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B. Chakravarthy

H. P. Cherukuri

R. G. Wilhelm

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Prediction of Soakout Time Using Analytical Models

B. Chakravarthy, H. P. Cherukuri, R. G. Wilhelm

Department of Mechanical Engineering and Engineering Science, University of North Carolina at Charlotte, Charlotte, North Carolina, USA

Corresponding author – H. P. Cherukuri, telephone 1-704-687-4371, fax 1-704-687-3246, email hcheruku@uncc.edu

Abstract
In precision manufacturing enterprises, machine parts at nonstandard temperatures are often soaked to standard temperature prior to making any dimensional measurements. The soakout times are usually determined using lumped heat-transfer models where the part temperatures are assumed to be uniform. This article discusses conditions under which lumped model assumptions are valid by comparing lumped analyses for various shapes and materials with the more general finite element results. In addition, the effect of ambient temperature cycling on part response is also studied.

Keywords: dimensional measurements, thermal errors, soakout time, metrology, lumped models, finite element methods

1. Introduction

The size and form of mechanical parts are specified according to a standard temperature [1] and, in precision manufacturing enterprises, verification procedures are often carried out at this standard temperature. Commonly, however, mechanical parts are manufactured or operated in a nonstandard thermal environment. While thermal compensation techniques can be used for verification of part size and form at nonstandard temperatures, a simpler and more repeatable approach is to bring the mechanical part into a standard thermal environment and allow the part to soak until it is at the standard temperature. The soakout time is the time required for the temperature of the part to cool to within some acceptably small range of the standard temperature.

Verification procedures are exposed to many different sources of uncertainty. Thermal distortions due to nonstandard temperature are an uncertainty source that can be controlled by soaking each part to the standard temperature. An optimal soakout time minimizes the uncertainty due to thermal distortion while also minimizing the time and resources required for the verification procedure. To balance these two competing objectives, the soakout time is chosen so that any expected thermal distortion is small with respect to the tolerances that are evaluated during the verification procedure. For example, for a part with larger tolerances, some distortion can be allowed and verification time and resources can be reduced. To use this approach of balancing soakout time and uncertainty, an accurate prediction of the soakout time is required.

Two possibilities exist for predicting the soakout time of parts of arbitrary shapes and arbitrary cooling conditions. The first method involves using the finite element method to solve the 3-dimensional heat-conduction problem. However, this method is time consuming and cannot be easily used to predict soakout times in real-time. The second method, also applicable to bodies of arbitrary shapes, is the lumped model where it is assumed that the entire body is at a uniform temperature at any given time. Clearly this is a gross approximation to the actual temperature in the part during soakout. However, an attractive feature of the lumped model approach is that analytical expressions for soakout time can be obtained and used to predict soakout times for parts of arbitrary shapes almost instantaneously.

The purpose of this paper is to investigate the conditions under which lumped models can be used to predict soakout times accurately. Analytical expressions for temperature in parts of arbitrary shapes are derived and formulae for soakout times (appropriately defined) are provided. Several examples involving various geometries and materials with both forced and natural convective cooling conditions are considered. The lumped model results are compared with the results from the commercial finite element software package ANSYS. The results from ANSYS are assumed to be exact. Comparisons of analytical and FEA results with experiments and issues related to dimensional uncertainties and the degree to which thermal uncertainties affect the overall uncertainties will be discussed in a forthcoming paper.

In addition, the problem of the effect of environmental temperature cycling on a part is also considered in the context of lumped models. Specifically, the environmental temperature is assumed to vary about a mean temperature in a sinusoidal fashion. Part response to this variation is considered in detail and conditions under which the temperature cycling does not have significant effect on the part are investigated.

Several excellent prior investigations related to the subject matter of this paper exist in the literature. A comprehensive study of the effects of thermal variations on the accuracy of machined parts was first presented in the doctoral dissertation of McClure [2]. In [3], a thorough investigation of various factors affecting the thermal response of mechanical parts under a variety of conditions was provided. The article [4] reviews the field of thermal error research till 1990. The ANSI article [5] provides description of procedures for controlling environment and for estimating the magnitudes of errors associated with dimensional measurements made at nonstandard temperatures. The report [6] examines the effects of uncertainties in temperature and material properties on the dimensional measurements made at nonstandard temperatures. An excellent example illustrating the uncertainties in dimensional measurements due to nonstandard thermal environments is considered in [7] where a profile measuring device is used to measure the profile of a space shuttle solid rocket motor.

The primary purpose of the above mentioned references was to examine the dimensional errors and uncertainties involved in measurements made at nonstandard temperatures. The subject of soakout time, which is the subject of this paper, has not been explicitly considered previously in the literature.

2. Determination of soakout time

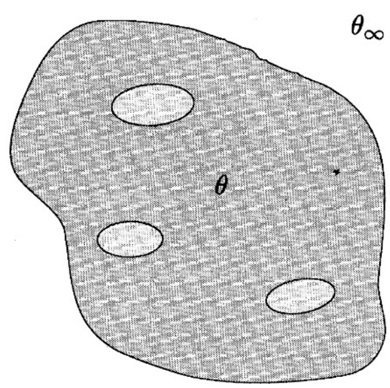

Figure 1. Lumped heat transfer model

Let us consider a body of an arbitrary shape as shown in Figure 1. We assume that the body is at a uniform initial temperature θ_0 and that it is placed in an environment where the temperature is θ_∞ and the heat transfer coefficient is h. Let us now assume that the change in the temperature of the body is uniform, i.e., the temperature in the body is a function only of time t. An application of the balance of energy (assuming uncoupled thermomechanical behavior) leads to

$$\int_V \dot{U} dV = -\int_A q dA \tag{1}$$

where V is the volume, A is the surface area over which there is heat-transfer, U is the internal energy per unit volume, and q is the heat flux normal to the surface. The dot on U represents differentiation with respect to time. Noting that

$$\dot{U} = \rho c \dot{\theta}$$

and $q = h(\theta - \theta_\infty)$, the earlier equation can be rewritten as

$$\rho c V\dot{\theta} = -hA(\theta - \theta_\infty) \tag{2}$$

or

$$\dot{\theta} = -\beta(\theta - \theta_\infty) \tag{3}$$

where

$$\beta = \frac{hA}{\rho c V}. \tag{4}$$

Here, c is the specific heat and ρ is the mass density of the part under consideration.

Upon solving Equation (3) for θ, we find

$$\theta(t) = \theta_\infty + (\theta_0 - \theta_\infty)e^{-\beta t} \tag{5}$$

or

$$\frac{\theta(t) - \theta_\infty}{\theta_0 - \theta_\infty} = e^{-\beta t} \tag{6}$$

Next, let us define soakout time t_s as the time required for the temperature of the body to cool to within ε of the ambient temperature, i.e., to cool to $\theta(t) = \varepsilon + \theta_\infty$. Then Equation (6) leads to

$$t_s = \frac{1}{\beta} ln\left[\frac{\theta_0 - \theta_\infty}{\varepsilon}\right] \tag{7}$$

The above relation can be generalized to the case where the heat-transfer rate is different on different faces. Let us assume that the body surface area consists of n different areas with n different heat-transfer coefficients. Let A_i denote the i^{th} area and h_i the heat-transfer coefficient on this area. Assuming that the body temperature is a function only of time, balance of energy can then be shown to lead to

$$\dot{\theta} = -(\theta - \theta_\infty)(\beta_1 + \beta_2 + \ldots + \beta_n) \tag{8}$$

where

$$\beta_i = \frac{h_i A_i}{\rho c V}, \text{ with } i = 1, 2, \ldots, n. \tag{9}$$

The solution of Equation (8) is given by

$$\theta(t) = \theta_\infty + (\theta_0 - \theta_\infty)e^{-(\beta_1 + \beta_2 + \ldots \beta_n)t} \tag{10}$$

The soakout time t_s in this case is given by

$$t_s = \frac{1}{(\beta_1 + \beta_2 + \ldots + \beta_n)} \ln\left[\frac{\theta_0 - \theta_\infty}{\varepsilon}\right] \tag{11}$$

3. Environmental temperature cycling

Let us now consider the case when the environmental temperature is varying periodically in time, i.e.,

$$\theta_\infty(t) = \theta_\infty^0 + \xi \sin \omega t \tag{12}$$

where ω is the frequency of variation and ξ is the amplitude of the variation. In this case, the differential equation (3) becomes

$$\dot{\theta} = -\beta(\theta - \theta_\infty(t)) \tag{13}$$

The solution of equation (13) can be written as the sum of a steady part $\theta^S(t)$ and a transient part $\theta^T(t)$

$$\theta(t) = \theta^S(t) + \theta^T(t) \tag{14}$$

where

$$\theta^S(t) = \theta_\infty^0 + \frac{\xi\beta}{\sqrt{\beta^2 + \omega^2}} \sin(\omega t - \varphi), \text{with } \varphi = \arctan\frac{\beta}{\omega} \tag{15}$$

and

$$\theta^T(t) = \frac{e^{-\beta t}}{\beta^2 + \omega^2}[\xi\beta\omega + (\theta_0 - \theta_\infty^0)(\omega^2 + \beta^2)]. \tag{16}$$

The transient term $\theta^T(t) \to 0$ as $t \to \infty$. Therefore, at late times,

$$\theta(t) \to \theta^S(t) = \theta_\infty^0 + \frac{\xi\beta}{\sqrt{\beta^2 + \omega^2}} \sin(\omega t - \varphi). \tag{17}$$

The result shows that the part temperature lags behind the ambient temperature by a phase angle φ with a decreased amplitude. The amplitude ratio of the part temperature variation with respect to the ambient temperature variation is given by

$$\text{amplitude ratio} = \frac{\beta}{\sqrt{\beta^2 + \omega^2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\beta}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{T\beta}\right)^2}} \tag{18}$$

where T is the time period of the environmental temperature fluctuation. The steady-state temperature in the context of a thin slab of a material was also given in [3]. Here, we have included the transient part in our solution and extended the validity of the model to bodies of arbitrary shapes.

3.1. Limitations of the lumped analysis model

As mentioned earlier, the lumped models assume that the temperature in the body is uniform at all times. Such an assumption is valid only in the limiting case of infinite thermal conductivity or in the trivial case where the body temperature is constant. However, we can expect the lumped models to give reasonably accurate results when the thermal conductivity is large or when the heat-transfer from the surface is small. In non-dimensional terms, one can show [8] that the temperature variation across the body is negligible when the Biot number Bi is small compared to unity, i.e.,

$$Bi = \frac{hL}{k} \ll 1$$

where L is an appropriately chosen length-scale of the body and k is the coefficient of thermal conductivity. Thus, for a given body, lumped models are expected to give more accurate results for natural convective conditions than for forced convective conditions. Nevertheless, we show below that even when there is significant amount of forced convection, the error involved in predicting temperature using lumped models is of the order no more than 20% for moderately large bodies.

4. Results

In this section, several numerical examples are presented that illustrate the theory presented above and its limitations.

4.1. Prediction of soakout time for various geometries

The range of validity of the lumped models developed above is examined by considering various geometries of practical interest and comparing the lumped analysis results with those predicted by finite element analyses. The results from finite element analyses are treated as exact and the relative errors involved in using lumped models for the prediction of temperature, soakout time and dimensional variations are calculated. The finite element analyses are conducted using the commercial package ANSYS v5.4. The meshes used are sufficiently fine to give very accurate temperatures at the nodal points.

The geometries considered are listed in Table 1. These geometries are chosen in order to study the accuracy of lumped models compared with the exact models under both natural and forced cooling conditions, small and large volumes and surface areas, and for materials with high and low thermal conductivities. For all the cases, it is assumed that the heat-transfer to the environment is through convection.

Table 1. List of different cases considered in the present work

Case	Geometry	Cooling Conditions
1	Steel sphere of 76.2 mm diameter	Natural convection with h = 30W/(m^2 – K)
2	Steel sphere of 500 mm diameter	Forced convection with h = 300W/(m^2 – K)
3	Granite sphere of 500 mm diameter	Forced convection with h = 300W/(m^2 – K)
4	Steel Rectangular Block (254 mm × 25.4 mm × 25.4 mm)	Natural convection with h = 30W/(m^2 – K)
5	Steel Rectangular Block with 4 holes (254 mm × 25.4 mm × 25.4 mm) Holes are perpendicular to the length with a diameter of 12.7 mm each.	Natural convection with h = 30W/(m^2 – K)
6	Gear pump housing	Forced convection with h = 300W/(m^2 – K)

The primary interest is in the verification of soakout times predicted by the lumped models. From equation (7), we recall that the soakout time is given by

$$t_s = \frac{1}{\beta}\ln\left[\frac{\theta_0 - \theta_\infty}{\varepsilon}\right] \tag{19}$$

where ε is the temperature difference between the part and the environment at soakout time. Here we consider two different values 0.5°C and 0.1°C for ε. The soakout time based on ε = 0.5°C is denoted by t^1_s and the soakout time based on ε = 0.1°C is denoted by t^2_s, i.e.,

$$t_s^1 = \frac{1}{\beta}\ln\left[\frac{\theta_0 - \theta_\infty}{0.5}\right] \text{ and } t_s^2 = \frac{1}{\beta}\ln\left[\frac{\theta_0 - \theta_\infty}{0.1}\right] \tag{20}$$

The large value of ε = 0.5°C is deliberately chosen to study the validity of the lumped models even when there may be thermal gradients present in the body. In a typical measurement application, ε is much smaller than 0.5°C.

In the numerical examples considered below, it is assumed that the initial part temperature is 25°C whereas the environment is at the standard temperature of 20°C. Heat transfer between the part and the environment is assumed to take place through convection. The finite element analyses consider the problem of unsteady heat conduction for each of the cases and take into account the symmetry of the problem along with the appropriate boundary conditions. It is worth noting that the temperature predicted by the FEA analyses are both spatially and temporally nonuniform whereas the lumped model predictions are only nonuniform in time. In each case, the part temperature predicted by the lumped model is compared with the temperature evolution at two points in the part as predicted by ANSYS. The two points in the FEA models correspond to the points where the maximum and minimum temperatures occur in the part during cooling. The point with the minimum temperature is denoted by P_s and is on the surface of the part. The point where the maximum temperature occurs is denoted by P_c and is an interior point. For each of these points, the error involved in using lumped models is defined as

$$\text{Relative Error} = \frac{\theta^{\text{Lumped}} - \theta^{\text{ANSYS}}}{\theta^{\text{ANSYS}}} \times 100 \tag{21}$$

The results for each of the cases listed in Table 1 are shown in Figures 2 through 9. For each case, the temperature predictions by lumped analysis and ANSYS results for the points P_s and P_c are plotted. The corresponding relative errors as given by Equation (21) are also calculated for these two points. The soakout times t^1_s and t^2_s are marked by circles and squares, respectively.

First we consider the effect of Biot number Bi. In Figure 2, the cooling of a steel sphere of diameter 76.2 mm is considered. The cooling is by natural convection ($h = 30\text{W}/(\text{m}^2 - \text{K})$) at the outer surface. In this case, $Bi = 0.0189 \ll 1$ and hence the temperature predicted by lumped models is expected to be very accurate. This is confirmed by Figure 2 according to which the relative error is at most 0.7% and the error in temperature at the soakout times t^1_s and t^2_s is even smaller.

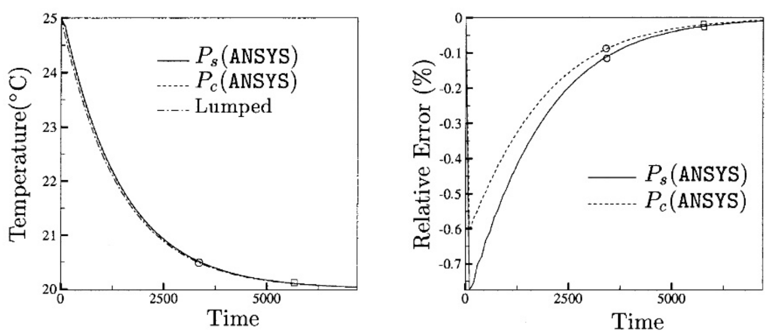

Figure 2. Comparison of ANSYS and lumped analysis results for temperature distribution for case 1: (left). Temperature (right). Relative error.

Next, we consider the cooling of a steel sphere of diameter 500mm. The cooling is assumed to be by forced convection with $h = 300\text{W}/(\text{m}^2 - \text{K})$. In this case, $Bi \approx 1.2395$. The temperature and relative error plots shown in Figure 3 indicate that the error involved is at most 6% and at the two soakout times, the errors are about 3% and 1%, which are higher than that for the smaller sphere considered above.

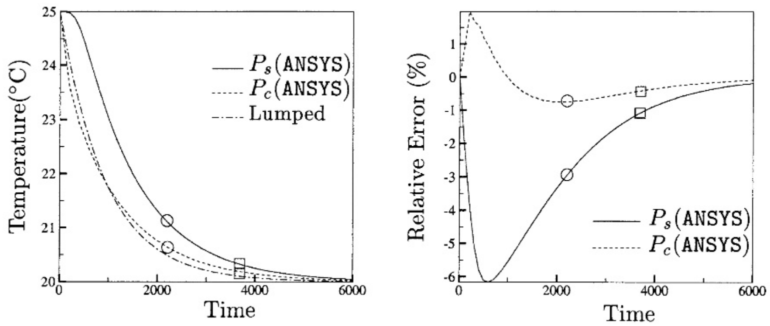

Figure 3. Comparison of ANSYS and lumped analysis results for temperature distribution for case 2: (left). Temperature (right). Relative error.

When the material is different from steel, the relative error due to the lumped models will be smaller if the thermal conductivity k of the material is larger than that of steel. Several commonly used materials such as Aluminum and its alloys, Copper and its alloys, many types of steels, Nickel, Tungsten, and Zinc fall in this category. For those materials for which the thermal conductivity is much smaller than that of steel, the relative error in temperature due to lumped models will be larger. This is illustrated in Figure 4 by considering a sphere made of granite and of diameter 500mm. Cooling is again by forced convection with $h = 300\text{W}/(\text{m}^2 - \text{K})$. In this case $Bi \approx 27 \gg 1$. Consequently, as the plots in Figure 4 show, the maximum relative error is quite large (20%). It is interesting to note that the temperature in the granite sphere is highly nonuniform at the soakout times t^1_s and t^2_s, the definitions of which are based on the lumped model. Although $t^2_s \approx 2500$s, we note that the ANSYS results indicate that the difference between the temperatures at the surface and at the center is approximately 2°C even at times as late as 10000s. Thus, we note that in the cases where $Bi \gg 1$, lumped models grossly underestimate the soakout times.

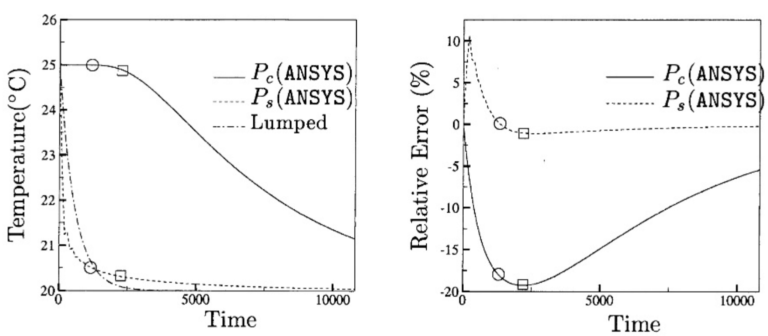

Figure 4. Comparison of ANSYS and lumped analysis results for temperature distribution for case 3: (left). Temperature (right). Relative error.

It is to be noted, however, that when natural convective cooling is used in place of forced convection, the heat-transfer coefficient will be substantially smaller and therefore Bi will be smaller as well. Consequently, lumped models yield more accurate results for natural convection than for forced convection with all the other parameters remaining the same. In fact, our results indicate that when natural convection (with h = 30W/(m^2 – K)) is used for the granite sphere considered above, the agreement between lumped and ANSYS is significantly better.

Next, we consider the lumped model temperature predictions for the cooling of a multiply-connected region. In particular, we consider a steel rectangular block of dimensions (254 mm × 25.4 mm × 25.4 mm) with four holes, each of diameter 12.7 mm (see Fig. 5). For comparison purposes, we consider a rectangular solid block of identical external dimensions.

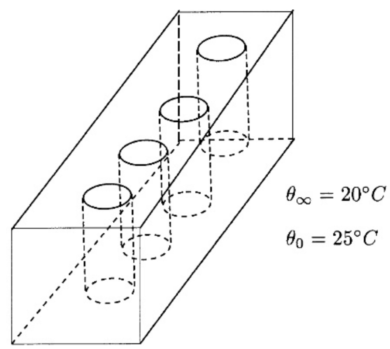

Figure 5. Cooling of a rectangular block with 4 holes.

The temperature predictions of lumped model along with ANSYS results for the solid rectangular block and the block with holes are shown in Figures 6 and 7, respectively.

Comparing the two cases in figures 6 and 7, we see that the soakout times for the block with holes are smaller than those for the solid block. Further, we note from the error plots that at the soakout times, the relative error for the block with holes is also smaller. This can be explained by noting from Equation (7) that the soakout time is inversely proportional to the time constant β given by

$$\beta = \frac{hA}{\rho c V} \tag{22}$$

where A is the surface area and V is the volume. When holes are introduced in a solid block, the surface area increases while the total volume decreases. Consequently, when the two blocks are at the same initial temperature, the block with holes has less total heat energy stored with a larger surface area to lose heat through. Thus, the block with holes soaks faster than the solid block. Further, due to the presence of holes, the effective Biot number Bi will now be smaller than that for the solid block and hence the lumped model is more accurate for the block with holes.

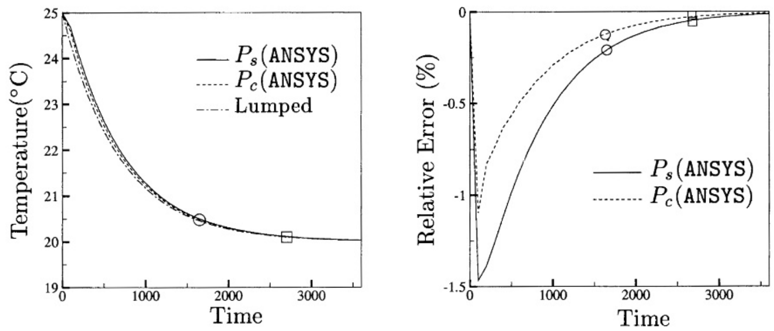

Figure 6. Comparison of ANSYS and lumped analysis results for temperature distribution for case 4: (left). Temperature (right). Relative error.

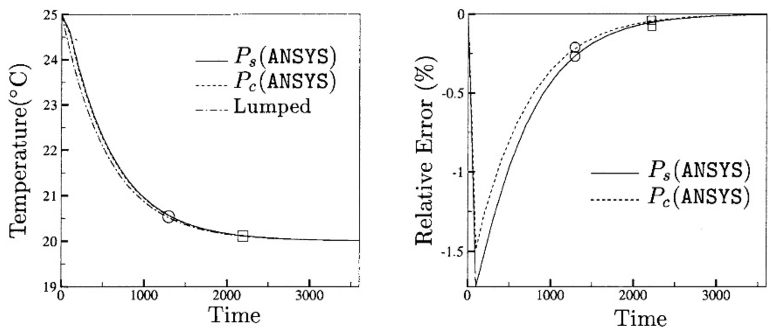

Figure 7. Comparison of ANSYS and lumped analysis results for temperature distribution for case 5: (left). Temperature (right). Relative error.

We conclude this section by considering an industrial part, the gear pump housing shown in Figure 8. Only the essential dimensions are shown in the figure for clarity. ANSYS and lumped analysis results for this part when cooled by forced convection with h = 300W/(m^2 – K) are shown in Figure 9. As the results indicate, the maximum error in using the lumped model is approximately 1.5% and at the soakout times, the error is even smaller.

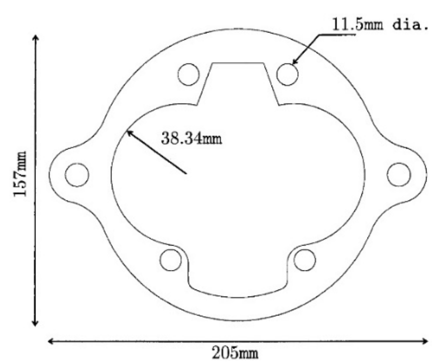

Figure 8. A gear-pump housing considered in the present work. The sketch is drawn to scale and not all the dimensions are shown for the sake of clarity.

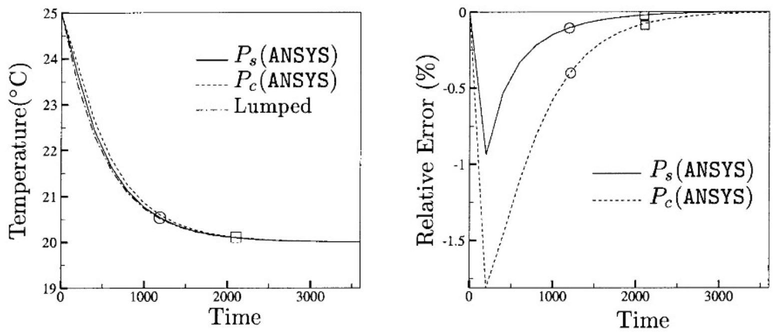

Figure 9. Comparison of ANSYS and lumped analysis results for temperature distribution for case 6: (left). Temperature (right). Relative error.

4.2. Periodic environmental temperature variations

Next, we consider the response of a part to periodic temperature variations of the environment. Here we use the lumped analysis model developed in section 3 for this purpose.

We will consider a steel block of dimensions 254 mm × 25.4 mm × 25.4 mm. As before, the block is assumed to be at a constant initial temperature of 25°C and the environmental temperature is assumed to be of the form

$$\theta_\infty(t) = \left(20 + 0.5 \sin\frac{2\pi t}{T}\right) °\mathrm{C} \tag{23}$$

where T is the time-period. Thus, in the example considered here, the environment temperature oscillates about the standard temperature with an amplitude of 0.5C and a time period of T. The heat transfer is assumed to be by natural convection with h = 30W/(m^2 – K). In figures 10 and 11, the part response given by Equation (14) is plotted against time. As can be seen from these figures, in each case, the initial transient period where the contribution of $\theta^T(t)$ given by Equation (16) to T is significant. The part temperature reaches the steady-state given by Equation (15) once the transients die out. The amplitude ratio of the part temperature variation with respect to the ambient temperature variation given by Equation (18) indicates that as T decreases, the amplitude ratio decreases as well. This is evident from a comparison of the Figures 10 and 11 corresponding to T = 15 minutes and T = 1 hour, respectively. As these figures suggest, the part temperature variation can be kept below a certain acceptable value (for example, 0.1°C) by keeping T, i.e., the time-period of ambient temperature variation, sufficiently small. A similar conclusion was also reached in [3].

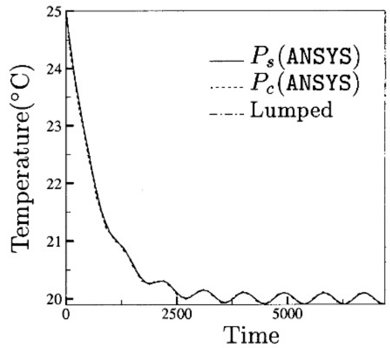

Figure 10. Comparison of ANSYS and lumped analysis results for temperature distribution in a rectangular block with the ambient temperature varying with time with a period of 15 minutes.

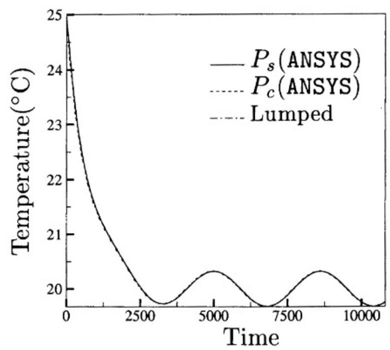

Figure 11. Comparison of ANSYS and lumped analysis results for temperature distribution in a rectangular block. Ambient temperature is varying with time with a period of 1 hour.

In the numerical example considered here, the Biot number is very small compared to unity. Hence, the lumped model is expected to accurately predict the temperature in the block. This is confirmed by the ANSYS results also shown in Figures 10 and 11.

From the expression for amplitude ratio given in Equation (18), one can obtain an estimate for an upper bound on the heat-transfer coefficient that can be used to keep the part temperature variation to within a prescribed limit δ. Thus, from Equation (18), we have

$$\delta \geq \frac{1}{\sqrt{1+\left(\frac{2\pi}{T\beta}\right)^2}} \tag{24}$$

or

$$\beta = \frac{hA}{\rho cV} \leq \frac{\omega\delta}{\sqrt{1-\delta^2}} \approx \omega\delta \text{ for } \delta \ll 1. \tag{25}$$

Clearly, if $h = 0$, that is, if the part surface is insulated, then the amplitude ratio would be zero. A method of achieving this is to add a thermal insulation to the part surface (see [3]). However, we note that using thermal insulation would lead to inordinately long periods of time for the part to soak to the standard temperature, since in this case the transient part of the temperature given by Equation (16) takes a long time to die out.

5. Conclusions

An expression for the estimation of soakout time for a body of arbitrary shape has been derived. The soakout time given by Equation (7) and restated here for convenience,

$$t_s = \frac{\rho cV}{hA} \ln\left[\frac{\theta_0 - \theta_\infty}{\varepsilon}\right], \tag{26}$$

is based on lumped analysis where the entire body is assumed to be at a uniform temperature that varies only with time. Based on the examples considered in the previous sections, the following observations can be made.

- When $Bi \ll 1$, the lumped model predictions differ from the more exact FEA models by less than 1%. This is the case for small parts made of materials with high thermal conductivity in the presence of natural convection.
- When $Bi = O(1)$, the lumped model predictions differ from the more exact FEA models by less than 5%. This is the case for moderately large parts (length scale in the range of 1/2 meter to 1 meter) made of materials with high thermal conductivity in the presence of natural convection or forced convective cooling in air.
- When $Bi = O(10)$, the difference between lumped model and FEA predictions increases and the relative error can be as large as 20%–30%. This is the case for moderately large parts (length scale in the range of 1/2 meter to 1 meter) made of materials with low thermal conductivity in the presence forced convective cooling in air.
- In all the cases, the soakout time given by Equation (26) underestimates the actual soakout time. However, the error is relatively small for many practical cases in natural or forced convective cooling in air.
- Equation (26) provides a means to quickly estimate the soakout time for a given part without resorting to the costly exercise of finite element modeling.
- As indicated by Equation (26), the soakout time t_s is directly proportional to the time constant T given by

$$\tau = \text{time constant} = \frac{1}{\beta} = \frac{\rho c V}{hA}$$

Thus, we note that the time constant alone is not sufficient to predict the soakout time. Depending on the magnitude of ε, the soakout time can be several orders of magnitude larger than τ.

In addition, thermal response of a part of arbitrary shape to periodic environmental temperature cycling has been studied. Our results indicate that the temperature changes in the part can be kept below a certain value by increasing the frequency of variation of the environment temperature and by decreasing the heat-transfer between the part surface and the environment. This can be achieved by placing a thermal insulation over the part as was originally suggested in [3].

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