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ABSTRACT OF DISSERTATION

Jennifer Ann Eli

The Graduate School
University of Kentucky

2009

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GRADES TEACHERS' MATHEMATICAL CONNECTIONS WHILE COMPLETING
INVESTIGATIVE TASKS IN GEOMETRY

ABSTRACT OF DISSERTATION

A dissertation submitted in partial fulfillment of the
requirements for the degree of Doctor of Philosophy in the
College of Education
at the University of Kentucky

By
Jennifer Ann Eli

Lexington, Kentucky

Co-Directors: Dr. Margaret J. Mohr-Schroeder, Assistant Professor of Mathematics
Education
and Dr. Carl W. Lee, Professor of Mathematics

Lexington, Kentucky

2009

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With the implementation of *No Child Left Behind* legislation and a push for reform curricula, prospective teachers must be prepared to facilitate learning at a conceptual level. To address these concerns, an exploratory mixed methods investigation of twenty-eight prospective middle grades teachers' mathematics knowledge for teaching geometry and mathematical connection-making was conducted at a large public southeastern university. Participants completed a diagnostic assessment in mathematics with a focus on geometry and measurement (CRMSTD, 2007), a mathematical connections evaluation, and a card sort activity. Mixed methods data analysis revealed prospective middle grades teachers' mathematics knowledge for teaching geometry was underdeveloped and the mathematical connections made by prospective middle grades teachers were more procedural than conceptual in nature.

KEYWORDS: Mathematics Education, Prospective Teacher Education, Middle Grades Mathematics, Mathematical Connections, Mathematics Knowledge for Teaching

Jennifer Ann Eli

July 23, 2009

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I would like to dedicate this work to my family and friends.

ACKNOWLEDGMENTS

The following dissertation, while an individual work, benefited from the insights, direction, and encouragement of many others. First, I would like to thank the co-chairs of this dissertation, Dr. Margaret Mohr-Schroeder and Dr. Carl Lee. I am grateful for their mentorship and guidance throughout my doctoral studies. Dr. Margaret Mohr-Schroeder, thank you for your patience, encouragement, and unwavering support. Dr. Carl Lee, thank you for encouraging me through both my undergraduate and graduate studies. Your passion for mathematics and mathematics education has been inspirational and an integral component to completing this work. You have helped me to persevere in even the direst of circumstances, and for that I am grateful. I would like to thank the other members of my dissertation committee-Dr. Willis Johnson, Dr. Joan Mazur, Dr. Mary Shake, and Dr. Jeffery Osborn whose insights and thought provoking comments guided my thinking in completing this dissertation.

I have also received support from many others and I am especially indebted to Dr. Kelly Bradley, and the members of “Team B”: Jessica Cunningham, Lingling Ma, Kenny Royal, Bill Harris, and Jenny Weber. Dr. Kelly Bradley, thank you for showing me by example and through challenging coursework how to think, reflect, and conduct educational research. You and the members of Team B encouraged me to conduct studies, submit proposals, present at conferences, and to publish! You and the team have helped me to become a better researcher by showing me the importance of taking an interdisciplinary approach to educational research. I appreciate the guidance and friendship!

The completion of this work would not have been possible without my network of incredibly supportive friends. You have kept me sane through what has been a difficult time both professionally and personally. I would like to express my thanks to Jakayla Robbins, Jessica Cunningham, JoAnn Garrett, JeNell Winchester, Jennifer Webster, David Feinauer, Deddie Quillen, Patrick Quillen, Olga Mendoza, Missy Pate, Dax Harris, and Lingling Ma.

I would also like to thank the prospective teachers who participated in this study and the Appalachian Mathematics and Science Partnership (AMSP) for funding a portion of this dissertation research. Through the AMSP project and its many summer institutes, I have gained invaluable professional and practical experiences in mathematics teaching

and research. I send my sincerest thanks to everyone involved with the AMSP project for giving me the opportunity to work with some of the most dedicated and creative mathematics and science educators.

Most of all, I would like to thank my family. My parents Larry and Nongpanga Rice for showing me how discipline and determination can take you anywhere you want to go in life. You have shown me the power of unconditional love, strength, and courage. I would like to thank my sister Amanda and brother Tim for always putting life into perspective. Thank you for always believing in me! Most of all, I would like to thank my husband Rob. Where do I even begin? You are my rock and source of inspiration. You have been with me through twelve years of ups and downs. Your voice of reason and patience has helped me navigate the rocky terrain of life. This work would not have been possible without your love and support.

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CHAPTER I

INTRODUCTION

Mathematics is the study of relationships among objects both real and abstract; it is a discipline of study that can be seen in every facet of life regardless of occupation (Devlin, 2000; National Council for Teachers of Mathematics [NCTM], 2000). Effective competition in a rapidly growing global economy places demands on a society to produce individuals capable of higher-order critical thinking and creative problem solving. In response to these demands, the NCTM (1989) published the *Curriculum and Evaluation Standards for School Mathematics* followed by the *Principles and Standards for School Mathematics* (NCTM, 2000), the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006), and the *Guiding Principles for Mathematics Curriculum and Assessment* (NCTM, 2009). Within the executive summary of the 2000 document is the guiding principle “students must learn mathematics with understanding, actively building new knowledge from experience and previous knowledge” (p. 2). The *PSSM* also highlights the importance of problem solving and establishing connections.

By solving mathematical problems, students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom...when students connect mathematical ideas, their understanding is deeper and more lasting, and they come to view mathematics as a coherent whole. (p. 4)

Our prospective middle grades teachers have been charged with the demanding task of helping middle grades students construct mathematical knowledge, establish mathematical connections, and develop mathematical habits of mind needed for problem solving (Conference Board of Mathematical Sciences [CBMS], 2001). Mathematical habits of mind encompass the skills needed to reason mathematically, communicate understanding of mathematics to others, and the ability to make connections not only within various strands of mathematics but to other disciplines. However, beginning teachers rarely make connections during instruction, or their connections are imparted in an implicit rather than explicit manner (Bartels, 1995; Eisenhart, Borko, Underhill, Brown, Jones, & Agard, 1993; Hiebert, 1989). We must look to our teacher education programs to help prospective teachers build the mathematical habits of mind that promote

a conceptually indexed, broad-based foundation of *mathematics knowledge for teaching* (Hill, Rowan, & Ball, 2005) which encompasses the establishment and strengthening of mathematical connection making for problem solving. In particular,

...the curriculum of teacher preparation programs must include helping preservice teachers make connections between mathematics concepts and between concepts and representations for the concepts. The teacher with this preparation should leave these programs with a well-developed, interconnected, and accessible knowledge base effective for teaching mathematics. (Bartels, 1995, p. 25)

If prospective middle grades teachers are expected to construct, emphasize, integrate, and make use of mathematical connections, then they must acquire an understanding of mathematics that is fluid, supple, and interconnected (Evitts, 2005). Prospective teachers must learn to access and unpack their mathematical knowledge in a connected, effective manner. Furthermore, prospective teachers must not only be able to do the mathematics they will teach but must possess a deep conceptual understanding of the mathematics. “Effective teaching requires an understanding of the underlying meaning and justifications for the ideas and procedures to be taught and the ability to make connections among topics” (Ball, Ferrini-Mundy, Kilpatrick, Millgram, Schmid, & Schaar, 2005, p. 1058). A deep understanding of connections between and among mathematical ideas is one of the four main characteristics of a teacher who has a *profound understanding of fundamental mathematics* (PUFM). As Ma (1999) states:

A teacher with PUFM has a general intention to make connections among mathematical concepts and procedures, from simple and superficial connections between individual pieces of knowledge to complicated and underlying connections among different mathematical operations and subdomains. When reflected in teaching, this intention will prevent students’ learning from being fragmented. Instead of learning isolated topics, students will learn a unified body of knowledge. (p. 122)

Without understanding the connections among the important, functional concepts in mathematics, prospective teachers cannot effectively engage middle grades students in mathematical connection making, reasoning, and problem solving. Given the increased attention by the NCTM (1989, 2000) standards and NCTM (2006) *Curriculum Focal Points* stressing the importance of mathematical connection making, an exploratory research study focused on the mathematical connections of middle grades prospective teachers as they engage in mathematical tasks is warranted.

Statement of the Problem

With the passage of the *No Child Left Behind* legislation and the pressure of high-stakes testing, what knowledge must a mathematics teacher possess to successfully educate the youth of today? To address this concern, researchers have begun to examine prospective teachers' mathematics knowledge for teaching and how such knowledge may impact student achievement. Traditionally measurements of teachers' knowledge have been assessed using variables such as coursework, degree(s) earned, certification routes, PRAXIS scores, and years taught. As a result, the empirical evidence establishing a connection between teachers' mathematics knowledge for teaching and student achievement has been limited (Wilson, Floden, & Ferinni-Mundy, 2001). To address this concern, Bush, Karp, McGatha, Ronau, and Thompson (2004) and Hill, Rowan, and Ball (2005) have developed valid and reliable mathematics assessments for middle and elementary teachers' mathematics knowledge for teaching, respectively. An exploratory study of mathematics knowledge for teaching geometry and its relationship to the mathematical connections made by prospective middle grades teachers while engaged in mathematical tasks is needed to inform scholars who wish to establish or refine such instruments at the middle grades level. Furthermore, the study will inform curriculum developers and program evaluators who wish to revisit their education programs for prospective middle grades teachers. Mathematics knowledge for teaching not only requires "making visible the connection to the kinds of mathematical thinking, judgment [and] reasoning one has to do in teaching" (Ball, 2008, p. 41), but also "unpacking the mathematics sufficiently and convincingly helping them [prospective teachers] see what there is to learn and do" (p. 41).

Purpose of the Study

The purpose of this sequential exploratory mixed methods study was to examine prospective middle grades teachers' mathematics knowledge for teaching geometry and the connections made while completing tasks meant to probe mathematical connections. In addition, the study investigated prospective middle grades teachers' coursework and its impact on their mathematics knowledge for teaching geometry and mathematical connections.

Research Questions

This study examined the mathematics knowledge for teaching geometry and the mathematical connections held by prospective middle grades teachers while engaged in investigative mathematical tasks. Specifically, this research study addressed the following questions:

1. What types of mathematical connections do prospective middle grades teachers make while completing tasks meant to probe mathematical connections?
2. What is the relationship between prospective middle grades teachers' mathematics knowledge for teaching geometry and the types of mathematical connections made while completing tasks meant to probe mathematical connections?

Ancillary Questions:

1. How does prospective middle grades teachers' coursework impact their mathematical connections?
2. How does prospective middle grades teachers' coursework impact their mathematics knowledge for teaching geometry?

The following (Table 1.1) lists the data sources for addressing these research questions.

Table 1.1. *Mapping Data Sources to Research Questions*

Research Question	Data Source		
	Diagnostic Teacher Assessments in Mathematics and Science	Mathematical Connections Evaluation	Card Sort
1. What types of mathematical connections do prospective middle grades teachers make while completing tasks meant to probe connections?		✓	✓
2. What is the relationship between prospective middle grades teachers' mathematics knowledge for teaching geometry and the types of mathematical connections made while completing tasks meant to probe mathematical connections?	✓	✓	✓
A-1. How does prospective middle grades teachers' coursework impact their mathematical connections?		✓	✓
A-2. How does prospective middle grades teachers' coursework impact their mathematics knowledge for teaching geometry?	✓		

Significance of Study

Although research has provided insights into the mathematics knowledge needed for teaching at the elementary level, there is little to no literature on assessing prospective middle grades teachers' mathematics knowledge for teaching geometry. Little research has been completed exploring the role of mathematical connections on teachers' mathematics knowledge for teaching geometry. This study will contribute to the literature examining mathematics knowledge for teaching geometry and the role of mathematical connections at the middle grades level.

Prospective middle grades mathematics teachers are typically trained in three types of programs: elementary, secondary, and those that directly prepare middle grades teachers. The *Mathematics Teaching in the 21st Century* (MT21) (2007) report, a cross-national study of the preparation of prospective middle school teachers, found that future

U.S. middle school mathematics teachers who were prepared through an elementary program had stronger pedagogical preparation, while those prepared through a secondary program had a stronger mathematics preparation (Schmidt et al., 2007). However, in contrast to prospective middle grades teachers in other countries, future U.S. middle school teachers prepared through a middle grades program had weaker preparation in both mathematics and pedagogy (Schmidt et al., 2007). Thus, prospective middle grades teachers prepared through a middle grades program in the United States need stronger mathematical and pedagogical preparation. The aforementioned MT21 report finding is of particular significance to this research study, which took place at a university where prospective middle grades teachers are prepared through a middle grades certification program.

Given the importance of mathematics knowledge for teaching, an exploratory mixed methods investigation of prospective middle grades teachers' mathematical connection making would inform mathematics educators and researchers seeking further understanding behind effective and ineffective prospective middle grades teacher preparation. Furthermore, by providing descriptions of prospective middle grades teachers' mathematics knowledge for teaching geometry and their mathematical connections, this study will aid those wishing to construct mathematical tasks for explicit connection making. Such tasks may include creating opportunities for prospective middle grades teachers to analyze errors, and to evaluate alternative methods or representations. This study aspires to add to the knowledge base of what we know about prospective middle grades teachers' mathematical connections and mathematics knowledge for teaching geometry.

Theoretical Framework

In the last quarter century, mathematics education reform and research on the learning and teaching of mathematics has been largely influenced by constructivist theory. Constructivism is grounded in the idea that all knowledge is constructed. A major tenet of constructivist theory posits that the learner constructs meaning from experiences by integrating prior knowledge with new knowledge. Through a constructivist lens, "mathematical knowledge is constructed, at least in part, through a process of reflective abstraction" (Noddings, 1990, p. 10). Constructing and understanding mathematical

concepts, ideas, facts, or procedures involves making connections between old and new knowledge. Hiebert and Carpenter (1992) suggest, “Many of those who study mathematics learning agree that understanding involves recognizing relationships between pieces of information” (p. 67). A constructivist perspective can provide an understanding of how prospective middle grades teachers construct, link, or bridge together relationships between mathematical concepts, ideas, and/or representations when engaged in tasks meant to probe mathematical connections. A constructivist theory of learning mathematics provides a supportive foundation for this study as the researcher attempted to understand and describe the types of mathematical connections prospective middle grades teachers make while engaged in tasks meant to probe mathematical connections.

Constructing, unpacking, and understanding connections are fundamental in carrying out the work of teaching mathematics. Mathematics teachers must “hold unpacked mathematical knowledge because teaching involves making features of particular content visible to and learnable by students” (Ball, Thames, & Phelps, 2008, p. 400). By constructing, decompressing, and unpacking their mathematical knowledge, teachers are better equipped to respond to students’ “why” questions, evaluate student conjectures, ask productive mathematical questions, and make connections to mathematics across the span of the curriculum. What is the mathematics knowledge entailed by teaching? Hill, Rowan, and Ball (2005) formally defined the mathematics knowledge entailed by teaching as *mathematics knowledge for teaching* (MKT).

By “mathematics knowledge for teaching”, we mean the mathematical knowledge used to carry out the work of teaching mathematics. Examples of this “work of teaching” include explaining terms and concepts to students, interpreting students’ statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs .(p. 373)

Figure 1.1 is a visual description of MKT and the specific subdomains implied by this definition.

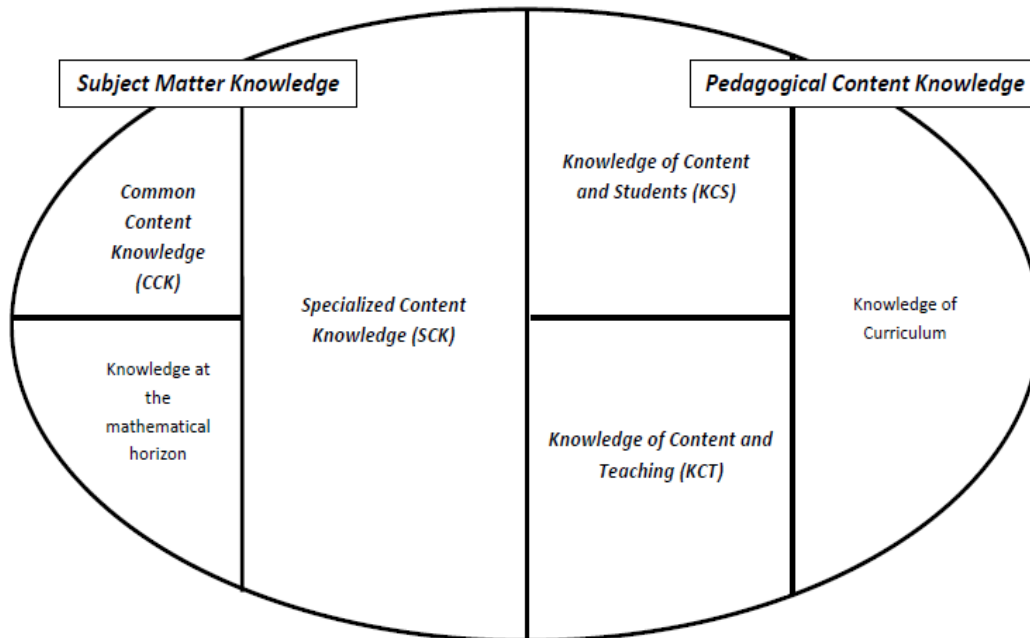


Figure 1.1. *Mathematics Knowledge for Teaching Framework (Ball, 2006).*

Common Content Knowledge (CCK) is defined as the mathematical knowledge and skill that is known by most educated adults. Specialized Content Knowledge (SCK) refers to the mathematical knowledge and skill unique to teaching. Knowledge at the mathematical horizon refers to knowledge of how mathematical topics are related across the curriculum. Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) refers to knowledge that combines knowledge of mathematics with knowledge of students and teaching, respectively. Explicit examples for each subdomain are discussed in Chapter 2. The MKT framework provides a lens for recognizing and classifying various mathematical connections made by prospective middle grades teachers.

Definition of Terms

The following are a list of terms and definitions that will be used throughout this study. Further explanation and applicability in use of these terms will be discussed in Chapter 2.

Common Content Knowledge: is mathematical knowledge that “any well-educated adult should have” (Ball, Hill & Bass, 2005, p. 22). It is “[mathematical] knowledge of a kind used in a wide variety of settings—in other words, not unique to teaching” (Ball, Thames, & Phelps, 2008, p. 399).

Constructivism: the building, integration, or assimilation of new knowledge within prior knowledge structures.

Geometry: the branch of mathematics focusing on properties of space, including points, lines, curves, planes and surfaces in space, as well as the figures which bound them.

Mathematical Connection: is a link (or bridge) in which prior or new knowledge is used to establish or strengthen an understanding of relationship(s) between or among mathematical ideas, concepts, strands, or representation.

Mathematics Knowledge for Teaching: the mathematical knowledge “used to carry out the work of teaching mathematics. Examples of this “work of teaching” include explaining terms and concepts to students, interpreting students’ statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs” (Hill, Rowan, & Ball, 2005, p. 373).

Prospective middle grades teacher: undergraduate middle grades education major enrolled in a program of studies leading to certification with a specialization in mathematics.

Mixed Methods: “*Mixed method* research is a research design with philosophical assumptions as well as methods of inquiry. As a methodology, it involves philosophical assumptions that guide the direction of the collection and analysis of data and the mixture of qualitative and quantitative approaches in many phases in the research process. As a method, it focuses on collection, analyzing, and mixing both quantitative and qualitative data in a single study or series of studies. Its central premise is that the use of quantitative and qualitative approaches in combination provides a better understanding of research problems than either approach alone” (Creswell & Plano-Clark, 2007, p. 5).

Pedagogical Content Knowledge: “Pedagogical content knowledge identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (Shulman, 1987, p. 4).

Specialized Content Knowledge: is “mathematical knowledge that is ‘specialized’ to the work of teaching and that only teachers need to know” (Ball, Hill, & Bass, 2005, p. 22).

Assumptions

1. The participants provided accurate information.
2. The participants did not receive outside help (i.e., other persons, textbooks, etc.) when completing the mathematical connections evaluation, card sort activity, and diagnostic teacher assessment in mathematics and science with focus in geometry and measurement.

Limitations

It was not possible to evaluate *all* prospective middle grades teachers. Thus, the study was limited to the number of prospective teachers available to the researcher. The current study focused on 28 prospective middle grades teachers engaged in a card sort activity as well as a mathematical connections evaluation utilizing a semi-structured interview format. These numerical values greatly limit the generalizability of the findings to larger groups of prospective middle grades teachers.

Organization of the Study

A goal of this dissertation study was produce two manuscripts for research publication, and as such, an articles formatted dissertation was chosen. An articles formatted dissertation is structured as follows:

- I. Introduction
- II. Review of Literature
- III. Methodology
- IV. Article 1: Exploring the web of connections: An investigation of prospective middle grades teachers' mathematical connection making through task-based interviews.
- V. Article 2: Prospective middle grades teachers' mathematical connections and its relationship to their mathematics knowledge for teaching.
- VI. Discussion, Conclusions, and Implications

The first chapter of the dissertation provided an introduction to the main ideas of the research topic. The second chapter provides a review of the research literature for this study. The third chapter describes the methodology for this study. The two research articles are presented in Chapters IV and V. The first article is a mixed methods analysis of the types of mathematical connections prospective middle grades teachers made while

completing tasks meant to probe mathematical connections. This article involves a Mathematical Connections Evaluation (MCE), Card Sort Activity (CSA), prospective middle grades teachers' content and methods coursework, and addresses research question 1 and ancillary research question 1. The second article is a mixed methods analysis examining the relationship between prospective middle grades teachers' MKT geometry and the types of mathematical connections made while completing tasks meant to probe mathematical connections. This article involves the use of a *Diagnostic Teacher Assessment in Mathematics and Science* (DTAMS) with focus in geometry and measurement for examining prospective middle grades teachers' MKT geometry and its relationship to their coursework and performance on MCE. The focus of article 2 is to address research question 2 and ancillary research question 2. The final chapter ties together the results and discussion along with implications, limitations, and recommendations for future research.

CHAPTER II

REVIEW OF LITERATURE

In this chapter, we review the research literature on constructivism, knowledge for teaching mathematics in general, mathematics knowledge for teaching geometry, and mathematical connections. This review of literature provides the foundation for the research study.

Constructivism

Constructivist theory has had a substantial impact on mathematics education in the last quarter century. The emergence of constructivism in education can be attributed to “dissatisfaction with information-processing theory, concerns that students are acquiring isolated, decontextualized skills and are unable to apply them in real-world situations and an interest in Vygotsky’s cultural-historical theory” (Gredler, 2005, p. 89). The basic tenet of constructivist theory is that a cognitive subject will respond to perturbations generated by conflict within their environment in such a way as to create and maintain their equilibrium. In other words, constructivist theory argues that when a learner is exposed to a new concept her goal is to reconstruct and build upon prior knowledge in order to “fit” this new knowledge within pre-existing notions about that concept. Within a constructivist paradigm, “knowledge is not passively received but actively built up by the cognizing subject” (Ernest, 1996, p. 336).

The key developers of constructivist theory include, but are not limited to, Jerome Bruner, Jean Piaget, and Leont’ev Vygotsky. “Vygotsky was deeply interested in the role of the social environment, included tools and cultural objects, as well as people, as agents in developing thinking” (National Research Council, 2000, p. 80). Vygotsky’s social constructivism is one form of educational constructivism. Social constructivism is rooted in the belief that knowledge is socially constructed and learning is attained through collaborative *assimilation* – how one transforms new information so that it makes sense within their knowledge structure and *accommodation* - referring to the change in cognitive structures in order to understand the new information received. “One of the basic tenets of the Vygotskian approach to education is the assumption that individual learning is dependent on social interaction” (van Oers, 1996, p. 93). Thus, learning is facilitated by integrating students into a knowledge community where student-centered

rather than teacher-centered approaches can be allowed to thrive. Knowledge cannot simply be transferred from one person to another; thus, the role of the teacher is that of a mentor responsible for 1) guiding peer interactions and 2) facilitating the continuity of building upon known concepts. From a social constructivist perspective, learning is both interactive and dialogic. In particular, mathematical learning is a “cognitive activity constrained by social and cultural process and a sociocultural phenomenon that is constituted by a community of actively cognizing individuals” (Wood, Cobb, & Yackel, 1995, p. 402).

Perhaps one of Vygotsky’s greatest contributions is his *zone of proximal development*: distance between the developmental level of the cognizing subject as ascertained via independent problem solving and the potential developmental level of problem solving via facilitator guidance, or in collaboration with more knowledgeable peers (Goos, Galbraith, & Renshaw, 2002). The zone of proximal development has direct implications for prospective teacher education as prospective middle grades mathematics teachers will have to continually assess their students’ mathematical understanding.

Role of Constructivism in Mathematics Education Research

Students are more likely to develop mathematical proficiency when they engage in mathematics as a community of learners rather than as isolated individuals (Kilpatrick, Swafford, & Findell, 2001). In *Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning* (Boaler & Humphreys, 2005) the authors illustrate how social constructivist theory in building a “community of learners” evolves through classroom practice. The authors demonstrate the powerful link between collaboration of practitioners and researchers as well as provide examples of a social constructivist approach to learning mathematics.

A major theme throughout the book focuses on the importance of small-group and whole-class discussions for connecting mathematical ideas and developing mathematical proficiency. As Boaler points out,

Mathematical discussions are extremely important, for a number of reasons. When students discuss a mathematical idea, they come to know that mathematics is more than a collection of rules and methods set out in books; they realize that mathematics is a subject that they can have ideas about, a subject that can invoke different perspectives and methods and one that is connected through organizing concepts and themes. (p. 83).

In chapter eight 8, the authors describe a lesson on discovering the volume of a cylinder by extending students' prior knowledge on the volume of a rectangular prism.

Humphreys begins by giving each group of students a picture of a rectangular prism constructed from unit cubes. She then asks each student to figure out mentally how many cubes are required to build the rectangular prism and then instructs the students to explain to their group members how they arrived at their answer. According to Vygotsky's social activity theory (which is under the umbrella of social constructivism), humans use tools that develop from a culture, such as speech, writing, and objects, to mediate their social environments. In this example, we see the "cultural tools" used to mediate student understanding of the volume of a rectangular prism include 1) the image of a rectangular prism constructed from unit cubes, 2) the mathematical writing and/or symbolism used to communicate an algebraic representation for the volume of a rectangular prism, and 3) the oral communication of how they arrived at a particular answer. Humphreys circulates around the room listening to student ideas and discovers that many of the students applied a formula they had learned before coming to seventh grade. She then asks the class as a whole, "Why does the length times width times height ($l \times w \times h$) make sense as a way of finding volume of a rectangular prism?" (p. 93). Humphreys uses her role as a facilitator to ask students to explain why their method makes sense. For many students, explaining why a formula works is a difficult task that requires them to make connections in their knowledge structures between the "how" and "why" the formula works. Through carefully constructed social interactions guided by the teacher, students were able to explain to their peers why the formula for the volume of a rectangular prism makes sense. One of the students, as a result of social interaction with their classmates, came up with the following written explanation:

$L \times W \times H$ makes sense because the $L \times W$ part of the formula gets you a flat face. Then you must multiply it by height, because a rectangular prism is 3 dimensional, so you must get the 3rd dimension, which is also the # of flat faces put on each of each-other. (p. 94)

Each student's explanation included the notion of figuring out the number of cubes in the bottom layer and then multiplying that number by the number of layers. Once students had arrived at this conclusion, Humphreys asked, "Is there anything about rectangular

prisms that could help us have a theory about how to find the volume of one of these [a cylinder]” (p. 96)? The question posed requires students to build upon prior knowledge (volume of a rectangular prism) for constructing new knowledge (volume of a cylinder). The acquisition of this new knowledge is constructed by building on prior knowledge through social interactions with peers through written and oral communication as well as mediated through other cultural artifacts such as concrete images representing cylindrical cans.

As research continues to provide good examples of instruction that help children learn important mathematics, there will be better understanding of the roles that teachers’ knowledge, beliefs, and goals play in their instructional thinking and actions...selection of tasks is highly dependent on teachers’ knowledge of mathematics, pedagogical content knowledge, and knowledge of students in general. (NRC, 2000, p. 171)

The aforementioned example illustrates how social constructivist theory has been applied by mathematics education researchers and practitioners.

Impact of Constructivism on Mathematics Curriculum

“Current reform in mathematics education has included discussion of and inquiry into the nature of mathematics, mathematics learning, and mathematics teaching. Reform efforts have been shaped by a number of influences including constructivist views on mathematics learning” (Simon, 1994, p. 71). Constructivist influence has had a substantial impact on a number of national curricular documents, in particular, the NCTM (1989, 2000) standards, the *Curriculum Focal Points* (NCTM, 2006), and the *Guiding Principles for Mathematics Curriculum and Assessment* (NCTM, 2009) In particular, “Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge” (NCTM, 2009, p. 2).

These documents, which are grounded in social constructivist principles, support a vision of classroom mathematics where students explore mathematical situations by engaging in both written and oral communication of mathematical ideas. These ideas are transmitted through social interaction where they are then validated or modified. Hence, students assume the role of mathematicians actively participating in a community effort for thinking, learning, creating, and evaluating mathematics. However, prospective teachers are not always afforded the opportunity to engage in such practice. If you were

to walk into a typical university mathematics course, what might you see? Would you see students working together in collaborative fashion actively engaging in mathematical conversation to solve problems or would you see a professor lecturing to a group of arguably attentive students? More likely than not, you would encounter the latter rather than the former. Nunn (1996) found that nearly 80% of class time is spent in lecture while only 14% of the time is devoted to class participation (the other 6% spent on teacher questions, praise, and criticism).

In the last decade several reform textbooks, funded by the National Science Foundation (NSF), have been integrated into schools. Prospective teachers are now faced with the demanding task of implementing these materials into their classrooms. These materials are grounded in the constructivist theory which posits students learn better when they are allowed to discover mathematics by interacting with other students. Teachers are often expected to teach mathematical topics and skills in ways substantially different from the ways in which they themselves learned the content (Ball, Lubienski, & Mewborn, 2001; Fennema, & Franke, 1992; Hiebert & Carpenter, 1992). Thus, these reform curricula pose a challenge to those involved with prospective teacher preparation. Our prospective teachers must not only possess a strong understanding of mathematics content and pedagogy but should make explicit the mathematical connections between and among mathematical concepts. These reform curricula place a focus on K-12 students' ability to make mathematical connections and thus, prospective teachers must be flexible in facilitation and integration of these reform curricula in their classroom.

As mathematics educators must prepare prospective middle grades teachers for integration into a community of practice saturated by standards-based reform curricula, whose roots are grounded in social constructivism, it was appropriate to focus on this particular variety of constructivism for this research study.

Knowledge for Teaching Mathematics

Teacher education programs are being challenged as never before to prepare prospective mathematics teachers in ways that will enhance teaching and learning of mathematics well into the 21st century. Research suggests that teachers' mathematics knowledge, knowledge of teaching, and knowledge of students' thinking, and general beliefs about teaching influence what is taught and ultimately what students learn (Ball &

Bass, 2003; Ball, Lubienski, & Mewborn, 2001; Ball & McDiarmid, 1990; Fennema & Franke, 1992; Hiebert & Carpenter, 1992; Putnam & Borko, 2000).

Teacher knowledge continues to be a topic of debate among mathematicians and mathematics educators. Historically, when it comes to teacher preparation, mathematicians have placed emphasis on content knowledge whereas mathematics educators have placed focus on pedagogy. In recent years, scholars have come to realize subject matter knowledge and pedagogy are inseparable. This indissoluble relationship between subject matter knowledge and pedagogical content knowledge is called *pedagogical content knowledge* (Shulman, 1986).

Pedagogical content knowledge identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. (Shulman, 1987, p. 4)

Researchers have begun to explore the idea that “teaching quality might not relate so much to performance on standard tests of mathematics achievement as it does to whether teachers’ knowledge is procedural or conceptual, whether it is connected to big ideas or isolated into small bits...”(Hill & Ball, 2004, p. 332). With this in mind, Hill, Rowan, and Ball (2005) refined Shulman’s (1986) concept of pedagogical content knowledge for teaching by focusing on the subject-specific nature of this type of knowledge. In particular, they adapted his definition to the field of mathematics education by introducing the notion of *mathematics knowledge for teaching* (MKT).

By “mathematical knowledge for teaching,” we mean the mathematical knowledge used to carry out the work of teaching mathematics. Examples of this “work of teaching” include explaining terms and concepts to students, interpreting students’ statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs. (Hill, Rowan, & Ball, 2005, p. 373)

Teaching mathematics effectively requires prospective teachers to 1) have a deep understanding not only of the mathematics they will be teaching but of the mathematics their students will encounter as they move through the educational system; and 2) have a

deep conceptual understanding of the subject matter along with the ability to make connections between and within disciplines. This allows teachers to make informed decisions about the appropriate pedagogy to use in their classrooms (Ball et al., 2005; CBMS, 2001; Fennema & Franke, 1992; Ma, 1999). As Lampert (2001) points out,

One reason teaching is a complex practice is that many of the problems a teacher must address to get students to learn occur simultaneously, not one after another. Because of this simultaneity, several different problems must be addressed in a single action. And a teacher's actions are not taken independently; they are interactions with students, individually and as a group. A teacher acts in different social arrangements in the same time frame. A teacher also acts in different time frames and at different levels of ideas with individuals, groups, and the class to make each lesson coherent, to link one lesson to another, and to cover a curriculum over the course of a year. Problems exist across social, temporal, and intellectual domains, and often the actions that need to be taken to solve problems are different in different domains. (p. 2)

Prospective middle grades teachers connection making is not only an essential component to the development and strengthening of their MKT but is vital in addressing the “simultaneity” that occurs when carrying out the work of teaching.

Mathematics Knowledge for Teaching

To better prepare prospective middle grades teachers to facilitate learning of mathematics within a K-12 system saturated by reform curricula that is grounded in constructivist theory, an understanding of the mathematics knowledge entailed by teaching is essential. Most scholars would agree that an understanding of content matters for teaching. However, what constitutes this content knowledge for teaching has been widely debated. In an effort to understand content knowledge needed for teaching, Ball and her colleagues have developed a framework of *mathematics knowledge for teaching* (MKT). Figure 2.1 is a visual representation of the MKT framework (Ball, 2006) framework and its components.

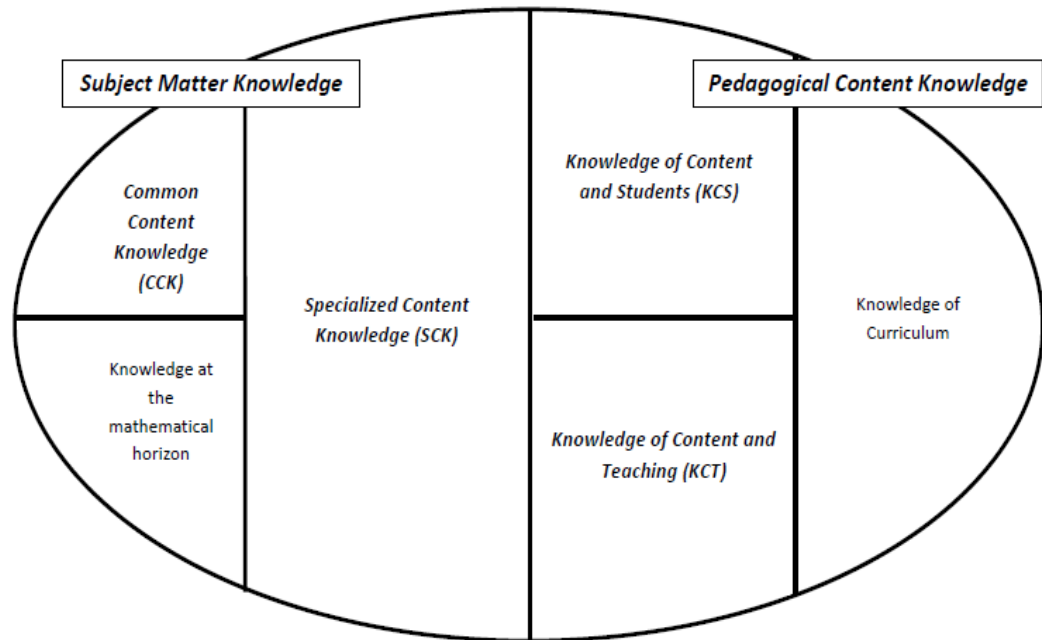


Figure 2.1. *Mathematics Knowledge for Teaching Framework (Ball, 2006).*

The framework is divided into two major components, subject matter knowledge and pedagogical content knowledge, each containing three subdomains. The subject matter knowledge component consists of Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and knowledge at the mathematical horizon. CCK refers to the mathematical knowledge “expected to be known by any well educated adult” (Bass, 2005, p. 429). CCK is “[mathematical] knowledge of a kind used in a wide variety of settings—in other words, not unique to teaching” (Ball, Thames, & Phelps, 2008, p. 399). An example of CCK would include the identification of various regular polygons such as a square, equilateral triangle or pentagon.

SCK refers to mathematical knowledge and skill that is “particular to the work of teaching, yet not required or known, in other mathematically intensive professions (including mathematics research)” (Bass, 2005, p. 429). SCK is mathematical knowledge, not pedagogy (Hill, Rowan, & Ball, 2005). SCK is considered to be “applied content knowledge that may be developed through the work of teaching” (Hill & Lubienski, 2007, p. 753). An example of SCK includes the recognition and analysis of non-standard solutions, explanations, representations, or approaches to solving a particular problem. A teacher is using SCK when developing a geometric justification for finding the area of a

regular n -sided polygon by dissecting the polygon into triangles and then summing up the area of the triangles to find the area of the regular n -sided polygon.

The third subdomain, knowledge at the mathematical horizon, is “an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (Ball, Thames, & Phelps, 2008, p. 403). A teacher is exercising knowledge at the mathematical horizon when they are aware of the interconnectedness of mathematics knowledge and its impact on learning mathematics later in a student’s mathematical career. An example of knowledge at the mathematical horizon is being aware that dissecting the regular n -sided polygon into triangles and then summing the area of the triangles to find the area of the polygon anticipates the extension of using calculus to find area enclosed by curves described by polar coordinates.

The pedagogical content knowledge component consists of Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and knowledge of the curriculum. KCS and KCT involve the intersection of knowledge of mathematics with knowledge of students and knowledge of teaching, respectively (Ball, 2006). KCS includes knowledge about student misconceptions, interpretation of student thinking that may have lead to misconceptions or errors, and the anticipation of what students will do when given a specific mathematical task. KCT includes the appropriate sequencing for instruction as well as recognizing the advantages or disadvantages of various manipulatives or representations for facilitating the understanding of a particular mathematical concept (Ball, 2006).

The MKT framework heavily grounded in constructivism provided the researcher a lens by which to recognize and classify various mathematical connections prospective middle grades teachers made while completing tasks meant to probe mathematical connections.

Mathematics Knowledge for Teaching Geometry

Numerous national educational groups consisting of mathematicians, mathematics educators, and classroom teachers have offered recommendations for the preparation of prospective mathematics teachers in the area of geometry (CBMS, 2001; NCTM, 2000; National Mathematics Advisory Panel, 2008; RAND Mathematics Study Panel, 2003). Geometry is one of the most interesting areas of mathematics to teach not only for its

appeal to the visual senses but for its historical significance in the development of mathematics. Geometry lends itself well to making “rich connections with the rest of mathematics, including topics and themes in discrete and continuous mathematics as combinatorics, algorithmic thinking, geometric series, optimization, functions, limits, trigonometry and more” (Goldenberg, Cuoco, & Mark, 1998, p. 23). Geometry is one of the focus areas for the NCTM (2000) content standards and NCTM (2006) *Curriculum Focal Points* and as such, prospective teachers must be prepared to effectively teach this subject. As Grover and Conner (2000) point out,

The college geometry course is especially important for prospective secondary teachers. In the United States, these students studied geometry only once in secondary school, and they will encounter geometric concepts only once more in college before they are certified to teach. Not only does the college geometry course need to lay a strong foundation for the content they will teach, but it is also one of the few courses that might develop the preservice teachers’ ability to create and present proofs. (p. 48)

The above statement not only holds for prospective secondary teachers but is applicable to prospective middle grades teachers. Cooney (2003) echoes these sentiments in his invited commentary on *The Trends in International Mathematics and Science Study (TIMSS) 1999 Video Study and the Reform of Mathematics Teaching*,

...the fact that only 22 percent of problems per U.S. lesson focused on geometry, suggests that some U.S. students may not be getting much geometry, including both two-and three-dimensional geometry. The role of school geometry in the United States, particular at the middle school level, deserves careful consideration in developing teacher education programs for both preservice and inservice teachers. (¶ 16)

The National Assessment of Educational Progress (NAEP) and TIMMS identified weaknesses in the performance of U.S. students on mathematics concepts, in particular geometry concepts, as compared to students in other countries (Gonzales et al., 2000).

Battista (1999) found that

U.S. students seemed to do better on items that were straightforward but formal in nature and not as well on spatial visualization and problem solving. Overall, the results suggest that U.S. students need more experience with spatial visualization, solving geometric problems and three-dimensional geometry. (p. 367)

A contributing factor to U.S. students’ weak performance on geometric concepts as compared to students in other countries could be attributed to the mathematical

knowledge of geometric concepts held by teachers. The *Mathematics Teaching in the 21st Century* (MT21) report, a cross-national study of the preparation of middle school teachers, found that prospective teachers' mathematics knowledge in the areas of algebra and geometry to be weak in comparison to prospective teachers in other countries (Schmidt et al., 2007). Evidence from the research literature suggests prospective teachers may not possess the subject-matter knowledge and pedagogical content knowledge needed to effectively teach geometrical concepts (Grover & Conner, 2000; Swafford, Jones, & Thorton, 1997).

In carrying out the mathematical tasks of teaching, prospective teachers must be prepared to unpack mathematical knowledge. This “unpacking” requires prospective teachers to make mathematical connections between and among mathematical ideas, concepts, strands, and representations (Ball, Thames, & Phelps, 2008; Hiebert & Carpenter, 1992; Fennema & Franke, 1992). Examining prospective middle grades teachers' mathematical connection making may provide additional insight into the strengths and weaknesses of prospective middle grades teachers' MKT. With this in mind, we turn our attention to the research literature on mathematical connections.

Mathematical Connections

What is a *mathematical connection*? Ma (1999) describes a mathematical connection in terms of a *concept knot* which links together underlying key concepts to a particular mathematical idea or representation. These concept knots are part of an interconnected web of *knowledge packages* consisting of key concepts for understanding and developing relationships among mathematical ideas, concepts, and procedures. Hiebert and Carpenter (1992) described mathematical connections as part of a mental network structured like a spider's web.

The junctures, or nodes, can be thought of as the pieces of represented information, and threads between them as the connections or relationships. All nodes in the web are ultimately connected, making it possible to travel between them by following established connections. Some nodes, however, are connected more directly than others. The webs may be very simple, resembling linear chains, or they may be extremely complex, with many connections emanating from each node. (p. 67)

Mathematical connections can also be described as components of a schema or connected groups of schemas within a mental network. A schema is a “memory structure that

develops from an individual's experiences and guides the individual's response to the environment" (Marshall, 1995, p. 15). Marshall posits that a defining feature of schema is the presence of connections. The strength and cohesiveness of a schema is dependent on connectivity of components within the schema or between groups of schemata. This model suggests that prospective middle grades teachers learn mathematics through assimilating or connecting new information into their mental networks, forming new connection(s) between existing knowledge components by accommodating or reorganizing their schemata to address perturbations in their knowledge structure and to correct misconceptions. Although mathematical connections have been defined, described, or categorized in various ways the common thread is the idea of a mathematical connection as a link or bridge between mathematical ideas. For the purposes of this study, a *mathematical connection* is a link (or bridge) in which prior or new knowledge is used to establish or strengthen an understanding of relationship(s) between or among mathematical ideas, concepts, strands, or representations.

Mathematics education literature supports the belief that mathematical understanding requires students to make connections between mathematical ideas, facts, procedures, and relationships (Hiebert & Carpenter, 1992; Ma, 1999; Moschkovich, Schoenfeld, & Arcavi, 1993; Skemp, 1978; Skemp, 1989). This belief is further supported by the creation of the NCTM (1989, 2000) standards which explicitly state the importance of mathematical connections in the school curriculum. According to these documents, mathematical connections are 'tools' for problem solving. As Hodgson (1995) points out,

...the investigation of problem situations leads naturally to the establishment and use of connections. In turn, the use of connections to solve problems brings about the need for their establishment. Connections are not seen as merely interesting mathematical facts but as integral components of successful problem solving" (p. 18)

Throughout the research literature a common theme emerges to explain why some students excel at problem solving while others do not. While there are numerous factors that may contribute to student learning within the problem solving process, consensus among researchers is that organization of knowledge plays a primary role (Anderson, 1990; Chinnapan & Lawson, 1996, 2000; Prawat, 1989; Pugalee, 2001; Sabella &

Cochran, 2003; Sabella & Redish, 2004). Successful problem solvers are those individuals who can readily access organized knowledge and thereby make appropriate connections within their knowledge schema. “For a knowledge structure to be useful in problem-solving, its components must be linked together not just exist as isolated facts and pieces of knowledge” (Sabella & Redish, 2004, p. 3).

Although students may have the components necessary to solve a problem, it is their inability to access these components in connection with other vital components that stymie their growth in problem solving (Chinnappan & Lawson, 2000; Livingston & Borko, 1990; Prawat, 1989; Sabella & Redish, 2004). In fact it has been argued that the knowledge possessed by an expert is not all different from a novice, but rather the distinction lies in the expert’s ability to make appropriate connections. The novice learner may have the relevant knowledge needed to solve a problem but is unable to access or use the knowledge in an effective manner (Lawson & Chinnappan, 1994). Thus, prospective middle grades teachers must be prepared to make connections between the content to be learned and their students’ understanding. By developing an understanding of the *mathematics knowledge for teaching* (Hill, Rowan, & Ball, 2005), mathematics educators will be able to help prospective teachers access and unpack knowledge in a connected, effective manner.

Although there are a few studies examining mathematical connections of prospective teachers at the elementary and secondary level (Bartels, 1995; Donigan, 1999; Evitts, 2005; Hau, 1993; Roddy, 1992; Wood; 1993), there is little to no research on mathematical connections made by prospective middle grades teachers.

Conclusions

Current reform movements and numerous national curricular documents on what teachers should know and be able to do have been heavily influenced by constructivist theory. When constructing mathematical knowledge, prospective teachers try to make connections within their knowledge structures by integrating new knowledge with prior knowledge. Mathematical connections are critical component of school mathematics, yet little research has been completed in this area. “If connections constitute the nervous system of understanding, then they surely deserve more attention and a research agenda” (Evitts, 2005, p. 112). Although, recognizing and understanding connections are

important aspects of developing MKT, there is little to no research examining the relationship between prospective middle grades teachers' mathematical connections and MKT. Thus, there is need for exploring the types of mathematical connections prospective middle grades teachers make and its relationship to their MKT.

CHAPTER III METHODOLOGY

This chapter describes the research design and methodology implemented. The purpose of this sequential exploratory mixed methods study was to examine prospective middle grades teachers' mathematics knowledge for teaching geometry and the connections made while completing tasks meant to probe mathematical connections.

Mixed Methods Research Design

Broadly speaking, there are three approaches or methods to conducting educational research: qualitative methods, quantitative methods, and mixed methods (Creswell, 2003; Creswell & Plano-Clark, 2007; Tashakkori & Teddlie, 1998; Teddlie & Tashakkori, 2009). Quantitative methods for educational research were adopted from the natural and/or physical sciences. The greatest strength associated with quantitative research is that its methods produce reliable and quantifiable data that can potentially be generalized to a large population. However, quantitative methods are not without their weaknesses. One of the greatest weaknesses of quantitative methods is they do not always address the “why” of a phenomenon. Quantitative methods can decontextualize the role of human behavior and in doing so variables that could help explain a phenomenon are left out of the statistical model. For example, suppose a Dean of a large public university has put tremendous pressure on its mathematics department to restructure its college algebra and elementary calculus courses due to a high rate of failure or withdraws from a course. A peer tutoring intervention program is then put into place with the hopes that student achievement in these courses will improve. Suppose through statistical analysis it is shown that participating in the intervention program does not have a statistically significant impact on student achievement in these courses. What explains the quantitative results of the study? To address this question, it might be beneficial to collect observational data of these peer tutoring sessions or even conduct student and instructor interviews to gauge their perception on the intervention program. As Creswell and Plano-Clark (2007) point out, quantitative research is “weak in understanding the context or setting in which people talk...the voices of participants are not directly heard in quantitative research. Further, quantitative researchers are in the

background, and their own personal biases and interpretations are seldom discussed” (p. 9).

Qualitative research is grounded in the theory that reality is constructed by an individual as they interact with the social environment. Qualitative researchers are interested in exploring and/or explaining social phenomenon as they occur in the natural setting. Qualitative research methods are designed to provide the researcher a means of understanding a social phenomenon by observing or interacting with the participants of the study.

Qualitative research involves the studied use and collection of a variety of empirical materials-case study; personal experience; introspection; life story; interview; artifacts; cultural texts and productions; observational, historical, interactional and visual texts-that describe routine and problematic moments and meanings in individuals’ lives. (Denzin & Lincoln, 2008, p. 4)

In qualitative research, the researcher becomes the instrument of data collection where hypotheses are generated through data collection and analysis. One of the greatest strengths of qualitative methods is that they have the potential to generate rich descriptions of the participants’ thought processes and tend to focus on reasons “why” a phenomenon has occurred. However, qualitative research methods are not without their weaknesses. As Creswell and Plano-Clark (2007) point out, “qualitative research is seen as deficient because of the personal interpretations made by the researcher, the ensuing bias created by this, and the difficulty in generalizing findings to a large group because of the limited number of participants studied” (p. 9). Although qualitative research methods have become increasingly popular, it has not yet been fully accepted by all members of the educational community.

By combining qualitative and quantitative methods the weaknesses in one method can be offset by the strengths in the other method (Creswell, 2003; Creswell & Plano-Clark, 2007). In particular, as Creswell and Plano-Clark (2007) explain,

A problem exists when the quantitative results are inadequate to provide explanations of outcomes, and the problem can best be understood by using qualitative data to enrich and explain the quantitative results in the words of the participants. Situations in which this problem occurs are those in which the quantitative results need further interpretation as to what they mean or when more detailed views of select participants can help to explain the quantitative results. (p. 35)

In other words, mixed methods research helps answer questions that cannot be answered using only qualitative or quantitative methods alone. Mixed methods provide a “more complete picture by noting trends and generalizations as well as in-depth knowledge of participants’ perspectives” (p. 33).

A mixed methods study involves the collection or analysis of both quantitative and/or qualitative data in a single study in which the data are collected concurrently or sequentially, are given a priority, and involve the integration of the data at one or more stages in the research process. (Creswell, Plano-Clark, Gutmann & Hanson, 2002, p. 212)

As this research study involved collecting and analyzing both quantitative and qualitative data, a mixed methods approach was needed to address the research questions. The following definition of *mixed method research* posited by Creswell and Plano-Clark (2007) was utilized for this study.

Mixed method research is a research design with the philosophical assumptions as well as methods of inquiry. As a methodology, it involves philosophical assumptions that guide the direction of the collection and analysis of data and the mixture of qualitative and quantitative approaches in many phases in the research process. As a method, it focuses on collecting, analyzing, and mixing both quantitative and qualitative data in a single study or series of studies. Its central premise is that the use of quantitative and qualitative approaches in combination provides a better understanding of research problems than either approach alone. (Creswell & Plano-Clark, 2007, p. 5)

A sequential exploratory mixed methods design of combining both qualitative and quantitative approaches served as a model for this study. Figure 3.1 provides a diagram of the sequential exploratory mixed methods design being used for this study.

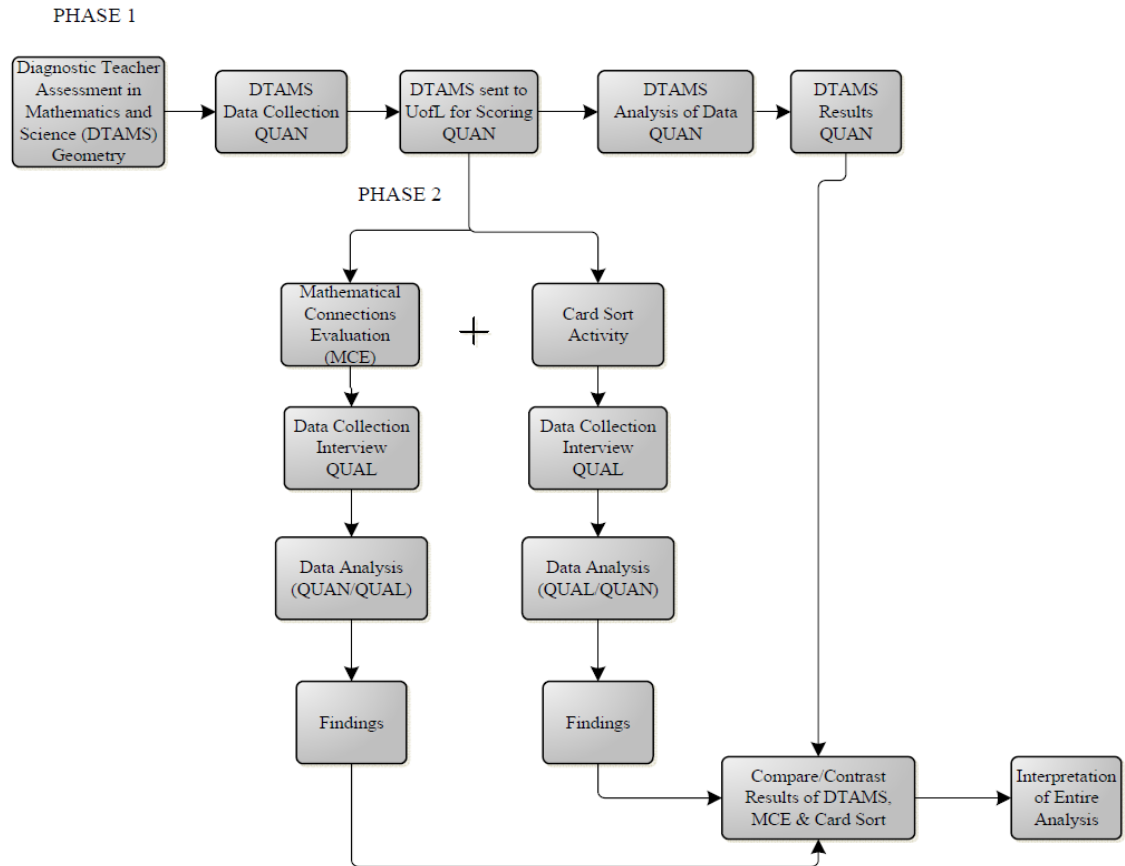


Figure 3.1. *Sequential Exploratory Design* (adapted from Creswell, 2003, p. 214)

The design is sequential as quantitative and qualitative data collection and analyses were implemented in two distinct phases. Quantitative data collection via the *Diagnostic Teacher Assessment in Mathematics and Science* (DTAMS) preceded qualitative data collection via the *Mathematical Connection Evaluation* (MCE) and *Card Sort Activity* (CSA). This research study is exploratory in nature as it “generates information about unknown aspects of a phenomenon” (Teddlie & Tashakkori, 2009, p. 25). In this case, (a) the types of mathematical connections prospective middle grades teachers made while engaged in tasks meant to probe mathematical connections and (b) how these connections are related to prospective middle grades’ teachers mathematics knowledge for teaching geometry. Unlike a traditional sequential exploratory design, the quantitative results of the DTAMS assessment (phase 1) did not directly inform or drive the construction of the MCE and CSA (phase 2) instruments. The quantitative data from the DTAMS and the qualitative data from the MCE and CSA were analyzed separately; results and findings merged during interpretation of entire analysis.

Population

The targeted population for this study was prospective middle grades teachers at a large mid-south university. The sampling frame was derived from a comprehensive list of prospective middle grades teachers meeting the following criteria: (a) declared middle school education major, and (b) actively pursuing a middle school certification in two content areas, one of which was mathematics. All prospective middle school teachers meeting both criteria were contacted for voluntary participation in this study. All 58 eligible participants were contacted, of which, 28 elected to participate. Of the 28 participants, 22 (78.6%) were female, 14 (50%) were juniors, 14 (50%) were seniors, and 6 (21.4%) were student teachers.

Instrumentation

There were three data collection instruments administered to prospective middle grades teacher; a Diagnostic Teacher Assessment in Mathematics and Science (DTAMS) with a focus on geometry and measurement, a Mathematical Connections Evaluation (MCE), and a Card Sort Activity (CSA).

Diagnostic Teacher Assessments in Mathematics and Science

The first instrument is the *Diagnostic Teacher Assessments in Mathematics and Science* (DTAMS) from the University of Louisville's Center for Research in Mathematics and Science Teacher Development [CRMSTD]. The DTAMS is comprised of four content domains: number and computation, geometry and measurement, probability and statistics, and algebraic ideas. For the purposes of this study, the DTAMS focused on the domain of geometry and measurement was selected. The domain of geometry and measurement consists of the following subcategories: two-dimensional geometry, three-dimensional geometry, transformational/coordinate geometry, and measurement. The 20-item assessment is composed of 10 multiple choice and 10 open response. In particular, the assessment measures four types of mathematics knowledge: (1) memorized knowledge, (2) conceptual knowledge, (3) problem solving and reasoning, and (4) pedagogical content knowledge (see Appendix A). The assessment contains five items in each of the four types of mathematical knowledge measured by DTAMS. Assessment items were developed by teams of mathematicians, mathematics educators, and classroom teachers who not only conducted extensive literature reviews on what

mathematics middle school teachers and students should know but also utilized national recommendations along with national and international test objectives in the development of research-based appropriate items. These content-valid items have been repeatedly tested and implemented in several institutions across the United States. As a measure of internal consistency the instrument has Cronbach's alpha $\alpha=.87$ with number of cases $n=429$. Inter-scorer reliability estimates were established "using percents of agreements among three graduate students who developed and used the scoring guides for scoring open response items and eventually scored all field tests" (CRMSTD, 2007, ¶ 8). The instrument was administered to participants prior to the interviews involving the mathematics connection evaluation and card sort activity. The DTAMS instrument served as a quantitative measure of prospective middle grades teachers MKT geometry. To strengthen the validity in use of the DTAMS assessment as a quantitative measure of prospective middle grades teachers' MKT, each item on the DTAMS was mapped to a subcategory of the MKT framework. The DTAMS assessments were scored by professional staff at the University of Louisville's CRMSTD.

Mathematical Connections Evaluation

The *Mathematical Connections Evaluation* (MCE) (see Appendix B) consisted of two components, a demographic survey followed by a series of mathematics problems. The purpose of this evaluation was two-fold: 1) to explore prospective middle grades teachers' mathematical connection making while engaged in tasks meant to probe mathematical connections, and 2) illuminate prospective middle grades teachers' mathematics content knowledge for teaching geometry. Utilizing a semi-structured clinical interview format, participants were asked to explain their thinking and thought processes as they solved each mathematics problem. The researcher developed a protocol of questions/probes for the semi-structured clinical interviews (see Appendix C). To strengthen the reliability and validity of the instrument, MCE items were constructed in cooperation with and reviewed by mathematicians and mathematics educators. Constructions of items were based on and aligned to national recommendations, in particular, *Recommendations for the Mathematical Education of Teachers* (CBMS, 2001), *Principles and Standards for School Mathematics* (NCTM, 2000), and *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence*

(NCTM, 2006) (see Appendix D). All MCE interviews were videotaped. In an effort to make the participants more comfortable and candid with their responses, their faces were not videotaped. The videotaped data focused on participants' written work, oral responses, and hand movements.

Card Sort Activity

Upon completion of the MCE interview, participants completed a *Card Sort Activity* (CSA). The CSA consisted of 20 cards labeled with various mathematical terms, concepts, definitions, and problems. Construction of the cards were based on and aligned to national recommendations, in particular, *Recommendations for the Mathematical Education of Teachers* (CBMS, 2001), *Principles and Standards for School Mathematics* (NCTM, 2000), and *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006) (see Appendix E). The card sort activity was also videotaped in the same manner as the MCE interviews. The videotaped data focused on participants' selection of subsets of cards from a 4 by 5 array of 20 cards displayed on a table.

The cards also underwent *quality review* (Halff, 1993; Tessmer, 1993) in which expert mathematicians and mathematics educators reviewed the cards for appropriateness and alignment to national recommendations. An expert quality review is an evaluation of a product (in this case the card sort activity) on the basis of content accuracy and design quality. Expert reviews consist of an expert or experts (in this case mathematicians and mathematics educators) reviewing a rough draft of the CSA to determine its strengths and weaknesses. The feedback/comments provided by the expert reviewers were analyzed and subsequent modifications were made to the CSA in order to improve the quality of the instrument. For instance, experts recommended that no more than 20 cards be used for the card sort activity. This recommendation was implemented as it was consistent with findings from card sort literature; in particular, Rugg and McGeorge (2005) found "the maximum number of entities which is conveniently manageable for repeated single-criterion sorts is about 20 or 30, though it is possible to use significantly more in some circumstances" (p. 98).

The purpose of the CSA was to examine the types of connections prospective middle grades teachers make between various mathematical concepts, definitions, and

problems. Participants were asked to complete a repeated single criterion open card sort and closed sort (Fincher & Tenenber, 2005; Rugg & McGeorge, 2005). For the closed card sort, five particular pairs of cards were chosen based on national recommendations (CBMS, 2001; NCTM 2000, 2006) on what middle school teachers and students should know and be able to do. The cards chosen were also influenced by content from the reform middle school curriculum textbook series *Connected Mathematics 2: Grade 6* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006), *Connected Mathematics 2: Grade 7* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006), and *Connected Mathematics 2: Grade 8* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006). The particular pairs of cards for the closed sort were also selected in consultation with mathematicians.

A card sort activity was a chosen data collection tool since sorting techniques are “aligned with the constructivist approach” (Rugg & McGeorge, 2005, p. 95). Furthermore, as suggested by Fincher and Tenenber (2005), “there is evidence to suggest that the way in which participants categorize entities *externally* reflects their *internal*, mental representations of these concepts” (p. 90). The participants sorted the cards based on a single criterion: their perceived notion of how the statements on the cards were connected. The researcher developed a protocol of interview questions focused on students’ mathematical connections (see Appendix F). The design of the protocols was influenced by the recommendations of Rugg and McGeorge (2005) for carrying out card sorting techniques:

The maximum number of entities which is conveniently manageable for repeated single-criterion sorts is about 20 or 30, though it is possible to use significantly more in some circumstances...Cards should likewise be all the same size. We usually use small filing cards, with the words word processed onto paper and then stuck onto the cards. This reduces problems with illegible handwriting, and avoids the issue of trying to get filing cards through a borrowed typewriter ...We usually encourage the respondents to look at all the items at the start of the session before they do any sorting, so that they are fully aware of the range of items to be sorted...We advise the use of a tape recorder (for respondents’ comments if problems occur). It is also worth considering using a Polaroid-type camera (for quick backup of record of groupings). If using a camera [or video camera], it is advisable to check beforehand that the photographs [video] can catch enough detail to allow all entities to be easily identified. (pp. 98-100)

The CSA interviews provided invaluable insight into students’ thinking and level of understanding that is not necessarily exhibited in written performance.

Contributions of the Pilot Study

Prior to the full study, pilot interviews were conducted with two student teachers (one secondary and one elementary), two prospective elementary teachers, and one in-service elementary teacher. The pilot interviews allowed the researcher to gain additional experience conducting interviews and to become more familiar with the logistical considerations of data collection and management.

As a result of the pilot, the MCE and CSA interview protocols were refined leading to an overall improvement of data collection procedures and subsequent analyses. For example, the pilot study informed the researcher whether to implement a reflective or concurrent think-aloud strategy when asking participants to respond to MCE items. During the pilot study, the researcher found that a reflective think-aloud approach allowed the participants to become more comfortable and open to giving mathematical explanations in an interview format. Arguably less comfortable (for participants) and more “on the spot”, a concurrent think-aloud approach allowed participants to report their thinking and understanding as they developed, which gave the researcher more insight into the explicit connection-making of participants that is not always exhibited in written explanations.

The pilot study revealed a semi-structured clinical interview format where there was a mixture of reflective and concurrent think-aloud strategies to the MCE items would be the best approach for improving the richness of data to be collected. In this study, a reflective think-aloud strategy was used for MCE items 1 (a)-(c) and (d)-(e), and for MCE items 2-4 (see Appendix B), a concurrent think-aloud strategy was used.

Collection of Data

Approval for this research study was granted by the University of Kentucky’s Institutional Review Board (IRB) through the Office of Research Integrity on February 8, 2008 prior to data collection (see Appendix G).

Data was collected via the DTAMS assessment, MCE, and CSA. The pool of eligible participants, i.e., prospective middle grades teachers actively pursuing a middle grades education major leading to certification in mathematics, fell into two groups—those enrolled in a problem solving course for prospective middle grades teachers and those not enrolled in the course. The researcher contacted the instructor for the problem solving

course at a large mid-south public university to arrange a time to solicit volunteers for the research study. Upon initial contact with course instructor, the researcher visited the course providing a detailed description of the research study and purpose (see Appendix H). Each potential participant was provided a copy of the informed consent letter (see Appendix I). The researcher carefully reviewed the informed consent letter with all potential participants stressing that participation in the study was purely voluntary and would have no negative effect on their course grade. The course instructor was not given access to the identity of students who consented to participate in the study. Once written consent had been obtained, the course instructor administered the DTAMS assessment. The instructor elected to use the DTAMS instrument in his course as a means of formative assessment and class discussion. Only data from consenting participants were used in this research study. The course instructor also elected to use the MCE and CSA in his course as a means of formative assessment and classroom discussion. Upon completion of the DTAMS assessment, participants scheduled an interview time with the researcher to complete the MCE and CSA.

Potential participants not enrolled in the problem solving course were contacted via a general email announcement (see Appendix J). The informed consent form was sent as an attachment in the email announcement (see Appendix K). Potential participants were asked to review the informed consent form. If they chose to participate in the study they were asked to schedule two meetings-one for taking the DTAMS assessment and the other an interview session for completing the MCE and CSA.

The DTAMS assessments were administered two weeks prior to the interview session. For those enrolled in the problem solving course, the DTAMS assessment was administered by the course instructor during the class period. For those not enrolled in the problem solving course, the DTAMS was administered by the researcher. All participants were given approximately 75 minutes to complete the DTAMS assessment. All participants (enrolled and not enrolled in the problem solving course) had completed the DTAMS assessment within the same two week time period. The MCE and CSA were conducted outside of class and after completion of the DTAMS assessment. After completing the DTAMS assessment, all participants were provided with a two-week block for scheduling the MCE and CSA interviews. All participants (enrolled and not

enrolled in the problem solving course) engaged in two separate sessions on two different days-one for taking the DTAMS assessment and the other for MCE/CSA interviews. The procedures and content for the interview session for all participants were identical. During the interview sessions, participants took approximately 45-60 minutes to complete the MCE. A semi-structured clinical interview format where there was a mixture of reflective and concurrent think-aloud strategies to the MCE items was implemented (see Appendix C). Participants worked independently on MCE problems 1(a)-(c) (see Appendix B). The researcher sat at another table in the same room. The participant was given as much time as they needed to complete MCE problems 1(a)-(c). Participants were asked to let the researcher know when they had completed MCE problems 1(a)-(c). The participant was then interviewed. This reflective think-aloud approach was repeated for MCE problems 1(d)-(e). A concurrent think-aloud strategy where the participant was interviewed as they solved the problems was implemented for MCE items 2-4 (see Appendix B).

Upon completion of the MCE interview, participants began the CSA and interview. In the open card sort, 20 cards were arranged on a table in a 4 by 5 array. The cards were arranged the same way for each participant. The arrangement of cards is illustrated in Figure 3.2 below.

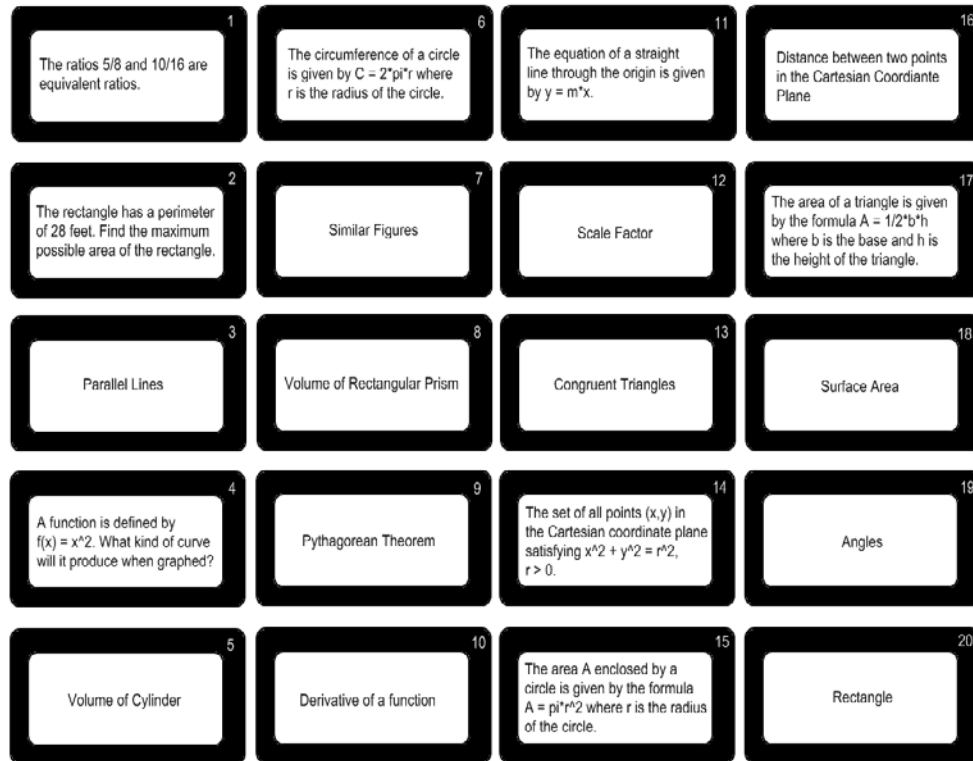


Figure 3.2. Arrangement of Cards for CSA

Participants were asked to select a subset of two or more cards they felt were related or connected. They were then asked to explain why the cards they had selected were related or connected. After giving an explanation, participants returned the cards back to the 4 by 5 array. They were then asked to select another subset of cards they felt were related or connected from the 4 by 5 array. This procedure allowed participants to re-use cards. This process was repeated until the participant indicated they could not make any more subsets.

In the closed card sort, the researcher selected five pairs of cards and asked if each pair of cards were related or connected and, if so, why? The five pairs chosen were cards 6 and 11, cards 2 and 4, cards 15 and 17, cards 4 and 15, and cards 9 and 16. The pairs of cards are illustrated in Figure 3.3 below.

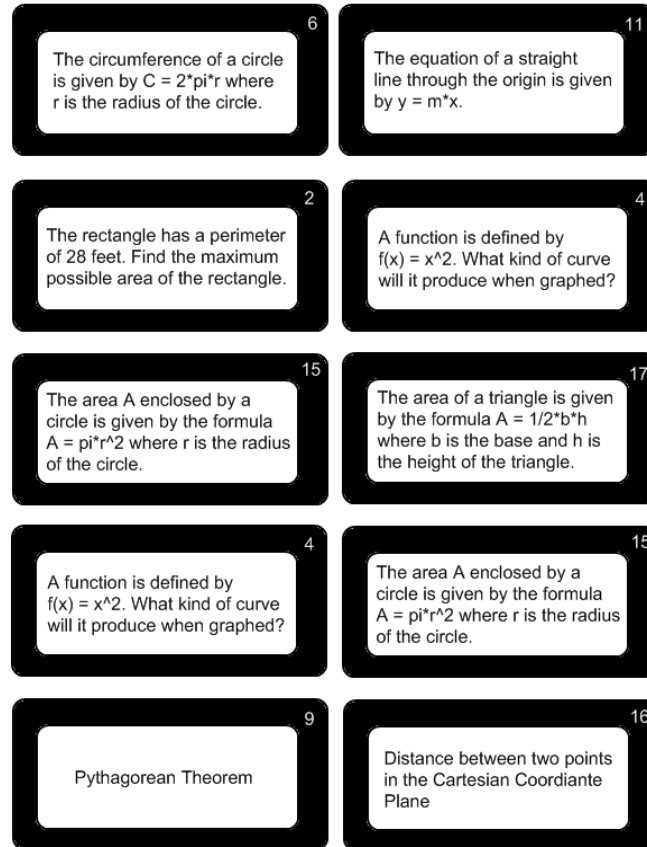


Figure 3.3. CSA Closed Sort Pairings

The researcher selected the first pair of cards, 6 and 11, from the 4 by 5 array (see Figure 3.2) and placed them in front of the participant. The participant was then asked if the pair of cards were related or connected and, if so, why? Once the participant provided a response, cards 6 and 11 were returned to the 4 by 5 array. This procedure was carried out for each of the aforementioned pairings of cards, and in the order listed. The CSA interviews took approximately 30-45 minutes.

The MCE and CSA interviews were audio and video recorded. All interview data was transcribed. All transcribed interview data remain confidential. The interviewees depicted in the transcribed data were given fictitious names as to conceal their true identity. The researcher has stored hard copy data in her office within a securely locked cabinet. Electronic data have been stored on a secure sequel server database. The audio and video interviews and their transcriptions have been kept in a securely locked cabinet. At anytime, participants could withdraw from the study and request that their data be permanently removed and/or destroyed.

MCE Scoring Rubric

The researcher used a deductive approach to the method of constant comparison in which codes are identified prior to the analysis (of participants' responses) and then looked for in the data (Leech & Onwuegbuzie, 2007) in order to identify the types of mathematical connections that were necessary or likely to be made as part of correctly solving the problems presented in the MCE. In order to generate these connection types *a priori*, the researcher used the guiding question, "What would the participant need to know or be able to do to solve this problem?" This question was used as a guide for each item on the MCE. A list of what was necessary or likely to be needed as part of correctly solving each MCE problem was generated. Each item on the list was given a descriptive code. Each new item on the list was compared to previously generated codes, so similar items could be labeled with the same descriptive code. After all the items on the list had been coded, the codes were grouped by similarity that represented a unique theme, i.e., mathematical connection type. Once these connections types were identified, two expert mathematicians were consulted to provide feedback and comments. Five types of mathematical connections were identified: *procedural*, *characteristic/property*, *algebraic/geometric*, *derivational*, and *2-D/3-D*. A scoring rubric using the aforementioned five types of mathematical connections was developed (see Appendix L).

Rubrics are used to determine if participants presented the correct information in their written responses to open-ended items. Rubrics are rating scales with systematic guidelines for assessing responses to open-ended questions, performances on tasks, and products related to the topic of interest. Rubrics typically include a set of criteria for assessing a written response, performance, or product plus a series of corresponding points on a numeric scale. In most cases, researchers use the numeric scales to summarize results across participants, thereby quantizing the original information. (Teddlie & Tashakkori, 2009, p. 237)

The scoring rubric was constructed using a numeric scale in which a participant received a score of 2 points if they correctly made a particular connection, 1 point if they made a partial connection and 0 points otherwise. The scoring rubric allowed the researcher a means by which to quantize the types of connections participants were and were not able to make.

Researcher Bias

In qualitative research, the investigator is the primary instrument for gathering and analyzing data (Creswell, 2003; Denzin & Lincoln, 2008; Merriam, 1988). As the primary investigator for this study, the researcher conducted all 28 individual interviews involving the MCE and CSA instruments. As qualitative research is “interpretative research, with the inquirer typically involved in a sustained and intensive experience with participants” (Creswell, 2003, p. 184) there is a need for the primary investigator to “explicitly identify their biases, values, and personal interests about their research topic and process” (p. 184).

As the primary investigator, I brought certain biases to the study as an experienced mathematics instructor for nearly eight years at the site where this study was conducted. My experiences as a mathematics educator have not only shaped my views on what prospective middle grades teachers should know and be able to do, but also expanded my own aptitude for recognizing, appreciating, developing, and understanding mathematical connections. Of the 28 participants in this study, 22 were former students who had taken a geometry course for prospective middle grades teachers for which I was the primary instructor.

At the core of the prospective teacher courses that I have taught is a vision of classroom mathematics where students explore mathematical situations by engaging in both written and oral communication of ideas. These ideas are then transmitted through social interaction, as interaction is one of the most important components of any learning experience. In such interactive classrooms, students assume the dual role of a mathematician and mathematics teacher by actively participating in a community effort for thinking, learning, creating, connecting, and evaluating mathematics. By building learning communities focused on both small and whole group discussion, students are responsible not only to themselves, but to other members of the community.

I view my role as a teacher of mathematics through a constructivist lens where I am a facilitator rather than dictator of knowledge. With every course I teach, I take great care in establishing an atmosphere of trust and rapport with my students by creating an open classroom environment in which students are encouraged to ask questions and add to discussion. I stress to students that it is acceptable to make mistakes; mistakes are just

opportunities to learn and grow. To create a safe learning environment for mathematical discovery and connection making, I ensure every student comment or question is met with a positive response.

I believe the rapport and trust I had established with my former students contributed to the overall comfort level of student involvement and openness during the MCE and CSA interviews. As a former instructor for a majority of the participants in this study, I inherently formed opinions and/or biases towards the students as individuals as well as groups. In an effort to address and minimize potential researcher bias and error, once data had been collected, participants' names were replaced by randomly generated three-digit numeric codes. Throughout the research process, I remained conscious of potential biases and attempted to minimize them.

Analysis of Data

The purpose of this sequential exploratory mixed methods study was to examine prospective middle grades teachers' MKT geometry and the mathematical connections made while completing tasks meant to probe mathematical connections. Data were analyzed both quantitatively and qualitatively.

Research Question 1

What types of mathematical connections do prospective middle grades teachers make while completing tasks meant to probe mathematical connections?

Mathematical Connections Evaluation (MCE).

In order to address research question 1, all MCE videotaped interview data were transcribed. The interview data from the MCE instrument were analyzed both qualitatively and quantitatively.

As described above, there were five types of mathematical connections identified for use in scoring the MCE: *procedural*, *characteristic/property*, *algebraic/geometric*, *derivational*, and *2-D/3-D* (see Appendix L). MCE data were then *quantitized* (Teddlie & Tashakkori, 2009) using the scoring rubric. A participant received a score of 2 points if they correctly made a particular connection, 1 point if they made a partial connection and 0 points otherwise. There were 7 procedural connections identified on the MCE for a possible maximum score of 14 points. There were 5 algebraic/geometric connections identified on the MCE for a possible maximum score of 10 points. There was 1

characteristic/property connection identified on the MCE for a possible maximum score of 2 points. There were 3 derivational connections identified on the MCE for a possible maximum score of 12 points. The maximum possible overall MCE score was 44 points. The researcher and an outside consultant scored the MCE using the aforementioned rubric. The second scorer was a mathematician at the site where the study was being conducted and who has taught mathematics content courses for prospective middle grades teachers. The researcher and consultant scored 2 of the MCEs together in order to become more familiar with the rubric and to help establish consistency in the scoring. The outside consultant independently scored a randomly selected sample of 35% ($n=10$) of the MCEs. Inter-rater reliability analysis as assessed by Pearson correlation analysis was .969.

The MCE interviews were qualitatively analyzed using a method of constant comparison. This method of constant comparison involved reviewing transcribed video data and participants' written work multiple times to become familiar with them. Next, these responses were "chunked" such that each meaningful phase was categorized into a unit. These units of information represented participants' approaches to solving each problem on the MCE. These units of information were then compared with one another, grouping similar units of information under a unique category. Each unit of information under each category was then analyzed further using the connection types developed for the MCE scoring rubric. There were two guiding questions for this part of the analysis:

1. What types of connections are inherent in this unit of information?
2. Does this unit of information represent a complete connection, partial connection, or no connection?

The process described above was carried out for each MCE item, focusing on one MCE item at a time. These analyses would allow the researcher to provide rich descriptions of participants' approaches to solving each problem and the mathematical connections that were and were not made.

Card Sort Activity (CSA)

To address research question 1, participant responses for each open card sort were analyzed using an inductive approach to the method of constant comparison (Denzin & Lincoln, 2000). This method of constant comparison involved reviewing videotape and

subsequent transcribed videotape data of participants' explanations for each card sort they had constructed. These explanations were "chunked" so that each meaningful phrase or sentence could be categorized with a descriptive code. Each new chunk of data was compared with previously generated descriptive codes, so that similar chunks could be labeled with the same descriptive code (Leech & Onwuegbuzie, 2007). After all the data had been coded, the codes were grouped by similarity, which represented a unique emergent theme, i.e., mathematical connection type. There were five types of mathematical connection themes that emerged from participants' responses to the open card sort: *categorical*, *procedural*, *characteristic/property*, *derivation*, and *curricular*.

The researcher and an outside consultant coded the open card sorts using a coding guide (see Appendix M). The coding guide provided a description of each of the five emergent mathematical connection types along with examples for each type. The second coder was a mathematician at the site where the study was being conducted and who has taught mathematics content courses for prospective middle grades teachers. The researcher and consultant together categorized 12 open card sorts (with each mathematical connections type represented at least twice) in order to become more familiar with the description for each mathematical connection type and to help establish consistency in the coding. The second coder independently coded a randomly selected sample of approximately 53% of the open card sorts ($n=137$). Inter-rater reliability analysis using a kappa statistic (Cohen, 1960) was performed to determine consistency among coders. The level of agreement among coders was found to be "substantially strong" (Landis & Koch, 1997, p. 165) with $\kappa=.74$. The CSA open sort data were quantitized by tallying the number of open sorts that fell into each mathematical connection category.

Unlike the open card sort, the closed card sort consisted of five pairs of preselected cards. Participants were asked to explain if and why each pair of cards was related or connected. Participants' responses for each pair of cards were analyzed using an inductive approach to the method of constant comparison (Denzin & Lincoln, 2000) for extracting themes. The method of constant comparison carried out for the open card sort was the same for each pair of cards in the closed sort. The CSA closed sort data were quantitized by tallying the number of responses that fell within each theme.

Research Question 2

What is the relationship between prospective middle grades teachers' mathematics knowledge for teaching geometry and the types of mathematical connections made while completing tasks meant to probe mathematical connections?

In order to address research question 2, the CSA closed sort data and MCE data were *quantitized* (Teddlie & Tashakkori, 2009) so that statistical analysis could be performed. Bivariate correlation analysis via Pearson product-moment correlations were used to examine the relationship between prospective middle grades teachers MKT geometry and MCE connections. Pearson product-moment correlations were used to examine the relationship between prospective middle grades teachers MKT geometry and the types of CSA closed sort connections made.

Ancillary Research Question 1

How does prospective middle grades teachers' coursework impact their mathematical connections?

To address ancillary research question 1, participants were divided into three distinct non-overlapping groups A, B, and C based on their coursework. As this data was collected within the last three weeks of the semester, currently enrolled courses were treated as courses that had been completed when placing participants into groups. All mathematics content courses are taught through a department of mathematics at the site where the study was conducted. There are six mathematics content courses in the middle school program. All participants had completed a calculus course. All participants had completed two mathematics content courses for elementary teachers. The first course focused on sets, numbers, and operations, problem solving, and number theory and the second course focused on algebraic reasoning, introductory probability and statistics, geometry, and measurement. All participants had completed a problem solving course for middle grades teachers. The remaining two mathematics content course requirements included a finite mathematics course (MATH I) and a geometry course for prospective middle grades teachers (MATH II). There were 20 participants who had completed MATH II and eight who had not. These same 20 participants had completed MATH I and the same eight participants had not completed MATH I. There were no cases where

participants had taken MATH II and not taken MATH I, or vice versa. Participants had either completed both MATH I and MATH II or had not completed both courses.

There are two methods courses in the middle school program at the site where this study was conducted. These methods courses are taught through a department of curriculum and instruction. METH I is a teaching mathematics in the middle school course. METH II is a student teaching in the middle school course. METH II is not mathematics methods specific. There were six participants who had completed METH I and 22 who had not. These same six participants were currently enrolled in METH II while the remaining 22 had not completed and were not currently enrolled in METH II. There were no cases where participants had completed or were currently enrolled in METH I and had not taken or were not currently enrolled in METH II, and vice versa. Participants had either completed or were currently enrolled in both METH I and METH II or they had not completed or were currently enrolled in both courses.

Participants were placed in Group A if they had completed all mathematics content and methods courses. Participants were placed in Group B if they had completed all mathematics content courses but had not taken mathematics methods courses. Participants were placed in Group C if they had not completed all content courses (in this case had not completed MATH I and II) and had not taken mathematics methods courses. The groups with their respective number can be found in Table 3.1.

Table 3.1. *Group Design for Data Analysis*

Group	Courses	Participants
A	MATH I Finite Mathematics (Completed)	6
	MATH II Geometry (Completed)	
	METH I Teaching Mathematics in the Middle School (Completed)	
	METH II Student Teaching in the Middle School (Currently Enrolled)	
B	MATH I Finite Mathematics (Completed)	14
	MATH II Geometry (Completed)	
	METH I Teaching Mathematics in the Middle School (Not completed & Not currently enrolled)	
	METH II Student Teaching in the Middle School (Not completed & Not currently enrolled)	
C	MATH I Finite Mathematics (Not completed & Not currently enrolled)	8
	MATH II Geometry (Not completed & Not currently enrolled)	
	METH I Teaching Mathematics in the Middle School (Not completed & Not currently enrolled)	
	METH II Student Teaching in the Middle School (Not completed & Not currently enrolled)	

To explore the impact of prospective middle grades teachers' content coursework on MCE a univariate analysis was conducted using a linear regression model. The participants in Groups B and C were utilized for this analysis because participants in Group B had completed all required mathematics courses while those in Group C had not. Participants in Groups B and C had not completed methods courses. The number of participants in this regression was 22. Qualitative variables, unlike quantitative variables, cannot be measured on a numerical scale. Therefore, coding of the qualitative variables

(in this case “Group”) into numbers is needed to fit the linear regression model. A process known as dummy coding was used to create a dichotomous variable from the categorical variable “Group”. Participants were coded as “1” if they belonged to the group and coded “0” otherwise. To assess the relationship between prospective middle grades teachers’ mathematics content coursework and performance on MCE, a linear regression analysis was conducted with MCE score as the dependent variable and mathematics content coursework as the independent variable.

To explore the impact of prospective middle grades teachers’ methods coursework on MCE a univariate analysis was conducted using a linear regression model. The participants in Groups A and B were utilized for the analysis because participants in Group A had completed all of the required methods courses while those in Group B had not. Participants in Groups A and B had completed all mathematics content courses. The number of participants in this regression was 20. Again, the process of dummy coding was used to create a dichotomous variable from the categorical variable “Group”. Participants were coded as “1” if they belonged to the group and coded “0” otherwise. To assess the relationship between prospective middle grades teachers’ methods coursework and performance on MCE, a linear regression analysis was conducted with MCE score as the dependent variable and methods coursework as the independent variable.

Ancillary Research Question 2

How does prospective middle grades teachers’ coursework impact their mathematics knowledge for teaching geometry?

To explore the impact of prospective middle grades teachers’ content coursework on their mathematics knowledge for teaching geometry, a univariate analysis was conducted using a linear regression model. The participants in Groups B and C were utilized for this analysis because participants in Group B had completed all required mathematics courses while those in Group C had not. Participants in Groups B and C had not completed methods courses. The number of participants in this regression was 22. Again, the process of dummy coding was used to create a dichotomous variable from the categorical variable “Group”. Participants were coded as “1” if they belonged to the group and coded “0” otherwise. To assess the relationship between prospective middle grades teachers’ mathematics content coursework and mathematics knowledge for

teaching, a linear regression analysis was conducted with DTAMS as the dependent variable and mathematics content coursework as the independent variable.

To explore the impact of prospective middle grades teachers' methods coursework on their mathematics knowledge for teaching geometry, a univariate analysis was conducted using a linear regression model. The participants in Groups A and B were utilized for the analysis because participants in Group A had completed all of the required methods courses while those in Group B had not. Participants in Groups A and B had completed all mathematics content courses. The number of participants in this regression was 20. Again, the process of dummy coding was used to create a dichotomous variable from the categorical variable "Group". Participants were coded as "1" if they belonged to the group and coded "0" otherwise. To assess the relationship between prospective middle grades teachers' methods coursework and mathematics knowledge for teaching geometry, a linear regression analysis was conducted with DTAMS score as the dependent variable and methods coursework as the independent variable.

Summary of Research Procedures

There were three instruments used to collect data for this research study, namely, the *Diagnostic Teachers Assessments in Mathematics and Science* (DTAMS) with a focus in geometry and measurement, the Mathematical Connection Evaluation (MCE), and the Card Sort Activity (CSA). These data were analyzed qualitatively and quantitatively. The DTAMS assessment served as a quantitative measure of prospective middle grades teachers' (MKT) in the domain of geometry and measurement. All interview data from the MCE and CSA were audio and video recorded. All interview data were transcribed. The MCE interview data were analyzed both quantitatively and qualitatively. The videotapes were reviewed and the MCE data were quantitized by grading the MCE using a scoring rubric. The MCE data were qualitatively analyzed. For each item on the MCE, transcript and video data were reviewed multiple times across all participants to provide rich descriptions of participants approach to solving each problem and the mathematical connections that were and were not made. The CSA data were analyzed both qualitatively and quantitatively. The CSA data were analyzed using a constant comparative method (Denzin & Lincoln, 2000) in which participant's interview responses to the card sort were identified, and unifying commonalities grouped into

metacategories. Once the metacategories of connection types had been identified, the CSA data were quantitized by tallying the types of connections made. In summary, data were qualitatively analyzed using constant comparative analysis (Denzin & Lincoln, 2000). The data were quantitatively analyzed using descriptive statistics, bivariate correlations, and linear regression.

CHAPTER IV

ARTICLE I: EXPLORING THE WEB OF CONNECTIONS: AN EXPLORATORY INVESTIGATION OF PROSPECTIVE MIDDLE GRADES TEACHERS' MATHEMATICAL CONNECTION MAKING THROUGH TASK-BASED INTERVIEWS

Mathematics education literature supports the belief that mathematical understanding requires students to make connections between mathematical ideas, facts, procedures and relationships (Hiebert & Carpenter, 1992; Ma, 1999; Moschkovich, Schoenfeld, & Arcavi, 1993; Skemp, 1978; Skemp, 1989). This belief is further supported by the formulation of the NCTM (1989, 2000) standards which explicitly state the importance of mathematical connections in the school curriculum. According to these documents, mathematical connections are 'tools' for problem solving. As Hodgson (1995) points out,

...the investigation of problem situations leads naturally to the establishment and use of connections. In turn, the use of connections to solve problems brings about the need for their establishment. Connections are not seen as merely interesting mathematical facts but as integral components of successful problem solving" (p. 18)

Prospective middle grades teachers must be prepared to help middle grades students construct mathematical knowledge, establish mathematical connections, and develop mathematical habits of mind needed for problem solving (CBMS, 2001). However, beginning teachers rarely make connections during instruction, or their connections are imparted in an implicit rather than explicit manner (Bartels, 1995; Eisenhart, Borko, Underhill, Brown, Jones, & Agard, 1993; Hiebert, 1989). If prospective middle grades teachers are expected to construct, emphasize, integrate, and make use of mathematical connections, then they must acquire an understanding of mathematics that is fluid, supple, and interconnected (Evitts, 2005). Prospective teachers must learn to access and unpack their mathematical knowledge in a connected, effective manner. Furthermore, prospective teachers must not only be able to do the mathematics they will teach but must possess a deep conceptual understanding of the mathematics. "Effective teaching requires an understanding of the underlying meaning and justifications for the ideas and procedures to be taught and the ability to make connections among topics" (Ball, Ferrini-Mundy, Kilpatrick, Millgram, Schmid, & Schaar, 2005, p.1058). Without

understanding the connections among the important functional concepts in mathematics, prospective teachers cannot effectively engage middle grades students in mathematical connection making, reasoning, and problem solving.

Given the increased attention by the NCTM (1989, 2000) standards and NCTM (2006) *Curriculum Focal Points* stressing the importance of mathematical connection making, an exploratory mixed methods study focused on the types of mathematical connections prospective teachers make as they engage in mathematical tasks may provide some insight into prospective middle grades teachers' accessing, unpacking, and connecting mathematical knowledge.

Mathematical Connections

What is a *mathematical connection*? Heibert and Carpenter (1992) described mathematical connections as part of a mental network structured like a spider's web.

The junctures, or nodes, can be thought of as the pieces of represented information, and threads between them as the connections or relationships. All nodes in the web are ultimately connected, making it possible to travel between them by following established connections. Some nodes, however, are connected more directly than others. The webs may be very simple, resembling linear chains, or they may be extremely complex, with many connections emanating from each node. (p. 67)

Mathematical connections can also be described as components of a schema or connected groups of schemas within a mental network. A schema is a "memory structure that develops from an individual's experiences and guides the individual's response to the environment" (Marshall, 1995, p. 15). Marshall posits that a defining feature of schema is the presence of connections. The strength and cohesiveness of a schema is dependent on connectivity of components within the schema or between groups of schemata. This model suggests that prospective middle grades teachers learn mathematics through assimilating or connecting new information into their mental networks, forming new connection(s) between existing knowledge components, accommodating or reorganizing their schemata to address perturbations in their knowledge structure and to correct misconceptions. Although mathematical connections have been defined, described, or categorized in various ways the common thread is the idea of a mathematical connection as a link or bridge between mathematical ideas. For the purposes of this study, a

mathematical connection is a link (or bridge) in which prior or new knowledge is used to establish or strengthen an understanding of relationship(s) between or among mathematical ideas, concepts, strands or representations.

Theoretical Framework

Ernest (1996) stated “constructivism is emerging as perhaps the major research paradigm in mathematics education” (p. 335). The basic tenet of constructivist theory is that a cognitive subject will respond to perturbations generated by conflict within their environment in such a way as to create and maintain their equilibrium. In other words, constructivist theory argues that when a learner is exposed to a new concept her goal is to reconstruct and build upon prior knowledge in order to “fit” this new knowledge within pre-existing notions about that concept. Thus, when prospective middle grades teachers are making mathematical connections they are trying to construct an understanding between and among mathematical ideas, concepts or representations by integrating new knowledge and with prior knowledge.

Constructivist influence has had a substantial impact on a number of national curricular documents; in particular, the NCTM (1989, 2000) standards and NCTM (2006) *Curriculum Focal Points*. These documents, which are grounded in social constructivist principles, explicitly state mathematical connections as a vital component for K-12 student learning of mathematics. In the last decade, several reform textbooks shaped by constructivist views on mathematics learning and placing emphasis on mathematical connections have been integrated into K-12 schools. Prospective middle grades teachers are now faced with the task of implementing these materials into their classrooms. These materials are grounded in the theory that students learn better when they are allowed to discover mathematics by interacting with other students. However, teachers are often expected to teach mathematical topics and skills in ways substantially different from the ways in which they themselves learned the content (Ball, Lubienski, & Mewborn, 2001; Fennema & Franke, 1992; Hiebert & Carpenter, 1992) and thus, these reform curricula focused on mathematical connection making pose a challenge to those involved with prospective teacher preparation. Our prospective teachers must not only possess a strong understanding of mathematics content and pedagogy but should make explicit the mathematical connections between and among mathematical concepts. These reform

curricula place a focus on K-12 students' ability to make mathematical connections and thus, prospective teachers must be flexible in facilitation and integration of these reform curricula in their classroom.

Constructivism examines how one constructs meaning from experience. Using such a perspective may provide an understanding of how prospective middle grades teachers construct, link, or bridge together relationships between mathematical concepts, ideas, and/or representations when engaged in tasks meant to probe mathematical connections. A constructivist theory of learning mathematics provides a supportive foundation for this study as the researcher attempted to understand and describe the types of mathematical connections prospective middle grades teachers make while engaged in tasks meant to probe mathematical connections.

Purpose of Study

The purpose of this study was to investigate the types of mathematical connections prospective middle grades teachers make when engaged in tasks meant to probe their mathematical connections. In addition, the study investigated prospective middle grades teachers' coursework and its impact on mathematical connections. Specifically the following questions were investigated:

1. What types of mathematical connections do prospective middle grades teachers make while completing tasks meant to probe mathematical connections?
2. How does prospective middle grades teachers' coursework impact their mathematical connections?

Mixed Methods Research Design

A *sequential exploratory* mixed methods design of combining both qualitative and quantitative approaches served as a model for this study. The following definition of *mixed method research* posited by Creswell and Plano-Clark (2007) was utilized for this study.

Mixed method research is a research design with philosophical assumptions as well as methods of inquiry. As a methodology, it involves philosophical assumptions that guide the direction of the collection and analysis of data and the mixture of qualitative and quantitative approaches in many phases in the research process. As a method, it focuses on collecting, analyzing, and mixing both quantitative and qualitative data in a single study or series of studies. Its central premise is that the use of quantitative and qualitative approaches in combination

provides a better understanding of the research problems than either approach alone. (p. 5)

Mixed methods research helps answer questions that cannot be answered using only qualitative or quantitative alone. Mixed methods research provides a “more complete picture by noting trends and generalizations as well as in-depth knowledge of participants’ perspectives” (p. 33). This mixed methods research study is exploratory as it “generates information about unknown aspects of a phenomenon” (Teddle & Tashakkori, 2009, p. 25), in this case, the types of mathematical connections prospective middle grades teachers make when engaged in tasks meant to probe mathematical connections. Figure 4.1 reveals a diagram of the sequential exploratory mixed methods design being used for this study.

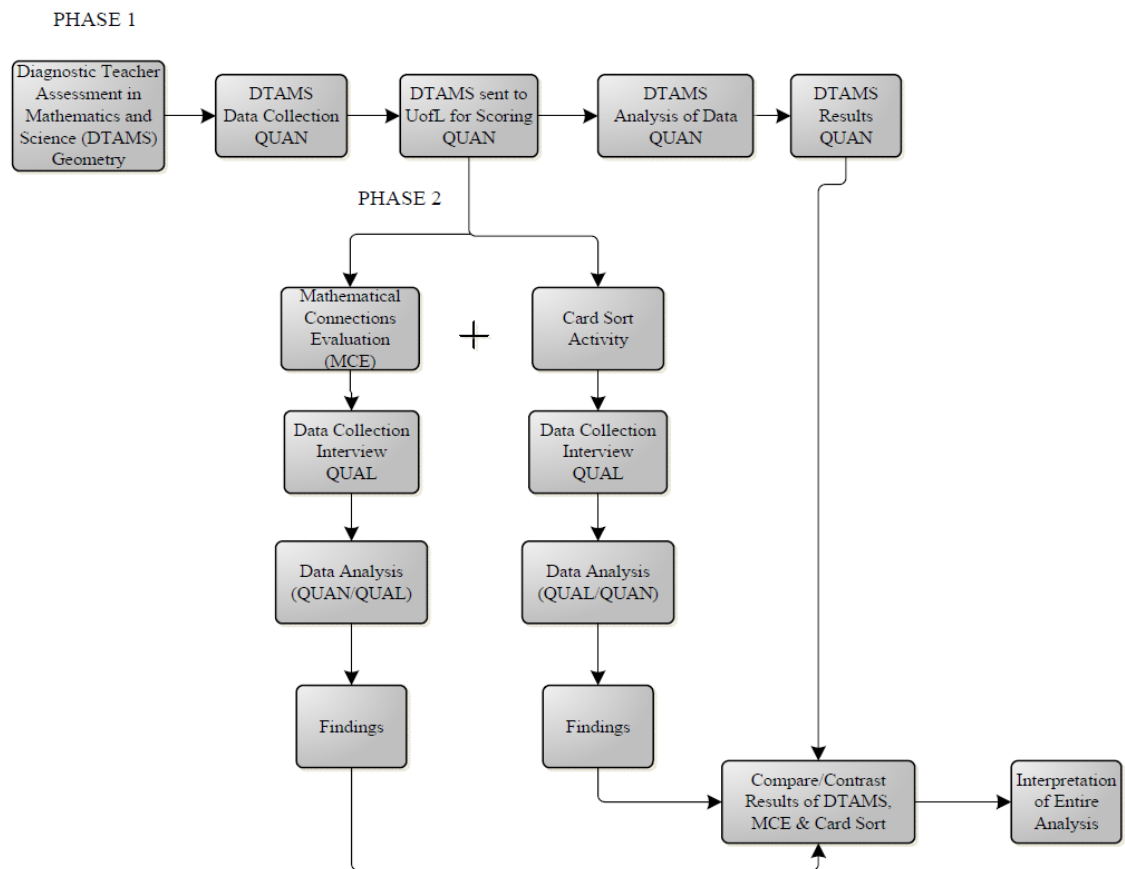


Figure 4.1. *Sequential Exploratory Mixed Methods Design*

Unlike a traditional sequential exploratory design, the quantitative results of the DTAMS assessment (phase 1) did not directly inform or drive the construction of the MCE and CSA (phase 2) instruments. In order to address the research questions at hand,

this article will focus its discussion on Phase 2 as seen in Figure 4.1. Qualitative data in the form of videotaped semi-structured interviews were collected from two instruments, a Mathematical Connections Evaluation (MCE) and a Card Sort Activity (CSA). The videotapes and transcribed interview data were analyzed both qualitatively and quantitatively. Constant comparative strategies (Denzin & Lincoln, 2000) were used to discover what types of mathematical connections prospective middle grades teachers made while engaged in the MCE and CSA. The qualitative data were then quantitized. Quantitizing data is “the process of converting QUAL data into numbers that can be statistically analyzed” (Teddlie & Tashakkori, 2009, p. 27). Phase 2 can be thought of as a *conversion mixed methods design* (p. 149) embedded in an overall sequential mixed methods design since “mixing occurs when one type of data is transformed and analyzed both qualitatively and quantitatively; this design answers related aspects of the same [research] questions” (p. 151). A conversion mixed methods design is one in which “data (e.g., QUAL) are gathered and analyzed using one method and then transformed and analyzed using the other method (e.g., QUAN)” (p. 155).

Population

The targeted population for this study was prospective middle grades teachers at a large mid-south university. The sampling frame was derived from a comprehensive list of prospective middle grades teachers meeting the following criteria: (a) declared middle school education major, and (b) actively pursuing a middle school certification in two content areas, one of which was mathematics. All prospective middle school teachers meeting both criteria were contacted for voluntary participation in this study. All 58 eligible participants were contacted, of which, 28 elected to participate. Most participants were female ($n=22$, 78.6%). There were 14 juniors (50%) and 14 were seniors (50%). There were 6 student teachers (21.4%) in the study.

Instrumentation

There were two data collection instruments administered to prospective middle grades teacher; a Mathematical Connections Evaluation (MCE), and a Card Sort Activity (CSA).

Mathematical Connections Evaluation

The *Mathematical Connections Evaluation* (MCE) (see Appendix B) consisted of two components, a demographic survey followed by a series of mathematics problems. A semi-structured clinical interview format in which participants used both concurrent and reflective think-aloud strategies when asked to explain their thinking and thought processes for solving each problem was implemented. Protocols were created for the semi-structured clinical interviews (see Appendix C). To strengthen the reliability and validity of the instrument, MCE items were constructed in cooperation with and reviewed by mathematicians and mathematics educators. Constructions of items were based on and aligned to national recommendations, in particular, *Recommendations for the Mathematical Education of Teachers* (CBMS, 2001), *Principles and Standards for School Mathematics* (NCTM, 2000), and *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006) (see Appendix D).

Card Sort Activity

Upon completion of the MCE interview, participants completed a *Card Sort Activity* (CSA). The CSA consisted of 20 cards labeled with various mathematical terms, concepts, definitions and problems. Figure 4.2 below illustrates the arrangement of cards for the open sort.



Figure 4.2. Arrangement of Cards for CSA

Construction of the cards was based on and aligned to national recommendations, in particular, *Recommendations for the Mathematical Education of Teachers* (CBMS, 2001), *Principles and Standards for School Mathematics* (NCTM, 2000), *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006) (see Appendix E). The purpose of the CSA was to examine the types of connections prospective middle grades teachers make between various mathematical concepts, definitions, and problems. Participants were asked to complete a repeated single criterion open card sort and closed card sort (Fincher & Tenenberg, 2005; Rugg & McGeorge, 2005).

In the closed card sort, five particular pairs were chosen based on national recommendations (CBMS, 2001; NCTM 2000, NCTM 2006) on what middle school teachers and students should be able to know and do. The cards chosen were also influenced by content from the reform middle school curriculum textbook series *Connected Mathematics2: Grade 6* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006), *Connected Mathematics 2: Grade 7* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006),

and *Connected Mathematics 2: Grade 8* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006). The particular pairs of cards chosen for the closed card sort were also selected in consultation with mathematicians. The participants sorted the cards based on a single criterion: their perceived notion of how the statements on the cards were connected. The researcher developed a protocol of interview questions for both the open and closed card sorts that focused on students' mathematical connections (see Appendix F). The design of the protocols was influenced by the recommendations of Rugg and McGeorge (2005) for carrying out card sorting techniques:

The maximum number of entities [cards] which is conveniently manageable for repeated single-criterion sorts is about 20 or 30, though it is possible to use significantly more in some circumstances...Cards should likewise be all the same size. We usually use small filing cards, with the words word processed onto paper and then stuck onto the cards. This reduces problems with illegible handwriting, and avoids the issue of trying to get filing cards through a borrowed typewriter...We usually encourage the respondents to look at all the items at the start of the session before they do any sorting, so that they are fully aware of the range of items to be sorted...We advise the use of a tape recorder (for respondents' comments if problems occur). It is also worth considering using a Polaroid-type camera (for quick backup of record of groupings). If using a camera [or video camera], it is advisable to check beforehand that the photographs [video] can catch enough detail to allow all entities to be easily identified. (pp. 98-100)

Figure 4.3 below illustrates the five closed sort pairings.

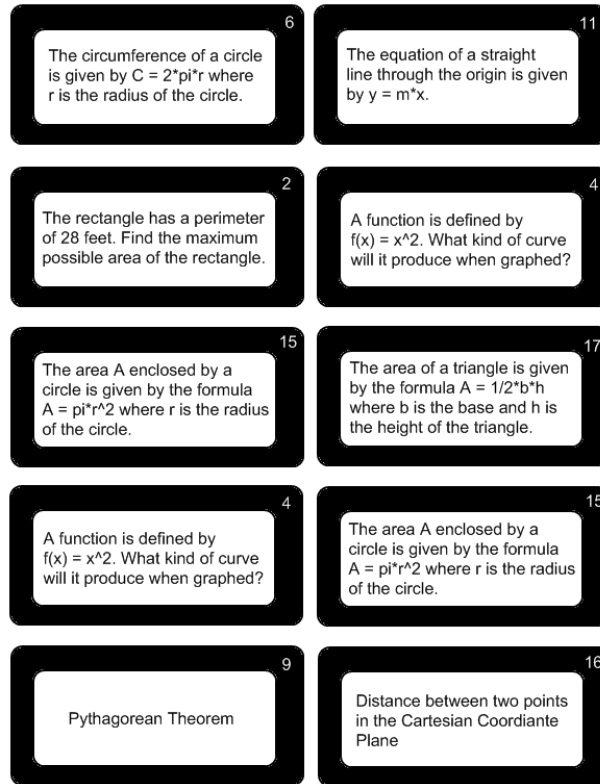


Figure 4.3. *CSA Closed Sort Pairings*

Quality Review

The MCE and CSA instruments underwent a *quality review* (Halff, 1993; Tessmer, 1993) to further strengthen the validity of each instrument. An expert quality review is an evaluation of a product (in this case the CSA and MCE instruments and protocols) on the basis of appropriateness, content accuracy, and design quality. Expert reviews consist of an expert or experts (in this case mathematicians and mathematics educators) reviewing a rough draft of each instrument along with interview protocols to determine strengths and weaknesses. The feedback and comments provided by the expert reviewers were analyzed and subsequent modifications were made to the MCE and CSA instruments in order to improve the quality of each instrument and interview protocols.

Analysis

What types of mathematical connections do prospective middle grades teachers make while completing tasks meant to probe mathematical connections?

Mathematical Connections Evaluation (MCE)

All MCE videotaped interview data were transcribed. The interview data from the MCE instrument were analyzed both qualitatively and quantitatively. A deductive approach to the method of constant comparison in which codes are identified prior to the analysis (of participants' responses) and then looked for in the data (Leech & Onwuegbuzie, 2007) was undertaken in order to identify the types of mathematical connections that were necessary or likely to be made as part of correctly solving the problems presented in the MCE. The result of this analysis was used to create the rubric for scoring the MCE. In order to generate these connection types *a priori*, the researcher used the guiding question, "What would the participant need to know or be able to do to solve this problem?" This question was used as a guide for each MCE item. A list of what was necessary or likely to be needed as part of correctly solving each MCE problem was generated. Each item on the list was given a descriptive code. Each new item on the list was compared to previously generated codes, so similar items could be labeled with the same descriptive code. After all the items on the list had been coded, the codes were grouped by similarity, which represented a unique theme (i.e., mathematical connection type). Once these connection types were identified, two expert mathematicians were consulted to provide feedback and comments.

In consultation with expert mathematicians, there were five types of mathematical connections identified *a priori*: *procedural*, *characteristic/property*, *algebraic/geometric*, *derivational*, and *2-D/3-D*. A scoring rubric using the aforementioned five types of mathematical connections was constructed (see Appendix L). The MCE data were then quantitized using the scoring rubric. A participant received a score of 2 points if they correctly made a particular connection, 1 point if they made a partial connection and 0 points otherwise. There were 7 procedural connections identified on the MCE for a possible maximum score of 14 points. There were 5 algebraic/geometric connections identified on the MCE for a possible maximum score of 10 points. There was 1 characteristic/property connection identified on the MCE for a possible maximum score of 2 points. There were 3 derivational connections identified on the MCE for a possible maximum score of 6 points. There were six 2-D/3-D connections identified on the MCE for a possible maximum score of 12 points. The maximum possible overall MCE score

was 44 points. The researcher and an outside consultant scored the MCE using the aforementioned rubric. The second scorer was a mathematician at the site where the study was being conducted and who has taught mathematics content courses for prospective middle grades teachers. The researcher and consultant scored 2 of the MCEs together in order to become more familiar with the rubric and to help establish consistency in the scoring. The outside consultant independently scored a randomly selected sample of 35% ($n=10$) of the MCEs. Inter-rater reliability as assessed by Pearson correlation analysis was .969.

The MCE interview data were qualitatively analyzed. For each item on the MCE, transcript and video data were reviewed multiple times across all participants to provide rich descriptions of participants approach to solving each problem and the mathematical connections that were and were not made.

Card Sort Activity (CSA)

Participant responses for each open card sort were analyzed using an inductive approach to the method of constant comparison (Denzin & Lincoln, 2000). This method of constant comparison involved reviewing videotape and subsequent transcribed videotape data of participants' explanations for each card sort they had constructed. These explanations were "chunked" so that each meaningful phrase or sentence could be categorized with a descriptive code. Each new chunk of data was compared with previously generated descriptive codes, so that similar chunks could be labeled with the same descriptive code (Leech & Onwuegbuzie, 2007). After all the data had been coded, the codes were grouped by similarity which represented a unique emergent theme, i.e., mathematical connection type. There were five types of mathematical connection themes that emerged from the data: *categorical*, *procedural*, *characteristic/property*, *derivation*, and *curricular*.

The researcher and an outside consultant coded the open cards sorts using a coding guide (see Appendix M). The coding guide provided a description of each of the five emergent mathematical connection types along with examples for each type. The second coder was a mathematician at the site where the study was being conducted and who has taught mathematics content courses for prospective middle grades teachers. The researcher and consultant together categorized 12 open cards sorts (with each

mathematical connection type represented at least twice) in order to become more familiar with the descriptions for each mathematical connection type and to help establish consistency in the coding. The second coder independently coded a randomly selected sample of approximately 53% of the open card sorts ($n=137$). Inter-rater reliability analysis using a kappa statistic (Cohen, 1960) was performed to determine consistency among coders. The level of agreement among coders was found to be “substantially strong” (Landis & Koch, 1997, p. 165) with kappa =.74. The CSA open sort data were quantitized by tallying the number of open sorts that fell into each mathematical connection category.

In the closed card sort, the researcher selected five pairs of cards and asked if each pair of cards were related or connected and, if so, why? The five pairs of cards chosen were cards 6 and 11, cards 2 and 4, cards 15 and 17, cards 4 and 15, and cards 9 and 16 (see Figure 4.3). The researcher selected the first pair of cards, 6 and 11, and placed them in front of the participant. The participant was then asked if the pair of cards were related or connected and, if so, why? Once the participant provided a response, cards 6 and 11 were returned to the 4 by 5 array (see Figure 4.2). This procedure was carried out for each of the aforementioned pairings of cards, and in the order listed. Participants were asked to explain if and why each pair of cards was related or connected. Participants’ responses for each pair of cards were analyzed using an inductive approach to the method of constant comparison (Denzin & Lincoln, 2000) for extracting themes. The method of constant comparison carried out for the open card sort was the same for each pair of cards in the closed sort. The CSA closed sort data were quantitized by tallying the number of responses that fell within each theme.

Coursework

How does prospective middle grades teachers’ coursework impact their mathematical connections?

To address this research question, participants were divided into three distinct non-overlapping groups A, B, and C based on their coursework. As this data was collected within the last three weeks of a semester, currently enrolled courses were treated as courses that had been completed when placing participants into groups. All mathematics content courses are taught through the department of mathematics at the site

where this study was conducted. There are six mathematics content courses in the middle school program. All participants had completed a calculus course. All participants had completed two mathematics content courses for elementary teachers—one focused on sets, numbers and operations, problem solving, and number theory; the other focused on algebraic reasoning, introductory probability and statistics, geometry, and measurement. All participants had completed a problem solving course for middle grades teachers. The remaining two mathematics content course requirements included a finite mathematics course (MATH I) and a geometry course for prospective middle grades teachers (MATH II). There were 20 participants who had completed MATH II and eight participants who had not. These same 20 participants had completed MATH I and the same eight participants had not completed MATH I. There were no cases where participants had taken MATH II and not taken MATH I or vice versa. Participants had either completed both MATH I and MATH II or not completed both courses.

There are two methods courses in the middle school program at the site where this study was conducted. These methods courses are taught through a department of curriculum and instruction. METH I is a teaching mathematics in the middle school course. METH II is a student teaching in the middle school course. METH II was not specific to mathematics. There were six participants who had completed METH I and 22 who had not. These same six participants were currently enrolled in METH II and the remaining 22 had not completed and were not currently enrolled in METH II. There were no cases where participants had completed or were currently enrolled in METH I and had not taken or were not currently enrolled in METH II, and vice versa. Participants had either completed or were currently enrolled in both METH I and METH II or they had not completed or were currently enrolled in both courses. Participants were placed in Group A if they had completed all mathematics content and methods courses. Participants were placed in Group B if they had completed all mathematics content courses but had not taken mathematics methods courses. Participants were placed in group C if they had not completed all content courses (in this case had not completed MATH I and II) and had not taken mathematics methods courses. There were six participants placed in Group A, 14 participants in Group B, and eight participants in Group C.

To assess the relationship between prospective middle grades teachers' content coursework and their performance on the MCE a univariate analysis was conducted using a linear regression model. The participants in Groups B and C were utilized for this analysis because participants in Group B had completed all required mathematics courses while those in Group C had not. Participants in Groups B and C had not completed methods courses. The number of participants in this regression was 22. Qualitative variables, unlike quantitative variables, cannot be measured on a numerical scale. Therefore, coding of the qualitative variables (in this case "Group") into numbers is needed to fit the linear regression model. A process known as dummy coding was used to create a dichotomous variable from the categorical variable "Group". Participants were coded as "1" if they belonged to the group and coded "0" otherwise. The linear regression analysis was conducted with MCE score as the dependent variable and mathematics content coursework as the independent variable.

To assess the relationship of prospective middle grades teachers' methods coursework and their performance on the MCE a univariate analysis was conducted using a linear regression model. The participants in Groups A and B were utilized for the analysis because participants in Group A had completed all of the required methods courses while those in Group B had not. Participants in Groups A and B had completed all mathematics content courses. The number of participants in this regression was 20. Again, the process of dummy coding was used to create a dichotomous variable from the categorical variable "Group". The linear regression analysis was conducted with MCE score as the dependent variable and methods coursework as the independent variable.

Results

The purpose of this exploratory mixed methods study was to investigate the types of mathematical connections prospective middle grades teachers make when engaged in tasks meant to probe their mathematical connections. The sample for this study consisted of 28 prospective middle grades teachers actively pursuing middle grades certification in two areas, one of which was mathematics. Each participant engaged in two semi-structured clinical interviews, one involving the MCE and the other involving the CSA. The results of the analysis involving these two instruments are discussed in this section.

MCE Connection Types

What types of mathematical connections do prospective middle grades teachers make while completing tasks meant to probe mathematical connections?

The interview data from the MCE instrument were analyzed both qualitatively and quantitatively. Qualitative analysis using a deductive approach to the method of constant comparison, in which codes are identified prior to the analysis (of participants' responses) and then looked for in the data, was undertaken to identify the types of mathematical connections that were necessary or likely to be made as part of correctly solving the MCE problems. The researcher in consultation with two expert mathematicians developed and refined five types of mathematical connections:

procedural, characteristic/property, algebraic/geometric, derivational and 2-D/3-D.

Table 4.1 provides a description and example for each MCE connection type.

Table 4.1. *Description of MCE Connections Types and Examples*

MCE Type	Description	Example
Procedural	A mathematical connection is called a procedural connection if the link (or bridge) used to establish or strengthen an understanding between mathematical ideas, concepts, strands, or representations is a procedure, method, or algorithm.	A participant is making a procedural connection when identifying the use of a table of values for graphing the line $y=3x$ in the Cartesian Coordinate Plane.
Algebraic/ Geometric	A mathematical connection is called an algebraic/geometric connection if it is a link (or bridge) used to establish or strengthen an understanding between geometric mathematical ideas, concepts, and/or representations with algebraic mathematical ideas, concepts, and/or representations.	A participant is making an algebraic/geometric connection when they are able to explain that the solution to the following linear system $\{y=3x; x=5\}$ is the intersection point of the line $y=3x$ and the line $x=5$ graphed in the Cartesian Coordinate Plane.

Table 4.1 (continued)

MCE Type	Description	Example
Characteristic/ Property	A mathematical connection is called a characteristic/property connection if the link (or bridge) used to establish or strengthen an understanding between mathematical ideas, concepts, strands, or representations involves using the mathematical properties and/or characteristics to describe, identify, or classify particular mathematical ideas, concepts, or representations.	A participant is making a characteristic/property connection when describing a rectangle as a quadrilateral with four interior 90 degree angles; opposite sides parallel and congruent.
Derivational	A mathematical connection is called a derivational connection if the link (or bridge) used to establish or strengthen an understanding between mathematical ideas, concepts, strands, or representations involves the justification or motivation for a particular mathematical theorem, formula, or procedure.	A participant is making a derivational connection when they are able to provide a justification or motivation for why the surface area, S , of a cylinder is given by $2\pi(\text{radius})^2 + 2\pi(\text{radius})(\text{height})$.
2-D/3-D	A mathematical connection is called a 2-D/3-D connection if it is a link (or bridge) used to establish or strengthen an understanding between 2-D mathematical ideas, concepts, or representations with 3-D mathematical ideas, concepts or representations.	Consider the region in the Cartesian Coordinate plane bounded by the lines $y=2$, $x=1$, the x -axis, the y -axis. The bounded region is a rectangle with vertices $(0, 0)$, $(2, 0)$, $(1, 0)$, and $(1, 2)$. Suppose the bounded region is rotated about the x -axis. A participant is making a 2-D/3-D connection if they are able to identify the 3-D object as a cylinder where the length and width of the rectangle correspond to the height and radius, respectively, of the cylinder.

The result of this *a priori* analysis was to create a scoring rubric using the aforementioned five types of mathematical connections (see Appendix L). The MCE data were then quantitized using the scoring rubric. A participant received a score of 2 points if they correctly made a particular connection, 1 point if they made a partial connection and 0 points otherwise. Descriptive statistics including means, standard deviations, minimums, and maximums for the MCE are reported in Table 4.2.

Table 2.2. *Descriptive Statistics for MCE (n=28)*

	Minimum	Maximum	Mean	SD
Procedural	5	14	10.18	3.175
Algebraic/Geometric	5	10	8.86	1.627
Characteristic/Property	1	2	1.89	.315
Derivational	0	6	2.96	2.117
2-D/3-D	0	12	9.54	3.305
Overall MCE Score	14	44	33.43	8.492

Graphing and Finding Area

In MCE problems 1(a)-(c), participants were asked to sketch the region bounded by the x -axis, the line $y=3x$ and the line $x=5$. Participants were then asked to describe the shape and find the area of this bounded region. A mathematical connection was deemed a procedural connection if the link (or bridge) used to establish or strengthen an understanding between mathematical ideas, concepts, strands, or representations was a procedure, method or algorithm. In the case of the MCE problem 1(a), a participant was said to have made a procedural connection if they could identify, explain, and carry out a correct procedure, method, or algorithm for graphing the lines $y=3x$ and $x=5$. Most participants were able to correctly graph the line $x=5$ (a vertical line passing through the

point (5, 0) in the Cartesian coordinate plane). The following is a typical representative response across participants with regard to graphing the line $x=5$.

First I was thinking graphing lines, um, cuz I knew you had to graph the x and the y and draw a Cartesian coordinate [plane]. Then to graph the x equals 5 that was easy because you just count over 5 on the x -axis here and it's a straight line [participant points to sketch of vertical line passing through the point (5,0)]. (P678, MCE Transcript, line 5)

There were two predominant methods exhibited by participants for graphing the line $y=3x$. The first involved the use of a table of values. The following is a representative response across participants for graphing the line $y=3x$ using a table of values.

This one is the y equals $3x$ line and I found it by picking these points, like when x is 0, plug it in, y is 0 because 3 times 0 is 0, when x is 1, plug it in and y will be 3, when x is 2, y will be 6, and then I made my line from those points. (P137, MCE Transcript, line 8)

The other predominant method for graphing the line $y=3x$ involved using the “rise over run” approach to plotting points on the Cartesian coordinate plane. In this approach, participants identified the slope of the line as having value 3 and the y -intercept as (0, 0). They would then use this information to plot two points and then draw the line passing through those two points. The following is a representative response across participants for graphing the line $y=3x$ using a “rise over run” approach.

Yeah, you just start at (0, 0) since b [y -intercept] is 0 and you go up 3 and over 1, because m [slope] is 3. You line up the line between (0, 0) and that point you just got to, so, which in this case would be (1, 3), and then you draw the line through those two points. (P113, MCE Transcript, line 22)

The “rise over run” approach produced both accurate and inaccurate sketches. There were two cases in which the participants had indicated that the line $y=3x$ is a line with slope 3 crossing the y -axis at (3, 0).

Um, okay, so from what I know about this line [$y=3x$], I know it crosses the y -axis at 3 and then I'm pretty sure, I just remember down 3 and over 1, it's kind of like how you get your next point, or up 3 over 1. (P263, MCE Transcript, line 35)

If I'm right in that $y=x$ is a diagonal line through the origin, then the line $y=3x$ will be a diagonal going through 3 on the y -axis. I'm not positive that I'm right about how to draw $y=3x$. (P962, MCE Transcript, line 20).

There was one case where a participant initially thought that the line $y=3x$ was a horizontal line passing through the point (15, 0).

Yes, I started with that first, because I had the value for x [points to the equation $x=5$], and then since I had a value for x here [points to the equation $y=3x$], I just plugged it in, multiplying out to get 15, and then used it as a horizontal line. (P130, MCE Transcript, line 25)

A mathematical connection was deemed a characteristic/property connection if the link (or bridge) used to establish or strengthen and understanding between mathematical ideas, concepts, strands, or representations involved using the mathematical properties and/or characteristics to describe, identify, or classify particular mathematical ideas, concepts, or representations. In the case of problem 1(b) participants had to describe the shape of the bounded region. In nearly all cases, participants were able to correctly identify the bounded region as a right triangle. The following are representative responses across participants for identifying the bounded region.

The shape of the bounded region is a right triangle, with three vertices connected at (0, 0), (5, 0), and (5, 15). It is a right triangle because a right angle (90 degrees) is formed at (5, 0), and the shape is a bounded region with 3 sides and 3 points forming 3 angles. (P305, MCE Transcript, line 30)

It has three sides, and, I said that the x -axis and then this line [$x=5$] formed a right angle, and then this was the hypotenuse [participant points to line segment connecting the point (0, 0) to the point (5, 15)], so it formed a right triangle...this is a vertical line [$x=5$] and this [x -axis] is a horizontal line, so they're perpendicular, they form a 90 degree angle. (P691, MCE transcript, lines 44-46)

A mathematical connection was deemed an algebraic/geometric connection if it was a link (or bridge) used to establish or strengthen an understanding between geometric mathematical ideas, concepts, and/or representations with algebraic mathematical ideas, concepts and/or representations. In sketching the bounded region in the Cartesian coordinate plane, participants would need to identify the point of intersection between the line $y=3x$ and $x=5$. This identification of this point of intersection was also needed to address problem 1(c) when finding the area of the triangle. There were two main approaches to finding the point of intersection-algebraically and graphically. In most cases, participants took an algebraic approach to finding the point of intersection. The

following are representative responses across participants for identifying the bounded region.

So I chose this as the base and I know that it's five because on my, um... on the x equals 5, it is five out from the origin which is where this line intersects. So that's my base and then my height I had to count up. I actually had to find where these two lines intersect and like I know from past classes that to find where two lines intersect, you have to set them equal to each other. But since one was a y equals formula...um... y equals $3x$ and the other one was x equals 5 so you can't exactly set them equal to each other. So I actually....you can either plug in 5 to find y or you can change the y equals $3x$ to an, um, an equation where you have x equals to something with y . Like I did x equals y divided by 3 and then I plugged or I substituted 5 in for x and solved for y . And then once I did that I put, I found that y equals 15 is where they intersect, which means that would be your height [of the triangle] because the height is on the y -axis because I chose this base [points to line segment connecting (0,0) to (5,0)]. (P486, MCE Transcript, lines 46-58)

Um, I knew that at this point, or uh, I knew that this line was x equals 5 and then I wanted to find out how tall the triangle would be and I knew that its base was 5 units long, um, but I wanted to find out how tall it would be, so I had to figure out what the value of y would be at x equals 5, so I plugged in 5 and solved for y . So at the point of intersection it would have been, y would have been equal to 15 so that was my height. (P496, MCE Transcript, lines 18-20)

There were a few cases where participants did not find an exact point of intersection but rather used a graphical approach by “counting” grid marks to estimate a point of intersection. The following are representative responses for estimating the point of intersection of the lines $y=3x$ and $x=5$.

And I knew the base was 5 because the line x equals 5 gave me that and then the height because my like it [participant points to sketch in coordinate plane] was off a little bit, I wasn't really sure, 'cuz I'm use to having like perfect graph paper where I can like see that, so I was like, I'm going to guess 13 [for the height of triangle] See, I drew this and I go well this is equal to 3, and you can see my little 3s, this is equal to 3, this is equal to 3, this is equal to 3, that looks like one more would give me the height, that's how I got 13. I knew it wasn't going to be any more than 15 but it couldn't be less than 12, because anymore than 15 would have gotten me past this line [referring to sketch of the line $x=5$ in the coordinate plane], it would have gotten me like, uh, up 1, 2, 3, over 1, it would have gotten me here and I needed this point of intersection here. (P633, MCE Transcript, lines 27-31).

Um, okay, so from what I know about this line [$y=3x$], I know it crosses the y -axis at 3 and then I'm pretty sure, I just remember down 3 over 1, it's kind of how you get your next point, or up 3 over 1. So I continued this pattern and here is

where it's on the 12th unit of the y-axis and it wasn't quite crossed so I thought on the 13th it looked like it would cross this line right here [$x=5$]. So that is where I got 12 from. (P263, MCE Transcript, lines 37-40)

Nearly all participants (92%) were able to make an algebraic/geometric connection by identifying a base and a height of the triangle along with correct measurements or measurements consistent with their sketch from problem 1(a). Nearly all participants identified the line segment connecting the point (0, 0) to (5, 0) as a base, b , of the triangle and the line segment connecting the point (5, 0) to the point (a, b) (where (a, b) is the point of intersection of the lines $y=3x$ and $x=5$, i.e., (5, 15)), as a height, h , of the triangle. In this case, $b=5$ units and $h=15$ units. All participants made a procedural connection for problem 1(c) by identifying the formula for the area, A , of a triangle as $A = (1/2)*(base)*(height)$.

Although all participants were able to make this procedural connection, not everyone was able to make a derivational connection. A mathematical connection was deemed a derivational connection if the link (or bridge) used to establish or strengthen an understanding between mathematical ideas, concepts, strands, or representations involved the justification or motivation for a particular mathematical theorem, formula, or procedure. When asked to give a justification and/or motivation for why the area of triangle can be found by taking half the base multiplied by the height, the results varied. Eight of the participants (29%) had indicated the formula for the area of a triangle was something that they had memorized.

I remembered the formula for the area of a triangle is one-half base times height. (P512, MCE Transcript, line 37) I remembered it from high school. (P512, MCE Transcript, line 43) I, I think I've probably seen it, but I don't remember where the formula derives from; I just remembered it off the top of my head. (P512, MCE Transcript line 47)

I mostly just have it memorized in my head, I just know, like I don't really spend time thinking about how to find the area of a triangle because I already have the formula stuck in my head because I use it so much lately, in my math classes. (P578, MCE Transcript, line 144)

The other two most common responses justifying and/or motivating the formula for the area of a triangle involved statements that the area of a triangle is one-half the area of a

rectangle. In some cases, participants' explanations pertained explicitly to the particular right triangle with base length 5 units and height 15 units.

Well, um, [pause], if this [pointing to the sketch of triangle for problem 1(a)] if it was like a full rectangle, to find the area is the base times the height, basically, and a triangle is half of a rectangle, so that's where the one-half comes from....you can see [points to the diagonal of "full rectangle"] it cuts it exactly in half. (P113, MCE Transcript, lines 50-54)

Because a triangle is half of a rectangle; and you know that the area of a rectangle is base times height. (P137, MCE Transcript line 43) [Participant draws in a dotted line segment from the point (5, 15) to (0, 15) to make a "rectangle".] Because these are congruent, because opposite sides of a like rectangle are congruent, and since, [pause], and those [pause], yeah, so these two sides are congruent [points to the two sides of the rectangle with length 5 units], and these two sides are congruent [points to the two sides of the rectangle with length 15 units], and this is the same for the both of them [points to the diagonal for each triangle inside the rectangle], they share that side, so it's going to be congruent to itself, and from side-side-side congruence you know that those two triangles are the same so the they have the same area which is one half the base times the height. (P137, MCE Transcript, lines 47-50)

When participants were asked, "When you went through this problem what kinds of things came to mind, like, what were you thinking as you went through this problem?" nearly 60% of the participants made a procedural connection to the Pythagorean Theorem when engaged in MCE problem 1(a)-(c). However, as many participants later explained, the Pythagorean Theorem was not needed to address the questions posed in parts (a)-(c).

Umm, [pauses], I [pauses] at first I was starting to do you see 5 squared, I was like ooh, Pythagorean Theorem! And then I'm like, she didn't even ask for this, so why am I even doing this [referring to carrying out a calculation using the Pythagorean Theorem] so that's why it's marked out [referring to the work participant did on paper]. (P633, MCE Transcript, lines 117-119)

Um, and this would be, using, [pause], you know, I could find the dimensions, or the length of this line [participants points to the hypotenuse of right triangle sketch on paper] by using the Pythagorean Theorem. 'Cuz I know that with the Pythagorean Theorem, to find this line which on my triangle would be the hypotenuse, given my base and my height, I could use the Pythagorean theorem to find the length of my hypotenuse, because looking at my uh, [pause], my uh coordinate plane here. (P130, MCE Transcript, lines 20-24)

Visualizing Revolutions and Finding Volumes

In MCE problems 1(d) and 1(e), participants were asked to generate a three-dimensional [3-D] object by revolving the bounded region about the x -axis. They were then asked to sketch the 3-D shape and determine its volume. A mathematical connection was deemed a 2-D/3-D connection if it was a link (or bridge) used to establish or strengthen an understanding between 2-D mathematical ideas, concepts, or representations with 3-D mathematical ideas, concepts, or representations. In the first part of problem 1 (d), participants were said to have made a 2-D/3-D connection if they could correctly identify and explain why the resulting 3-D shape was a cone. Almost all participants were able to identify the resulting 3-D shape as a cone, though the explanations among participants varied. The following are representative responses where participants' explanations involved the use of particular points on the graph of the bounded region.

I just kind of tried to visualize it, what it was that, that it would create by using, like I had said before, kind of imagine it as kind of chalk all along here and just circling all the way around, and visualize what type of shape it would be. I would just kind of, thought about grabbing it by this point here, this (5, 15) point because it says right here to do it along the x -axis, which I know is that [participant points to x -axis in sketch of coordinate plane], take the (5, 15) and just, and just imagine there's a hinge [participant points to the line segment connecting the point (0, 0) to the point (5,0) in sketch of bounded region on coordinate plane] that will do 360 degrees and just kind of circle around and shade everything in. And I would rotate it all the way around, and when I did that I came up with, this isn't really to scale, but it would kind of be like a really open cone where the opening was much bigger than the depth of it. That's kind of how I came up with that it was a cone. (P130, MCE Transcript, lines 174-189)

Yes, I looked back at the bounded region I had before, and I drew the triangle again on a different axis where the height of it was 15 and the base was 5, and then if I was going to rotate it, I kind of visualized it in my head, where if I were to take this top point [participant points to the point (5,15) on sketch of bounded region] and to kind of move it in a circular motion all the way around that this point on the origin would stay there and this one [referring to the point (5,15)] would swoop around creating a circle and the hypotenuse of the triangle would just create like, a, [pause], a continuous sloping side connecting the origin point to the circular base and that makes a cone, so I came up with a cone. (P137, MCE Transcript, lines 91-97)

In some cases, to visualize what the shape would look like after it had been revolved about the x -axis, participants would sketch a mirror image of the shape on the opposite side of the axis of revolution, and from there would sketch a basic cylindrical outline through the original shape and its mirror image.

Although most participants were able to identify the resulting 3-D shape as a cone, there were seven cases (25%) in which participants indicated the need of a physical manipulative in order to visualize the revolution. Each of the seven participants had tried to construct a physical manipulative by making a triangle out of scrap paper. This need for a physical manipulative arose during the pilot study. As a result, the researcher constructed a physical manipulative consisting of a cardboard triangle where one of the legs of the triangle was spiral bound. By putting a pencil inside the spiral, participants could simulate the revolution with a handheld object. The physical manipulative is depicted in Figure 4.4.



Figure 4.4. *Physical Manipulative for Simulating Revolution*

Using the physical manipulative, 6 of the 7 participants were able to correctly identify the resulting 3-D shape as a cone. These participants also referred back to the MCE demonstration of revolving a rectangle (see Appendix B).

Well, first, I really, [pause], its kind hard to visualize a 3-D object on a 2-D piece of paper, it's really hard to visualize it because I've really never had to draw anything like 3-D, except, I mean, [pause], unless it's trying to find like a box [participant begins to draw a cube on paper by overlaying two squares and connecting vertices]. So, it is really frustrating, because I couldn't do it, even when I made a paper triangle. [Interviewer hands over blue spiral bound triangle manipulative to participant]. It's the same thing [referring to the paper triangle she had constructed], I get the idea [participant puts pencil into spiral and begins spinning the manipulative] I'm trying to see if you did it really fast enough if you can really get the shape. I think it's a cone, [pause], because this is always going to be flat [points to the side of the triangle], as you spin around here and, [pause],

since this is the outer portion, the cone shape is going to be like this [participant makes a cone shape with her hands]. It would be a short cone. (P546, MCE Transcript, lines 84-101)

I say that it forms a cone, but I'm not sure if I'm right. Well, because, like a cone has like a, [pause], triangular like top and so when I was looking at this triangle, when it comes around, this [point to the triangle sketch at the top of page 1] is going to form into a shape of a cone... 'cuz when you um, [pause], like for the cylinder, [points to demonstration], you said if you did a cross section of it, that it looks like a rectangle, I feel like if you cut through a cross section of a cone, then you would see a triangle as your 2-D, and since that's what I have, I feel like it's a cone. (P758, MCE Transcript, lines 102-106)

There was one case, where the participant identified the resulting shape as a “wedge of cheese”. After spending a few minutes simulating the revolution using the physical manipulative illustrated in Figure 4.4, the participant stated, “I would still go back to this [participant points to the wedge of cheese drawn on paper] because I don't see it by doing this [referring to use of physical manipulative]; I don't really see anything with this” (P226, MCE Transcript, lines 89-91).

A participant was said to have made a 2-D/3-D connection if they correctly identified the relationship between the dimensions of the triangle (2-D object) with the dimensions of the cone (3-D object). That is, a correct mapping of the “pieces” of the triangle to the “pieces” of the cone. In all cases, when identifying a base and a height for the triangle, participants chose the line segment joining (0, 0) and (5, 0) on the x -axis as the base and the line segment joining (5, 0) to (5, 15) as the height. The majority of participants were able to correctly map the “height” of the triangle to the “radius” of the cone and the “base” of the triangle to the “height” of the cone.

...I knew the height was 5 because, um, the height was 5 here in this drawing [points to the sketch of triangle on coordinate plane] and the radius of the base [referring to the cone] is going to be 15 because like I found before the height of the triangle was 15 which meant the radius [of the cone] was 15... (P113, MCE Transcript, lines 103-104)

Well, this side is 15 and this side is 15, so all together this, um, base [participant points to the diameter of the base of the cone] would be 30. The base of a circular would be the diameter would equal 30 and half the diameter is r [radius] so r would be 15 and then the height of the cone would be 5. The height of the cone

would be right here on the x -axis which was 5 so that would be the height if you stood it up. (P252, MCE Transcript, lines 189-195)

...I turned it this way [rotates sketch to view upright cone] and I revolved it around that and then I'm like hey, that 5 is the height and this [participant points to sketch] since it goes all the way around is the, ah, [pauses] oh no, [pauses], this would be 15. I put the whole things as 15, but when I just told you, like, [pauses], this is revolving around here [participant points to triangle and uses pencil to show revolution of triangle about x -axis], it would be double it, so the radius would be 15. I had whole thing as 15 [referring to the diameter of the cone] but that's not right, and uh just one side is 15 so the diameter is 30. (P633, MCE Transcript, lines 179-185).

However, there were 5 cases in which the participants "switched" dimensions, by mapping the "height" of the triangle to the "height" of the cone and the "base" of the triangle to the "radius" of the cone. The following is a representative response across these particular cases.

Well, it has a circular base and then I said it had a radius of 5, because that's the length of the base of the triangle and then the height is 15, [participant points to the upright cone drawn in lower right hand corner of paper] because that was the height of the triangle, and this [points to the slant height of upright cone drawn in lower right hand corner of paper] would be equal to the hypotenuse of the triangle. (P691, MCE Transcript, lines 110-114)

After generating the 3-D shape, participants were asked to find its volume. Participants were said to have made a procedural connection if they could identify and explain a correct procedure, method, or algorithm for finding the volume of the cone. Less than half were able to identify a correct procedure, method, or algorithm for calculating the volume of a cone. The majority of participants indicated that they either did not know or could not remember a formula for finding the volume of a cone. Participants were said to have made a derivational connection if they could provide justification and/or motivation for why the volume of a cone could be found by taking one-third the volume of a cylinder. The most common justification and/or motivational explanation involved the comparison of a cylinder and a cone of the same radius and height. The following are some representative responses.

Well, the volume of pretty much everything that we learned about in the geometry class was, um, the volume of the base, [pause], well that, [pause], yeah, the [area of] base times height, like how much does it take to fill up one layer, then how many layers is it...and so...that I just, I just thought about the shape of the

cylinder and what shape was the base, it's a circle. And so that's how I got the base and then the height is just going to be the height, and I remembered it [the volume of a cone] was one-third the volume of cylinder because the rice and the geometric shape thingies it took three of the cones, three of the fillings of rice in the cone to fill up the cylinder. (P137, MCE Transcript, lines 111-113)

Um, but I did the cylinder and then I, like outlined the cone in the cylinder and looked at it, and was like well, it's about one-third, one-third seems right, so I did one third...and um, [pause], just kind of looked at the space that was left over, and realized that you're also going to have space in the front and back too, so it's not just these two spots, but these two as well behind it and in front of it, so that would make one-third of it, [pause], that's like just there for the cone and nothing else. So one-third, I knew had to be there, and then to find the volume of a cylinder, you need to do the base times the height, and the base, the area of the base, and the area of the base is a circle, or the base is a circle, so the area of a circle is πr squared, then times the height which is h , so I just did one-third πr squared times h . (P113, MCE Transcript 116-119)

There were some cases where such comparisons to the cylinder did not result in a correct formula for the volume of a cone. In the first excerpt the participant determines the volume of a cone by taking the area of a triangle and multiplying by the area of a circle. In the second excerpt, the participant determines the volume of a cone by comparing the relationship between area of a rectangle and the area of a triangle to establish a relationship between the volume of a cylinder and the volume of a cone.

So the base of a cylinder is a circle and in order to fill the cylinder you want to know how many times can the base of it, how many circles can you get to fill it up which would be the height, so you would take the area of a circle and multiple that times the height to find the volume of that, so I did that for the volume of the cylinder, but I don't think it right [referring to using this same procedure for getting the volume of a cone]. I did the area of a circle and then I thought you can't do it times the height because it doesn't, a triangle, it doesn't fill up, the height doesn't go all the way up both sides, its uneven, I guess, so then I multiplied times the area of a triangle. Um, I was just thinking that you have to, it has to be that you have to multiple the base times the height so I was thinking obviously that's the base of the cone; to get the height and since it forms a triangle I just figured you would use the formula for the area of a triangle. 'Cuz you can't fill a cone all the way, you can't fill a cone because it's not all the way around it's not a circle for both bases there's only one base. (P678, MCE Transcript, lines 113-127]

Um, because, okay, like where a triangle its one-half the base times the height because the triangle you know if you went ahead and just multiplied base times height you would have a rectangle or a square and that's why its [referring to

area] one-half base times height because it's actually a triangle and cuts off the other half, if that makes sense. And then I was thinking [along] the same lines, like I said I was drawing a blank on the volume of a cone, so I was like, okay, so same thing, I mean, if we left it as [area of the] base times height which would be the area of the base which would be a circle, would be the volume of the cylinder, and then I thought of a cone as being half the volume of a cylinder. Yeah, like, relating them like to a triangle and a rectangle, kind of the same idea, if that makes sense. So I was thinking the volume of a cone would be like half the volume of a cylinder. (P496, MCE Transcript, lines 76-83)

In MCE problem 2, participants were asked to revolve the bounded region about the y -axis and describe the resulting 3-D shape. Participants were said to have made a 2-D/3-D connection if they could correctly identify the 3-D shape. A majority of participants were able to describe the resultant 3-D shape as a "cylinder with a cone removed".

Since this is the pivot point [participant points to the origin] of the slant, you have a diagonal and a straight line, the straight line would create some kind of, the vertical line would create some kind of cylinder shape and then the diagonal line would create the cone, by how its slanted and its basically just going around so that's where it creates the cone shape. And this is vertical [participant points to the vertical line $x=5$] so it creates the circle but still has the vertical shape so it creates the cylinder. It's a cylinder outside but then it kind of dips in or if you had a cylinder and you just take a cone and stick it inside a cylinder, if it was clear you would see a cone, you would see it's a cone inside if you had a clear cylinder. (P291, MCE Transcript, lines 117-121)

Yeah, it would make, um, it would make a cylinder, but it would have like a cut out, a cone cut out of the top. Okay, so you would have...here's your cylinder [begins sketching a cylinder] that's what it would make, like if you ignore this um hypotenuse and that it's a triangle, if you pretend it is a uh rectangle now, you get a cylinder, but then after you rotate it, there's going to be this cone that's missing [sketches "missing cone" inside the cylinder] because that's the blank space, the blank triangle space between the y -axis and the uh, I guess the other boundary line, so we would have...the part that's shaded [begins shading outside the cone within the cylinder]. (P137, MCE Transcript, 140-145)

...You could pretend this was a rectangle and rotate it all the way around and that would become a cylinder, but I just made it all a rectangle but then if you just rotate the triangle around its goin' make a, [pause], oh okay, wait, so you made a cylinder but then if you imagine this triangle that makes up the other part of the rectangle, imagine rotating that around that would be the empty space, so then that would make a cone, so then you would have a cone inside of a cylinder, I guess, yeah. (P678, MCE Transcript, lines 169-176)

However, there were 5 cases in which participants were unable to make this 2-D/3-D connection. In some cases, they described the 3-D object as a cone or combination of cones.

It would still be a cone, it would be taller. It would be a cone because this bottom one would make a circle [rotates manipulative], right here, it would just be taller this time because it would have the height this side; it would still be a cone shape, but it would just be a taller cone, it would have greater height, instead of 5 it would be 15. The circular base would also be smaller, it would have a radius of 5 this time, the height and the radius and height would just be flipped on the new cone. (P263, MCE Transcript, lines 195-200)

So that would probably form a cone, wouldn't it? I don't know [laughs], because I feel like, if you're revolving this face around the axis it's going to form a circle and the triangle still stays up so it's going to form a cone. (P758, MCE Transcript, lines 324-328)

It's a cylinder with a cone in the middle but it's not filled. On the outside these are filled. There are two other cones [participant points to the shaded region inside the cylinder indicating the cylinder is made up of three cones, one that is empty and two that are filled]. (P252, MCE Transcript, lines 294-296)

In MCE problem 3, participants were asked to find the volume of the 3-D shape found in problem 2 (the volume of a "cylinder with cone removed") Participants were said to have made a procedural connection if they could identify and explain a correct procedure or method for finding the volume of a "cylinder with cone removed". The majority of participants (81%) who described the 3-D object as a cylinder with a cone missing were able to describe a procedure for finding the volume, by taking the volume of a cylinder minus the volume of a cone. Participants were said to have made a derivational connection if they could give a correct justification, motivation, and/or explanation for the volume of the "cylinder minus cone" shape.

Umm, I realized that if I could find the volume of the whole shape, this cylinder as a whole, then subtract out a cone inside it, in our image the inside of the cone isn't filled, we would want to find, [pause], the area outside the cone. I would subtract the volume of the cone since nothing is there. So I just found the volume of the cylinder, the volume of the cone and subtract it, what I got for the cone from the cylinder. (P291, MCE Transcript, lines 124-126)

Um, I guess you could find the volume of the cylinder and then subtract the volume of the [pause] cone, because that's what's missing out of it. 'Cuz if you

were just to find the volume of the shape itself, that would be really complicated, I think, it's just a random shape, like an odd shape, it would be hard to figure out, so, [pause], but the volume of the cylinder, there is a formula for, so you could just find that, and then if this is what's missing out of, if that's what your open space is equal to is that cone, then just find the volume of the cone and subtract it. (P691, MCE Transcript, lines 257-261)

Participants were said to have made a procedural connection if they could correctly calculate the volume of the 3-D shape. There were a few participants who correctly described how to find the volume of the 3-D shape but could not carry out a calculation because they could not remember explicit formulas for the volume of a cylinder and the volume of a cone. There were a couple of cases where participants used the volume of the cone found in problem 1 (i.e., as the volume of the “missing” cone). They did not recognize that the cone generated in problem 1 was different from the “missing” cone generated in problem 2.

I would find the volume of the cylinder first and write that down, and then I would use this “white” triangle and its going to have the same base and height, because as I showed you earlier, um, the two triangles are congruent when you make a rectangle out of it, [pause] so then I would find the volume of the cone, which actually we already have [referring to the calculation for the volume of the cone carried out in problem 1], because if the base and the height are the same, then it's going to have the same volume, so you take the volume of the whole cylinder and subtract the volume of the cone. (P137, MCE Transcript, lines 166-168)

In MCE problem 4, participants were presented with 2-D objects sketched in the x - y plane. They were then asked to revolve each object about the x -axis and describe the resulting 3-D shape. Participants were said to have made a 2-D/3-D connection if they could correctly describe the 3-D object generated from the revolution of the 2-D object about the x -axis. There were 17 participants (61%) who were able to make a 2-D/3-D connection for both 2-D objects. When describing the resulting 3-D shape for each object, participants tended to provide descriptions that related the 3-D object to a “real world” object. In the case of revolving the first object, the descriptions included such “real world” objects like a donut, Cheerios® cereal, sledding tube, bracelet, slinky, and ring.

Like a donut? Um, [pause], yeah. But like the centers gone, that's this little area right here [participant points to the space between x -axis and the solid circle].

This area right here would be like the hole in the donut, like if you had this [participant draws picture of a donut] (P678, MCE Transcript, lines 370-374)

Um, it would be like a donut or ring, one of those rings because if you were to rotate and its filled in its going to rotate all the way around in a circle and since you have this little area here [participant points to empty space between object and x -axis] there's going to be a hole in the middle. (P486, MCE Transcript, lines 224-226)

In the case of revolving the second object, the descriptions included such “real world” objects like a Charms Blow Pop® without the bubble gum, watermelon without the pink inside, plastic globe, earth and its core, an orange with the inside removed, a cell and its nucleus, a basketball, and a tennis ball.

Some kind of sphere with a hole in the middle because you're going to go around and this part [participant points to shaded region] will still be filled in and that [participant points to the white space between the shaded region and the x -axis] part won't...I kind of think of it like the earth too, where this [participant points to the space between the shaded region and x -axis] is the core and the earth is around so it's kind of two spheres, a smaller sphere inside of a bigger sphere, but taking the smaller sphere out there because it's an empty space. (P291, MCE Transcript, lines 143-150)

It would be a sphere but it would be hollow inside this hole. You wouldn't be able to tell from the rotation, unless, you could cut it like a cross section, because this [point to shaded region] would sweep all the way around, well, yeah, this would sweep all the way around, and then this [participant points to space between shaded region and x -axis] would be left empty inside. (P263, MCE Transcript, lines 220-222)

There were 5 participants (18%) who could make the 2-D/3D connection with the first object but not the second and 3 participants (11%) who could make the 2-D/3-D connection with the second object but not the first object. There were 2 participants (7%) who indicated that they just could not visualize what the 3-D object would be in either case. They both said that revolving the first object might result in a sphere, but indicated that this was a guess. They both said that revolving the second object would result in a sphere, similar to the object obtained in revolving the first object, but with a “hole” in it. The following is a representative response across these two participants.

I don't know, [pause], a sphere? [Referring to revolution of the first object] I can't visualize what it would form. I don't really see anything. Why don't I think in 3-D? [Participant notably frustrated]. This one [referring to revolution of second

object] might be a sphere with a hole in it...umm...I don't know what I'm thinking...because you have a whole circle here [participant points to the first object], kind of half a circle here [participant points to second object], so like if you revolve a whole circle around it would form a sphere and if you did half of one, it would be a sphere with a hole in it....I don't know if that's right, I was just taking a guess. (P758, MCE Transcript, lines 375-394)

We now turn our attention to the types of mathematical connections prospective middle grades teachers made while engaged in the CSA task.

Card Sort Activity (Open Sort)

There were a total of 258 open card sorts. On average each participant made 9 open card sorts. The unique emergent themes (i.e., the types of mathematical connections made by prospective middle grades teachers during the open cards sort) resulting from an inductive analysis of participants' responses using the method of constant comparison were as follows: *categorical*, *procedural*, *characteristic/property*, *derivation*, and *curricular* (see Appendix M). A mathematical connection was deemed *categorical* if the participant's explanation relied upon the use of surface features primarily as a basis for defining a group or category. A participant who put cards 9 and 14 together, explaining "The formulas look similar. The a would be the x and b would be your y so c would be your r " (P252, CSA Transcript, Sort 4) would be making a categorical connection. A mathematical connection was considered *procedural* if the participant's explanation for the sort involved relating ideas based on a mathematical procedure or algorithm possible through the construction of an example; which may include a description of the mechanics involved in carrying out the procedure rather than the mathematical ideas embedded in the procedure. A participant who stated the following as a reason for putting cards 4 and 10 together was making a procedural connection.

The derivative is move the exponent in front and subtract exponent by 1, so the derivative of f of x equals x squared is $2x$. Whenever I've seen derivative they always use f of x equals x squared or whatever and f prime of x is the derivative. I've had experience taking the derivative of things that look like this. (P291, CSA Transcript, Sort 4)

A mathematical connection was deemed *characteristic/property* if the participant's explanation for the sort involved defining the characteristics or describing the properties of concepts in terms of other concepts. A participant who grouped cards 19, 20, and 3

together because, “A rectangle has two sets of parallel sides and four ninety degree angles” (P876, CSA Transcript, Sort 7) was making characteristic/property connection. A mathematical connection was considered a *derivation* connection if the participants’ explanation for the sort involved knowledge of one concept(s) to build upon or explain other concept(s); included but not limited to the recognition of the existence of a derivation. A participant who stated the following as a reason for grouping cards 5, 15, 18, 8, and 6 together was making a derivational connection.

I can derive the formula for the volume and surface area of a cylinder using the area of a circle and circumference of a circle. To find the volume of a cylinder you take the area of the base times its height, which is the number of layers you stack, and since the base of a cylinder is a circle, then you know the area of circle which is πr^2 . Then to find the surface area of the cylinder you would take area of both its bases plus unroll cylinder would give you a rectangle. The length of the rectangle would be circumference of circular base. You could also do the same to find the volume and surface area of a rectangular prism. (P758, CSA Transcript, Sort 1)

A mathematical connection was considered *curricular* if the participant’s explanation for the sort involved relating ideas or concepts in terms of the impact to curriculum, including but not limited to, the order in which one would teach concepts or topics. A participant who stated the following as a reason for grouping cards 15 and 6 together was making a curricular connection.

If you were going to teach a lesson on circles you would have to teach them [middle grades students] area and circumference rules. They would fall in the same lesson you would teach them. They would have to understand π and radius for both of them. The circumference of a circle its perimeter...thinks like triangle and rectangle so my students would understand what circumference is. (P678, CSA Transcript, Sort 9)

Although there were 258 open card sorts, there were 287 mathematical connections made that fell into one or more of the aforementioned categories. A participant’s response for grouping particular cards together could fall into one or more of the five types of mathematical connections categories. Table 4.3 lists the number of connections that fell into each mathematical connection category. Table 4.4 displays a count of CSA open sort connections broken down by MCE Scores.

Table 4.3. *CSA Open Sort Counts by Connection Category (n=28)*

Mathematical Connection Type	Count	Frequency
Categorical	97	34%
Procedural	68	23%
Characteristic/Property	51	18%
Curricular	36	13%
Derivational	35	12%
Totals	287	100%

Table 4.4. *Counts of CSA Open Sort Connections by MCE Score (n=28)*

MCE Score	No. of Participants	CSA Categorical	CSA Char/Prop	CSA Curricular	CSA Procedural	CSA Derivational
40-44	8	21	12	19	25	15
35-39	8	31	13	13	16	7
30-34	3	18	2	2	1	1
25-29	3	5	8	2	12	5
<24	6	22	16	0	15	7
Totals	28	97	51	36	68	35

Card Sort Activity (Closed Sort)

In the closed card sort, five particular pairs of cards were selected: cards 6 and 11; cards 2 and 4; cards 15 and 17; cards 4 and 15; cards 9 and 16 (see Figure 4.3).

Participant explanations were qualitatively analyzed using an inductive approach to the method of constant comparison for each closed sort pairing. For the closed sort pairing of cards 6 and 11 the following themes emerged: *yarn explanation*; *radius as a “line”*; *both are formulas*; *both are equations*; *both are linear functions*; *none*. These themes, exemplars, and the frequencies with which each occurred are shown in Tables 4.5.

Table 4.5. *Themes and Exemplars for Closed Sort Pair 6 and 11 (n=28)*

Themes	Exemplars of Participant Responses	Count	Frequency
Yarn Explanation	If you take a piece of yarn at a certain point around the circle and brought it all the way around, then straightened it out, it would make a straight line that you could lay against a ruler.	6	21%
Radius as a "line"	If you were to graph the circle on the coordinate plane, the line $[y=mx]$ could be the radius of that circle".	7	25%
Both are Formulas	Right off the bat, I think they are both formulas. It's kind of one of the second nature formulas that you just know. Hopefully, your teachers help you derive it and you know what they are. I think this is another case like with the last two, I wouldn't teach together. From a teacher's perspective they are kind of unrelated in terms of how I would teach it.	3	11%
Both are Equations	They are both equations. I don't really know if find the slope of a straight line would help you find the circumference of a circle, but they are both equations. They are both equations. This $y=mx$ gives you a line and the other gives you a circle.	2	7%
Both are Linear Functions	I think they can be related because they are both functions, really. Well, the x I would just think of it relating C the circumference can be a function of the radius. If you change the radius, it will change the circumference. Whenever you change the x value it's going to change the y, the output. They are both input/output. They are both lines.	1	4%
None	I don't think they are related because that [card 6] has to do with a shape [a circle] and this [card 11] has to do with a line.	9	32%
Totals		28	100%

Note. Card 6 read "The circumference of a circle is given by $C=2\pi r$ where r is the radius of the circle". Card 11 read "The equation of a straight line through the origin is given by $y=mx$ ".

For the closed sort pairing of cards 2 and 4 the following themes emerged: *max area most square like*; *calculus problem*; *derivative to find max*; *graphing possibilities*; *none*. These themes, exemplars, and frequencies are shown in Table 4.6.

Table 4.6. *Themes and Exemplars for Closed Sort Pair 2 and 4 (n=28)*

Themes	Exemplars of Participant Responses	Count	Frequency
Max Area Most Square Like	I'm trying to find the max possible area of the rectangle. I think it relates because the max possible area of rectangle is going to be given by length times width which is 7 times 7 so you could say 7 squared so the is some kind of connection to x squared.	3	11%
Calculus Problem	Here I think about, there is some calculus interwoven in this, when trying to find the maximum area with a given perimeter. When you do the arithmetic, the math is going to create a parabola and that maximum value....I would need to flush this one out, but they are related.	3	11%
Derivative To Find Max	I think these are related. I think you have to take the derivative to find the maximum. We did problems like this last semester where sometimes it was undefined and sometimes a maximum. I need my notes for this one.	1	4%
Graphing Possibilities	To find the maximum area of a rectangle you can graph it which is usually going to be a parabola and this is the equation that gives you a parabola. You could graph every possibility and the graph would look like this [participant uses hands to indicate a downward opening parabola] which is a parabola.	5	17%
None	I don't see how finding the max area of a rectangle has to do with a parabola...nope...nothing.	16	57%
Totals		28	100%

Note. Card 2 read, "A rectangle has perimeter 28 feet. Find the maximum possible area of the rectangle". Card 4 read, "A function is defined by $f(x) = x^2$. What kind of curve will it produce when graphed?"

For the closed sort pairing of cards 15 and 17 the following themes emerged: *both area formulas*; *geometric/relational*; *volume of cone*; *none*. These themes, exemplars, and the frequencies with which each occurred are shown in Table 4.7.

Table 4.7. *Themes and Exemplars for Closed Sort Pair 15 and 17 (n=28)*

Theme	Exemplars of Participant Responses	Count	Frequency
Both Area Formulas	That's just going back to area because you are trying to find area in each. If you want to find the area of a triangle you use this formula and if you want to find area of circle you use this one and that's how they are related. They are formulas for area but just different objects.	17	60%
Geometric/Relational	They're both area, just of different shapes. I'm trying to figure out how much more I can relate them than that. I guess if you have your circle and you make it into a bunch of different pie pieces which is kind of similar to a triangle you could end up using this formula [card 17] to roughly get to this one [card 15]. The more triangles you put into the circle, the closer it will get to the area of a circle.	9	32%
Volume of Cone	If you go by what I said earlier about multiplying the area of a triangle times the area of a circle, then it might be volume of a cone.	1	4%
None	There is something there but I can't remember what it is, I can't put my finger on it. It is something I've done and I don't remember when and where.	1	4%
Totals		28	100%

Note. Card 15 read, "The area A enclosed by a circle is given by the formula $A=\pi r^2$ where r is the radius of the circle". Card 17 read, "The area of a triangle is given by the formula $A=1/2bh$ where b is the base and h is the height of the triangle".

For the closed sort pairing of cards 4 and 15 the following themes emerged: *both have “squares”; both are quadratic functions; invalid geometric; none*. The themes, exemplars, and the frequencies with which each occurred are shown in Table 4.8.

Table 4.8. *Themes and Exemplars for Closed Sort Pair 4 and 15 (n=28)*

Theme	Exemplars of Participant Responses	Count	Frequency
Both have “Squares”	The variable in both formulas is squared. They both have “squares” in them.	10	35%
Both are Quadratic Functions	You have two functions squared. You could substitute πx for r . They are both even quadratic functions.	2	7%
Invalid Geometric	Again, I’m going to go with they are connected because area squared and this [function] is squared. This one says what kind of curve will it produce when graphed and we know what kind of curve a circle is going to produce. I guess half of it is going to be a parabola. The function is going upward like a U-shape. If it continued or if you flip it, rotate it, then you could find the area of a circle. This gives you like a parabola which is kind of like a half-circle...And maybe if that was like a half-circle and the parabola was laying on the x -axis and you want to know the area of that specific function or half circle then you would need to know how to find the area of a full circle in order to find the area of x squared laying on the x -axis.	8	29%
None	I’m not sure I can think of a relationship between 4 and 15. This [card 4] could be the area of a wedge of a circle, but that is pretty obscure.	8	29%
Totals		28	100%

Note: Card 4 read, “A function is defined by $f(x) = x^2$. What kind of curve will it produce when graphed?” Card 15 read, “The area A enclosed by a circle is given by the formula $A = \pi r^2$ where r is the radius of the circle.

For the closed sort pairing of cards 9 and 16 the following themes emerged: *given triangle; create triangle; distance formula looks like Pythagorean Theorem; Pythagorean theorem is the distance formula; none*. These themes, exemplars, and the frequencies with which each occurred are shown in Table 4.9.

Table 4.9. *Themes and Exemplars for Closed Sort Pair 9 and 16 (n=28)*

Theme	Exemplars of Participant Responses	Count	Frequency
Given Triangle	These are connected because if you have a right triangle on the coordinate plane you can figure out, easily figure out, the base and the height and then you could use the Pythagorean theorem to figure out the hypotenuse.	15	54%
Create Triangle	Like, I'm picturing if I want to find this line and I wanted to find the distance between these two points, I could make a triangle out of that. I would put two points in the plane, I was picturing a line between the two points, and then so I was picturing to draw a triangle. Then finding the distance between these two points would be like finding this line. If this was my triangle and this was my right angle then using the Pythagorean theorem to find the line.	5	17%
DF looks like PT	Yeah [indicating the statement on the two cards are related], because the Pythagorean theorem is pretty much the distance formula. Because a squared plus b squared equals c squared and square root all that to find c by itself which is the distance equal to the square root of a squared plus b squared. The a 's could be the x 's, the b 's could be the y 's and so square root of a squared plus b squared is square root of $(x^2 \text{ minus } x1)^2$ squared plus $(y^2 \text{ minus } y1)^2$ squared which equals the distance which equals c .	3	11%
PT is DF	The Pythagorean theorem is the distance formula in the coordinate plane. Here I thought about the Pythagorean theorem, actually...because I have never been able to remember the distance formula and I've learned in two classes this year that you can use the Pythagorean theorem to find the distance between two points instead of having to memorize the distance formula which I found to be really helpful.	4	4%
None	I'm not sure if they are related. I can't remember right now.	1	4%
Totals		28	100%

Note: Card 9 read, "Pythagorean Theorem". Card 16 read, "Distance between two points in the Cartesian Coordinate Plane".

Impact of Coursework on Mathematical Connections

How does prospective middle grades teachers' coursework impact their mathematical connections?

In order to examine prospective middle grades teachers' coursework and their impact on mathematical connections, the MCE data were analyzed quantitatively. The fit of the MCE data to the normal curve is necessary for carrying out parametric analysis. Skewness and kurtosis were computed and a deficiency of extreme elevated skewness and kurtosis was noted. The fit of the MCE total scores to a normal distribution was assessed through a Shapiro-Wilk Test ($W=.926$ with $N=28$ and $p=.058$). Since $p>0.05$, the null hypothesis was not rejected and the data fits the normality assumption. Cronbach's alpha, α , was used to assess the internal consistency reliability for the MCE instrument. The coefficient alpha of $\alpha=.892$ suggests that the questions comprising the MCE instrument for this sample were internally consistent. The mean MCE score of the 28 participants was 33.43 out of a possible score of 44 ($SD=8.492$; $range=14-44$).

The analysis of this question involved placing participants into distinct non-overlapping groups based on their coursework. Group A consisted of participants who had completed all mathematics content and methods courses. Group B consisted of participants who had completed all mathematics content courses but had not taken methods courses. Group C consisted of those participants who had completed all but two mathematics content courses and had not taken methods courses.

To assess the relationship of prospective middle grades teachers' methods coursework and performance on the MCE a univariate analysis was conducted using a linear regression model. The participants in Groups A and B were utilized for the analysis because participants in Group A had completed all of the required methods courses while those in Group B had not. Participants in Groups A and B had completed all mathematics content courses. The number of participants in this regression was 20. The analysis was conducted with methods coursework as the independent variable and MCE score as the dependent variable. Table 4.10 reveals the linear regression estimates of the effects of methods coursework on MCE performance.

Table 4.10. *Linear Regression Estimates of the Effects of Methods Coursework on MCE*
(*n=20*)

Dependent Variable: MCE Score

Variable	B	SE B	β	Sig.
Constant	35.214	1.821		<.001
Methods Coursework	4.119	3.325	3.325	.231

Note. $R^2 = .079$ (Adjusted $R^2 = .027$). B indicates unstandardized regression coefficient. β indicates standardized regression coefficient.

Fitting the linear regression yields $Y=35.214 + 4.119X$. The intercept which is equal to 35.214 is the mean MCE score for participants in Group B, i.e., those who have not taken any methods course. The mean MCE score for participants in Group A, i.e. those who had taken methods courses was 4.119 points higher. There was no statistically significant effect of methods course work on MCE performance.

To assess the relationship of prospective middle grades teachers' mathematics content coursework and performance on MCE a univariate analysis was conducted using a linear regression model. The participants in Groups B and C were utilized for this analysis because participants in Group B had completed all required mathematics courses while those in Group C had not. Participants in Groups B and C had not completed methods courses. The number of participants in this regression was 22. The analysis was conducted with mathematics content coursework as the independent variable and MCE score as the dependent variable. Table 4.11 reveals a statistically significant effect ($p=.009$) of mathematics content coursework on MCE performance.

Table 4.11. *Linear Regression Estimates of the Effects of Content Coursework on MCE*
($n=22$)
Dependent Variable: MCE Score

Variable	B	SE B	β	Sig.
Constant	25.875	2.595		<.001
Mathematics Content Coursework	9.339	3.253	.540	.009*

Note. $R^2 = .292$ (Adjusted $R^2 = .256$). B indicates unstandardized regression coefficient. β indicates standardized regression coefficient.
* $p < 0.05$

The mean MCE score for participants in Group B, i.e., those who had taken all mathematics content courses was 9.339 points higher than those participants in Group C who had not completed all mathematics content coursework. Effect sizes were medium (Huck, 2004). Therefore, adjusted R squared = .256 meant that 25.6% of the variability in MCE scores can be explained or accounted for by mathematics content coursework.

Discussion

The five types of mathematical connections identified in the MCE by the researcher, prior to the analysis of participants' MCE responses were as follows: *procedural, algebraic/geometric, characteristic, derivational, and 2-D/3-D*. Many mathematics educators would consider MCE problems 1(a)-(c) to be fairly routine problems, ones that prospective middle grades teachers should have little difficulty answering. A majority (79%) of the prospective middle grades teachers was able to make the procedural, algebraic/geometric, and characteristic/property connections associated with problem 1(a)-(c). In problem 1(a) the majority of participants' procedural connection making was restricted to an algebraic approach for graphing the lines $x=5$ and $y=3x$. Researchers have posited a contributing factor influencing participants' preference for an algebraic approach stems from traditional curriculum and instructional emphasis on procedures where students are typically asked to construct graphs from given equations by computing functional values to create a table of ordered pairs for plotting points in the coordinate plane (Dugdale, 1993; Knuth, 2000; Leinhardt, Zaslavsky, &

Stein, 1990). As Knuth (2000) points out, this procedural algebraic approach “is often perceived as being mathematical straightforward, and in short time, students are expected to have mastered the equation-to-graph connections” (p. 506). However, there were a few interesting cases in problem 1(a) where procedural and algebraic/geometric misconceptions (participants were unable to make connection(s) that would lead to correct solution to the problem) occurred. In these cases, participants tried to graph the line $y=3x$ by incorrectly applying “transformation of graph” rules. These participants believed the “3” in the line $y=3x$ indicated the graph of the line would intersect the y -axis at $(3, 0)$. That is, they believed the graph of the line $y=3x$ was a vertical shift upwards 3 units of the graph of $y=x$, resulting in a “diagonal line going through 3 on the y -axis” (P962, MCE Transcript, line 20). They also indicated the “3” represented slope, so once they knew the graph of the line $y=3x$ crossed the y -axis at $(3, 0)$, they would use “up 3 over 1” (interpretation of the slope m in $y=mx$) to plot other points in order to graph the line. These participants failed to make a connection between “a particular feature of a function in one representation to the same feature in another representation” (Leinhardt et al., 1990, p. 24). Specifically, these participants did not make a connection between the y -intercept of the graph [geometric representation] and the b in the equation $y=3x+b$ (in this case $b=0$) [algebraic representation]. The approach taken by these participants represents an algebraic/geometric misconception between the symbolic and graphical representation of the line.

Nearly all participants were able to sketch the bounded region and make a characteristic/property connection by identifying the bounded region as a triangle. The majority of participants were also able to make a procedural connection for finding the area of a triangle by stating and applying a correct formula. These two connections appeared to be the easiest for participants to make. This finding is not surprising considering basic knowledge of 2-D shapes, such as finding area, occurs with great frequency in K-16 curriculum materials and in national documents specifying what students need to know and be able to do (NCTM, 1989, 2000, 2006). In order to use the formula for the area of a triangle, it was necessary to determine the intersection point of the lines composing the bounded region. In nearly all cases, participants were able to make an algebraic/geometric connection by recognizing the x -coordinate of the point of

intersection of the graph of the lines $x=5$ and $y=3x$ as having value 5. Substituting $x=5$ into the equation $y=3x$, yielded the intersection point (5, 15). Thus, if the base of the triangle was identified as the line segment connecting the origin to the point (5, 0), then the height of the triangle was the line segment connecting the point (5, 0) to (5, 15) and the value for the height of the triangle was represented by the expression $h=15$. However, there were a few participants who were only able to calculate the area through geometric estimation (i.e., counting grid marks for estimating the height of the triangle) and did not make a connection to an algebraic approach that would yield a precise intersection point. These participants failed to make an algebraic/geometric connection between the point of intersection as a common solution to the system of linear equations, $y=3x$ and $x=5$. These data indicate that some prospective middle grades teachers may not have developed meaningful connections between algebraic and geometric representations of linear functions.

Although most participants made a procedural connection for finding the area of a triangle using the formula mentioned previously, the same cannot be said for making a derivational connection. Half the participants stated either that the formula was just something they had memorized or stated that the area of a triangle is one-half the area of a rectangle without providing further detail or explanation. This lack of a derivational connection was also exhibited in participant responses for the volume of the cone generated in 1(d). Over half the participants failed to make a derivational connection stating one of the following: 1) they did not know the formula for the volume of a cone; 2) the formula was something they had memorized; or 3) they tried to make comparisons to a cylinder that resulted in an incorrect formula for the volume of a cone. This finding is consistent with CBMS (2001) sentiments that “prospective [middle grades] teachers have some basic knowledge about shapes and about how to calculate areas and volumes of common shape, but many will not have explored the properties of these shapes or know why the area and volume formulas are true” (p. 33). Another reason for this lack of a derivational connection may be attributed to traditional geometry curriculum. Battista (2007) suggests, “many traditional curricula prematurely teach numerical procedures for geometric measurement, students have little opportunity to think about the appropriateness of the numerical procedures they apply...in fact, the traditional

premature instructional focus on computational formulas seems to interfere with students' concept development in geometric measurement" (p. 892). These sentiments are further echoed by Boaler and Humphreys (2005) in their work with middle school students,

Middle school students usually experience surface area and volume by learning and applying formulas. Most textbooks approach these measurements ideas by showing pictures of two-or three-dimensional figures, introducing a formula with diagrams to show why the formula works, and following up with examples and exercises. And while accurate and efficient use of formulas is an essential tool in mathematics, students who have not had an opportunity to think deeply about what these concepts mean, or to experience the mathematical relationships involved, often apply formulas blindly and inappropriately. (p. 91)

In MCE problem 1(d), 2, and 4, participants were asked to revolve various 2-D shapes about an axis in the x - y plane and describe the resulting 3-D shape. Such tasks required spatial visualization to make a 2-D/3-D connection. Spatial visualization refers to "the mental manipulation of spatial information to determine how a given spatial configuration would appear if portions of that configuration were to be rotated, folded, repositioned, or otherwise transformed" (Salthouse, Babcock, Skovronek, Mitchell, & Plamon, 1990, p. 128). In the case of MCE problems 1(d), 2, and 4, spatial visualization involved "imagining the rotations of objects and their parts in 3-D space in a holistic as well as piece by piece fashion" (Olkun, 2003, p. 2). For problem 1(d) the majority of participants were able to identify the resulting 3-D shape as a cone. However, these participants fell into two distinct groups. The first group was those participants who readily carried out a mental manipulation of the 2-D object and then presented an external representation of the 3-D object through either a pictorial form (sketch), and/or through verbal descriptions involving the extensive use of their hands to demonstrate the revolution. The other group consisted of those participants who had difficulty mentally manipulating the 2-D object to create the 3-D object and required the use of a physical manipulative, a concrete image. The use of a physical manipulative provided these participants with a tangible object that could facilitate their mental manipulation of 3-D objects in both a holistic and piece-by-piece fashion. Participants were said to have made a 2-D/3-D connection if they could correctly identify and/or describe the relationship between the "pieces" of the 2-D object with the "pieces" of the 3-D object. In all cases, participants making the 2-D/3-D connection depended on two factors: 1) the sketch of the

triangle where the measurement of the side lengths was represented by the algebraic equation $b=5$ and $h=15$, where b is the base of the triangle and h is the height of the triangle and 2) the orientation of the sketch of the cone. Every participant who used a physical manipulative to visualize the revolution of the 2-D object provided a sketch of an upright cone as the 3-D object. The sketch of the upright cone along with the sketch of the aforementioned triangle led these participants to an incorrect mapping of the dimensions of the triangle to the dimensions of the cone. In each sketch the height of each object was labeled with an h and since the height of the triangle was 15 units, it must be that the height of the cone is also 15 units. The participants who did not require the use of a physical manipulative were able to make the 2-D/3-D connection for mapping the dimensions of the triangle to the dimensions of the cone. In MCE problem 2, participants were asked to revolve the bounded region found in 1(b) about the y -axis. Participants who made the 2-D/3-D connection in problem 1 also made the 2-D/3-D connection in problem 2 by correctly identifying the 3-D shape as a “cylinder with a cone removed”. Furthermore, these participants who correctly identified the 3-D shape also made a 2-D/3-D connection by correctly describing the relationship between the dimensions of the triangle and the dimensions of the “cylinder with cone removed”. The participants who required the use of a physical manipulative to visualize the revolution in problem 1 (i.e. the cone) also used the physical manipulative when trying to visualize the revolution of the bounded region about the y -axis in problem 2. The participants who used the physical manipulative to visualize the revolution in problem 2 did not make a 2-D/3-D connection. They did not identify the resultant 3-D shape as a “cylinder with cone removed”. Upon reflection, the use of the physical manipulative in problem 2 proved to be problematic. When the participant would use the physical manipulative to revolve the object, they would simulate a revolution about one of its legs rather than by simulating the revolution about the y -axis. As a result, each of these participants identified the 3-D shape as a cone with the same height and base as found in problem 1. This finding highlights the importance of both the position of a 2-D object in the coordinate plane and the axis of revolution in determining the resultant 3-D object. There were a number of instances where the importance of the position and axis of revolution was not considered by the participants which led to some interesting misconceptions.

- If the revolution of a triangle in problem 1 results in a cone, then revolution of the triangle in problem 2 should also be a cone.
- If the area of a triangle is half the area of a rectangle, and the revolution of a triangle and rectangle results in a cone and cylinder, respectively, then the volume of the cone must be half the volume of a cylinder.
- The volume of the “cylinder minus cone” shape in problem 3 can be found by taking the volume of the cylinder minus the volume of the cone found in problem 2.
- If this object is a filled in circle [problem 4a] then when it’s revolved it will be a sphere. Since the second object [problem 4b] is half of a filled circle, then when it’s revolved it will be half of a sphere.

Card Sort Activity (Open Sort)

An inductive analysis of participants’ responses to the open card sort using the method of constant comparison resulted in the emergence of five types of mathematical connections: *categorical*, *procedural*, *characteristic/property*, *derivation*, and *curricular* (see Appendix M). In the open card sort, each participant was asked to select a subset of cards they felt were related or connected. After each sort, participants were asked, “Can you make another subset?” This question was repeated after each sort. The researcher assumed if the participant responded “no”, then they had exhausted all possibilities for creating subsets. However, there were two cases in which participants indicated they could “make connections all day” and thus, these participants theoretically did not exhaust all possibilities for creating subsets. Since the researcher did not explicitly instruct each participant to “make as many subsets as possible”, it could be argued that some participants may have been able to make more subsets if they had more time.

Methodologically speaking this is a limitation of the research design as it could be argued that not all participants went through the EXACT same procedures for the open card sort. If we make the assumption that the participants had exhausted all possibilities for creating subsets, then Tables 4.3 and 4.4 can be reasonably interpreted. As a group, the prospective middle grades teachers made more categorical and procedural connections and far fewer derivational and curricular connections. Since the card sorting technique is “an advanced level sorting task that can be used to identify how concepts in

a content area are organized in a learner's knowledge structures" (Johnnassen, Beissner, & Yacci, p. 45) the number of sorts under each connection type provides a glimpse into how these prospective middle grades teachers tend to unpack, relate, and connect the concepts presented in the open card sort. It is not surprising that the majority of card sorts made by prospective middle grades teachers were categorical and procedural in nature for three potential reasons: 1) the majority of participants had never engaged in a card sort activity and thus, may have related or connected the cards based on the most "obvious" relationships or links between the mathematical concepts, ideas, and terms presented on the cards, 2) the majority of participants' experiences with learning mathematics has been dominated by traditional curriculum focused on instrumental rather than relational understanding of mathematics (Skemp, 1978), and 3) the majority of participants had not yet taken mathematics methods courses so perhaps they did not think about creating subsets from the perspective of what a future middle school teacher should know and be able to do. Another potential reason why the majority of participants made fewer curricular and derivational connections may reside in the order in which the MCE and CSA were conducted. All participants engaged in the CSA immediately following the MCE. The MCE was more focused on mathematical content connections and less on pedagogical connections and thus, participants may not have been in the frame of mind to create subsets from the perspective of what a future middle school teacher should know and be able to do.

However, the fact that nearly 25% of the subsets were curricular and/or derivational in nature (see Table 4.3) is an encouraging result. Faculty at the site where the study was conducted currently use and draw upon NSF reform curriculum emphasizing a constructivist approach to learning and teaching mathematics in the prospective middle grades teacher content and methods courses. The development, improvement, and refinement of these prospective teacher courses include a focus on how to make visible the connections to the kinds of mathematical thinking, judgment, and reasoning one has to do in teaching (Ball, 2008). Overall, the results here suggest that some progress is being made towards improving prospective middle grades teachers' making of mathematical connections.

Card Sort Activity (Closed Sort)

In the closed card sort, five particular pairs of cards were selected: cards 6 and 11; cards 2 and 4; cards 15 and 17; cards 4 and 15; cards 9 and 16. (see Figure 4.3). Each participant was asked to decide if each pair of cards were related or connected, and if so, explain why? They were also told not to assume that each pair of cards were connected or related.

For card pairing 6 and 11, only one participant (4%) was able to identify the expression in both cards as linear functions represented algebraically. There were 18% who used the surface features of the cards as a basis for their connection. In particular, these participants focused on the equal sign on both cards and said the cards were related because both represented equations or formulas. Nearly a third of the participants said that the two cards were not related or connected. The remainder of the participants (46%) tried to make a connection between the two cards by focusing on a visual or graphical representation for the statement on each card. When talking about circumference of a circle, participants tended to draw a picture of a circle, labeling the distance from the center of the circle to a point on the circle, r , for radius. When looking at card 11 they tended to focus on the visual representation of a line, rather than the equation given on the card. They would use the pictorial representation of a circle to build a connection to a pictorial representation of a line. The participants who gave the “yarn explanation” (see Table 4.5) indicated that you could take a piece of yarn, wrap it around the circle and then you could straighten out the piece of yarn and it would be a “line”. The participants, who gave the “radius as a line” (see Table 4.5) explanation, indicated that the radius could be thought of as a straight line. For this closed sort pairing, the majority of participants either did not make a connection, the connection was based purely on the surface features of the card, or they made an algebraic/geometric misconnection.

For card pairing 2 and 4, more than half the participants said there was no connection between the two cards (see Table 4.6). These participants tended to focus on a geometric representation for the statement on each card. For card 2 they focused on a geometric representation of a rectangle and for card 4 they focused on a geometric representation of a parabola. These participants said there was no connection because

they could not see how the graph of a parabola had any relationship to finding the maximum area of a rectangle.

For card pairing 15 and 17, the majority of participants indicated that the two cards were related. The most popular response being that the two cards were both area formulas for two different objects. These participants focused on the surface features of the statements on the card to make a connection. However, nearly a third of participants were able to go beyond the surface in making a connection between the two cards. These participants tried to make more of a derivational connection in relating the two cards. In particular, they focused on how to use the area of a triangle to motivate the area of a circle. The following illustrates how participants made a derivational connection by making connections between the algebraic and geometric representations of the area of a triangle and the area of a circle.

They're both area, just of different shapes. I'm trying to figure out how much more I can relate them than that. I guess if you have your circle and you make it into a bunch of different pieces which is kind of similar to a triangle you could end up using this formula [card 17] to roughly get to this one [card 15]. The more triangles you put into the circle, the closer it will get to the area of a circle. (P137, CSA Closed Sort Pairing 15, 17)

Both are finding area of two dimensional shapes. If you cut the circle along the radius, and then unfold it, then it kind of forms a triangle. Then you could use the area of a triangle to show the area of a circle is πr squared. (P496, CSA Closed Sort Pairing 15, 17)

For card pairing 4 and 15, over a third of the participants indicated that the cards were connected because they “both have squares”, referring to the exponent of the variable for each equation on each card. Similar to previous closed sort pairings, participants focused solely on the surface features of the card resulting in a superficial rather than mathematical connection. Nearly a third of the participants tried to make a connection between the two cards by relating what they indicated to be geometric representation for each equation. For card 4, participants would describe the graph of the function $f(x)$ as a parabola or “U”-shape. For card 15, participants associated the equation for the area of a circle with the geometric representation of a circle by saying that the “curve” for card 15 was a circle. In other words, participants indicated that in card 4 the

“curve” is a U-shape and in card 15 the “curve” is a circle. They would then try to establish a connection between the two cards by comparing the geometric representations, i.e., the “curves”. The following illustrates how participants tried to establish a connection between the two cards by comparing the “curves”.

The function is going upward like a U-shape. If it continued or if you flip it, rotate it, then you could find the area of a circle. (P252, CSA Closed Sort Pairing 4, 15)
Again, I’m going to go with they are connected because area squared and this [function] is squared. This one says what kind of curve will it produce when graphed and we know what kind of curve a circle is going to produce. I guess half of it is going to be a parabola. (P876, CSA Closed Sort Pairing 4, 15)

This gives you like a parabola which is kind of like a half-circle. And maybe if that was like a half-circle and the parabola was laying on the x -axis and you want to know the area of that specific function or half circle then you would need to know how to find the area of a full circle in order to find the area of x squared laying on the x -axis. (P860, CSA Closed Sort Pairing 4, 15)

There were only two participants (7%) who recognized the equations on both cards were algebraic representations of particular quadratic functions, that is, when graphed in the Cartesian coordinate plane each equation would produce the graph of a parabola. The remaining eight participants (29%) could not make a connection between the two cards. In some of these cases, the participants indicated that they could not see a connection between the two cards because one card was describing the area of a circle, while the other card was focused on the graph of a particular curve.

I don’t see how they are related because this number 4 is talking about curves on the graph and number 15 is the area of a circle. (P421, CSA Closed Sort Pairing 4, 15)

While other participants indicated they were not related because of where the topics typically fall within K-12 curriculum.

I would say they’re not related. Again, they are far apart. I feel like area is such a basic math that you really have to understand that before you can move on to understand the x - y coordinate plane. Before you ever get to sketching curves you have to understanding what this was [participant points to card 15]. The area of a circle has nothing to do with knowing how to sketch a curve. But I feel like this [participant points to card 15] is something you have to understand before you every get to understand this [participant points to card 4]. This one [participant points to card 15] is something you learn in middle school where as this one

[participant points to card 4] is something you learn to do in high school. (P190, CSA Closed Sort Pairing 4, 15)

When I think of x squared now, I think of calculus, I think of area under a curve. I think of area. In here [participant points to card 15] this is the area of a circle, so again, what is inside something. Again, thinking from a teacher's perspective these two are not really related. I wouldn't teach them together. Area under a curve is not something I would teach to middle school students, its higher level like high school or college. (P806, CSA Closed Sort Pairing 4, 15)

The previous statements are of particular interest when thinking about prospective middle grades teacher preparation, mathematical connection making, K-12 curriculum, and “horizon knowledge” (Ball, 1993). Ball, Thames, & Phelps (2008) describe horizon knowledge as an “awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (p. 403). Knowledge at the mathematical horizon is “useful in seeing connections to much later mathematical ideas” (p. 403). Prospective middle grades teachers' ability to unpack mathematics and make insightful connections between mathematics learned in college courses to the mathematics they will teach may be related to the extent to which their knowledge of mathematics is connected. With respect to the preparation of prospective middle grades teachers in this study, perhaps greater care must be taken toward explicitly demonstrating how certain geometric concepts, themes, or topics their future middle school students will encounter will again reappear and be examined in greater depth and complexity as their future students move into high school and beyond.

For card pairing 9 and 16, all but one participant (96%) indicated the two cards were connected which is not surprising given the Pythagorean Theorem is arguably the most popular and remembered mathematical statement from high school geometry. As seen in the majority of responses here, the Pythagorean Theorem is often remembered as “ a squared plus b squared equals c squared”, and when prompted participants usually recalled a , b , and c represent the lengths of the legs and hypotenuse, respectively, of a right triangle. More than half the participants' responses for relating the two cards fell under the “given triangle” theme. That is, given a right triangle in the Cartesian coordinate plane, the Pythagorean Theorem could be applied to find the distance between the two endpoints of the hypotenuse. During their explanations, participants would sketch

a right triangle oriented in the coordinate plane with one leg of the right triangle parallel to the x -axis and the other leg parallel to the y -axis. Given this orientation, participants indicated that finding the length of the legs of the right triangle was a matter of counting grid marks and once these lengths had been found, the Pythagorean Theorem could be applied.

These are connected because if you have a right triangle on the coordinate plane you can figure out, easily figure out, the base and height and then you could use the Pythagorean Theorem to figure out the hypotenuse. Because you can't just count the points like you did on the base and height because they are not exact. Like on a grid it would go through just a corner of a box or half of a box or three-quarters of a box, it wouldn't be accurate. (P876, CSA Closed Sort Pairing 9, 16)

For the Pythagorean theorem a squared plus b squared equals c squared, the distance from each point on the triangle, each vertex, so for instance if you had a triangle and you had vertex 1, 2 and 3. The vertex 1 and 3 is c and vertex 1 and 2 could be a , and vertex 2 and 3 could be b . You would have to be given a couple or two to be able to count boxes on a grid to use Pythagorean Theorem or if you were given the vertices you could use the distance formula to get the length of sides. (P226, CSA Closed Sort Pairing 9, 16)

This finding has some interesting mathematical and pedagogical implications. What if the right triangle was not oriented in the way described above but was rotated 30 degrees? How would these participants have responded to a situation in which simply counting grid marks would not yield a precise solution?

In contrast to those participants who indicated the need to be given a triangle in the coordinate plane in order to apply the Pythagorean Theorem, there were only 3 participants (11%) who made a connection to finding the distance between two points in the coordinate plane by creating a triangle and then applying the Pythagorean Theorem. The following is a representative response.

Oh, I would relate those but I didn't put them together at all earlier, but now that I look at it that the distance between two points would be the same thing as kind of finding that third side. Like I'm picturing if I want to find this line [participant draws line between two points] and I wanted to find the distance between these two points I could make a triangle out of that. Put two points in the plane, I was picturing a line between the two points, and then so I was picturing to draw a triangle. Then finding the distance between these two points would be like finding this line. If this was my triangle and this was my right angle then using the Pythagorean Theorem to find the line. You could use either one of them [distance

formula or Pythagorean Theorem] to find the line. I'm not positive. I would have to work out a bunch of different examples. (P190, CSA Closed Sort Pairing 9, 16)

There were approximately 25% of participants who made a procedural connection to card 16 by stating there was a formula for the distance between two points in the Cartesian coordinate plane. These participants then used this procedural connection to make a connection to the Pythagorean Theorem. In some cases, participants described how the distance formula looks like the Pythagorean Theorem while others made the connection that the distance formula is just an application of the Pythagorean Theorem in the coordinate plane.

Yeah [indicating the statement on the two cards are related], because the Pythagorean Theorem is pretty much the distance formula. Because a squared plus b squared equals c squared and square root all that to find c by itself which is the distance equal to the square root of a squared plus b squared. The a 's could be the x 's, the b 's could be the y 's and so the square root of a squared plus b squared is square root of x_2 minus x_1 squared plus y_2 minus y_1 squared which equals the distance which equals c . (P137, CSA Closed Sort Pairing 9, 16)

Oh yeah, like I said before the Pythagorean Theorem is pretty much the same as the distance formula. In the distance formula you're taking the square root of the square of the difference of the two y values plus the square of the difference between the x values. So you could find the Pythagorean Theorem in that where the difference between the x and y values could be your a and b and you're trying to find c . (P305, CSA Closed Sort Pairing 9, 16)

If you look at the distance between two points, um, you have two points and you're looking at (x_1, y_1) and the second point (x_2, y_2) to find the distance you take the square root of x_2 minus x_1 squared plus y_2 minus y_1 squared which if you look at the Pythagorean theorem, the distance could be c and you're using those two points since square root of a squared plus b squared equals c . So you have a triangle and you put it into the Cartesian plane then the hypotenuse is going to be the distance between the two points. (P291, CSA Closed Sort Pairing 9, 16)

The Pythagorean Theorem is the distance formula in the coordinate plane. Here I thought about the Pythagorean Theorem, actually...because I have never been able to remember the distance formula and I've learned in two classes this year that you can use the Pythagorean Theorem to find the distance between two points instead of having to memorize the distance formula which I found to be really helpful. Because you already have to know the Pythagorean Theorem anyway so well just use it for that [participants points to card 16] too. (P914, CSA Closed Sort Pairing 9, 16)

The findings exhibited in participant statements above are encouraging because we are beginning to see how prospective middle grades teachers are able to move away from a rote memorization of formulas to making connections between algebraic and geometric representations of distance in order to reason out and explain why the distance formula is just an application of the Pythagorean Theorem in the coordinate plane.

Relationship between MCE and CSA

Data analysis revealed participants who made more curricular connections tended to have higher MCE scores (see Table 4.4). To further investigate this relationship, the researcher reviewed MCE interview data (both transcripts and videos) of those students who made more curricular connections during the open card sort looking for further evidence of why this might be the case. Participants who made more curricular connections during the open card sort tended to provide correct solutions to MCE problems that exhibited elements of pedagogical content knowledge, a subcategory of MKT (Ball, 2006). These particular participants provided solutions that involved how to explain, model, or demonstrate a solution to an MCE problem to someone who did not understand. In most cases, these particular participants referenced how they would explain, model, or demonstrate their solution to a middle grades student or peer. Furthermore, many of these participants made reference to the appropriateness of a particular MCE problem for a middle grades student and how they might modify such a problem. The explanations and comments made by participants during the MCE interview demonstrated knowledge of mathematics and middle grades students, knowledge of mathematics and teaching, as well as knowledge of the middle grades curriculum. In each case, the participant was not explicitly prompted by the researcher to provide such explanations or comments but rather did so of their own accord. Thus, it is a reasonable predication that these participants would have made more curricular connections during the open card sort since they seemed to be viewing the activities from the perspective of what a middle grades teacher should know and be able to do.

As we can see from Table 4.4, the relationship between MCE scores and the other types of connections that emerged from the open card sort are fairly random. The participants who had higher MCE scores made just as many categorical, characteristic/property, procedural, and derivational connections as participants who had

lower MCE Scores. This “randomness” could be explained by the nature of the MCE and CSA activities. The structure of the MCE was such that it was necessary to make certain mathematical connections in order to solve each problem correctly. However, with the CSA activities participants could make any type of connection or connections between cards. Thus, participants may have opted to make connections during the CSA that were more “surface level”. This was certainly the case during the closed sort activity. The participants who had higher MCE scores tended to make just as many “surface level” connections during the closed card sort as participants who had lower MCE scores. However, participants who had lower MCE scores tended to make more “misconnections” or “none” (meaning no connection) during the closed card sort. For example, participants who gave the “yarn explanation” for closed sort pair 6 and 11 tended to have lower MCE scores. Participants who provided an “invalid geometric” connection or “none” for card sort pairing of cards 4 and 15 also had lower MCE scores. In each of these cases, interview data revealed that participants who had difficulty making mathematically correct valid connections during the closed card sort also struggled to make connections that were needed to get through the MCE problems.

Impact of Coursework on Mathematical Connections

How does prospective middle grades teachers’ coursework impact their mathematical connections?

Mathematics content coursework had a statistically significant impact on prospective middle grades teachers’ mathematical connection making on MCE. Effect sizes were medium (Huck, 2004) with adjusted R squared equal to .256. The group that had not completed all mathematics content coursework still needed to take MATH I (finite mathematics course) and MATH II (geometry course for prospective middle grades teachers). Given the heavy focus on 2-D and 3-D geometry in Math II, a reasonable prediction would be scores on the MCE would increase after successfully completing MATH II. MATH II was recently redesigned to incorporate several national recommendations (CBMS, 2001; NCTM 1989; NCTM 2000; NCTM 2006) on what prospective middle grades teachers should know and be able to do with regard to geometry and measurement. Specific attention to connections in mathematics can be found in some of the objectives for MATH II: tracing and making connections on how

geometric concepts are developed in middle school and beyond; and approaching geometry from an investigative constructivist stance by building small learning communities focused on mathematical communication, exploration, and problem solving as well as formulating, proving or disproving conjectures. Methods coursework did not have a statistically significant impact on prospective middle grades teachers' MCE score. This finding is not surprising given the items on the MCE were more focused on content knowledge than pedagogical knowledge.

Conclusions and Implications

This exploratory mixed methods study describes the types of mathematical connections prospective middle grades teachers made while engaged in tasks meant to probe mathematical connections. One task focused on connection making in the context of solving mathematics problems, while the other focused on connection making in the context of card sorting. Findings from the problem solving task suggest participants had difficulties making derivational connections. The lack of derivational connection making supports national recommendations that “formulas for measuring area and volume should be developed in such a way that a teacher could later derive a formula if it is not remembered” (CBMS, 2001, p. 101). While a majority of participants were able to make algebraic/geometric and 2-D/3-D connections, the fluency and ease with which they made these connections is questionable and in some cases, participants failed to make these connections at all. These findings have implications for prospective middle grades teacher preparation. As the NCTM (2009) points out,

Too often individuals perceive mathematics as a set of isolated facts and procedures. Through curricular and everyday experiences, students should recognize and use connections among mathematical ideas. Of great importance are the infinite connections between algebra and geometry. These two strands of mathematics are mutually reinforcing in terms of concept development and the results that form the basis for much advanced work in mathematics as well as in applications. Such connections build mathematical conceptual understanding based on interrelationships across earlier work in what appear to be separate topics. (p. 3)

Before coming to college, most prospective middle grades teachers have taken an Algebra I, Geometry, and Algebra II course. Algebra and geometry are typically viewed by prospective middle grades teachers as separate, distinct fields of study. Maintaining

the study of algebra and geometry as two distinct courses will only perpetuate the difficulties prospective middle grades teachers have in making mathematical connections between strands. Prospective middle grades teachers' algebraic/geometric and 2-D/3-D connection making may be strengthened by creating a two semester course sequence focused on the interrelationships between algebra and geometry. In this study, the fundamental algebraic/geometric and 2-D/3-D misconceptions made by participants suggest that a two semester course sequence specifically designed for prospective middle grades teachers should include 1) making visible and explicit the connections between algebraic/geometric concepts and 2-D/3-D representations, 2) providing prospective middle grades teachers more opportunities to explore the "equation to graph" and "graph to equation" relationships, and 3) creating opportunities for prospective middle grades teachers to develop spatial visualization skills by working with and comparing components of 2-D and 3-D models, visualizing movements of objects in space, and matching corresponding parts of images and pre-images resulting from revolutions or rotations of 2-D and 3-D objects.

In the open card sorting task, five types of mathematical connections were identified: *categorical*, *procedural*, *characteristic/property*, *curricular*, and *derivational*. The majority of the open card sorts were categorical and procedural. The majority of responses to the closed card sort were also predominantly categorical in nature as prospective middle grades teachers tended to focus mainly on the surface features of the cards when relating each preselected pairing. It is probable that the majority of the participants' experiences in mathematical connection making have been limited to exploring the more "obvious" and "surface level" relationships between mathematical concepts. Perhaps these participants' mathematical experiences have been dominated by traditional curriculum placing focus on procedural fluency rather than conceptual understanding of mathematics. There were very few (13%) subsets made in the open card sort that were curricular. The majority of participants (79%) had yet to take methods courses and thus, may have not considered making subsets from the perspective of what a future teachers need to know and be able to do in the context of teaching. Given that the majority of participants had yet to take methods courses, perhaps this lack of curricular connection making could be improved by integrating more pedagogy into all

mathematics content courses for teachers. By infusing pedagogy in content courses, mathematicians and mathematics educators could help to make visible the connections to the kinds of mathematical thinking, judgment, and reasoning one has to do in teaching (Ball, 2008).

What can mathematics educators do to bring derivational and curricular connections to the forefront of prospective middle grades teachers thinking? How might card sorting techniques be adapted in prospective teacher courses to facilitate enhancing prospective middle grades teachers' mathematics knowledge for teaching?

The results of this study also have implications for K-12 and prospective middle grades teachers' methods preparation. In methods courses, prospective middle grades mathematics teachers focus on lesson planning, instructional strategies, and assessment. However, prospective middle grades mathematics teachers are rarely afforded the opportunity in their methods courses to reflect on the role mathematical connections play in lesson planning, instructional strategies, and assessments. The MCE and CSA activities along with the MCE rubric construction and implementation could serve as a model for both formative and summative assessment techniques for mathematical connection making that could be implemented during K-12 classroom instruction and lesson planning. By constructing such rubrics, prospective teachers will have more opportunities to reflect on the role and importance mathematical connections plays in carrying out the work of teaching. In understanding the role mathematical connections play in carrying out the work of teaching, prospective middle grades teachers will also be better prepared to carry out best mathematical practices addressed in the recently released draft of *College and Career Readiness Standards for Mathematics* (2009). According to this document,

Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems (p.5).

By strengthen prospective middle grades teachers' mathematical connection making and its role in carrying out the work of teaching, mathematics educators will be helping these

future teachers implement and carry out college and career readiness standards needed to succeed in an ever changing competitive 21st century marketplace.

Overall, the findings of this study are particularly useful to mathematics educators, curriculum developers, and researchers seeking further understanding behind effective and ineffective teacher preparation. This study will aid those wishing to construct mathematics tasks for explicit connection making with the intent to strengthen prospective teachers' conceptual understanding of underlying mathematical concepts and mathematics knowledge for teaching.

Limitations

This exploratory sequential mixed methods study focused on 28 prospective middle grades teachers solving mathematics problems and engaging in a card sort activity for approximately 2 hours utilizing a semi-structured interview format. It is not possible to evaluate *all* prospective middle grades teachers. Thus, the study was limited to the number of prospective middle grades teachers available to the researcher. These numerical values greatly limit the generalizability of findings to larger groups of prospective teachers. While the 2 hour interview offered plenty of time for participants to make connections, the findings presented here represent only a snapshot for the types of connections prospective middle grades teachers make use of in problems meant to probe mathematical connections.

Future Research

The question of how prospective middle grades teachers come to make mathematical connections in a variety of contexts remains an issue of great importance and is deserving of future research. This study focused its attention on prospective middle grades teachers. Future studies should include other populations such as inservice middle school teachers. Are there particular courses or aspects of teacher preparation that explicitly help prospective middle grades teachers develop and build mathematical connections? A longitudinal study following a cohort of prospective middle grades teachers through their undergraduate studies on into their 1st and 2nd year of teaching could potentially reveal how connections are developed over time. Future studies should include increasing the number of participants so that more sophisticated statistical analyses can be carried out on the data. This would allow the researcher to strengthen

both the reliability and validity of both the instruments and rubrics. Replication and longitudinal studies would also help to refine the data collection instruments and protocols that should be adapted for other studies.

The card sorting techniques used in this study should be adapted and integrated into prospective teacher courses. Future research studies should include making comparisons between prospective middle grades teachers open and closed card sorts to “expert” sorts, such as those sorted by mathematicians, mathematics educators, and inservice teachers. These card sort comparison studies could provide insight into the “gap” between expert and novice mathematical connection making and offer recommendations on how to bridge this “gap”. Finally, the types of connections that were identified or emerged from this study offer a beginning point from where future studies could refine or expand on the types of connections prospective teachers make in other contexts.

CHAPTER V

ARTICLE II: PROSPECTIVE MIDDLE GRADES TEACHERS' MATHEMATICAL CONNECTIONS AND ITS RELATIONSHIP TO THEIR MATHEMATICS KNOWLEDGE FOR TEACHING

Effective competition in a rapidly growing global economy places demands on a society to produce individuals capable of higher-order critical thinking, creative problem solving, connection making, and innovation. In response to these demands, the National Council of Teachers of Mathematics (NCTM) (1989) published the *Curriculum and Evaluation Standards for School Mathematics* (CESSM) followed by the *Principles and Standards for School Mathematics* (NCTM, 2000), and the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006). Within the executive summary of the 2000 document is the guiding principle “students must learn mathematics with understanding, actively building new knowledge from experience and previous knowledge” (p. 2). The *PSSM* also highlights the importance of problem solving and establishing connections.

By solving mathematical problems, students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom...when students connect mathematical ideas, their understanding is deeper and more lasting, and they come to view mathematics as a coherent whole. (p. 4)

We must look to our teacher education programs to help prospective teachers build the mathematical habits of mind that promote a conceptually indexed, broad-based foundation of mathematics knowledge for teaching which encompasses the establishment and strengthening of mathematical connections. In particular,

...the curriculum of teacher preparation programs must include helping preservice teachers make connections between mathematics concepts and between concepts and representations for the concepts. The teacher with this preparation should leave these programs with well-developed, interconnected, and accessible knowledge base effective for teaching mathematics. (Bartels, 1995, p. 25)

If prospective teachers are expected to construct, emphasize, integrate and make use of mathematical connections, then they must acquire an understanding of mathematics that is fluid, supple, and interconnected (Evitts, 2005). Prospective teachers must learn to access and unpack their mathematical knowledge in a connected, effective manner.

Furthermore, prospective teachers must not only be able to do the mathematics they will teach but must possess a deep conceptual understanding of the mathematics. Without understanding the connections among the important, functional concepts in mathematics, prospective teachers will be ill equipped to effectively engage middle grades students in mathematical connection making, reasoning, and problem solving. Given the increased attention by the NCTM (1989, 2000) standards and recent publication of the NCTM (2006) *Curriculum Focal Points* stressing the importance of mathematical connection making, an exploratory mixed methods study of prospective middle grades teachers' mathematics knowledge for teaching and its relationship to the mathematical connections is warranted.

To provide a foundation for this exploratory mixed methods study, an overview of literature on knowledge for teaching mathematics, mathematics knowledge for teaching geometry and mathematical connections is presented.

Knowledge for Teaching Mathematics

Teacher education programs are being challenged as never before to prepare prospective mathematics teachers in ways that will enhance teaching and learning of mathematics well into the 21st century. Research suggests that teachers' mathematics knowledge, knowledge of teaching, and knowledge of students' thinking and general beliefs about teaching influence what is taught and ultimately what students learn (Ball & Bass, 2003; Ball, Lubienski, & Mewborn, 2001; Ball & McDiarmid, 1990; Fennema & Franke, 1992; Hiebert & Carpenter, 1992; Putnam & Borko, 2000). Scholars have come to realize subject matter knowledge and pedagogy are inseparable. This indissoluble relationship between subject matter knowledge and pedagogical content knowledge is called *pedagogical content knowledge* (Shulman, 1986).

Pedagogical content knowledge identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. (Shulman, 1987, p. 4)

Researchers have begun to explore the idea that "teaching quality might not relate so much to performance on standard tests of mathematics achievement as it does to

whether teachers' knowledge is procedural or conceptual, whether it is connected to big ideas or isolated into small bits..."(Hill & Ball, 2004, p. 332). With this in mind, Hill, Rowan, and Ball (2005) refined Shulman's (1986) concept of pedagogical content knowledge for teaching by focusing on the subject-specific nature of this type of knowledge. In particular, they adapted his definition to the field of mathematics education by introducing the notion of *mathematics knowledge for teaching* (MKT).

By "mathematical knowledge for teaching," we mean the mathematical knowledge used to carry out the work of teaching mathematics. Examples of this "work of teaching" include explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and effects of teachers' mathematical knowledge on student achievement providing students with examples of mathematical concepts, algorithms, or proofs. (Hill, Rowan, & Ball, 2005, p. 373)

Teaching mathematics effectively requires prospective teachers to (a) have a deep understanding not only of the mathematics they will be teaching but of the mathematics their students will encounter as they move through the educational system; and (b) have a deep conceptual understanding of the subject matter along with the ability to make connections between and within disciplines. This allows teachers to make informed decisions about the appropriate pedagogy to use in their classrooms (Hill et. al, 2005; CBMS, 2001; Fennema & Franke, 1992; Ma, 1999). As Lampert (2001) points out,

One reason teaching is a complex practice is that many of the problems a teacher must address to get students to learn occur simultaneously, not one after another. Because of this simultaneity, several different problems must be addressed in a single action. And a teacher's actions are not taken independently; they are interactions with students, individually and as a group. A teacher acts in different social arrangements in the same time frame. A teacher also acts in different time frames and at different levels of ideas with individuals, groups, and the class to make each lesson coherent, to link one lesson to another, and to cover a curriculum over the course of a year. Problems exist across social, temporal, and intellectual domains, and often the actions that need to be taken to solve problems are different in different domains. (p. 2)

Prospective middle grades teachers' connection making is not only an essential component to the development and strengthening of their MKT but is vital to addressing the "simultaneity" that occurs when carrying out the work of teaching. By developing an

understanding of MKT, mathematicians and mathematics educators will be able to help prospective teachers access and unpack knowledge in a connected, effective manner.

Mathematics Knowledge for Teaching Geometry

Numerous national educational groups consisting of mathematicians, mathematics educators, and classroom teachers have offered recommendations for the preparation of prospective mathematics teachers in the area of geometry (CBMS, 2001; NCTM, 2000; National Mathematics Advisory Panel [NMAP], 2008; RAND Mathematics Study Panel, 2003). Geometry is one of the most interesting areas of mathematics to teach not only for its appeal to the visual senses but for its historical significance in the development of mathematics. Geometry lends itself well to making “rich connections with the rest of mathematics, including topics and themes in discrete and continuous mathematics as combinatorics, algorithmic thinking, geometric series, optimization, functions, limits, trigonometry and more” (Goldenberg, Cuoco, & Mark, 1998, p. 23). Geometry is one of the focus areas for the NCTM (2000) content standards and NCTM (2006) *Curriculum Focal Points and* as such, prospective teachers must be prepared to effectively teach this subject. As Grover and Conner (2000) point out,

The college geometry course is especially important for prospective secondary teachers. In the United States, these students studied geometry only once in secondary school, and they will encounter geometric concepts once more in college before they are certified to teach. Not only does the college geometry course need to lay a strong foundation for the content they will teach, but it is also one of the few courses that might develop the preservice teachers’ ability to create and present proofs. (p. 48)

The above statement not only holds for prospective secondary teachers but is applicable to prospective middle grades teachers. Cooney (2003) echoes these sentiments in his invited commentary on *The Trends in International Mathematics and Science Study (TIMSS) 1999 Video Study and the Reform of Mathematics Teaching*,

...the fact that only 22 percent of problems per U.S. lesson focused on geometry, suggests that some U.S. students may not be getting much geometry, including both two- and three-dimensional geometry. The role of school geometry in the United States, particular at the middle school level, deserves careful consideration in developing teacher education programs for both preservice and inservice teachers. (¶ 16)

The National Assessment of Educational Progress (NAEP) identified weaknesses in the performance of U.S. students on mathematics concepts, in particular geometry concepts, as compared to students in other countries (Gonzales et al., 2000). Battista (1999) found

U.S. students seemed to do better on items that were straightforward but formal in nature and not as well on spatial visualization and problem solving. Overall, the results suggest that U.S. students need more experience with spatial visualization, solving geometric problems and three-dimensional geometry. (p. 367)

A contributing factor to U.S. students' weak performance on geometric concepts as compared to student in other countries could be attributed to the mathematical knowledge for teaching geometric concepts held by teachers. The *Mathematics Teaching in the 21st Century* (MT21) report, a cross-national study of the preparation of middle school teachers, found prospective middle grades teachers' mathematics knowledge in the areas of algebra and geometry to be weak in comparison to prospective middle grades teachers in other countries (Schmidt et al., 2007). Evidence from the research literature suggests prospective teachers may not possess the subject-matter knowledge and pedagogical content knowledge needed to effectively teach geometric concepts (Grover & Conner, 2000; Swafford, Jones, & Thorton, 1997).

Mathematical Connections

What is a *mathematical connection*? Ma (1999) describes a mathematical connection in terms of a *concept knot* which links together underlying key concepts to a particular mathematical idea or representation. These concept knots are part of an interconnected web of *knowledge packages* consisting of key concepts for understanding and developing relationships among mathematical ideas, concepts and procedures.

Hiebert and Carpenter (1992) described mathematical connections as part of a network structured like a spider's web.

The junctures, or nodes, can be thought of as pieces of represented information, and threads between them as the connections or relationships. All nodes in the web are ultimately connected, making it possible to travel between them by following established connections. Some nodes, however, are connected more directly than others. The webs may be very simple, resembling linear chains, or they may be extremely complex, with many connections emanating from each node. (p. 67)

Mathematical connections can also be described as components of a schema or connected groups of schemas within a mental network. A schema is a “memory structure that develops from an individual’s experiences and guides the individual’s response to the environment” (Marshall, 1995, p. 15). Marshall posits that a defining feature of schema is the presence of connections. The strength and cohesiveness of a schema is dependent on connectivity of components within the schema or between groups of schemata. This model suggests that prospective middle grades teachers learn mathematics through assimilating or connecting new information into their mental networks, forming new connection(s) between existing knowledge components, accommodating or reorganizing their schemata to address perturbations in their knowledge structure and to correct misconceptions.

Although mathematical connections have been defined, described, or categorized in various ways the common thread is the idea of a mathematical connection as a link or bridge between mathematical ideas. For the purposes of this study, a *mathematical connection* is a link (or bridge) in which prior or new knowledge is used to establish or strengthen an understanding of relationship(s) between or among mathematical ideas, concepts, strands or representations.

Mathematics education literature supports the belief that mathematical understanding requires students to make connections between mathematical ideas, facts, procedures, and relationships (Hiebert & Carpenter, 1992; Ma, 1999; Moschkovich, Schoenfeld, & Arcavi, 1993; Skemp, 1989). This belief is further supported by the creation of the NCTM (1989, 2000) standards which explicitly state the importance of mathematical connections in the school curriculum. According to these documents, mathematical connections are ‘tools’ for problem solving. As Hodgson (1995) points out,

...the investigation of problem situations leads naturally to the establishment and use of connections. In turn, the use of connections to solve problems brings about the need for their establishment. Connections are not seen as merely interesting mathematical facts but as integral components of successful problem solving. (p. 18)

Thus, prospective middle grades teachers must be prepared to make connections between the content to be learned and their students’ understanding. Although there are a few studies examining mathematical connections of prospective teachers at the elementary

and secondary level (Bartels, 1995; Donigan, 1999; Evitts, 2005; Hau, 1993; Roddy, 1992; Wood; 1993), there is little to no research on mathematical connections made by prospective teachers at the middle grades level.

Theoretical Framework

“Current reform in mathematics education has included discussion of and inquiry into the nature of mathematics, mathematics learning, and mathematics teaching. Reform efforts have been shaped by a number of influences including constructivist views on mathematics learning” (Simon, 1994, p. 71). Constructivist influence has had a substantial impact on a number of national curricular documents, in particular, the NCTM (1989, 2000) standards and the *Curriculum Focal Points* (NCTM, 2006). These documents, which are grounded in social constructivist principles, support a vision of classroom mathematics where students explore mathematical situations by engaging in both written and oral communication of mathematical ideas. These ideas are transmitted through social interaction where they are then validated or modified. Hence, students assume the role of mathematicians actively participating in a community effort for thinking, learning, creating and evaluating mathematics. However, prospective teachers are not always afforded the opportunity to engage in such practice. If you were to walk into a typical university mathematics course, what might you see? Would you see students working together in collaborative fashion actively engaging in mathematical conversation to solve problems or would you see a professor lecturing to a group of arguably attentive students? More likely than not, you would encounter the latter rather than the former. Nunn (1996) found that nearly 80% of class time is spent in lecture while only 14% of the time is devoted to class participation (the other 6% spent on teacher questions, praise and criticism).

In the last decade, K-12 public school systems across the country have been inundated with reform curricula encompassing connection-rich material that is grounded in a constructivist theory which posits students learn best when they are allowed to discover and build mathematics while interacting with other students. Teachers are often expected to teach mathematical topics and skills in ways substantially different from the ways in which they themselves learned the content (Ball, Lubienski, & Mewborn, 2001; Fennema & Franke, 1992; Hiebert & Carpenter, 1992) Thus, these reform curricula pose

a challenge to those involved with prospective teacher preparation. Our prospective teachers must not only possess a strong understanding of mathematics content and pedagogy but should exhibit mathematical connections between and among mathematical concepts. These reform curricula place a focus on K-12 students' ability to make mathematical connections and thus, prospective teachers must be flexible in facilitation and integration of these reform curricula in their classroom.

Mathematics Knowledge for Teaching

To better prepare prospective middle grades teachers to facilitate learning of mathematics within a K-12 system saturated by reform curricula that is grounded in constructivist theory, an understanding of the mathematics knowledge entailed by teaching is essential. Most scholars would agree that an understanding of content matters for teaching. However, what constitutes this content knowledge for teaching has been widely debated. In an effort to understand content knowledge needed for teaching, Ball and her colleagues have developed a framework of *mathematics knowledge for teaching*. Figure 5.1 is a visual representation *mathematics knowledge for teaching* (Hill, Rowan, & Ball, 2005) framework and its components.

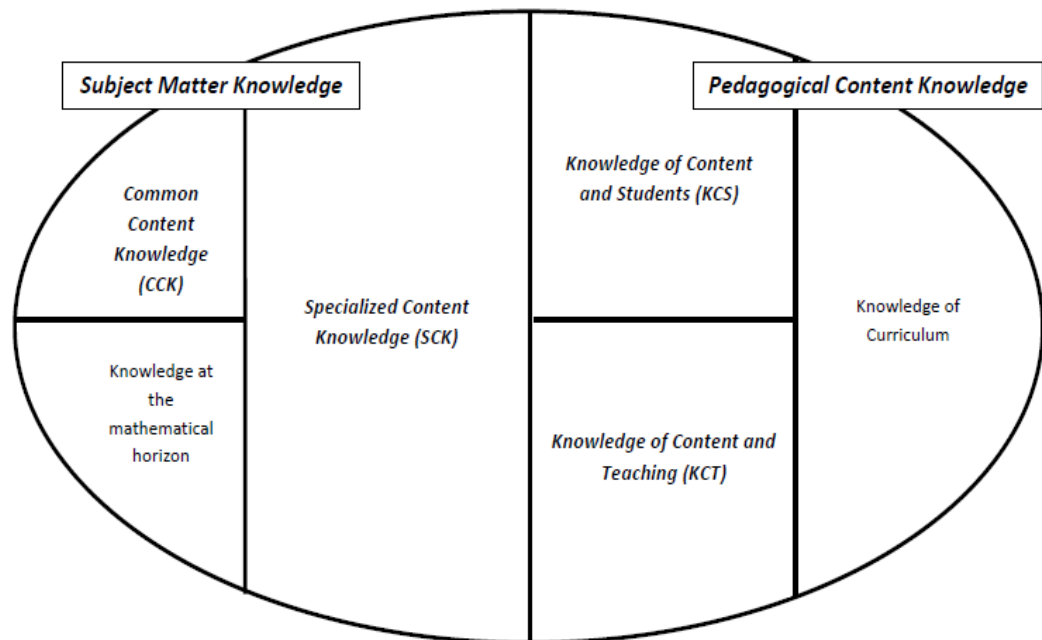


Figure 5.1. *Mathematics Knowledge for Teaching* (Ball, 2006).

The MKT framework is divided into two major components, subject matter knowledge and pedagogical content knowledge, each containing three subdomains. The subject matter knowledge consists of common content knowledge (CCK), specialized content knowledge (SCK), and knowledge at the mathematical horizon. CCK refers to mathematical knowledge “expected to be known by any well educated adult” (Bass, 2005, p. 429). CCK is “[mathematical] knowledge of a kind used in a wide variety of settings-in other words, not unique to teaching” (Ball, Thames & Phelps, 2008, p. 399). An example of CCK would include the identification of various regular polygons such as a square, equilateral triangle, or pentagon.

SCK refers to mathematical knowledge and skill that is “particular to the work of teaching, yet not required or known, in other mathematically intensive professions (including mathematics research)” (Bass, 2005, p. 429). SCK is mathematical knowledge, not pedagogy (Hill, Rowan & Ball, 2005). SCK is considered to be “applied content knowledge that may be developed through the work of teaching” (Hill & Lubienski, 2007, p. 753). An example of SCK includes the recognition and analysis of non-standard solutions, explanations, representations, or approaches to solving a particular problem. A teacher is using SCK when developing a geometric justification for finding the area of a regular n -sided polygon by dissecting the polygon into triangles and then summing up the area of the triangles to find the area of the regular n -sided polygon.

The third subdomain, knowledge at the mathematical horizon, is “an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (Ball, Thames, & Phelps, 2008, p.403). A teacher is exercising knowledge at the mathematical horizon when they are aware of the interconnectedness of mathematics knowledge and its impact on learning mathematics later in a student’s mathematical career. An example of knowledge at the mathematical horizon is being aware that dissecting the regular n -sided polygon into triangles and then summing the area of the triangles to find the area of the polygon anticipates the extension of using calculus to find area enclosed by curves described by polar coordinates..

The pedagogical content knowledge component consists of Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and knowledge of the curriculum. KCS and KCT involve the intersection of knowledge of

mathematics with knowledge of students and knowledge of teaching, respectively (Ball, 2006). KCS includes knowledge about student misconceptions, interpretation of student thinking that may have lead to misconceptions or errors, and the anticipation of what students will do when given a specific mathematical task. KCT includes the appropriate sequencing for instruction, recognizing the advantages or disadvantages of various manipulatives or representations for facilitating the understanding of a particular mathematical concept (Ball, 2006).

The MKT framework heavily grounded in constructivism provided the researcher a lens by which to recognize and classify various mathematical connections prospective middle grades teachers made while completing tasks meant to probe mathematical connections.

Purpose of Study

The purpose of this sequential exploratory mixed methods study was to examine prospective middle grades teachers' mathematics knowledge for teaching geometry and the connections made while completing tasks meant to probe mathematical connections. In addition, the study examined prospective middle grades teachers' coursework and its impact on their mathematics knowledge for teaching geometry. Specifically, the following questions were investigated:

1. What is the relationship between prospective middle grades teachers' mathematics knowledge for teaching geometry and the types of mathematical connections made while completing tasks meant to probe mathematical connections?
2. How does prospective middle grades teachers' coursework impact their mathematics knowledge for teaching geometry?

Mixed Methods Research Design

A sequential exploratory mixed methods design of combining both qualitative and quantitative approaches served as a model for this study. The design is sequential as quantitative and qualitative data collection and analyses were implemented in two distinct phases. The following definition of *mixed method research* posited by Creswell and Plano-Clark (2007) was utilized for this study.

Mixed method research is a research design with the philosophical assumptions as well as methods of inquiry. As a methodology, it involves philosophical assumptions that guide the direction of the collection and analysis of data and the mixture of qualitative and quantitative approaches in many phases in the research process. As a method, it focuses on collecting, analyzing, and mixing both quantitative and qualitative data in a single study or series of studies. Its central premise is that the use of quantitative and qualitative approaches in combination provides a better understanding of research problems than either approach alone. (p. 5)

By combining qualitative and quantitative methods the weaknesses in one method can be offset by the strengths in the other method (Creswell, 2003; Creswell & Plano-Clark, 2007). In particular, as Creswell and Plano-Clark explain,

A problem exists when the quantitative results are inadequate to provide explanations of outcomes, and the problem can best be understood by using qualitative data to enrich and explain the quantitative results in the words of the participants. Situations in which this problem occurs are those in which the quantitative results need further interpretation as to what they mean or when more detailed views of select participants can help to explain the quantitative results. (p. 35)

Mixed methods research helps answer questions that cannot be answered using only qualitative or quantitative methods alone. Mixed methods can provide a “more complete picture by noting trends and generalizations as well as in-depth knowledge of participants’ perspectives” (p. 33). Figure 5.2 reveals a diagram of the sequential exploratory mixed method design being used for this study.

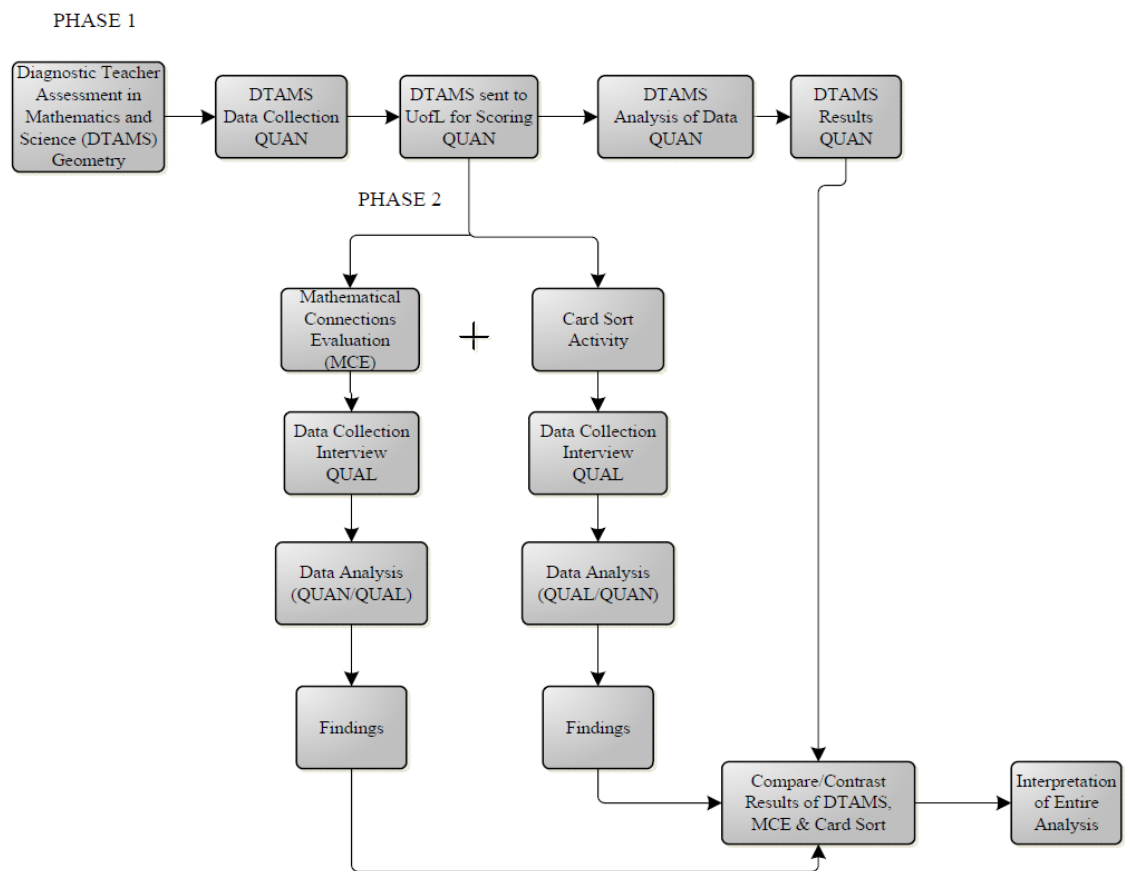


Figure 5.2. *Sequential Exploratory Mixed Methods Design*

The research study is exploratory in nature as it “generates information about unknown aspects of a phenomenon” (Teddlie & Tashakkori, 2009, p. 25), in this case, (a) the types of mathematical connections prospective middle grades teachers make when engaged in tasks meant to probe mathematical connections and (b) how these connections are related to prospective middle grades teachers’ MKT. As illustrated in Figure 5.2, the design is sequential as research methods were implemented in two distinct phases. The quantitative data collection via the *Diagnostic Teacher Assessment in Mathematics and Science* (DTAMS) preceded qualitative data collection via the *Mathematical Connections Evaluation* (MCE) and *Card Sort Activity* (CSA). Unlike a traditional sequential exploratory design, the quantitative results of the DTAMS assessment (phase 1) did not directly inform or drive the construction of MCE and CSA (phase 2) instruments. The quantitative data from the DTAMS and the qualitative data from the MCE and CSA were analyzed separately; results and findings merged during interpretation.

Population

The targeted population for this study was prospective middle grades teachers at a large mid-south university. The sampling frame was derived from a comprehensive list of prospective middle grades teachers meeting the following criteria: (a) declared middle school education major, and (b) actively pursuing a middle school certification in two content areas, one of which was mathematics. All prospective middle school teachers meeting both criteria were contacted for voluntary participation in this study. All 58 eligible participants were contacted, of which, 28 elected to participate. Most participants were female ($n=22$, 78.6%). There were 14 juniors (50%) and 14 were seniors (50%). There were 6 student teachers (21.4%) in the study.

Instrumentation

There were three data collection instruments administered to prospective middle grades teachers; a Diagnostic Teacher Assessment in Mathematics and Science (DTAMS) focused on geometry and measurement, a Mathematical Connections Evaluation (MCE), and a Card Sort Activity (CSA).

Diagnostic Teacher Assessments in Mathematics and Science

The first instrument was the *Diagnostic Teacher Assessments in Mathematics and Science* (DTAMS) from the University of Louisville's Center for Research in Mathematics and Science Teacher Development [CRMSTD]. The DTAMS is comprised of four content domains: number and computation, geometry and measurement, probability and statistics, and algebraic ideas. For the purposes of this study, the DTAMS focused on the domain of geometry and measurement was selected. The domain of geometry and measurement consists of the following subcategories: two-dimensional geometry, three-dimensional geometry, transformational/coordinate geometry, and measurement. The 20-item assessment is composed of 10 multiple choice and 10 open response. In particular, the assessment measures four types of mathematics knowledge: (1) memorized knowledge, (2) conceptual knowledge, (3) problem solving and reasoning, and (4) pedagogical content knowledge (see Appendix A). The assessment contains five items in each of the four types of mathematical knowledge measured by DTAMS. Assessment items were developed by teams of mathematicians, mathematics educators, and classroom teachers who not only conducted extensive literature reviews on what

mathematics middle school teachers and students should know but also utilized national recommendations along with national and international test objectives in the development of research-based appropriate items. These content-valid items have been repeatedly tested and implemented in several institutions across the United States. As a measure of internal consistency the instrument has Cronbach's alpha $\alpha=.87$ with number of cases $n=429$. Inter-scorer reliability estimates were established "using percents of agreements among three graduate students who developed and used the scoring guides for scoring open response items and eventually scored all field tests" (CRMSTD, 2007, ¶ 8). The instrument was administered to participants prior to the interviews involving the mathematics connection evaluation and card sort activity.

The DTAMS instrument served as a quantitative measure of prospective middle grades teachers' MKT geometry. To strengthen the validity in use of the DTAMS assessment as a quantitative measure of prospective middle grades teachers' MKT, each item on the DTAMS instrument was mapped to a subcategory of the MKT framework. The researcher in consultation with mathematicians and mathematics educators mapped each DTAMS item to the most appropriate subcategory in the MKT framework. The DTAMS was scored out of total of 40 points by professional staff at the University of Louisville's CRMSTD. The MKT framework is divided into two major components- subject matter knowledge and pedagogical content knowledge (see Figure 5.1). Those items that were mapped into the subject matter knowledge component represented 30 of the 40 points (75%). Those items mapped into the pedagogical content knowledge component represented 10 of the 40 points (25%). In particular, items that were mapped into the CCK subcategory represented 11 out of 40 points (27.5%). Items that were mapped into the SCK subcategory represented 19 out of 40 points (47.5%). Items that were mapped into KCS and KCT categories represented 10 out of 40 points (25%). There were no items on the DTAMS that could be mapped into the "knowledge at the mathematical horizon" and "knowledge of the curriculum" subcategories. To date there is no empirical evidence on how knowledge at the mathematical horizon and knowledge of curriculum play a role in MKT which could explain why there are no questions of this nature on the DTAMS assessment. The DTAMS instrument serves as a good measure of MKT for this particular study by allowing the researcher to examine the relationship(s)

between particular subcategories of MKT (CCK, SCK, KCT, and KCS) and types of mathematical connections prospective middle grades teachers make when engaged in tasks meant to probe mathematical connections.

Mathematical Connections Evaluation

The Mathematical Connections Evaluation (MCE) (see Appendix B) consisted of two components, a demographic survey followed by a series of mathematics problems. A semi-structured clinical interview format in which participants used both concurrent and reflective think-aloud strategies when asked to explain their thinking and thought processes for solving each problem was implemented. Protocols were created for the semi-structured clinical interviews (see Appendix C). To strengthen the reliability and validity of the instrument, MCE items were constructed in cooperation with and reviewed by mathematicians and mathematics educators. Constructions of items were based on and aligned to national recommendations, in particular, *Recommendations for the Mathematical Education of Teachers* (CBMS, 2001), *Principles and Standards for School Mathematics* (NCTM, 2000), and *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006) (see Appendix D). The author, in consultation with mathematicians and mathematics educators, created a rubric to quantize the MCE data by applying a deductive approach to the method of constant comparison. The mathematical connections category types that emerged were: *procedural*, *algebraic/geometric*, *characteristic/property*, *derivational*, and *2-D/3-D*. A participant received a score of 2 points if they correctly made a connection, 1 point if they made a partial connection and 0 points otherwise.

All MCE interviews were videotaped. In an effort to make the participants more comfortable and candid with their responses, their faces were not videotaped. The videotaped data focused on participants' written work, oral responses, and hand movements.

Card Sort Activity

Upon completion of the MCE interview, participants completed a Card Sort Activity (CSA). The CSA consisted of 20 cards labeled with various mathematical terms, concepts, definitions, and problems (see Appendix F). Construction of the cards were based on and aligned to national recommendations, in particular, *Recommendations for*

the Mathematical Education of Teachers (CBMS, 2001), *Principles and Standards for School Mathematics* (NCTM, 2000), and *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006) (see Appendix E). The purpose of the CSA was to examine the types of connections prospective middle grades teachers' make between various mathematical concepts, definitions, and problems. Participants were asked to complete a repeated single criterion open card sort and closed card sort (Fincher & Tenenberg, 2005; Rugg & McGeorge, 2005). In the closed card sort, five particular pairs were chosen based on national recommendations (CBMS, 2001; NCTM 2000, 2006) on what middle school teachers and students should know and be able to do. The cards chosen were also influenced by content from the reform middle school curriculum textbook series *Connected Mathematics2: Grade 6* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006), *Connected Mathematics 2: Grade 7* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006), and *Connected Mathematics 2: Grade 8* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006). The particular pairs of cards chosen for the closed card sort were also selected in consultation with mathematicians. For the open card sort, participants sorted the cards based on a single criterion: their perceived notion of how the statements on the cards were connected. The researcher developed a protocol of interview questions for both the open and closed card sorts that focused on students' mathematical connections (see Appendix F). The design of the protocols was influenced by Rugg and McGeorge (2005) recommendations for carrying out card sorting techniques:

The maximum number of entities which is conveniently manageable for repeated single-criterion sorts is about 20 or 30, though it is possible to use significantly more in some circumstances...Cards should likewise be all the same size. We usually use small filing cards, with the words word processed onto paper and then stuck onto the cards. This reduces problems with illegible handwriting, and avoids the issue of trying to get filing cards through a borrowed typewriter ... We usually encourage the respondents to look at all the items at the start of the session before they do any sorting, so that they are fully aware of the range of items to be sorted... We advise the use of a tape recorder (for respondents' comments if problems occur). It is also worth considering using a Polaroid-type camera (for quick backup of record of groupings). If using a camera [or video camera], it is advisable to check beforehand that the photographs [video] can catch enough detail to allow all entities to be easily identified. (pp. 98-100)

Quality Review

The MCE and CSA instruments underwent a quality review (Half, 1993; Tessmer, 1993) to further strengthen the validity of each instrument. An expert quality review is an evaluation of a product (in this case the CSA and MCE instruments and protocols) on the basis of appropriateness, content accuracy, and design quality. Expert reviews consist of an expert or experts (in this case mathematicians and mathematics educators) reviewing a rough draft of each instrument along with interview protocols to determine strengths and weaknesses. The feedback and comments provided by the expert reviewers were analyzed and subsequent modifications were made to the MCE and CSA instruments in order to improve the quality of each instrument and interview protocols.

Collection of Data

Data was collected via the DTAMS, MCE, and CSA. The pool of eligible participants, i.e., prospective middle grades teachers actively pursuing a middle grades education major leading to certification in mathematics, fell into two groups—those enrolled in a problem solving course for prospective middle grades teachers and those not enrolled in the course. The researcher contacted the instructor for the problem solving course at a large mid-south public university to arrange a time to solicit volunteers for the research study. The researcher carefully reviewed the informed consent letter with all potential participants stressing that participation in the study was purely voluntary and would have no negative effect on their course grade. The course instructor was not given access to the identity of students who consented to participate in the study. Once written consent had been obtained, the course instructor administered the DTAMS assessment. Upon completion of the DTAMS assessment, participants scheduled an interview time with the researcher to complete the MCE and CSA.

Potential participants not enrolled in the problem solving course were contacted via a general email announcement (see Appendix J). The informed consent form was sent as an attachment in the email announcement (see Appendix K). Potential participants were asked to review the informed consent form. If they chose to participate in the study they were asked to schedule two meetings—one for taking the DTAMS assessment and the other an interview session where they will complete the MCE and CSA.

The DTAMS assessments were administered prior to the interview session. For those enrolled in the problem solving course, the DTAMS assessment was administered by the course instructor during the class period. For those not enrolled in the problem solving course, the DTAMS was administered by the researcher. All participants were given approximately 75 minutes to complete DTAMS assessment. All participants (enrolled and not enrolled in the problem solving course) completed the DTAMS assessment within the same two week time period. The MCE and CSA were conducted outside of class and after completion of the DTAMS assessment.

After completing the DTAMS assessment, all participants were provided with a two-week block for scheduling the MCE and CSA interviews. All participants (enrolled and not enrolled in the problem solving course) engaged in two separate sessions on two different days-one for taking the DTAMS assessment and the other for MCE/CSA interviews. The procedures and content for the MCE/CSA interview session for all participants were identical. During the interview sessions, participants took approximately 45-60 minutes to complete the MCE. Upon completion of the MCE interview, participants took approximately 30-45 minutes to complete the CSA. The MCE and CSA interviews were audio and video recorded

In the open card sort, 20 cards were arranged on a table in a 4 by 5 array. Figure 5.3 below illustrates the arrangement of cards for the open sort.

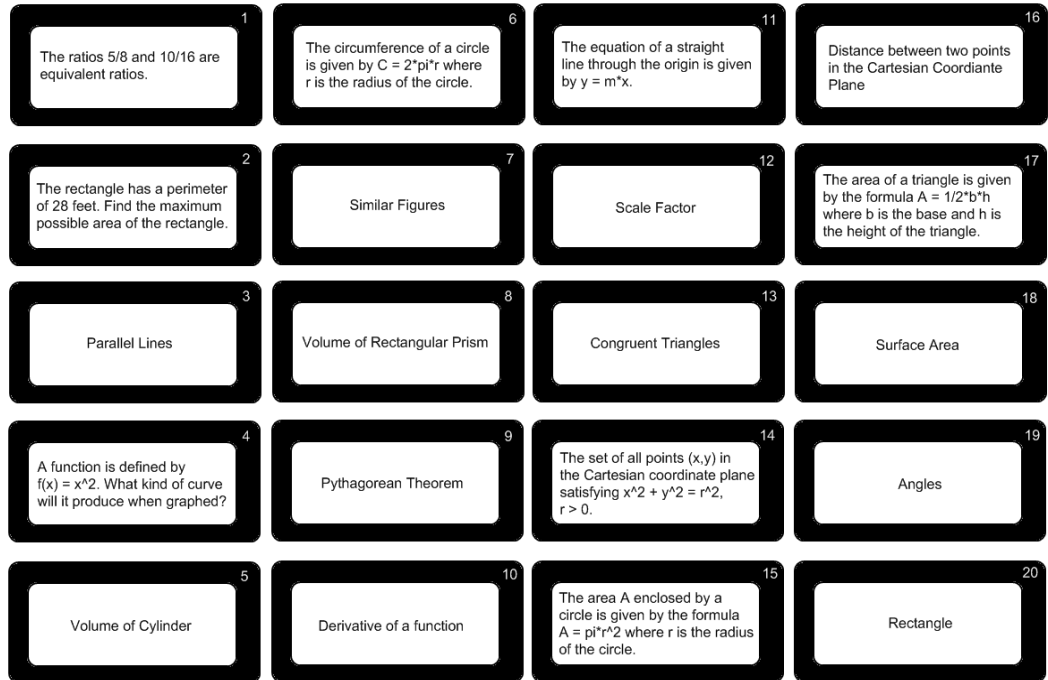


Figure 5.3. Arrangement of Cards for CSA

The cards were arranged the same way for each participant. Participants were asked to select a subset of two or more cards they felt were related or connected. They were then asked to explain why the cards they had selected were related or connected. After giving an explanation, participants returned the cards back to the 4 by 5 array. They were then asked to select another subset of cards they felt were related or connected from the 4 by 5 array. This procedure allowed participants to re-use cards. This process was repeated until the participant indicated they could not make any more subsets.

In the closed card sort, the researcher selected five pairs of cards and asked if each pair of cards were related or connected and, if so, why? The five pairs of cards chosen were cards 6 and 11, cards 2 and 4, cards 15 and 17, cards 4 and 15, and cards 9 and 16. Figure 5.4 provides an illustration of the five closed sort pairings.

Figure 5.4. *CSA Closed Sort Pairings*

The researcher selected the first pair of cards, 6 and 11, and placed them in front of the participant. The participant was then asked if the pair of cards were related or connected and, if so, why? Once the participant provided a response, cards 6 and 11 were returned to the 4 by 5 array (see Figure 5.3). This procedure was carried out for each of the aforementioned pairings of cards, and in the order listed.

Analysis

What is the relationship between prospective middle grades teachers' mathematics knowledge for teaching geometry and the types of mathematical connections made while completing tasks meant to probe mathematical connections?

A deductive approach to the method of constant comparison in which codes are identified prior to the analysis and then looked for in the data (Leech & Onwuegbuzie, 2007) was undertaken in order to identify the types of mathematical connections that were necessary or likely to be made as part of correctly solving the problems presented in the MCE. The result of this data analysis was used to create the rubric to score the MCE. In order to generate these connection types *a priori*, the researcher used the guiding

question, “What would the participant need to know or be able to do to solve this problem?” This question was used as a guide for each MCE item. A list of what was necessary or likely to be needed as part of correctly solving each MCE problem was generated. Each item on the list was given a descriptive code. Each new item on the list was compared to previously generated codes, so similar items could be labeled with the same descriptive code. After all the items on the list had been coded, the codes were grouped by similarity, which represented a unique theme (i.e., mathematical connection type). Once these connection types were identified, two expert mathematicians were consulted to provide feedback and comments. In consultation with expert mathematicians, there were five types of mathematical connections used to construct the scoring rubric: *procedural*, *characteristic/property*, *algebraic/geometric*, *derivational*, and *2-D/3-D* (see Appendix L).

In contrast to the MCE, participant’s responses in the CSA were qualitatively analyzed using an inductive approach to the method of constant comparison (Denzin & Lincoln, 2000) for extracting themes. To address the research question, the CSA and MCE data were quantitized (Teddle & Tashakkori, 2009) so that statistical analysis could be performed. The CSA data were quantitized by tallying the types of connections made; the connections categories utilized were those found through an inductive approach to the method of constant comparison (see Appendix M). The MCE was quantitized by scoring the evaluations using a rubric. The participant received a score of 2 points if they made a particular connection, 1 point if they made a partial connection, and 0 points otherwise.

These quantitative data were analyzed using bivariate correlation analysis via Pearson correlations to examine the relationship between prospective middle grades teachers MKT geometry and MCE connections. Pearson correlations were also used to examine the relationship between prospective middle grades teachers MKT geometry and the types of CSA connections made.

How does prospective middle grades teachers’ coursework impact their mathematics knowledge for teaching geometry? To address the research question, participants were divided into three distinct non-overlapping groups A, B, and C based on their coursework. As this data was collected within the last three weeks of the semester,

currently enrolled courses were treated as courses that had been completed when placing participants into groups. All mathematics content courses are taught through a department of mathematics at the site where the study was being conducted. There are six mathematics courses in the middle school program. All participants had completed a calculus course. All participants had completed two mathematics content courses for elementary teachers. The first course focused on sets, numbers, and operations, problem solving and number theory and the second course focused on algebraic reasoning, introductory probability and statistics, geometry, and measurement. All participants had completed a problem solving course for middle grades teachers. The remaining two mathematics content course requirements included a finite mathematics course (MATH I) and a geometry course for prospective middle grades teachers (MATH II). There were 20 participants who had completed MATH II and eight who had not. These same 20 participants had completed MATH I and the same eight participants had not completed MATH I. There were no cases where participants had taken MATH II and not taken MATH I, or vice versa. Participants had either completed both MATH I and MATH II or had not completed both courses.

There are two mathematics methods courses in the middle school program at the site where this study was conducted. These methods courses are taught through a department of curriculum and instruction. METH I is a teaching mathematics in the middle school course. METH II is a student teaching in the middle school course. METH II is not specific to the teaching of mathematics. There were six participants who had completed METH I and 22 who had not. These same six participants were currently enrolled in METH II and 22 who had not completed and were not currently enrolled in METH II. There were no cases where participants had completed or were currently enrolled in METH I and had not taken or were not currently enrolled in METH II, and vice versa. Participants had either completed or were currently enrolled in both METH I and METH II or they had not completed or were currently enrolled in both courses.

Participants were placed in Group A if they had completed all mathematics content and methods courses. Participants were placed in Group B if they had completed all mathematics content courses but had not taken mathematics methods courses. Participants were placed in Group C if they had not completed all content courses (in this

case had not completed MATH I and II) and had not taken mathematics methods courses. There were six participants in Group A, 14 participants in Group B, and eight participants in Group C.

To explore the impact of prospective middle grades teachers' mathematics content coursework on their MKT geometry, a univariate analysis was conducted using a linear regression model. The participants in Groups B and C were utilized for this analysis because participants in Group B had completed all required mathematics courses while those in Group C had not. Participants in Groups B and C had not completed methods courses. The number of participants was 22. A process known as dummy coding was used to create a dichotomous variable from the categorical variable "Group". Participants were coded as a "1" if they belonged to the group and coded "0" otherwise. To assess the relationship between prospective middle grades teachers' mathematics content coursework and mathematics knowledge for teaching geometry, a linear regression analysis was conducted with DTAMS score as the dependent variable and mathematics content coursework as the independent variable.

To explore the impact of prospective middle grades teachers' mathematics methods coursework on their MKT geometry, a univariate analysis was conducted using a linear regression model. The participants in Groups A and B were utilized for the analysis because participants in Group A had completed all of the required methods courses while those in Group B had not. Participants in Groups A and B had completed all mathematics content courses. The number of participant in this regression was 20. Again, the process of dummy coding was used to create a dichotomous variable from the categorical variable "Group". Participants were coded as "1" if they belonged to the group and coded "0" otherwise. To assess the relationship between prospective middle grades teachers' mathematics methods coursework and mathematics knowledge for teaching geometry, a linear regression analysis was conducted with DTAMS score as the dependent variable and methods coursework as the independent variable.

Results

What is the relationship between prospective middle grades teachers' mathematics knowledge for teaching geometry and the types of mathematical connections made while completing tasks meant to probe mathematical connections?

Diagnostic Teacher Assessments in Mathematics and Science

The DTAMS assessment served as a quantitative measure of prospective middle grades teachers' MKT geometry. In order to make comparisons of prospective middle grades teachers' MKT geometry and mathematical connections, the DTAMS data had to be analyzed. The data from the DTAMS were analyzed quantitatively. The fit of the DTAMS data to the normal curve is necessary for carrying out parametric analysis. Skewness and kurtosis were computed and a deficiency of extreme elevated skewness and kurtosis was noted. The fit of the DTAMS total scores to a normal distribution was assessed through a Shapiro-Wilk Test ($W=.946$ with $N=28$ and $p=.153$). Since $p>0.05$, the null hypothesis was not rejected and the data fits the normality assumption. Cronbach's alpha, α , was used to assess the internal consistency reliability for the DTAMS assessment. The coefficient alpha of $\alpha=.768$ suggests that the questions comprising the DTAMS instrument for this sample were internally consistent. The mean DTAMS score of the 28 participants was 27.79 out of a possible score of 40 ($SD=5.971$; $range=18-38$).

Mathematical Connection Evaluation

In order to make comparisons between prospective middle grades teachers' MCE mathematical connections and MKT geometry, the MCE data were analyzed quantitatively. The fit of the MCE data to the normal curve is necessary for carrying out parametric analysis. Skewness and kurtosis were computed and a deficiency of extreme elevated skewness and kurtosis was noted. The fit of the MCE total scores to a normal distribution was assessed through a Shapiro-Wilk Test ($W=.926$ with $N=28$ and $p=.058$). Since $p>0.05$, the null hypothesis was not rejected and the data fits the normality assumption. Cronbach's alpha, α , was used to assess the internal consistency reliability for the MCE instrument. The coefficient alpha of $\alpha=.892$ suggests that the questions comprising the MCE instrument for this sample were internally consistent. The mean MCE score of the 28 participants was 33.43 out of a possible score of 44 ($SD=8.492$; $range=14-44$).

Types of MCE Connections

The interview data from the MCE instrument were analyzed both qualitatively and quantitatively. As stated above, a rubric was created to quantize the MCE data. The

five types of mathematical connections used to grade the evaluation were: *characteristic/property*, *algebraic/geometric*, *derivational*, and *2-D/3-D*. Table 5.1 provides a description and example for each MCE connection type.

Table 5.1. *Description of MCE Connection Types and Examples*

MCE Type	Description	Example
Procedural	A mathematical connection is called a procedural connection if the link (or bridge) used to establish or strengthen an understanding between mathematical ideas, concepts, strands, or representations is a procedure, method, or algorithm.	A participant is making a procedural connection when identifying the use of a table of values for graphing the line $y=3x$ in the Cartesian Coordinate Plane.
Algebraic/ Geometric	A mathematical connection is called an algebraic/geometric connection if it is a link (or bridge) used to establish or strengthen an understanding between geometric mathematical ideas, concepts, and/or representations with algebraic mathematical ideas, concepts, and/or representations.	A participant is making an algebraic/geometric connection when they are able to explain that the solution to the following linear system $\{y=3x; x=5\}$ is the intersection point of the line $y=3x$ and the line $x=5$ graphed in the Cartesian Coordinate Plane.
Characteristic/ Property	A mathematical connection is called a characteristic/property connection if the link (or bridge) used to establish or strengthen an understanding between mathematical ideas, concepts, strands, or representations involves using the mathematical properties and/or characteristics to describe, identify, or classify particular mathematical ideas, concepts, or representations.	A participant is making a characteristic/property connection when describing a rectangle as a quadrilateral with four interior 90 degree angles; opposite sides parallel and congruent.

Table 5.1 (continued)

MCE Type	Description	Example
Derivational	A mathematical connection is called a derivational connection if the link (or bridge) used to establish or strengthen an understanding between mathematical ideas, concepts, strands, or representations involves the justification or motivation for a particular mathematical theorem, formula, or procedure.	A participant is making a derivational connection when they are able to provide a justification or motivation for why the surface area, S , of a cylinder is given by $2\pi(\text{radius})^2 + 2\pi(\text{radius})(\text{height})$.
2-D/3-D	A mathematical connection is called a 2-D/3-D connection if it is a link (or bridge) used to establish or strengthen an understanding between 2-D mathematical ideas, concepts, or representations with 3-D mathematical ideas, concepts or representations.	Consider the region in the Cartesian Coordinate plane bounded by the lines $y=2$, $x=1$, the x -axis, the y -axis. The bounded region is a rectangle with vertices $(0, 0)$, $(2, 0)$, $(1, 0)$, and $(1, 2)$. Suppose the bounded region is rotated about the x -axis. A participant is making a 2-D/3-D connection if they are able to identify the 3-D object as a cylinder where the length and width of the rectangle correspond to the height and radius, respectively, of the cylinder.

A participant received a score of 2 points if they correctly made a particular connection, 1 point if they made a partial connection and 0 points otherwise. The researcher and an outside consultant scored the MCE using the aforementioned rubric. The second scorer was a mathematician at the site where the study was being conducted and who has taught mathematics content courses for prospective middle grades teachers. The researcher and consultant scored 2 of the MCEs together in order to become more familiar with the rubric and to help establish consistency in the scoring. The outside consultant independently scored a randomly selected sample of 35% ($n=10$) of the MCEs. Inter-rater reliability assessed by correlation analysis was .969. Descriptive

statistics including means, standard deviations, minimums, and maximums for the MCE are reported in Table 5.2.

Table 5.2. *Descriptive Statistics for MCE (n=28)*

	Minimum	Maximum	Mean	SD
MCE Procedural	5	14	10.18	3.175
MCE Algebraic/Geometric	5	10	8.86	1.627
MCE Characteristic/Property	1	2	1.89	.315
MCE Derivational	0	6	2.96	2.117
MCE 2-D/3-D	0	12	9.54	3.305
Overall MCE Score	14	44	33.43	8.492

Relationship of MKT Geometry and Types of MCE Connections

In order to address the research question, bivariate analysis using Pearson product-moment correlations were calculated between each MCE connection type and DTAMS scores.

No statistically significant correlations were found between characteristic/property and 2-D/3D connections with DTAMS scores. However, there were statistically significant correlations between procedural, algebraic/geometric, and derivational connections with DTAMS scores. There was a statistically significant moderate positive correlation (Visual Statistics Studio, 2006) between MCE algebraic/geometric connection type and DTAMS score ($r=.546, p<.05, n=28$). There was a statistically significant high positive correlation (Visual Statistics Studio, 2006) between MCE procedural connection type and DTAMS score ($r=.754, p<.05, n=28$).

There was a statistically significant high positive correlation (Visual Statistics Studio, 2006) between MCE derivational connection type and DTAMS score ($r=.709, p<.05, n=28$).

Relationship of MKT Geometry and MCE

Pearson product-moment correlations were calculated between DTAMS score and overall MCE score. There was a statistically significant positive high correlation (Visual Statistics Studio, 2006) between DTAMS score and overall MCE score ($r=.705, p<.05, n=28$). Pearson product-moment correlations were also calculated between MKT subcategories as measured by the DTAMS and MCE scores. There was a statistically significant positive high correlation (Visual Statistics Studio) between subcategory CCK and overall MCE score ($r=.741, p<.05, n=28$). There was a statistically significant moderate positive correlation between subcategory SCK and overall MCE score ($r=.648, p<.05, n=28$). There were no statistically significant correlations between MKT subcategories KCT and KCS with overall MCE score.

Types of CSA Open Sort Connections

There were a total of 258 open card sorts. The categories that emerged from an inductive analysis using constant comparative methods on participant's interview responses to the open sort were identified, and unifying commonalities were grouped into metacategories. The types of mathematical connections made by prospective middle grades teachers during the CSA open sort fell into the following five metacategories: *categorical, procedural, characteristic/property, derivational, and curricular*. The researcher and an outside consultant coded the open card sorts using a coding guide (see Appendix M). The coding guide provided a description of each of the five emergent mathematical connections types along with examples for each type. The second coder was a mathematician at the site where the study was being conducted and who has taught mathematics content course for prospective middle grades teachers. The researcher and consultant together categorized 12 open card sorts (with each mathematical connection type represented at least twice) in order to become more familiar with the descriptions for each mathematical connection type and to help establish consistency in the coding. The second coder independently coded a randomly selected sample of approximately 53% of the open card sorts ($n=137$). Inter-rater reliability analysis using a kappa statistic (Cohen,

1960) was performed to determine consistency among coders. The level of agreement among coders was found to be “substantially strong” (Landis & Koch, 1997, p. 165) with $\kappa=.74$.

Relationship of MKT Geometry and Types of CSA Open Sort Connections

Although there were 258 open card sorts, there were 287 mathematical connections made that fell into one or more of the CSA open sort connection categories. A participant’s response for grouping particular cards together could fall into one or more of the aforementioned mathematical connection categories. To further investigate the research question, bivariate analysis using Pearson correlations were calculated between each connection type in the CSA open sort and DTAMS scores. Table 5.3 displays a count of CSA open sort connection types broken down by DTAMS scores.

Table 5.3. *Counts of CSA Open Sort Connections by DTAMS Score (n=28)*

DTAMS Score	No. of Participants	CSA Categorical	CSA Char/Prop	CSA Curricular	CSA Procedural	CSA Derivational
18-21	4	15	8	0	7	3
22-25	7	23	13	3	15	15
26-29	5	20	9	7	17	4
30-33	8	27	19	14	24	8
34-38	4	12	2	12	5	5
Totals	28	97	51	36	68	35

There were no statistically significant correlations found between categorical, characteristic/property, procedural, and derivational connection types with DTAMS

scores. However, a statistically significant positive moderate correlation (Visual Statistics Studio, 2006) was found between the curricular connection type and DTAMS scores ($r=.520, p<.05, n=28$).

Types of CSA Closed Sort Connections

In the CSA closed sort, five particular pairs of cards were selected: cards 6 and 11; cards 2 and 4; cards 15 and 17; cards 4 and 15; cards 9 and 16 (see Figure 5.4). Participant responses were qualitatively analyzed using an inductive approach to the method of constant comparison for each closed sort pairing. For the closed sort pairing of cards 6 and 11 the following themes emerged: *yarn explanation; radius as a “line”; both are formulas; both are equations; both are linear functions; none*. For closed sort pairing of cards 2 and 4 the following themes emerged: *max area most square like; calculus problem; derivative to find max; graphing possibilities; none*. For closed sort pairing of cards 15 and 17 the following themes emerged: *both area formulas; geometric/relational; volume of cone; none*. For closed pairing of cards 4 and 15 the following themes emerged: *both have “squares”; both are quadratic functions; invalid geometric; none*. For closed sort pairing of cards 9 and 16 the following themes emerged: *given triangle; create triangle; distance formula looks like the Pythagorean Theorem; Pythagorean Theorem is the distance formula; none*. The themes and exemplars for each closed sort pairing are provided in Tables 5.4-5.8.

Table 5.4. *Themes and Exemplars for Closed Sort Pair 6 and 11 (n=28)*

Themes	Exemplars of Participant Responses	Count	Frequency
Yarn Explanation	If you take a piece of yarn at a certain point around the circle and brought it all the way around, then straightened it out, it would make a straight line that you could lay against a ruler.	6	21%
Radius as a "line"	If you were to graph the circle on the coordinate plane, the line $[y=mx]$ could be the radius of that circle".	7	25%
Both are Formulas	Right off the bat, I think they are both formulas. It's kind of one of the second nature formulas that you just know. Hopefully, your teachers help you derive it and you know what they. I think this is another case like with the last two, I wouldn't teach together. From a teacher's perspective they are kind of unrelated in terms of how I would teach it.	3	11%
Both are Equations	They are both equations. I don't really know if find the slope of a straight line would help you find the circumference of a circle, but they are both equations.	2	7%
	They are both equations. This $y=mx$ gives you a line and the other gives you a circle.		
Both are Linear Functions	I think they can be related because they are both functions, really. Well, the x I would just think of it relating C the circumference can be a function of the radius. If you change the radius, it will change the circumference. Whenever you change the x value it's going to change the y, the output. They are both input/output. They are both lines.	1	4%
None	I don't think they are related because that [card 6] has to do with a shape [a circle] and this [card 11] has to do with a line.	9	32%
Totals		28	100%

Note: Card 6 read, "The circumference of a circle is given by $C=2*\pi*r$ where r is the radius of the circle". Card 11 read, "The equation of a straight line through the origin is given by $y=m*x$ ".

Table 5.5. Themes and Exemplars for Closed Sort Pair 2 and 4 (n=28)

Themes	Exemplars of Participant Responses	Count	Frequency
Max Area Most Square Like	I'm trying to find the max possible area of the rectangle. I think it relates because the max possible area of rectangle is going to be given by length times width which is 7 times 7 so you could say 7 squared so the is some kind of connection to x squared.	3	11%
Calculus Problem	Here I think about, there is some calculus interwoven in this, when trying to find the maximum area with a given perimeter. When you do the arithmetic, the math is going to create a parabola and that maximum value....I would need to flush this one out, but they are related.	3	11%
Derivative To Find Max	I think these are related. I think you have to take the derivative to find the maximum. We did problems like this last semester where sometimes it was undefined and sometimes a maximum. I need my notes for this one.	1	4%
Graphing Possibilities	To find the maximum area of a rectangle you can graph it which is usually going to be a parabola and this is the equation that gives you a parabola. You could graph every possibility and the graph would look like this [participant uses hands to indicate a downward opening parabola] which is a parabola.	5	17%
None	I don't see how finding the max area of a rectangle has to do with a parabola...nope...nothing.	16	57%
Totals		28	100%

Note. Card 2 read, “A rectangle has perimeter 28 feet. Find the maximum possible area of the rectangle”. Card 4 read, “A function is defined by $f(x) = x^2$. What kind of curve will it produce when graphed?”

Table 5.6. Themes and Exemplars for Closed Sort Pair 15 and 17 (n=28)

Theme	Exemplars of Participant Responses	Count	Frequency
Both Area Formulas	That's just going back to area because you are trying to find area in each. If you want to find the area of a triangle you use this formula and if you want to find area of circle you use this one and that's how they are related. They are formulas for area but just different objects.	17	60%
Geometric/Relational	They're both area, just of different shapes. I'm trying to figure out how much more I can relate them than that. I guess if you have your circle and you make it into a bunch of different pie pieces which is kind of similar to a triangle you could end up using this formula [card 17] to roughly get to this one [card 15]. The more triangles you put into the circle, the close it will get to the area of a circle.	9	32%
Volume of Cone	If you go by what I said earlier about multiplying the area of a triangle times the area of a circle, then it might be volume of a cone.	1	4%
None	There is something there but I can't remember what it is, I can't put my finger on it. It is something I've done and I don't remember when and where. I'll remember at some point, it may be tomorrow or the next day, but I'll remember what this is and where I did it. I feel like I should know this but I don't.	1	4%
Totals		28	100%

Note. Card 15 read, “The area A enclosed by a circle is given by the formula $A=\pi r^2$ where r is the radius of the circle”. Card 17 read, “The area of a triangle is given by the formula $A=1/2bh$ where b is the base and h is the height of the triangle”.

Table 5.7. Themes and Exemplars for Closed Sort Pair 4 and 15 (n=28)

Theme	Exemplars of Participant Responses	Count	Frequency
Both have “Squares”	<p>The variable in both formulas is squared.</p> <p>They both have “squares” in them.</p>	10	35%
Both are Quadratic Functions	You have two functions squared. You could substitute πx for r . They are both even quadratic functions.	2	7%
Invalid Geometric	<p>Again, I’m going to go with they are connected because area squared and this [function] is squared. This one says what kind of curve will it produce when graphed and we know what kind of curve a circle is going to produce. I guess half of it is going to be a parabola.</p> <p>The function is going upward like a U-shape. If it continued or if you flip it, rotate it, then you could find the area of a circle.</p> <p>This gives you like a parabola which is kind of like a half-circle...And maybe if that was like a half-circle and the parabola was laying on the x-axis and you want to know the area of that specific function or half circle then you would need to know how to find the area of a full circle in order to find the area of x squared laying on the x-axis.</p>	8	29%
None	I’m not sure I can think of a relationship between 4 and 15. This [card 4] could be the area of a wedge of a circle, but that is pretty obscure.	8	29%
Totals		28	100%

Note: Card 4 read, “A function is defined by $f(x) = x^2$. What kind of curve will it produce when graphed?” Card 15 read, “The area A enclosed by a circle is given by the formula $A = \pi r^2$ where r is the radius of the circle.

Table 5.8. *Themes and Exemplars for Closed Sort Pair 9 and 16 (n=28)*

Theme	Exemplars of Participant Responses	Count	Frequency
Given Triangle	These are connected because if you have a right triangle on the coordinate plane you can figure out, easily figure out, the base and the height and then you could use the Pythagorean theorem to figure out the hypotenuse.	15	54%
Create Triangle	Like, I'm picturing if I want to find this line and I wanted to find the distance between these two points, I could make a triangle out of that. I would put two points in the plane, I was picturing a line between the two points, and then so I was picturing to draw a triangle. Then finding the distance between these two points would be like finding this line. If this was my triangle and this was my right angle then using the Pythagorean theorem to find the line.	5	17%
DF looks like PT	Yeah [indicating the statement on the two cards are related], because the Pythagorean theorem is pretty much the distance formula. Because a squared plus b squared equals c squared and square root all that to find c by itself which is the distance equal to the square root of a squared plus b squared. The a 's could be the x 's, the b 's could be the y 's and so square root of a squared plus b squared is square root of $(x^2 \text{ minus } x1)^2$ squared plus $(y^2 \text{ minus } y1)^2$ squared which equals the distance which equals c .	3	11%
PT is DF	The Pythagorean theorem is the distance formula in the coordinate plane. Here I thought about the Pythagorean theorem, actually....because I have never been able to remember the distance formula and I've learned in two classes this year that you can use the Pythagorean theorem to find the distance between two points instead of having to memorize the distance formula which I found to be really helpful.	4	4%
None	I'm not sure if they are related. I can't remember right now.	1	4%
Totals		28	100%

Note: Card 9 read, "Pythagorean Theorem". Card 16 read, "Distance between two points in the Cartesian Coordinate Plane".

Relationship of MKT Geometry and Types of CSA Closed Sort Connections

To further investigate research question 1, bivariate analysis using Pearson correlations were calculated between each CSA closed sort theme and DTAMS scores. No statistically significant correlations were found between DTAMS scores and extracted themes from CSA closed sort pairings (6, 11), (4, 15), and (15, 17). However, there were statistically significant correlations between DTAMS scores and extracted themes from CSA closed sort pairings (2, 4), and (9, 16). There was a statistically significant positive moderate correlation (Visual Statistics Studio, 2006) between the extracted theme “graphing possibilities” for the closed sort pair (2, 4) and DTAMS score ($r=.510, p<.05, n=28$). There was a statistically significant negative moderate correlation (Visual Statistics Studio, 2006) between the extracted theme “none” for the closed sort pair (2, 4) and DTAMS score ($r=-.499, p<.05, n=28$). There was a statistically significant negative moderate correlation between the extracted theme “given triangle” for the closed sort pair (9, 16) and DTAMS score ($r=-.510, p<.05, n=28$).

Relationship of MKT Geometry and Teachers’ Coursework

The analysis of research question 2 involved placing participants into distinct non-overlapping groups based on their coursework. Group A ($n=6$) consisted of participants who had completed all mathematics content and methods courses. Group B ($n=14$) consisted of participants who had completed all mathematics content courses but had not taken mathematics methods courses. Group C ($n=8$) consisted of those participants who had completed all but two mathematics content courses and had not taken methods courses.

To assess the relationship between prospective middle grades teachers’ methods coursework and their MKT geometry, a univariate analysis was conducted using a linear regression model. The participants in Groups A and B were utilized for the analysis because participants in Group A had completed all required methods courses while those in Group B had not. Participants in Groups A and B had completed all mathematics content courses. The number of participants in this regression was 20. The analysis was conducted with methods coursework as the independent variable and DTAMS score as the dependent variable. Table 5.9 reveals the linear regression estimates of the effects of methods coursework on mathematics knowledge for teaching geometry.

Table 5.9. *Linear Regression Estimates of the Effects of Methods Coursework on MKT Geometry (n=20)*
 Dependent Variable: DTAMS Score

Variable	B	SE B	β	Sig.
Constant	29.071	1.372		<.001
Methods Coursework	3.262	2.505	.293	.209

Note. R squared =.086 (Adjusted R squared =.035). B indicates unstandardized regression coefficient. β indicates standardized regression coefficient.

Fitting the linear regression yields $Y=29.071+3.262X$. The intercept which is equal to 29.071 is the mean DTAMS score for participants in Group B, i.e., those who have not taken any methods courses. The mean DTAMS score for participants in group A, i.e. those who had taken methods courses was 3.262 points higher than those participants in Group B. There was no statistically significant effect of mathematics methods coursework on DTAMS performance.

To assess the relationship of prospective middle grades teachers' mathematics content coursework and their MKT geometry, a univariate analysis was conducted using a linear regression model. The participants in Groups B and C were utilized for this analysis because participants in Group B had completed all required mathematics courses while those in Group C had not. Participants in Groups B and C had not completed methods courses. The number of participants in this regression was 22. The analysis was conducted with mathematics content coursework as the independent variable and DTAMS score as the dependent variable. Table 5.10 illustrates the linear regression estimates of the effects of mathematics content coursework on MKT geometry.

Table 5.10. *Linear Regression of the Effects of Content Coursework on MKT Geometry (n=22)*

Dependent Variable: DTAMS Score

Variable	B	SE B	β	Sig.
Constant	22.125	1.620		<.001
Mathematics Content Coursework	6.946	2.031	.608	.003*

Note. R squared = .369 (Adjusted R squared = .338). B indicates unstandardized regression coefficient. β indicates standardized regression coefficient.

* $p < .05$

There was a statistically significant effect ($p = .003$) of mathematics content coursework on MKT geometry. The mean DTAMS score for participants in Group B (those who had taken all mathematics content courses) was 6.946 points higher than those participants in Group C. Effect sizes were medium (Huck, 2004). Therefore, adjusted R squared = .338 meant that 33.8% of the variability in DTAMS scores can be explained or accounted for by mathematics content coursework.

Discussion

Data analysis revealed statistically significant high positive correlations between MCE derivational connections with DTAMS scores, suggesting participants who had more developed MKT geometry made more derivational connections. The mathematics knowledge for teaching geometry involves being able to ask and answer the “how” and “why” questions behind mathematical ideas, concepts, and procedures. It seems reasonable that if participants are able to make more derivational connections, (they are able to use mathematical knowledge of one concept to build upon and explain the “how” and “why” other concepts), then they would have more developed MKT, and thus a higher DTAMS score. Data analysis revealed statistically significant high positive correlations between MCE procedural connections with DTAMS scores, suggesting participants who had more developed MKT geometry made more procedural connections. This result aligns well with the previous result linking more derivational connections with higher DTAMS scores. In many cases, participants’ derivational connections were built upon carrying out, explaining, and justifying a procedure. Thus, a

procedural connection could be thought of as a possible building block for making derivational connections.

Data analysis revealed statistically significant moderate correlations between CSA curricular connections and DTAMS scores, suggesting that more curricular connections would yield higher DTAMS scores. One of the fundamental components of the MKT framework is pedagogical content knowledge which is comprised of 1) knowledge of content and students, 2) knowledge of content and teaching, and 3) knowledge of the curriculum. Participants who made more curricular connections tended to sort the cards from the perspective of a middle school teacher. These participants tended to sort the cards by applying their knowledge of mathematics and teaching, knowledge of mathematics and middle grades students, and knowledge of middle grades mathematics curriculum. Thus, it seems reasonable that participants who made more curricular connections during the open card sort would have higher DTAMS scores.

Data analysis revealed statistically significant correlations between DTAMS scores and CSA closed sort pairings (2, 4) and (9, 16). For card sort pairing (2, 4) there was a statistically significant negative moderate correlation between the extracted theme “none” and DTAMS scores. Those participants who believed there was no connection or relation between card 2 and 4 tended to have lower DTAMS scores. A large number of participants whose responses fell within the “none” theme had difficulties making the algebraic/geometric connections during the MCE problem solving task. Thus, given the statistically significant positive high correlation between MCE and DTAMS scores, it seems reasonable that these participants would have lower DTAMS scores. For card sort pairing (2, 4) there was a statistically significant positive moderate correlation between the extracted theme “graphing possibilities” and DTAMS scores. The “graphing possibilities” theme represents participants’ attempts to relate the two cards by thinking about how to solve the problem on card 2 for finding the maximum possible area of a rectangle with a fixed perimeter. Participants whose responses fell within this theme tended to be those prospective middle grades teachers who exhibited such problem solving attributes as persistence, reflection, and sense making during the MCE problem solving task. Given the statistically significant positive high correlations between MCE and DTAMS scores, along with the observation of these participants exhibiting attributes

of successful problem solvers, it seems reasonable that these participants would tend to have higher DTAMS scores.

Data analysis revealed a statistically negative moderate correlation between the extracted theme “given triangle” and DTAMS score. For closed card sort pairing (9, 16), all but one participant indicated the two cards were connected which is not surprising given the Pythagorean Theorem is arguably the most popular and remembered mathematical statement from high school geometry. The Pythagorean Theorem is often remembered as “ a squared plus b squared equals c squared”, and when prompted participants usually recalled a , b , and c represent the lengths of the legs and hypotenuse, respectively, of a right triangle. More than half the participants’ responses for relating the two cards fell under the “given triangle” theme. Given a right triangle in the Cartesian coordinate plane, the Pythagorean Theorem could be applied to find the distance between the two endpoints of the hypotenuse. The previous description is a typical problem that prospective middle grades teachers have encountered on several occasions in middle school, high school and post secondary education. The participants’ “given triangle” explanation is dependent upon the recollection of a procedure that they have carried out on numerous occasions. Arguably, participants’ “given triangle” explanation was heavily reliant upon procedural rather than conceptual understanding of the relationship between the Pythagorean Theorem and the distance between any two points in the Cartesian coordinate plane. Well-developed MKT geometry requires both procedural and conceptual understanding of why and how particular mathematical ideas are related and/or connected to other mathematical ideas. The heavy focus of CCK and SCK items on the DTAMS instrument coupled with the lack of conceptual understanding of the mathematical ideas presented on cards 9 and 16 may explain why these participants tended to have lower DTAMS scores.

Mathematics methods coursework did not have a statistically significant impact on DTAMS performance. At first glance this finding may seem alarming. How could mathematics methods courses have no statistically significant effect on a measure for MKT geometry when a large component of the MKT framework is pedagogical content knowledge? The participants for this analysis had completed all mathematics content coursework. The mathematics content courses at the site where the study was conducted

has undergone considerable curricular changes. The participants who had taken MATH II were some of the first to partake in these changes were the author served as the primary instructor. MATH II was recently redesigned to incorporate several national recommendations (CBMS, 2001; NCTM 1989; NCTM, 2000; NCTM 2006) on what prospective middle grades teachers need to know and be able to do. Specific attention to the mathematical connection making and mathematical tasks of teaching as part of the MKT framework can be found in some of the objectives for MATH II: tracing and making connections on how geometric concepts are developed in the middle school and beyond (knowledge at the mathematical horizon), approaching geometry from an investigative constructivist stance by building small learning communities focused on mathematical communication, exploration, and problem solving as well as formulating, proving, or disproving conjectures. MATH II sought to help prospective middle grades teachers unpack, decompose, and make explicit the mathematical ideas and connections central to school mathematics curricula. As the instructor, the author used middle school curriculum materials as the foundation for the MATH II course and supplemented these materials with mathematically rigorous problems and activities. In an effort to build a fluid transition between prospective middle grades teachers' mathematics content and methods courses, mathematical problems were framed in the tasks of teaching. The tasks of teaching included: video analysis of middle school instruction on a particular mathematics topic, examining textbook treatments of mathematics problems, analyzing routine and non-routine student solutions, creating opportunities to analyze errors, and evaluating alternative methods or representations. The efforts made to build a fluid transition between content and methods courses by infusing pedagogy within a content course may have contributed to the strengthen of prospective middle grades teachers' pedagogical content knowledge before entering their mathematics methods courses, which in turn, may explain why mathematics methods coursework had no statistically significant impact on DTAMS scores. Another plausible reason for no statistically significant effect of methods coursework on prospective middle grades teachers' MKT geometry resides in the composition of items on the DTAMS instrument. The items on the DTAMS were more heavily weighted toward subject matter knowledge (75%) than pedagogical content knowledge (25%).

Mathematics content coursework had a statistically significant impact on prospective middle grades teachers' MKT geometry. The group that had not completed all mathematics content coursework still needed to take MATH I (finite mathematics course) and MATH II (geometry for prospective middle grades teachers). Given the DTAMS assessment was focused on measuring MKT within the domain of geometry and measurement, the items on the DTAMS were more heavily weighted toward subject matter knowledge, along with the redesign of MATH II (as described in the previous section), a reasonable prediction would be scores on the DTAMS would increase after successfully completing MATH II.

Conclusions and Implications

This sequential exploratory mixed methods study described the types of connections prospective middle grades teachers make when engaged in tasks meant to probe mathematical connections and its relationship to MKT geometry. The statistically significant relationships discovered between MCE procedural connections, MCE derivational connections, CSA curricular connections, and MKT geometry are particularly encouraging. Both mathematicians and mathematics educators at the site where the study was conducted currently use and draw upon NSF reform curriculum emphasizing a constructivist approach to learning and teaching mathematics in prospective middle grades teacher content and methods courses. The development, improvement, and refinement of these prospective teacher courses include a focus on how to make visible the connections to the kinds of mathematical thinking, judgment, and reasoning one has to do in teaching (Ball, 2008). However, the below average scores for mathematics knowledge for teaching geometry, as measured by the DTAMS assessment, support the findings of the MT21 report that future U.S. middle school teachers prepared through a middle grades program need stronger mathematical and pedagogical preparation (Schmidt et. al, 2007).

Implications for Content Courses

Findings from the MCE task suggest that participants had difficulties making derivational connections. Furthermore, those who had difficulty also tended to have lower DTAMS scores. The lack of derivational connection making supports national recommendations that “formulas for measuring area and volume should be developed in

such a way that a teacher could later derive a formula if it is not remembered (CBMS, 2001, p. 101). While the majority of participants were able to make algebraic/geometric, 2-D/3-D, and procedural connections, the fluency and ease with which they made these connections is questionable and in some cases, participants failed to make these connections at all. These findings have implications for prospective middle grades teachers content course preparation. As the NCTM (2009) points out,

Too often individuals perceive mathematics as a set of isolated facts and procedures. Through curricular and everyday experiences, students should recognize and use connections among mathematical ideas. Of great importance are the infinite connections between algebra and geometry. These two strands of mathematics are mutually reinforcing in terms of concept development and the results form the basis for much work in mathematics as well as in applications. Such connections build mathematical conceptual understanding on interrelationships across earlier work in what appear to be separate topics (p. 3)

Before coming to college, most prospective middle grades teachers have taken an Algebra I, Geometry, and Algebra II course. Algebra and geometry are typically viewed by prospective middle grades teachers as distinct fields will only perpetuate the difficulties prospective middle grades teacher have in making mathematical connections between strands. Prospective middle grades teachers' algebraic/geometric, 2-D/3-D, and derivational connection making may be strengthened by creating a two semester course sequence focused on the interrelationships between algebra and geometry. In this study, the fundamental misconceptions made by participants suggest that a two semester sequence specifically designed for prospective middle grades teachers should be developed through the MKT framework with particular focus on 1) making visible and explicit the connections between algebraic/geometric concepts, 2-D/3-D representations as well as how to derive such connections, 2) providing middle grades teachers more opportunity to explore the "equation to graph" and "graph to equation" relationships, 3) creating opportunities for prospective teachers to develop spatial visualization skills by working with and comparing components of 2-D and 3-D models.

Curricular connection making could be improved in such integrated courses by infusing more pedagogy within these content courses. By infusing more pedagogy in these integrated courses, mathematicians and mathematics educators will help prospective teachers make more explicit connections to the kinds of mathematical

thinking, judgment, and reasoning one has to do in carrying out the work of teaching mathematics in a K-12 setting (Ball, 2008).

Implications for Methods Courses

In methods courses, prospective middle grades teachers' focus on curriculum, lesson planning, instructional strategies, and assessment. However, prospective middle mathematics teachers are rarely afforded the opportunity in their methods courses to reflect upon the role mathematical connections play in curriculum development, lesson planning, instructional strategies, and assessments with the goal of improving their MKT. Findings from this study suggest that higher MCE scores are associated with higher DTAMS scores, resulting in more developed MKT. The MCE activities and CSA activities along with the MCE rubric construction and implementation could serve as a model for getting prospective middle grades teachers to think about various forms of summative and formative assessment that could be implemented during K-12 classroom instruction and lesson planning. By developing lesson plans that include a focus on mathematical connections as an explicit objective and thinking about how to assess such connection making, prospective middle grades teachers will begin to strengthen both their mathematical and pedagogical connection making as well as MKT. The MCE and CSA instruments and rubrics could also serve as a starting point for helping prospective middle grades teachers think about how to sequence their instruction, reflect upon the goals of instruction, reflect upon and state the connections expected to be made by their students, and develop appropriate assessments that help to measure the objectives set forth in their lesson plans.

Implications for Researchers

Although there are a few studies that have examined the mathematical connections of prospective middle grades teachers at the elementary and secondary level (Bartels, 1995; Donigan, 1999; Evitts, 2005; Hau, 1993; Roddy, 1992; Wood, 1993) there is little to no research on the mathematical connections made by prospective middle grades teachers at the middle grades level. The findings of this study resulted in the development of several mathematical connection types, a set in the context of problem solving and the other in context of card sorting. The mathematical connection categories that emerged from this study should be used as a starting point for evaluating the types of

connections practicing teachers are or are not making during instruction. The mathematical connection categories and rubrics developed in this study should be adapted and refined for use in other contexts. By understanding the types of mathematical connections inservice teachers make during instruction, mathematicians and mathematics educators will be better informed on the types of mathematical connections that need to be focused on during prospective teacher preparation for strengthen and developing MKT.

The design of this study provides a unique contribution to mixed methods research by providing an example of non-traditional sequential exploratory mixed methods design in which the quantitative results from the first phase of the study did not directly inform the development of instruments and procedures carried out in the second phase of the study. As Teddlie and Tashakkori (2009) posit,

You may want to select the best available MM research design for your study, but you realize that you may have to eventually generate your own. It is important to recognize that it is impossible to enumerate all possible MM designs. Therefore, you should look for the most appropriate or single best available research design, rather than the “perfect fit”. You may have to combine existing designs, or create new designs, for your study. (p. 163)

Furthermore, this study will contribute to the utility and fruitfulness of data that can be gleaned through mixed methods research in the context of mathematics education. As Hart, Smith, Swars, and Smith (2009) found only 29% of 701 mathematics education articles published between 1995 and 2005 utilized mixed methods for integrating qualitative and quantitative approaches to research.

Future Research

Prospective middle grades teachers must learn to access and unpack their mathematical knowledge in a connected, effective manner. Future studies should include research on the development of mathematical tasks for explicit connection making in order to make visible the connections to the kinds of mathematical thinking and judgment one has to do in teaching. How can we prepare prospective middle grades teachers to unpack and decompose mathematical ideas in a connected explicit manner? Future studies should include research of effective ways for constructing mathematics tasks that

help prospective middle grades teachers learn to create questions and questioning techniques that will strengthen their mathematics knowledge for teaching.

Future research studies should include the careful construction of assessments that not only explicitly address the relationship between mathematical concepts and topics, but value the making and learning of connections. A longitudinal study following a cohort of prospective middle grades teachers through their undergraduate studies on into their 1st and 2nd year of teaching could potentially reveal how mathematical connections and MKT are developed over time. Future studies should involve the development and implementation of an integrated mathematics content course sequence for prospective middle grades teachers. What would be the effects of an integrated mathematics content course sequence on prospective middle grades teachers' mathematical connection making and MKT? The current research study is only an exploratory preliminary study that highlights the mathematical connections prospective middle grades teachers make and its relationship to mathematical knowledge for teaching geometry. This study serves as a foundation for future studies examining the relationship between the mathematics knowledge needed for teaching and the mathematical connections prospective teachers make within other domains and contexts.

CHAPTER VI
DISCUSSIONS, CONCLUSIONS, and RECOMMENDATIONS

Discussion

The purpose of this sequential exploratory mixed methods study was to describe the types of connections prospective middle grades teachers make when engaged in tasks meant to probe mathematical connections and the relationship to their MKT geometry. One task focused on connection making in the context of solving mathematics problems, while the other focused on connection making in the context of card sorting.

In the MCE there were five types of mathematical connections: *procedural*, *algebraic/geometric*, *characteristic/property*, *derivational*, and *2-D/3-D*. The majority of participants were able to make the procedural, algebraic/geometric, and characteristic/property connections associated with MCE problem 1(a)-(c). However, there were a few interesting cases where participants had difficulty carrying out a procedure for correctly graphing the lines $y=3x$ and $x=5$ and finding the precise intersection point of the two lines. Most mathematicians and mathematics educators would consider this a routine problem that is commonly found throughout the middle school, high school, and college curriculum. While there were only a few cases where this occurred, it still gives rise for concern that prospective middle grades teachers need to develop more meaningful connections between algebraic and geometric representations of simple linear functions.

Nearly all participants (96%) were able to sketch and identify the bounded region as a triangle, and carry out a procedure for finding the area. However, half of the participants failed to make a derivational connection, indicating that the formula was something that they have memorized. This lack of derivational connection was also inherent in the majority of responses for finding the volume of a cone in MCE problem 1(e). Unlike the case of the triangle, more than half of the participants (57%) were unable carry out a procedure for finding the volume of a cone, indicating that they did not know a formula for the volume of a cone nor how to derive it. This finding is consistent with CBMS (2001) sentiments that “prospective [middle grades] teachers have some basic knowledge about shapes and about how to calculate areas and volumes of common shapes, but many will not have explored the properties of these shapes or know why the

area and volume formulas are true” (p. 33). Battista (2007) in his work with middle school students found similar results applied to other 2-D and 3-D shapes. He found that “most students who correctly use the formulas for the area of a rectangle or volume of right rectangular prism in standard problem contexts, neither understand why the formulas work nor apply the formulas appropriately in nonstandard contexts” (p. 892).

In the MCE, participants were asked to revolve various 2-D shapes about an axis in the Cartesian coordinate plane. Such tasks required spatial visualization to make a 2-D/3-D connection. Spatial visualization involved “imagining the rotations of objects and their parts in 3-D space in a holistic as well as piece by piece fashion” (Olkun, 2003, p. 2). The majority of participants were able to identify the resulting 3-D shape as a cone in problem 1(d). However, these participants fell into two distinct groups—those who required a manipulative to visualize the revolution and those who did not. The use of a physical manipulative proved problematic when participants were asked to revolve the triangle about the y -axis. Participants who used a physical manipulative simulated a revolution about the leg of a triangle rather than by the y -axis, making it difficult to see the resulting 3-D shape as a “cylinder with cone removed”. This particular finding highlights the importance of both the position of a 2-D object in the coordinate plane and the axis of revolution for determining the resultant 3-D object. While the majority of participants were able to make the 2-D/3-D connection for identifying the resultant 3-D shape, many had difficulties articulating the relationship between the measurements of the 2-D shape to the measurements of the resultant 3-D shape.

There were five types of mathematical connections that emerged from an inductive analysis of participants’ responses to the open card sort. The five types of mathematical connections were as follows: *categorical*, *procedural*, *characteristic/property*, *derivation*, and *curricular*. As a group, the prospective middle grades teachers made more categorical and procedural connections and far fewer derivational and curricular connections. This finding is consistent with findings from the MCE activity where participants were able to make procedural connections, but in many cases were unable to make derivational connections. These results may be indicative of participants’ experiences learning mathematics through a traditional curriculum focused on instrumental rather than relational understanding of mathematics (Skemp, 1978).

Data analysis revealed participants who made more curricular connections tended to have higher MCE scores. Participants who made more curricular connections during the open card sort tended to provide correct solutions to MCE problems that exhibited elements of pedagogical content knowledge, a subcategory of MKT (Ball, 2006). These particular participants provided solutions that involved how to explain, model, or demonstrate a solution to an MCE problem to someone who did not understand. In most cases, these particular participants referenced how they would explain, model, or demonstrate their solution to a middle grades student or peer. Furthermore, many of these participants made reference to the appropriateness of a particular MCE problem for a middle grades student and how they might modify such a problem. The explanations and comments made by participants during the MCE interview demonstrated knowledge of mathematics and middle grades students, knowledge of mathematics and teaching, as well as knowledge of the middle grades curriculum. In each case, the participant was not explicitly prompted by the researcher to provide such explanations or comments but rather did so of their own accord. Thus, it is a reasonable predication that these participants would have made more curricular connections during the open card sort since they seemed to be viewing the activities from the perspective of what a middle grades teacher should know and be able to do.

However, less than 25% of the subsets created in the open sort were curricular. The majority of participants (78%) had not yet taken mathematics method courses so perhaps they did not think about creating subsets from the perspective of what a future middle school teacher should know and be able to do. However, it could be argued since mathematics methods courses did not have a statistically significant impact on the MCE they might not have had an impact on the card sort activity.

The relationship between MCE scores and the other types of connections that emerged from the open card sort are fairly random. The participants who had higher MCE scores made just as many categorical, characteristic/property, procedural, and derivational connections as participants who had lower MCE Scores. This “randomness” could be explained by the nature of the MCE and CSA activities. The structure of the MCE was such that it was necessary to make certain mathematical connections in order to solve each problem correctly. However, with the CSA activities participants could

make any type of connection or connections between cards. Thus, participants may have opted to make connections during the CSA that were more “surface level”. Making more categorical types of connections between cards was prevalent in many of the responses for the closed sort activity. Participants who had higher MCE scores tended to make just as many “surface level” connections during the closed card sort as participants who had lower MCE scores. However, participants who had lower MCE scores tended to make more “misconnections” or “none” (meaning no connection) during the closed card sort. For example, participants who gave the “yarn explanation” for closed sort pair 6 and 11 tended to have lower MCE scores. There were several cases where participants tried to relate the cards based on geometric representations they associated with the statement on the card. In many cases such associations resulted in a response of no connection or a misconnection. For instance, let us consider the card sort pairing of cards 4 and 15. For card 4, participants would describe the graph of the function $f(x)$ as a “U”-shape. For card 15, participants associated the equation for the area of a circle with the geometric representation of a circle by saying that the “curve” for card 15 was a circle. These participants then tried to relate the curves by stating that the “U” shape could represent half of a circle. Participants who provided an “invalid geometric” connection or “none” for card sort pairing of cards 4 and 15 also had lower MCE scores. In each of these cases, interview data revealed that participants who had difficulty making mathematically correct valid connections during the closed card sort also struggled to make connections that were needed to get through the MCE problems.

The DTAMS assessment served as a quantitative measure of prospective middle grades teachers’ MKT geometry. Data analysis revealed statistically significant positive high correlation between DTAMS scores and overall MCE scores. In particular, data analysis revealed statistically significant moderate to high positive correlations between MCE procedural connections and DTAMS scores, MCE derivational connections and DTAMS scores, and CSA curricular connections and DTAMS scores. MKT for geometry involves being able to ask and answer the “how” and “why” questions behind mathematical ideas, concepts, and procedures. It follows that if participants were able to make more derivational connections (they are able to use mathematical knowledge of one concept to build upon and explain the “how” and “why” of other concepts), then they

would have more developed MKT geometry, and thus a higher DTAMS score. The statistically significant relationship between MCE procedural connections with DTAMS scores may be directly linked to the statistically significant relationships between MCE derivational connections and DTAMS scores, given the nature of the MCE items. In many cases, participants' MCE derivational connections were built upon carrying out, explaining, and justifying of a procedure. Thus, a procedural connection could be thought of as a possible building block for making derivational connections. MKT geometry involves an understanding mathematical ideas and concepts in the context of how they would be taught. Participants who made CSA curricular connections provided explanations that demonstrated a knowledge of geometry and teaching, knowledge of geometry and students and knowledge of the curriculum. Thus, it seems reasonable that participants who had higher DTAMS scores were able to make more CSA curricular connections.

Well-developed MKT geometry requires both procedural and conceptual understanding of mathematical knowledge. Results of this study found the types of mathematical connections that prospective teachers made were more procedural than conceptual in nature. Thus, the below average scores for MKT geometry, as measured by the DTAMS, may be due to the lack of conceptual connection making exhibited in participants' responses to the MCE and CSA.

Conclusions and Implications

This sequential exploratory mixed methods study described the types of connections prospective middle grades teachers make when engaged in tasks meant to probe mathematical connections and the relationship to mathematics knowledge for teaching geometry. The statistically significant relationships discovered between MCE procedural connections, MCE derivational connections, CSA curricular connections, and mathematics knowledge for teaching geometry are particularly encouraging. Both mathematicians and mathematics educators at the site where the study was conducted currently use and draw upon NSF reform curriculum emphasizing a constructivist approach to learning and teaching mathematics in their prospective middle grades content and methods courses. The development, improvement, and refinement of these prospective teacher courses include a focus on how to make visible the connections to the

kinds of mathematical thinking, judgment, and reasoning one has to do in teaching (Ball, 2008). Overall, the results of this study suggest that some progress is being made towards improving prospective middle grades teachers making of mathematical connections.

Implications for Prospective Teacher Preparation

The results of this study have implications for prospective teacher preparation. The results of this study support the findings of the MT21 report that future U.S. middle school teachers prepared through a middle grades program need stronger mathematical and pedagogical preparation. Mathematics educators need to make the connections within mathematics and between mathematics and teaching more made explicit. The lack of algebraic/geometric connections by prospective teachers could be improved by creating an integrated algebra/geometry course sequence. Before coming to college, most prospective middle grades teachers have taken an Algebra I, Geometry, and Algebra II course. Algebra and geometry are typically viewed by prospective middle grades teachers as separate, distinct fields of study. Maintaining the study of algebra and geometry as two distinct courses will only perpetuate the difficulties prospective middle grades teachers have in making mathematical connections between strands. Prospective middle grades teachers' algebraic/geometric and 2-D/3-D connection making may be strengthened by creating a two semester course sequence focused on the interrelationships between algebra and geometry. In this study, the fundamental algebraic/geometric and 2-D/3-D misconceptions made by participants suggest that a two semester course sequence specifically designed for prospective middle grades teachers should include 1) making visible and explicit the connections between algebraic/geometric concepts and 2-D/3-D representations, 2) providing prospective middle grades teachers more opportunities to explore the "equation to graph" and "graph to equation" relationships, and 3) creating opportunities for prospective middle grades teachers to develop spatial visualization skills by working with and comparing components of 2-D and 3-D models, visualizing movements of objects in space, and matching corresponding parts of images and pre-images resulting from revolutions or rotations of 2-D and 3-D objects.

The results of this study have implications for K-12 and prospective middle grades teachers' methods preparation. In methods courses, prospective middle grades mathematics teachers' focus on lesson planning, instructional strategies, and assessment.

However, prospective middle grades mathematics teachers are rarely afforded the opportunity in their methods courses to reflect on the role mathematical connections play in lesson planning, instructional strategies, and assessments. The MCE and CSA activities along with the MCE rubric construction and implementation could serve as a model for both formative and summative assessment techniques for mathematical connection making that could be implemented during K-12 classroom instruction and lesson planning. By constructing such rubrics, prospective teachers will have more opportunities to reflect on the role and importance mathematical connections plays in carrying out the work of teaching. Findings from this study suggest that higher MCE scores are associated with higher DTAMS scores, resulting in more developed MKT. Using the MCE and CSA activities as a guide, prospective middle grades teachers could learn to develop lesson plans which focus on mathematical connections as an explicit objective. The development of the instruments used in this study provide a model for how prospective middle grades teachers could assess such connection making in their K-12 classrooms, thus helping prospective middle grades teachers strengthen both their mathematical and pedagogical connection making as well as MKT. The MCE and CSA instruments and rubrics could also serve as a starting point for helping prospective middle grades teachers think about how to sequence their instruction, reflect upon the goals of instruction, reflect upon and state the connections expected to be made by their future students, and develop appropriate assessments that help to measure the objectives set forth in their lesson plans.

Implications for Researchers

Although there are a few studies that have examined the mathematical connections of prospective middle grades teachers at the elementary and secondary level (Bartels, 1995; Donigan, 1999; Evitts, 2005; Hau, 1993; Roddy, 1992; Wood, 1993) there is little to no research on the mathematical connections made by prospective middle grades teachers at the middle grades level. The findings of this study resulted in the development of several mathematical connection types, a set in the context of problem solving and the other in context of card sorting. The mathematical connection categories that emerged from this study should be used as a starting point for evaluating the types of connections practicing teachers are or are not making during instruction. The

mathematical connection categories and rubrics developed in this study should be adapted and refined for use in other contexts. By understanding the types of mathematical connections inservice teachers make during instruction, mathematicians and mathematics educators will be better informed on the types of mathematical connections that need to be focused on during prospective teacher preparation for strengthen and developing MKT.

What can we do as mathematics educators to bring derivational and curricular connections to the forefront of prospective middle grades teachers thinking? How might card sorting techniques be adapted in prospective teacher courses to facilitate enhancing prospective middle grades teachers' mathematics knowledge for teaching? The findings of this study are particularly useful to mathematics educators, curriculum developers, and researchers seeking further understanding behind effective and ineffective teacher preparation. This study will aid those wishing to construct mathematics tasks for explicit connection making with the intent to strengthen prospective teachers' conceptual understanding of underlying mathematical concepts and mathematics knowledge for teaching.

The design of this study provides a unique contribution to mixed methods research by providing an example of non-traditional sequential exploratory mixed methods design in which the quantitative results from the first phase of the study did not directly inform the development of instruments and procedures carried out in the second phase of the study. As Teddlie and Tashakkori (2009) posit,

You may want to select the best available MM research design for your study, but you realize that you may have to eventually generate your own. It is important to recognize that it is impossible to enumerate all possible MM designs. Therefore, you should look for the most appropriate or single best available research design, rather than the "perfect fit". You may have to combine existing designs, or create new designs, for your study. (p. 163)

This study will contribute to the utility and fruitfulness of data that can be gleaned through mixed methods research in the context of mathematics education. As Hart, Smith, Swars, and Smith (2009) found, only 29% of 701 mathematics education articles published between 1995 and 2005 utilized mixed methods for integrating qualitative and quantitative approaches to research.

Recommendations

Prospective middle grades teachers must learn to access and unpack their mathematical knowledge in a connected, effective manner. Future studies should include research on the development of mathematical tasks for explicit connection making in order to make visible the connections to the kinds of mathematical thinking and judgment one has to do in teaching. How can we prepare prospective middle grades teachers to unpack and decompose mathematical ideas in a connected explicit manner? Future studies should include research of effective ways for constructing mathematical tasks that help prospective middle grades teachers learn to create questions and questioning techniques that will strengthen their MKT. Future research studies should also include the careful construction of assessments that not only explicitly address the relationship between mathematical concepts and topics, but value the making and learning of connections.

Future studies should involve the development and implementation of an integrated mathematics content course sequence for prospective middle grades teachers. What would be the effects of an integrated mathematics content course sequence on prospective middle grades teachers' mathematical connection making and MKT? The current research study is only an exploratory preliminary study that highlights the mathematical connections prospective middle grades teachers make and its relationship to MKT geometry. This study serves as a foundation for future studies examining the relationship between the mathematics knowledge needed for teaching and the mathematical connections prospective teachers make within other domains and contexts.

This study focused its attention on prospective middle grades teachers. Future studies should include other populations such as inservice middle school teachers. Are there particular courses or aspects of teacher preparation that explicitly help prospective middle grades teachers develop and build mathematical connections? A longitudinal study following a cohort of prospective middle grades teachers through their undergraduate studies on into their first and second year of teaching could potentially reveal how connections and MKT are developed over time. Replication and longitudinal studies would also help to refine data collections instruments and protocols that could be adapted for other studies. Future studies should include larger populations so that more

sophisticated statistical analysis can be performed for strengthen the reliability and validity of the instruments.

The card sorting techniques used in this study should be adapted and integrated into prospective teacher courses. Future research studies should include making comparisons between prospective middle grades teachers open and closed card sorts to “expert” sorts, such as those sorted by mathematicians, mathematics educators, and inservice teachers. These card sort comparison studies could provide insight into the “gap” between expert and novice mathematical connection making and offer recommendations on how to bridge this “gap”. Finally, the types of connections that were identified or emerged from this study offer a beginning point from where future studies could refine or expand on the types of connections prospective teachers make in other contexts.

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APPENDIX A

Description of Types of Knowledge Measured by DTAMS (CRMSTD, 2007)

Type I: *Memorized Knowledge*

This mathematics knowledge is rote learned and employs memorization. It includes memorized knowledge of definitions, procedures, or rules. Teachers with this knowledge can rote perform skills, apply rules, and give definitions.

Type II: *Conceptual Understanding*

This mathematics knowledge is conceptual in nature. It includes a deep understanding of mathematical concepts, procedures, laws, principles, and rules. It is knowledge of connections and relationships among concepts. It is often associated with meaning. Teachers with this knowledge can give examples/non-examples and identify properties/characteristics of mathematical concepts. They can compare and contrast and represent mathematical concepts and generalizations in multiple ways. They can explain and create mathematical procedures and represent them in multiple ways.

Type III: *Problem Solving & Reasoning*

This mathematics knowledge is higher order in nature. It includes applying knowledge to solve problems and real-world applications. Teachers with this knowledge can reason informally and formally, conjecture, validate, analyze, and justify. They can use deductive, inductive, proportional, and spatial reasoning to solve problems.

Type IV: *Pedagogical Content Knowledge*

This mathematics knowledge is unique to teaching mathematics. It represents the mathematics knowledge that teachers use in the act of teaching. It includes knowledge of the most regularly taught topics in mathematics, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations. Teachers with this knowledge can identify student misconceptions about mathematics and provide strategies to correct them. Teachers can derive activities that promote understanding, reasoning, and proficiency. They can provide examples, analogies, models, or representations to help students understand mathematical concepts or procedures.

APPENDIX B

Mathematical Connections Evaluation (Demographics)

1. Name: _____

2. What is your gender?
 Female
 Male

3. What is your major? Please mark one.
 Elementary Education
 Middle School Education
 Other (please specify): _____

4. What is your current grade level?
 Freshman
 Sophomore
 First Semester Junior
 Second Semester Junior
 First Semester Senior
 Second (or more) Semester Senior
 Other (please specify): _____

5. If you are a middle school education major, what are your area(s) of specialization?
 English & Communication
 Mathematics
 Science
 Social Studies
 Other (please specify) _____
 I am not a middle school education major

6. I have taken the following mathematics course (mark all that apply):
 MA 109 **College Algebra**
 MA 113 **Calculus I**
 MA 123 **Elementary Calculus & Its Applications**
 MA 162 **Finite Mathematics & Its Applications**
 MA 201 **Mathematics for Elementary Teachers I**
 MA 202 **Mathematics for Elementary Teachers II**
 MA 241 **Geometry for Middle School Teachers**
 MA 310 **Mathematical Problem Solving for Teachers**

Other (please specify): _____

7. I am currently enrolled in the following mathematics courses (mark all that apply):

MA 109 *College Algebra*

MA 113 *Calculus I*

MA 123 *Elementary Calculus & Its Applications*

MA 162 *Finite Mathematics & Its Applications*

MA 201 *Mathematics for Elementary Teachers I*

MA 202 *Mathematics for Elementary Teachers II*

MA 241 *Geometry for Middle School Teachers*

MA 310 *Mathematical Problem Solving for Teachers*

Other (please specify): _____

8. I **HAVE TAKEN** the following education courses (mark all that apply):

EDP 202 *Human Development and Learning*

EDP 203 *Teaching Exceptional Learners in Regular Classrooms*

EPE 301 *Education in American Culture*

EDC 317 *Introduction to Instructional Media*

EDC 329 *Teaching Reading and Language Arts*

EDC 341 *Middle School Curriculum & Instruction*

EDC 330 *Designing a Reading & Language Arts Program for the Middle School*

EDC 343 *The Early Adolescent Learner: Practicum*

EDC 345 *Teaching Mathematics in the Middle School*

EDC 349 *Student Teaching in the Middle School*

Other (please specify) _____

9. I am **CURRENTLY ENROLLED** in the following education courses (mark all that apply):

EDP 202 *Human Development and Learning*

EDP 203 *Teaching Exceptional Learners in Regular Classrooms*

EPE 301 *Education in American Culture*

EDC 317 *Introduction to Instructional Media*

EDC 329 *Teaching Reading and Language Arts*

EDC 341 *Middle School Curriculum & Instruction*

EDC 330 *Designing a Reading & Language Arts Program for the Middle School*

EDC 343 *The Early Adolescent Learner: Practicum*

EDC 345 *Teaching Mathematics in the Middle School*

EDC 349 *Student Teaching in the Middle School*

Other (please specify) _____

Course Descriptions

MA 113 Calculus I

A course in one-variable calculus, including topics from analytic geometry. Derivatives and integrals of elementary functions (including the trigonometric functions) with applications.

MA 123 Elementary Calculus and Its Applications

An introduction to differential and integral calculus, with applications to business and the biological and physical sciences.

MA 162 Finite Mathematics and Its Applications

Finite mathematics with applications to business, biology, and the social sciences. Linear functions and inequalities, matrix algebra, linear programming, probability. Emphasis on setting up mathematical models from stated problems.

MA 201 Mathematics for Elementary Teachers I

Sets, numbers and operations, problem solving and number theory.

MA 202 Mathematics for Elementary Teachers II

Algebraic reasoning, introduction to statistics and probability, geometry, and measurement.

MA 241 Geometry for Middle School Teachers

A course in plane and solid geometry designed to give middle school mathematics teachers the knowledge needed to teach a beginning geometry course.

MA 310 Mathematical Problem Solving for Teachers

Heuristics of problem solving. Practice in solving problems from algebra, number theory, geometry, calculus, combinatorics and other areas.

EDP 202 Human Development and Learning

Theories and concepts of human development, learning, and motivation are presented and applied to interpreting and explaining human behavior and interaction in relation to teaching across the developmental span from early childhood to adulthood. A field experience in a school or other educational agency is a required and basic part of course.

EDP 203 Teaching Exceptional Learners in Regular Classrooms

An introduction to the characteristics and instructional needs of exceptional learners is presented with an overview of principles, procedures, methods, and materials for adapting educational programs to accommodate the integration of exceptional children in

regular classrooms, when appropriate. A field experience in a school or other education agency is a required and basic part of the course.

EPE 301 Education in American Culture

Critical examination of contending views, past and present, regarding the nature and role of education institutions in American society as well as proposed purposes and policies for schools and other educational agencies.

EDC 317 Introduction to Instructional Media

An introductory instructional media experience including basic production and utilization techniques for media materials and operation of commonly used educational media equipment. Topics include graphic preservation, transparency production, audio materials, motion pictures, 35mm photographic techniques, and an introduction to videotape television.

EDC 329 Teaching Reading and Language Arts

Development of competencies for teaching of reading and other language arts to groups. Course will also provide an overview of the nature of reading and language arts development from grade K-8.

EDC 330 Designing a Reading and Language Arts Program for the Middle School

A study of materials and techniques useful in the diagnostic teaching of reading and other language arts with students in grades 5-8. The course will emphasize materials, techniques, and procedures with diagnose individual strengths and weaknesses, and prescriptive instruction based upon the diagnosis.

EDC 341 Middle School Curriculum and Instruction

This course is designed to acquaint teachers of early adolescents with the rationale behind the middle school concept, and, in particular, the techniques of teaching as an individual and as a member of an interdisciplinary team. The development of generic teaching skills such as planning, implementing, managing, and evaluating learning programs is emphasized.

EDC 343 The Early Adolescent Learner: Practicum

This course is designed to extend and apply knowledge of the social, emotional, intellectual, and physical characteristics of the early adolescent learning through observation and interaction in school settings. The course format will include a weekly seminar and a supervised field placement in a middle school setting.

EDC 345 Teaching Mathematics in the Middle School

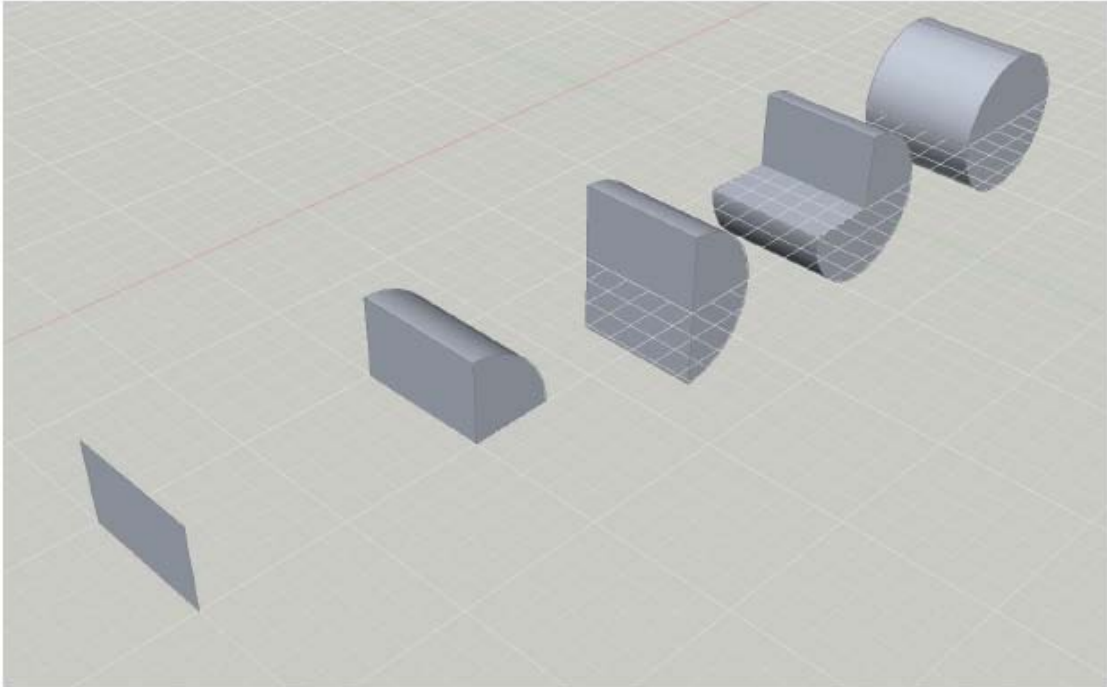
A study of theoretical models and methodological strategies for teaching arithmetic, informal geometry, and introductory algebra at the middle school level. The course will include a critical analysis of a variety of objectives, instructional materials and strategies and evaluation techniques. Consideration will be given to addressing the individual needs of a diverse student population.

EDC 349 Student Teaching in the Middle School

This course is designed to give the student experience teaching within a middle school setting. Weekly seminars will be held to discuss issues relevant to the student teacher's experience.

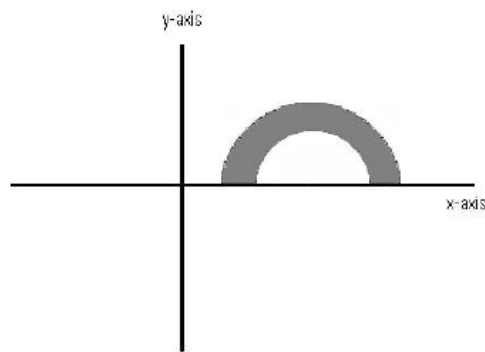
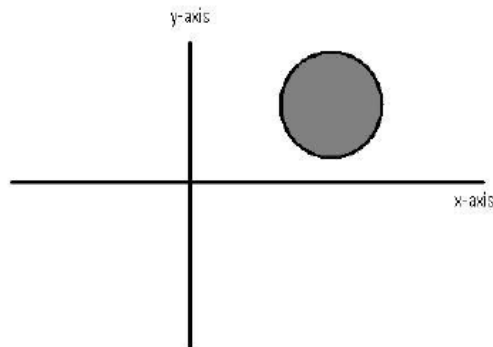
Mathematical Connections Evaluation (Demonstration)

A mechanical engineer is evaluating new 3-D modeling software. As a learning exercise, the engineer decides to model a simple peg in the shape of a cylinder. To generate the cylindrical peg, the software requires the user to sketch a cross section of the object on a 2-D plane and then revolve the cross section about an axis, sweeping out the 3-D object.



Mathematical Connections Evaluation (Interview Problems)

1.
 - a. In the Cartesian coordinate plane, sketch the region bounded by the x -axis, the line $y=3x$, and the line $x=5$.
 - b. What is the shape of the bounded region?
 - c. What is the area of the bounded region?
 - d. Generate a three dimensional object by revolving the bounded region about the y -axis.
 - i. What three-dimensional shape do you get?
 - ii. Sketch this three-dimensional shape.
 - e. What is the volume of the three-dimensional shape you found in part (c)?
2. What if you revolved the region in part 1(b) about the y -axis? Describe the resulting three-dimensional shape.
3. What is the volume of the three-dimensional shape you just generated? Explain.
4. In each of the problems below a two-dimensional object is revolved about the x -axis to sweep out a three-dimensional object. In each case, describe the three-dimensional object that is generated.



APPENDIX C

Mathematical Connections Evaluation Interview Protocol

1. Interviewee will review informed consent for participation (5-10 minutes)
 - a. The researcher will spend a few minutes asking participant about how their semester is going. The researcher will spend a few minutes discussing the purpose of the study by focusing on how the participant can help the researcher strengthen her own area of weakness in understanding how and what prospective teachers are thinking when they are engaged in mathematical tasks. The purpose of these discussions is to help make the participants more comfortable and open about telling the researcher what they are thinking as they are working on mathematical problems.
 - b. Participants will be given a copy of the consent form and reminded that they can elect to withdraw from the study at anytime.

2. Participants will complete Mathematical Connections Evaluation (MCE) (45-60 minutes)
 - a. Participant will fill out demographic information (5 minutes).
 - b. Researcher will begin audio and video recording.
 - c. Researcher will demonstrate what it means to “revolve” an object on a two-dimensional plane to sweep out a three-dimensional object (5 minutes).
 - d. Participant will be asked to complete problem 1 (a)-(c) independent of the researcher.
 - i. Researcher will sit at another table until participant has completed 1 (a)-(c). Participant will let the researcher know when they have completed 1(a)-(c).
 - ii. Researcher will engage in interview using the following probes:
 1. What were your first thoughts after reading the problem?
 2. When you began to solve the problem, what were you thinking?
 3. Explain what you are doing from this step to this step.
 4. How did you know to....?
 5. Are there other things you tried or thought about before your final chosen method?
 6. Could you have chosen this as your base and this as your height (point to sketch)?

7. Could you have used the hypotenuse of the right triangle as the base of the triangle?
 8. What did you use from your mathematical/experience toolbox to help you with this problem?
- e. Participant will be asked to complete problem 1 (d)-(e) independent of the researcher.
- i. Researcher will sit at another table until participant has completed 1 (d)-(e).
Participant will let researcher know when they have completed 1 (d)-(e).
 - ii. Researcher will engage in interview using the following probes:
 1. What were your first thoughts after reading the problem?
 2. When you began to solve the problem, what were you thinking?
 3. What did you use from your mathematical/experience toolbox to help you with this problem?
 4. Explain what you are doing from this step to this step.
 5. How did you know to....?
 6. Can you talk a little more about what you were thinking....?
 7. What mental images or visualization do you get when thinking about.....?
 - a. If participant has problems visualizing the three-dimensional object generated, researcher will use a manipulative.
8. What did you use from your mathematical and/or experience toolbox to help you with the volume of a cone?
- a. If participant does not remember the formula for the volume of a cone, the researcher will ask the following:
 - i. Can you think of a shape that is close to a cone for which you know how to calculate the volume?

- f. Researcher will now ask about problems 2 and 3, using a talk-aloud strategy. Participants will engage in problems 2 and 3 alongside the researcher. Researcher will utilize the following probes to tease out participants' "in the moment" thinking.
 1. What were your first thoughts after reading the problem?
 2. What are you thinking?
 3. What are you using your mathematical/experience toolbox to help you with this problem?
 4. What mental images or visualizations do you get when thinking about this problem?
 5. Does this problem remind you of anything you've seen before?

 - g. Researcher will now ask about problem 4, using a talk-aloud strategy. Participants will engage in problem 4 alongside the researcher. Researcher will utilize the following probes to tease out participants' "in the moment" thinking.
 1. Can you describe the mental images or visualization you get when thinking about the three-dimensional object that is generated?
 2. Does the three-dimensional object resemble something you've seen before?
 3. Would you expect the three-dimensional objects in each case to be the same?
3. The researcher will ask the participant to reflect back on the entire mathematical connection evaluation in order to answer the following question, "Why do you think I chose this problem to ask a prospective middle grades teacher"?
 - a. If participant gives a response that alludes to a particular concept, approach or idea that is "important to know", the researcher will probe the participants' thinking on why they feel it is "important".

 4. Researcher will offer participants a 5 minute break before beginning card sort activity.
 - a. Researcher will place all participant artifacts in participant folder.
 - b. Researcher will clear off table for card sort activity.
 - c. Researcher will put in new video tape for card sort activity.

APPENDIX D

Alignment of MCE to National Recommendations

Question	CBMS (2001) MET	NCTM (2000) PSSM	NCTM (2006) Curriculum Focal Points
(a) In the Cartesian coordinate plane, sketch and shade the region bounded by the x-axis, the line $y=3x$, and the line $x=5$.	-Connect geometry to other mathematical topics	-Use coordinate geometry to represent and examine properties of geometric shapes	-Students use linear functions, linear equations, and systems of linear equations to represent, analyze, and solve a variety of problems
(b) What is the shape of the bounded region?	-Identify common two-and three-dimensional shapes and list their basic characteristics and properties	-Analyze characteristics and properties of two – dimensional shapes	-Identify, name, and describe a variety of shapes, such as squares, triangles, circles, rectangles, etc.
(c) What is the area of the shaded region?	-Understand, derive, and use measurement techniques and formulas	-Use geometric models to represent and explain numerical and algebraic relationships	-Students extend their understanding of properties of two-dimensional shapes as they find areas of polygons.
(d) Generate a three dimensional object by revolving the bounded region about the x-axis. What three dimensional shape do you get? Sketch this three dimensional shape.	-Demonstrate ability to visualize and solve problems involving two-and three-dimensional objects	-Use two-dimensional representations of three dimensional objects to visualize and solve problems such as those involving surface area and volume.	-Describing three-dimensional shapes and analyzing their properties, including volume and surface area -Students related two-dimensional shapes to three – dimensional shapes and analyze properties
(e) What is the volume of the three dimensional object you found in part (d)?	-Understand, derive, and use measurement techniques and formulas	-Use two-dimensional representations of three dimensional objects to visualize and solve problems such as those involving surface area and volume.	-Problems involve areas and volumes calling on students to find areas or volumes from lengths...these problems extend student's work in grade 5 on area and volume and provide a context for applying new work with equations -Developing and understanding of using formulas to determine surface areas and volumes of three-dimensional shapes
(f) What if you revolved the shape in part (b) about the y-axis? Describe the resulting three dimensional shape.	-Demonstrate ability to visualize and solve problems involving two-and three-dimensional objects	-Use two-dimensional representations of three dimensional objects to visualize and solve problems such as those involving surface area and volume.	Describing three-dimensional shapes and analyzing their properties, including volume and surface area -Students related two-dimensional shapes to three – dimensional shapes and analyze properties
(e) What is the volume of the three dimensional shape generated in part (f)?	-Strategies for decomposing and recomposing figures	-Use geometric models to represent and explain numerical and algebraic relationships	-Analyze two-and three-dimensional space and figures by using distance and angle

APPENDIX E

Alignment of CSA to National Recommendations

Card	CBMS (2001) MET	NCTM (2000) PSSM	NCTM (2006) Curriculum Focal Points
Card 1: The ratios $\frac{10}{16}$ and $\frac{15}{24}$ are equivalent ratios.	p.27-30	N&O ¶2	p.18-19, 35-36
Card 2: A rectangle has a perimeter of 28 feet. Find the maximum possible area of the rectangle.	p. 32-34	Geo ¶4, Alg ¶1	p.19, 36-37
Card 3: Parallel lines	p.32-34	Geo ¶4	p.19-20, 37-39
Card 4: A function defined by $f(x) = x^2$. What kind of curve will it produce when graphed?	p.31-32	Alg ¶2, 4	p. 20, 36, 39-40
Card 5: Volume of a Cylinder	p. 32-34, 111	Geo ¶4	p.19, 36-37
Card 6: The circumference of a circle is given by $C = 2\pi r$ where r is the radius of the circle.	p.30-31	Alg ¶2, 4	p. 19-20, 37-38
Card 7: Similar Figures	p.32-34, 111	Geo ¶1	p.19-20, 31, 36-40
Card 8: Volume of a Rectangular Prism	p. 32-34, 111	Geo ¶4	p.19, 36-37
Card 9: Pythagorean Theorem	p.32-34, 111	Geo ¶1	p. 20, 37, 39
Card 10: Derivative of a function	p.118		p.20, 40
Card 11: The equation of a straight line through the origin is given by $y = mx$.	p.31-32	Geo ¶1	p.20, 36, 39
Card 12: Scale Factor	p.32-34, 111	Geo ¶1, 3	p. 19, 37-38
Card 13: Congruent Triangles	p.32-34	Geo ¶1, 3	p. 26, 32, 39
Card 14: The set of all points (x, y) in the Cartesian coordinate plane satisfying $x^2 + y^2 = r^2$, $r > 0$.	p. 118	Geo ¶2	p. 20, 36, 40
Card 15: The area A enclosed by a circle is given by $A = \pi r^2$ where r is the radius of the circle.	p.32-34, 111	Geo ¶4	p.19, 36-37
Card 16: Distance between two points in the Cartesian Coordinate Plane.	p.32-34	Geo ¶2	p. 20, 39
Card 17: The area of a triangle is given by the formula $A = \frac{1}{2}bh$ where b is the base and h is the height of the triangle.	p.32-34, 111	Geo ¶4	p.19, 36-37
Card 18: Surface Area	p. 32-34, 111	Geo ¶4	p.19, 36-37
Card 19: Angles	p. 32-34	Geo ¶1	p.20, 39
Card 20: Rectangle	p. 32-34	Geo ¶1	p. 36-37

APPENDIX F

Card Sort Activity Interview Protocol

Researcher: “I have a series of cards which contain mathematical ideas, concepts, terms, definitions and problems. I would like you to go through and read each of these cards. When you have finished reading each card, please hand them back to me”.

Card 1: The ratios $\frac{5}{8}$ and $\frac{10}{16}$ are equivalent ratios.

Card 2: A rectangle has a perimeter of 28 feet. Find the maximum possible area of the rectangle.

Card 3: Parallel Lines

Card 4: A function is defined by $f(x) = x^2$. What kind of curve will it produce when graphed?

Card 5: Volume of Cylinder

Card 6: The circumference of a circle is given by $C = 2\pi r$ where r is the radius of the circle.

Card 7: Similar Figures

Card 8: Volume of Rectangular Prism

Card 9: Pythagorean Theorem

Card 10: Derivative of a function

Card 11: The equation of a straight line through the origin is given by $y = mx$.

Card 12: Scale Factor

Card 13: Congruent Triangles

Card 14: The set of all points (x, y) in the Cartesian coordinate plane satisfying $x^2 + y^2 = r^2, r > 0$.

Card 15: The area of a circle is given by the formula $A = \pi r^2$ where r is the radius of the circle.

Card 16: Distance between two points in the Cartesian Coordinate Plane

Card 17: The area of a triangle is given by the formula $A = \frac{1}{2}bh$ where b is the base and h is the height of the triangle.

Card 18: Surface Area

Card 19: Angles

Card 20: Rectangle

(Hand cards over to participant.)

(Ask for participant's initial thoughts upon reading each card)

Researcher: “What were some of your first thoughts after reading card number ____?”

(When they have finished giving initial thoughts about each card, the researcher will lay the card on the table in columns by number on the card. The arrangement is four columns with five cards in each column.)

Researcher: I would like you to select a group of more than 1 card that you believe are related. Do not assume that any of these topics are connected or that they are all connected—just select your subsets as you see fit.

1. Participant will select a subset of cards he/she feels are connected.
2. **Researcher:** “How are these concepts or ideas connected? What were you thinking when you selected these cards?”
3. Participant will give an explanation.
4. **Researcher:** “Any other cards you would add to this group?”
5. **Researcher:** Please return the cards. Now select another subset of cards that you feel are related.
6. Repeat steps 1-5 for approximately 8 sorts (the average number of sorts from pilot study)
7. **Researcher:** “Can you make any more subsets?”

Researcher: “If you could make up your own cards, what kinds of cards would you make to help create additional subsets of related cards? Or what kinds of cards would you create to add to the subsets you already selected?”

Researcher: “Are there any cards here that you believe are connected to or related to what is on card 14? Are there any cards here that you believe are connected or related to what is on card 10”? (Card 10 and 14 were selected as these particular cards were less frequently selected during the pilot study)

1. If response is NO, then researcher will ask “What kind of cards would you create that would show a connection or relation to this particular card?”

Researcher: “I’m going to select a couple of cards and would like to know if you think these cards are connected or related in some way. Do not assume that the cards I select are related or connected. I just want to hear your thoughts.” (Researcher selects two cards, paired as follows: card 6 and 11; card 15 and 4; card 16 and 9; card 17 and 15; card 2 and 4. These particular pairs were chosen in consultation with expert mathematicians).

(Once participants have completed the entire card sort activity, the researcher will ask the following reflective questions below)

Researcher: “What did you think of the card sort activity?”

Researcher: “What are some advantages or disadvantages to doing a card sort activity?”

Researcher: “What do you think is the purpose of this particular activity?”

Researcher: “Why would I do an activity like this with a prospective middle grades teacher?”

APPENDIX G

Institutional Review Board Approval Letter



Office of Research Integrity

University Veterinarian, IRB, RDRC, IACUC
315 Kinkead Hall
Lexington, KY 40506-0057
859 257-9428
fax 859 257-8995

Initial Review

Approval Ends
January 23, 2009

Project Ends
May 31, 2008

IRB Number
07-0828-F4S

TO: Jennifer A Eli, M.S.
Education Curriculum & Instr
715 Patterson Office Tower
CAMPUS 0027 (859)396-8213

FROM: Chairperson/Vice Chairperson *NWT/PS*
Non-medical Institutional Review Board (IRB)

SUBJECT: Approval of Protocol Number 07-0828-F4S

DATE: February 8, 2008

On February 8, 2008, the Non-medical Institutional Review Board approved minor revisions requested at the convened meeting on January 25, 2008 for your protocol entitled:

*An Exploratory Investigation of Prospective Middle Grades Teachers'
Mathematical Connections during Problem Solving in Geometry*

Approval is effective from January 25, 2008 until January 23, 2009. This approval extends to any consent/assent document unless the IRB has waived the requirement for documentation of informed consent. If applicable, attached is the IRB approved consent/assent document(s) to be used when enrolling subjects. [Note, subjects can only be enrolled using consent/assent forms which have a valid "IRB Approval" stamp unless special waiver has been obtained from the IRB.] Prior to the end of this period, you will be sent a Continuation Review Report Form which must be completed and returned to the Office of Research Integrity so that the protocol can be reviewed and approved for the next period.

In implementing the research activities, you are responsible for complying with IRB decisions, conditions and requirements. The research procedures should be implemented as approved in the IRB protocol. It is the principal investigator's responsibility to ensure any changes planned for the research are submitted for review and approval by the IRB prior to implementation. Protocol changes made without prior IRB approval to eliminate apparent hazards to the subject(s) should be reported in writing immediately to the IRB. Furthermore, discontinuing a study or completion of a study is considered a change in the protocol's status and therefore the IRB should be promptly notified in writing.

For information describing investigator responsibilities after IRB approval has been obtained, download and read the document "PI Guidance to Responsibilities, Qualifications, Records and Documentation of Human Subjects Research" from the Office of Research Integrity's Guidance/Policy Documents web page [<http://www.research.uky.edu/ori/human/guidance.htm#PIresp>]. Additional information regarding IRB review, federal regulations, and institutional policies may be found through ORI's web site [<http://www.research.uky.edu/ori/>]. If you have questions, need additional information, or would like a paper copy of the above mentioned document, contact the Office of Research Integrity at (859) 257-9428.

Norman Van Tubingen, Ph.D./PS
Chairperson/Vice Chairperson

APPENDIX H

Researcher Script for Class Recruitment

Hello. My name is Jennifer Eli. I am a Mathematics Education Doctoral student in the Department of Curriculum and Instruction under the direction of Drs. Margaret Mohr and Xin Ma here at the University of Kentucky. I am conducting a research study exploring prospective middle grades teachers' mathematics knowledge for teaching with a focus on mathematical connection making during the problem solving process. You are being invited to participate in this research study because you are a prospective middle grades teacher.

By conducting this study, we hope to learn more about prospective middle grades teachers' mathematics knowledge for teaching and the mathematical connections made during the problem solving process in light of reform curricula. This research will contribute to the improvement of mathematics education courses at the University of Kentucky. .

(Distribute written consent forms)

If, after reviewing the consent form, you agree to participate, please sign and date the last page of the consent form. Please review the written consent form before the next class meeting. At the next class meeting an independent third party will collect the **ALL** consent forms (signed or unsigned). Your consent to participate in this research study will have no impact on your course grade. Your course instructor will **NOT** know the identity of students who did or did not consent to participate in this study.

Your consent to participate in this study is voluntary. You will receive a gift certificate to amazon.com for participating in the research study. .

If you have any questions concerning the research study, please do not hesitate to email me or call.

(Put contact information on the board)

Jennifer A. Eli, M.A.
918 Patterson Office Tower
Lexington, KY 40506-0027
jeli@ms.uky.edu
jennifer.eli@uky.edu
(859) 514-3121
(859) 396-8213

Thank you for your time and consideration.

APPENDIX I

MA 310 Student Consent to Participate in a Research Study

AN EXPLORATORY INVESTIGATION OF PROSPECTIVE MIDDLE GRADES TEACHERS' MATHEMATICAL CONNECTIONS DURING PROBLEM SOLVING IN GEOMETRY

WHY ARE YOU BEING INVITED TO TAKE PART IN THIS RESEARCH?

You are being invited to take part in a research study about prospective middle grades teachers' mathematics knowledge for teaching focusing on mathematical connection making during the problem solving process. You are being invited to take part in this research study because you are a prospective middle grades teacher. If you volunteer to take part in this study, you will be one of about 32 people at the University of Kentucky to do so.

WHO IS DOING THE STUDY?

The person in charge of this study is Jennifer A. Eli of University of Kentucky Department of Curriculum and Instruction. She is being guided in this research by Dr. Xin Ma and Dr. Margaret Mohr both of the University of Kentucky Department of Curriculum and Instruction. There may be other people on the research team assisting at different times during the study.

WHAT IS THE PURPOSE OF THIS STUDY?

The purpose of this study is to explore prospective middle grades teachers' mathematics knowledge for teaching geometry and the mathematical connections made during the problem solving process in light of reform curricula. By doing this study, we hope to learn more about prospective middle grades teachers' mathematics knowledge for teaching and the mathematical connections that evolve during problem solving. This research will contribute to the improvement of mathematics education courses at the University of Kentucky.

WHERE IS THE STUDY GOING TO TAKE PLACE AND HOW LONG WILL IT LAST?

The research procedures will be conducted at University of Kentucky. You will need to come to 918 Patterson Office Tower. As part of your MA 310 course you will be asked to complete a diagnostic mathematics assessment as well as engage in a two-hour interview session where you will be asked to complete a mathematical connections evaluation and card sort activity. Each of these activities is a required portion of your MA 310 course. You will not be asked to commit any additional time beyond what is required for satisfying the MA 310 course requirements.

WHAT WILL YOU BE ASKED TO DO?

As part of your MA 310 course requirements you will be asked to complete an in-class diagnostic mathematics assessment as well as engage in a two-hour interview session where you will be asked to complete a mathematical connections evaluation and card sort activity. If you agree to be in the study, the interview session the interview session will be audio-and/or video recorded. In addition, the researcher will be collecting observational data on your learning of mathematics throughout the semester. In class, prior to these interviews, you will be asked to complete a diagnostic assessment that should take no longer than one hour and fifteen minutes to complete. Although the mathematical connections evaluation, card sort activity and diagnostic mathematics assessment are required course activities, your consent to have the data from these activities to be used for research is strictly voluntary and will have no impact on your course grade. Your course instructor will not know if you did or did not consent to participate in this study. Again, you will not be asked to commit any additional time beyond what is required for satisfying the MA 310 course requirements.

ARE THERE REASONS WHY YOU SHOULD NOT TAKE PART IN THIS STUDY?

If you are not a prospective middle grades teacher you should not participate in this study.

WHAT ARE THE POSSIBLE RISKS AND DISCOMFORTS?

To the best of our knowledge, the things you will be doing have no more risk of harm than you would experience in everyday life.

WILL YOU BENEFIT FROM TAKING PART IN THIS STUDY?

There is no guarantee that you will get any benefit from taking part in this study. However, it is anticipated that you will learn some mathematics as a result of participating in this study. Your willingness to take part in this study, may, in the future, help university faculty and curriculum developers shape content and pedagogy courses for prospective teachers.

DO YOU HAVE TO TAKE PART IN THE STUDY?

If you decide to take part in the study, it should be because you really want to volunteer. . You will not lose any benefits or rights you would normally have if you choose not to volunteer. . You can stop at any time during the study and still keep the benefits and rights you had before volunteering. . If you decide not to take part in this study, your decision will have no effect. Your course instructor will not know if you volunteered to participate in the study.

IF YOU DON'T WANT TO TAKE PART IN THE STUDY, ARE THERE OTHER CHOICES?

If you do not want to be in the study, there are no other choices except not to take part in the study.

WHAT WILL IT COST YOU TO PARTICIPATE?

There are no costs associated with taking part in the study.

WILL YOU RECEIVE ANY REWARDS FOR TAKING PART IN THIS STUDY?

You will not receive any rewards or remuneration for taking part in the study.

WHO WILL SEE THE INFORMATION THAT YOU GIVE?

Your information will be combined with information from other people taking part in the study. When we write about the study to share it with other researchers, we will write about the combined information we have gathered. You will not be personally identified in these written materials. We may publish the results of this study; however, we will keep your name and other identifying information private.

We will keep private all research records that identify you to the extent allowed by law. However, there are some circumstances in which we may have to show your information to other people. We may be required to show information which identifies you to people who need to be sure we have done the research correctly; these would be people from the University of Kentucky. We will make every effort to prevent anyone who is not on the research team from knowing that you gave us information, or what that information is. If you agree to participate in this research study your responses will remain completely confidential and you will not be able to be identified in any way in any published work that may come from the analysis of the data. All interview tapes, evaluations, assessments, and observational data will be kept in a locked cabinet only accessible to the researcher.

CAN YOUR TAKING PART IN THE STUDY END EARLY?

If you decide to take part in the study you still have the right to decide at any time that you no longer want to continue. You will not be treated differently if you decide to stop taking part in the study. The individuals conducting the study may need to withdraw you from the study. This may occur if you are not able to follow the directions they give you, if they find that your being in the study is more risk than benefit to you, or if the agency funding the study decides to stop the study early for a variety of scientific reasons. .

WHAT IF YOU HAVE QUESTIONS, SUGGESTIONS, CONCERNS, OR COMPLAINTS?

Before you decide whether to accept this invitation to take part in the study, please ask any questions that might come to mind now. Later, if you have questions, suggestions, concerns, or complaints about the study, you can contact the investigator, Jennifer Eli at (859)514-3121 or jennifer.eli@uky.edu. If you have any questions about your rights as a volunteer in this research, contact the staff in the Office of Research Integrity at the University of Kentucky at 859-257-9428 or toll free at 1-866-400-9428. We will give you a signed copy of this consent form to take with you.

Signature of person agreeing to take part in the study

Date

Printed name of person agreeing to take part in the study

Name of [authorized] person obtaining informed consent

Date

APPENDIX J

General Announcement Email Recruitment Script

Hello. My name is Jennifer Eli. I am a Mathematics Education Doctoral student in the Department of Curriculum and Instruction under the direction of Drs. Margaret Mohr and Xin Ma here at the University of Kentucky. I am conducting a research study exploring prospective middle grades teachers' mathematics knowledge for teaching with a focus on mathematical connection making during the problem solving process. You are being invited to participate in this research study because you are a prospective middle grades teacher.

By conducting this study, we hope to learn more about prospective middle grades teachers' mathematics knowledge for teaching and the mathematical connections made during the problem solving process in light of reform curricula. This research will contribute to the improvement of mathematics education courses at the University of Kentucky. .

The research procedures will be conducted at the University of Kentucky. The total amount of time you will be asked to volunteer for this study is no more than 4 hours between the dates of January 9, 2008 through May 31, 2008.

Your participation in this study is voluntary. You will receive a gift certificate to amazon.com for participating in the research study. .

Attached to this email you will find a written consent form to participate in this research study. The written consent form provides a detailed description of the study. If you would like to volunteer to participate in this study please email me at jeli@ms.uky.edu or jennifer.eli@uky.edu. If you have any questions concerning the research study, please do not hesitate to email me or call. This research study has been approved by the University of Kentucky's Institutional review Board (IRB).

Thank you for your time and consideration,

Jennifer A. Eli, M.A.
918 Patterson Office Tower
Lexington, KY 40506-0027
jeli@ms.uky.edu
jennifer.eli@uky.edu
(859) 514-3121
(859) 396-8213

APPENDIX K

Student Consent to Participate in a Research Study

AN EXPLORATORY INVESTIGATION OF PROSPECTIVE MIDDLE GRADES TEACHERS' MATHEMATICAL CONNECTIONS DURING PROBLEM SOLVING IN GEOMETRY

WHY ARE YOU BEING INVITED TO TAKE PART IN THIS RESEARCH?

You are being invited to take part in a research study about prospective middle grades teachers' mathematics knowledge for teaching focusing on mathematical connection making during the problem solving process. You are being invited to take part in this research study because you are a prospective middle grades teacher. If you volunteer to take part in this study, you will be one of about 32 people at the University of Kentucky to do so.

WHO IS DOING THE STUDY?

The person in charge of this study is Jennifer A. Eli of University of Kentucky Department of Curriculum and Instruction. She is being guided in this research by Dr. Xin Ma and Dr. Margaret Mohr both of the University of Kentucky Department of Curriculum and Instruction. There may be other people on the research team assisting at different times during the study.

WHAT IS THE PURPOSE OF THIS STUDY?

The purpose of this study is to explore prospective middle grades teachers' mathematics knowledge for teaching geometry and the mathematical connections made during the problem solving process in light of reform curricula. By doing this study, we hope to learn more about prospective middle grades teachers' mathematics knowledge for teaching and the mathematical connections that evolve during problem solving. This research will contribute to the improvement of mathematics education courses at the University of Kentucky.

WHERE IS THE STUDY GOING TO TAKE PLACE AND HOW LONG WILL IT LAST?

The research procedures will be conducted at University of Kentucky. You will need to come to 918 Patterson Office Tower. The total amount of time you will be asked to volunteer for this study is no more than 4 hours from January 9, 2008 through May 31, 2008.

WHAT WILL YOU **BE ASKED TO DO?**

If you agree to be in the study, you will be asked to participate in (2) two-hour sessions. In the first session you will be asked to complete a diagnostic mathematics assessment.

The diagnostic assessment typical takes no longer than one hour and fifteen minutes to complete. The second session is an interview session where you will be asked to complete a mathematical connections evaluation and card sort activity. The interview session will be audio-and/or video recorded.

ARE THERE REASONS WHY YOU SHOULD NOT TAKE PART IN THIS STUDY?

If you are not a prospective middle grades teacher you should not participate in this study.

WHAT ARE THE POSSIBLE RISKS AND DISCOMFORTS?

To the best of our knowledge, the things you will be doing have no more risk of harm than you would experience in everyday life.

WILL YOU BENEFIT FROM TAKING PART IN THIS STUDY?

There is no guarantee that you will get any benefit from taking part in this study. However, it is anticipated that you will learn some mathematics as a result of participating in this study. Your willingness to take part in this study, may, in the future, help university faculty and curriculum developers shape content and pedagogy courses for prospective teachers.

DO YOU HAVE TO TAKE PART IN THE STUDY?

If you decide to take part in the study, it should be because you really want to volunteer. You will not lose any benefits or rights you would normally have if you choose not to volunteer. You can stop at any time during the study and still keep the benefits and rights you had before volunteering. If you decide not to take part in this study, your decision will have no effect.

IF YOU DON'T WANT TO TAKE PART IN THE STUDY, ARE THERE OTHER CHOICES?

If you do not want to be in the study, there are no other choices except not to take part in the study.

WHAT WILL IT COST YOU TO PARTICIPATE?

There are no costs associated with taking part in the study.

WILL YOU RECEIVE ANY REWARDS FOR TAKING PART IN THIS STUDY?

Participants will receive a twenty-dollar gift certificate to amazon.com for participating in the research study. Participants will receive the aforementioned compensation at the conclusion of the Mathematical Connections Evaluation and Card Sort Activity Interview session. Participants will receive compensation no later than May 31, 2008. If the

participant elects to withdraw before the completion of the research study but has completed the DTAMS assessment, they will receive a prorated compensation in the form of a ten-dollar gift certificate to amazon.com.

WHO WILL SEE THE INFORMATION THAT YOU GIVE?

Your information will be combined with information from other people taking part in the study. When we write about the study to share it with other researchers, we will write about the combined information we have gathered. You will not be personally identified in these written materials. We may publish the results of this study; however, we will keep your name and other identifying information private.

We will keep private all research records that identify you to the extent allowed by law. However, there are some circumstances in which we may have to show your information to other people. We may be required to show information which identifies you to people who need to be sure we have done the research correctly; these would be people from the University of Kentucky. We will make every effort to prevent anyone who is not on the research team from knowing that you gave us information, or what that information is. If you agree to participate in this research study your responses will remain completely confidential and you will not be able to be identified in any way in any published work that may come from the analysis of the data. All interviews tapes, evaluations, assessments, and observational data will be kept in a locked cabinet only accessible to the researcher.

CAN YOUR TAKING PART IN THE STUDY END EARLY?

If you decide to take part in the study you still have the right to decide at any time that you no longer want to continue. You will not be treated differently if you decide to stop taking part in the study. The individuals conducting the study may need to withdraw you from the study. This may occur if you are not able to follow the directions they give you, if they find that your being in the study is more risk than benefit to you, or if the agency funding the study decides to stop the study early for a variety of scientific reasons.

WHAT IF YOU HAVE QUESTIONS, SUGGESTIONS, CONCERNS, OR COMPLAINTS?

Before you decide whether to accept this invitation to take part in the study, please ask any questions that might come to mind now. Later, if you have questions, suggestions, concerns, or complaints about the study, you can contact the investigator, Jennifer Eli at (859)514-3121 or jennifer.eli@uky.edu. If you have any questions about your rights as a volunteer in this research, contact the staff in the Office of Research Integrity at the University of Kentucky at 859-257-9428 or toll free at 1-866-400-9428. We will give you a signed copy of this consent form to take with you.

Signature of person agreeing to take part in the study

Date

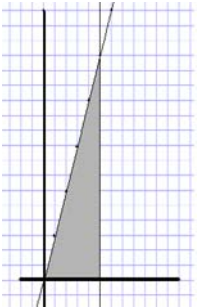
Printed name of person agreeing to take part in the study

Name of [authorized] person obtaining informed consent

Date

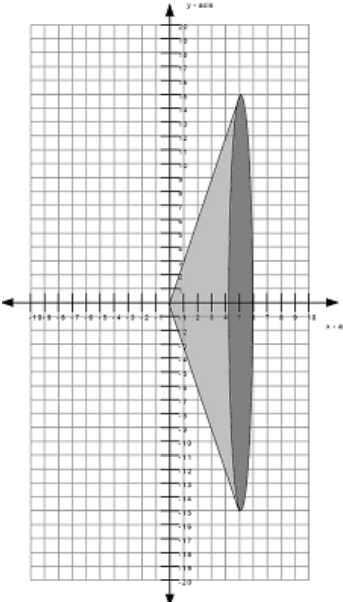
APPENDIX L

Scoring Rubric for Mathematical Connections Evaluation

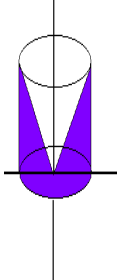
Problem	2 point	1 point	0 points	Score
1(a)	<p>Procedural Connection: identifies, explains and carries out a correct procedure, method or algorithm for graphing the lines $y=3x$ and $x=5$.</p>	<p>Partial Procedural Connection: identifies, explains and carries out a partially correct procedure, method or algorithm for graphing either the line $y=3x$ or the line $x=5$.</p>	<p>Did not make the procedural connection.</p>	
	<p>Algebraic/Geometric Connection: identifies the correct intersection point of the two lines. (The two lines intersect at the point (5,15))</p>	<p>Partial Algebraic/Geometric Connection: identifies an approximate intersection point of the two lines. This includes finding the intersection point through “counting” or “estimating”.</p>	<p>Did not make the algebraic/geometric connection.</p>	
	<p>Algebraic/Geometric Connection: Correctly sketched the appropriate bounded region (right triangle with vertices (0, 0), (5, 0) and (5, 15)).</p> <div style="text-align: center;">  </div> <p><i>Sketch is consistent with 1(a) Procedural Connection.</i></p>	<p>Partial Algebraic/Geometric Connection: gives a partially correct sketch of the bounded region. This may include a sketch of a triangle with 1-2 incorrect vertices.</p>	<p>Did not make the algebraic/geometric connection.</p>	
1(b)	<p>Characteristic/Property Connection: correctly</p>	<p>Partial Characteristic/Property</p>	<p>Did not make the</p>	

	identifies the shape of the bounded region (right triangle).	<p>Connection: correctly identifies the shape of the region as a three-sided polygon but does not identify shape of bounded region as a right triangle.</p> <p><i>Includes not initially recognizing the bounded region as a triangle but did recognize after being asked “how” or “why”.</i></p>	characteristic /property connection.	
1(c)	<p>Procedural Connection: identifies and explains a correct procedure, method or algorithm for computing area of the triangle using the formula $A = (1/2) * (\text{base}) * (\text{height})$.</p>	<p>Partial Procedural Connection: identifies and explains a partially correct procedure, method or algorithm for computing the area of a triangle using the formula $A = (1/2) * (\text{base}) * (\text{height})$.</p> <p><i>Includes use of an incorrect formula such as $(1/3) * (\text{base}) * (\text{height})$.</i></p>	Did not make the procedural connection.	
	<p>Algebraic/Geometric Connection: identifies a base and height of the triangle along with their correct measurements.</p> <p><i>Identifies a base and height of triangle that is consistent with 1(a).</i></p>	<p>Partial Algebraic/Geometric Connection: identifies a base and height of the triangle but with one of the dimensions with incorrect measurement.</p>	Did not make the algebraic/geometric connection.	

	<p>Derivational Connection: gives a correct justification/motivation for utilizing a particular procedure, method or algorithm explaining why the area of a triangle can be found by taking half the base multiplied by height.</p>	<p>Partial Derivational Connection: gives a partially correct justification/motivation for utilizing a particular procedure, method, or algorithm explaining why the area of a triangle can be found by taking half the base multiplied by the height.</p> <p><i>Includes the statement that “area of triangle is just $\frac{1}{2}$ the area of a rectangle” without further detail or explanation.</i></p>	<p>Did not make the derivational connection.</p> <p><i>Includes the statement that “I just had it memorized”.</i></p>	
	<p>Procedural Connection: correctly calculates the area of the bounded region (Area = 37.5 square units)</p> <p><i>Includes a correct calculation that is consistent with $I(a)$.</i></p>	<p>Partial Procedural Connection: gives a partially correct calculation for the area of the bounded region.</p>	<p>Did not make the procedural connection.</p>	
	<p>2-D/3-D Connection: correctly identifies and explains why the resulting three-dimensional shape is a cone.</p>	<p>Partial 2-D/3-D Connection: identifies the three dimensional shape as a cone but has difficulty articulating/explaining/justifying why the resulting three-dimensional shape is a cone.</p>	<p>Did not make the 2-D/3-D connection.</p>	

1(d) i	<p>2-D/3-D Connection: correctly identifies the relationship between the dimensions of the triangle (2-D object) with the dimensions of the cone (3-D object). That is, a correct mapping of the “pieces” of the triangle to the “pieces” of the cone. <i>Includes dimensions that are consistent with 1(a).</i></p>	<p>Partial 2-D/3-D Connection: partially identifies the relationship between the dimensions of the triangle (2-D object) with the dimensions of the cone (3-D object). This may include one incorrect mapping of dimensions of 2-D object to the dimensions of the 3-D object. <i>Includes a “switching” of dimensions.</i></p>	Did not make the 2-D/3-D connection.	
1 (d) ii	<p>Algebraic/Geometric Connection: correctly <u>sketches</u> and/or <u>describes</u> the three dimensional using the coordinate plane.</p> 	<p>Partial Algebraic/Geometric Connection: gives a partially correct sketch and/or description of the three dimensional shape in the Cartesian coordinate plane.</p>	Did not make the algebraic/geo metric connection.	

1(e)	<p>Algebraic/Geometric Connection: identifies the radius and height of the cone along with its correct measurements.</p> <p><i>Identifies a radius and height of cone that is consistent with 1(d).</i></p>	<p>Partial Algebraic/Geometric Connection: identifies the radius and height of the cone but either radius or height is given with incorrect measurement.</p>	<p>Did not make the algebraic/geometric connection.</p>	
	<p>Procedural Connection: identifies and explains a correct procedure, method or algorithm for computing the volume of a cone using the formula $V = \frac{1}{3}\pi r^2 h$.</p>	<p>Partial Procedural Connection: identifies and explains a partially correct procedure, method or algorithm for computing the volume of a cone using the formula $V = \frac{1}{3}\pi r^2 h$</p> <p><i>Includes explanation for an “incorrect” formula, such as $\frac{1}{2}$ the volume of a cylinder or $\frac{2}{3}$ the volume of a cylinder.</i></p>	<p>Did not make the procedural connection.</p>	
	<p>Derivational Connection: gives a correct justification/motivation for why the volume of a cone can be found by taking one-third the volume of a cylinder.</p>	<p>Partial Derivational Connection: gives a partially correct justification/motivation for why the volume of the cone can be found by taking one-third the volume of a cylinder.</p>	<p>Did not make the derivational connection.</p>	
	<p>Procedural Connection: correctly calculated the volume of the three dimensional shape (Volume = 375π cubic units)</p> <p><i>Includes a correct calculation that is consistent with 1(d).</i></p>	<p>Partial Procedural Connection: gives a partially correct calculation for the volume of the three-dimensional shape.</p>	<p>Did not make the procedural connection.</p>	

2	<p>2-D/3-D Connection: correctly identifies the three dimensional shape (cylinder with cone removed)</p> <p>(Radius of cylinder and cone is 5 units; height of cylinder and cone is 15 units)</p> 	<p>Partial 2-D/3-D Connection: identifies the three dimensional shape as a “cylinder with cone removed” but has difficulty articulating/justifying/explaining why the resulting three dimensional shape is a “cylinder with cone removed”.</p>	Did not make the 2-D/3-D connection.	
	<p>2-D/3-D Connection: correctly identifies the relationship between the dimensions of the triangle (2-D object) with the dimensions of the cylinder with cone removed (3-D object).</p> <p><i>Includes dimensions that are consistent with 1(a).</i></p>	<p>Partial 2-D/3-D Connection: partially identifies the relationship between the dimensions of the triangle (2-D object) with the dimensions of the cylinder with cone removed (3-D object). This may include one incorrect mapping of the dimensions of 2-D object to the dimensions of the 3-D object.</p> <p><i>Includes “switching” of dimensions.</i></p>	Did not make the 2-D/3-D connection.	

3	<p>Procedural Connection: identifies and explains a correct procedure, method or algorithm for computing the volume of a (cylinder with cone removed) using the formulas $V(\text{cylinder}) = \pi r^2 h$ for the volume of a cylinder and $V = \frac{1}{3} \pi r^2 h$ for the volume of a cone.</p> <p><i>Includes formula for volume of cone that is consistent with 1(e).</i></p>	<p>Partial Procedural Connection: identifies and explains a partially correct procedure, method, or algorithm for computing the volume of the “cylinder with cone removed” using the formulas $V(\text{cylinder}) = \pi r^2 h$ and $V = \frac{1}{3} \pi r^2 h$</p> <p><i>Includes recognition that the volume of a cylinder and volume of a cone are needed to find the total volume, but does not recognize how to get the total volume.</i></p>	Did not make the procedural connection.	
	<p>Derivational Connection: gives a correct justification/motivation/explanation for the volume of the “cylinder minus cone” shape.</p>	<p>Partial Derivational Connection: gives a partially correct justification/motivation/explanation for the volume of the “cylinder minus cone” shape.</p>	Did not make the derivational connection.	
	<p>Procedural Connection: correctly calculates the volume of the three dimensional shape (Volume=250 π cubic units)</p> <p><i>Includes a correct calculation that is consistent with 2.</i></p>	<p>Partial Procedural Connection: gives a partially correct calculation for the volume of the three-dimensional shape.</p>	Did not make the procedural connection.	
4(a)	<p>2-D/3-D Connection: correctly describes the three dimensional object (solid torus; donut); may give a description of a “real world” object.</p>	<p>Partial 2-D/3-D Connection: gives a partially correct description of the three dimensional object.</p>	Did not make the 2-D/3-D connection.	
	<p>2-D/3-D Connection: correctly describes the</p>	<p>Partial 2-D/3-D Connection: gives a</p>	Did not make the 2-D/3-D	

4(b)	three dimensional object (spherical shell; gumball); may give a description of a “real world” object.	partially correct description of the three dimensional object.	connection.	
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TOTAL SCORE _____/44

****Partially Correct Response****

Incorrect initially, but realized needed to do something else when asked “why” or “how”; however score of “0” given if it appears that the “connection” was a result of the researcher having to “guide” or “scaffold” the participant through problem.

APPENDIX M

Description of Mathematical Connections for Coding Open Card Sort

- **Categorical:** *use of surface features primarily as a basis for defining a group or category.*
 - Example: Card 9 and 14
“The formulas look similar. The a would be the x and b would be your y so c would be your r .”

- **Procedural:** *relating ideas based on a mathematical procedure or algorithm possibly through construction of an example; may include description of the mechanics involved in carrying out procedure rather than the mathematical ideas embedded in the procedure.*
 - Example: Card 4 and 10
“The derivative is move the exponent in front and subtract exponent by 1. So the derivative of $f(x) = x^2$ is $2x$. Whenever I've seen derivative they always use $f(x) = x^2$ or whatever and f prime of x is the derivative. I've had experience taking the derivative of things that look like this.”

- **Characteristic/Property:** *defining characteristics or describing the properties of concepts in terms of other concepts.*
 - Example: Card 19, 20, and 3
“A rectangle has two sets of parallel sides and four ninety degree angles.”

- **Derivation:** *knowledge of one concept(s) to build upon or explain another concept(s); including but not limited to the recognition of the existence of a derivation.*
 - Example: Card 5, 15, 18, 8, and 6
“I can derive the formula for the volume and surface area of a cylinder using the area of a circle and circumference of a circle... [Participant gives detailed explanation/justification]”

- **Curricular:** *relating ideas or concepts in terms of impact to the curriculum, including the order in which one would teach concepts/topics.*
 - Example: Card 15 and 6
“If you were going to teach a lesson on circles you would have to teach them area and circumference rules. They would fall in the same lesson you would teach them. They would have to understand pi and radius for both of them. The circumference of a circle its perimeter; think like triangle and rectangle so my students would understand what circumference is.”

Vita

Jennifer Ann Eli

Date of Birth: June 28, 1978

Place of Birth: Henderson, Kentucky

EDUCATIONAL INSTITUTIONS

University of Kentucky
Masters of Arts Degree, 2002
Area of Concentration: Mathematics

University of Kentucky
Bachelors of Science Degree, 2000
Major: Mathematics

PROFESSIONAL POSITIONS

- Research Assistant, Geometry Assessments for Secondary Teachers (GAST) NSF Grant, University of Kentucky, Lexington, Kentucky (*Fall 2008-Present*)
- Teaching Assistant, Department of Curriculum and Instruction, University of Kentucky, Lexington, Kentucky (*Spring 2008*)
- Teaching Assistant, Department of Mathematics, University of Kentucky, Lexington, Kentucky (*Fall 2006; Fall 2007*)
- Research Assistant, Appalachian Mathematics Science Partnership (AMSP) NSF Grant, University of Kentucky, Lexington, Kentucky (*Summer 2005; Fall 2005; Spring 2007; Summer 2007*)
- Research Assistant, Appalachian Collaborative Center for Learning, Assessment, and Instruction in Mathematics (ACCLAIM) NSF Grant, University of Kentucky, Lexington, Kentucky (*Summer 2004-Spring 2007*).

SCHOLASTIC AND PROFESSIONAL HONORS

- University of Kentucky Women's Club Endowed Fellowship (2007-2008)
- Kappa Delta Pi International Honor Society Louisa A. Oriente Scholarship (2007-2008)

- University of Kentucky College of Education George Denmark Scholarship (2007-2008)
- University of Kentucky Association of Emeriti Faculty Endowed Fellowship (2006-2007)
- Kappa Delta Pi International Honor Society-Alpha Gamma Chapter Nominee for the *Hollis L. Caswell Laureate Doctoral Scholarship in Curriculum and Instruction* (May 2006).
- University of Kentucky College of Education John Edwin Partington and Gwendolyn Gray Partington Scholarship (2004-2005; 2005-2006)

PROFESSIONAL PUBLICATIONS

Bradley, K.D., Royal, K.D., Cunningham, J.C., Weber, J., & Eli, J.A. (2007). What constitutes good educational research? A consideration of ethics, methods and theory. *Mid-Western Educational Researcher*.

Royal, K.D., Eli, J.A., & Bradley, K.D. (Accepted for Publication). A pilot study exploring community college faculty perceptions regarding study outcomes. *Community College Journal of Research and Practice*.

PROFESSIONAL PRESENTATIONS

Eli, J.A., & Schroeder, M.J. (2009, April). *An exploratory study of prospective middle grades teachers' mathematical connections while completing tasks in geometry*. Paper presented at the American Educational Research Association (AERA) Conference, San Diego, CA.

Eli, J.A., & Schroeder, M.J. (2009, April). *Prospective middle grades teachers' mathematical connections in geometry*. Poster presented at the National Council of Teachers of Mathematics (NCTM) Research Pre-session, Washington, D.C.

Bradley, K.D., Cunningham, J.C., & Eli, J.A. (2009, February). *The relationship between primary methodology and perceptions of good educational research*. Paper presented at the 61st annual meeting of the American Association of College for Teacher Education (AACTE), Chicago, Illinois.

Eli, J.A., & Schroder, D.C. (20067, November). *Alpha gamma's road to success: Tips for establishing a scholarship review process*. Poster presented at the 46th biennial convocation of the Kappa Delta Pi International Honor Society, Louisville, KY.

- Bradley, K.D., Royal, K.D., Cunningham, J.C., Weber, J., Eli, J.A., Baas, T., & Harris, W.E. (2007, October). *A consideration of ethics, methods, and theory: Is there consensus on what constitutes good education research?* Paper presented at the annual meeting of the Mid-Western Educational Research Association, St. Louis, Missouri.
- Schroeder, D.C., Mohr, M.J., Goldsby, D., & Eli, J.A. (2007, March). *Preservice teachers' mathematics knowledge for teaching: A comparison.* Poster presented at the annual meeting of the National Council of Teachers of Mathematics, Atlanta, GA.
- Eli, J.A., Royal, K.D., & Bradley, K.D. (2005, October). *A pilot study evaluating community college educators' attitudes and perceptions regarding student outcomes: An investigation into differences by discipline.* Paper presented at the annual Mid-Western Educational Research Conference, Columbus, OH.