Texture Analysis Using Rényi's Generalized Entropies

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Published in:
EPRINTS-BOOK-TITLE

TEXTURE ANALYSIS USING RÉNYI'S GENERALIZED ENTROPIES

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ABSTRACT

We propose a texture analysis method based on Rényi's generalized entropies. The method aims at identifying texels in regular textures by searching for the smallest window through which the minimum number of different visual patterns is observed when moving the window over a given texture. The results show that any of Rényi's entropies can be used for texel identification. However, the second order entropy, due to its robust estimation, is the most reliable. The main advantages of the proposed method are its robustness and its flexibility. We illustrate the usefulness and the effectiveness of the method in a texture synthesis application.

1. INTRODUCTION

Only rarely have real world objects surfaces of uniform intensity; most of the time they are textured. The word texture is used to refer to a number of commonly encountered visual patterns in natural scenes, such as foliage, grass, pebbles, clouds, etc. While there is no proper definition of texture, it is widely accepted that the term generally refers to a repetition of certain basic elements, sometimes called *texels*. The textures exhibiting such a repetition of basic primitives are commonly referred to as structural textures. Although the structural approach to texture analysis is an old research direction in the field of texture analysis, it was less productive than other areas in the same field, due to various difficulties encountered [1, 2].

Structural texture analysis focuses primarily on identifying periodicity in texture or on identifying texels and their placement rules. Such methods are mainly based on Fourier analysis [3], cooccurrence matrices [4], and autocorrelation functions [1, 2]. While widely used, these methods prove to be fragile when it comes to natural texture analysis due to the imperfection in the regularity of these textures [1, 4].

Whether they deal directly with texel identification or they search for periodicity in texture, the structural texture analysis methods try to find the basic texture unit - the texel. This step is important for texture analysis not only in the context of structural approaches, it can also be used when other methods are employed for texture study. Next to being a tool for texture analysis [2, 4], texel identification is also useful in other fields of image processing, computer vision, and computer graphics such as texture synthesis [4], texture compression [4], image database retrieval [1], and 3D vision [5].

In this paper, we propose a texel identification method based on a concept borrowed from information theory - Rényi's generalized entropies. Rényi's entropy family is one of the most popular generalizations of Shannon entropy [6]. Similar to the Shannon entropy, it measures the amount of uncertainty in predicting the output of a probabilistic event [6]: when the uncertainty is reduced, the entropy decreases. This property is used in the method proposed here. More precisely, the texel identification method described in the following sections searches for the smallest window through which the minimum number of different visual patterns – different in a given sense – is observed when moving the window over a given texture. Such a window has the property of minimizing Rényi's generalized entropies.

2. RÉNYI'S GENERALIZED ENTROPIES

Rényi's generalized entropies were introduced in [7] as a family of measures that characterize the distribution of a random variable. They are defined as follows: if a random variable ξ takes the values $[x_i]_{i=1\dots N}$ with probabilities $[p_i = P(\xi = x_i)]_{i=1\dots N}$, then the generalized entropy of order q of ξ is defined as

$$H_q = \begin{cases} \dfrac{\log \sum\limits_{i=1}^{N} p_i^q}{1-q} & \text{for } q \neq 1, \\ \lim\limits_{q\to 1} \left(\dfrac{\log \sum\limits_{i=1}^{N} p_i^q}{1-q} \right) = -\sum\limits_{i=1}^{N} p_i \log p_i & \text{for } q = 1, \\ -\log \max\limits_{i=1\dots N} (p_i) & \text{for } q \to \infty. \end{cases} \quad (1)$$

The zero order entropy counts the number of values x_i for which p_i is nonzero[1]. The entropy of order one is the Shannon entropy. The entropy of order two, also called the *quadratic entropy* [8], gives the probability that ξ takes twice the same value. The order of the entropy can be seen as a weight of the contribution of each x_i to the value of the entropy. For the limit case $q \to \infty$, the entropy H_∞, also called the *min-entropy* [9], depends only on the probability of occurrence of the most frequent x_i.

3. RÉNYI'S GENERALIZED ENTROPIES OF TEXTURE

Let us consider an artificial texture image (Fig. 1) comprising a checkerboard pattern whose black and white blocks are squares $k \times k$ pixels wide. Such a texture can be described as consisting of square texels of size $2k \times 2k$. If we observe this texture locally, through a square shaped window of size $2k \times 2k$, we will see pieces of the texture that are, in fact, circularly shifted versions of the same pattern. A similar thing will happen if the size of the observation window is a multiple of $2k \times 2k$, *i.e.* $4k \times 4k$,

[1] When using the standard convention that $0^0 = 0$.

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$6k \times 6k$, etc. However, if the size of the observation window is not a multiple of $2k \times 2k$, we see a number of distinct patterns, *i.e.* patterns that are not merely circularly shifted versions of each other. In general, the number of observed distinct patterns depends on the window size and on the complexity of the analyzed texture.

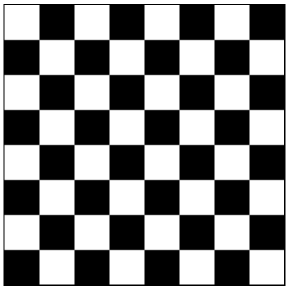
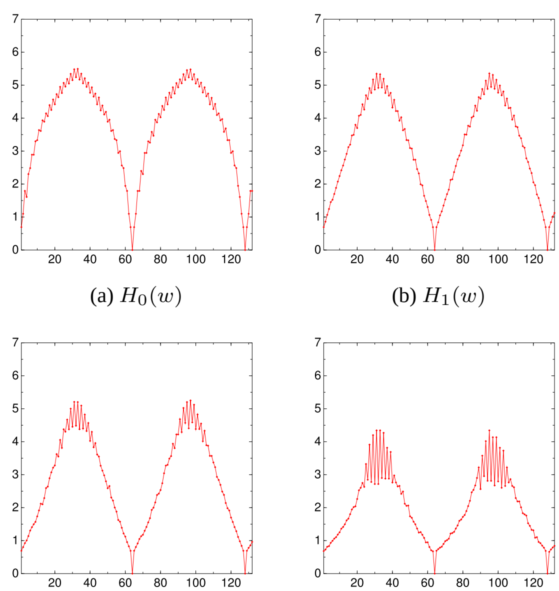


Fig. 1. A synthetic texture.

Let us call *visual event* a certain pattern that is seen through a square shaped observation window of a given size. When we observe the same pattern or patterns that are circularly shifted versions of each other, we say that we encounter identical visual events. Otherwise, we consider the patterns as different visual events. When moving such a window over a texture image we encounter a number of events. The number of different visual events encountered and their probabilities of occurrence are characteristics of the analyzed texture and they can be used to identify the texture.



(a) $H_0(w)$ (b) $H_1(w)$ (c) $H_2(w)$ (d) $H_\infty(w)$

Fig. 2. Rényi's entropies of the texture in Fig. 1, for $k = 32$. The values on the abscissa represent the window size w, while the values on the ordinate represent $H_q(w)$.

A concise way of describing these characteristics is through Rényi's generalized entropies computed from the probability of occurrence of each distinct visual event – we call them Rényi's entropies of the texture.

For a given texture, the texture entropies depend on the size of the observation window. We illustrate this property by considering a simple case, the synthetic texture from Fig. 1. We compute the distribution of visual events observed in a square window of size $w \times w$ and we plot its corresponding $H_q(w)$ as a function of w. For deciding whether two patterns represent the same visual event or not, we compare the histograms of the gray levels in the two observation windows. We consider two patterns as being the same event if the histograms associated with the two corresponding observation windows are identical.

From the family of generalized entropies we choose to study four representatives to see how the order affects the dependence of Rényi's entropy on the observation window size. We selected H_0 because of its counting property. H_∞ is interesting because it shows how likely it is to encounter the most frequent pattern. Further, we consider the Shannon entropy, H_1, to see how informative it is compared with the other entropies and the quadratic entropy, H_2, because it can be robustly estimated [6].

The zero order entropy $H_0(w)$, Fig. 2a, indicates how many different visual events are encountered for a given window size. For example, for a window of size 1×1, there are only two events that can be observed in the texture shown in Fig. 1 ($H_0(1) = 0.69$) – a white or a black pixel. For $w = 2$, the eight patterns encountered in the analyzed texture are grouped in three visual events (Fig. 3) leading to $H_0(2) = 1.01$. When observed through a window of 2×2, six of the patterns encountered in Fig. 1 are considered the same visual event because they have the same histogram.

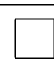
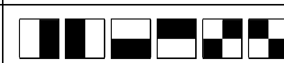
Event 1	Event 2	Event 3
		

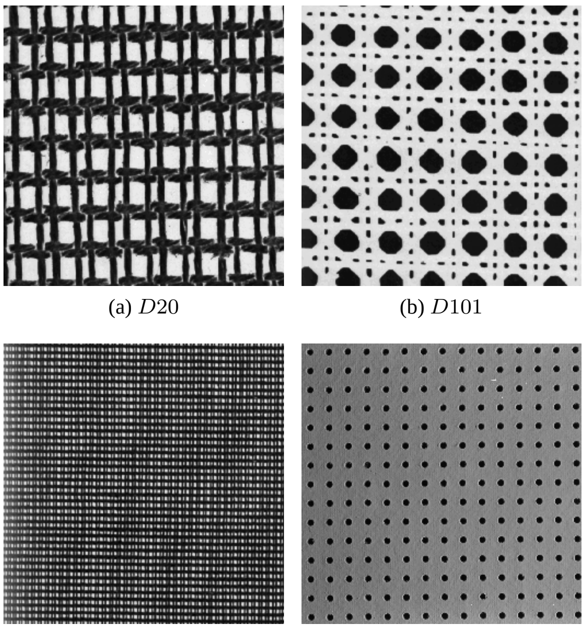
Fig. 3. The three classes of patterns encountered in Fig. 1 when observed through a window of size 2×2.

While in a general context the classification of the patterns in the third column of Fig. 3 in the same group could be a mistake, in the context of texture analysis such a classification is not necessarily wrong since all these patterns could be regarded as translated or rotated versions of the same pattern. When classifying textures, assigning rotated or translated version of the same texture to one class is desirable. However, for segmentation purposes the lack of discrimination between texture patches that are rotated versions of each other is undesirable. Nonetheless, in the current context of texture entropy computation, the rotation and translation invariance can be added or removed by choosing the right type of pattern comparison method for grouping different patterns in one event.

The zero order entropy reaches its minimum value when the observation window has a size multiple of $2k \times 2k$. Similarly, the other entropies plotted in Fig. 2 reach their minima for windows whose sizes are multiples of $2k \times 2k$. In other words, the window sizes for which the texture entropies reach their minima depend on the size of the texels of the texture and do not depend on the entropy's order. The same thing cannot be said about the maximum.

4. RÉNYI'S ENTROPIES OF NATURAL TEXTURES

The results displayed in Fig. 2 show that Rényi's entropies of a texture could be used to find the texel size in a regular texture. They also show that Rényi's entropies of any order can be used for this purpose. The next question is how such entropies behave for natural textures. Unlike artificial textures, natural textures do not consist of identical texels, even when they are highly regular. In order to see how this difference affects Rényi's entropy behavior, in the following, we present experiments with textures from Brodatz album [10] and from VisTex texture database [11]. We selected a number of regular textures containing square texels of various sizes (Fig. 4). The test images are of size 256×256 pixels.



(a) $D20$ (b) $D101$

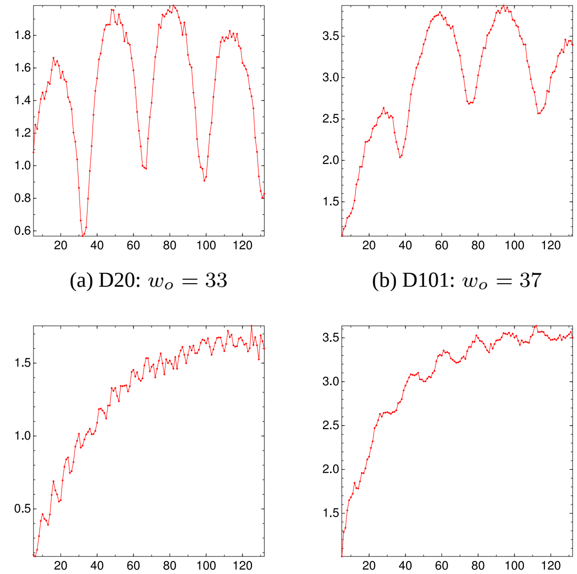
(c) $D21$ (d) $Tile9$

Fig. 4. Natural textures consisting of square shaped texels.

To account for the variability of the texels in real textures, we compare the histograms of the patterns observed in two windows of the same size by means of the two-sample Kolmogorov-Smirnov statistical test [12]. Applying Kolmogorov-Smirnov test to histogram comparison is a natural choice if one takes in consideration that histograms are estimates of the probability distribution functions of the gray levels. In such a case, computing the p_i in (1) is done using the box-counting schemes described in [13].

In Fig. 5, we display the results obtained for the textures in Fig. 4. For conciseness reasons, we present here only the plots obtained for the quadratic entropy. Similar plots were obtained for the other entropies. These results were obtained using the Kolmogorov-Smirnov test at a significance level 0.01, a significance level often used in statistics. Similar results were obtained at significance levels 0.05 and 0.10. We computed the texture entropies for window sizes between 4×4 and 133×133. We used window sizes greater than 4×4 pixels in order to have enough samples when performing the Kolmogorov-Smirnov test. We limited the size of the windows to 133×133 because using bigger windows would reduce the number of observed events in the image making the entropy computation less robust while it would not make the plots more informative.

In Section 3, we saw that in the case of a simple synthetic texture, Rényi's entropies can be used to determine the size of its texels, when the texels were square shaped. The entropies of such a texture are minimal around the window size that is equal to the size of the texel. Moreover, the entropy plots have deep valleys around such points.



(a) D20: $w_o = 33$ (b) D101: $w_o = 37$

(c) D21: $w_o = 6$ (d) Tile9: $w_o = 14$

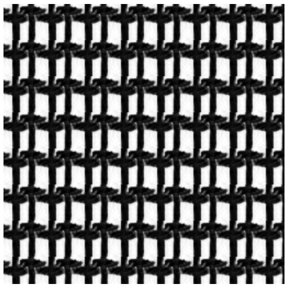
Fig. 5. Rényi's quadratic entropies of the textures in Fig. 4. The values on the abscissa represent the window size w, while the values on the ordinate represent the entropy $H_2(w)$. The caption of each plot gives the texture name and the size of the texel in the original texture as computed from that plot.

Similar to the synthetic texture case, the generalized entropies of natural textures have deep valleys around the window size equal to the size of the texel. However, the texture entropies are not zero, neither are they minimal at such points. Typically, they do not reach the zero value at all and they are minimal for the smallest window size. Nonetheless, the sizes of the texels can be determined if one looks at the position of the deep valleys in the entropy plots. More precisely, the texel size is given by the window size for which the smallest local minimum[2] is reached. The captions of each of the plots in Fig. 5 give the size of the texel as computed according to this rule.

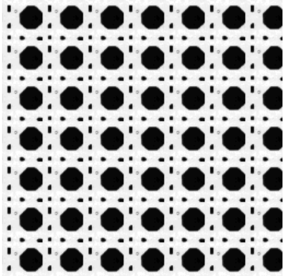
5. APPLICATION TO TEXTURE SYNTHESIS

In this section, we demonstrate an application of the texel size determination in texture synthesis by tiling. Such a synthesis method has the advantages of being fast and able to preserve the regularity of the texture sample. In Fig. 6, we present artificially generated counterparts of the textures in Fig. 4. Each artificial texture was generated by tiling with a square window randomly selected from the sample image. The size of that window was computed according to the method presented in Section 4.

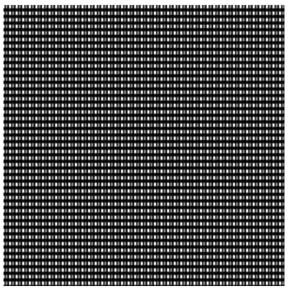
[2]In the current context, $H_q(w)$ is considered to have a local minimum for window size w if $H_q(w) \leq H_q(w-1)$ and $H_q(w) \leq H_q(w+1)$.



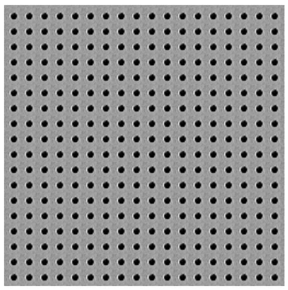
(a) D20: $w_o = 33$



(b) D101: $w_o = 37$



(c) D21: $w_o = 6$



(d) Tile9: $w_o = 14$

Fig. 6. The synthesized counterparts of the texture in Fig. 4. The caption of each image gives the size of the texel used in the synthesis of that texture.

6. SUMMARY AND CONCLUSIONS

We study here an application of Rényi's generalized entropies to texture analysis. More precisely, we show that these entropies can be used to identify square shaped texels in regular textures. The proposed method is based on the property of the generalized entropies of the distribution of a stochastic variable of being small when the uncertainty in the outcome of that stochastic variable is small. In the case of texture analysis, the stochastic variable is the histogram of the pattern observed through a square window of a given size. We show that, for regular textures with square shaped texels, Rényi's entropies depend on the size of the observation window and that they are small when the size of the observation window matches the size of the texels in the analyzed texture.

Unlike most of the structural texture analysis methods, the method proposed here does not use cooccurrence matrices, Fourier analysis, or autocorrelation functions. These classical methods exhibit some drawbacks, such as (i) sensitivity to slight distortions and noise and (ii) high computational demands. Our method, while being computationally tractable, is robust to distortions and noise in the analyzed texture. Another advantage of the proposed method is its flexibility. The method can easily incorporate different texture models by simply adapting the way in which two visual patterns observed through two different windows are compared. For illustration, we compare here the gray level histograms in the considered windows.

The pattern comparison method used in our experiments is a statistical approach to texture analysis. It uses the gray level histogram of a window as a descriptor. The main reasons for such a choice are the invariance of the histogram to circular shift and its computational simplicity, while being a good discriminator between different textures. We are well aware of the limitations of such descriptors [14], however, in the current context such limitations are not felt.

The proposed texel identification method is able to identify only square texels. Texels with different shapes can be identified if windows with different profiles are used. Our choice of a square window was inspired by the *time delay embedding* technique used in dynamic systems for time series analysis [6]. Using an extension of the time delay embedding for texture analysis is a suitable approach because texture is a neighborhood property. However, extending 1D techniques to 2D situations proves to be a difficult problem with no optimal solution. In this paper, we used the simplest way of extending a 1D technique to a 2D problem. Studying other extension alternatives remains a task for future work.

7. REFERENCES

[1] J.G. Leu, "On indexing the periodicity of image textures," *Image and Vision Computing*, vol. 19, no. 13, pp. 987–1000, 2001.

[2] S.R. Jan and Y.C. Hsueh, "Window-size determination for granulometrical structural texture classification," *Pattern Recognition Letters*, vol. 19, no. 5-6, pp. 439–446, 1998.

[3] T. Matsuyama, S.I. Miura, and M. Nagao, "Structural analysis of natural textures by Fourier transformation," *Computer Vision Graphics and Image Processing*, vol. 25, pp. 347–362, 1983.

[4] G. Oh, S. Lee, and S. Yong Shin, "Fast determination of textural periodicity using distance matching function," *Pattern Recognition Letters*, vol. 20, no. 2, pp. 191–197, 1999.

[5] D. Blostein and N. Ahuja, "Shape from texture: integrating texture-element extraction and surface estimation," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 11, no. 12, pp. 1233–1251, 1989.

[6] H. Kantz and T. Schreiber, *Nonlinear Time Series Analysis*, Cambridge Nonlinear Science series. Cambridge University Press, 1999.

[7] A. Rényi, "On measures of entropy and information," in *4th Berkeley Symp.Math. Stat. and Prob.*, 1961, vol. 1, pp. 547–561.

[8] E. Gokcay and J.C. Principe, "Information theoretic clustering," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 24, no. 2, pp. 158–171, 2002.

[9] C. Cachin, "Smooth entropy and Rényi entropy," *Lecture Notes in Computer Science, Advances in Cryptology: EUROCRYPT '97*, vol. 1233, pp. 193–208, 1997.

[10] P. Brodatz, *Textures*, Dover, 1966.

[11] Media Laboratory at MIT, "Vistex vision texture database," http://www-white.media.mit.edu/ vismod/ imagery/ VisionTexture/ vistex.html, 1995.

[12] B.W. Lindgren, *Statistical theory*, Chapman & Hall, fourth edition, 1993.

[13] T. S. Parker and L. O. Chua, *Practical Numerical Algorithms for Chaotic Systems*, Springer-Verlag, New York, 1989.

[14] B. Julesz, E.N. Gilbert, L.A. Shepp, and H.L. Frisch, "Inability of humans to discriminate between visual textures that agree in second-order statistics - revisited," *Perception*, vol. 2, pp. 391–405, 1973.