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Skewness as a probe of baryon acoustic oscillations

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ABSTRACT

In this study, we show that the skewness S_3 of the cosmic density field contains a significant and potentially detectable and clean imprint of baryonic acoustic oscillations (BAOs). Although the BAO signal in the skewness has a lower amplitude than second-order measures like the two-point correlation function and power spectrum, it has the advantage of a considerably lower sensitivity to systematic influences. Because it lacks a direct dependence on bias if this concerns simple linear bias, skewness will be considerably less beset by uncertainties due to galaxy bias. Also, it has a weaker sensitivity to redshift distortion effects. We use perturbation theory to evaluate the magnitude of the effect on the volume-averaged skewness, for various cosmological models. One important finding of our analysis is that the skewness BAO signal occurs at smaller scales than that in second-order statistics. For a lambda cold dark matter spectrum with *Wilkinson Microwave Anisotropy Probe* 7 normalization, the BAO feature has a maximum wiggle amplitude of 3 per cent and appears at a scale of $\sim 82\,h^{-1}$ Mpc. We conclude that the detection of BAO wiggles in future extensive galaxy surveys via the skewness of the observed galaxy distribution may provide us with a useful, and potentially advantageous, measure of the nature of dark energy.

Key words: cosmological parameters – cosmology: theory – dark energy – large-scale structure of the Universe.

1 INTRODUCTION

We live in the era of precision cosmology. The last 20 yr of tremendous progress in astronomical observations allowed us to establish the standard cosmological model, the lambda cold dark matter model (LCDM). The cosmological parameters characterizing the model have been determined to an accuracy of better than a per cent.

Notwithstanding the success of these measurements, we are left puzzled at finding the Universe whose dynamics and fate are dominated by dark energy whose identity remains a mystery. It is not even sure whether it really concerns an energy component to be associated with a new species in the Universe, a cosmological constant or a modification of gravity itself. It is far from trivial to constrain the nature of dark energy, due to the relatively weak imprint of the equation of state of dark energy, in combination with sizeable observational errors (Frieman, Turner & Huterer 2008). Its principal influence is the way in which it affects the expansion of the Universe. In turn, this determines the growth of the cosmic density perturbations.

Since the discovery of the accelerated expansion of the universe on the basis of the luminosity distance of supernovae (Riess et al. 1998; Perlmutter et al. 1999), a range of alternative probes have provided additional constraints on the nature of dark energy. The temperature perturbations in the cosmic microwave background (CMB), weak gravitational lensing by the large-scale matter distribution and the structure formation growth rate are well-known examples of dark energy probes.

The particular probe that we wish to address in this study are baryonic acoustic oscillations (BAOs), the residual leftover in the baryonic and dark matter mass distribution of the primordial sound waves in the photon–baryon plasma in the pre-recombination universe. Following the decoupling between matter and radiation, the primordial sound waves devolve into slight and subtle wiggles in the distribution of galaxies, the 'baryonic wiggles' (Peebles & Yu 1970; Sunyaev & Zeldovich 1970). These have a characteristic size of the order of the sound horizon at decoupling or, more accurately, the baryon drag epoch at which baryons are released from the Compton drag of photons. The value of the latter is around $\sim 150\,h^{-1}$ Mpc for the standard LCDM cosmology.

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The BAO wiggles were first detected in the power spectrum of galaxies in the 2dF Galaxy Redshift Survey (2dFGRS) (Cole et al. 2005) and in the two-point correlation function of the Sloan Digital Sky Survey (SDSS) galaxy redshift surveys (Eisenstein et al. 2005). In the meantime, this has been followed up by a flurry of studies (e.g. Hütsi 2006; Tegmark et al. 2006; Padmanabhan et al. 2007; Percival et al. 2007), and have led to the initiation of a number of large observational programme. Notable examples of surveys that (partially) probe dark energy by means of BAOs are the WiggleZ Dark Energy Survey (Drinkwater et al. 2010), the Baryon Oscillation Spectroscopic Survey (BOSS) (Ross et al. 2010) and the Dark Energy Survey, along with the ambitious ESA Euclid Mission project (Laureijs et al. 2011).

The basic idea behind the use of BAOs as probe of dark energy is to use the sound horizon at decoupling as a standard ruler. Its physical size is known precisely from first principle, and its angular size was measured accurately in the angular spectrum of the microwave background temperature anisotropies. By assessing the size of the BAO characteristic scale over a range of (lower) redshifts, we may directly measure the angular diameter distance as a function of z, and hence constrain the nature of dark energy. The BAO scale can be inferred from the imprint of the sound horizon in the spatial distribution of galaxies, and can be inferred from the correlation function, power spectrum or other measures of spatial clustering (Eisenstein, Hu & Tegmark 1998; Cooray et al. 2001; Blake & Glazebrook 2003; Hu & Haiman 2003; Seo & Eisenstein 2003; Zhan & Knox 2006). It was also suggested that the BAO signature could be measured with a use of higher order statistics. However, this mainly concentrated on bispectrum or three-point function alone (Sefusatti et al. 2006; Gaztañaga et al. 2009). Albeit, while the theoretical underpinning of BAOs is quite straightforward, the practical limitations and complications are considerable. A range of systematic effects, ranging from non-linearities to redshift space distortions and unknown galaxy biasing effects, severely complicates the analysis.

To alleviate some of the practical complications, in this paper we advocate the idea to look for the BAO wiggles in the reduced third moment of the density field, the skewness S_3. According to perturbation theory, the S_3 is sensitive to the second-order logarithmic derivative of the matter power spectrum $P(k)$. The direct implication of this is the imprint of the BAO wiggles in the skewness. This forms a potentially powerful means of probing the BAOs. Unlike the more conventional second-order probes, i.e. two-point correlation function and power spectrum, the skewness is insensitive to the bias of galaxies in case this involves simple linear or local bias. As important is the fact that skewness has been found to be rather marginally affected, in comparison to two-point correlation function or power spectrum, by redshift space distortions (Bouchet et al. 1992, 1995; Hivon et al. 1995).

These observations suggest that the skewness may offer a cleaner probe of the BAOs in the galaxy distribution, less ridden by the systematic uncertainties that still beset the present BAO experiments. In this study, we present the basic results for measuring the signature and scale of BAOs. We assess the BAO signature in the skewness of the cosmic matter density field, by invoking the theoretical power spectrum for a universe filled with a given baryonic fraction of matter. In Section 2, we present the perturbation theoretical basis of the use of skewness as BAO probe, including a discussion of the influence of non-linearities on the skewness estimates. This is followed by a presentation of the results in Section 3 towards the feasibility of relating the skewness to the scale and amplitude of the BAO. Prospects and complications are discussed in Section 4.

2 BAO IN WEAKLY NON-LINEAR PERTURBATION THEORY

In this section, we first outline the basic perturbation theory concepts underlying our BAO analysis, followed by a presentation of the formalism to incorporate the baryonic acoustic signatures in the power spectrum to evaluate the skewness S_3 of the field.

2.1 Skewness and perturbation theory

In our discussion on the skewness S_3 of the density field, we focus on the volume-averaged second- and third-order moments. Invoking the ergodic theorem [1] or fair-sample hypothesis, we can express the volume-averaged J-point correlation function as

$$\bar{\xi}_J = V_W^{-J} \int_S \mathrm{d}\boldsymbol{x}_1 \cdots \mathrm{d}\boldsymbol{x}_J W(\boldsymbol{x}_1) \cdots W(\boldsymbol{x}_J) \xi_J(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_J), \tag{1}$$

where $\boldsymbol{x}_i$ is the comoving separation vector, $W(\boldsymbol{x})$ is a window function with volume

$$V_W = \int_S \mathrm{d}\boldsymbol{x}\, W(\boldsymbol{x}), \tag{2}$$

and the integral covers the entire volume S. Because of the fair-sample hypothesis, $\bar{\xi}_J$ does not depend on the location $\boldsymbol{x}$ and is a function of the window volume V_W only (Peebles 1980) .

The skewness S_3 is the normalized ratio of third-order to second-order moment, defined as

$$S_3(R) = \frac{\bar{\xi}_3(R)}{\bar{\xi}_2^{\,2}(R)} = \frac{\bar{\xi}_3}{\sigma^4}. \tag{3}$$

The skewness of the cosmic density field has been assessed within the context of a perturbation theory analysis (Peebles 1980; Juszkiewicz, Bouchet & Colombi 1993; Bernardeau 1994a). In the linear and quasi-linear regime, Juszkiewicz et al. (1993) and Bernardeau (1994a) found that the skewness S_3 of the field, smoothed by a spherical top-hat window filter of scale R, is given by

$$S_3(R) = \frac{34}{7} + \gamma_1(R). \tag{4}$$

The factor 34/7 would have been obtained if smoothing were not taken into account, and was inferred by Peebles for an $\Omega = 1$ cosmology. The smoothing introduces a dependence on scale, through the logarithmic derivative $\gamma_1(R)$ of the variance,

$$\gamma_1(R) = \frac{\mathrm{d} \log \sigma^2(R)}{\mathrm{d} \log R}, \tag{5}$$

where $\sigma^2(R)$ is the variance within the spherical top-hat window $W_{\mathrm{TH}}(x)$,

$$\sigma^2(R) = \frac{1}{2\pi^2} \int_0^\infty \mathrm{d}k\ k^2 P(k)\, W_{\mathrm{TH}}^2(kR). \tag{6}$$

In terms of the effective slope n_{eff} of the power spectrum at scale R, $\gamma_1(R)$ may be written as

$$\gamma_1(R) = -(n_{\mathrm{eff}} + 3). \tag{7}$$

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[1] The ergodic theorem states that the ensemble average of statistical quantities over a large number of different realizations of a given probability distribution is equal to the average of the same quantity over a sufficiently large volume, representing a statistically fair sample of the statistical process.

Several authors confirmed that equation (4) can be used as a fair estimator of the skewness of the top-hat-filtered density field on linear and weakly non-linear scales (e.g. Juszkiewicz, Bouchet & Colombi 1993; Bernardeau 1994a,b; Baugh, Gaztanaga & Efstathiou 1995; Szapudi et al. 1999).

2.2 Skewness and power spectrum

Perturbation theory may be used towards calculating the expected shape and amplitude of the BAO signal for a given cosmological model with power spectrum $P(k)$. On the basis of a few straightforward mathematical operations, one may infer that the γ_1 factor equation (5) can be expressed in terms of an integral over the power spectrum $P(k)$ of the density field (we present the derivation in Appendix A):

$$\gamma_1(R) = 2 - \frac{2 \int_0^\infty dk\, P(k) j_1(kR) kR \left[j_0(kR) - 2 j_2(kR) \right]}{3 \int_0^\infty dk\, P(k) j_1^2(kR)} , \qquad (8)$$

where j_0, j_1 and the j_2 are the spherical Bessel functions of the zeroth, first and second order.

To obtain insight into the behaviour of γ_1 as a function of cosmology, we use the expressions by Eisenstein & Hu (1998, 1999) to model a linear power spectrum that incorporates BAO wiggles,

$$P(k) = A \cdot k^{n_s} \cdot T(k)^2 , \qquad (9)$$

where n_s is the primordial power spectrum slope, usually close to 1.0. The transfer function $T(k)$ encapsulates the details of the cosmological model, including the baryonic wiggles,

$$T(k) = f_b T^b(k) + (1 - f_b) T^{cdm}(k) , \qquad (10)$$

where $f_b \equiv \Omega_b / \Omega_0$ is the fraction that baryons contribute to the full matter density. The effects of the baryons are multifold. In addition to the relatively small acoustic oscillations, it also includes major effects like Silk damping. To investigate the relative importance of the baryonic oscillations, we compare the value of S_3 for power spectra based on a baryonic transfer function T_b that includes the acoustic oscillations with those of 'no-wiggles' power spectra where these signatures are averaged out.

The amplitude A of the power spectrum is normalized on the basis of the value of the (linear) σ_8 parameter at $z = 0$,

$$A = \sigma_8^2 \Big/ \frac{1}{2\pi^2} \int_0^\infty k^{2+n_s} T^2(k) W_{TH}(k \cdot R_{TH}) , \qquad (11)$$

for $R_{TH} = 8\,h^{-1}$ Mpc. In our study, we have adopted the LCDM model at $z = 0$ as our reference cosmology, using the *Wilkinson Microwave Anisotropy Probe* 7 (*WMAP*7) best-fitting values for the cosmological parameter estimates (data wmap7+bao+h0; Komatsu & et al. 2010): $\Omega_0 h^2 = 0.134$, $\Omega_b h^2 = 0.0226$, $\Omega_\lambda = 0.728$, $\sigma_8 = 0.809$, $n_s = 0.963$, $h = 0.704$.

2.3 Non-linear influences

The linear power spectrum expression of equation (9) will provide us with an impression of the influence of baryonic wiggles on the skewness S_3, via the integral expression of equation (8), for any given set of cosmological parameters.

However, even on scales as large as those characteristic for BAO wiggles, $R \sim 100\,h^{-1}$ Mpc, the effects of non-linear evolution of the density field become noticeable. Non-linear amplitude growth and mode coupling will affect the BAO signal, strongly on non-linear scales and subtly but significantly on quasi-linear scales.

A range of studies have demonstrated that non-linearities do indeed represent a major influence. On the basis of the analytical fitting formula by Smith et al. (2003), Guzik, Bernstein & Smith (2007) showed that the per cent-level shift of the acoustic peak location in the two-point correlation function may be expected. Significant shifts were also found by the study of Smith, Scoccimarro & Sheth (2007), on the basis of perturbation theory and on the basis of the power spectrum of haloes in simulations. Following a similar route, Angulo et al. (2008) looked at the impact of non-linearities on the power spectrum of galaxies in semi-analytical models. They conclude that per cent-level shifts cannot be excluded. The thorough analytical study by Crocce & Scoccimarro (2008) has provided particularly strong insight into this issue, supporting the claims for a non-negligible influence.

To account for the non-linear power spectrum evolution, we have decided to use the well-known HALOFIT formula of Smith et al. (2003). We are aware of the fact that this approach may not be the most accurate. However, we can treat this procedure as a first-order approximation of the non-linear effects. The full study of the impact of the non-linearities calls for *N*-body experiments and will be presented in the accompanying paper.

2.4 BAO impact on power spectra

Fig. 1 compares the power spectra – for both linear as well as non-linear situations – that incorporate the BAO wiggles with those of 'no-wiggle' ones where these are smoothed out. To compare these we plot the ratio of the spectrum with BAO wiggles to the 'no-wiggle' one, so that the unity baseline represents the corresponding 'no-wiggle' spectrum.

We compare three pairs of spectra. The first pair compares the linear BAO wiggle spectrum to the corresponding linear 'no-wiggle'
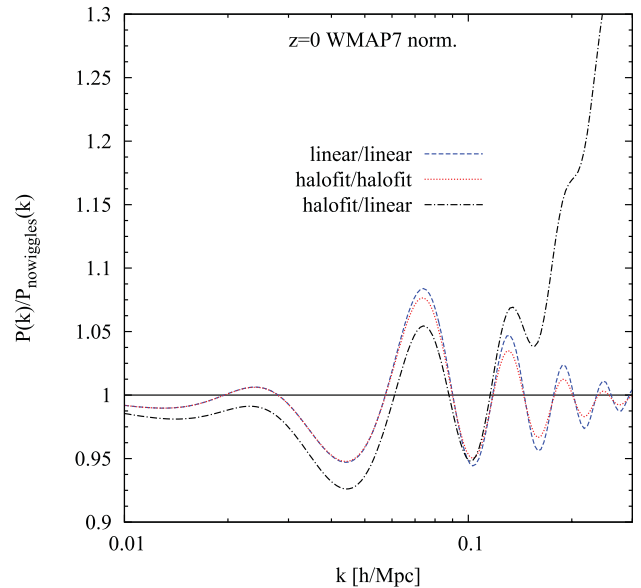
Figure 1. BAO imprint in power spectrum. To highlight the effect of the acoustic oscillations, we plot the ratio of the spectrum including the BAO wiggles to that of the corresponding power spectrum without BAO wiggles. Dashed line: ratio of linear power spectra with and without BAO. Dotted line: ratio of non-linear HALOFIT-corrected power spectra with and without BAO. Dash–dotted line: ratio of non-linear HALOFIT-corrected power spectrum to linear 'no-wiggle' spectrum. The latter highlights the effects of non-linearities.

one. For both, we use the outcome from the Eisenstein and Hu expression equation (10). The second pair consists of the non-linear spectrum with BAO wiggles and the corresponding non-linear one with wiggles smoothed out. Both spectra are obtained from the corresponding linear spectra, subsequently transformed via the HALOFIT formula. The third pair compares the full non-linear, HALOFIT-corrected spectrum with baryonic wiggles to that of the no-wiggles linear spectrum.

The first pair nicely illustrates the imprint of the BAOs. The impact of non-linearities is most starkly exemplified by the third pair, revealing a dramatic rise of the spectral amplitude at high frequencies. However, the effect of acoustic wiggles is not dramatically changed when comparing the corresponding non-linear spectrum to the non-linearly evolved 'no-wiggle' one. Its principal effect is merely a lowering of amplitude, of not more than a few per cent and mainly visible at higher order harmonics.

2.5 Skewness and non-linearities

On the basis of the power spectra in Fig. 1, we may expect that non-linearities will slightly affect the estimates of the skewness S_3. The lowering of the amplitude of the BAO wiggles in the non-linear regime should lead to a weakening of the BAO signal in S_3.

Fig. 2 plots the skewness, computed from equations (4) and (8), for linear and non-linear power spectra. To enhance our appreciation of the effect that the BAOs have on the result, we plot the ratio of the skewness obtained for a power spectrum with the acoustic wiggles to that of the corresponding 'no-wiggle' spectrum. The dashed line marks the ratio of $S_3(R)/S_3^{\text{nowiggles}}(R)$ for the linear power spectra, the dotted line for the non-linear HALOFIT-corrected spectra.

The first observation is that we indeed find a clear imprint of the baryonic oscillations in the skewness S_3, in both the linear and the non-linear situations at the 2–3 per cent level. The amplitude of the BAO signal in S_3 is slightly smaller for the non-linear situation than the linear one. Of real importance is the observation that the
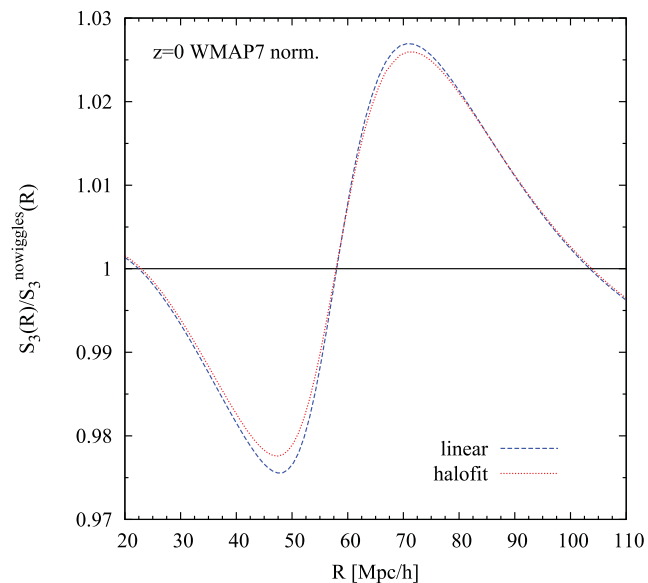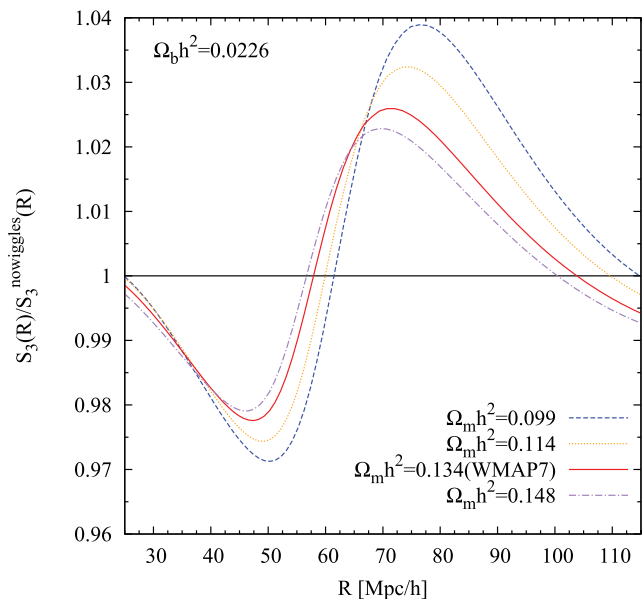
Figure 3. The acoustic oscillations imprint in the skewness S_3. In all cases, we show the skewness computed using non-linear $P(k)$ with wiggles divided by the skewness computed for 'no-wiggles' power spectrum. The lines depict models with different values of $\Omega_m h^2$: 0.099 – the dashed line, 0.114 – the dotted line, 0.134 (*WMAP*7) – the solid line and 0.148 marked by dash–dotted line. For all models the baryon density has the same value $\Omega_b h^2 = 0.0226$.

non-linear evolution of the power spectrum does not lead to a shift in the scale dependence of the skewness signal. The scale at which we find the peak and the minimum in the BAO impact on S_3 remains the same. Also, there is no change in the scale at which the BAO effect changes from negative to positive. As we will discuss later, the latter will be of crucial significance.

3 RESULTS

Having established the basic ingredients of our study, we next turn to the principal results of this study.

3.1 Baryonic fraction

Fig. 3 plots the ratio of the skewness for a spectrum with and without wiggles, $S_3(r)/S_3^{\text{nowiggles}}(r)$, for a range of LCDM cosmologies in which baryons constitute a different fraction of the non-relativistic matter content of the Universe. The baryon density is the same for all models, $\Omega_b h^2 = 0.0226$, a value which is tightly constrained by the CMB temperature anisotropy power spectrum (e.g. Komatsu & et al. 2010). The total matter density varies from $\Omega_m h^2 = (0.099, 0.114, 0.134, 0.148)$.

In all cosmologies, the BAO skewness signal consists of a characteristic pattern. Starting from small scales, we observe a decrease of the skewness as the scale increases, reaching a minimum at scales of roughly 45–50 h^{-1} Mpc. Subsequently, it climbs towards values larger than unity, which it reaches at a scale of ∼55–60 h^{-1} Mpc, after which it increases towards a peak value of 2–4 per cent at a scale of around 75–80 h^{-1} Mpc. Towards larger scales it gradually falls off towards unity.

We immediately observe that the BAO skewness signal increases strongly as Ω_m is lower and baryons constitute a more prominent fraction of matter content of the Universe. While the amplitude

Figure 2. Reduced skewness and non-linearities. The lines depict the ratios of skewness computed using spectra including BAO wiggles to that computed with 'no-wiggles' spectra. Dashed line: skewness following from linear power spectra. Dotted line: skewness following from non-linear HALOFIT-corrected power spectra.

of the BAO skewness signal is responding systematically to the cosmic baryon fraction, it does not seem to exceed the $\sim$3–4 per cent level. In this sense, the BAO imprint is smaller than that seen in the corresponding power spectrum of density perturbations, where the wiggles may amount to a $\sim$10 per cent effect.

3.2 Skewness scale and sound horizon

We also make the interesting observation that the main wiggle in the skewness appears at a considerably smaller scale than that of the BAO peak in the two-point correlation function $\xi_2(r)$ (e.g. Crocce & Scoccimarro 2008). This goes along with the finding that the BAO imprint on the skewness is visible over a considerable larger range of scales than that in the two-point correlation function. While for the latter, the BAO signal is noticeable on scales from roughly 80 to 130 h^{-1} Mpc, the skewness is affected over a wider range of 20–100 h^{-1} Mpc.

While the amplitude change of the skewness is closely related to the effective baryon fraction f_b of a cosmological model, the scales associated with the wiggle should be correlated with the scale of the acoustic horizon. The scale of the acoustic horizon r_s is a fundamental aspect of the physics of BAOs. It is the physical scale of the largest acoustic oscillations at the epoch of recombination, and as such is the scale imprinted in the BAOs visible in the distribution of galaxies and baryons in the Universe. Determining the apparent BAO scale over a wide range of redshifts, and relating it to the sound horizon, is the goal of a large number of current and future galaxy redshift surveys (Meiksin, White & Peacock 1999; Seo & Eisenstein 2003; White 2005; Seo & Eisenstein 2007).

To enable the exploitation of the information content of the skewness on the acoustic oscillations, it is therefore of fundamental importance to establish the connection of the sound horizon scale with the characteristic scales we find in the BAO feature in S_3. To this end, we compare the sound horizon scale r_s for a range of cosmologies to the crossing scale r_c at which the effect of BAO wiggles on the skewness reverses from negative to positive (see Figs 2 and 3). This scale is clearly a characteristic for the BAO imprint on skewness, and may be formally defined as the scale r_c at which

$$\Delta_{S_3}(r_c) \equiv \frac{S_3(r_c)}{S_3^{\text{nowiggles}}(r_c)} = 1. \qquad (12)$$

In other words, it is the scale at which $\Delta_{S_3}(r) - 1$ changes its sign from negative to positive. We have also checked that the r_c scale is corresponding with the location of the inflection point of the S_3 curve near that scale.

We have measured the characteristic crossing scale for all models, and related them to the corresponding sound horizon scales and redshift of the baryon drag epoch – z_d (Eisenstein & Hu 1999). We list the measured values in Table 1. To obtain a good under-

Table 1. The sound horizon scale r_s at the baryon drag epoch z_d and the corresponding crossing scale ion S_3 signal r_c. All models are computed with fixed $\Omega_b h^2 = 0.0226$. Values of r_c are obtained with use of $h = 0.704$.

$\Omega_m h^2$	$r_s(z_d)$ (Mpc)	r_c (Mpc)	z_d
0.099	164.82	87.24	1016.04
0.114	159.48	85.13	1018.14
0.134	152.95	82.24	1020.48
0.148	149.1	80.54	1021.67
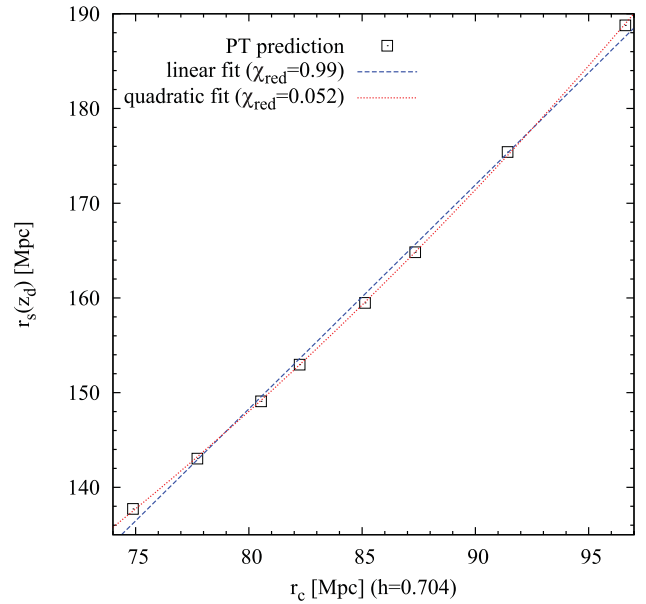
Figure 4. The dependence of the sound horizon scale (r_s) on the crossing scale (r_c). The boxes mark the values predicted by PT skewness values (see Fig. 3). Two lines show our fit to this scaling: the linear fit (the dashed line) and quadratic fit (the dotted line). The values of r_c where rescaled to Mpc using the value of $h = 0.704$.

standing of the systematic relation between the two quantities, we have also evaluated their values for a few cosmological models that are currently disfavoured by observations, involving very low and very high Ω_m values. In Fig. 4, we have plotted the values of the sound horizon scale $r_s(z_d)$ versus the measured characteristic skewness crossing scale r_c. The figure shows that the relation is close to linear.

In order to assess whether a higher order function would fit the relation between r_c and r_s better, we evaluated a first-order and a second-order fit,

$$r_s = a_l \cdot r_c + b_l \qquad \text{(linear)}, \qquad (13)$$

$$r_s = a_q \cdot r_c^2 + b_q \cdot r_c + c_q \qquad \text{(quadratic)}. \qquad (14)$$

For the relation shown in Fig. 4, we have found $a_l = 2.365$ and $b_l = -40.92$ for the linear fit, with $\chi_{\text{red}}^2 = 0.99$. The quadratic fit appears to be considerably better, and has a $\chi_{\text{red}}^2 = 0.052$ at parameters $a_q = 0.0187$, $b_q = -0.934$ and $c_q = 95.538$.

The important conclusion from this result is that it is indeed possible to determine the sound horizon scale at the baryon drag epoch, $r_s(z_d)$, if we can succeed in successfully measuring the characteristic skewness crossing scale r_c.

3.3 Skewness BAO signature

As a final aspect of our analysis, in Fig. 5 we plot the behaviour of the reduced skewness as a function of (top-hat) scale R, at scales where the BAO feature is most prominent. The red solid line represents the skewness, as predicted by perturbation theory, for a power spectrum with BAO wiggles. For comparison we also plot the skewness resulting from the power spectrum with a 'no-wiggle' transfer function.

For the spectrum with wiggles, we find a characteristic shoulder at around the crossing scale. The conclusion from this comparison
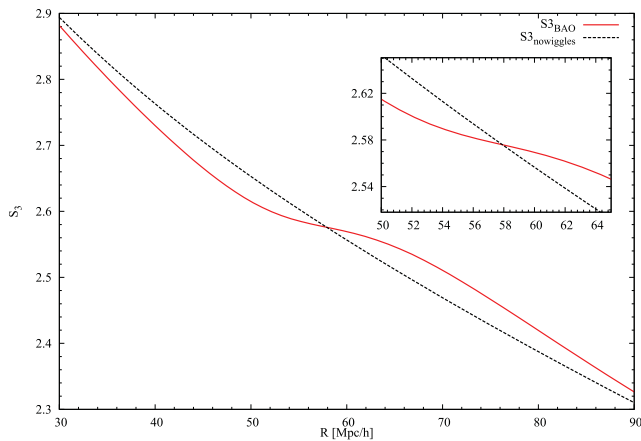
Figure 5. Skewness BAO feature. The red solid line represents the volume-averaged skewness as function of top-hat scale. It shows the characteristic skewness BAO feature, a mild shoulder around the characteristic crossing scale r_c. For comparison, the dotted line represents the skewness for a 'no-wiggle' power spectrum. The inset zooms in on the shoulder at the crossing scale r_c.

is that it should indeed be possible, at the 1–2 per cent level, to find the imprint of BAOs. In an idealized survey, without systematic errors and bias, it should therefore be possible to detect the BAO feature.

4 SUMMARY AND DISCUSSION

In this report, we have studied the imprint of BAOs on the skewness of the density field, i.e. its reduced third moment. The skewness of the density field has been found earlier to be a robust test of the gravitational instability mechanism on cosmological scale. Amongst others, it may function as a probe of the nature of gravity (e.g. Hellwing, Juszkiewicz & van de Weygaert 2010), and of the initial conditions itself (White 1999; Durrer et al. 2000). This has prodded us to investigate whether the skewness could also help towards inferring cosmological relevant information via BAOs.

To investigate the question of the BAO sensitivity of skewness, we have resorted to perturbation theory. From this, we obtain an integral expression for the skewness, dependent on the power spectrum. To model the BAOs, we have used the linear power spectra expressions of Eisenstein & Hu (1998, 1999). To take account of the non-linearities in the evolving matter density field, we use the approximate analytical expressions of Smith et al. (2003).

We find that the BAO skewness signal is characteristic and may not only be detectable in the observational reality, but may even offer a potentially powerful alternative to existing means of probing BAOs. The BAO skewness signal appears to have an amplitude of the order of 3–4 per cent. Interestingly, it stretches out over a substantial range of scales, from ∼20 to $100\,h^{-1}$ Mpc and occurs at much smaller scales than the BAO imprint on power spectrum and correlation function. Perhaps most importantly, we have established a strict, near-linear relation between the sound horizon scale at the baryon drag epoch and the scale at which the BAO skewness signal crosses from negative to positive.

Even though the amplitude of the BAO skewness signal is somewhat lower than that of the BAO wiggles in the power spectrum and correlation function, it is far less sensitive to several systematic effects that still represent a major challenge for inferring cosmological parameters on the basis of BAO measurements in observational

surveys. One major advantage of the BAO skewness signal over that in second-order measures, like the two-point correlation function and power spectrum, is that it is less sensitive to the bias of the galaxy population with respect to the mass distribution. If the galaxy density is a local function of the mass density, the relation between the skewness and the variance of the density field is preserved (Fry & Gaztanaga 1993; Juszkiewicz et al. 1995). In other words, the shape of the S_3-reduced skewness as a function of scale is preserved when one concerns linear and local biasing of the density field. A second, and perhaps even more prominent advantage, is that the skewness – in the weakly non-linear regime – is relatively insensitive to redshift space distortions (Bouchet et al. 1992, 1995; Hivon et al. 1995).

While this short publication is meant to establish the feasibility of using skewness to probe BAOs, there are numerous issues and details that will be discussed and evaluated in an upcoming study. One major issue is the influence of non-linearities in the density field. Subtle non-linear effects may go beyond the scope of what the non-linear HALOFIT power spectra may model. Also, the complexities of galaxy bias may only be adequately modelled by means of large N-body simulations and realistic galaxy formation models. On the other hand, the current observational estimates of S_3 from the SDSS (Ross, Brunner & Myers 2007) and 2dFGRS (Croton et al. 2004, 2007) are characterized by relative big errors. This suggests that detection of the BAO feature will be difficult. In the end, proper modelling of hydrodynamical and radiative processes will be essential for a truly full treatment of non-linear and biasing effects (Guillet, Teyssier & Colombi 2010).

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APPENDIX A: DERIVATION OF THE ANALYTICAL FORM FOR THE $\gamma_1(R)$

To derive expression (8) for the γ_1 parameter, we proceed as follows. The γ_1 parameter is defined as

$$\gamma_1 \equiv -\frac{\mathrm{d}\log\sigma^2(R)}{\mathrm{d}\log R}, \qquad (A1)$$

with $\sigma^2(R) \equiv \langle\delta^2(R)\rangle$ the second moment of the density contrast smoothed on (comoving) scale R. For a top-hat filter, the latter is defined as

$$\delta_R(\boldsymbol{x}) = \int \delta(\boldsymbol{x}')W_R(|\boldsymbol{x} - \boldsymbol{x}'|)\,\mathrm{d}^3x', \qquad (A2)$$

where the expression for the top-hat filter $W_R(x)$ is

$$W_R(x) \equiv \begin{cases} 3/4\pi R^3, & x \le R, \\ 0, & x > R. \end{cases} \qquad (A3)$$

Using the integral expression for the density contrast, we find the following integral expression for the second moment of the density field (Peebles 1980):

$$\langle\delta_R^2(\boldsymbol{x})\rangle = \iint \mathrm{d}^3x'\mathrm{d}^3x''\langle\delta(\boldsymbol{x}')\delta(\boldsymbol{x}'')\rangle W_R(|\boldsymbol{x}-\boldsymbol{x}'|)W_R(|\boldsymbol{x}-\boldsymbol{x}''|)$$
$$= \iint \mathrm{d}^3x'\mathrm{d}^3x''\xi(\boldsymbol{x}'-\boldsymbol{x}'')W_R(\boldsymbol{x}')\cdot W_R(\boldsymbol{x}''). \qquad (A4)$$

It is convenient to evaluate this integral expression in Fourier space. To this end, we use the following Fourier conventions:

$$\delta(\boldsymbol{x}) = (2\pi)^{-3/2}\int \delta_k \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}\mathrm{d}^3k,$$
$$\delta_k = (2\pi)^{-3/2}\int \delta(\boldsymbol{x})\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\mathrm{d}^3x. \qquad (A5)$$

Recasting the double integral in equation (A4) to Fourier space yields

$$\sigma^2(R) = (2\pi)^{-3}\iint \mathrm{d}^3k\mathrm{d}^3k'\langle\delta_k\delta_{k'}\rangle\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}+i\boldsymbol{k}'\cdot\boldsymbol{x}}W(kR)W(k'R), \quad (A6)$$

leading to the following expression in terms of the power spectrum $P(k)$,

$$\sigma^2(R) = (2\pi)^{-3}\int \mathrm{d}^3k\,P(k)W^2(kR)$$
$$= \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k\,k^2 P(k)W^2(kR), \qquad (A7)$$

where the power spectrum is defined as the Fourier transform of the variance,

$$\langle\delta_k\delta_{k'}\rangle = P(k)\delta_D(\boldsymbol{k}+\boldsymbol{k}'), \qquad (A8)$$

in which δ_D is the Dirac delta function. The Fourier expression for the top-hat filter, $W(kR)$, is given by

$$W(kR) = \frac{3}{(kR)^3}(\sin kR - kR\cos kR) = \frac{3}{x}j_1(x), \qquad (A9)$$

with $j_1(x)$ the first-order spherical Bessel function.

Differentiation of $\sigma^2(R)$ by R yields

$$\frac{\mathrm{d}\sigma_R^2}{\mathrm{d}R} = \frac{\mathrm{d}\sigma_R^2}{\mathrm{d}R} = \frac{1}{\pi^2}\int_0^\infty \mathrm{d}k\,P(k)k^3 W(kR)\frac{\mathrm{d}W(kR)}{\mathrm{d}(kR)}, \qquad (A10)$$

where we have used the fact that

$$\frac{\mathrm{d}W^2(kR)}{\mathrm{d}R} = 2kW(kR)\frac{\mathrm{d}W(kR)}{\mathrm{d}(kR)}. \qquad (A11)$$

From this, we may immediately find the expression for γ_1,

$$\gamma_1 = -\frac{\mathrm{d}\log\sigma^2(R)}{\mathrm{d}\log R}$$
$$= -\frac{2\int_0^\infty \mathrm{d}k\,P(k)k^2(kR)W(kR)\frac{\mathrm{d}W(kR)}{\mathrm{d}(kR)}}{\int_0^\infty \mathrm{d}k\,P(k)k^2 W^2(kR)}. \qquad (A12)$$

Recalling the fact that the spherical top-hat filter is directly related to the first-order Bessel function, $W(x) = \frac{3}{x}j_1(x)$, and using the derivation relations for Bessel functions, we find for the derivative of the top-hat function,

$$\frac{\mathrm{d}W}{\mathrm{d}x} = -\frac{3}{x^2}j_1(x) - \frac{3}{x}j_1'(x) = -\frac{3}{x^2}j_1(x) + \frac{1}{x}\left[j_0(x) - 2j_2(x)\right], \qquad (A13)$$

and inserting this relation into equation (A12), after some manipulation immediately leads to the following expression:

$$\gamma_1(R) = 2 - \frac{2\int_0^\infty \mathrm{d}k\,P(k)j_1(kR)kR\left[j_0(kR) - 2j_2(kR)\right]}{3\int_0^\infty \mathrm{d}k\,P(k)j_1^2(kR)}. \qquad (A14)$$

This relation for γ_1 is the central expression of this paper.

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