

University of Groningen

## Structure and kinematics of edge-on galaxy discs - IV. The kinematics of the stellar discs

Kregel, M.; van der Kruit, P. C.

*Published in:*  
Monthly Notices of the Royal Astronomical Society

*DOI:*  
[10.1111/j.1365-2966.2005.08811.x](https://doi.org/10.1111/j.1365-2966.2005.08811.x)

**IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.**

*Document Version*  
Publisher's PDF, also known as Version of record

*Publication date:*  
2005

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*  
Kregel, M., & van der Kruit, P. C. (2005). Structure and kinematics of edge-on galaxy discs - IV. The kinematics of the stellar discs. *Monthly Notices of the Royal Astronomical Society*, 358(2), 481-502. <https://doi.org/10.1111/j.1365-2966.2005.08811.x>

### Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

### Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

# Structure and kinematics of edge-on galaxy discs – IV. The kinematics of the stellar discs

M. Kregel and P. C. van der Kruit<sup>★</sup>

*Kapteyn Astronomical Institute, University of Groningen, PO Box 800, 9700AV Groningen, The Netherlands*

Accepted 2005 January 2. Received 2004 December 10; in original form 2004 June 6

## ABSTRACT

The stellar disc kinematics in a sample of 15 intermediate- to late-type edge-on spiral galaxies are studied using a dynamical modelling technique. The sample covers a substantial range in maximum rotation velocity and deprojected face-on surface brightness and contains seven spirals with either a boxy- or peanut-shaped bulge. Dynamical models of the stellar discs are constructed using the disc structure from *I*-band surface photometry and rotation curves observed in the gas. The differences in the line-of-sight stellar kinematics between the models and absorption-line spectroscopy are minimized using a least-squares approach. The modelling constrains the disc surface density and stellar radial velocity dispersion at a fiducial radius through the free parameter  $\sqrt{M/L} (\sigma_z/\sigma_R)^{-1}$ , where  $\sigma_z/\sigma_R$  is the ratio of vertical and radial velocity dispersion and  $M/L$  is the disc mass-to-light ratio. For 13 spirals a transparent model provides a good match to the mean line-of-sight stellar velocity dispersion. Models that include a realistic radiative transfer prescription confirm that the effect of dust on the observable stellar kinematics is small at the observed slit positions. We discuss possible sources of systematic error and conclude that most of these are likely to be small. The exception is the neglect of the dark halo gravity, which has probably caused an overestimate of the surface density in the case of low surface brightness discs.

**Key words:** galaxies: fundamental parameters – galaxies: kinematics and dynamics – galaxies: spiral – galaxies: structure.

## 1 INTRODUCTION

The stellar kinematics in discs of spiral galaxies contain valuable information on their structure. The fact that H I rotation curves that extend well beyond the stellar discs are generally flat has provided evidence that spiral galaxies are embedded within massive dark matter haloes (Bosma 1978; Begeman 1987). The contribution of the disc gravitational field to the circular velocities in the inner part of these galaxies is unknown without an independent measurement of the disc surface density. In the absence of such knowledge the ‘maximum disc hypothesis’ has been formulated (van Albada et al. 1985), which states that the disc is as massive as allowed by the rotation curve. A promising tool suited for lifting this disc–halo degeneracy is the dynamical modelling of the stellar disc kinematics (van der Kruit & Freeman 1986). In addition dynamical modelling provides important information on the stellar disc velocity dispersion (van der Kruit & Freeman 1986; Bottema 1993), which allows investigation of the Toomre criterion  $Q$  for local stability (Toomre 1964). Furthermore, if we were able to secure information on the ratio of vertical to radial stellar velocity dispersion (Gerssen,

Kuijken & Merrifield 1997, 2000; van der Kruit & de Grijs 1999; Shapiro, Gerssen & van der Marel 2003) we would be in a position to obtain information on the actual mechanisms that gave rise to the secular evolution of the stellar motions, the so-called ‘heating’ giving rise to an increase in the velocity dispersions (Jenkins & Binney 1990).

This is the fourth paper in a series<sup>1</sup> in which we aim to provide new constraints on the dynamics of spiral galaxy discs through an observational synthesis of the global stellar disc structure and kinematics. The series is preceded by a re-analysis (Kregel, van der Kruit & de Grijs 2002, hereafter referred to as KKG) of the surface photometry in the *I* band of the sample of edge-on galaxies from de Grijs (1997, 1998). In Paper I in the series (Kregel, van der Kruit & Freeman 2004a) we presented optical spectroscopy to study the stellar kinematics in 17 edge-on galaxies taken from this sample. In Paper II (Kregel, van der Kruit & de Blok 2004b) we presented H I synthesis observations of 15 of these galaxies and derived rotation curves using the envelope-tracing method. In Paper III (Kregel & van der Kruit 2004) we introduced an automated method to derive

<sup>★</sup>E-mail: vdkruit@astro.rug.nl

<sup>1</sup> The series contains adapted versions of most chapters in the PhD thesis of MK (Kregel 2003).

H I rotation curves by fitting the full position–velocity diagrams and applied that to eight of these galaxies with sufficient signal-to-noise ratio. The rotation curves from the H I observations were combined with the emission line kinematics from the optical spectra in Paper I to estimate the full rotation curves. In this paper we will analyse the stellar kinematics data of Paper I and correct for the line-of-sight effects using these rotation curves combined with the stellar disc structure from KKG. In the final paper (Kregel, van der Kruit & Freeman 2005; hereafter Paper V) we will present a general discussion.

In this paper the stellar disc kinematics are investigated through dynamical modelling of 15 intermediate- to late-type edge-on spirals. The stellar disc kinematics are modelled according to the semi-empirical approach introduced by van der Kruit & Freeman (1986). This approach has been applied successfully to a dozen intermediate- to late-type spirals of various inclinations (van der Kruit & Freeman 1986; Bottema 1993, 1999). The stellar disc model is self-gravitating and three-dimensional and allows for an anisotropic stellar velocity ellipsoid. The model further includes a radiative transfer prescription, which makes it suitable for modelling highly inclined spirals.

Edge-on galaxies were chosen first because of their higher surface brightness, such that good-quality absorption-line spectra are more easily obtained for larger samples. Secondly, this effect also facilitates the study of discs with a low *face-on* surface brightness. Thirdly, in edge-on spirals the disc scaleheight required for a dynamical estimate of the disc mass can be directly determined. Fourthly, the edge-on orientation ensures that the entire galaxy plane is sampled by a single slit, such that any local deviations in the stellar structure and kinematics tend to be averaged out. Finally, young stellar populations and dust extinction can be largely avoided by placing the slit away from the galactic plane.

This paper is organized as follows. The selection and properties of the sample are discussed in Section 2. In Section 3 the dynamical stellar disc model is described. Using a realistic radiative transfer prescription this model is then used to investigate the effects of line-of-sight projection and dust extinction at orientations close to

edge-on (Section 3.4). Using the stellar disc structure (KKG) and the observed rotation curves (Paper III), the model is then fitted to the observed stellar kinematics (Section 4). The effect of the neglect of dust extinction and possible systematic errors are addressed in Sections 4.3 and 4.4. The adopted distances of the galaxies are based on the Virgo-centric velocities from the Lyon/Meudon Extragalactic Data base (LEDAs) (which uses the flow model described in Bottinelli et al. 1986) and a Hubble constant of  $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

## 2 THE SAMPLE

### 2.1 Properties

The galaxy sample for which the stellar kinematics have been observed (Paper I) consists of 12 edge-on spirals taken from the sample of de Grijs (1998) plus the three large edge-on spirals NGC 891, 5170 and 5529. For this sample the stellar kinematics are available out to a projected radius between one and three scale-lengths, well within the disc-dominated region. Two galaxies, ESO 416-G25 and 509-G19, had to be removed from the sample of Paper I because of the limited radial range of the stellar kinematics. Information regarding the sample selection, the spectroscopic observations and the extraction of the stellar kinematics can be found in Paper I. The determination of the rotation curves from the combined H I and optical emission line kinematics is described in Paper III.

The sample mainly consists of intermediate- to late-type spirals (Table 1). The exception is ESO 437-G62 which is classified Sb in LEDA but is in view of its very low H I mass probably a lenticular (see table I in KKG). In three galaxies the emission lines show a ‘figure-of-eight’ signature: ESO 240-G11, NGC 5529 (appendix A in Paper III) and ESO 487-G02 (Bureau & Freeman 1999). These plus four additional spirals show a boxy- or peanut-shaped bulge and may be barred (marked by a dagger in Table 1). Note that the radial velocities of most galaxies are large,  $v_{\text{vir}} > 2000 \text{ km s}^{-1}$ , indicating that distance errors due to peculiar motion are small, probably less

**Table 1.** Sample properties.

Galaxy	Type	Distance	$A_I$	$\mu_{0,I}$	$\pm$	$i$	$\pm$	$v_{\text{max}}$	$\pm$	
(1)	(LEDA)	(Mpc)	(mag)	(mag arcsec <sup>-2</sup> )	(5)	(deg)	(6)	(km s <sup>-1</sup> )	(7)	
ESO 142-G24	IC 4872	Scd	23.2	0.09	22.09	0.19	85	1	121	4
ESO 157-G18	IC 2058	Scd	14.7	0.00	22.02	0.22	88	2	88	6
ESO 201-G22		Sc	50.9	0.00	21.60	0.27	87.5	1	169	5
ESO 240-G11		Sc†	35.4	0.01	21.29	0.48	87	1	273	7
ESO 269-G15	NGC 4835A	Sc	42.5	0.17	21.02	0.41	87	1.5	167	8
ESO 288-G25		Sbc	31.1	0.00	19.77	0.50	86	2	175	5
ESO 435-G14		Sc	33.1	0.07	20.11	0.50	82	2	136	9
ESO 435-G25*	IC 2531	Sc†	30.5	0.09	20.83	0.11	89.5	0.5	232	10
ESO 437-G62	NGC 3390	Sb†	38.1	0.11	20.63	0.50	89.5	0.5	277	20
ESO 446-G18		Sb†	62.1	0.08	21.79	0.15	87	1	198	7
ESO 487-G02	NGC 1886	Sbc†	20.8	0.01	20.40	0.33	88.5	0.5	167	18
ESO 564-G27		Sc	26.9	0.20	21.98	0.28	88	1	160	6
NGC 891*		Sb†	9.2	0.13	20.09	0.11	89.8	0.5	227	5
NGC 5170		Sc	19.1	0.15	20.93	0.44	86.5	1	250	4
NGC 5529*		Sc†	40.9	0.02	19.63	0.11	87.5	1	284	8

Columns. (1) Galaxy, an asterisk indicates that the disc structure according to Xilouris et al. (1999) was used. (2) Morphological type (LEDA), a dagger indicates spirals with a boxy- or peanut-shaped bulge. (3) Adopted distance. (4) Galactic extinction in the *I* band, as in de Grijs (1998). For galaxies marked with an asterisk the Schlegel, Finkbeiner & Davis (1998) values were taken. (5) Deprojected face-on central surface brightness of the disc according to the best-fitting exponential disc model (KKG, for galaxies marked with an asterisk from Xilouris et al. 1999), corrected for Galactic extinction. (6) Inclination (see the text). (7) Maximum rotational velocity (Paper III).

**Table 2.** Group membership and nearest neighbours.

Galaxy (1)	Group (2)	Number (3)	Nearest neighbour (4)	Projected distance (Mpc) (5)
ESO 142-G24	F	–	NGC 6810	0.63
ESO 157-G18	NOGG 214 (Dorado)	12	NGC 1553	0.07
ESO 201-G22	F	–	–	–
ESO 240-G11	F	–	ESO 240-G10	0.14
ESO 269-G15	F	–	ESO 269-G30	0.86
ESO 288-G25	NOGG 1018	4	NGC 7166	0.29
ESO 435-G14	NOGG 450	2	NGC 3089	0.25
ESO 435-G25	F	–	ESO 435-G19	0.35
ESO 437-G62	NOGG 495	5	ESO 437-G65	0.22
ESO 446-G18	F	–	–	–
ESO 487-G02	F	–	ESO 487-G17	0.82
ESO 564-G27	F	–	NGC 2758	0.87
NGC 891	NGC 1023 group	13	NGC 1003	0.55
NGC 5170	NOGG 724	2	MCG –03-34-082	0.18
NGC 5529	NOGG 777	5	NGC 5544/45	0.33

Columns: (1) Galaxy; (2) group membership, NOGG, group identification in the Nearby Optical Galaxy sample (Giuricin et al. 2000, according to their P1 algorithm), F, field galaxy (not assigned to a group). For the NGC 1023 group see Tully (1980); (3) number of galaxies in the group; (4) neighbour with smallest projected distance to the main galaxy; (5) projected distance according to the adopted distance of the main galaxy.

than 15 per cent. Exceptions are ESO 157-G18 and NGC 891. ESO 157-G18 is a member of the Dorado group (Table 2), for which the central elliptical NGC 1549 is at a distance of 19.7 Mpc ( $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ) according to the Dn- $\sigma$  method (Tonry et al. 2001). For NGC 891 the adopted distance is consistent with results from the planetary nebulae luminosity function ( $9.9 \pm 0.7$  Mpc, Ciardullo, Jacoby & Harris 1991) and surface brightness fluctuations ( $8.4 \pm 0.6$  Mpc, Tonry et al. 2001).

The sample covers an appreciable range in deprojected central surface brightness, from 19.6 to 22.1  $I$  mag arcsec $^{-2}$ . The disc inclination listed in Table 1 was estimated from the curvature of the dust lane in the  $B$ -band images of de Grijs (1998). For NGC 891, the inclination was determined by comparing dusty models with the stellar kinematics observed at the vertical slit position (Section 4.2). Finally, the galaxies also cover a substantial range in the observed maximum rotational velocity ( $v_{\text{max}} \simeq 90\text{--}280 \text{ km s}^{-1}$ ). For most galaxies this quantity was taken from the rotation curves (table 2 of Paper III). For two galaxies no optical or H I rotation curve is available and  $v_{\text{max}}$  was determined either from the H I global profile (ESO 487-G02) or by modelling the stellar kinematics (ESO 437-G62, see Section 4.2).

## 2.2 Environment

The majority of the galaxies were selected from the sample of de Grijs (1998) to be regular and non-interacting (see Paper I). Table 2 summarizes the environment of each galaxy, listing group membership, the number of neighbouring galaxies and the closest neighbour (in projection, ignoring small companions). Only two galaxies have large neighbours at a relatively small separation (in projection): ESO 157-G18 and 240-G11. The remaining galaxies are located either in the field or in small loose groups. This general picture of relatively undisturbed discs is confirmed by the H I observations (Paper II). These indicate that there are no massive H I companions, and that the H I is distributed in regular, moderately warped layers. The exception is NGC 5529. This massive spiral has a strongly warped H I layer and two nearby satellites connected to the main galaxy via H I bridges.

## 3 THE DISC MODEL

### 3.1 Stellar emissivity

The stellar disc luminosity density is assumed, as in KKG, to be axisymmetric, and exponential in both the radial and vertical directions:

$$L(R, z) = L_0 e^{-R/h_R} e^{-|z|/h_z}, \quad (1)$$

where  $(R, z)$  are the usual cylindrical coordinates,  $L_0$  is the central luminosity density and  $h_R$  and  $h_z$  are the scalelength and the scaleheight, respectively. In those cases where a truncation was detected (KKG) an infinitely sharp truncation was added [i.e.  $L(R, z) = 0$  for  $R > R_{\text{max}}$ ]. The vertical exponential distribution is adopted following the results of photometric studies of edge-on spiral galaxies in the near-infrared. These reveal vertical disc light distributions intermediate between exponential and sech( $z$ ), although uncertainties due to seeing and inclination imply that the observations are consistent with the vertical distribution being exponential (Wainscoat, Freeman & Hyland 1989; de Grijs, Peletier & van der Kruit 1997). Star counts in the solar neighbourhood (Gilmore & Reid 1983; Chen et al. 2001) and the Galactic near-infrared emission (Kent, Dame & Fazio 1991; Drimmel & Spergel 2001; López-Corredoira et al. 2002) also favour an exponential distribution for the old disc light. As the contribution of young red supergiants in the near-infrared is probably small (Jones et al. 1981; Rhoads 1998), the vertical exponential distribution mainly refers to the old stellar population. The scaleheight is taken to be constant with radius (van der Kruit & Searle 1981; de Grijs & Peletier 1997; Bizyaev & Mitronova 2002).

### 3.2 Stellar kinematics

The Poisson and Jeans equations yield simple analytical expressions for the stellar kinematics under the fundamental assumptions that the stellar discs are self-gravitating and plane-parallel (i.e. the radial and vertical stellar motions are decoupled). The appropriateness of these assumptions will be addressed in Section 4.4. The vertical stellar velocity dispersion follows from the vertical Jeans

equation (i.e. hydrostatic equilibrium). For a disc which is exponential in both the radial and vertical direction (cf. van der Kruit 1988):

$$\sigma_z(R, z) = \sqrt{\pi G h_z (2 - e^{-z/h_z})(M/L) \mu_0} e^{-R/2h_R}, \quad (2)$$

where  $M/L$  is the disc mass-to-light ratio,  $\mu_0$  is the face-on disc central surface brightness (in linear units) and  $G$  is the gravitational constant. Equation (2) states that for discs with a constant mass-to-light ratio the vertical velocity dispersion declines exponentially with radius, with a scalelength twice that of the luminosity density. Although there is no strong evidence for the constancy of the disc  $M/L$  within galaxies, an approximately constant  $M/L$  is consistent with the observations of the stellar kinematics in the inner parts of face-on and intermediately inclined spirals (van der Kruit & Freeman 1984, 1986; Bottema 1993; Gerssen et al. 1997, 2000) and the modest radial colour gradients (de Jong 1996b). Here it is assumed that the  $M/L$  is simply constant. The effect of realistic deviations from constant  $M/L$  on the modelling results will be discussed in Section 4.4. Note that the vertical velocity dispersion has a minimum at  $z = 0$ , such that  $\sigma_z(R, 0) = 1/\sqrt{2} \sigma_z(R, \infty)$ . Such a minimum is present for all mass distributions steeper than the  $\text{sech}^2(z)$  (isothermal) distribution (van der Kruit & Searle 1981; van der Kruit 1988).

Assuming a constant velocity anisotropy  $\sigma_z/\sigma_R$ , the radial velocity dispersion becomes

$$\sigma_R(R, z) = \sqrt{\pi G h_z (2 - e^{-z/h_z})(M/L) \mu_0} \times (\sigma_z/\sigma_R)^{-1} e^{-R/2h_R}. \quad (3)$$

Theoretical arguments suggest that a constant velocity anisotropy is a fair approximation in the inner parts of galaxy discs (Cuddeford & Amendt 1992; Famaey, van Caelenberg & Dejonghe 2002). An observational argument for the approximate constancy of the velocity anisotropy is provided by the ages and kinematics of 182 F and G dwarf stars in the solar neighbourhood (Edvardsson et al. 1993). These indicate that the anisotropy was set after an early heating phase and, though the Galaxy has probably changed much over its lifetime, the anisotropy has remained constant throughout the life of the old disc (Freeman 1991).

The tangential velocity dispersion follows from the epicyclic approximation:

$$\sigma_\phi(R, z) = (1/\sqrt{2}) \sigma_R \left( 1 + \frac{\partial \ln v_c}{\partial \ln R} \right)^{1/2}, \quad (4)$$

where  $v_c$  is the circular velocity curve. Theoretical studies of realistic distribution functions indicate that the epicycle approximation is at least 20 per cent accurate down to a radius of approximately half a scalelength (Cuddeford & Binney 1994; Batsleer & Dejonghe 1995).

The mean streaming velocity of the stars can now be calculated from the radial Jeans equation (i.e. the asymmetric drift equation):

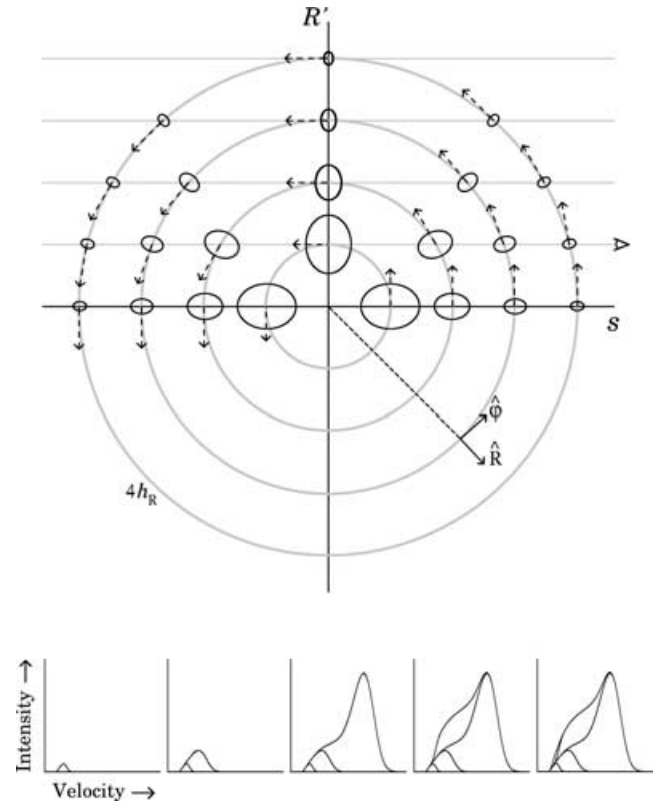
$$v_*^2(R, z) = v_c^2 + \sigma_R^2 \left[ \frac{\partial \ln(\rho \sigma_R^2)}{\partial \ln R} + \left( 1 - \frac{\sigma_\phi^2}{\sigma_R^2} \right) + \frac{R}{\sigma_R^2} \frac{\partial \sigma_{Rz}^2}{\partial z} \right], \quad (5)$$

where  $\rho = L(R, z) M/L$  is the density in the disc. The final term refers to the tilt of the ellipsoid. In the plane-parallel case, as assumed here, this term is zero. In Section 4.4 it will be shown that the influence of this term on the results is small. As the sum of the remaining two terms between brackets is negative, stars lag the circular velocity  $v_c$  by an amount proportional to the radial velocity dispersion – the well-known asymmetric drift.

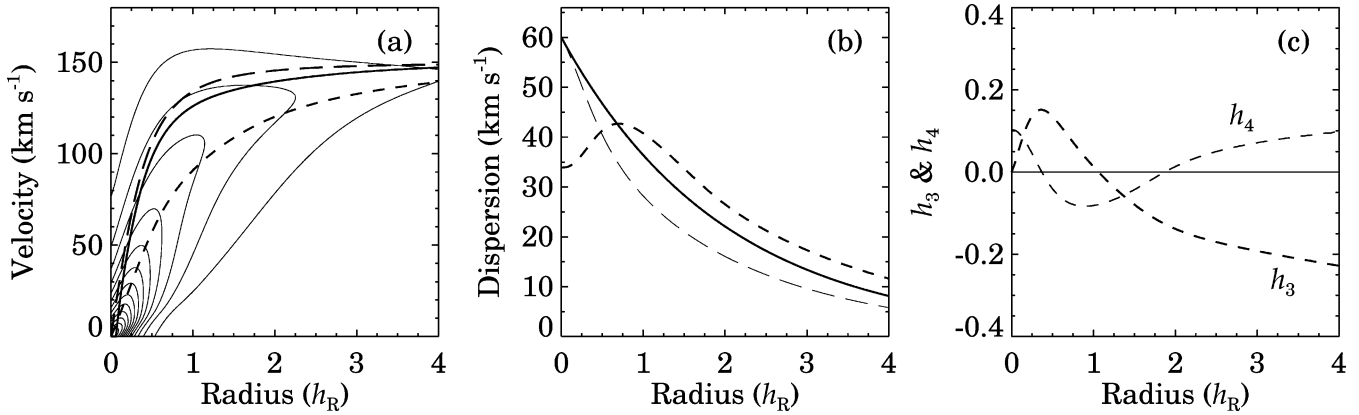
Thus, given the observed rotation curve (Paper III) and the disc luminosity density (KKG), equations (2)–(5) predict the stellar velocity ellipsoid and the stellar mean streaming velocity as a function of the position within the disc. In the general case this model has two free parameters,  $M/L$  and  $\sigma_z/\sigma_R$ . For an edge-on disc, however, only  $\sigma_R$ ,  $\sigma_\phi$  and  $v_*$  enter in the observable stellar kinematics. In that case the product  $\sqrt{M/L} (\sigma_z/\sigma_R)^{-1}$  acts as a single free parameter. Fitting this parameter directly fixes the disc radial velocity dispersion (via equation 3), and through an assumed  $\sigma_z/\sigma_R$  the disc surface density. It is important to note that unlike the surface density of the disc, the inferred  $M/L$  also depends on the accuracy of  $\mu_0$  ( $M/L = \Sigma/\mu_0$ ). As for edge-on systems the determination of the disc  $\mu_0$  (and luminosity) is rather uncertain (e.g. KKG), the dynamical model is unable to provide tight constraints on the disc  $M/L$ .

### 3.3 Radiative transfer

The combined effects of absorption and scattering during the interaction of photons with interstellar dust modify the line-of-sight velocity distribution (LOSVD). Further on it will be shown that in edge-on systems this effect can be significantly reduced by positioning the slit outside the dust lane. Yet, several galaxies in the present sample do not clearly show a dust lane and even away from a central dust lane extinction may affect the observed stellar kinematics. To



**Figure 1.** Sketch of the galaxy plane illustrating the build-up of the LOSVD in an edge-on disc. Top, view of the disc plane. Circles mark galactocentric radii of 1–4 disc scalelengths. The horizontal lines represent sightlines crossing the receding side of the disc. The arrows and ellipses represent the stellar velocity and velocity dispersion, respectively. The symbol size indicates the amplitude of the rotation/dispersion. Bottom, LOSVD build-up along the line of sight as marked by the eye in the top panel, from far behind the line of nodes (left) to the near side (right).



**Figure 2.** The effect of the line-of-sight projection on the stellar kinematics for a representative disc model. (a) Line-of-sight projected (short-dashed line) and true (solid line) stellar rotation curve overlaid on the position–velocity diagram (contours). The long-dashed line denotes the circular velocity curve. (b) Line-of-sight projected velocity dispersion (short-dashed line) and true radial velocity dispersion (solid line). The long-dashed line denotes the tangential velocity dispersion. (c) The  $h_3$  (skewness) and  $h_4$  (kurtosis/peakedness) parameters.

investigate the role of dust extinction a radiative transfer prescription has been included in the model (cf. Bottema, van der Kruit & Valentijn 1991).

The radiative transfer equation is solved using the iteration method of Kylafis & Bahcall (1987), generalized to include the kinematic information. For brevity the absorption-only case is described here. The scattering treatment is described in Appendix A. The dust is distributed smoothly with an extinction coefficient:

$$\kappa(R, z) = \kappa_0 e^{-R/h_{R,d}} e^{-|z|/h_{z,d}}, \quad (6)$$

where  $\kappa_0$  is the extinction coefficient at the galaxy centre and  $h_{R,d}$  and  $h_{z,d}$  are the dust scalelength and scaleheight, respectively. The amount of dust is parametrized by the face-on central optical depth  $\tau_0 = 2\kappa_0 h_{z,d}$ . In reality dust distributions are of course clumpy, resulting in lower effective dust extinction (Witt & Gordon 1996). On the other hand, for highly inclined galaxies a line of sight intersects a large number of clumps such that the effect of the clumpiness on the global observed surface brightness distribution may be much reduced (Kuchinski et al. 1998). Note that the choice for the vertical form of the dust distribution is ad hoc. However, Baes & Dejonghe (2001) find that the difference in extinction between an exponential and a sech<sup>2</sup> dust distribution is small, whereas the ratio  $h_{z,d}/h_z$  on the other hand is crucial.

An observed stellar velocity distribution is an emission-weighted integral of the stellar kinematics encountered at each position along a sight line. Given the galaxy inclination  $i$ , each position along a line of sight can be written in terms of the cylindrical galaxy coordinates  $(R, \varphi, z)$ . The unattenuated velocity distribution at a single location along the line of sight is then

$$f(R, \varphi, z, v) = \frac{L(R, z)}{\sqrt{2\pi}\sigma_{\text{los}}} \exp\left[-(v - v_{\text{los}})^2/2\sigma_{\text{los}}^2\right], \quad (7)$$

where the mean line-of-sight projected rotational velocity and velocity dispersion follow from the geometry,

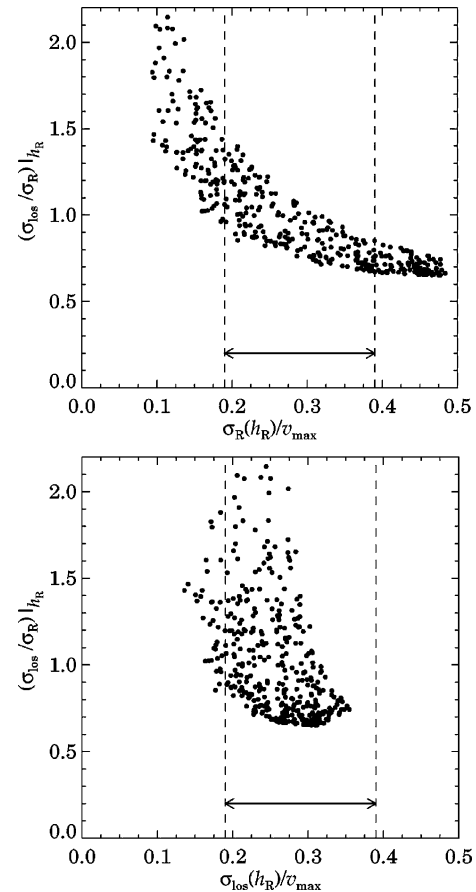
$$v_{\text{los}} = v_*(R, z) \sin i \cos \varphi, \quad (8)$$

$$\sigma_{\text{los}}^2 = [\sigma_R(R, z) \sin i \sin \varphi]^2 + [\sigma_\varphi(R, z) \sin i \cos \varphi]^2 + [\sigma_z(R, z) \cos i]^2, \quad (9)$$

and the local velocity distribution is approximated by a trivariate Gaussian. The observed absorption-only LOSVD is simply an integral sum of these local velocity distributions along the line of sight,

each of them weighted according to the optical depth at that position (cf. Kylafis & Bahcall 1987, equation 9):

$$f_0(v) = \int_0^{s_0} ds f(R, \varphi, z, v) \exp\left[-\int_{s'}^{s_0} ds' \kappa(R, z)\right], \quad (10)$$



**Figure 3.** Comparison of the line-of-sight projected and the intrinsic velocity dispersion for the ensemble of models. Top, ratio of the line-of-sight projected dispersion and the intrinsic radial dispersion at one disc scalelength versus the disc ‘temperature’. The dashed lines bracket the range of  $\sigma_R(h_R)/v_{\text{max}}$  found by Bottema (1993). Bottom, as above but versus the observed ratio  $\sigma_{\text{los}}(h_R)/v_{\text{max}}$ .

where  $s$  is the line-of-sight coordinate and  $s_0$  is the location of the observer. By calculating this LOSVD for a number of positions on the galaxy one can construct position–velocity diagrams or a ‘data cube’ of the model.

### 3.4 The edge-on view

In edge-on discs a single sight line samples the density and kinematics across a large range in galactocentric radius. This projection effect is considered in the following, first for a transparent and then for a dusty stellar disc.

#### 3.4.1 Line-of-sight projection

The line-of-sight projection affects the stellar velocity distribution in a way similar to that of the H I (fig. 1 in Paper III). For a stellar disc the projection is, however, somewhat more complex because of the non-constant, anisotropic velocity ellipsoid. In this case both the distributions of line-of-sight projected velocities and line-of-sight projected velocity dispersions enter in the projection. This is illustrated in Fig. 1.

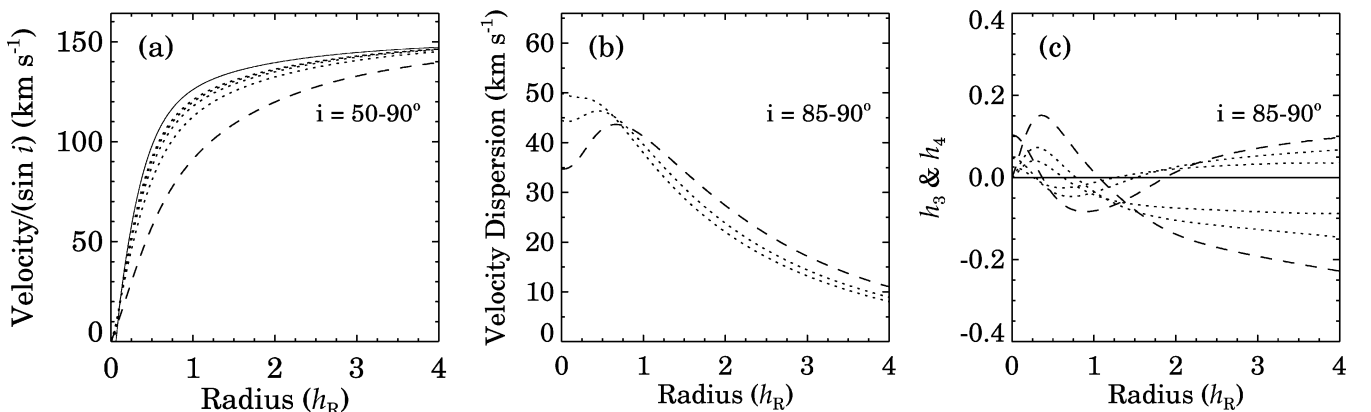
To investigate the projection, major axis stellar position–velocity diagrams were calculated for an ensemble of 400 model discs having a rotation curve of the form  $v_c(R) = v_{\max} R/\sqrt{R^2 + d^2}$ . Different values of  $v_{\max}$ ,  $d$ ,  $h_R$  and  $\sigma_R(0)$  were assigned to each model. These values were randomly drawn from uniform distributions chosen to roughly match the properties of spirals:  $50 < v_{\max} < 350 \text{ km s}^{-1}$ ,  $0.0125v_{\max} < h_R < 0.0625v_{\max}$  (from KKG),  $0.09v_{\max} < \sigma_R(h_R) < 0.49v_{\max}$  (Bottema 1993) and  $0.25h_R < d < 1.25h_R$ . As with the observations, each of the LOSVDs in the calculated position–velocity diagrams was fitted with a truncated Gauss–Hermite series (cf. Paper I).

As a representative example, consider first a single model with  $v_{\max} = 150 \text{ km s}^{-1}$ ,  $\sigma_R(0) = 60 \text{ km s}^{-1}$  and  $d = h_R/2$  (Fig. 2). Clearly, the line-of-sight projected stellar rotation curve is lower and less steep than the true stellar rotation curve. For the velocity dispersion curve the effect of the projection is less severe; at projected radii beyond one scalelength the line-of-sight projected stellar velocity dispersion is of a similar amplitude and shape as the true radial velocity dispersion. This similarity arises because the rotation curve is flat at these radii and yields a constant ratio  $\sigma_\phi/\sigma_R$ . At smaller radii, the line-of-sight projected dispersion shows a drop, reaching a minimum at  $R = 0$ . This is due to a continuous decrease in the range of line-of-sight projected velocities toward small radii

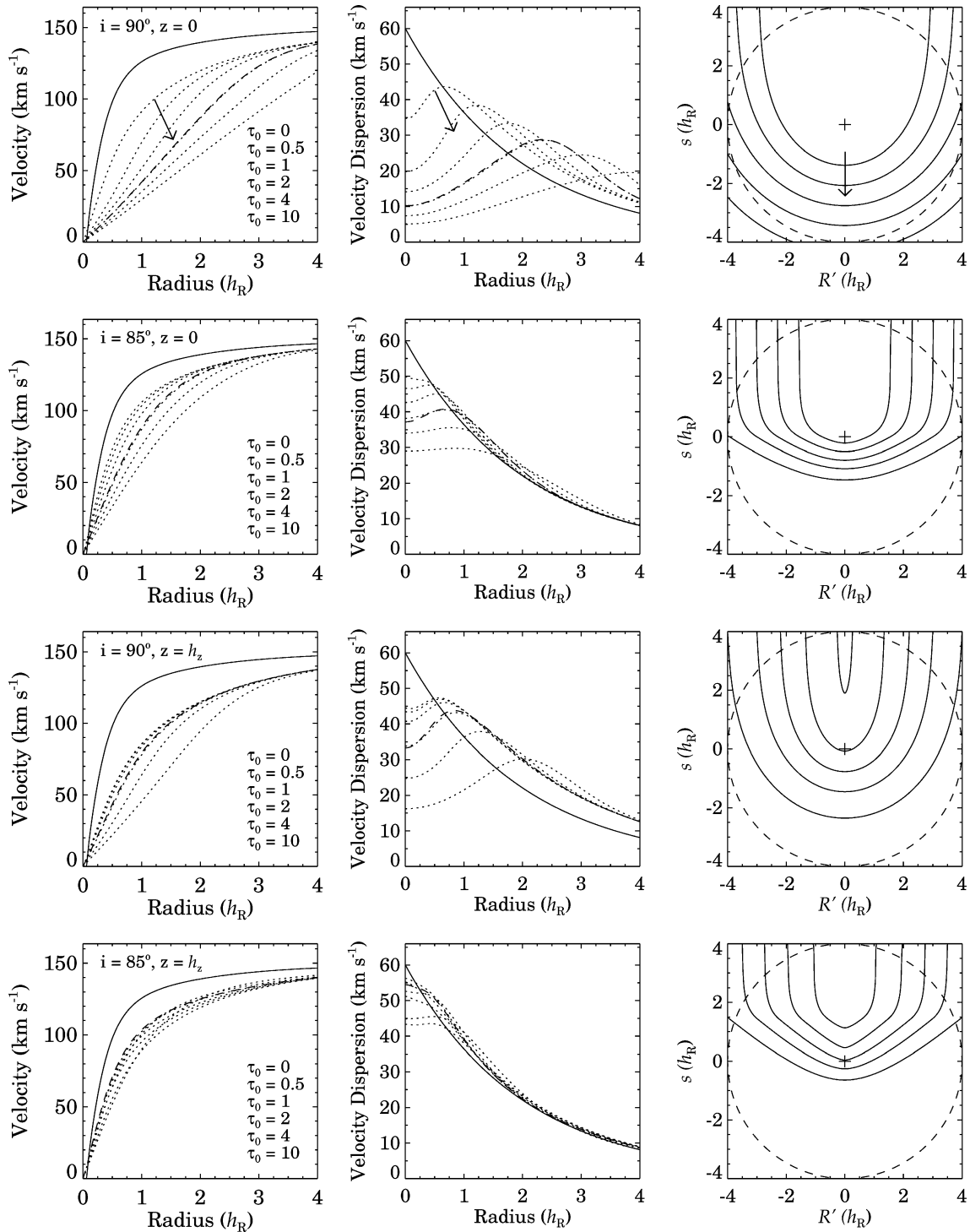
(velocity crowding), until at  $R = 0$  all line-of-sight projected velocities are zero and only the radial velocity dispersion contributes to the LOSVD. At radii very close to  $R = 0$  the asymmetric drift equation predicts an imaginary mean streaming velocity and the model breaks down. This occurs when the velocity dispersion becomes comparable to the mean streaming velocity ( $R \lesssim 0.2h_R$  in the example).

The projection causes a pronounced asymmetry in the LOSVDs, leaving a clear signature in the  $h_3$  and  $h_4$  curves. At large radii the LOSVDs are more sharply peaked than Gaussian ( $h_4 > 0$ ) and show a retrograde tail ( $h_3 < 0$ , i.e. toward systemic). This tail is formed by the lower-density regions behind and in front of the line of nodes. Toward smaller projected radii the LOSVDs change markedly, with  $h_3$  and  $h_4$  changing sign once and twice, respectively. This behaviour, such as the drop in the line-of-sight projected dispersion, is due to the rapidly increasing fraction of stars with small line-of-sight velocities toward smaller radii. As a result of this velocity crowding, the stars with lower line-of-sight velocity (away from the line of nodes) start to dominate the LOSVD at small projected radii, whereas the stars with high line-of-sight velocity (close to the line of nodes) form only a prograde tail. The pattern seen in  $h_3$  and  $h_4$  is common to the entire ensemble, with the strength of the asymmetries and the positions of the sign changes depending on the model parameters. The projection causes pure axisymmetric discs to exhibit asymmetric LOSVDs.

The calculations for the entire ensemble confirm that the line-of-sight projected velocity dispersion is comparable to the true radial velocity dispersion. Fig. 3(a) shows the ratio of the line-of-sight projected dispersion and the true radial dispersion at one scalelength versus  $\sigma_R(h_R)/v_{\max}$ , the disc ‘temperature’. This dispersion ratio decreases toward dynamically hotter discs. At constant  $v_{\max}$ , a disc with higher  $\sigma_R$  (= higher  $\sigma/v_{\max}$ ) shows a broader LOSVD. For the hottest discs the projection effect becomes negligible, with the line-of-sight projected velocity dispersion approaching the tangential dispersion; the ratio  $\sigma_{\text{los}}/\sigma_R$  approaches  $\sigma_\phi/\sigma_R(1/\sqrt{2})$  for a flat rotation curve). At constant  $\sigma_R/v_{\max}$  the ratio  $\sigma_{\text{los}}/\sigma_R$  has a rather narrow range (Fig. 3a). In particular, for  $\sigma_R(h_R)/v_{\max} = 0.29 \pm 0.10$  (Bottema 1993) the ratio is roughly unity. Hence, it is expected that if a correlation between  $\sigma_R(h_R)$  and  $v_{\max}$  exists, it would be visible in a plot of  $\sigma_{\text{los}}(h_R)$  versus  $v_{\max}$ . This is indeed the case (Paper I). However, for discs that are dynamically hotter,  $\sigma_R(h_R)/v_{\max} \gtrsim 0.4$ , or colder,  $\sigma_R(h_R)/v_{\max} \lesssim 0.2$ , a correlation between  $\sigma_{\text{los}}(h_R)$  and  $v_{\max}$  is introduced artificially. This is shown in



**Figure 4.** (a) Line-of-sight projected stellar rotation curves for the example of Fig. 2 at inclinations of  $90^\circ$  (dashed) and  $80^\circ$ ,  $70^\circ$ ,  $60^\circ$  and  $50^\circ$  (dotted) using  $h_R/h_z = 8$ . The solid curve shows the true stellar rotation. The  $1/\sin i$  correction allows a direct comparison of the curves. (b) Line-of-sight projected stellar velocity dispersion curves for the same model at inclinations of  $90^\circ$  (dashed),  $87.5^\circ$  and  $85^\circ$  (dotted). (c) The  $h_3$  and  $h_4$  parameters, curves are as in (b).



**Figure 5.** Line-of-sight projected stellar rotation (left) and velocity dispersion (middle) curves for six different values of  $\tau_0$  at inclinations of  $85^\circ$  and  $90^\circ$  and at projected heights  $z = 0$  and  $h_z$  (on the far side). In the top panels arrows indicate the direction of increasing  $\tau_0$ . For  $\tau_0 = 2$  calculations including scattering are also shown (dashed lines). The solid lines indicate the intrinsic curves. The right-hand panels show the loci of unit optical depth projected on to the plane defined by the line of sight and line of nodes. The dashed line denotes a circle with a radius of four scalelengths.

Fig. 3: model discs with widely different  $\sigma_R(h_R)/v_{\max}$  in panel (a) show similar  $\sigma_{\text{los}}(h_R)/v_{\max}$  ( $\sim 0.2\text{--}0.3$ ) in panel (b). This stresses the need for deprojecting the stellar kinematics of edge-on spirals.

Fig. 4 shows the effect of the projection at inclinations away from edge-on, using the average disc flattening  $h_R/h_z = 8$  (KKG). For the rotation curve the line-of-sight projection is still significant at

inclinations as low as  $\sim 60^\circ$ . For discs that are less flattened the effect is even larger because of the longer path-length through the disc. For inclinations close to edge-on the line-of-sight projected stellar velocity dispersion is rather insensitive to the inclination (Fig. 4b). The deviations from Gaussianity weaken with decreasing inclination (Fig. 4c).



### 3.4.2 Dust extinction

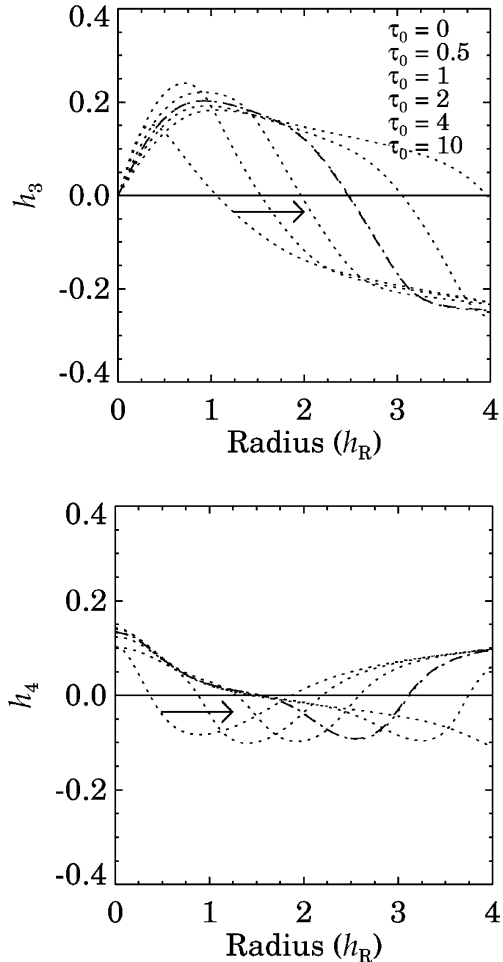
The intrinsic kinematics can be further cloaked by dust extinction. To study this effect a smooth exponential dust distribution (equation 6) was embedded in the reference model of Fig. 2, using  $h_R/h_z = 8$ . The scalelength of the dust distribution was set to the scalelength of the stellar light and for the dust scaleheight  $h_z$ ,  $d = 0.5 h_z$  was taken (Xilouris et al. 1999). Position–velocity diagrams were calculated for various amounts of dust and, in the first instance, excluding scattering. Fig. 5 displays the results. In addition to the line-of-sight projected velocity and velocity dispersion the figure also shows the loci at which the optical depth along the line of sight reaches unity. The area on the near side of these curves (at negative  $s$ ) corresponds to the part of the disc which still makes a significant contribution to the LOSVD.

Starting with the first row of Fig. 5 it is obvious that increasing  $\tau_0$  leads to a further decrease of the line-of-sight projected stellar rotation. At  $\tau_0 \sim 10$  the line-of-sight projected stellar rotation curve has a solid-body shape ( $\tau_0^{\text{edge-on}} = \tau_0 h_{R,d}/h_{z,d} \sim 160$ ). In that extreme case only the outskirts of the near side of the disc contribute to the LOSVD (see the loci of unit optical depth). The line-of-sight projected dispersion also decreases with increasing  $\tau_0$ , because regions with larger line-of-sight velocity dispersions are becoming hidden from view. At large projected radii, the dispersion actually first increases at small  $\tau_0$  before decreasing at very large  $\tau_0$ . Here, the contribution of the region around the line of nodes to the LOSVD first becomes less, causing the LOSVD to flatten, and eventually, at large  $\tau_0$ , becomes negligible.

At lower inclination, the path-length through the dust layer decreases, thereby lessening its effect on the line-of-sight projected stellar kinematics. This is shown in the second row of Fig. 5 for  $i = 85^\circ$ . While overall the line-of-sight projected velocity dispersion is lower because of the inclination (cf. Fig. 4), the decrease in velocity dispersion due to dust extinction is now limited to the inner parts. At one projected stellar scaleheight on the far side of the galaxy (third and fourth rows) the effect of dust extinction is already much smaller. In fact, for central face-on optical depths up to  $\tau_0 = 1$ , the line-of-sight projected velocity and velocity dispersion curves are practically indistinguishable, even at  $i = 90^\circ$ . This gives support to the strategy employed in Paper I of positioning the slit parallel to the major axis at one stellar scaleheight in cases of a strong dust lane.

The behaviour of the  $h_3$  and  $h_4$  parameters for various amounts of dust extinction is shown in Fig. 6. The effect of the dust extinction is a radial ‘stretching’ of the curves as more and more of the stellar disc becomes hidden from view. This causes the positions of sign reversal to shift towards larger radii. The dependence is rather sensitive, suggesting that it may form a useful tool for assessing the amount of dust extinction. Figs 5 and 6 also show the results obtained when including scattering for the  $\tau_0 = 2$  case (dashed lines). The addition of the scattered photons to the LOSVD counteracts the effect of absorption for all LOSVD parameters, but the reduction is negligible and scattering can be safely ignored.

Although the question of the opacity of spiral galaxies is still not completely settled, the most widely supported view comes from studies of individual edge-on galaxies. Both radiative transfer modelling of multiband photometry (Kylafis & Bahcall 1987; Xilouris et al. 1999) and comparisons of the kinematics of the gaseous component at different wavelengths (Bosma et al. 1992; Bosma 1995) indicate that massive edge-on spirals with strong dust lanes have a face-on central optical depth  $\tau_0$  around unity in the  $V$  band (see Kuchinski et al. 1998, for a discussion). Here we showed



**Figure 6.** The  $h_3$  and  $h_4$  parameters for various amounts of dust extinction (dotted lines) at  $i = 90^\circ$ ,  $z = 0$  (Fig. 5, top row). The arrows indicate the direction of increasing  $\tau_0$ . The dashed lines show calculations for  $\tau_0 = 2$  that include scattering.

that at central optical depths of unity and for  $z = h_z$ , the line-of-sight projected kinematics of the model with smoothly distributed dust are practically indistinguishable from the dust-free case. At  $z = 0$  dust extinction becomes important if the inclination is close to  $90^\circ$ . These results are in general agreement with the recent study of Baes et al. (2003).

## 4 MODELLING THE OBSERVED STELLAR KINEMATICS

### 4.1 Approach

A least-squares approach was used to determine the transparent stellar disc model (Section 3) that best matches the stellar kinematics for each galaxy. In Section 4.3 it will be shown that using dusty models instead does not significantly change the results. First, position–velocity (XV) diagrams were calculated for a sequence of transparent models. The models cover a sufficient range in the free parameter,  $\sqrt{M/L} (\sigma_z/\sigma_R)^{-1}$ , and use the observationally determined  $I$ -band disc luminosity distribution (KKG), the rotation curve (Paper III) and the inclination (Table 1). The XV diagrams were calculated for the exact same positions as sampled by the slit (table 4 in Paper I). Then, each of the simulated stellar XV diagrams

was first smoothed to match the seeing during the observations, and subsequently spatially averaged using the same binning scheme as applied to the observations. In further analogy with the observations, the velocity profile in each bin was then fitted with a parametrized LOSVD, using either a Gaussian or a truncated Gauss–Hermite series. Finally, the best-fitting model was determined by comparing the parametrized stellar kinematics of the models with the observations in a least-squares analysis. Both the line-of-sight projected stellar velocity and velocity dispersion were fitted simultaneously in case of a Gaussian, whereas for a truncated Gauss–Hermite series the line-of-sight velocity, dispersion and the  $h_3$  and  $h_4$  parameters were fitted.

For the ESO–LV galaxies and NGC 5170 the 2D bulge-disc decomposition parameters were used for the disc luminosity distribution (KKG; for NGC 5170 see appendix A in Paper I). For ESO 435-G25, NGC 891 and 5529 the  $I$ -band parameters determined by Xilouris et al. (1999) were taken. Xilouris et al. (1999) have studied multiband surface photometry to constrain the global luminosity and dust distributions, employing the same model luminosity and dust distributions as adopted here. This allows a check on the importance of dust extinction for these three galaxies (Section 4.3). Xilouris et al. (1999) quote formal errors of a few per cent for the disc luminosity parameters of ESO 435-G25, NGC 891 and 5529. Here, a more conservative 10 per cent error for their derived face-on central surface brightness and disc scale parameters was adopted.

The model is inapplicable at small galactocentric radii. First, the radial force gradient, neglected in the plane-parallel approximation, becomes comparable to the disc density (Section 3.4). By monitoring the ratio of radial force gradient to disc density ( $C$ ) the accuracy of the plane-parallel approximation can be assessed (Bottema, van der Kruit & Freeman 1987). Therefore, the ratio of radial force gradient to disc luminosity density,  $C M/L$ , was calculated as a function of radius for each galaxy. The inner radial range where the ratio is substantial was not included in the fits. Secondly, a bulge restricts the radial range at which the disc model can be applied. This was one of the reasons for selecting intermediate- to late-type galaxies. For galaxies with an appreciable bulge, data inside the projected radius corresponding to a bulge-to-disc surface brightness ratio of

1/10 was excluded (ESO 240-G11, 435-G25, 437-G62, 446-G18, NGC 891, 5170 and 5529).

## 4.2 Results

In Table 3 the fit parameters are gathered, listing the product  $\sqrt{M/L} (\sigma_z/\sigma_R)^{-1}$  and the intrinsic stellar kinematics at one disc scalelength. The density weighted velocity dispersion,  $\langle \sigma_R(h_R) \rangle_z = \sqrt{3/2} \sigma_R(h_R, 0)$ , allows a direct comparison with Bottema (1993). The errors were obtained by quadratically adding the formal fitting error to the errors introduced by the uncertainties in the face-on central surface brightness, the disc scale parameters, the inclination (Table 1) and the position of the dynamical centre (table 4 in Paper I). Note that we have not added a column listing the disc  $M/L$ , as the mass-to-light ratio depends on the uncertain disc surface brightness (see Section 3.2). The disc mass-to-light ratios will be addressed briefly in Paper V.

In most cases the fits are well behaved, with five galaxies having a reduced  $\chi^2$  below 2, and 12 having a  $\chi^2$  below 6. The exceptions are ESO 437-G62 and NGC 5529, for which the line-of-sight projected velocity dispersion and especially the  $h_3$  and  $h_4$  cannot be reproduced. In general, the lower quality of some of the fits reflects the fact that our model is based on global, simplified assumptions that can only approximate the local details. This translates into uncertainties in the fitted parameters (see the grey lines in Fig. 7) that may be different from galaxy to galaxy. Still, in general the disc model based on the  $I$ -band luminosity distribution and a constant  $M/L$  provides a good match. Note that none of the sample galaxies shows the drop in the line-of-sight projected dispersion at small radii expected for a pure disc system (Section 3.4). In most cases this is probably due to the bulge, which will have a larger line-of-sight projected velocity dispersion than the disc. For galaxies with no clear bulge component, such as ESO 157-G18, 142-G24 and 269-G15, the lack of a drop can be explained by the inclination (cf. Fig. 4). An interesting feature is that the range in the derived products  $\sqrt{M/L} (\sigma_z/\sigma_R)^{-1}$  is only a factor of 2. This will be discussed in Paper V. For NGC 891 and 5170 the results are consistent with earlier studies (Bottema et al. 1987, 1991). In the following the fits are described for each galaxy.

**Table 3.** Results of the least-squares fits to the stellar kinematics.

Galaxy	Side	$\chi^2$	$\sqrt{M/L} (\sigma_z/\sigma_R)^{-1}$	$\pm$	$v_*(h_R, 0)$	$\pm$	$\sigma_R(h_R, 0)$	$\pm$	$\langle \sigma_R(h_R) \rangle_z$	$\pm$
(1)	(2)	(3)	$[(M/L)^{1/2} \odot]$	(4)	(5)	(5)	(6)	(6)	(7)	(7)
ESO 142-G24	b	1.5	3.0	+1.0 -1.0	98	+4 -4	23	+8 -9	28	+9 -11
ESO 157-G18	r	1.4	2.0	+0.9 -1.0	80	+3 -4	14	+7 -6	17	+8 -7
ESO 201-G22	r	1.2	2.8	+0.7 -1.6	156	+6 -8	35	+16 -17	43	+20 -21
ESO 240-G11	b	1.7	2.2	+1.3 -1.3	267	+9 -5	29	+17 -18	36	+21 -22
ESO 269-G15	b	4.3	1.9	+0.6 -1.0	141	+6 -7	36	+11 -19	44	+13 -23
ESO 288-G25	b	6.7	1.9	+0.7 -1.0	156	+8 -15	39	+10 +4	48	+12 -26
ESO 435-G14	b	5.0	1.8	+0.4 -0.5	107	+6 -5	40	+21 -6	49	+5 -7
ESO 435-G25	b	2.5	2.4	+1.2 -0.7	207	+14 -9	39	+14 -11	48	+17 -13
ESO 437-G62	b	9.2	1.9	+1.1 -0.7	270	+11 -18	49	+34 -11	60	+42 -13
ESO 446-G18	b	3.2	3.2	+0.8 -1.6	186	+4 -4	34	+6 -18	42	+7 -22
ESO 487-G02	r	0.1	4.2	+0.7 -1.3	144	+16 -9	69	+10 -22	85	+12 -27
ESO 564-G27	b	3.6	5.7	+1.7 -1.4	130	+8 -8	45	+10 -12	55	+12 -15
NGC 891	a	5.3	2.9	+0.6 -0.8	208	+4 -4	54	+8 -10	66	+10 -12
NGC 5170	r	3.6	3.1	+1.7 -1.1	184	+4 -7	58	+15 -14	71	+18 -17
NGC 5529	a	16	3.3	+1/2 -0.8	223	+23 -21	92	+8 -16	113	+10 -20

Columns. (1) Galaxy. (2) Side(s) fitted, b, both, a, approaching, b, receding. (3) Reduced  $\chi^2$ . (4) Best-fitting product, with  $M/L$  referring to the  $I$  band. (5) Stellar velocity at ( $R = h_R, z = 0$ ). (6) Radial stellar velocity dispersion at ( $R = h_R, z = 0$ ). (7) As (6) but weighted with the vertical density.

**ESO 142-G24.** The model provides a rather good fit, even at radii smaller than the inner fitting boundary. The low inclination (Table 1) suggests that the scaleheight may have been overestimated, causing a product  $\sqrt{M/L} (\sigma_z/\sigma_R)^{-1}$  which is perhaps somewhat too low (Section 4.4). The  $C M/L$  parameter is rather large in fitted radial range, indicating that the plane-parallel approximation is rather poor.

**ESO 157-G18.** The approaching side of this dwarf system shows anomalous kinematics, both for the stars and the gas (see Paper III). This side was therefore not included in the fit. As in ESO 142-G24, the rotation curve rises slowly such that  $C M/L$  parameter is rather large in fitted range. The inferred intrinsic velocity dispersion is the lowest in the sample.

**ESO 201-G22.** The approaching side, for which the ionized gas velocities lie far below the H I envelope (see Paper III), was not included in the fits. The stellar kinematics, both observed and intrinsic, are similar to those of ESO 269-G15 and 288-G25.

**ESO 240-G11.** To obtain a reasonable fit to the data from the Double Beam Spectrograph at the Siding Spring 2.3-m telescope (the DBS data), the circular velocity in the inner 20 arcsec needs to be higher than at larger radii. A lower circular velocity curve in the inner parts leads to a severe underprediction of the observed stellar velocities. The adopted rotation curve has a maximum at  $R \sim 7$  arcsec, consistent with the bulge luminosity profile (KKG) and the H I and [N II] XV diagrams (see Paper III, appendix). The amplitude of the maximum was chosen to yield the best fit to the observed stellar kinematics, but the results do not depend sensitively on this choice. The solution corresponds to the ‘coldest’ disc in the sample in terms of  $\sigma_R(h_R)/v_{\max}$ . The asymmetric kinematics observed in the data from the FORS2 instrument on the VLT (the FORS2 data) taken at 2 arcsec from the major axis (Paper I) cannot be reproduced by this solution. Perhaps this is caused by its low inclination in conjunction with a strongly non-axisymmetric disc (see Paper I).

**ESO 269-G15.** The observed stellar velocity is slightly asymmetric. Simultaneously, fitting both the observed velocity and dispersion provides a rather poor match to the data. A larger-amplitude rotation curve (by  $\sim 10 \text{ km s}^{-1}$ ) would improve the fit but is inconsistent with the H I and H II data. A slightly less steeply rising rotation curve is allowed by the data (see Paper III). However, that would yield a very poor fit.

**ESO 288-G25.** In the outermost bin the observed stellar velocity and dispersion exceed those implied by the model. The model can be made consistent with the outermost data points if the rotation curve is allowed to be higher. However, such a rotation curve would undo the match obtained at smaller radii. The uncertainty in the inner rotation curve (Paper III) is included in the errors quoted in Table 3.

**ESO 435-G14.** The best-fitting model is well determined because the line-of-sight projection effect is small due to the low inclination. The model cannot reproduce the outermost data points; the observed velocities and velocity dispersions are larger by up to  $10 \text{ km s}^{-1}$  than those predicted by the model. These discrepancies may be due to the pronounced spiral arms which cross the slit at these positions.

**ESO 435-G25.** As a result of a deficiency of gas in the inner parts the circular velocities are unknown at  $R \lesssim 50$  arcsec. Therefore, models were constructed for two extreme cases, taking a solid-body and a flat rotation curve in this region (down to  $R = 0$ ). The best-fitting model parameters found for the two cases are consistent to within the errors (the inner parts are largely excluded from the fits). The solid-body rotation is adopted here because it better matches the stellar rotation at radii smaller than the inner fitting boundary. The  $h_3$  and  $h_4$  parameters are also reproduced rather well. These results, obtained using the luminosity distribution according to Xilouris et al. (1999), are consistent with those obtained using the distribution derived in KKG (not shown).

**ESO 437-G62.** This lenticular remained undetected in the H I (Paper II) and in the H  $\beta$  and O III emission lines (Paper III). Instead, the systemic velocity was estimated by symmetrizing the stellar velocity curve (Paper I). For the rotation curve, the function  $v_c(R) = v_{\max} R/\sqrt{R^2 + d^2}$  was adopted, adjusting the parameters such that the best fit of the stellar kinematics is obtained. This yielded  $v_{\max} = 277 \pm 10 \text{ km s}^{-1}$ ,  $d < 1$  arcsec, essentially a flat rotation curve. Overall, the model yields a good fit. It fails, however, to closely match the line-of-sight projected stellar velocity in the outermost bins and to reproduce the  $h_4$  parameter.

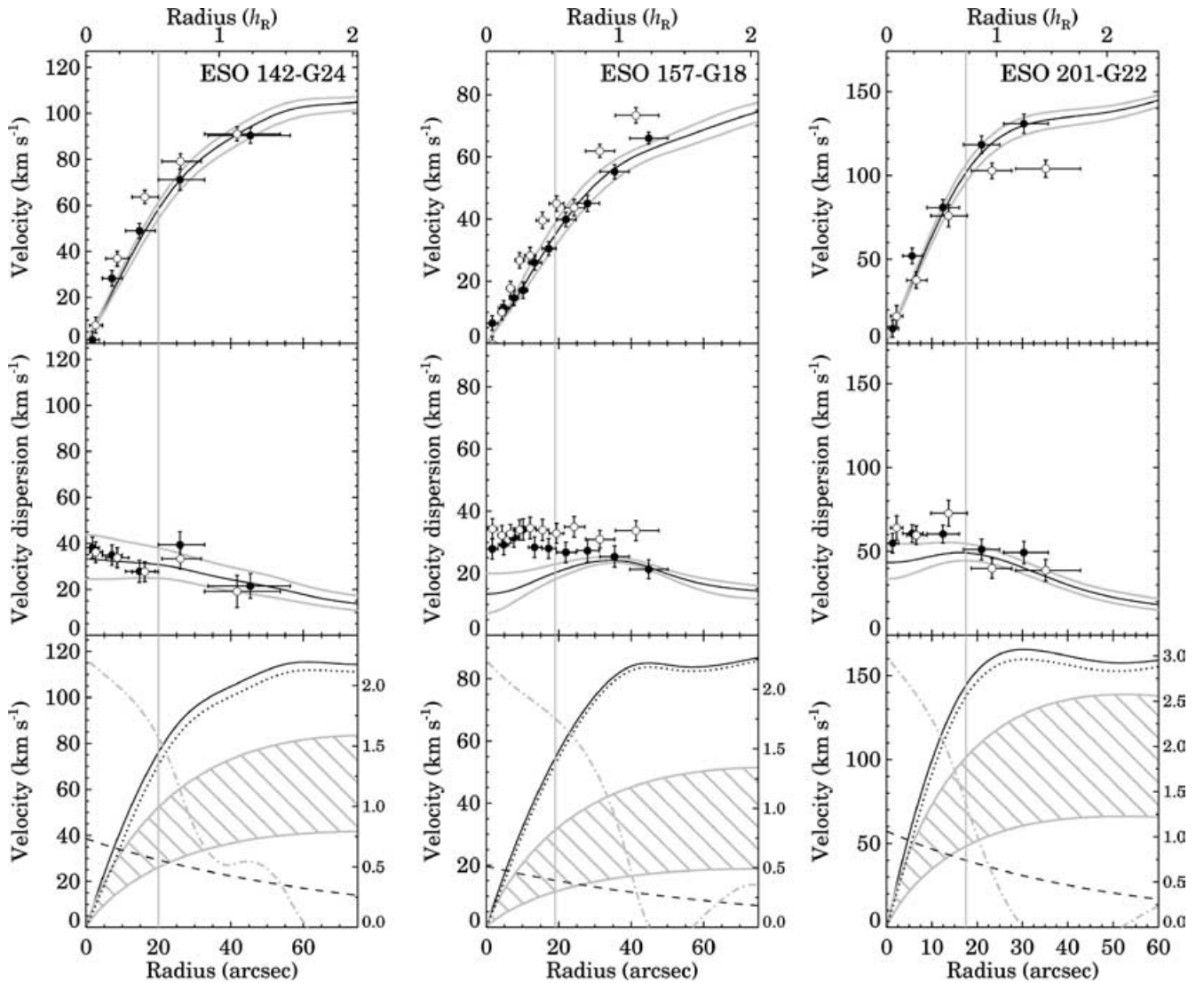
**ESO 446-G18.** In the inner parts the rotation curve is poorly constrained by the H I and H II observations (Paper III). It may well rise more steeply than the adopted curve. The effect of a steeper rotation curve is small, however, and well within the errors. As the adopted rotation curve rises slowly, the  $CM/L$  parameter is rather large in the fitted range.

**ESO 487-G02.** For the rotation a flat curve at a level of  $167 \pm 10 \text{ km s}^{-1}$  was used, based on the H I linewidth (Theureau et al. 1998, corrected for instrumental broadening and random motions according to Verheijen & Sancisi 2001). Models were also calculated for  $\tau_0 = 1.0$ , using the prescription of Section 3.3. The quoted errors take this possibility into account.

**ESO 564-G27.** The fit is of poor quality, with the model overpredicting the observed velocity and dispersion. None of the other galaxies show this feature. Fitting the dispersion only would yield a substantially lower product  $\sqrt{M/L} (\sigma_z/\sigma_R)^{-1} \simeq 3.6$  but a line-of-sight projected rotation much larger than observed. This behaviour is expected when dust extinction plays a role (Section 4.3), but may also be due to strong non-axisymmetry.

**NGC 891. Parallel to the major axis.** The transparent model provides a good match, except for the outermost bins. Taking a rotation curve which does not have the inner maximum but instead is ‘solid body’ up to 60 arcsec does not significantly affect the results.

*Perpendicular to the major axis.* Fig. 7 shows the best-fitting dusty model, obtained using only the data at  $z \geq h_z$ . This model is consistent with that determined from the parallel slit position. At and above  $z = h_z$  dust extinction is clearly unimportant. Remarkably, the model also provides a good match to the velocity dispersions below  $h_z$ . This shows that the drop in the observed stellar velocity and dispersion below one stellar scaleheight is very likely to be due to dust extinction. At low  $z$  the predicted stellar velocities of the dusty model are too low. This may indicate that a smooth dust model overestimates the effect of extinction on the velocity. Interestingly, the position of the minimum in the line-of-sight projected velocity and dispersion is very sensitive to the inclination.



**Figure 7.** Fits to the line-of-sight stellar kinematics, and the intrinsic kinematics of the best-fitting disc model. Top, these panels display the line-of-sight projected stellar velocity curve, folded across the dynamical centre. Dots denote the receding side and circles the approaching side. The best-fitting model is shown by the solid line, the grey lines bracket the  $1\sigma$  error. The vertical grey line marks the adopted inner fitting boundary (goodness of fit should be judged from the region beyond this boundary). For slit positions parallel to the minor axis the spatial axis is as in fig. 5 in Paper I. Middle, these panels show the line-of-sight projected stellar velocity dispersion. Symbols and lines are as in the top panel. Bottom, for slit positions parallel to the major axis the bottom panel shows the intrinsic kinematics of the best-fitting model. The solid and dotted lines indicate the rotation curve and the mean stellar rotation curve, respectively. The dashed line denotes the radial stellar velocity dispersion at  $z = 0$ . The grey line (dot-dash) shows the product  $C M/L$  (Section 4.1), its scale is added to the right. The hatched area indicates the range of probable disc contributions to the rotation curve assuming  $\sigma_z/\sigma_R = 0.6$  (see Paper V for a justification of this value). In cases where the Gauss–Hermite parametrization was used the  $h_3$  and  $h_4$  curves are also shown.

The adopted inclination,  $i = 89.8 \pm 0.5^\circ$ , is based on model comparisons at various inclinations (dotted lines, for  $i < 90$  the far side is to the northwest).

**NGC 5170.** The bulge surface brightness is negligible compared with that of the disc at both slit positions, but at the inner position (position A) the  $I$ -band image shows signs of extinction (Paper I). Therefore, position B was modelled. It is reassuring that the model which best fits the data at B roughly agrees with the amplitudes of the line-of-sight projected velocities and dispersions at A (solid lines in Fig. 7). The grey lines in the plot for position A indicate the behaviour when a dust distribution is included with  $\tau_0 = 0.6$ ,  $h_R$ ,  $d = h_R$  and  $h_z$ ,  $d = 0.5 h_z$ . This simple model roughly reproduces the observed velocity, but fails for the dispersion at positive  $z$ .

**NGC 5529.** The inner fitting boundary was positioned beyond the region that shows strong non-circular motions in  $H\alpha$  (see Paper III). The circular velocity curve is unknown in this region. Therefore, models were constructed for two extreme cases, taking a solid-body and a flat rotation curve (down to  $R = 0$ ). The solid-body rotation is adopted here because it better matches the stellar rotation at radii smaller than the inner fitting boundary. The results for the flat curve yield a product  $\sqrt{M/L} (\sigma_z/\sigma_R)^{-1} = 3.6$  and  $\sigma_R(h_R) = 98 \text{ km s}^{-1}$ , consistent with the values for the solid-body curve (Table 3). The models overpredict the velocity dispersions measured in the outermost bins and cannot reproduce the strong asymmetries ( $h_3$  and  $h_4$ ). These features in conjunction with the high disc ‘temperature’  $\sigma_R(h_R)/v_{\text{max}} \simeq 0.4$  constitute further evidence that the disc is strongly perturbed.

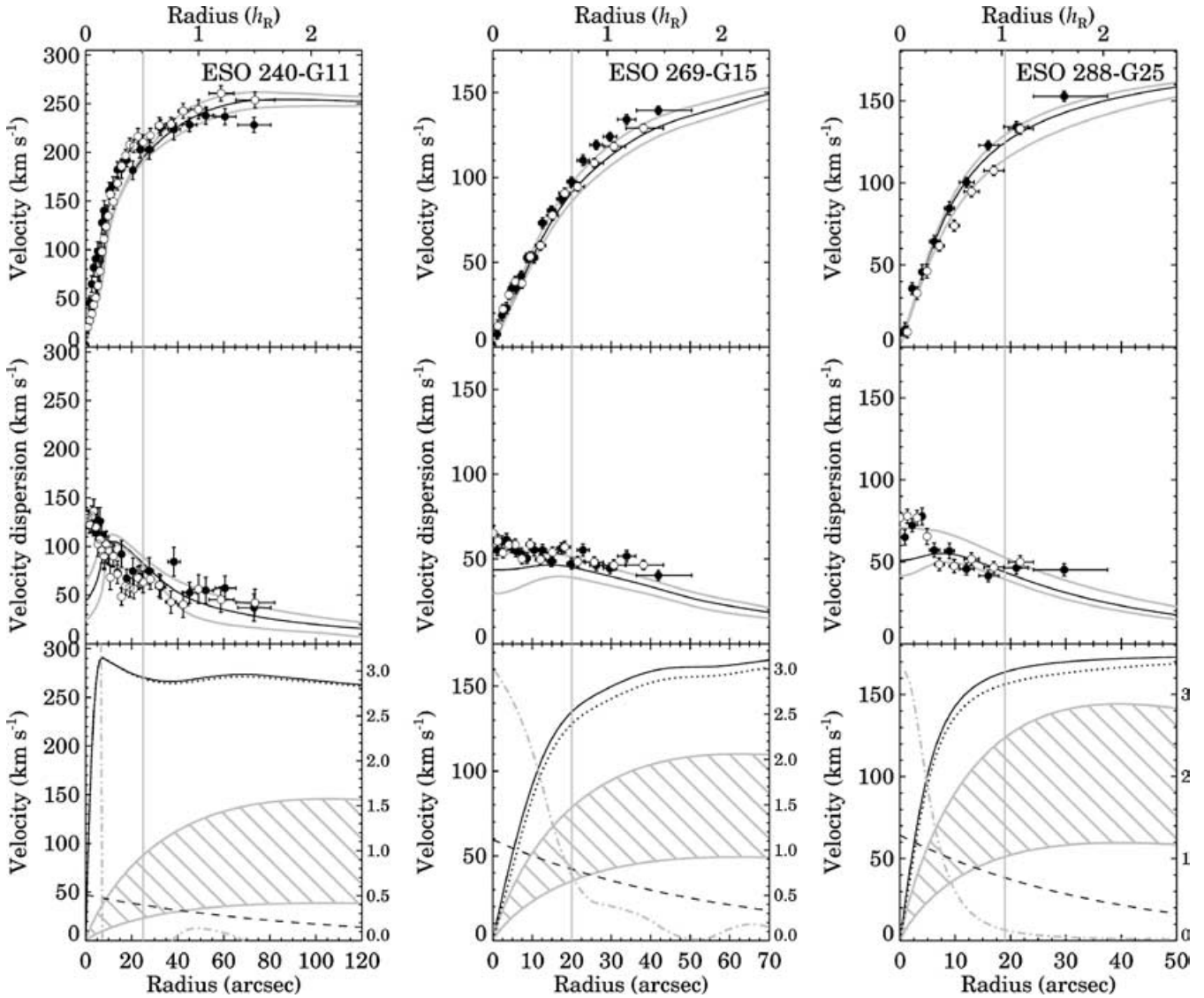


Figure 7 – continued

### 4.3 Dust extinction

For the three galaxies in common with Xilouris et al. (1999) the fits were repeated after including a smooth dust distribution (Section 3.3). These models used the Xilouris et al. (1999) dust distribution in the  $V$  band, which corresponds most closely to the 4800–5700 Å region used to measure the stellar kinematics. For the disc luminosity distribution the  $I$ -band parameters were retained. The results for these three massive spirals are listed in Table 4 and are shown in Fig. 7 (dashed lines). For ESO 435-G25, which has the smallest face-on central optical depth, the effect of dust is negligible. For NGC 891 and 5529 the derived products  $\sqrt{M/L}(\sigma_z/\sigma_R)^{-1}$  and dispersions are somewhat smaller, although still within the errors derived for the transparent models. The goodness of fit obtained for the dusty models of NGC 891 and 5529 is substantially poorer than for the transparent models. This suggests that the smooth dust models are overestimating the effect of dust extinction. A clumpy dust distribution (Matthews & Wood 2001) or a dust distribution shallower than exponential may perhaps offer a solution. It can be concluded that the effect of dust is small in these three cases. Note

that in these cases the slit was positioned away from the major axis (see Paper I).

Although direct estimates of the effect of the dust extinction in the remainder of the galaxies are not available, several arguments support the view that it is small. First, for all galaxies except ESO 201-G22, 564-G27 and 437-G62 the slit was positioned away from the dust lane. The simulations show that this avoids the bulk of the dust (Section 3.4). Indeed, a comparison of the H I and H II kinematics (Paper III) shows that at the slit positions studied the galaxies are transparent beyond approximately one scalelength. Secondly, the adopted inner fitting boundary ensures that the central parts ( $R \lesssim 0.5 h_R$ ), those expected to be the most affected, are not included in the fits. A third reason is that as the effect of dust is to reduce both the stellar velocity and velocity dispersion (Section 3.4), a transparent model will no longer be able to fit dust-affected kinematics. Briefly, in the transparent model a lower intrinsic velocity dispersion means a smaller asymmetric drift and hence is associated with a *higher* observed stellar velocity. As this is contrary to what is observed for dust-affected kinematics, the least-squares routine will compensate and pick a velocity and velocity dispersion which

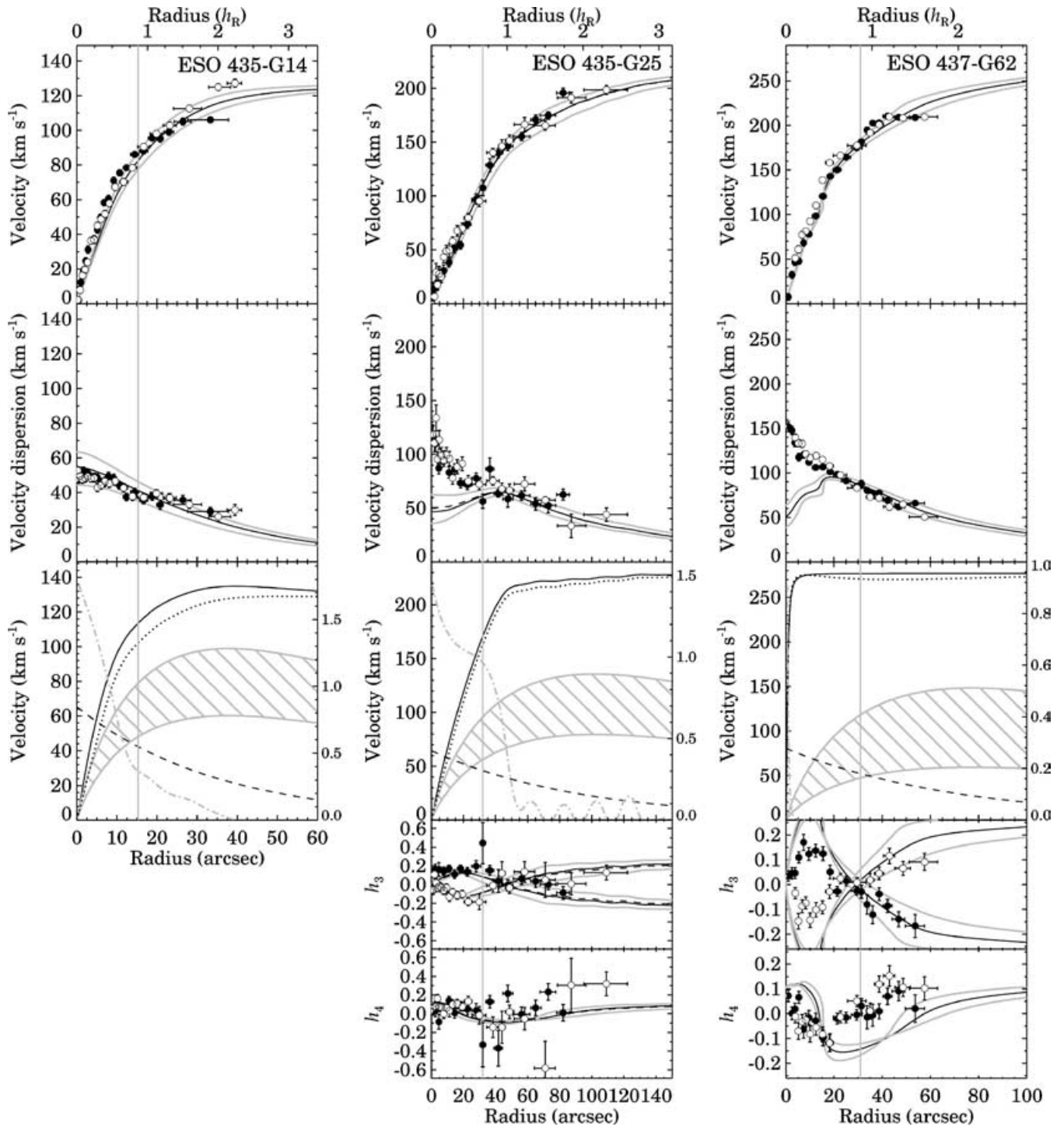


Figure 7 – continued

are higher than observed. In this case, the product  $\sqrt{M/L}(\sigma_z/\sigma_R)^{-1}$  and the intrinsic velocity dispersion will be overestimated. None of the transparent model fits show such a signature, except for ESO 564-G27. Fourthly, the three galaxies for which the dust was included (Table 4) and shown to have a small effect are among the most massive systems. Several studies indicate that in massive systems the amount of dust is relatively large (Giovanelli et al. 1995; Tully et al. 1998; Matthews & Wood 2001). This is also consistent with our observation (Paper II) that the slope of the luminosity–

linewidth relation is shallower for edge-on galaxies than that of the small-inclination sample from the *HST* Key Project. It is therefore likely that the effect of dust extinction is also small in the remaining, smaller systems. A fifth argument against a dust extinction effect is that for those galaxies for which the Gauss–Hermite parametrization of the LOSVD was used, such as ESO 437-G62, the fitted  $h_3$  and  $h_4$  are similar to those of the observations. There is no need for the radial ‘stretching’ seen in the dusty models (Section 3.4). Finally, the contribution of dusty patches is naturally reduced because

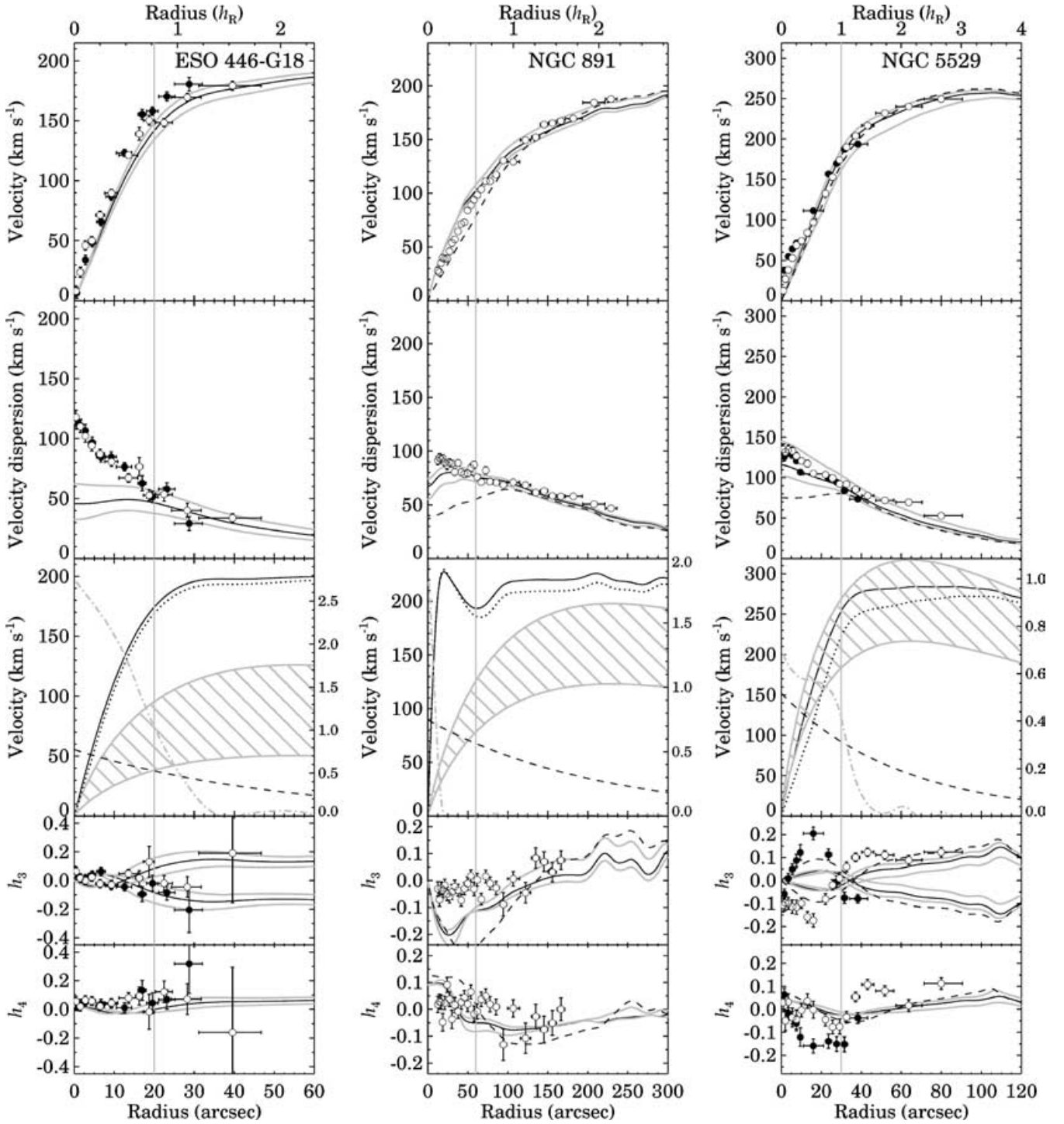


Figure 7 – continued

the spatial binning scheme used to extract the stellar kinematics is intensity weighted.

In short, from Paper III there was already strong evidence based on a detailed comparison of H I and H II kinematics that most of the galaxies are transparent beyond approximately one disc scalelength. Here, a radiative transfer prescription was included to model three massive spirals, showing that the effect of dust extinction on the stellar kinematics is small. The fact the slit was positioned away from the dust lane in the remaining galaxies plus a number of additional

arguments based on simulated dusty galaxies (Section 3.4) shows that the effect of dust extinction is probably small for the remainder of the galaxies. Possible exceptions are ESO 201-G22 and 564-G27.

#### 4.4 Systematic errors

The important assumptions underlying our disc model which have not yet been addressed are the plane-parallel approximation, the

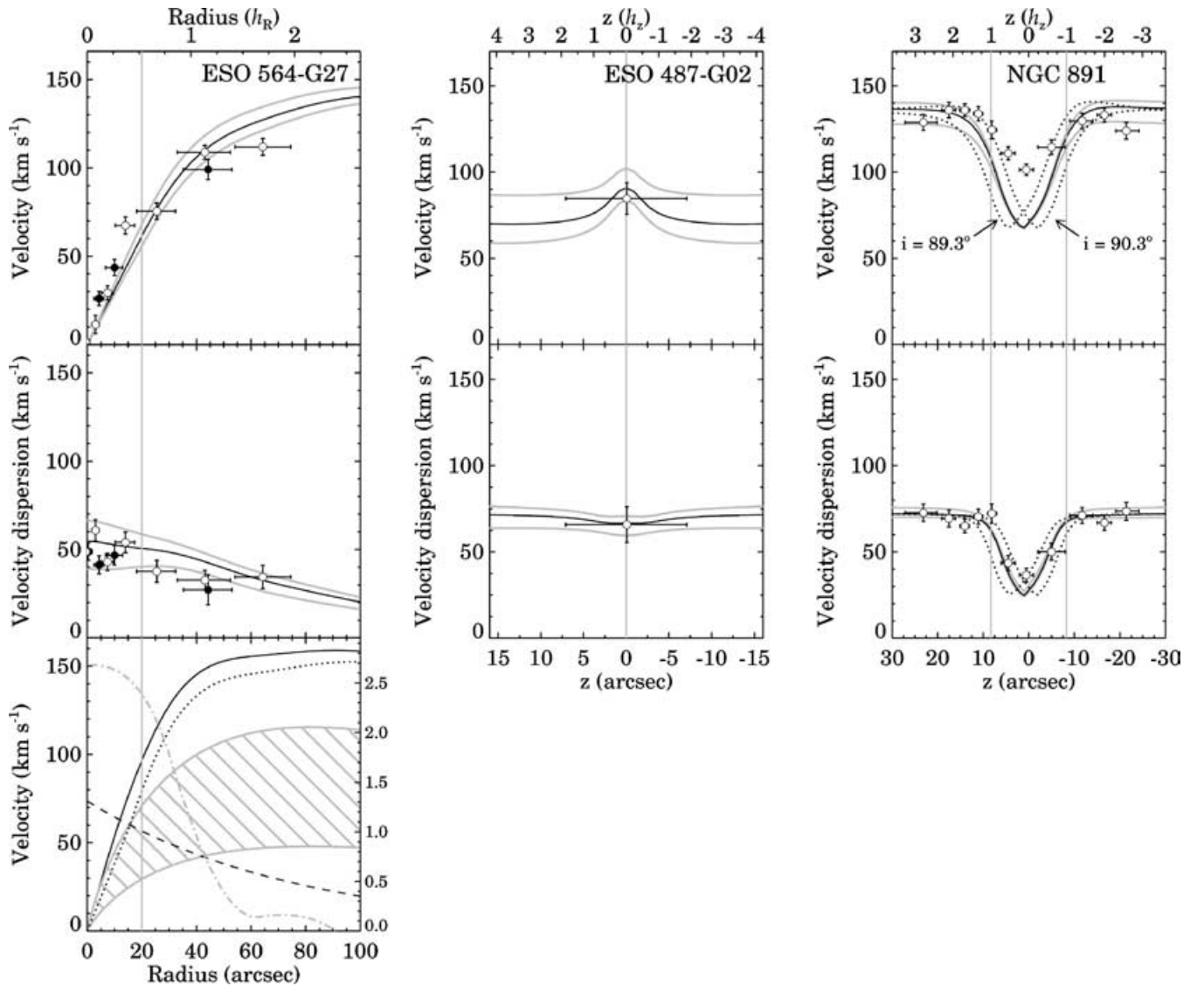


Figure 7 – continued

constancy of the disc  $M/L$ , the disc self-gravity and the shape of the vertical density distribution.

To monitor the plane-parallel approximation the ratio of radial force gradient to disc luminosity density ( $CM/L$ ) was calculated for each galaxy (Section 4.1). The calculated  $CM/L$  parameter is much less than unity when the gradient in the rotation curve is shallow (Fig. 7). Hence, in most cases the  $C$  parameter is small, even for a low value of the disc  $M/L$ . In these cases the use of the plane-parallel approximation is well justified. For galaxies for which the rotation curve is rising in the fitted region, such as ESO 142-G24, 157-G18 and 564-G27, the radial force gradient is appreciable. To assess the importance of deviations from the plane-parallel approach in these cases, the spherical approximation for the tilt term in the radial Jeans equation (equation 5) was used (Oort 1965) and the fits were repeated. The differences with the plane-parallel approach are insignificant, suggesting that the plane-parallel approximation is also justified for galaxies with more slowly rising rotation curves.

The mass-to-light ratio of the disc is assumed to be constant. The effect of deviations from this assumption can be investigated by fitting simulated galaxies having non-constant  $M/L$  with the constant

$M/L$  model. Clues regarding the radial  $M/L$  behaviour are given by nearly face-on spirals, for which radial colour gradients (de Jong 1996b) and the ratios of disc scalelengths in different bandpasses (Peletier et al. 1994) indicate that discs are bluer at larger galactocentric radii. If this outward bluing is primarily driven by lower mean ages and metallicities as suggested by spectrophotometric models of disc evolution (de Jong 1996b; Bell & de Jong 2000), the stellar  $M/L$  will drop with galactocentric radius. A simple prescription for such a decreasing  $M/L$  is obtained when both the disc mass and luminosity decrease exponentially with radius but each with a different scalelength:

$$M/L = \frac{\Sigma_0 e^{-R/h_{R,M}}}{\mu_0 e^{-R/h_{R,L}}} = (M/L)_0 e^{-R/h_{R,M/L}}, \quad (11)$$

where  $\Sigma_0$  is the central surface mass density,  $h_{R,M}$  is the mass scalelength,  $h_{R,L}$  is the luminosity scalelength and  $h_{R,M/L}$  is the corresponding  $M/L$  scalelength. It follows that the disc  $M/L$  decreases exponentially in the case where  $h_{R,L} < h_{R,M}$ .

Fig. 8(a) shows three different  $M/L$  gradients:  $h_{R,M/L} = , 8h_{R,L}$  and  $2h_{R,L}$  (or  $h_{R,L}/h_{R,M} = 1, \frac{9}{8}$  and  $\frac{3}{2}$ ). Using these  $M/L$  gradients, three versions of the reference model of Section 3.4 were created, all



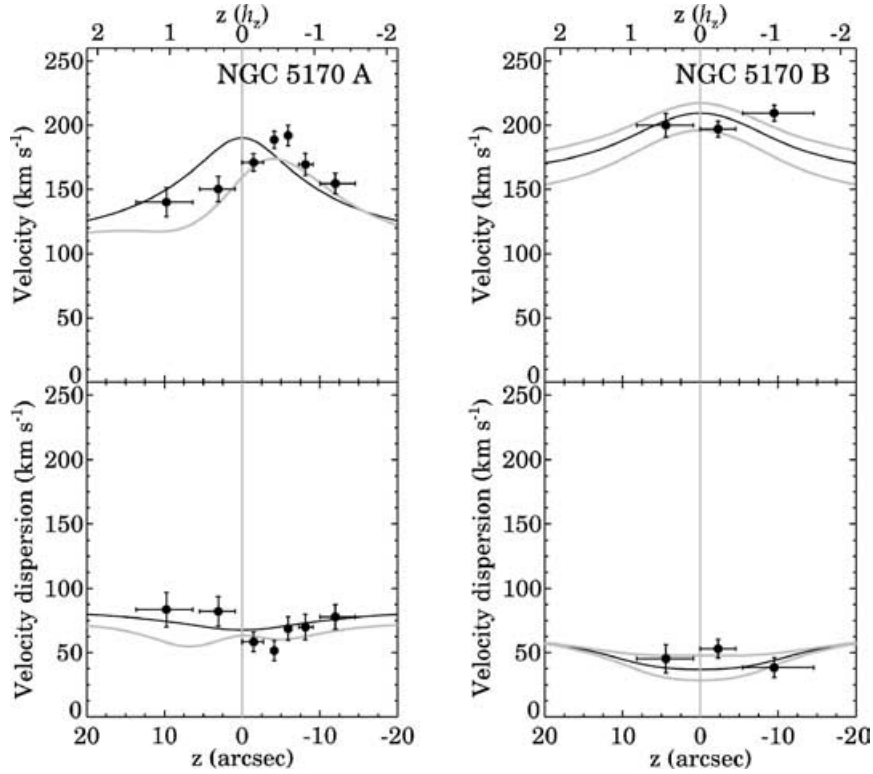


Figure 7 – continued

having the same luminosity scalelength but different mass scalelengths. Fig. 8(b) shows their line-of-sight projected stellar kinematics. At small  $h_{R,M/L}$  the observed stellar velocity increases and the observed dispersion decreases because of the lower surface densities. A parallel with the observable scalelengths can be drawn recognizing that the stellar kinematics were studied in the  $V$  band (approximately) and assuming that the disc  $M/L$  is truly constant in the  $K$  band. In that case  $h_{R,L}/h_{R,M} = h_{R,V}/h_{R,K}$ . As spirals with  $h_{R,V}/h_{R,K} > \frac{3}{2}$  are rare (de Jong 1996a) it follows that  $h_{R,M/L} = 2h_{R,L}$  is an extreme case. Fig. 8(b) shows that for this case the differences with a constant  $M/L$  case are less than 10 per cent (compare the dashed and solid lines). The actual fit of a constant  $M/L$  model to the  $h_{R,M/L} = 2h_{R,L}$  case at  $R < 2h_{R,L}$  is shown by the grey lines. The fit underestimates (overestimates) the  $M/L$  and intrinsic dispersions at small (large) radii, such that the inferred  $M/L$  is close to the average  $M/L$  in the fitted range (Fig. 8a). The inferred radial dispersion at  $h_{R,L}$  is close to the true dispersion (Fig. 8c). Hence, it can be concluded that the effect of deviations from a constant  $M/L$  on the derived stellar kinematics at  $h_{R,L}$  is small, probably within 10 per cent.

The disc is assumed to be self-gravitating. The neglect of the halo gravity is addressed in Appendix B, based on solutions for the vertical disc structure and kinematics in a disc+halo potential. It is shown that a stellar disc embedded within a halo has a larger vertical velocity dispersion compared with an isolated disc with the same surface density and energy. Therefore, the surface density inferred from an observed velocity dispersion using a self-gravitating disc model is an overestimate of the true surface density. The overestimate in surface density is negligible for high surface brightness discs,  $\sim 5$  per cent, but may become appreciable for very low surface brightness discs,  $\sim 20$  per cent. This would affect ESO 142-G24, 157-G18, 201-G22 and 446-G18. The neglect of the gravity of the gas layer is treated in Appendix C. There numerical solutions of the joint

potential of stars and H I gas are used to investigate the difference between the actual stellar disc surface density and the surface density inferred assuming a self-gravitating stellar disc. For the spirals studied here the disc surface density at one scalelength appears to be slightly overestimated, on average by  $\sim 10$  per cent (Table C2 in Appendix C). The exception is the dwarf spiral ESO 157-G18 for which the gas fraction is large,  $\Sigma_{\text{H I}} \sim 0.6$ , indicating that the stellar disc surface density is overestimated by almost 50 per cent.

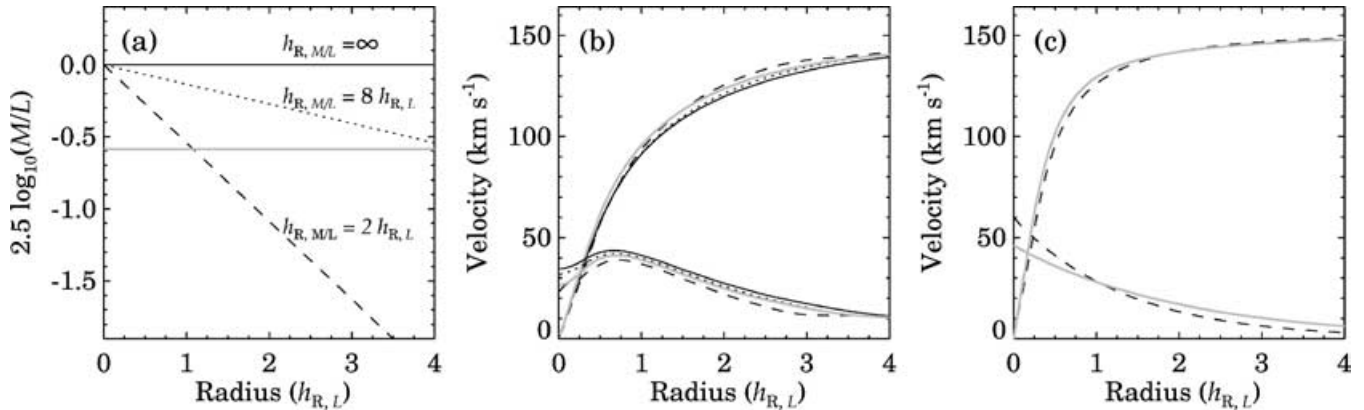
An exponential form is used for the vertical density distribution. This distribution is not well known close to the galaxy plane, because of uncertainties in the interpretation of the observations of edge-on spirals (dust, inclination, seeing) and the exact contribution of young red supergiants in the near-infrared, although probably small (Jones et al. 1981; Rhoads 1998). To gauge the effect of a shallower vertical distribution, the fits were repeated for a sech( $z$ ) form (van der Kruit 1988). The face-on central surface brightnesses were calculated by requiring that the sech( $z$ ) profile matches that of the exponential model at large  $z$ , giving  $\mu_0^{\text{sech}} = (\pi/4) \mu_0^{\text{exp}}$ . The inferred radial velocity dispersions  $\langle \sigma_R(h_R) \rangle_z$  are  $\sim 15$  per cent smaller for discs with  $i < 89^\circ$ . At higher inclination the shape of the vertical distribution does not enter in the line-of-sight projection, and the  $\langle \sigma_R(h_R) \rangle_z$  are the same as those obtained with the exponential form (within the errors). The products  $\sqrt{M/L} (\sigma_z/\sigma_R)^{-1}$  obtained with the sech( $z$ ) distribution are smaller for inclinations  $i < 89^\circ$  by, on average, 12 per cent. For galaxies with higher inclinations the product is unchanged.

At relatively low inclinations ( $i < 87^\circ$ ) the two-dimensional bulge–disc decomposition tends to overestimate the disc scaleheight (KKG). Only one-third of the sample has an inclination  $i < 87^\circ$  (Table 1). In a worst case scenario, the scaleheights of some of these discs may have been overestimated by 100 per cent (fig. 2.2 in Kregel 2003). To estimate its effect, the scaleheights of these galaxies were halved, and the fits repeated. The inferred dispersions are

**Table 4.** Fitting results for three massive spirals using dusty models.

Galaxy	$\tau_0$	$h_{R,d}^*$ (kpc)	$h_{z,d}^*$ (kpc)	$\sqrt{M/L} (\sigma_z/\sigma_R)^{-1}$ [[ $(M/L)^{1/2} \odot$ ]]	$\chi^2$	$v_*(h_R, 0)$ (km s $^{-1}$ )	$\sigma_R(h_R, 0)$ (km s $^{-1}$ )	$\langle \sigma_R(h_R) \rangle_z$ (km s $^{-1}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ESO 435-G25	0.30	11.4	0.33	2.5	2.4	207	41	50
NGC 891	0.85	7.5	0.28	2.0	16	213	39	48
NGC 5529	0.65	9.8	0.44	2.6	22	236	73	89

Columns. (1) Galaxy. (2) Face-on optical depth. (3) Dust scalelength. (4) Dust scaleheight. (5) Best-fitting product. (6) Reduced  $\chi^2$ . (7) Stellar velocity. (8) Radial stellar velocity dispersion. (9) Radial velocity dispersion weighted with the vertical density.



**Figure 8.** The effect of non-constant  $M/L$  on the derived stellar kinematics, illustrated using the reference model of Section 3.4. (a)  $M/L$  versus radius for three models (arbitrary scale, see the text for details). (b) Line-of-sight projected stellar velocity (top) and velocity dispersion (bottom) for the three models. Lines are as in (a). (c) Intrinsic stellar velocity and velocity dispersion for  $h_{R,M/L} = 2 h_{R,L}$  (dashed line). In each panel the grey lines indicate the constant  $M/L$  fit to the  $h_{R,M/L} = 2 h_{R,L}$  model.

somewhat larger, to compensate for the smaller line-of-sight projection effect for smaller scaleheights. Even for this extreme case the effect is small though,  $\sim 10$  per cent. The product  $\sqrt{M/L} (\sigma_z/\sigma_R)^{-1}$  was larger by a factor of  $\simeq 1.3$ . This can be easily understood from equation (3): the product has to compensate for the reduction of  $h_z$  while the best-fitting  $\sigma_R$  is still roughly the same.

The product  $\sqrt{M/L} (\sigma_z/\sigma_R)^{-1}$  scales inversely with the square root of the distance. For example, for  $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Freedman et al. 2001) the derived products are 3 per cent smaller. In individual cases deviations from the adopted Virgo-centric velocity model may have introduced an error somewhat larger than this (5 per cent for a residual peculiar velocity of  $\sim 200 \text{ km s}^{-1}$  at a radial velocity of  $2000 \text{ km s}^{-1}$ ).

Finally, the model contains several approximations such as axisymmetry and an exponential disc. These approximations probably hold to first order, introducing small errors that average out given a substantial sample. For example, the symmetry of most of the observed stellar velocity curves indicates that non-axisymmetric components such as spiral structure have a small effect on the overall results. There are of course exceptions such as NGC 5529 for which the effect is difficult to quantify. This is also true for deviations from the exponential disc. These are probably less important here than for modelling rotation curves in terms of mass distributions (Sellwood 1999). Although we have not investigated this systematically we have for a few cases used a deprojected radial profile instead of the exponential fit as a test. This resulted in differences in the fitted parameters within the present uncertainties.

In summary, most of the systematic uncertainties are probably small. In order of decreasing importance the effects are: (i) neglect

of the halo gravity, (ii) a possible scaleheight overestimate (for at most one-third of the sample), (iii) steepness of the vertical density distribution at low  $z$ , (iv) neglect of the gas gravity and (v) constant disc  $M/L$ . For the dispersions the effects are less than 10 per cent and tend to cancel one another. The disc surface density is possibly overestimated by  $\sim 20$  per cent for the lowest surface brightness galaxies due to the neglect of the halo gravity. This will be taken into account in Paper V.

## 5 CONCLUSIONS

The stellar kinematics of 15 intermediate- to late-type edge-on spiral galaxies were modelled to study the dynamical properties of their stellar discs. The sample covers a substantial range in both galaxy maximum rotational and deprojected face-on disc surface brightness. Seven spirals show either a boxy- or peanut-shaped bulge which probably indicates that these are barred. Most of these spirals are regular in the optical and the H I ranges, have sufficiently high recessional velocities for distance uncertainties to be relatively small, and are located either in the field or in small loose groups.

Realistic stellar disc models show that the effects of projection and internal dust extinction on the stellar kinematics depend sensitively on the optical depth, inclination and the position with respect to the major axis. However, a large face-on optical depth ( $\tau_0 \sim 10$ ) and an inclination within a few degrees of edge-on are needed to produce an apparent solid-body velocity curve and a significant decrease in the observable stellar dispersion. For realistic face-on optical depths

around unity the line-of-sight projected kinematics are practically dust-free at a projected distance of one scaleheight from the major axis, even at an inclination of  $90^\circ$ .

In most cases the dynamical model provides a good match to the observed stellar disc kinematics, without the need for dust extinction. In fact, by including a smooth dust distribution for three massive spirals the results are unchanged. Several indirect arguments support the view that the effect of extinction is similarly small in the remaining galaxies.

Possible systematic errors resulting from our assumptions and approximations have been discussed, including the plane-parallel approximation, the constancy of the mass-to-light ratio, the neglect of the halo gravitational field, the form of the vertical density distribution and the axisymmetry and the exponential nature of the disc. Most of these errors are probably small. The exception is the error introduced by the neglect of halo gravity, which can be as much as 20 per cent for the surface density in LSB discs. Future modelling should therefore attempt to include the dark halo and the gas layer in a self-consistent manner.

## ACKNOWLEDGMENTS

We would like to thank Erwin de Blok and Rob Swaters for valuable comments which helped to improve this paper. We have made use of the LEDA data base, recently incorporated in HyperLeda.

## REFERENCES

- Auer L. H., van Blerkom D., 1972, *ApJ*, 178, 175  
 Baes M., Dejonghe H., 2001, *MNRAS*, 326, 733  
 Baes M. et al., 2003, *MNRAS*, 343, 1081  
 Bahcall J. N., 1984, *ApJ*, 276, 156  
 Batsleer P., Dejonghe H., 1995, *A&A*, 294, 693  
 Begeman K. G., 1987, PhD thesis, Univ. Groningen  
 Bell E. F., de Jong R. S., 2000, *MNRAS*, 312, 497  
 Binney J., Tremaine S., 1987, *Galactic Dynamics*. Princeton Univ. Press, Princeton  
 Bizyaev D., Mitronova S., 2002, *A&A*, 389, 795  
 Bosma A., 1978, PhD thesis, Univ. Groningen  
 Bosma A., 1995, in Davies J. I., Burstein D., eds, *Proc. NATO Advanced Study Institute (ASI) Series C, Vol. 469, The Opacity of Spiral Disks*. p. 317  
 Bosma A., Byun Y., Freeman K. C., Athanassoula E., 1992, *ApJ*, 400, 21  
 Bottema R., 1993, *A&A*, 275, 16  
 Bottema R., 1999, *A&A*, 348, 77  
 Bottema R., van der Kruit P. C., Freeman K. C., 1987, *A&A*, 178, 77  
 Bottema R., van der Kruit P. C., Valentijn E. A., 1991, *A&A*, 247, 357  
 Bottinelli L., Gouguenheim L., Paturel G., Teerikorpi P., 1986, *A&A*, 156, 157  
 Bureau M., Freeman K. C., 1999, *AJ*, 118, 126  
 Chen B. et al., 2001, *ApJ*, 553, 184  
 Ciardullo R., Jacoby G. H., Harris W. E., 1991, *ApJ*, 383, 487  
 Cuddeford P., Amendt P., 1992, *MNRAS*, 256, 166  
 Cuddeford P., Binney J., 1994, *MNRAS*, 226, 273  
 de Grijs R., 1997, PhD thesis, Univ. Groningen (<http://www.ub.rug.nl/eldoc/dis/science/r.de.grijs>)  
 de Grijs R., 1998, *MNRAS*, 299, 595  
 de Grijs R., Peletier R. F., 1997, *A&A*, 320, L21  
 de Grijs R., Peletier R. F., van der Kruit P. C., 1997, *A&A*, 327, 966  
 de Jong R. S., 1996a, *A&AS*, 118, 557  
 de Jong R. S., 1996b, *A&A*, 313, 377  
 Di Bartolomeo A., Barbaro G., Perinotto M., 1995, *MNRAS*, 277, 1279  
 Drimmel R., Spergel D. N., 2001, *ApJ*, 556, 181  
 Edvardsson B., Gustafsson B., Nissen P. E., Andersen J., Lambert D. L., Tomkin J., 1993, in Hensler G., Theis C., Gallagher J. S., eds, *Panchromatic view of Galaxies; Proc. Int. Scientific Spring Meeting of the Astronomische Gesellschaft*. Editions Frontières, Gif-sur-Yvette, p. 401  
 Famaey B., van Caelenberg K., Dejonghe H., 2002, *MNRAS*, 335, 201  
 Freeman K. C., 1991, in Sundelius B., ed., *Dynamics of Disc Galaxies*. Göteborgs Univ., Göteborg, p. 15  
 Freedman W. L. et al., 2001, *ApJ*, 553, 47  
 Gerssen J., Kuijken K., Merrifield M. R., 1997, *MNRAS*, 288, 618  
 Gerssen J., Kuijken K., Merrifield M. R., 2000, *MNRAS*, 317, 545  
 Gilmore G., Reid N., 1983, *MNRAS*, 202, 1025  
 Giovanelli R., Haynes M. P., Salzer J. J., Wegner G., da Costa L. N., Freudling W., 1995, *AJ*, 110, 1059  
 Giuricin G., Marinoni C., Ceriani L., Pisani A., 2000, *ApJ*, 543, 178  
 Henyey L. G., 1937, *ApJ*, 85, 107  
 Henyey L. G., Greenstein J. L., 1941, *ApJ*, 93, 70  
 Jenkins A., Binney J., 1990, *MNRAS*, 245, 305  
 Jones T. J., Ashley M., Hyland A. R., Ruellas-Mayorga A., 1981, *MNRAS*, 197, 413  
 Kalnajs A. J., 1983, in Athanassoula E., ed., *Proc. IAU Symp. 100, Internal Kinematics of Galaxies*. Reidel, Dordrecht, p. 87  
 Kent S., Dame T., Fazio G., 1991, *ApJ*, 378, 131  
 Kregel M., 2003, PhD thesis, Univ. Groningen (<http://www.ub.rug.nl/eldoc/dis/science/m.kregel>)  
 Kregel M., van der Kruit P. C., 2004, *MNRAS*, 352, 787 (Paper III)  
 Kregel M., van der Kruit P. C., de Grijs R., 2002, *MNRAS*, 334, 646 (KKG)  
 Kregel M., van der Kruit P. C., Freeman K. C., 2004a, *MNRAS*, 351, 1247 (Paper I)  
 Kregel M., van der Kruit P. C., de Blok W. J. G., 2004b, *MNRAS*, 352, 768 (Paper II)  
 Kregel M., van der Kruit P. C., Freeman K. C., 2005, *MNRAS*, in press (doi:10.1111/j.1365-2966.2005.08855.x) (Paper V, this issue)  
 Kuchinski L. E., Terndrup D. M., Gordon K. D., Witt A. N., 1998, *AJ*, 115, 1438  
 Kylafis N. D., Bahcall J. N., 1987, *ApJ*, 317, 637  
 López-Corredoira M., Cabrera-Lavers A., Garzón F., Hammersley P. L., 2002, *A&A*, 394, 883  
 Matthews L. D., Wood K., 2001, *ApJ*, 548, 150  
 Narayan C. A., Jog C. J., 2002, *A&A*, 390, L35  
 Oort J. H., 1965, in Blaauw A., Schmidt M., eds, *Stars and Stellar Systems, Vol. V*. Univ. Chicago Press, Chicago, p. 455  
 Peletier R. F., Valentijn E. A., Moorwood A. F. M., Freudling W., 1994, *A&AS*, 108, 621  
 Press W. H., Teukolsky S. A., Vetterling W. T., Flannery B. P., 1992, *Numerical Recipes*, 2nd edn. Cambridge Univ. Press, New York  
 Rhoads J. E., 1998, *AJ*, 115, 472  
 Schlegel D. J., Finkbeiner D. P., Davis M., 1998, *ApJ*, 500, 525  
 Sellwood J. A., 1999, in Merritt D., Sellwood J. A., Valluri M., eds, *ASP Conf. Ser. Vol. 182, Galaxy Dynamics. A Rutgers Symp. Astron. Soc. Pac., San Francisco*, p. 351  
 Shapiro K. L., Gerssen J., van der Marel R. P., 2003, *AJ*, 126, 2707  
 Spitzer L., 1942, *ApJ*, 95, 329  
 Theureau G., Bottinelli L., Coudreau-Durand N. P., Gouguenheim L., Hallet N., Loulergue M., Paturel G., Teerikorpi P., 1998, *A&AS*, 130, 333  
 Tonry J. L., Dressler A., Blakeslee J. P., Ajhar E. A., Fletcher A. B., Luppino G. A., Metzger M. R., Moore C. B., 2001, *ApJ*, 546, 681  
 Toomre A., 1964, *ApJ*, 139, 1217  
 Tully R. B., 1980, *ApJ*, 237, 390  
 Tully R. B., Pierce M. J., Huang J. S., Saunders W., Verheijen M. A. W., Witchalls P. L., 1998, *AJ*, 115, 2264  
 van Albada T. S., Bahcall J. N., Begeman K., Sancisi R., 1985, *ApJ*, 295, 305  
 van der Kruit P. C., 1988, *A&A*, 192, 117  
 van der Kruit P. C., de Grijs R., 1999, *A&A*, 352, 129  
 van der Kruit P. C., Freeman K. C., 1984, *ApJ*, 278, 81  
 van der Kruit P. C., Freeman K. C., 1986, *ApJ*, 303, 556  
 van der Kruit P. C., Searle L., 1981, *A&A*, 95, 105

- van Houten C. J., 1961, *Bull. Astron. Inst. Neth.*, 16, 1  
 Verheijen M. A. W., 1997, PhD thesis, Univ. Groningen (<http://www.ub.rug.nl/eldoc/dis/science/m.a.w.verheijen>)  
 Verheijen M. A. W., Sancisi R., 2001, *A&A*, 370, 765  
 Wainscoat R. J., Freeman K. C., Hyland A. R., 1989, *ApJ*, 337, 163  
 Witt A. N., Gordon K. D., 1996, *ApJ*, 463, 681  
 Xilouris E. M., Byun Y. I., Kylafis N. D., Paleologou E. V., Papamastorakis J., 1999, *A&A*, 344, 868

## APPENDIX A: SCATTERING FORMALISM

When spirals are viewed at high inclination the effect of scattering on the brightness distribution is similar to that of absorption. Most scattered photons are sent out of the plane because of the relatively large optical depth in directions within the plane (van Houten 1961; Baes & Dejonghe 2001). Studies of the optical emission-line kinematics that include realistic dust distributions show that scattering has a negligible effect on the derived H $\alpha$  rotation curves (Bosma et al. 1992; Matthews & Wood 2001). In Section 3.4 it was shown that scattering has a similarly small effect on the stellar kinematics. There scattering was treated as follows.

The total intensity is written as a summation of partial intensities (Henyeey 1937; Kylafis & Bahcall 1987). The zeroth partial intensity  $I_0$  corresponds to the fraction of the light which has not been scattered, the first partial intensity  $I_1$  corresponds to the fraction of the light which has been scattered once, etc. The angular re-distribution of photons due to scattering is approximated by the Henyeey–Greenstein phase function (Henyeey & Greenstein 1941). The asymmetry parameter and the scattering albedo are assumed to be constant over the wavelength range 4800–5700 Å, used to derive the stellar kinematics. The  $V$ -band values of Di Bartolomeo, Barbaro & Perinotto (1995) were adopted for these parameters.

The LOSVD of the unscattered disc light is an integral sum of the local velocity distributions along the line of sight, weighted according to the local emissivity and optical depth (equation 10). Similarly, the LOSVD of the light which has been scattered  $n$  times before reaching the observer is (cf. Kylafis & Bahcall 1987, equation 10)

$$f_n(v) = \omega \int_0^{s_0} ds \exp \left[ - \int_{s'}^{s_0} ds' \kappa(R, z) \right] \kappa(R, z) \times \int_{\Omega} f_{n-1}(v, R, \varphi, z, \hat{n}) p(\hat{n}) \frac{d\Omega}{4\pi}, \quad (\text{A1})$$

where  $\omega$  is the dust albedo,  $f_{n-1}(v, R, \varphi, z, \hat{n})$  denotes the LOSVD of the fraction of the light received at position  $(R, \varphi, z)$  along the line of sight from direction  $\hat{n}$  that has been scattered  $n-1$  times and  $p(\hat{n})$  is the Henyeey–Greenstein phase function. Note that during scattering the velocity information received from direction  $\hat{n}$  is Doppler shifted according to the bulk motion of the dust grains (Auer & van Blerkom 1972). This velocity shift is estimated by assuming that the dust is in pure circular rotation. Finally, in analogy with Kylafis & Bahcall (1987), the total LOSVD  $f(v) = \sum_{n=0}^{\infty} f_n(v)$  is approximated as

$$f(v) \approx f_0(v) + f_1(v) + f_0(v) \frac{(I_1/I_0)^2}{1 - I_1/I_0}, \quad (\text{A2})$$

where  $I_0 = \int dv f_0(v)$  and  $I_1 = \int dv f_1(v)$  are the zeroth and first partial intensities, respectively.

## APPENDIX B: EMBEDDING A STELLAR DISC IN A DARK HALO

The model used (Section 3) strictly applies to a self-gravitating stellar disc. Here, the accuracy of this assumption is investigated by comparing the stellar velocity dispersion in an isolated stellar disc with that of the same disc in a dark halo. The treatment follows similar steps as in B93, which was not entirely complete. The gas gravity is treated separately in Appendix C. The complexity of the problem is much reduced by considering the vertical distribution and kinematics of a single isothermal population of stars (Bahcall 1984). For clarity a summary of the relevant equations is given.

The vertical stellar mass distribution  $\rho_d$  is related to the gravitational potential  $\Phi$  according to Poisson's equation. In cylindrical coordinates:

$$\frac{\partial^2 \Phi}{\partial z^2} = 4\pi G(\rho_d + \rho_h^{\text{eff}}), \quad (\text{B1})$$

where  $\rho_h^{\text{eff}}$  is the so-called effective halo density (Bahcall 1984):

$$\rho_h^{\text{eff}} = \rho_h - \frac{1}{4\pi G R} \frac{\partial}{\partial R} v_c^2, \quad (\text{B2})$$

with  $\rho_h$  being the halo density and  $v_c$  the circular velocity curve. The stellar disc further obeys the vertical Jeans equation. For an isothermal population in a plane-parallel geometry:

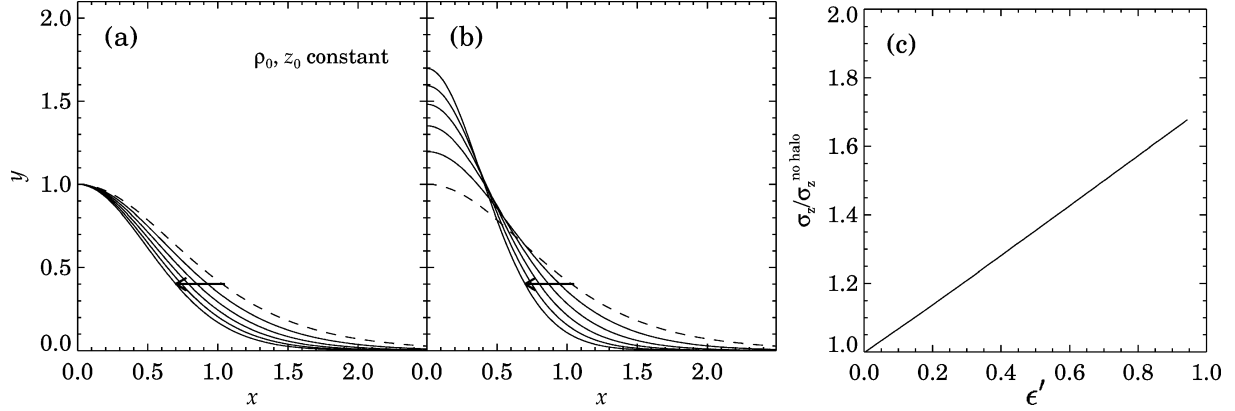
$$\sigma_z^2 \frac{\partial \rho_d}{\partial z} = - \frac{\partial \Phi}{\partial z} \rho_d. \quad (\text{B3})$$

By eliminating  $\Phi$  between equations (B1) and (B3) a second-order differential equation is obtained for the density:

$$\frac{dy^2}{dx^2} = \frac{1}{y} \left( \frac{dy}{dx} \right)^2 - 2y^2 - 2\epsilon y, \quad y(0) = 1, \quad \left. \frac{dy}{dx} \right|_0 = 0, \quad (\text{B4})$$

where the following definitions were used: the disc scale parameter  $z_0 = \sigma_z / \sqrt{2\pi G \rho_d(0)}$ ,  $x = z/z_0$ , the normalized density  $y(x) = \rho_d(z)/\rho_d(0)$  and the ratio of the effective halo density to the central disc density  $\epsilon(x) = \rho_h^{\text{eff}}(z)/\rho_d(0)$ . For an isolated disc, i.e.  $\epsilon = 0$  everywhere, the solution for the dimensionless disc density is  $y_{\epsilon=0}(x) = \text{sech}^2(x)$  (Spitzer 1942). For  $\epsilon \neq 0$ , solutions can be found numerically by rewriting equation (B4) as two first-order differential equations and using, for example, the Runge–Kutta method (Press et al. 1992). These solutions are shown in Fig. B1(a) for a range of halo densities. The halo densities were taken to be constant with  $z$ , i.e.  $\epsilon(x) = \epsilon$ , which is a good approximation except in the very central regions of a galaxy disc. If  $\rho_0$  and  $z_0$  are constant the vertical stellar density distribution becomes narrower and the disc contains less matter with increasing  $\epsilon$ .

To compare an isolated disc with the same disc within a dark halo, one needs to consider the solutions with the same disc surface density and energy. In B93 the energy conservation was not included. Conservation of mass and energy together yield a scaling relation for the solutions of Fig. B1(a) as follows. Suppose an isolated disc is placed, by some divine act, in a dark halo. Initially, the disc is out of equilibrium with parameters  $\sigma_z$ ,  $\rho_d(0) (= \rho_0)$ ,  $z_0$  and  $\epsilon_0$ . Gradually, the disc settles to a new equilibrium with parameters  $\sigma'_z$ ,  $\rho'_d(0) (= \rho'_0)$ ,  $z'_0$  and  $\epsilon'$ . Assuming that the halo is a static background potential, the halo drops out of the equations of mass and energy



**Figure B1.** Vertical density profiles of an isothermal disc embedded in a static halo (dimensionless units). (a) Density profiles for different central halo-to-disc density values  $\epsilon_0 = 0$  (dashed line) to 1 in steps of 0.2 (the arrow points to increasing  $\epsilon_0$ ). Solutions are for a constant disc central density and scale parameter and hence do not conserve mass. (b) Density profiles after scaling according to mass and energy conservation. A larger halo contribution leads to a narrower stellar disc density profile with a higher central density. (c) The ratio of the actual velocity dispersion to the dispersion inferred from the self-gravitating disc model as a function of  $\epsilon'$ .

conservation. Then for the disc surface density,

$$\begin{aligned}\Sigma_0 &= \int_{-\infty}^{\infty} \rho_0 y_{\epsilon=0}(z/z_0) dz = 2\rho_0 z_0, \\ \Sigma' &= \int_{-\infty}^{\infty} \rho'_0 y_{\epsilon'}(z/z'_0) dz = 2\rho'_0 z'_0 I_{\epsilon'}, \\ I_{\epsilon'} &= \int_0^{\infty} y_{\epsilon'}(x) dx,\end{aligned}\quad (\text{B5})$$

such that mass conservation,  $\Sigma_0 = \Sigma'$ , requires

$$\rho_0 z_0 = \rho'_0 z'_0 I_{\epsilon'}. \quad (\text{B6})$$

The kinetic energy of the disc is simply  $E_{\text{kin}} = \frac{1}{2} \Sigma_0 \sigma_z^2$  before settling and  $E'_{\text{kin}} = \frac{1}{2} \Sigma' \sigma_z'^2$  after settling. The disc potential energy is given by (e.g. Binney & Tremaine 1987, p. 34)

$$E_{\text{pot}} = \frac{1}{2} \int_{-\infty}^{\infty} \rho(z) \Phi(z) dz. \quad (\text{B7})$$

The disc potential follows from equation (B3):

$$\Phi(z) = -\sigma_z^2 \ln [\rho_d(z)/\rho_0], \quad (\text{B8})$$

in which the integration constant was fixed by requiring  $\Phi(0) = 0$ . Substituting equation (B8) in equation (B7) and integrating yields the potential energy of the unsettled disc:

$$E_{\text{pot}} = (2 - \ln 4) \rho_0 z_0 \sigma_z^2. \quad (\text{B9})$$

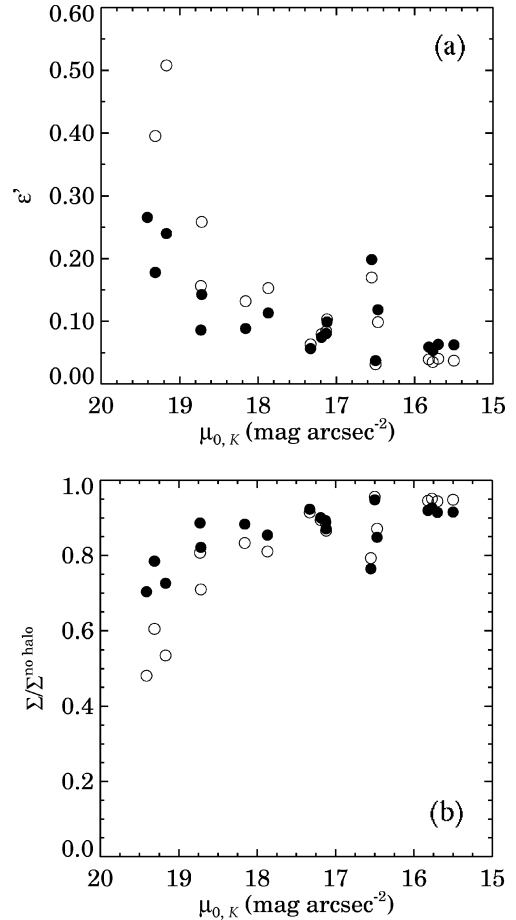
Similarly for the potential energy of the settled disc:

$$E'_{\text{pot}} = -K_{\epsilon'} \rho'_0 z'_0 \sigma_z'^2, \quad K_{\epsilon'} = \int_0^{\infty} y_{\epsilon'}(x) \ln y_{\epsilon'}(x) dx. \quad (\text{B10})$$

Finally, the conservation of disc energy  $E_{\text{kin}} + E_{\text{pot}} = E'_{\text{kin}} + E'_{\text{pot}}$  and mass (equation B6) together yield the following condition:

$$\frac{\epsilon'}{\epsilon_0} = (3 - \ln 4) \frac{I_{\epsilon'}^3}{I_{\epsilon_0} - K_{\epsilon'}}, \quad (\text{B11})$$

which can be solved to give  $\epsilon'$  and hence the settled disc parameters;  $\rho'/\rho_0 = \epsilon_0/\epsilon'$ ,  $z'/z_0 = I_{\epsilon_0}^{-1} \epsilon'/\epsilon_0$  and  $\sigma_z'/\sigma_z = I_{\epsilon_0}^{-1} \sqrt{\epsilon'/\epsilon_0}$ . The density distributions of the settled disc are compared with those of the unsettled disc in Fig. B1(b). While the velocity dispersion of an embedded disc is only slightly larger, the presence of the halo causes the disc to have a higher central density and to be significantly thinner.



**Figure B2.** The stellar surface density correction for the presence of a halo, according to mass modelling of the Ursa Major cluster spirals (Verheijen 1997). (a)  $\epsilon$  at two disc scalelengths versus disc central surface brightness for a constant disc flattening (circles) and a flattening increasing with decreasing surface brightness (dots). (b) The corresponding surface density correction versus disc central surface brightness. Symbols are as in (a).

Now consider an observer's strategy, who, magically, knows the disc surface density. The disc thickness is obtained from a fit of a  $\text{sech}^2(z)$  function to the vertical density profile, yielding an observed scale parameter  $z_0^{\text{obs}}$ . If the disc were isolated this parameter would equal the true value,  $z_0^{\text{obs}} = z_0$ . The velocity dispersion obtained from modelling the galaxy as a self-gravitating disc is then  $\sigma_z^2 = \pi G \Sigma z_0^{\text{obs}}$ . For a disc which is embedded in a dark halo, the observed scale parameter does not equal the true parameter. To relate the observed scale parameter to the physical parameter  $z_0$  in this case, a least-squares fit was made of a  $\text{sech}^2(z/z_0^{\text{obs}})$  to the curves in Fig. B1(b) yielding the ratio  $h(\epsilon') = z_0/z_0^{\text{obs}}$ . Thus, armed with the above theory an observer determines the scale parameter to be  $z_0 = z_0^{\text{obs}} h(\epsilon')$ . The velocity dispersion of an embedded disc is finally  $\sigma_z^2 = \sigma_z^{\text{no halo}2} I_{\epsilon'}^{-2} \epsilon' / \epsilon_0 = \pi G \Sigma z_0 I_{\epsilon'}^{-2} \epsilon' / \epsilon_0$ . Hence, the stellar disc velocity dispersion in a disc plus halo system can be obtained by multiplying the velocity dispersion of the best-fitting self-gravitating model by

$$\frac{\sigma_z}{\sigma_z^{\text{no halo}}} = \sqrt{\frac{\epsilon' h(\epsilon')}{\epsilon_0 I_{\epsilon'}^2}}. \quad (\text{B12})$$

This relation is shown in Fig. B1(c); for a constant disc surface density the velocity dispersion of a stellar disc embedded in a halo is larger.

Now consider the reverse situation where  $\Sigma$  is unknown. If a halo is present a lower disc surface density is needed to produce the same  $\sigma_z$  as in the isolated disc case. For a given  $\sigma_z$ , the ratio of the actual disc surface density in the presence of a halo to the surface density inferred using a self-gravitating disc model is

$$\frac{\Sigma}{\Sigma^{\text{no halo}}} = \left( \frac{\sigma_z}{\sigma_z^{\text{no halo}}} \right)^{-2} = \frac{\epsilon_0 I_{\epsilon'}^2}{\epsilon' h(\epsilon')}. \quad (\text{B13})$$

To estimate these correction factors, the theory was applied to the Ursa Major cluster sample of Verheijen (1997) for which halo and disc density estimates are available from mass modelling. The  $\epsilon$  and correction factors (equation B13) are shown in Fig. B2, adopting his 'Bottema disc' decompositions. The correction factors refer to a radius of two disc scalelengths. At this radius rotation curves are roughly flat and the second term in equation (B2) can be safely

ignored. The factors were calculated both for a constant disc flattening  $h_R/h_z = 8$  (circles) and an observationally motivated flattening increasing with decreasing surface brightness (see Paper V), fixing the relation by assuming  $h_R/h_z = 8$  at  $\mu_{0,K} = 17$  mag arcsec $^{-2}$ . This suggests that the self-gravitating disc model overestimates the disc surface density, by  $\sim 5$  per cent for high surface brightness discs up to  $\sim 20$  per cent at  $\mu_{0,K} \gtrsim 19$  mag arcsec $^{-2}$ .

## APPENDIX C: THE EFFECT OF THE GAS LAYER

Galaxy discs are multicomponent systems for which the stellar disc density distribution is influenced by the gas layer through the combined potential and vice versa. Adding an amount of gas to a pure stellar disc acts to reduce the stellar scaleheight. Hence, when the observed stellar kinematics of a galaxy disc are modelled using a single stellar component (Section 3), the reduced stellar scaleheight will be attributed entirely to the stellar disc. This leads to an overestimate of the stellar surface density. In the following the importance of this effect is quantified.

Multiple-component discs that are in steady state are governed by the combined Poisson equation and the equations of vertical hydrostatic equilibrium, one for each component. Narayan & Jog (2002) use an iterative scheme to simultaneously solve these equations for a plane-parallel disc consisting of isothermal components. Their scheme was adopted to calculate the vertical stellar density distribution in a two component system consisting of stars and H I gas. Stellar density distributions were calculated as a function of the H I mass fraction ( $\Sigma_{\text{H I}}$ ) and the stellar velocity dispersion ( $\sigma_z$ ). For the H I a velocity dispersion  $\sigma_{\text{H I}} = 10$  km s $^{-1}$  and a surface density  $\Sigma_{\text{H I}} = 4 M_{\odot} \text{ pc}^{-2}$  was used. A  $\text{sech}^2(z)$  function was fitted to these density distributions, yielding the 'observed' scaleheight,  $h_z$ . This observed scaleheight was used to calculate the surface density which would be inferred using a single component stellar disc model, i.e.  $\Sigma_*^{\text{no gas}} = \sigma_z^2 / (2 \pi G h_z)$ . The ratio of the true stellar surface density to the stellar surface density inferred using a single component ( $\Sigma_* / \Sigma_*^{\text{no gas}}$ ) is listed in Table C1 for a range of stellar

**Table C1.** The stellar surface density correction for the presence of gas.

$\sigma_z = 10 \text{ km s}^{-1}$		$\sigma_z = 20 \text{ km s}^{-1}$		$\sigma_z = 30 \text{ km s}^{-1}$		$\sigma_z = 50 \text{ km s}^{-1}$	
$\Sigma_{\text{H I}} / \Sigma_*$	$\Sigma_* / \Sigma_*^{\text{no gas}}$	$\Sigma_{\text{H I}} / \Sigma_*$	$\Sigma_* / \Sigma_*^{\text{no gas}}$	$\Sigma_{\text{H I}} / \Sigma_*$	$\Sigma_* / \Sigma_*^{\text{no gas}}$	$\Sigma_{\text{H I}} / \Sigma_*$	$\Sigma_* / \Sigma_*^{\text{no gas}}$
0.050	0.98	0.010	1.00	0.005	1.00	0.002	1.00
0.100	0.92	0.020	0.99	0.010	1.00	0.004	1.00
0.150	0.88	0.030	0.97	0.015	0.99	0.006	1.00
0.200	0.84	0.040	0.95	0.020	0.98	0.008	0.99
0.250	0.81	0.050	0.94	0.025	0.97	0.010	0.99
0.300	0.77	0.060	0.93	0.030	0.96	0.012	0.99
0.350	0.74	0.070	0.91	0.035	0.95	0.014	0.98
0.400	0.72	0.080	0.90	0.040	0.95	0.016	0.98
0.450	0.69	0.090	0.89	0.045	0.94	0.018	0.97
0.500	0.67	0.100	0.88	0.050	0.93	0.020	0.97
0.550	0.65	0.110	0.87	0.055	0.92	0.022	0.97
0.600	0.63	0.120	0.86	0.060	0.92	0.024	0.96
0.650	0.61	0.130	0.85	0.065	0.91	0.026	0.96
0.700	0.59	0.140	0.84	0.070	0.90	0.028	0.96
0.750	0.57	0.150	0.83	0.075	0.90	0.030	0.96
0.800	0.56	0.160	0.82	0.080	0.89	0.032	0.95
0.850	0.54	0.170	0.81	0.085	0.88	0.034	0.95
0.900	0.53	0.180	0.80	0.090	0.88	0.036	0.95
0.950	0.51	0.190	0.79	0.095	0.87	0.038	0.94
1.000	0.51	0.200	0.78	0.100	0.87	0.040	0.94

**Table C2.** The adopted correction factors.

Galaxy	$\Sigma_*/\Sigma_*^{\text{no gas}}$
ESO 142-G24	0.84
ESO 157-G18	0.63
ESO 201-G22	0.84
ESO 240-G11	0.90
ESO 269-G15	0.91
ESO 288-G25	0.95
ESO 435-G14	0.87
ESO 435-G25	0.92
ESO 437-G62	1.00
ESO 446-G18	0.93
ESO 487-G02	1.00
ESO 564-G27	0.93
NGC 891	0.98
NGC 5170	0.97
NGC 5529	1.00

velocity dispersions. The ratios are rather insensitive to the adopted values of  $\sigma_{\text{HI}}$  and  $\sigma_{\text{HI}}$ : similar calculations for  $\sigma_{\text{HI}} = 6\text{--}14 \text{ km s}^{-1}$  and  $\Sigma_{\text{HI}} = 1\text{--}10 \text{ M}_{\odot} \text{ pc}^{-2}$  give ratios within a few per cent of these reference values.

When the H I mass fraction  $\Sigma_{\text{HI}}$  and the vertical stellar dispersion are known, the corresponding ratio  $\Sigma_*/\Sigma_*^{\text{no gas}}$  in Table C1 can be used to correct the stellar surface density. Of course  $\Sigma_{\text{HI}}$ , such as  $\Sigma_*$ , is not known in the first place, but in practice one can use the uncorrected disc surface density  $\Sigma_*^{\text{no gas}}$  to estimate this ratio. In this way correction factors were determined for each galaxy studied, at a radius of one disc scalelength. For this the observed H I surface densities (Paper III), stellar velocity dispersions and the uncorrected disc surface densities were used, assuming  $\sigma_z/\sigma_R = 0.6$ . The correction factors are listed in Table C2.

This paper has been typeset from a  $\text{T}_{\text{E}}\text{X}/\text{L}_{\text{A}}\text{T}_{\text{E}}\text{X}$  file prepared by the author.