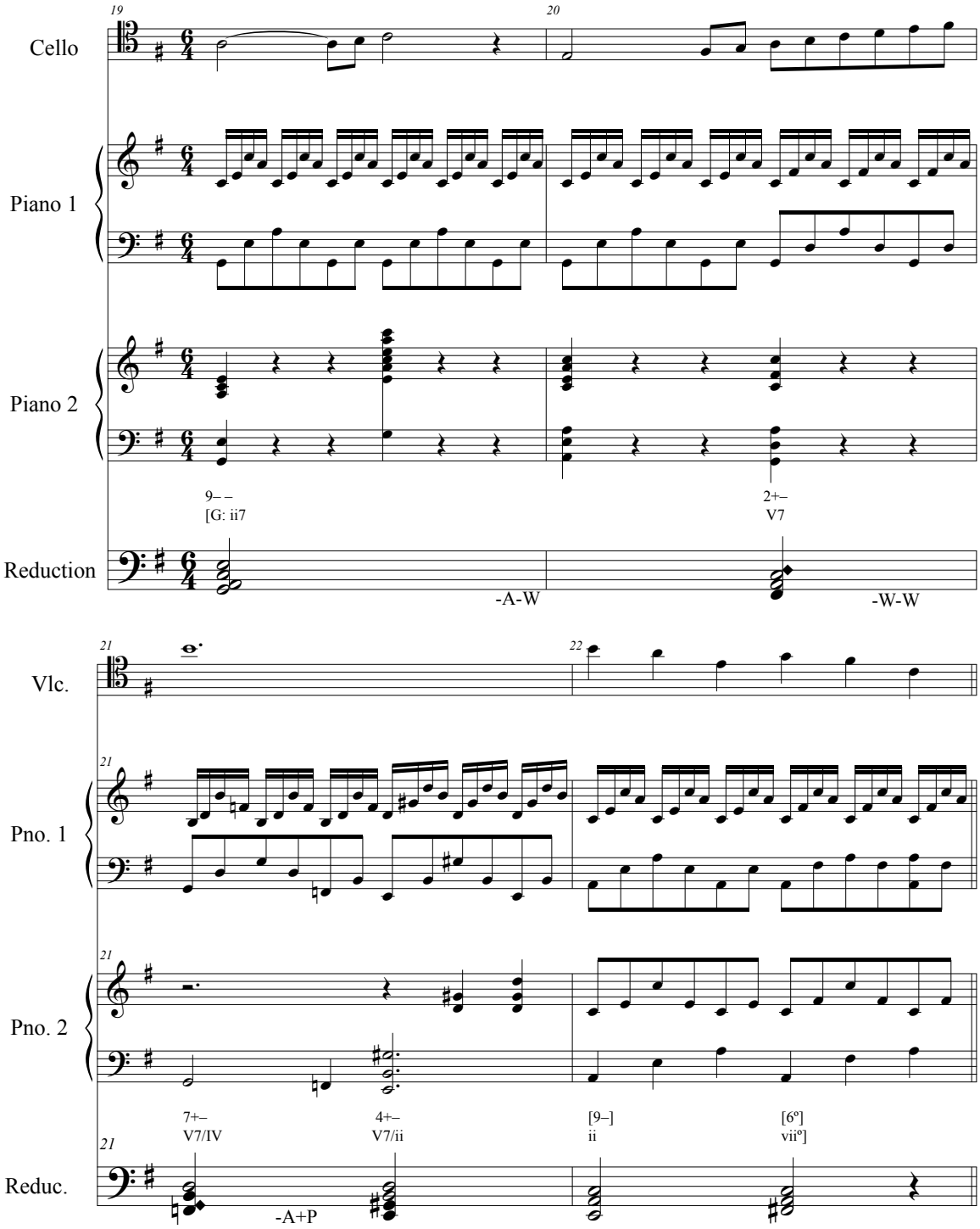
analyzed with the lattice. This passage is given in Example 5.2. Whereas the lattice may not be the only applicable model, it might serve to unite diatonic and non-diatonic sonorities within a single super-structure. Figure 5.3 graphs the passage. Its more extensive use of the *W* transformation in a short period of time suggests a significantly different harmonic syntax from what I typically found in Chopin's works. Figure 5.3 reflects this in its primarily vertical orientation, rather than the diagonal or zig-zag orientations graphs of Chopin's music. Still, there are similarities, such as the predominance of downward motion. All of these observations require further study to substantiate; and they raise questions regarding the applicability of the model to other passages of Saint-Saëns's repertory, how the composer navigated such passages, and how his syntax compares and contrasts with that of his contemporaries. A broader survey is required to answer these uncertainties.

3. Integration of Triadic Structures into the Lattice

Another example from Chopin illustrates a different sort of problem I hope to overcome— that of interspersed triads amid seventh sonorities. Clifton Callender's "Voice-Leading Parsimony in the Music of Alexander Scriabin"[69] and Julian Hook's "Cross-Type Transformations and the Path Consistency

[69] Callender, 219–33.

Example 5.2: Camille Saint-Saëns, "Le Cygne," *Carnaval des animaux*, mm. 19–22



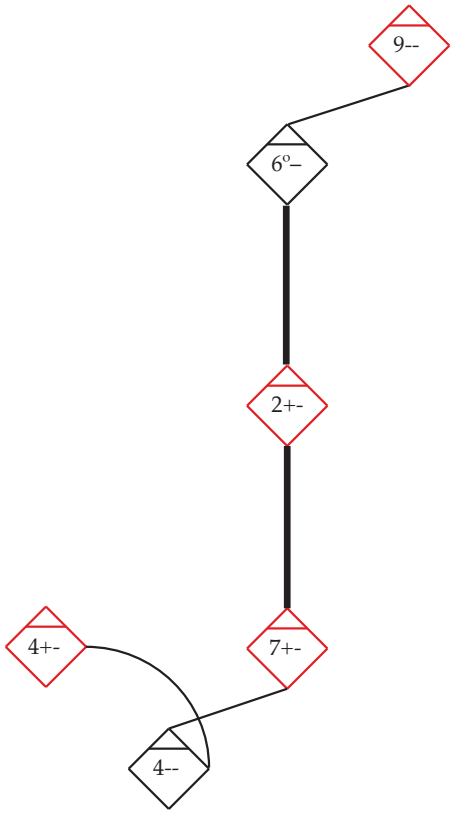


Figure 5.3: Map of "Le Cygne," mm. 19–22

Condition"[70] both treat the issue of transformations between triads and seventh chords. Although the geometric models I have developed are designed for seventh sonorities, I believe triads could be integrated easily, if given a systematic approach based on the groundwork Callender and Hook have already laid. Indeed, as I previously observed in Figure 3.3, the links between major-major sevenths, minor-minor sevenths, and consonant triads are already embedded within the neo-Riemannian *Tonnetz*. Callender's split and fuse

[70] Hook, 1–39.

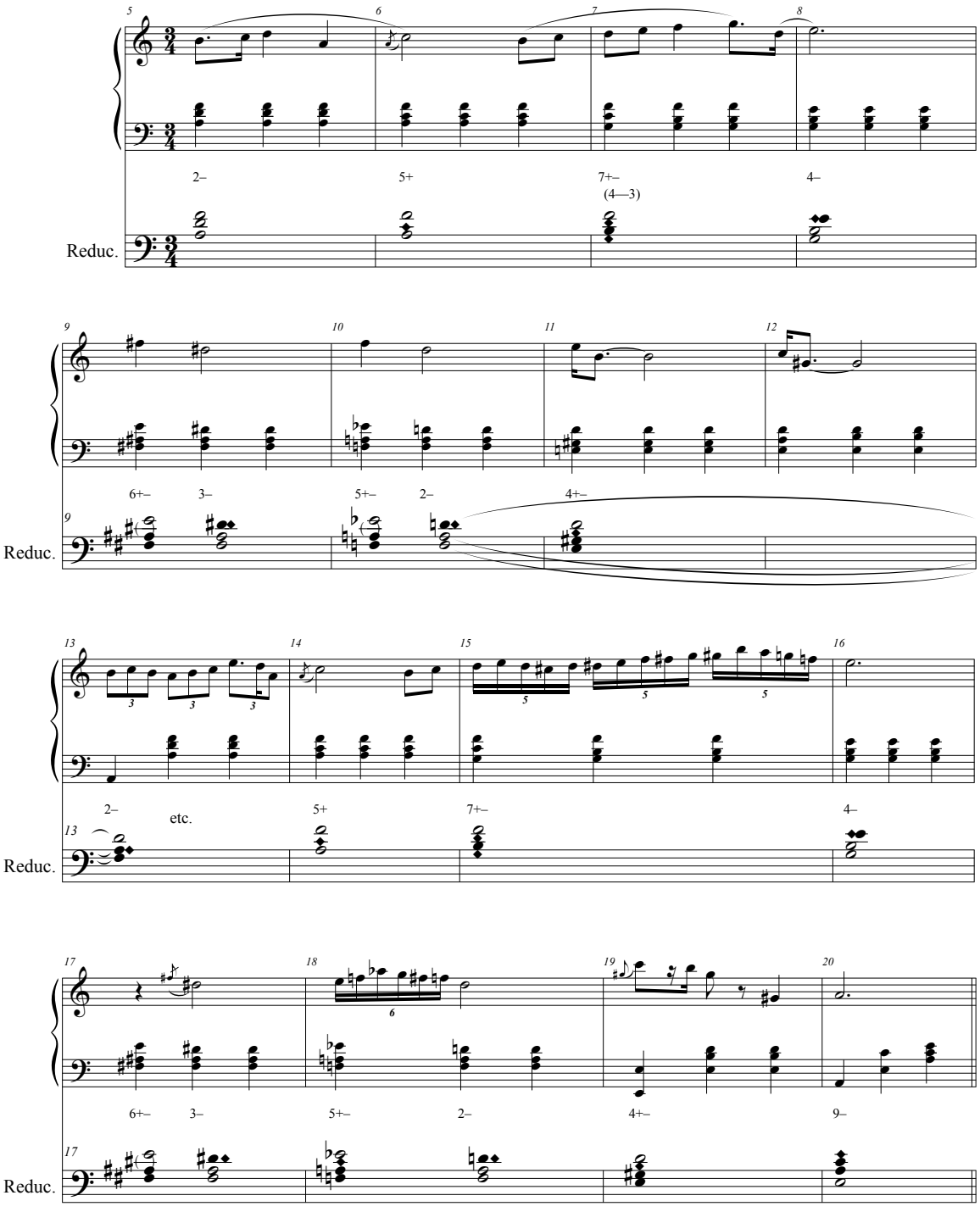
functions and Hook's cross-type transformations give me further launch point for investigation.

The following excerpt from Chopin, the Mazurka, Op. 17, No. 4 in Example 5.3, serves as an example of the sort of passage for which the current model is still ill-fitted. A thorough integration of triadic and seventh-sonority models would make better sense of this passage than the current model does on its own. I hope that with future, more comprehensive permutations of the lattice model I can better accommodate such passages.

4. Prescription for Composition and Improvisation

Finally, my model seventh-sonority parsimony could have prescriptive applications for composers and improvisers. My own unfamiliarity with jazz repertory and the incredible number of differences between various versions of the same song have inhibited my attempts to delve into that body of music, but that avenue is certainly open to those who possess the necessary background. I am certain the lattice model could find traction among jazz musicians as both an analytical tool and, perhaps, an improvisational one. Indeed, it could serve as a compositional tool to less improvisatory modes of composition as well. To illustrate this idea, I have included a short composition in the appendix with an analysis beneath.

Example 5.3: Chopin, Mazurka, Op. 17, No. 4, mm. 5–20



Summary

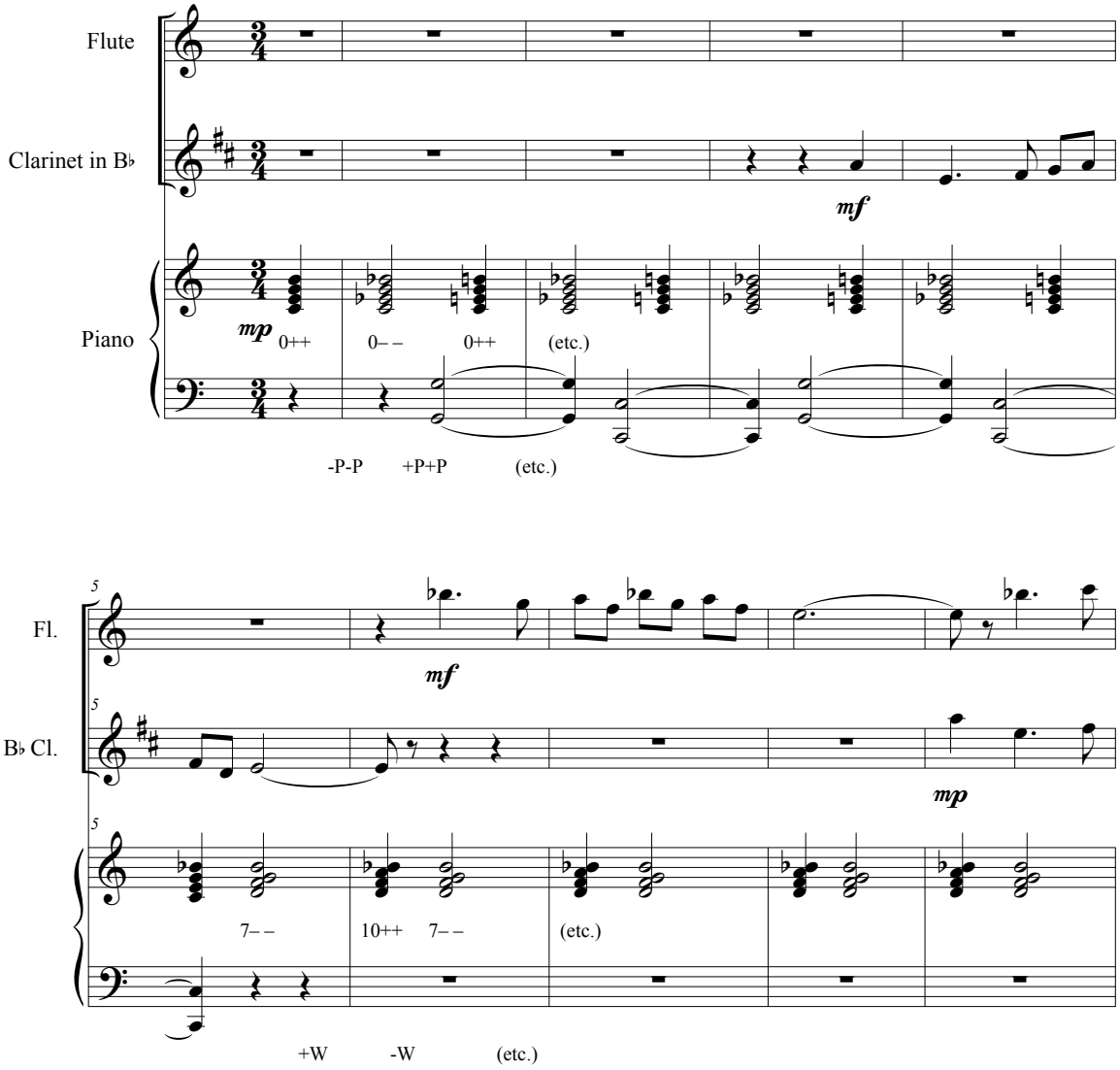
My purpose in this study has been to lay the foundation for these lines of inquiry, in both a theoretical way and a practical way. The theoretical foundation relies heavily on metaphors of space and distance to represent pitch-class relationships and harmonic relationships. As such, I arranged normative seventh sonorities to form a parsimonious geometric network in which greater parsimony was represented by greater proximity. The network wraps back upon itself along two axes, necessitating its visualization in three dimensions.

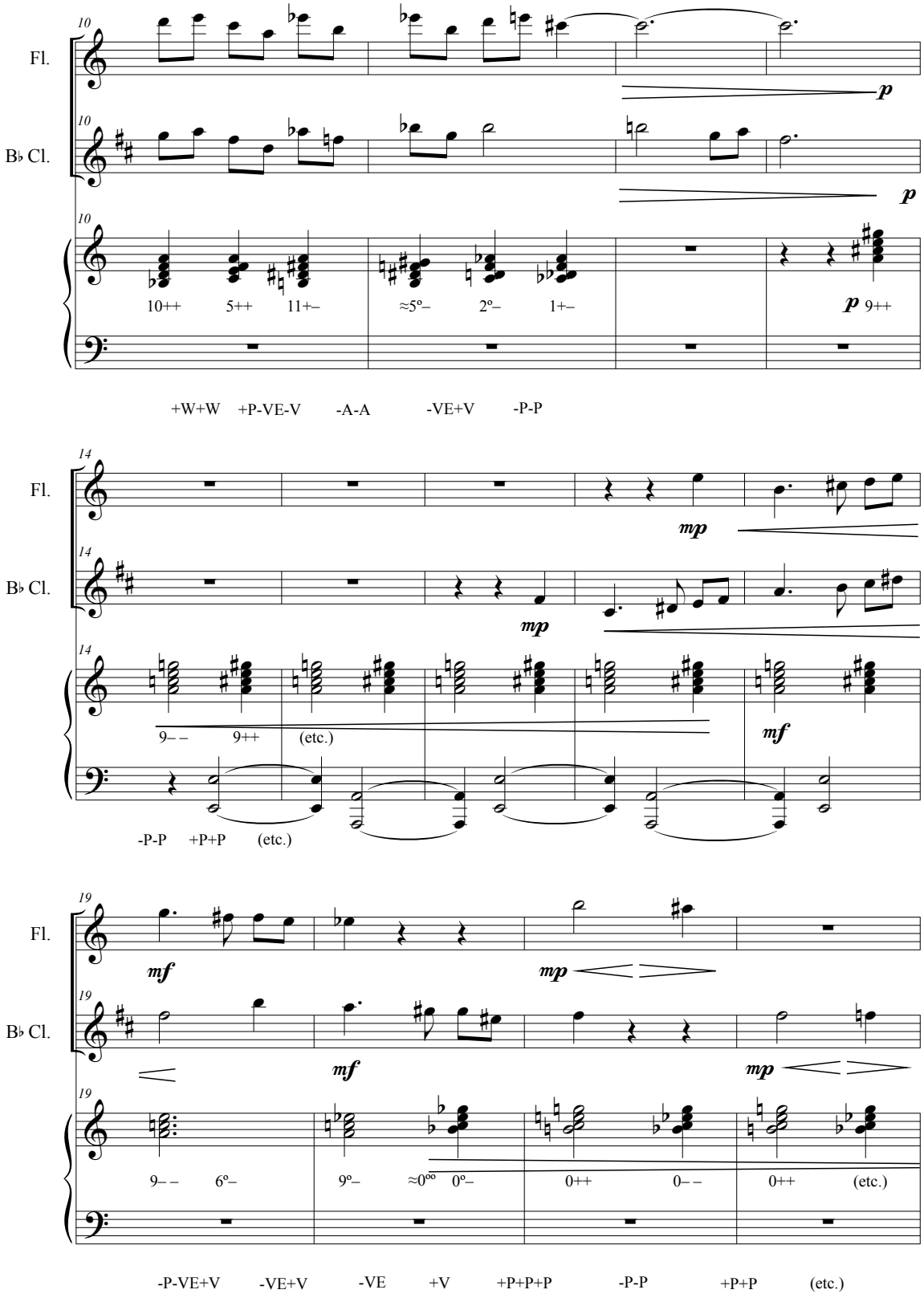
As a practical foundation, this study used the theory to demonstrate its validity and prove its worth as more than a thought experiment. I applied the network to musical passages, primarily from Chopin, using an analytical shorthand to summarize each pitch movement of a given transformation. These transformation labels were then translated into geometric maps of the harmonic network. The maps charted the paths taken through the theoretical parsimonious space, and they exhibited variety while simultaneously implying potential norms in the handling of such harmonic conditions. Through these analyses, new explorations into the construction of previously indefinable musical passages open up to the musician, particularly to the interested non-theorist.

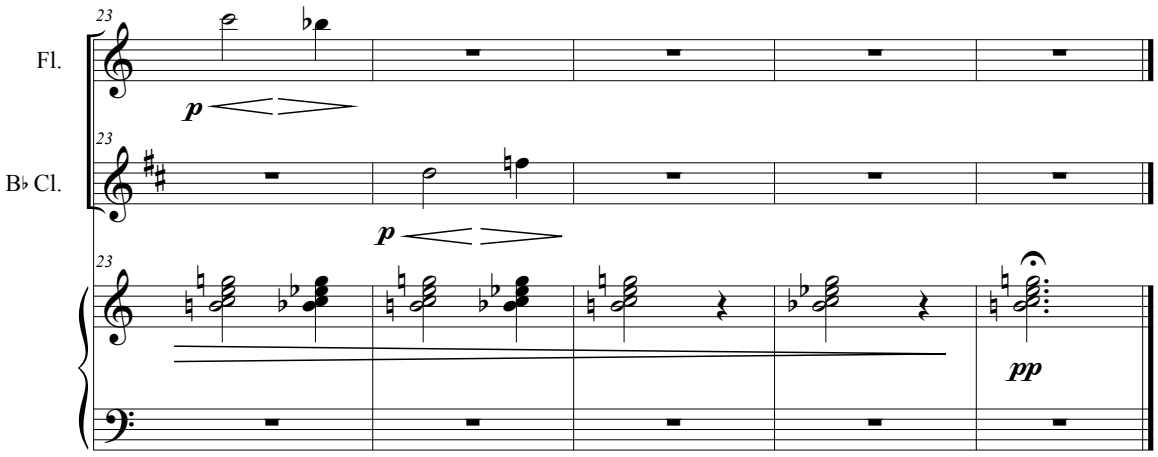
APPENDIX

EXAMPLE COMPOSITION: PRESCRIPTIVE APPLICATION OF THE LATTICE MODEL

E. S. A. JACOBUS




10
Fl.
B♭ Cl.
p
10++ 5++ 11+– ≈5º– 2º– 1+–
p 9++
+W+W +P-VE-V -A-A -VE+V -P-P
14
mp
9– – 9++ (etc.)
mf
-P-P +P+P (etc.)
19
mf
mp
9– – 6º– 9º– ≈0ºº 0º– 0++ 0– – 0++ (etc.)
-P-VE+V -VE+V -VE +V +P+P+P -P-P +P+P (etc.)



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