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BLACK HOLE FORMATION IN PRIMORDIAL GALAXIES: CHEMICAL AND RADIATIVE CONDITIONS

DOMINIK R. G. SCHLEICHER^{1,2}, MARCO SPAANS³, AND SIMON C. O. GLOVER⁴

¹ ESO Garching, Karl-Schwarzschild-Str. 2, 85748 Garching bei München, Germany; dschleic@eso.org

² Leiden Observatory, Leiden University, P.O. Box 9513, NL-2300 RA Leiden, The Netherlands

³ Kapteyn Astronomical Institute, University of Groningen, P.O. Box 800, 9700 AV, Groningen, The Netherlands

⁴ Zentrum für Astronomie der Universität Heidelberg, Institut für Theoretische Astrophysik, Albert-Ueberle-Str. 2, D-69120 Heidelberg, Germany

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ABSTRACT

In massive primordial galaxies, the gas may directly collapse and form a single central massive object if cooling is suppressed. H_2 line cooling can be suppressed in the presence of a strong soft-ultraviolet radiation field, but the role played by other cooling mechanisms is less clear. In optically thin gas, $\text{Ly}\alpha$ cooling can be very effective, maintaining the gas temperature below 10^4 K over many orders of magnitude in density. However, the large neutral hydrogen column densities present in primordial galaxies render them highly optically thick to $\text{Ly}\alpha$ photons. In this paper, we examine in detail the effects of the trapping of these $\text{Ly}\alpha$ photons on the thermal and chemical evolution of the gas. We show that despite the high optical depth in the Lyman series lines, cooling is not strongly suppressed, and proceeds via other atomic hydrogen transitions. At densities larger than $\sim 10^9 \text{ cm}^{-3}$, collisional dissociation of molecular hydrogen becomes the dominant cooling process and decreases the gas temperature to about 5000 K. The gas temperature evolves with density as $T \propto \rho^{\gamma_{\text{eff}}-1}$, with $\gamma_{\text{eff}} = 0.97\text{--}0.98$. The evolution is thus very close to isothermal, and so fragmentation is possible, but unlikely to occur during the initial collapse. However, after the formation of a massive central object, we expect that later-infalling, higher angular momentum material will form an accretion disk that may be unstable to fragmentation, which may give rise to star formation with a top-heavy initial mass function.

Key words: atomic processes – cosmology: theory – dark ages, reionization, first stars – molecular processes – stars: Population III

1. INTRODUCTION

Supermassive black holes with masses $M > 10^8\text{--}10^9 M_\odot$ are known to have existed at very early times in the history of the universe (Jiang et al. 2009), prompting one to ask how black holes of such a size could have formed so quickly. They may have started life as stellar-mass objects, the compact remnants of the first generation of stars, and they may have grown to their present size by accretion. However, forming supermassive black holes in this way presents a number of difficulties. The first stars are thought to have had masses on the order of $100 M_\odot$ or more, making them strong sources of ionizing radiation (Abel et al. 2002; Bromm & Larson 2004; Glover 2005; Yoshida et al. 2008). The dark matter halos in which these stars formed were small, with masses of only $10^6 M_\odot$, and models of the effects of the ionizing radiation produced by a primordial star within one of these small halos show that it readily photo-evaporates the bulk of the gas from the halo, leaving little gas available to be accreted by the black hole (Johnson & Bromm 2007; Milosavljevic et al. 2009; Alvarez et al. 2009). Although accretion is likely more efficient in the $5\text{--}6\sigma$ peaks that harbor the observed supermassive black holes, it is still constrained by the Eddington limit. As discussed by Shapiro (2005), seed masses of at least $10^5 M_\odot$ are required for a magnetohydrodynamic disk or a standard thin disk. Similarly, Schleicher et al. (2010) found that high Eddington ratios and unusually low mass–energy conversion efficiencies are needed for stellar progenitors.

Previous studies have therefore examined the possibility of forming these large seed black holes by direct gravitational collapse (Eisenstein & Loeb 1995; Koushiappas et al. 2004; Begelman et al. 2006; Spaans & Silk 2006). Gas falling into a halo with a mass $M > 5 \times 10^7 [(1+z)/10]^{-3/2} M_\odot$ (correspond-

ing to a virial temperature $T_{\text{vir}} > 10^4$ K) will be shock-heated up to T_{vir} , and so will become hot enough to cool via the electronic emission lines of atomic hydrogen. The fate of the gas in one of these so-called “atomic cooling” halos will then depend on its subsequent thermal evolution. If the gas can cool efficiently as it collapses, significantly lowering both its temperature and its Jeans mass, then it is very difficult to prevent it from fragmenting, and hence forming stars rather than a massive black hole (Bromm & Loeb 2003; Omukai et al. 2008; Clark et al. 2009). On the other hand, if cooling remains inefficient and the effective equation of state for the gas is stiffer than isothermal, then fragmentation is suppressed (Li et al. 2005; Lodato & Natarajan 2006) and the formation of a massive black hole is a plausible outcome. In gas that has been enriched with heavy elements and dust from the first generation of stars, it appears that cooling and fragmentation cannot be avoided (Omukai et al. 2008). On the other hand, in primordial gas the only effective low-temperature coolant is molecular hydrogen (H_2), and if enough of this can be destroyed by photodissociation, then cooling can be suppressed (Begelman et al. 2006; Dijkstra et al. 2008; Regan & Haehnelt 2009; Shang et al. 2010). The soft-UV background needs to be strong enough to suppress H_2 formation until densities of $\sim 10^5 \text{ cm}^{-3}$ are reached, following which collisional dissociation will suppress the H_2 abundance for gas temperatures greater than a few 1000 K. The required background field depends on the chemical model, the spectral shape, and the potential presence of additional electrons created in shock fronts. It is on the order of $J_{21} = 10^3\text{--}10^5$ (Omukai 2001; Bromm & Loeb 2003; Shang et al. 2010). These values are much larger than estimates of the mean strength of the background, which range from $J_{21} \sim 0.1$ (Johnson et al. 2008) to $J_{21} \sim 40$ (Dijkstra et al. 2008). However, recent work has shown that the radiation background is highly inhomogeneous, owing to the strong

clustering of the first generation of star-forming galaxies (Dijkstra et al. 2008; Ahn et al. 2009). In regions close to these galaxies, the ultraviolet background can be very large, and Dijkstra et al. (2008) have recently demonstrated that enough massive primordial galaxies with a local value of $J_{21} > 10^3$ may exist. In addition, early reionization in local patches of the universe may suppress star formation in some primordial halos in a similar fashion (Johnson 2009). They may thus remain pristine, but grow in mass until they can collapse.

In the absence of H_2 cooling, emission from the Lyman series lines of atomic hydrogen, especially $Ly\alpha$, plays a central role in regulating the temperature of the collapsing gas. Hydrodynamic models of the collapsing gas typically assume that the Lyman series lines remain effectively optically thin, allowing the optically thin form of the cooling rate to be used. These models find that the temperature of the collapsing gas decreases with increasing density, albeit very slowly, typically reaching $T \sim 6000\text{--}7000$ K at densities on the order of $n \sim 10^9 \text{ cm}^{-3}$. However, the large hydrogen column densities present in these protogalaxies produce very large optical depths in the Lyman series lines, and so the assumption that the optically thin form of the cooling rate can safely be used is highly questionable. Spaans & Silk (2006) examined the effects of $Ly\alpha$ photon trapping within an analytic framework for the evolution of the gas and showed that in the absence of additional cooling mechanisms, it gives rise to a temperature evolution described by

$$\gamma_{\text{eff}} = 1 + \frac{d \log T}{d \log \rho} \sim 1 - \frac{0.5 - 7/18 \cdot Bn^{7/18}}{\log Cn^{1/2} + Bn^{7/18}}, \quad (1)$$

where ρ is the mass density, T the temperature, n the number density, $B \sim 0.5\text{--}0.1 \text{ cm}^{7/6}$, and $C \sim 10^{-36} M_h / (10^7 M_\odot) \text{ cm}^{3/2}$, with M_h the halo mass. For halo masses in the range $10^7\text{--}10^9 M_\odot$, this yields $\gamma_{\text{eff}} - 1 \sim 0.01\text{--}0.5$, indicating an effective equation of state that is stiffer than isothermal, and hence a temperature that *increases* with increasing density. This result suggests that the neglect of the effects of photon trapping is not only a poor approximation, but also may lead to qualitatively incorrect results, as it is crucial for fragmentation whether the effective equation of state is harder than isothermal.

In this paper, we re-examine the issue of $Ly\alpha$ photon trapping, using a considerably more detailed treatment than in Spaans & Silk (2006). In particular, we address the issue of whether there are other cooling mechanisms that can compensate for $Ly\alpha$ if the latter is strongly suppressed.

2. METHOD

To follow the thermal and chemical evolution of the collapsing gas, we make use of a one-zone treatment in which the form of the density evolution is prescribed in advance. One-zone models are widely used for studying the chemistry and thermodynamics of primordial or low metallicity gas (e.g., Omukai 2001; Omukai et al. 2005; Cazaux & Spaans 2009; Glover & Savin 2009; Schleicher et al. 2009), as they enable one to model the chemistry in great detail and to include a large number of different cooling processes, without the computational efficiency concerns inherent to a three-dimensional treatment.

We extend the one-zone model developed by Glover & Savin (2009) and Schleicher et al. (2009) to include effects that become important at higher temperatures and in the presence of $Ly\alpha$ trapping. Near $T \sim 10^4$ K, H^- formation cooling may contribute to the overall cooling rate (Omukai 2001). We assume that a typical electron undergoing radiative attachment to form H^-

has an energy on the order of $k_B T$, where k_B is Boltzmann's constant, and hence write the H^- formation cooling rate as

$$\Lambda_{H^-} = k_{H^-} n_H n_e k_B T, \quad (2)$$

where k_{H^-} is the H^- formation rate coefficient, n_H is the atomic hydrogen number density, and n_e is the electron number density. To model the effects of $Ly\alpha$ trapping, we include different level populations as separate species in the code. In our model, we consider energy levels up to $n = 5$. For the first excited state, we distinguish between the $2s$ and the $2p$ states, as only decays from the $2p$ state to the ground state will produce $Ly\alpha$ photons; two-photon decays from the $2s$ state will produce continuum photons that will not be trapped. The other states are considered as averages over the angular momentum quantum numbers. Radiative decay rates A_{ij} , collisional excitation and de-excitation rates C_{ij} , collisional ionization and three-body recombination rates, and collisional transition rates between the $2s$ and $2p$ state are all adopted from Omukai (2001). We define the transition rate R_{ij} from level i to level j as

$$R_{ij} = A_{ij} \beta_{\text{esc},ij} (1 + Q_{ij}) + C_{ij}, \quad i > j, \quad (3)$$

$$R_{ij} = \frac{g_j}{g_i} A_{ji} \beta_{\text{esc},ji} Q_{ji} + C_{ij}, \quad i < j, \quad (4)$$

where $\beta_{\text{esc},ij}$ is the escape probability for the $i \rightarrow j$ transition, $g_i = 2i^2$ denotes the statistical weight of level i and $Q_{ij} = c^2 J_{\text{cont},ij} / (2h\nu_{ij}^3)$, where ν_{ij} is the frequency of the transition and $J_{\text{cont},ij}$ the average intensity of the background radiation field at this frequency. The spectral shape of the background radiation field is taken to be that of a blackbody with a temperature of 10^5 K (Glover & Savin 2009), and the normalization is set by fixing the value of J_{21} . We assume that UV photons more energetic than 13.6 eV are absorbed by atomic hydrogen in the intergalactic medium, and we do not consider the effects of an extragalactic X-ray background. For radiative or collisional transitions to the first excited state, we assume that the reaction products will be distributed according to the statistical weights of the $2s$ and $2p$ states. However, our results are not sensitive to this assumption, as collisional transitions between these two states occur rapidly in the conditions of interest, and so the ratio between the $2s$ and $2p$ level populations is always very close to equilibrium.

For the escape probability $\beta_{\text{esc},ij}$, we adopt the expression

$$\beta_{\text{esc},ij} = \frac{1 - \exp(-\tau_{ij})}{\tau_{ij}} \exp(-\beta t_{\text{ph}} / t_{\text{coll}}), \quad (5)$$

where τ_{ij} is the optical depth at line center of the $i \rightarrow j$ transition, t_{coll} the collapse time of the gas, and t_{ph} is the photon diffusion time, i.e., the time required for a photon to diffuse out of the optically thick gas, and β a geometrical factor. Even small deviations from spherical symmetry lead to escape along a preferred direction and $\beta = 3$ (Dijkstra et al. 2006; Spaans & Silk 2006). In our model, the collapse time is the free-fall time, corrected by a factor that takes into account the effective equation of state (see Omukai et al. 2005; Schleicher et al. 2009). For most line transitions, $t_{\text{ph}} \ll t_{\text{coll}}$, such that the exponential factor is negligible. For these lines, we follow Omukai (2000) and assume that the dominant contribution to the optical depth comes from material within one local Jeans length, and that the density, temperature, etc., do not vary significantly on this scale. This is justified provided that the collapse is close to isothermal,

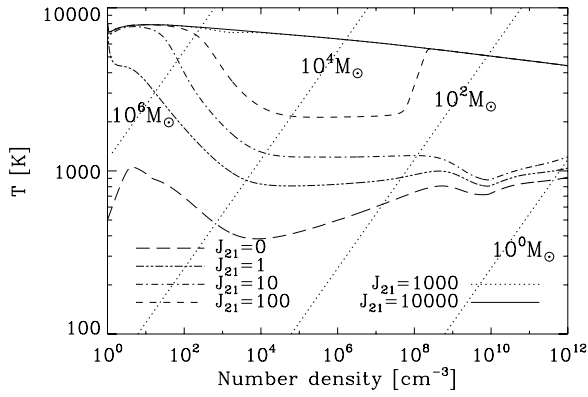


Figure 1. Temperature evolution as a function of density for different values of J_{21} . The thin dotted lines indicate lines of constant Jeans mass.

and that the initial mass of the collapsing gas is comparable to the Jeans mass, as in this case the density and velocity profiles of the gas will come to resemble the Larson–Penston similarity solution (Larson 1969; Penston 1969), which has just these properties. For Ly α photons and other direct transitions to the ground state, we compute the optical depth τ and the photon diffusion timescale following Spaans & Silk (2006), in order to take into account the significant line broadening that occurs during the diffusion process. For the Ly α line, this yields a timescale $t_{\text{ph}} = L(a\tau_{21})^{1/3}/c$ with the natural-to-thermal line width a and an optical depth $\tau_{21} = 1.04 \times 10^{-13} N_{\text{H}} T_4^{-0.5}$, with L the distance to the edge of the halo, and N_{H} the column density of atomic hydrogen. For the Ly α line, the natural-to-thermal line width is taken as $a = 4.7 \times 10^{-4} T_4^{-1/2}$ (Spaans & Silk 2006), while we correct this value for the increased lifetime in case of other line transitions to the ground state. To calculate the column density, we assume that the density profile of the gas scales with radial distance r as $r^{-2.2}$, as indicated by previous numerical simulations (e.g., Wise & Abel 2007). The cooling functions for the hydrogen lines are then evaluated based on the escape probability, the level populations and the strength of the background radiation.

3. RESULTS AND CONCLUSIONS

The temperature evolution that we find with our approach for several different values of J_{21} is plotted as a function of density in Figure 1. Even for $J_{21} = 1$, the temperature remains much higher during the collapse than in the radiation-free case, and never drops significantly below 1000 K.

As previously noted, much higher values of J_{21} can be obtained locally, in the presence of a luminous neighbor within a distance of 10 kpc or less (Dijkstra et al. 2008). As we increase J_{21} , we find systematically higher temperatures at each density; and for $J_{21} = 100$, we find that the effects of H₂ collisional dissociation start to become important. In this case, H₂ cooling is marginally effective at densities $n < 10^8 \text{ cm}^{-3}$, but at higher densities, collisional dissociation destroys the H₂, causing a sharp rise in the temperature. Such a radiation field may be present in a fraction of $10^{-5} - 10^{-3}$ of all atomic cooling halos (Dijkstra et al. 2008). For $J_{21} = 10^3$ and $J_{21} = 10^4$, we find, in common with previous studies, that H₂ cooling never becomes important, and the gas temperature remains high. Nevertheless, in contrast to the predictions of Spaans & Silk (2006), we find that the temperature does decline with increasing density, although it never drops below 5000 K over the range of densities examined here. With $\gamma_{\text{eff}} = 0.97-0.98$, we find that the effective

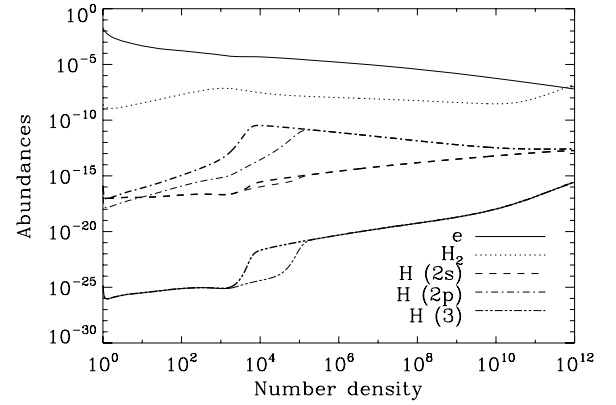


Figure 2. Fractional abundances for electrons, H₂, and the hydrogen level populations, plotted as a function of density, for the case where $J_{21} = 1000$. The values of the hydrogen level populations depend on the assumed size of the protogalaxy; values are plotted for virial radii of 500 pc (thin lines) and 3 kpc (thick lines).

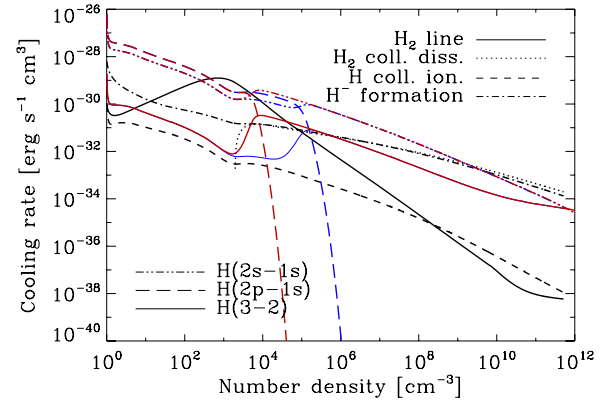


Figure 3. Main cooling functions for $J_{21} = 1000$ as a function of density. Lines are black if the contribution is not affected by Ly α trapping. For the hydrogen lines, blue lines correspond to a system with virial radius 0.5 kpc ($\sim 10^7 M_{\odot}$), and red lines to a virial radius of 3 kpc ($\sim 2 \times 10^9 M_{\odot}$). The H₂ collisional dissociation cooling rate is corrected for the effect of H₂ heating.

equation of state of the gas is just slightly softer than isothermal. We note, though, that this estimate provides just a lower limit for the value of γ_{eff} , which is strictly defined as $d \log T / d \log \rho$ at constant entropy.

In order to understand why the gas is still able to cool, we explore the case with $J_{21} = 1000$ in more detail in Figures 2 and 3. In Figure 2, we show the evolution of the fractional abundances of free electrons and H₂ molecules, as well as the fractional level populations of the 2s, 2p, and 3 states of atomic hydrogen (denoted henceforth as H(2s), H(2p), and H(3)). In Figure 3, we show the contributions that the main cooling processes make to the total cooling rate. The Ly α optical depth depends on the size of the protogalaxy, and we consider two examples, one with a virial radius of 500 pc (corresponding to a halo mass of $\sim 10^7 M_{\odot}$), and a second with a virial radius of 3 kpc (corresponding to a halo mass of $2 \times 10^9 M_{\odot}$). In each case, the abundance of the 2p state is initially very small, but increases roughly linearly with increasing density, owing to the increasing importance of Ly α photon trapping. However, collisional excitation to the H(3) state eventually comes to dominate over radiative decay from the 2p state, following which the abundance of the 2p state remains roughly constant.

The abundances of the H(2s), H(2p), and H(3) states are clearly larger in the case with the larger virial radius, as Ly α photons are trapped more effectively, directly boosting the

population of the $2p$ state. The population of the $2s$ state is then increased due to collisional coupling. The same holds true for the $H(3)$ state. However, the electron and H_2 abundances are not affected. Despite these changes in the hydrogen level populations, the total cooling rate does not change significantly. Cooling due to the $2p-1s$ line decreases, but this is balanced by additional cooling from the $2s-1s$ and $3-2$ transitions, due to the increased populations of these states. At densities higher than 10^8 cm^{-3} , collisional dissociation of H_2 becomes the dominant cooling process (even after correcting for H_2 formation heating) and cools the gas to about 5000 K. H^- formation cooling is found to contribute significantly in a broad range of densities.

Although we find that the effective equation of state of the gas may be softer than isothermal, it is nevertheless true that $\gamma_{\text{eff}} \sim 1$ throughout the collapse. Moreover, the temperature evolution of the gas is surprisingly similar to that obtained in previous three-dimensional simulations that assume optically thin $\text{Ly}\alpha$ cooling. The outcome of the initial collapse may thus be similar to that found in previous simulations, i.e., the formation of a single massive bound object at the center of the halo (see, e.g., Wise et al. 2008; Begelman & Shlosman 2009; Regan & Haehnelt 2009; Shang et al. 2010). However, it is also highly likely that gas that falls in later, with higher angular momentum, will begin to build up an accretion disk surrounding this object. The high gas temperatures imply a rapid accretion flow onto such a disk, which may quickly become gravitationally unstable. If the gas in the disk is also able to cool effectively, which our results suggest will be the case even if $\text{Ly}\alpha$ cooling is completely discounted, then we would expect it to fragment and form stars (Levin 2007). The latter may be drawn to the center by dynamical friction and merge with the central black hole (Devecchi & Volonteri 2009). Owing to the high temperatures, we would expect the stars formed in the disk to accrete more rapidly than standard Population III stars, by a factor of 10 or more, and hence their final masses may be very large, $M \sim 100-500 M_{\odot}$. However, recent works also indicate the possibility of quasi-stable cold self-gravitating accretion disks that do not fragment, which could feed the central object during the further evolution (Levine et al. 2008; Begelman & Shlosman 2009).

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