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Parallel importation in a supply Chain: The impact of gray market structure

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ABSTRACT

With the rapid development of global economic integration, the size of gray markets continues to expand. The purpose of this paper is to analyse the impact of different structures of gray markets on supply chain decisions and profits. Using game theory, we comprehensively analyze pricing and quantity decisions under monopoly parallel importation (either third-party parallel importation or retailer parallel importation), and duopoly parallel importation, including three different structures in which the retailer and the third-party parallel importation coexist in gray markets with different power structures. We obtain equilibrium results for each structure, compare the optimal strategies of these structures, and develop valuable insights.

1. Introduction

Parallel importation, also referred to as a "gray market", is defined as the sale of genuine-branded products outside authorized distribution channels. Accompanied by the trend of global economic integration over the past few decades, gray markets have proliferated in many industries ranging from consumer goods to industrial equipment. More recently, the growth of efficient global logistics networks and e-business have further boost gray markets at the operational level. For instance, approximately 1 million iPhones were reported to have been unlocked and diverted to China in a year (New York Times, 2008). Nearly 10% of the pharmaceutical market in the European Union is comprised of gray market goods (Danzon, 1998). In the IT industry, gray market products are worth approximately 58 billion dollars and account for about 8% of total global IT sales (KPMG and AGMA, 2008).

Third-party parallel importation is very common in the gray market. For example, in China, there is a significant business phenomenon called "Haitao" (overseas online shoppers) or "Daigou (shopping agents)". It refers to the Chinese nationals who take advantage of their stay or travel overseas to buy goods for "clients" in China. This is a type of third-party parallel importation, which has attracted much attention in international reports (Swanson, 2014). Booker (2015) reported that Chinese consumers made an estimated 70% of luxury purchases in 2014, either online (via third-party or Daigou agents) or on trips abroad. During "Singles Day," a major online shopping holiday in China, third-party parallel importers may temporarily empty a country's supply of a certain product, e.g., infant formula in Australia.

In practice, entities that engage in gray market activities are not limited to third-party agents who purchase from authorized retailer. In fact, well-established retailers such as Amazon, eBay, Kmart, and Costco may sell gray market goods (Schonfeld, 2010; Shulman, 2013; Iravani et al., 2016). However, manufacturers are limited with respect to the legal strategies available to them to deter gray markets. Recently, in a series of lawsuits on copyright infringement between manufacturers and retailers, US courts ruled

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in favour of unauthorized retailers being able to continue to sell gray market products (Greg, 2013; The Washington Post, 2015). Therefore, it is imperative to analyze the environment with gray markets from the perspective of a global supply chain (Antia et al., 2004, Meixell and Gargeya, 2005).

We refer to the above two major types of gray markets "third-party parallel importation" and "(authorized) retailer parallel importation." In practice, both types of parallel imports may coexist in the market. When both the third party and authorized retailer conduct gray market activities, there can be a variety of market structures where the parallel importers have different, relative power and make decisions in different sequences. In particular, if the retailer has relatively greater power, then it acts as a first mover in the gray market while the third party follows up. In contrast, a third party with relatively greater power may move ahead of a retailer. If the retailer and third party have similar power, then they may make decisions simultaneously in a Nash-fashion game.

To provide a real-life example of the interactions, we choose the example of the infant formula brand Enfami. Mead Johnson (the formula manufacturer owning the famous brand Enfami) makes quantity (or price) decision first. Then, in the gray market, participants may act in different time sequences. Parallel imports sourced from Hong Kong or the US flood into China immediately by third parties (Nguyen et al., 2017). Alternatively, famous authorized retailers such as Amazon or Costco (here Amazon and Costco sell products as a retailer though they also allow third parties to sell through their websites) may also sell parallel imports of Enfami sourced from Hong Kong or the US to Chinese consumers online. In short, the power structure in gray markets plays a crucial role in supply chain decision making and different structures will lead to different outcomes in supply chain decisions.

In this paper, we consider a gray market where both the retailer and third party exist and act as parallel importers. To the best of our knowledge, the current literature focuses on either retailer or third-party parallel importation but not both. However, the co-existence of the retailer and third party in the gray market will affect their own and the others' decisions and profits, and will consequently exert an impact on the manufacturer. Furthermore, due to different power structure between the retailer and third party, the game sequence in the gray market may also be different, which affects supply chain members' decisions and profits. To close the gap, we consider the cases where the third party and the retailer coexist in the gray markets with various market power structures between them, and aim to find the effect of different gray market structures on optimal strategies and profit performance.

Specifically, we consider a manufacturer who distributes a branded product in two markets. In addition to the authorized channels, there is a potential gray market where an authorized retailer and a third party may parallel import products between the two markets. The retailer and third party commit to quantity competition in the gray market where they have different types of power structures. To better reflect the reality, we study six possible gray market structures in the paper. We also consider three benchmark cases: no gray market, retailer parallel importation, and third-party parallel importation. Our main goal is to investigate how the manufacturer, retailer, and third party make their own decisions when facing different gray market structures and to analyze the impact of gray market structure on the optimal decisions and performance of the supply chain members.

Our main findings are as follows: When both the retailer and third party conduct parallel importation, compared with monopoly parallel importation, the quantity in the authorized channel in the high market increases while the retail price decreases, which negatively affects the manufacturer. However, in the low market where the retailer sells, the manufacturer is able to increase the wholesale price charged to the retailer. Overall, duopoly parallel importation leads to a lower profit of the manufacturer than monopoly parallel importation. Among the three cases of duopoly parallel importation, the manufacturer obtains the highest profit under a Nash game where the retailer's and third party's power is balanced, and the lowest profit under the third-party Stackelberg game where the third party's power is greater than the retailer's. Furthermore, the retailer obtains lower profits under duopoly parallel importation than under retailer parallel importation where the retailer enjoys the monopoly profit from the gray market. Similar to the manufacturer, both the retailer and third party prefer a Nash game among the three cases of duopoly parallel importation. In other words, the retailer and third party would rather forego the first-moving advantage even if they have greater power. The third party benefits from parallel importation even if it has less power than the retailer, as long as the retailer participates in parallel importation.

Our main contributions are threefold: First, we are the first to investigate a competitive gray market where both a retailer and a third party conduct parallel importation under different power structures. Second, we derive equilibrium in each game where the parallel importers have different power structures. Third, we show how market structure affects supply chain members' decisions and the impact on their profits. Furthermore, we examine the impact of different discount factors for different channels of parallel imports. We find that even when consumers know the source of parallel imports, our main results still hold.

The remainder of this paper is organized as follows. In Section 2, we provide a brief literature review. In Section 3, we specify the assumptions and model setting, and then derive the inverse demand functions. In Sections 4, we analyze three benchmark cases. In Section 5, we analyze the duopoly parallel importation cases. In Section 6, we compare the cases and examine the impact of gray market structure on supply chain decisions and performances. In Section 7, we discuss the extension to include different values of discount factors for parallel imports. Finally, we offer a summary of the results and several future research directions to conclude the paper in Section 8.

2. Literature review

Our research is closely related to the literature on gray markets and supply chain power structure. In the first literature stream, Bennato and Valletti (2014) propose a theoretic model to study the interaction between price control and parallel imports. Yun (2017) studies the gray market issue from the perspective of third-party parallel importation and finds that the manufacturer may obtain higher profit with parallel imports when value-added services are available only for the authorized products. There is also a number of research papers studying gray markets or parallel importation in the context of supply chain (Maskus and Chen, 2004,

Chen and Maskus, 2005, Dasu et al., 2012). Some researchers focus on third-party parallel importation occurs at the retail level (Ahmadi and Yang, 2000, Chen and Maskus, 2005, Autrey and Bova, 2011, Ahmadi et al., 2015, Kim and Park, 2016, Zhang and Feng, 2017). For example, motivated by the common practice of cross-border shopping in China, Zhang and Feng (2017) study the gray markets problems caused by price gap at the retail level. Another group studies retailer parallel importation occurs at the wholesale level (Ganslandt and Maskus, 2007, Raff and Schmitt, 2007, Zhang, 2016, Shao et al., 2016). For example, Shao et al. (2016) investigate both "local gray markets" and "bootlegging" and examine their different impacts on the manufacturer. Ahmadi et al. (2017) investigate the impact of gray markets under wholesale price and quantity discount contracts, and they find that the presence of gray markets degrades the performance of both contracts. Mostly close to us is Xiao et al. (2011) who find that the identity of parallel importers is crucial not only for the structure of gray markets but also for supply chain decisions. However, they study the third-party parallel importation and retailer parallel importation separately, and they focus on the channel structures whereas we focus on the gray market structures. They mainly compare chain members' profits under different channel structures while we compare the profits of all the players including parallel importers under different gray market structures. Even in monopoly parallel importation cases, Xiao et al. (2011) have not analyzed on the profits of parallel importer and the quantities of parallel imports because their focus is on channel structure. None of the extant research considers the cases where third-party parallel importation and retailer parallel importation coexist in the gray market. We are the first to comprehensively investigate the gray markets structures and consider the complex situations where third-party parallel importation and retailer parallel importation coexist in the gray market.

In the supply chain power structure literature, "power" is defined as an agent's ability to influence decision making by moving first in a non-cooperative game (Choi, 1996, Pan et al., 2010, Edirisinghe et al., 2011, Wu et al., 2012, Li et al., 2013, Chen et al., 2017). For instance, Choi (1991) examines different power structures between two manufacturers and one common retailer in three non-cooperative games, including manufacturer Stackelberg, retailer Stackelberg, and vertical Nash. Fang et al. (2018) study the impact of both channel leadership and echelon dominance in a supply chain with two manufacturers and one common retailer using Stackelberg games. Wu et al. (2012) consider vertical and horizontal competitions characterized by a Stackelberg or Bertrand model and analyze six types of power structures in a supply chain. Wei et al. (2013) study the pricing of complementary products in a supply chain composed of a monopolistic retailer and duopolistic manufacturers with consideration of different market power structures among the channel members. However, the above research does not investigate power structure in the context of parallel importation or gray markets.

3. Model description

Consider a manufacturer who sells a branded product in two markets (labeled market 1 and market 2). Following the related literature (e.g., Ganslandt and Maskus, 2007; Li and Robles, 2007; Autrey and Bova, 2011), we assume that the channel structure of the supply chain is as follows: In market 1, the manufacturer sells the product through a retailer, while selling directly in market 2. Our purpose is to build a model that not only captures the problems of the retailer and the third-party parallel importation as a whole, but also is simple enough to allow for analytical tractability. The setup of model allows us to focus on the issue of gray market structures from the perspective of supply chain. Fig. 1 illustrates the supply chain structure presumed in the presence or absence of gray markets.

Assume that the willingness-to-pay of consumers for the authorized product follows a uniform distribution over $[0,a_1]$ with density $\frac{1}{a_1}$. In market 2, consumers' willingness-to-pay is uniformly distributed over $[0,a_2]$ with density $\frac{1}{a_2}$. Suppose that $a_2 > a_1$; that is, market 1 is a "low" market and market 2 a "high" market. We use $a = \frac{a_2}{a_1}$ to reflect the demand difference between the two markets.

Consider two types of parallel importation: (1) "retailer parallel importation," where the authorized retailer in market 1 parallel imports the product into market 2 without the manufacturer's sanction, and (2) "third-party parallel importation," where a third party buys the product from market 1 at the retail price and resells in market 2.[1] Denote by δ $(0 < \delta < 1)$ the discount factor in market 2 consumers' willingness-to-pay for parallel imports due to a lack of product warranty or service package and possible compatibility problems.[2]

Table 1 specifies the notations in the rest of the model:

In the absence of parallel imports, we can derive the demand functions for authorized products in markets 1 and 2 as follows: $q_1 = 1-\frac{p_1}{a_1}$ and $q_2 = 1-\frac{p_2}{a_2}$. When parallel imports are present, the demand functions for authorized products in the two markets are $q_1 = 1-\frac{p_1}{a_1}$ and $q_2 = 1-\frac{p_2-p_3}{(1-\delta)a_2}$; and the demand for parallel imports in market 2 is $q_3 = \frac{\delta p_2 - p_3}{a_2(1-\delta)\delta}$. Since we assume that the retailer and third party compete in Cournot fashion (if both types of parallel importation exist), we derive the equilibrium solutions in the Cournot competition where the firms choose quantities rather than prices (Autrey and Bova, 2011, Autrey et al., 2014, Autrey et al., 2015, Matsui, 2014, Li et al., 2016). Therefore, we invert the demand functions and obtain $p_1 = a_1-a_1q_1$ and $p_2 = a_2-a_2q_2$ in the absence of parallel imports, and we obtain $p_1 = a_1-a_1q_1$, $p_2 = a_2(1-q_2-\delta q_3)$, and $p_3 = a_2\delta(1-q_2-q_3)$ in the presence of parallel imports.

[1] For tractability, we assume there is a third party in the gray market. We relax this assumption and model the situations in which there are multiple third-party parallel importers in the final market. The main findings of our paper have not been affected by allowing multiple third-party parallel importers.

[2] For brevity, we assume that the discount factor in consumer valuation for parallel imports is equal no matter sourced by the retailer or a third-party. Although consumers might trust the parallel imports sourced by the retailer more, it is very difficult for consumers to identify the source of parallel imports. We relax this assumption in an extension and the main results still hold.

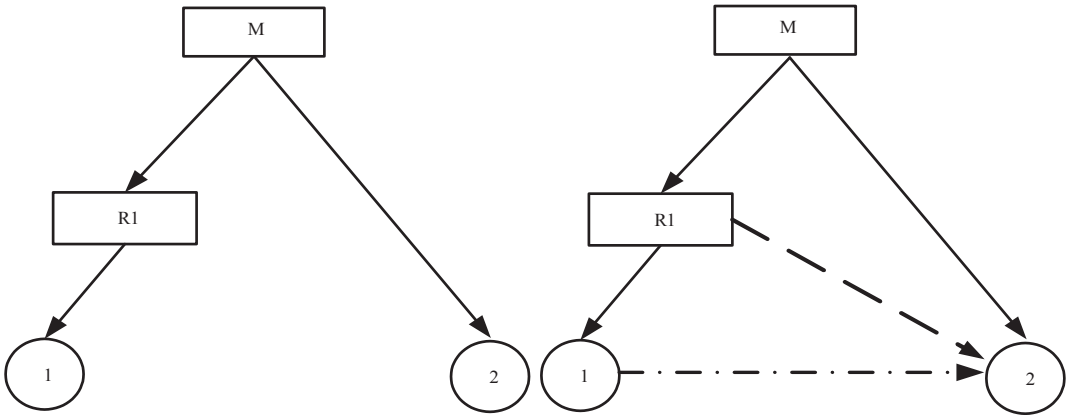

Fig. 1. Supply Chain without parallel importation (left) and with parallel importation (right). In the right, the dashed arrow indicates "retailer parallel importation" and the dash-dotted arrow indicates "third-party parallel importation".

Table 1
Notations.

Variable	Definition
q_i	Quantity of the product sold through the authorized channel in market i (i = 1,2)
q_3	Quantity of parallel imports
q_{3p}	Third-party parallel importation quantity
q_{3r}	Retailer parallel importation quantity
p_i	Retail price of the products in market i (i = 1,2)
p_3	Retail price of parallel imports
w_1	Wholesale price of authorized products in market 1
m	Retail margin enjoyed by the retailer
π_m	Profit of the manufacturer
π_r	Profit of the retailer
π_{3p}	Profit of the third-party parallel importer

In the following, we consider three scenarios: In Scenario 1, there is no gray market (NGM). In Scenario 2, which we refer to as "monopoly parallel importation," either the retailer or third party conducts parallel importation. Scenario 2 contains two cases: (1) "retailer parallel importation" (RPI), where only the retailer conducts parallel importation, and (2) "third-party parallel importation" (TPI), where only the third party conducts parallel importation.

In Scenario 3, which we refer to as "duopoly parallel importation," both the retailer and third party conduct parallel importation. Depending on the relative power between the retailer and third party, there can be three cases: (1) the retailer has relatively greater power and acts first in the gray market, which we refer to as the "retailer Stackelberg game" (RSG); (2) the third party has relatively greater power and acts first in the gray market, which we refer to as the "third-party Stackelberg game" (TSG); and (3) the retailer and third party have equal power and act simultaneously in the gray market, which we refer to as "Nash game" (NG).

Our main goal is to compare duopoly parallel importation with monopoly parallel importation and investigate the impact of power structure in the gray market. Therefore, in Section 4, we take the first three cases in Scenarios 1 and 2 as our benchmark cases and explore insights additional to the literature. Then we examine Scenario 3 in Section 5 and compare the benchmark cases in Section 6.

4. Benchmark cases

In this section, we examine the three benchmark cases in Scenarios 1 and 2. We first consider Scenario 1, and no parallel importation arises. We simply derive the market equilibrium as follows:

$$q_2 = \frac{1}{2}, w_1 = \frac{a_1}{2}, q_1 = \frac{1}{4}, p_1 = \frac{3}{4}a_1, p_2 = \frac{a_2}{2}, m = p_1 - w_1 = \frac{a_1}{4}. \tag{1}$$

Then, we can calculate the manufacturer's and retailer's profit as well as the total profit of the supply chain in equilibrium:

$$\pi_m = \frac{a_1}{8} + \frac{a_2}{4}, \pi_r = \frac{a_1}{16}, \pi_{total} = \frac{3a_1}{16} + \frac{a_2}{4}. \tag{2}$$

We then consider the two cases in Scenario 2. In retailer parallel importation, the sequence of the game is as follows: In stage 1, the manufacturer determines the wholesale price w_1 in market 1, and the quantity q_2 in market 2. In stage 2, the retailer determines

Table 2
Equilibrium in the cases of retailer parallel importation and third-party parallel importation.

	RPI	TPI
q_1	$\frac{a_1}{2(a_1+a_2\delta)}$	$\frac{1}{4}(\frac{3a_1}{a_1+2a_2\delta}+\frac{a_1(4-\delta)}{a_1(4-\delta)+4a_2(2-\delta)\delta})$
q_2	$\frac{1}{2}$	$\frac{1}{2}+\frac{a_1\delta}{4a_1+8a_2\delta-4a_2\delta^2-a_1\delta}$
w_1	$\frac{a_1a_2\delta}{a_1+a_2\delta}$	$\frac{3a_1a_2\delta}{2(a_1+2a_2\delta)}$
q_3	$\frac{1}{4}-\frac{a_1}{2(a_1+a_2\delta)}$	$\frac{3a_2\delta}{2(a_1+2a_2\delta)}-\frac{6-\delta}{8-2\delta}-\frac{4a_2\delta(2-\delta)}{(4-\delta)(a_1(4-\delta)+4a_2\delta(2-\delta))}$

the quantity q_1 to sell in market 1, and the quantity q_3 to parallel import to market 2. In third-party parallel importation, there are three stages: In stage 1, the manufacturer determines w_1 and q_2. In stage 2, the retailer determines the retail quantity q_1 in market 1. In Stage 3, a third party determines the quantity q_3 to buy from the retailer in market 1 and resell in market 2.

Based on the above game sequences, we can derive the equilibrium in each case, which is given in Table 2:

In the two monopoly parallel importation cases, we conduct comparative statics in terms of δ and a (recall that $a = a_2/a_1$) to examine the impact of consumers' acceptance for parallel imports and the difference between the two markets.

Lemma 1. In the case RPI, we have

(i) $\frac{\partial q_1}{\partial \delta} < 0$, $\frac{\partial q_2}{\partial \delta} = 0$, $\frac{\partial q_3}{\partial \delta} > 0$, $\frac{\partial w_1}{\partial \delta} > 0$;
(ii) $\frac{\partial q_1}{\partial a} < 0$, $\frac{\partial q_2}{\partial a} = 0$, $\frac{\partial q_3}{\partial a} > 0$.

See all the proofs in Appendix B.

Lemma 1(i) indicates that as consumers' acceptance for parallel imports δ increases, the quantity of parallel imports increases. As the retailer diverts some products from its authorized channel in market 1 to market 2 (parallel imports), the quantity in the authorized channel in market 1 reduces. However, overall, the total procurement of the retailer from the manufacturer increases, which pulls up the wholesale price. Furthermore, the sales in the authorized channel in market 2 remain unchanged.

Lemma 1(ii) examines the effect of the difference between the two markets. As the difference between the two markets increases, the arbitrage opportunity for parallel importation becomes larger. As a result, the quantity of parallel imports increases. Similar to (i), here the retailer also diverts some products from market 1 to market 2 to seek a high profit in market 2.

We now examine the case of third-party parallel importation:

Lemma 2. For the case TPI, we have (i) $\frac{\partial q_1}{\partial \delta} < 0$, $\frac{\partial q_2}{\partial \delta} > 0$, $\frac{\partial w_1}{\partial \delta} > 0$; (ii) $\frac{\partial q_1}{\partial a} < 0$, $\frac{\partial q_2}{\partial a} < 0$, $\frac{\partial q_3}{\partial a} > 0$.

In the case of third-party parallel importation, as the difference between the two markets increases, the quantity in the authorized channel in market 2 reduces. However, as consumers' acceptance for parallel imports δ increases, the quantity in the authorized channel in market 2 increases. This is because when the parallel imports become more competitive, the price of the authorized products sold in market 2 will be reduced to face the competition of parallel imports. Then the quantity in the authorized channel in market 2 will increase.

Lemma 3.

(i) $m^{RPI} < m^{NGM}$, $m^{TPI} < m^{NGM}$;
(ii) $\pi_m^{RPI} < \pi_m^{NGM}$; $\pi_r^{RPI} < \pi_r^{NGM}$ if $\frac{1}{\delta} < \frac{a_2}{a_1} < \frac{3}{\delta}$; $\pi_r^{RPI} > \pi_r^{NGM}$ otherwise.
(iii) $\pi_m^{RPI} + \pi_r^{RPI} < \pi_m^{NGM} + \pi_r^{NGM}$.
(iv) $\pi_m^{TPI} > \pi_m^{NGM}$ if $a_1^3\delta + 8a_1^2a_2\delta + 2a_1^2a_2\delta^2 + 24a_1a_2^2\delta^2 + 8a_2^3\delta^4 - 4a_1^3 - 14a_1a_2^2\delta^3 - 16a_2^3\delta^3 > 0$; $\pi_m^{TPI} < \pi_m^{NGM}$ otherwise; $\pi_r^{TPI} < \pi_r^{NGM}$.

In the case TPI, in the presence of gray markets, the retailer is willing to give up the high retail margin to indirectly reach market 2 by selling to the third-party parallel importer. Consequently, with a lower markup, the retailer can cover a larger market. Meanwhile, the manufacturer can increase the wholesale price due to the larger demand not only from consumers in market 1 but also from the gray market. This implies that the manufacturer can reduce the negative impact caused by the double marginalization in market 1 due to the presence of gray markets. In the case TPI, the profit of retailer is reduced by the presence of gray markets.

Similar to the case TPI, the double marginalization effect is also reduced when the retailer becomes a parallel importer. In the case RPI, the retailer chooses to sacrifice its price margin due to the large opportunity of profitable parallel importation. The double marginalization problem can be alleviated in both cases where either the retailer or the third party carries out parallel importation. However, the manufacturer always suffers from the retailer parallel importation and the overall profit across the entire channel is always lower than the case without gray markets even when parallel importation is profitable for the retailer.

As for the case RPI, Xiao et al. (2011) conclude, "The retailer benefits if and only if $\delta \in (0,0.8)$ and $\frac{a_2}{a_1} \in \left(\frac{1}{\delta} + \frac{\sqrt{(4-\delta)(1-\delta)}}{\delta(1-\delta)}, \frac{4}{\delta^2}\right)$". Xiao et al. (2011) do not provide a clear explanation. Different from their results, Lemma 3 clearly indicates that the retailer is worse off when $\frac{1}{\delta} < \frac{a_2}{a_1} < \frac{3}{\delta}$. The retailer can benefit from parallel importation when the demand difference between markets and the value

of the discount factor are big enough. It can be proved that parallel importation is beneficial to the retailer only when the high market is at least three times greater than the low market. If the demand difference or the discount factor is small, the retailer could be worse off. In addition, Xiao et al. (2011) compare the case RPI with the case TPI and note that the opportunity for the manufacturer to benefit from alleviated double marginalization in market 1 is eliminated in the case RPI. Hence, the manufacturer always gets hurt in the case RPI by comparing with the case TPI. However, by numerical analysis, we find that, in contrast to the case TPI, the double marginalization effect is likely to be further reduced and the parallel importation quantity increases greatly due to the low cost of parallel importation at the wholesale level in the case RPI, which leads to large-quantity ordering from the manufacturer and eventually high wholesale price. The retailer cannot keep high markup in market 1 in the presence of high wholesale price and the double marginalization effect is reduced. In fact, the manufacturer's profit is worse off than in the case TPI, which is mainly because of the competition caused by a large amount of parallel imports in market 2. In other words, it is the cannibalization effect rather than the double marginalization effect that eventually results in the decrease in the manufacturer's profit in contrast to the case TPI.

Using numerical tests with different values of parameters, we find that the manufacturer will benefit from the presence of gray markets only in the case TPI when some requirements are satisfied (e.g., when we set the values of parameters: $a_2 = 20000$, $a_1 = 7000$ and $\delta \in (0,1)$). These requirements are as follows: (i) The values of a_1 and a_2 cannot be significantly different, which means that the demand situations in two markets cannot be quite different. Otherwise, the profit of the manufacturer will suffer from the significant amount of parallel imports; (ii) The values of a_1 and a_2 cannot be too close, otherwise the parallel importer would quit the gray market; (iii) The parallel imports cannot be perceived close to the authorized products by consumers. This is reasonable because under this setting low-priced parallel imports can satisfy the demand of consumers with low willingness to pay and avoid competition with the authorized products for consumers with high willingness to pay. Eventually, the reduction of double marginalization in market 1 offsets the cannibalization effect caused by gray markets and the manufacturer's profit can be enhanced in the presence of gray markets.

We next compare the two cases where there is only one parallel importer in the gray markets with the case NGM (i.e., "No gray market"), and we have the following results:

Proposition 1. (i) $w_1^{RPI} > w_1^{TPI} > w_1^{NGM}$; (ii) $q_2^{TPI} > q_2^{RPI} = q_2^{NGM}$; (iii) $p_2^{NGM} > p_2^{RPI} > p_2^{TPI}$.

The explanation of Proposition 1 is as follows Because the retailer can carry out parallel importation at the wholesale level while the third party carries out parallel importation at the retail level, the parallel importation cost incurred by the retailer is much lower than that by the third party if the wholesale price is given. It is more profitable for the retailer to carry out parallel importation. Hence, to cope with competition from parallel imports, the wholesale price charged by the manufacturer in the case RPI is higher than that in the case TPI, which means that the wholesale pricing strategy to discourage the retailer parallel importation is more drastic than that to discourage third-party parallel importation. By increasing the wholesale price, the manufacturer can reduce the retailer's motivation to initiate parallel importation and gain higher markup from the retailer in the case RPI. In addition, the wholesale price charged by the manufacturer in the case TPI is higher than in the case NGM. By the same logic, the manufacturer can discourage the third-party parallel importation by increasing the cost of parallel importation and gain high markup from the retailer. In addition, the sale quantity of authorized products sold in market 2 in the case RPI is lower than that in the case TPI. The reason is that in contrast to the third-party parallel importer, the retailer parallel importer has an advantage in parallel importation and competes more aggressively with the manufacturer in market 2. Moreover, the price of authorized products sold in market 2 in the case NGM is the highest, which is intuitive because the manufacturer is the monopolist in the market 2 when gray market is absent. However, it is counterintuitive that the price of authorized product sold in market 2 in the case RPI is higher than that in the case TPI. This is because the price of the authorized product in market 2 is not only determined by the amount of parallel imports but also by the manufacturer's sale quantity of authorized product in market 2, as in the expression $p_2 = a_2(1-q_2-\delta q_3)$. Compared to the third-party parallel importer, the retailer parallel importer may transfer a larger amount of parallel imports into market 2. However, for the manufacturer's profit maximization, the decrease in the sale quantity of authorized products in market 2 is so large that p_2 eventually becomes higher in the case RPI despite a large amount of parallel imports.

5. Duopoly parallel importation

In this section, we consider three cases in duopoly parallel importation, i.e., retailer Stackelberg game, third-party Stackelberg game, and Nash game.

5.1. Retailer Stackelberg game (RSG)

In this case, the retailer has relatively greater power than the third party; so the retailer acts first in the game. The sequence of the game is as follows: In stage 1, the manufacturer determines the wholesale price w_1 and quantity q_2 in market 2. In stage 2, the retailer determines the quantity in the authorized channel in market 1 q_1, and the quantity of parallel imports q_{3r}. In stage 3, the third party determines the quantity of parallel imports q_{3p}. We derive the equilibrium of the three-stage game, which is given in Table 3.

Lemma 4. In the case RSG, we have:

(i) $\frac{\partial w_1}{\partial \delta} > 0$, $\frac{\partial q_2}{\partial \delta} > 0$;
(ii) $\frac{\partial q_2}{\partial a} < 0$, $\frac{\partial q_{3r}}{\partial a} > 0$, $\frac{\partial q_{3p}}{\partial a} > 0$.

Table 3
Equilibrium in the case of retailer Stackelberg.

	Retailer Stackelberg game (RSG)
q_1	$1-\frac{a_2\delta}{2(a_1+a_2\delta)}+\frac{a_2\delta(5+\frac{a_1\delta}{a_1(2-\delta)+a_2\delta(4-3\delta)})}{4(a_1+2a_2\delta)}$
q_2	$\frac{1}{2}+\frac{a_1\delta}{8a_2\delta-6a_2\delta^2+4a_1-2a_1\delta}$
w_1	$\frac{a_1a_2\delta}{a_1+a_2\delta}$
q_{3r}	$\frac{1}{4}(1-\frac{2a_1}{a_1+a_2\delta}-\frac{a_1\delta}{a_1(2-\delta)+a_2\delta(4-3\delta)})$
q_{3p}	$\frac{1}{8}(1-\frac{3a_1}{a_1+2a_2\delta}-\frac{2a_1(2-\delta)}{a_1(2-\delta)+a_2\delta(4-3\delta)})$

Combining with Lemma 1 and Lemma 2, we find that under various power structures of gray market (i.e., TPI, RPI, and RSG), the wholesale price in the low market increases with the discount rate (δ). With a higher δ, parallel imports are more competitive and the manufacturer raises the wholesale price to discourage retailer parallel importation. The high wholesale price not only improves the manufacturer's profit and but also indirectly deters the parallel importation by resulting in a high retail price. In addition, we use a to represent the degree of demand difference between markets 1 and 2. When the demand difference between markets becomes greater, the sale quantity of authorized products sold in the high market decreases. The logic underlying these propositions is explained as follows. The increasing demand difference provides more arbitrage opportunities for parallel imports and the increasing amount of parallel imports will cannibalize the sales of authorized products in the high market (market 2). With the increasing demand difference between markets, the sales of authorized products in the high market are cannibalized more severely by parallel imports diverted from the gray market.

5.2. Third-party Stackelberg game (TSG)

In this case, the third party has relatively greater power and acts as the Stackelberg leader in parallel importation. In stage 1, the manufacturer determines w_1 in market 1 and q_2 in market 2. In stage 2, the retailer determines the quantity q_1 in the authorized channel in market 1. In stage 3, a third party determines the quantity of parallel imports q_{3p}. In stage 4, the retailer determines the quantity of parallel imports q_{3r}. We derive the equilibrium in Table 4:

5.3. Nash game (NG)

In the case of Nash game, the retailer and third party have almost equal power. In stage 1, the manufacturer determines w_1 in market 1 and q_2 in market 2. In stage 2, the retailer determines q_1. In stage 3, the retailer and third party simultaneously determine the quantities of parallel imports q_{3r} and q_{3p}. We derive the equilibrium in Table 5:

6. Impact of gray market structure

In this section, we compare duopoly parallel importation with the benchmark cases to examine the impact of gray market structure. In Section 6.1, we take the retailer Stackelberg game as a representative case of duopoly parallel importation to compare with the benchmark cases analytically. In Section 6.2, we compare all cases through numerical study.

Table 4
Equilibrium strategies under Third-party Stackelberg game.

	Third-party Stackelberg game (TSG)
q_1	$\frac{6a_1^2-3a_1w_1+a_1a_2\delta-4a_2\delta w_1+3a_1a_2\delta q_2}{6a_1^2+8a_1a_2\delta}$
q_2	$\frac{a_1\delta(24a_1^2+70a_1a_2\delta+47a_2^2\delta^2)}{(144-72\delta)a_1^3+(528a_2\delta-312a_2\delta^2)a_1^2+(640a_2^2\delta^2-434a_2^2\delta^3)a_1+256a_2^3\delta^3-192a_2^3\delta^4}+\frac{1}{2}$
w_1	$\frac{(84a_2\delta-42a_2\delta^2)a_1^3+(202a_2^2\delta^2-127a_2^2\delta^3)a_1^2+(120a_2^3\delta^3-90a_2^3\delta^4)a_1}{(72-36\delta)a_1^3+(264a_2\delta-156a_2\delta^2)a_1^2+(320a_2^2\delta^2-217a_2^2\delta^3)a_1+(128a_2^3\delta^3-96a_2^3\delta^4)}$
q_{3r}	$\frac{1}{2}-\frac{q_{3p}}{2}-\frac{w_1}{2a_2\delta}-\frac{q_2}{2}$
q_{3p}	$\frac{w_1-2a_1+a_2\delta+2a_1q_1-a_2\delta q_2}{2a_2\delta}$

Table 5
Equilibrium strategies under Nash game.

	Nash game (NG)
q_1	$1-\frac{a_2\delta(14-5q_2)}{2(5a_1+9a_2\delta)}-\frac{w_1}{2a_1}$
q_2	$\frac{1}{2}(1-\frac{a_1\delta(15a_1^2+54a_1a_2\delta+44a_2^2\delta^2)}{25a_1^3(-2+\delta)+54a_2^3\delta^3(-3+2\delta)+10a_1^2a_2\delta(-23+13\delta)+2a_1a_2^2\delta^2(-171+107\delta)})$
w_1	$\frac{a_1a_2\delta(30a_1^2(-2+\delta)+51a_2^2\delta^2(-3+2\delta)+a_1a_2\delta(-193+113\delta))}{25a_1^3(-2+\delta)+54a_2^3\delta^3(-3+2\delta)+10a_1^2a_2\delta(-23+13\delta)+2a_1a_2^2\delta^2(-171+107\delta)}$.
q_{3r}	$\frac{a_1(1-q_1)+a_2\delta(1-q_2)-2w_1}{3a_2\delta}$
q_{3p}	$\frac{a_2\delta(1-q_2)-2a_1(1-q_1)+w_1}{3a_2\delta}$

6.1. Comparison between retailer Stackelberg game and benchmark cases

We take the retailer Stackelberg game as a representative case of duopoly parallel importation, and compare it with the benchmark cases. Due to the complex forms of the third-party Stackelberg game and Nash game, we delay the analyses of these two cases in Section 6.2 through numerical study. We first compare the wholesale prices in the retailer Stackelberg game and benchmark cases:

Proposition 2. The wholesale prices in the retailer Stackelberg game and benchmark cases have the following relationships: $w_1^{RSG} = w_1^{RPI} > w_1^{TPI} > w_1^{NGM}$.

Proposition 2 shows that compared with the no gray market case, the manufacturer's wholesale price is higher if parallel importation arises. This is because the manufacturer's total demand in the low market increases due to parallel importation. Furthermore, relative to the case where a third party carries out parallel importation, the manufacturer can charge an even higher wholesale price if the retailer is involved in parallel importation, as the retailer tends to parallel import greater quantity than the third party does. The reason is that in the third-party parallel importation, the retailer acts as an intermediary, which leads to double marginalization effect between the retailer and third party. As a result, the total quantity of parallel importation is lower, whereas when the retailer conducts parallel importation itself, the quantity of parallel imports is higher without the double marginalization effect. This enables the manufacturer to charge higher wholesale price. Furthermore, we find that the wholesale prices in the retailer Stackelberg game equals that in the retailer parallel importation case. This implies that the manufacturer will not change the wholesale price when a third party joins in duopoly parallel importation as a Stackelberg follower in contrast with monopoly parallel importation cases.

We next compare the quantity and retail price across the cases and present the following proposition.

Proposition 3. Comparing the quantity and retail price in the retailer Stackelberg and benchmark cases, we obtain the following relationships:

(i) $q_2^{RSG} > q_2^{TPI} > q_2^{RPI} = q_2^{NGM}$;
(ii) $p_2^{RSG} < p_2^{TPI} < p_2^{RPI} < p_2^{NGM}$.

Proposition 3 indicates that when parallel importation arises, the quantity always increases and the price decreases in the authorized channel in market 2 compared with the case of no gray market. Furthermore, comparing the two cases where a single party conducts parallel importation, we obtain that q_2 is even higher while p_2 is lower in the case of third-party parallel importation (TPI) than in the case of retailer parallel importation (RPI). This is because in the case of third-party parallel importation, due to the double marginalization effect between the third party and retailer, the third party will act more aggressively in market 2, which intensifies the competition between parallel imports and the authorized channel in market 2. As a result, the authorized channel in market 2 ends up with lower price and higher quantity, compared with those under the case of retailer parallel importation.

Furthermore, Proposition 3 shows that if both the retailer and third party conduct parallel importation (case RSG), the quantity will be even higher and the price will be lower in the authorized channel in market 2, compared with the monopoly parallel importation cases. This is because the competition in the gray market (between the goods parallel-imported by the retailer and third party) also intensifies the competition between parallel imports and authorized goods in market 2, which further drives down the price and increases the quantity in the authorized channel.

In summary, Propositions 2 and 3 provide the following implications: First, compared with the case of no gray market, the manufacturer is adversely affected in market 2 where its direct sales encounter competition from parallel imports. Specifically, the manufacturer's monopoly retail price in market 2 decreases as the quantity increases. However, in market 1, when parallel importation emerges, the manufacturer benefits from an increase in the wholesale price as the retailer will order more to feed the parallel importation. Second, comparing duopoly and monopoly parallel importation, we find that the above effects are further strengthened as both the retailer and the third party conduct parallel importation. Therefore, in the next subsection, we explore the net effect of duopoly parallel importation on the manufacturer by comparing duopoly parallel importation with the benchmark cases. We also examine the impact of gray market structure by comparing the three duopoly parallel importation cases.

6.2. Comparison between duopoly parallel importation and benchmark cases

Due to the complexity of the problem, we need to resort to numerical studies to compare the firms' profits across all the cases. We consider two numerical instances where the demand difference in the two markets is large and moderate (If the demand difference in the two markets is too low, parallel importation will not happen). Specifically, in Instance 1, we set $a_1 = 3000$, $a_2 = 20000$, and in Instance 2, we set $a_1 = 5000$, $a_2 = 20000$. Next, we study the profits of all the participants (i.e., manufacturer, retailer and third-party parallel importer) as well as the quantity of parallel imports in the above two numerical instances.

(1) Manufacturer's Profit

We calculate the manufacturer's profits under the six cases, and plot them against consumers' acceptance for parallel imports δ in the situations of moderate and large market difference respectively (see Figs. 2a and 2b). First, note that in both instances, the manufacturer is generally hurt by parallel importation. Recall that in Section 4 we find that the manufacturer benefits from parallel importation in market 1 since the retailer orders more to feed the parallel imports. Whereas in market 2, the manufacturer's margin in the direct authorized channel is hurt by parallel imports. Overall, the manufacturer is worse off as the gray market emerges.

Furthermore, comparing the manufacturer's profit under monopoly and duopoly parallel importation, we have the following observation:

Remark 1. The manufacturer obtains relatively lower profit under duopoly parallel importation than under monopoly parallel importation.

One may expect that competition between parallel importers would benefit the manufacturer, as they are in essence the manufacturer's downstream distributors, although unauthorized. However, Remark 1 shows a counterintuitive result. Notice that in the absence of parallel importation, in market 2, the manufacturer's authorized channel is a monopoly without any intermediary (manufacturer is direct-selling in market 2). The emergence of parallel importers adds intermediaries in market 2 which reduces the efficiency of the supply chain. What is more, the coexistence of duopoly parallel importers increases the sale quantity of parallel imports; this further squeezes out the sales in the authorized channel in market 2, which possesses a relatively higher margin. As a result, the manufacturer is even worse off under duopoly parallel importation than under monopoly parallel importation.

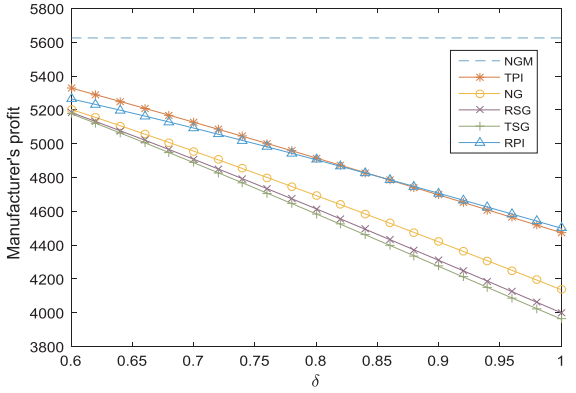

Fig. 2a. Manufacturer's profits in the six cases ($a_1 = 5000$, $a_2 = 20000$).

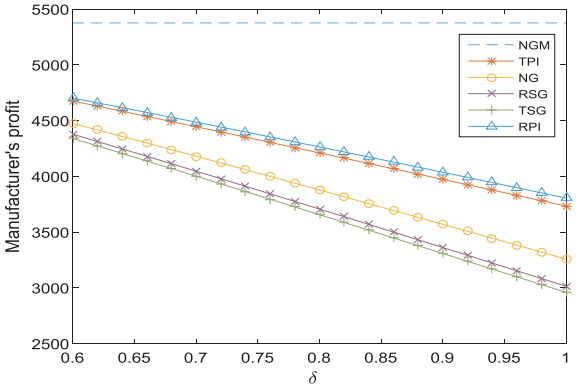

Fig. 2b. Manufacturer's profits in the six cases ($a_1 = 3000$, $a_2 = 20000$).

Remark 2. Among the three cases of duopoly parallel importation, the manufacturer obtains the highest profit under the Nash game and the lowest profit under the third-party Stackelberg game.

Remark 2 indicates that if the retailer and the third party coexist, then the manufacturer's loss from parallel importation is the lowest under the Nash game where the retailer's and the third party's power is balanced. The manufacturer's loss is greater if either one of the retailer and the third party has significantly higher power than the other does. This result is intuitive since when the retailer and the third party co-exist in the gray market, they both serve as downstream distributors of the manufacturer. If their power is balanced, the manufacturer can benefit from the competition between them. However, if one has greater power than the other does, the relative power of this party to the manufacturer becomes higher, which is in the disadvantage of the manufacturer.

Furthermore, if it has to be the case that the power of the retailer and third party is imbalanced, then the manufacturer prefers that the authorized retailer has greater power than the third-party parallel importer. This result is opposite to previous research that concludes that the manufacturer generally gets hurt from retailer parallel importation more severely than third-party parallel importation. Note that the retailer is one of the supply chain members in the authorized channel; so the retailer also takes into account its demand and profit in the authorized channel when making decision in the gray market. However, when a third party conducts parallel importation, it ignores the externality that its gray market activity will exert on the authorized channel, which can hurt the manufacturer more.

To summarize, the net effect of duopoly parallel importation on the manufacturer is negative: with two parallel importers in the gray market, the manufacturer suffers from greater profit loss compared with the cases where only the retailer or a third party conducts parallel importation. Therefore, if the manufacturer detects that both the retailer and third party are active in the gray market, it should try to prevent at least one of them from parallel importing. However, such an effort may not be successful due to the high cost and difficulty of preventing gray market activities. If so, the best outcome for the manufacturer is that the retailer's and third party's power is balanced so that they enter into a Nash game in the gray market, whereas the worst case is that the third party has greater power than the retailer when they both conduct parallel importation.

Further, we find that the manufacturer will give up the low market to block the gray market under the conditions that the parallel imports are competitive and the demand difference between markets is large. In the case TSG, the manufacturer is most likely to give up the low market than in the other cases. Unlike previous research that concludes that the manufacturer generally gets more severely hurt from retailer parallel importation than from third-party parallel importation, we find that when two markets are quite different, retailer parallel importation is more beneficial for the manufacturer compared with third-party parallel importation.

(2) Retailer's Profit

Figs. 3a and 3b demonstrate the retailer's profit in the six cases under the two instances. We find that in contrast to the manufacturer, the retailer may make higher profit as long as the retailer participates in parallel importation. This occurs even when the third party has greater power than the retailer (see Fig. 3b).

In general, the retailer obtains the highest profit in the case of retailer (only) parallel importation. This is intuitive since the retailer can conduct parallel importation as a monopolist. However, we observe from both instances (Figs. 3a and 3b) that the retailer may also be worse off from parallel importation, even under the case where it conducts parallel importation alone. This occurs when the market difference is moderate and the consumers' acceptance for parallel imports is low, i.e., a harsh condition for parallel imports. Under this situation, even when the retailer is involved in parallel importation, its profit could still be reduced in the presence of gray markets.

We then compare the retailer's profit under duopoly and monopoly parallel importation cases.

Remark 3. (a) When the demand difference is moderate (Fig. 3a), the retailer obtains lower profits under the duopoly parallel importation cases than under the monopoly parallel importation cases. (b) When the demand difference is large (Fig. 3b), the retailer

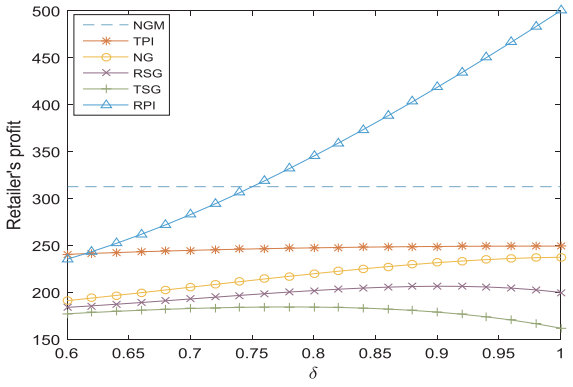

Fig. 3a. Retailer's profits in the six cases ($a_1 = 5000$, $a_2 = 20000$).

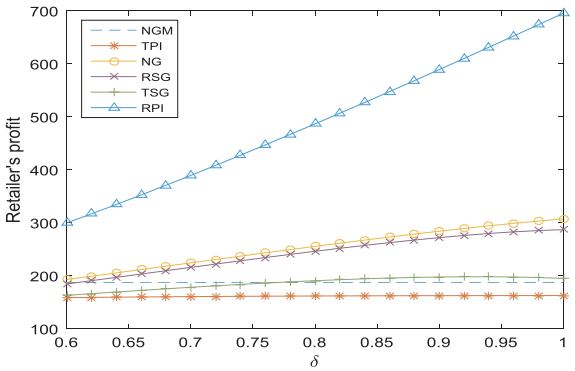

Fig. 3b. Retailer's profits in the six cases ($a_1 = 3000$, $a_2 = 20000$).

obtains lower profits under the duopoly parallel importation cases than under the retailer (only) parallel importation, and earns higher profits under the duopoly parallel importation cases than under the third party (only) parallel importation.

Remark 3 (b) is intuitive since the retailer prefers to take all gray market profit (retailer parallel importation) than to share gray market profit with a third party (duopoly parallel importation), whereas what leads to the lowest profit of the retailer is the case where the third party conducts parallel importation as a monopolist. However, surprisingly, Remark 3 (a) shows that the retailer can obtain a higher profit under third-party parallel importation than under any of the duopoly parallel importation cases where the retailer also participates in parallel importation. This occurs when the demand difference is moderate, which is as aforementioned, not a good market condition for parallel importation. Note that under such condition, the retailer is worse off under almost all the cases of parallel importation. The retailer's loss from parallel importation is thus relatively low if only the third party conducts parallel importation. The retailer would suffer even higher loss if the retailer participated in parallel importation together with the third party, which results in higher total quantity in the gray market.

We next compare the retailer's profit under the three duopoly parallel importation cases.

Remark 4. Among the three cases of duopoly parallel importation, the retailer obtains the highest profit under the Nash game and the lowest profit under the third-party Stackelberg game.

It is intuitive that among the cases where both the retailer and third party conduct parallel importation, the retailer obtains the lowest profit under the third-party Stackelberg game where the third party has greater power than the retailer. It is interesting that the preferences of manufacturer and retailer for the gray market structure under the duopoly parallel importation are consistent. That is to say, they can easily engage in collusion and take the same action to influence the gray market structure if possible. However, it is surprising that the retailer obtains the highest profit under the Nash game rather than in the retailer Stackelberg game. A possible explanation is that in any case where the two parallel importers' power is very imbalanced, the follower may respond by cutting the price of parallel imports severely, which hurts both parties. However, under the Nash game, both parties focus on sharing the benefit from parallel importation and hence the competition is less intense, which results in higher profit for the retailer.

From Remarks 3 and 4, we can obtain implications to the retailer: First, from Remark 4, even if the retailer has greater power than the third party does, it should forego its first-moving advantage to achieve relatively higher profit under the Nash game. Note that if the retailer does so, it will also benefit the manufacturer as indicated by Remark 2. Furthermore, the third-party Stackelberg game leads to the lowest profit of the retailer among all the duopoly parallel importation cases. One way for the retailer is to withdraw from the gray market while letting the third party be the monopolist in the gray market. This will also enable the manufacturer to achieve higher profit from Remark 1. However, withdrawing from the gray market is effective to the retailer only when the demand difference between the two markets is moderate (see Fig. 3a). If the demand difference between the two markets is large, the retailer's best option is to participate in the third-party Stackelberg game, as withdrawing from the gray market will lead to third party (only) parallel importation where the retailer obtains even lower profit (see Fig. 3b). Nonetheless, large demand difference is a condition favorable to gray markets; and the retailer can still make profit or almost break even under the third-party Stackelberg game.

(3) Third party's Profit

We also examine the third party's profit; however, we only need to compare four cases where the third party participates in parallel importation. The third party benefits from parallel importation even if it has less power than the retailer as long as the retailer participates in parallel importation. This is not surprising, since if not, it would voluntarily withdraw from parallel importation. It is also intuitive that the third party obtains the highest profit under third-party parallel importation among all the cases. Because the third party can always earn arbitrage profit, it would not voluntarily withdraw from parallel importation. Hence, combining our observations in (2), the retailer should not enter the gray market if the demand difference between markets is moderate and consumers' perceived value of parallel imports is very low, because the case TPI would be better for the retailer than any duopoly parallel importation cases.

What is more interesting is to compare the third party's profit under the three duopoly parallel importation cases as follows.

Remark 5. Among the three cases of duopoly parallel importation, the third party obtains the highest profit under the Nash game and the lowest profit under the retailer Stackelberg game.

Remark 5 provides another surprising observation that the third party obtains the highest profit under the Nash game, as we expect that the third party would most prefer the third-party Stackelberg game where it has greater power than the retailer in the gray market. However, as explained in (2), the Nash game incurs the least level of competition in the gray market, which also benefits the third party.

The implication for the third party is that it should also forego its first-mover advantage if it has greater power than the retailer does to enter into a Nash game. Furthermore, if the retailer has greater power, the third party obtains the lowest profit under the retailer Stackelberg game. However, as indicated by both Fig. 4a and Fig. 4b, the third party will still participate in parallel importation as it can make a positive profit.

From our observations in Remarks 1–5, all the participants (i.e., the manufacturer, retailer, and third party) prefer the Nash game to the other cases of duopoly parallel importation. As explained above, under the Nash game, the power of the two parallel importers is balanced, which leads to less intense competition in the gray market. This consequently alleviates the competition between parallel imports and the authorized goods. As a result, all members in the supply chain achieve relatively higher profit under the Nash game

Since both the retailer and the third party have an incentive to participate in the Nash game, it is possible that the one with the greater power will forego its power and enter the gray market at the same time as the other. If so, all supply chain members will benefit from the collusion between the retailer and the third party in the gray market. However, in practice, collusion may be difficult for various reasons. For example, a retailer with greater power may not trust an unknown third party; even if the retailer trusts the third party, the third party may not believe that the retailer would trust it.

If collusion cannot be achieved, then either a retailer or a third-party Stackelberg game will emerge in the gray market. If the retailer has greater power than the third party, both the manufacturer and retailer will suffer less loss from the retailer Stackelberg game than under a third-party Stackelberg game. Under certain market conditions, the retailer may further withdraw from parallel importation, which leads to even higher profits for both the manufacturer and the retailer. The worst situation for the manufacturer and the retailer is that the third-party parallel importer has greater power than the retailer does. What the manufacturer and retailer can do in this case is to jointly deter the third-party parallel importation.

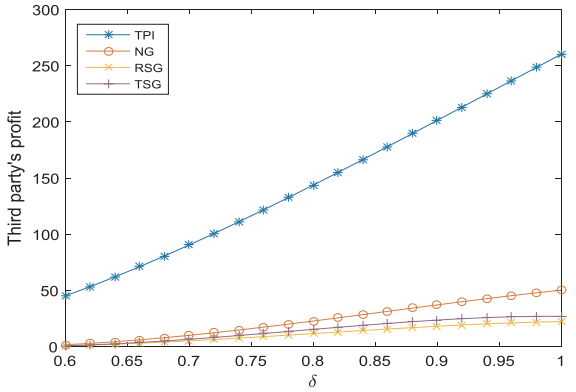

Fig. 4a. Third party's profits in the six cases ($a_2 = 20000$, $a_1 = 5000$).

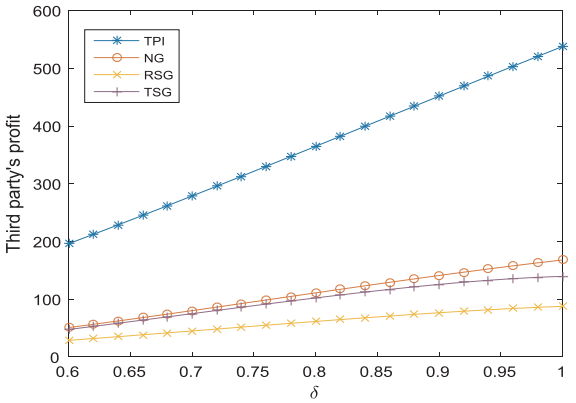

Fig. 4b. Third party's profits in the six cases ($a_1 = 3000$, $a_2 = 20000$).

(4) Quantity of Parallel Imports

We finally compare the quantity of parallel imports under the six cases (see Fig. 5a and Fig. 5b)

Remark 6. (a) When the demand difference between the two markets is moderate, quantities of parallel imports under the duopoly parallel importation cases are higher than under the third-party parallel importation, but lower than under the retailer parallel importation. (b) When the demand difference between the two markets is large, duopoly parallel importation leads to higher quantities of parallel imports than monopoly parallel importation.

We expect that under duopoly parallel importation, the quantity of parallel imports increases compared with that under monopoly parallel importation. This is true if the difference between the two markets is large. However, when the demand difference between the two markets is moderate, we observe that the quantity of parallel imports is the highest under the retailer parallel importation. This might be counterintuitive, but can be explained as follows. When the demand difference between markets is large, it provides a great opportunity for both retailer parallel importation and third-party parallel importation. Compared with the third-party parallel importer, the retailer has more advantage to carry out parallel importation. Therefore, retailer Stackelberg game can lead to the largest parallel importation quantity. Nevertheless, when demand difference is moderate, the opportunity for parallel importation is so limited that the competition between the retailer and the third party is fierce, which forces the retailer and the third party to make quantity decisions very cautiously. Hence, due to the advantage of retailer parallel importation and the limited opportunity in the gray market, the parallel importation quantity becomes larger when the retailer monopolizes the gray market compared with the duopolistic cases where the third party and the retailer coexist in the gray market. Consequently, the parallel importation quantity achieves the largest value in the case RPI.

Furthermore, we observe that the quantity of parallel imports is the smallest if the third party conducts parallel importation alone. This is because in this case, the third party acts as another intermediary and adds an echelon between the retailer and end-consumers in the supply chain. Due to the intensified double-marginalization effect (the third party also demands a margin), the prices will be high and the quantities will be low. If the retailer conducts parallel importation, it does not add any intermediary in the supply chain as the retailer is already an authorized intermediary. Parallel importation only expands the retailer's market. The competition

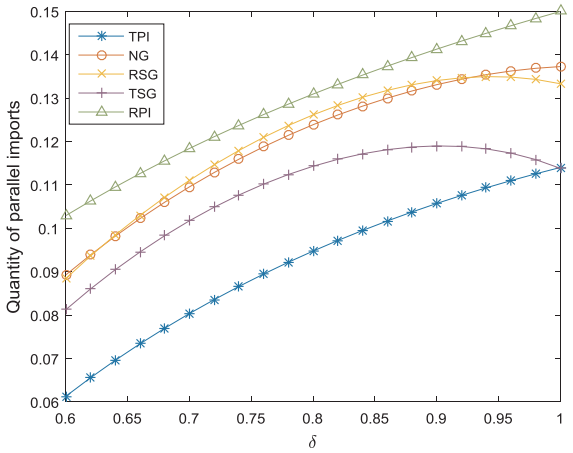

Fig. 5a. Quantity of parallel imports in the six cases ($a_1 = 5000$, $a_2 = 20000$).

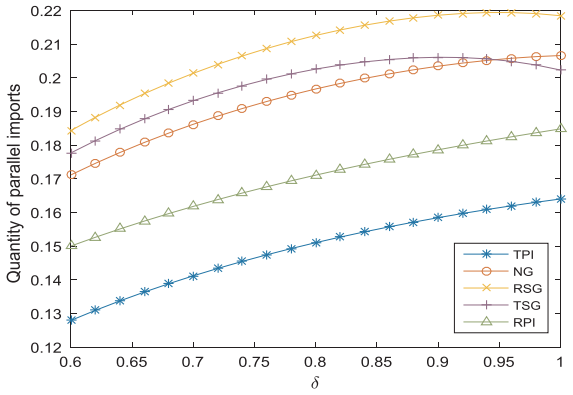

Fig. 5b. Quantity of parallel imports in the six cases ($a_1 = 3000$, $a_2 = 20000$).

between parallel imports and authorized products in market 2 drives down both the manufacturer's and retailer's margins and increase the total quantity in the market.

7. Extension: Differentiated discount factor in consumers' Willingness-To-Pay for parallel imports

In the main model, we assume that consumers cannot identify the source of parallel imports that they purchase. In this section, we relax the assumption and consider a case where the discount factors in consumers' willingness-to-pay for parallel imports are differentiated. Specifically, denote by θ the discount factor for the retailer's parallel imports, and δ the discount factor for the third party's parallel imports. Suppose that $\delta < \theta < 1$.

Consumers are thus segmented into four groups in market 2: (1) consumers with the highest willingness to pay purchase authorized products; (2) consumers with the second highest willingness to pay purchase parallel imports from the retailer; (3) consumers with the third highest willingness to pay purchase parallel imports from third-party parallel importer; (4) consumers with the lowest willingness to pay purchase nothing. We can derive the demand functions in market 2 and obtain the following:

$$q_2 = \frac{1}{a_2}\left(a_2 - \frac{p_2 - p_{3r}}{1-\theta}\right), q_{3r} = \frac{1}{a_2}\left(\frac{p_2 - p_{3r}}{1-\theta} - \frac{p_{3r} - p_{3p}}{\theta - \delta}\right), q_{3p} = \frac{1}{a_2}\left(\frac{p_{3r} - p_{3p}}{\theta - \delta} - \frac{p_{3p}}{\delta}\right).$$

The corresponding inverse demand functions are given by:

$$p_2 = a_2(1 - q_2 - \delta q_{3p} - \theta q_{3r}),$$

$$p_{3r} = a_2(\theta - \theta q_2 - \theta q_{3r} - \delta q_{3p}),$$

$$p_{3p} = a_2\delta(1 - q_2 - q_{3r} - q_{3p}).$$

Based on the inverse demand functions, following the same approach in the model, we can derive the equilibrium in each case. We then use numerical study to demonstrate the robustness of the results from the main model.

From Figs. 6, 7, 8 and 9, our numerical analysis shows that when the difference between the discount factors δ and θ is small (i.e., to the right of the figures), the results are almost the same as in the main model. If the difference between the discount factors δ and θ is large, the numerical analysis shows some different results. Moreover, it is interesting to find that the difference is most likely to occur when the demand difference is moderate.

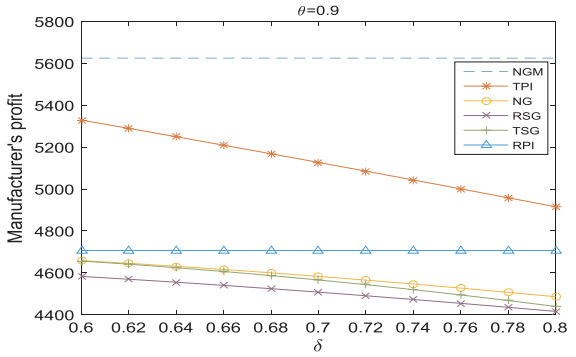

Fig. 6a. Comparison of manufacturer's profits ($a_1 = 5000$, $a_2 = 20000$).

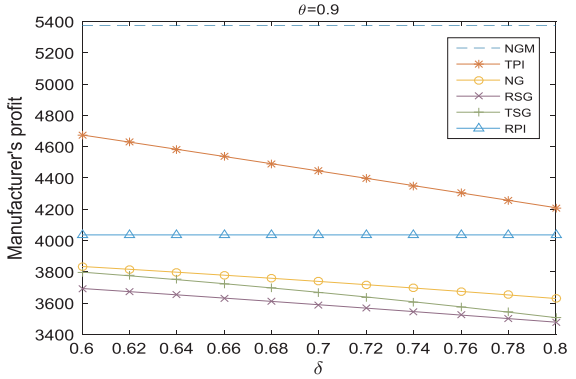

Fig. 6b. Comparison of manufacturer's profits ($a_1 = 3000$, $a_2 = 20000$).

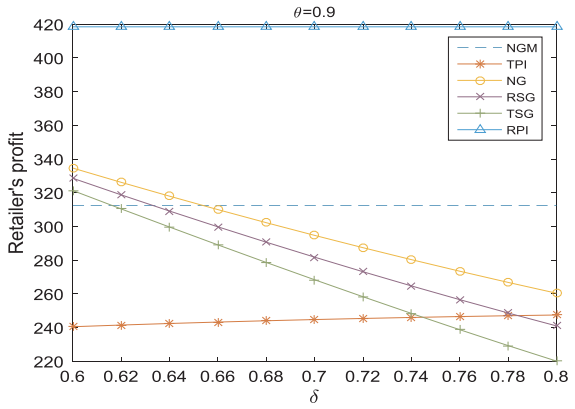

Fig. 7a. Comparison of retailer's profits ($a_1 = 5000$, $a_2 = 20000$).

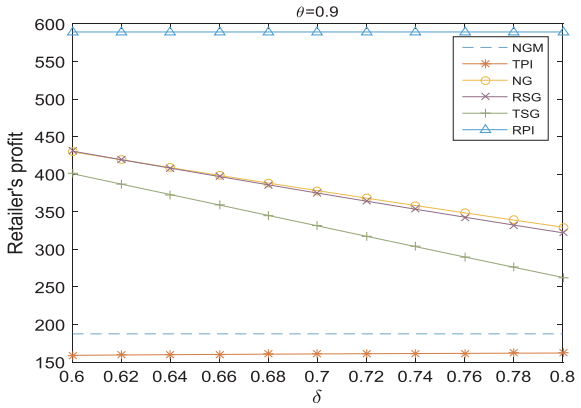

Fig. 7b. Comparison of retailer's profits ($a_2 = 20000$, $a_1 = 3000$).

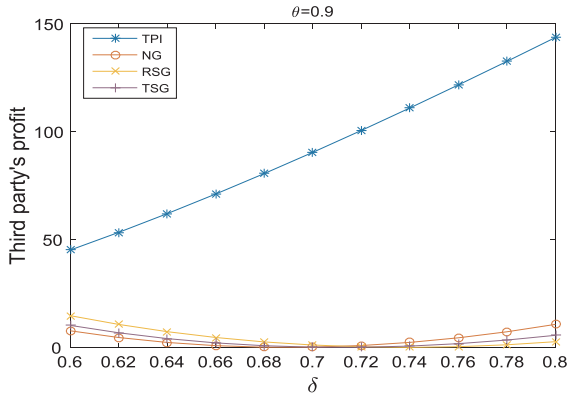

Fig. 8a. Comparison of third party's profits ($a_1 = 5000$, $a_2 = 20000$).

We summarize the important different findings as follows. (1) The retailer has a very strong advantage over the third party in the gray market and therefore, has more motivation to enter the gray market. On the other hand, when the retailer and the third party coexist in the gray market, the participants could still prefer "Nash game" to the other cases. (2) The manufacturer hurts from the gray market most severely in the case RSG. The case TPI is most beneficial for manufacturers among various gray markets settings. The manufacturer should thus deter the retailer parallel importation drastically. (3) When the demand difference is moderate and the value of the discount factor δ is small, the third-party parallel importer earns the highest profit in the case RSG and the lowest profit in the case NG.

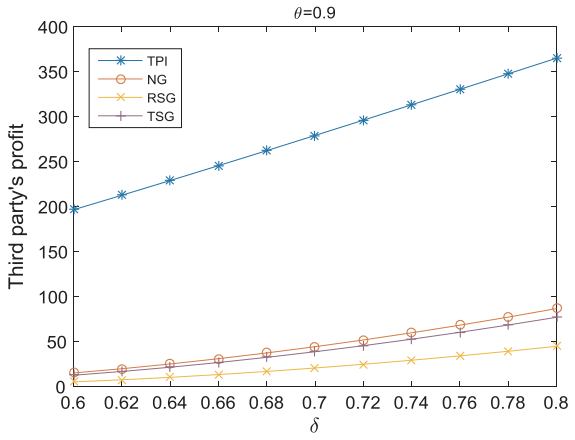

Fig. 8b. Comparison of third party's profits ($a_1 = 3000$, $a_2 = 20000$).

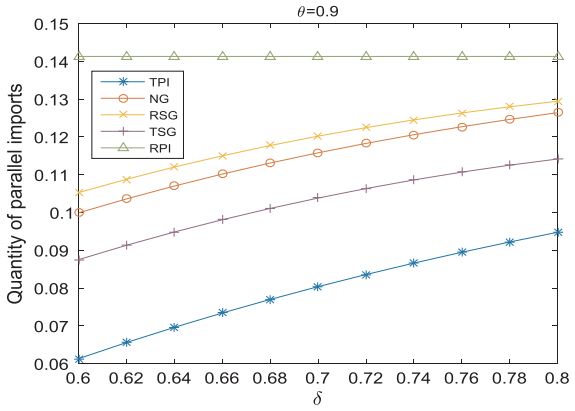

Fig. 9a. Comparison of quantity of parallel imports ($a_1 = 5000$, $a_2 = 20000$).

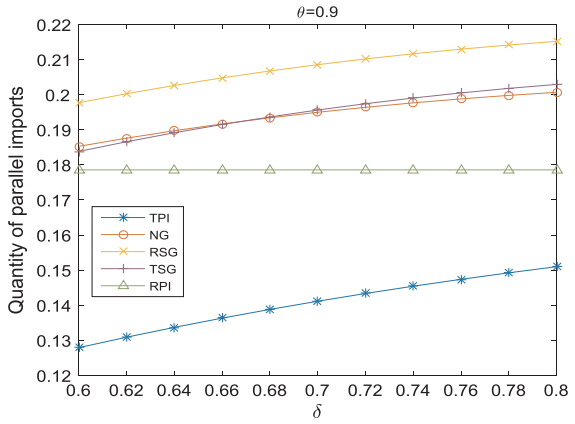

Fig. 9b. Comparison of quantity of parallel imports ($a_1 = 3000$, $a_2 = 20000$).

8. Conclusions

In this paper, we comprehensively investigate the gray markets structures and consider complex situations where third-party parallel importation and retailer parallel importation coexist in the gray market. Through game theory models, we consider three cases where the retailer and third party coexist in the gray market: retailer Stackelberg game, third-party Stackelberg game, and Nash game.

Our main findings are as follows. First, the manufacturer obtains lower profit under duopoly parallel importation than under monopoly parallel importation. Among the three cases of duopoly parallel importation, the manufacturer obtains the highest profit

under the Nash game and the lowest profit under the third-party Stackelberg game. Second, the retailer obtains lower profits under duopoly parallel importation than under retailer parallel importation where the retailer enjoys the monopoly profit from the gray market. However, the retailer may also prefer the third-party parallel importation to duopoly parallel importation. Among the three cases of duopoly parallel importation, the retailer obtains the highest profit under the Nash game and the lowest profit under the third-party Stackelberg game, which is the same for the manufacturer. Third, among the three cases of duopoly parallel importation, the third party obtains the highest profit under the Nash game and the lowest profit under the retailer Stackelberg game.

Our results shed light on various parties in a supply chain where gray markets are of concern: to all the participants including manufacturer, retailer, and third party, Nash game in the gray market result in the highest profits when both the retailer and third party conduct parallel importation. If the retailer's and third party's power is imbalanced, the one with greater power should forego its first-moving advantage, and collude with the other to enter Nash game. However, when collusion cannot be achieved, if the retailer has greater power, then the retailer can withdraw from parallel importation under certain market conditions, which increases the profits of both the manufacturer and the retailer. The worst case for the manufacturer and the retailer is that the third party has greater power; the manufacturer and the retailer should then try to jointly deter parallel importation by the third party.

Furthermore, we discover that even if consumers are aware of different sources of parallel imports with different values of discount factors, our conclusions hold under these following situations: (1) when the demand difference is large; (2) when the difference between two values of discount factor δ and θ is small. These findings are novel contributions to the literature related to gray markets. Further, we model the situations in which there are multiple third-party parallel importers in the final market. We extend TPI, NG, RSG, and TSG by assuming multiple third-party parallel importers. The major difference in terms of the results is that the manufacturer's profit under TPI becomes lower when the number of third parties increases. The reason is that when the number of the third parties increases, the amount of parallel imports in the markets increases due to fierce competition, which causes the decrease of the number of consumers who purchase the authorized products. As a result, the manufacturer suffers from a loss of the profit. The main findings of our paper have not been affected by allowing multiple third-party parallel importers.

This paper can be extended in several directions. First, it will be interesting to consider a supply chain with information asymmetry, such as asymmetry in information regarding the value discount of parallel imports and demand information. Second, one can also consider the coordination of decentralized supply chains under the gray market setting.

Acknowledgements

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Appendix A. Equilibrium solutions

A.1. Equilibrium solution for case NGM

We solve this Stackelberg leader-follower game by backward induction. The retailer chooses the value of q_1 to maximize his profit π_r.

$$\max_{q_1} \pi_r = (p_1 - w_1) q_1. \tag{3}$$

Solving the first-order condition from (3), we obtain

$$q_1 = \frac{a_1 - w_1}{2a_1}. \tag{4}$$

Next, we solve the first stage of the game. Anticipating the sale quantity chosen by the retailer in market 1, the manufacturer decides on the optimal wholesale price in market 1 and sale quantity in market 2 to maximize the total profit in both markets.

$$\max_{w_1, q_2} \pi_m = w_1 q_1 + p_2 q_2. \tag{5}$$

By solving the first conditions $\frac{\partial \pi_m}{\partial w_1} = 0$ and $\frac{\partial \pi_m}{\partial q_2} = 0$, we obtain

$$q_2 = \frac{1}{2}, \tag{6}$$

$$w_1 = \frac{a_1}{2}. \tag{7}$$

Substitution and calculation yields $q_1 = \frac{1}{4}$, $p_1 = \frac{3}{4}a_1$, $p_2 = \frac{a_2}{2}$, $\pi_m = \frac{a_1}{8} + \frac{a_2}{4}$, $\pi_r = \frac{a_1}{16}$. The overall profit across the entire channel is $\pi_{total} = \frac{3a_1}{16} + \frac{a_2}{4}$. The markup from the retailer is $m = p_1 - w_1 = \frac{a_1}{4}$, where m represents the retail margin of the retailer.

We assume $p_2 > p_1$ to guarantee that products be transferred from market 1 to market 2 due to the price gap. It is straightforward to obtain $a_1 < \frac{2}{3}a_2$, which means that average consumer willingness to pay for the product in market 1 should be low enough compared with that in market 2. Otherwise, the retail price in market 1 may become higher than in the market 2 through the double marginalization effect.

A.2. Equilibrium solution for case TPI

Using backward induction, we first maximize the profit of the third-party parallel importer as follows:

$$\max_{q_3} \pi_{P3} = (p_3 - p_1)q_3. \tag{8}$$

Solving the first-order condition from (, we have

$$q_3 = \frac{a_2\delta(1-q_2) - a_1(1-q_1)}{2a_2\delta}. \tag{9}$$

Considering the third-party parallel importer's sale quantity decision, the retailer faces the following problem:

$$\max_{q_1} \pi_r = (p_1 - w_1)(q_1 + q_3). \tag{10}$$

According to the first order condition, the optimal sale quantity is

$$q_1 = 1 - \frac{a_1\left(\frac{w_1}{2} + \frac{3a_2\delta}{2} - \frac{a_2\delta q_2}{2}\right) + a_2\delta w_1}{a_1(a_1 + 2a_2\delta)}. \tag{11}$$

Anticipating quantity responses from the retailer and the third party, the manufacturer chooses the optimal quantity and wholesale price to maximize the total profit in both market 1 and market 2 as follows:

$$\max_{w_1, q_2} \pi_m = w_1(q_1 + q_3) + p_2 q_2. \tag{12}$$

Solving the first-order condition, we obtain the equilibrium:

$$q_2 = \frac{1}{2} + \frac{a_1\delta}{4a_1 + 8a_2\delta - 4a_2\delta^2 - a_1\delta}, w_1 = \frac{3a_1a_2\delta}{2(a_1 + 2a_2\delta)}. \tag{13}$$

$m = p_1 - w_1 = \frac{a_1a_2(2-\delta)\delta}{4a_1 - a_1\delta + 8a_2\delta - 4a_2\delta^2}$, where m means the retail margin of the retailer.

Further, the other players' decisions can be derived by substitutions. For ease of analysis, the results for this scenario are summarized in Table 2.

A.3. Equilibrium solution for case RPI

Using backward induction, we first maximize the profit of the retailer as follows:

$$\max_{q_1, q_3} \pi_r = (p_3 - w_1)q_3 + (p_1 - w_1)q_1. \tag{14}$$

By solving (12), we obtain

$$q_3 = \frac{1}{2} - \frac{w_1}{2a_2\delta} - \frac{q_2}{2}, \tag{15}$$

$$q_1 = \frac{1}{2} - \frac{w_1}{2a_1}. \tag{16}$$

Taking the retailer's quantity decision into account, the manufacturer faces the following problem:

$$\max_{w_1, q_2} \pi_m = w_1(q_1 + q_3) + p_2 q_2. \tag{17}$$

Solving the first-order condition, we obtain the equilibrium:

$$q_2 = \frac{1}{2}, w_1 = \frac{a_1a_2\delta}{a_1 + a_2\delta}. \tag{18}$$

$m = p_1 - w_1 = \frac{a_1^2}{2a_1 + 2a_2\delta}$, where m is the retail margin of the retailer.

Note that $q_3 = \frac{1}{4} - \frac{a_1}{2(a_1 + a_2\delta)} > 0$ only when the average consumer willingness-to-pay for the products in market 2 is much higher than that in market 1 (i.e., $a_2\delta > a_1$ should hold). This implies that the gray market emerges in this scenario only if there is significant difference of the average consumer willingness-to-pay between markets.

Further, other players' decisions can be derived by substitutions. We summarize the equilibrium values of the players' decisions and profits in Table 2.

A.4. Equilibrium solution for case RSG

Using backward induction, we first maximize the profit of the third-party parallel importer as follows:

$$\max_{q_{3p}} \pi_{3p} = (p_3 - p_1)q_{3p}. \tag{19}$$

Moreover, the total sales of parallel imports come from the retailer parallel importation and the third-party parallel importation,

respectively. It can be expressed by $q_3 = q_{3p} + q_{3r}$. Substituting $q_3 = q_{3p} + q_{3r}$ into the objective function Eq. (23) and solving the first-order condition, we obtain

$$q_{3p} = \frac{a_2\delta(1-q_2-q_{3r})-a_1(1-q_1)}{2a_2\delta}. \tag{20}$$

Taking third-party parallel importer's quantity decision into account, the retailer faces the following problem:

$$\max_{q_1,q_{3r}} \pi_r = (p_3-w_1)q_{3r} + (p_1-w_1)(q_1 + q_{3p}). \tag{21}$$

The first-order conditions yield

$$q_{3r} = \frac{1}{2}-\frac{w_1}{2a_2\delta}-\frac{q_2}{2}, q_1 = \frac{2a_1^2 + a_1a_2\delta + a_1a_2\delta q_2}{2a_1(a_1 + 2a_2\delta)}-\frac{w_1}{2a_1}. \tag{22}$$

Having the information about the quantities decisions of the retailer and the third party, the manufacturer uses sales quantity in market 2 as well as the wholesale price to maximize the profit.

$$\max_{w_1,q_2} \pi_m = w_1(q_1 + q_3) + p_2q_2. \tag{23}$$

Solving the optimization problem with respect to sales quantity in market 2 q_1 and wholesale price w_1, we have the following equilibrium results:

$$q_2 = \frac{1}{2} + \frac{a_1\delta}{8a_2\delta-6a_2\delta^2 + 4a_1-2a_1\delta}, w_1 = \frac{a_1a_2\delta}{a_1 + a_2\delta}. \tag{24}$$

By substitution, we obtain $q_{3p} = \frac{a_2\delta - a_1 + \frac{a_2\delta}{2}(\frac{a_1\delta}{3a_2\delta^2 + a_1\delta - 4a_2\delta - 2a_1} - 1)}{2a_1 + 4a_2\delta}$ and $q_{3r} = \frac{a_1\delta}{4[3a_2\delta^2 + (a_1 - 4a_2)\delta - 2a_1]}-\frac{a_1}{2(a_1 + a_2\delta)} + \frac{1}{4}$.Finally, the rest of equilibrium of the quantities and profits for the supply chain members are then straightforward to derive.

A.5. Equilibrium solution for case TSG

Using backward induction, we first maximize the profit of the retailer with respect to the quantity of parallel imports q_{3r} as follows

$$\max_{q_{3r}} \pi_r = (p_3-w_1)q_{3r} + (p_1-w_1)(q_1 + q_{3p}). \tag{25}$$

The first-order condition yields

$$q_{3r} = \frac{1}{2}-\frac{q_{3p}}{2}-\frac{w_1}{2a_2\delta}-\frac{q_2}{2}. \tag{26}$$

Considering the retailer's parallel importation quantity decision, the third-party parallel importer faces the following problem

$$\max_{q_{3p}} \pi_{3p} = (p_3-p_1)q_{3p}. \tag{27}$$

Checking the first-order conditions, we have

$$q_{3p} = \frac{w_1-2a_1 + a_2\delta + 2a_1q_1-a_2\delta q_2}{2a_2\delta}. \tag{28}$$

After substituting Eqs. (30) and (32) into the expression $q_3 = q_{3p} + q_{3r}$, we obtain the total sales of parallel imports given by

$$q_3 = \frac{3a_2\delta + 2a_1q_1-2a_1-w_1-3a_2\delta q_2}{4a_2\delta}. \tag{29}$$

The retailer's profit function at the second stage is

$$\max_{q_1} \pi_r = (p_3-w_1)q_{3r} + (p_1-w_1)(q_1 + q_{3p}). \tag{30}$$

By substitutions and applying the first-order condition, we have

$$q_1 = \frac{6a_1^2-3a_1w_1 + a_1a_2\delta-4a_2\delta w_1 + 3a_1a_2\delta q_2}{6a_1^2 + 8a_1a_2\delta}. \tag{31}$$

Having the information about the quantities decisions of the retailer and the third party, the manufacturer sets optimal sales quantity in market 2 as well as the optimal wholesale price.

$$\max_{w_1,q_2} \pi_m = w_1(q_1 + q_3) + p_2q_2. \tag{32}$$

Solving the optimization problem with respect to sales quantity in market 2 q_1 and wholesale price w_1, we have the following equilibrium:

$$q_2 = \frac{a_1\delta(24a_1^2 + 70a_1a_2\delta + 47a_2^2\delta^2)}{(144-72\delta)a_1^3 + (528a_2\delta-312a_2\delta^2)a_1^2 + (640a_2^2\delta^2-434a_2^2\delta^3)a_1 + 256a_2^3\delta^3-192a_2^3\delta^4} + \frac{1}{2},$$

$$w_1 = \frac{(84a_2\delta-42a_2\delta^2)a_1^3 + (202a_2^2\delta^2-127a_2^2\delta^3)a_1^2 + (120a_2^3\delta^3-90a_2^3\delta^4)a_1}{(72-36\delta)a_1^3 + (264a_2\delta-156a_2\delta^2)a_1^2 + (320a_2^2\delta^2-217a_2^2\delta^3)a_1 + (128a_2^3\delta^3-96a_2^3\delta^4)}.$$

Finally, the rest of equilibrium of the quantities and profits for the supply chain members are then straightforward to derive.

A.6. Equilibrium solution for case NG

Using backward induction, we first maximize the profit of the third-party parallel importer and the retailer as follows:

$$\max_{q_{3p}} \pi_{3p} = (p_3-p_1)q_{3p}. \tag{33}$$

$$\max_{q_{3r}} \pi_r = (p_3-w_1)q_{3r} + (p_1-w_1)(q_1 + q_{3p}). \tag{34}$$

Jointly solving the first-order conditions, we obtain equilibrium quantities:

$$q_{3r} = \frac{a_1(1-q_1) + a_2\delta(1-q_2)-2w_1}{3a_2\delta},$$

$$q_{3p} = \frac{a_2\delta(1-q_2)-2a_1(1-q_1) + w_1}{3a_2\delta}.$$

Since the total sales of parallel imports come from the retailer parallel importation and third-party parallel importation respectively, it is given by

$$q_3 = q_{3p} + q_{3r}. \tag{35}$$

Then, in stage 2, the retailer faces the following problem:

$$\max_{q_1} \pi_r = (p_3-w_1)q_{3r} + (p_1-w_1)(q_1 + q_{3p}).$$

Solving the first-order condition, we obtain

$$q_1 = 1-\frac{a_2\delta(14-5q_2)}{2(5a_1 + 9a_2\delta)}-\frac{w_1}{2a_1}.$$

Having the information about the quantities decisions of the retailer and the third party, the manufacturer chooses the optimal selling quantity in market 2 and the wholesale price to maximize the total profit in both market 1 and market 2.

$$\max_{w_1,q_2} \pi_m = w_1(q_1 + q_3) + p_2q_2. \tag{36}$$

Solving the first-order condition yields the equilibriums:

$$q_2 = \frac{1}{2}\left(1-\frac{a_1\delta(15a_1^2 + 54a_1a_2\delta + 44a_2^2\delta^2)}{25a_1^3(-2 + \delta) + 54a_2^3\delta^3(-3 + 2\delta) + 10a_1^2a_2\delta(-23 + 13\delta) + 2a_1a_2^2\delta^2(-171 + 107\delta)}\right),$$

$$w_1 = \frac{a_1a_2\delta(30a_1^2(-2 + \delta) + 51a_2^2\delta^2(-3 + 2\delta) + a_1a_2\delta(-193 + 113\delta))}{25a_1^3(-2 + \delta) + 54a_2^3\delta^3(-3 + 2\delta) + 10a_1^2a_2\delta(-23 + 13\delta) + 2a_1a_2^2\delta^2(-171 + 107\delta)}.$$

Finally, the rest of equilibrium of the quantities and profits for the supply chain members are then straightforward to derive.

Appendix B. Proofs

Proof of Lemma 1. It is straightforward to obtain the results. □

Proof of Lemma 2. It is straightforward to obtain the results. □

Proof of Lemma 3. $m^{TPI}-m^{NGM} = \frac{a_1^2(-4+\delta)}{4(4a_1-a_1\delta+8a_2\delta-4a_2\delta^2)} < 0$; Since $a_2\delta > a_1$, we obtain $m^{RPI}-m^{NGM} = \frac{a_1^2-a_1a_2\delta}{4(a_1+a_2\delta)} < 0$.

$$\pi_m^{RPI}-\pi_m^{NGM} = -\frac{(a_1-a_2\delta)^2}{8(a_1 + a_2\delta)} < 0, \pi_m^{RPI} + \pi_r^{RPI}-(\pi_m^{NGM} + \pi_r^{NGM}) = \frac{a_1}{16}-\frac{a_2\delta}{16} < 0, \pi_r^{RPI}-\pi_r^{NGM} = \frac{3a_1^2-4a_1a_2\delta + a_2^2\delta^2}{16(a_1 + a_2\delta)}$$

.

$$\pi_m^{TPI}-\pi_m^{NGM} = \frac{a_1^3\delta + 8a_1^2a_2\delta + 2a_1^2a_2\delta^2 + 24a_1a_2^2\delta^2 + 8a_2^3\delta^4-4a_1^3-14a_1a_2^2\delta^3-16a_2^3\delta^3}{8(a_1 + 2a_2\delta)(4a_1-a_1\delta + 8a_2\delta-4a_2\delta^2)}$$

$$\pi_r^{TPI}-\pi_r^{NGM} = -\frac{a_1^2(16a_1-8a_1\delta + 32a_2\delta + a_1\delta^2-16a_2\delta^2)}{16(-4a_1 + a_1\delta-8a_2\delta + 4a_2\delta^2)^2} < 0.$$

□

Proof of Lemma 4. It is straightforward to obtain the results. □

Proof of Proposition 1.

$$w_1^{RPI} = \frac{a_1a_2\delta}{a_1 + a_2\delta} > \frac{a_1a_2\delta}{\frac{2}{3}(a_1 + 2a_2\delta)} = w_1^{TPI}, w_1^{TPI} = \frac{3a_1a_2\delta}{2(a_1 + 2a_2\delta)} = \frac{3a_1}{2(\frac{a_1}{a_2\delta} + 2)} > w_1^{NGM}.$$

$$p_2(RPI)-p_2(TPI) = \left[\frac{a_1}{2(a_1 + a_2\delta)} + \frac{3a_2\delta}{2(a_1 + 2a_2\delta)}-\frac{3}{4}\right]a_2\delta > 0, \frac{a_1a_2\delta}{2(a_1 + a_2\delta)}-\frac{1}{4}a_2\delta < 0,$$

$$p_2(TPI)-p_2(NGM) = \frac{a_2\delta}{2}-\frac{3a_2^2\delta^2}{2(a_1 + 2a_2\delta)} < 0.$$

In addition, $q_2(RPI) = q_2(NGM) = \frac{1}{2}$, $q_2(TPI) > \frac{1}{2}$.

This completes the proof. □

Proof of Proposition 2. Proof. Note that gray market only exists when $q_3 > 0$ in the case "Only retailer parallel importation", which requires that $a_2\delta > a_1$. Since $a_2\delta > a_1$, with some simple algebraic manipulations, we can verify that as follows:

$$w_1^{RPI} = w_1^{RSG} = \frac{a_1a_2\delta}{a_1 + a_2\delta} > \frac{a_1a_2\delta}{\frac{2}{3}(a_1 + 2a_2\delta)} = w_1^{TPI}, w_1^{TPI} = \frac{3a_1a_2\delta}{2(a_1 + 2a_2\delta)} = \frac{3a_1}{2(\frac{a_1}{a_2\delta} + 2)} > w_1^{NGM}.$$

This completes the proof. □

Proof of Proposition 3. Proof.

$$q_2^{RSG}-q_2^{TPI} = \frac{a_1\delta}{4a_2\delta^2 + (a_1-8a_2)\delta-4a_1}-\frac{a_1\delta}{6a_2\delta^2 + 2(a_1-4a_2)\delta-4a_1}.$$

Comparing the denominators, we obtain $q_2^{RSG}-q_2^{TPI} > 0$.

In addition, $q_2^{RPI} = q_2^{NGM} = \frac{1}{2}$, $q_2^{TPI} > \frac{1}{2}$. This completes the proof.

Since $a_2\delta > a_1$, with some simple algebraic manipulations, we can verify that as follows:

$$p_2^{RPI}-p_2^{TPI} = \left[\frac{a_1}{2(a_1 + a_2\delta)} + \frac{3a_2\delta}{2(a_1 + 2a_2\delta)}-\frac{3}{4}\right]a_2\delta > 0,$$

$$p_2^{RPI}-p_2^{NGM} = \frac{a_1a_2\delta}{2(a_1 + a_2\delta)}-\frac{1}{4}a_2\delta < 0,$$

$$p_2^{TPI}-p_2^{NGM} = \frac{a_2\delta}{2}-\frac{3a_2^2\delta^2}{2(a_1 + 2a_2\delta)} < 0,$$

$$p_2^{RSG}-p_2^{TPI} = \frac{3a_2^2\delta^2}{4(a_1 + 2a_2\delta)}-\frac{a_2^2\delta^2}{2(a_1 + a_2\delta)} < 0.$$

This completes the proof. □

Appendix B. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.tre.2018.04.006.

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