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## Variants of the block GMRES method for solving multi-shifted linear systems with multiple right-hand sides simultaneously

Sun, Donglin

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Donglin Sun

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# Variants of the block GMRES method for solving multi-shifted linear systems with multiple right- hand sides simultaneously

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Friday 15 February 2019 at 11.00 hours

by

**Donglin Sun**

born on 16 January 1989  
in Anhui, China

## **Supervisor**

Prof. R.W.C.P. Verstappen

## **Co-supervisor**

Prof. B. Carpentieri

## **Assessment Committee**

Prof. A.J. van der Schaft

Prof. C. Vuik

Prof. M. Bollhoefer

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