Limiting temperatures and the Equation of State of Nuclear Matter

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From experimental observations of limiting temperatures in heavy ion collisions we derive T_c, the critical temperature of infinite nuclear matter. The critical temperature is 16.6 ± 0.86 MeV. Theoretical model correlations between T_c, the compressibility modulus, K the effective mass, m^* and the saturation density, ρ_s, are exploited to derive the quantity $(K/m^*)^{\frac{1}{2}}\rho_s^{-\frac{1}{3}}$. This quantity together with calculations employing Skyrme and Gogny interactions indicates a nuclear matter incompressibility in moderately excited nuclei that is in excellent agreement with the value determined from Giant Monopole Resonance data. This technique of extraction of K may prove particularly useful in investigations of very neutron rich systems using radioactive beams.

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Improved knowledge of the nuclear equation of state and a coherent picture of the relationship between the properties of finite nuclei and bulk nuclear matter remains a key requirement in both nuclear physics and astrophysics. It is key, for example, to understanding nuclear structure, heavy ion collisions, supernova explosions and neutron star properties[1, 2, 3]. Significant effort has been devoted to the development of microscopic theoretical models which can provide reliable mathematical formulations of this equation of state[4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. Such calculations are usually specified for symmetric nuclear matter, a hypothetical system of equal numbers of neutrons and (uncharged) protons interacting through nuclear forces. Driven by the astrophysical problems and more recent laboratory excursions into the region of more exotic nuclei, the dependence of the equation of state on neutron-proton asymmetry has also become a subject of significant interest.[21, 22, 23]. In this letter we employ data from experimental measurements of caloric curves in nuclear collisions, together with systematic trends and correlations derived from a number of theoretical investigations of nuclear matter, to derive the critical temperature and incompressibility of symmetric nuclear matter. The techniques employed offer a natural method to extend such investigations to more asymmetric systems.

In a recent paper measurements of nuclear specific heats from a large number of experiments were employed to construct caloric curves for five different regions of nuclear mass[24]. Within experimental uncertainties each of these caloric curves exhibits a plateau region at higher excitation energy, i.e., a "limiting temperature" is reached. In Figure 1 these limiting temperatures from reference[24] are presented as a function of mass. As previously noted, they are observed to decrease with increasing mass. This decrease with increasing mass has long been predicted as resulting from Coulomb Instabilities of expanded and heated nuclei[25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35].

The results employed in reference[24] were based upon temperature determinations derived from double isotope yield ratios and from slope measurements of particle spectra. More recently the TAPS Collaboration has reported temperatures determined from a new technique, observations of "second chance" bremsstrahlung gamma ray emission for a series of reactions which span a wide range of mass[36, 37]. There are not yet sufficient data of this latter type to construct caloric curves for relatively narrow mass regions as was done for the previous temperature data. However, in each case studied with this technique the collisions lead to excitation energies which are above those identified as the starting points of the plateau regions identified in reference[24]. Thus it is reasonable to compare the temperatures determined from the thermal bremsstrahlung measurements with the earlier limiting temperature values. As seen in Figure 1, the reported second chance gamma temperatures and their mass dependence are in excellent agreement with the earlier results. We take this agreement as an independent confirmation of the earlier results and note that the new results extend the determination of the mass dependence to significantly higher mass.

A relatively large number of theoretical calculations of the critical temperature of semi- infinite nuclear matter (nuclear matter with a surface) have been reported in the literature [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35]. The different nuclear interactions employed in these calculations lead to large differences in the critical temperatures derived. Values from 13 to 24 MeV are reported in references 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35. The limiting temperatures plotted in Figure 1 are well below these calculated critical temperatures. This difference reflects finite size effects, Coulomb effects and isospin asymmetry effects for the finite nuclei studied. A first order estimate of the magnitude of these combined effects can be made by comparing the volume coefficient of the Liquid Drop Model Binding Energy Equation, -16.0 MeV, which represents the binding energy per nucleon in infinite nuclear matter, to typical nuclear binding energies, $\approx$ 8 MeV/nucleon. This suggests that limiting temperatures in nuclei should be ≈ 0.5 times the critical temperature of nuclear matter [5]. Given the wide variation in the calculated values of T_c it is not surprising that large variations result for the absolute values of limiting temperatures calculated for finite nuclei.

Employing a variety of Skyrme type interactions Song and Su have previously noted a mass dependent scaling of the Coulomb Instability temperatures with the critical temperature of nuclear matter (see Figure 6 of reference 28). These calculations were performed for nuclei along the line of beta stability. A similar scaling exists when other model interactions are employed.

Mean values of T_{lim}/T_c for five different masses which result from averaging the results of different calculations [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35] are shown in Figure 2. The estimated uncertainties are relatively small, $\approx 6\%$. For comparison, the figure also presents ratios of T_{lim}/T_c which are expected to result assuming only finite size effects as derived from a lattice calculation[38] and the ratio of the nuclear binding energy per nucleon along the line of beta stability to the bulk binding energy per nucleon, 16 MeV. We have employed the mean variation of T_{lim}/T_c with A, determined from commonly used microscopic theoretical calculations, together with the five experimental limiting temperatures reported in reference 24, to extract the critical temperature of nuclear matter. In doing so we treat the theoretical variation as if it were an experimental uncertainity. Since the various interactions employed have been "tuned" to other nuclear properties, we consider this a reasonable approach. The results are presented in Figure 3. Averaging the individual results we find 16.6 ± 0.86 MeV.

It is interesting to ask whether additional Equation of State information can be extracted from this result. Blaizot *et al.*[9] have argued that the most effective way to extract the incompressibility modulus of nuclear matter from experimental data is by comparison with microscopic calculations. This is usually done by comparison of the measured energies for the centroids of the strength distribution of Giant Monopole Resonances (GMR) with the calculated centroids. The generally accepted best current value of $K = 231 \pm 5$ MeV has been determined in such a fashion[39] by comparison with the calculated centroids using Gogny interactions[9]. We have adopted a similar comparison procedure using the present result for T_c determination.

We began by using a relation suggested by the work of Kapusta[40] and Lattimer and Swesty [41] who have pointed out that correlations between parameters used to describe nuclear matter are such that a relationship between the critical temperature, T_c, the incompressibility, K, the effective mass, m^* ($= m_{eff}/m$ where m_{eff} is the nucleon effective mass and m is the nucleon mass) and the saturation density, ρ_s, may be written as

$$T_c = C_T (K/m^*)^{\frac{1}{2}} \rho_s^{-\frac{1}{3}} \tag{1}$$

where C_T is a constant. Using this relationship, we have determined the constant C_T in this equation using published theoretical values for T_c calculated utilizing a number of different microscopic interactions [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. Results of calculations using interactions with $155 < K < 384$ MeV are depicted in Figure 4. A least squares fit to these data suggests a very slight decrease of C_T with increasing $(K/m^*)^{\frac{1}{2}} \rho_s^{-\frac{1}{3}}$. Using the present value for T_c and an iterative technique to establish C_T leads to

$$C_T = 0.484 \pm 0.074$$

and

$$(K/m^*)^{\frac{1}{2}} \rho_s^{-\frac{1}{3}} = 34.2 \pm 5.34 MeV^{\frac{1}{2}} fm.$$

The saturation density, ρ_s, is well established by charge density measurements to be 0.16 ± 0.005 fm^{-3}[13]. The standard deviation of the model values of ρ_s from 0.16 fm^{-3}, calculated with the different interactions, is 3.4%. Either of these uncertainties is very small compared to other uncertainties in the determination. Therefore T_c is a measure of K/m^*. It is important to recognize that K and m^* are not independent variables but are correlated[9, 13]. For Skyrme effective interactions Chabanet *et al.* have given expressions for K and m^* and discussed the resultant correlation between them [13]. Using the relationships discussed in that work m^* can be written as:

$$m^* = (1 + \frac{m}{8\hbar^2} \rho \Theta_s)^{-1} \tag{2}$$

where

$$\Theta_s = \frac{K - B - C\sigma}{D(1 - \frac{3}{2}\sigma)} \tag{3}$$

and σ is a parameter which ranges from 0 to 1 and controls the density dependence of the interaction. B, C, D are parameters directly related to e_∞, the energy per nucleon in infinite nuclear matter and e_F , the Fermi energy of infinite nuclear matter.

$$B = -9e_\infty + \frac{3}{5} e_F \tag{4}$$

$$C = -9e_\infty + \frac{9}{5} e_F \tag{5}$$

$$D = \frac{3}{20} \rho k_F^2 \tag{6}$$

Here k_F is the Fermi momentum.

It is clear then that for a given K the ratio K/m^* in equation 1 depends on the choice of σ, the parameter of the density dependent term. (In the Gogny interactions of reference 9 this parameter controlling the density dependent term is designated α.). As a result, determination of K from K/m^* is sensitive to the choice of this parameter. For example in reference 13 the relation between K and m^* is such that small values of σ dictate lower values of K. Also, for smaller values of σ, m^* decreases as K increases while for larger σ, m^* increases with increasing K.(See Fig 2, Reference 13).

For comparison to the data we present in Figure 5 a plot of K vs $(K/m^*)^{\frac{1}{2}}\rho_s^{-\frac{1}{3}}$, obtained using the Gogny interactions from reference 9 and various Skyrme interactions. The dashed lines in the plot show the trend of the generalized Skyrme interactions for $\sigma = \frac{1}{6}, \frac{1}{3}$ and 1, as obtained in reference [13]. The solid lines connect results for Gogny interactions with $\alpha = \frac{1}{3}$ and $\frac{2}{3}$[9]. As seen in the figure, a higher value of σ leads to a higher apparent K. It has been pointed out that maintaining K in a "reasonable" range of 200-300 MeV requires low values of σ (or α)[5, 9, 13]. In particular, the value of $K = 231 \pm 5$ MeV derived from the GMR data[39] was obtained by comparison of data for the breathing mode energy of five different nuclei with energies calculated employing the Gogny D1 ($\alpha = \frac{1}{3}$), D1S($\alpha = \frac{1}{3}$) and D250($\alpha = \frac{2}{3}$) interactions[9]. For three of these nuclei only the D1S interaction results were used. For the other two a fit to the trend in energies calculated from the three interactions was employed.

The value of $(K/m^*)^{\frac{1}{2}}\rho_s^{-\frac{1}{3}}$ derived from this work is also indicated on the Figure by the vertical line. We note that this line intersects the calculated values essentially at a point where the $\frac{1}{6}$ Skyrme and $\frac{1}{3}$ Gogny lines intersect. The different slopes of the Skyrme and Gogny lines in Figure lead to different uncertainties in the K value. Thus employing Skyrme interactions with the $\sigma = \frac{1}{6}$ parameterization[13], $K = 232 \pm 22$MeV. Using Gogny interactions with $\alpha = \frac{1}{3}$[9] leads to $K = 233 \pm 39$ MeV. These results for K lead respectively to m^* values of $0.674^{+.18}_{-.13}$ or $0.674^{+0.11}_{-0.09}$. The compressibility modulus determined from the critical temperature in this manner is then entirely consistent with that determined from the GMR measurements. Higher values of σ (or α) will lead to higher apparent K. Thus for the Skyrme $\sigma = \frac{1}{3}$ line a value of $K = 252$ would result. For the extension of the Gogny $\alpha = \frac{2}{3}$ line, $K = 242$ would be obtained). The calculated breathing mode energies are apparently less sensitive to the value of the parameter of the density dependent interaction.

In summary, from limiting temperature values obtained in five different mass regions we have determined a critical temperature of 16.6 ± 0.86 MeV for symmetric infinite nuclear matter. This has been used to derive both K, the incompressibility and m^*, the effective mass. Extracted by comparison with the same interactions as were employed to determine K from observations of the Giant Monopole Resonance at low excitation energy, the value of K, obtained here from properties of nuclei at moderate excitation energies, is found to be in excellent agreement with that GMR result[39]. The precision of the GMR measurement is better than that obtained from the present determination which incorporates data from a number of different experiments. The precision for the T_c measurement could be improved. However, given the relative complexity of the collision dynamics involved, the breathing mode measurements should remain as the standard. Nevertheless, using newly available radioactive beams the determination of limiting and critical temperatures may play a significant role in providing a means to establish the N/Z asymmetry dependence of the compressibility modulus and other important nuclear properties[42].

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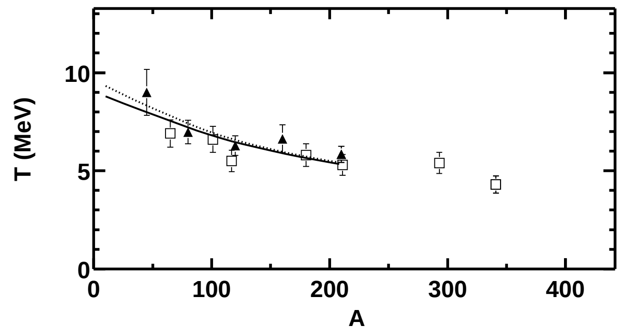

FIG. 1: Limiting Temperatures vs Mass. Limiting temperatures derived from double isotope yield ratio measurements are represented by solid triangles. Temperatures derived from thermal bremsstrahlung measurements are represented by open squares. Lines represent limiting temperatures calculated using interactions proposed by Gogny (dashed)[30] and Furnstahl *et al.*[35] (solid).

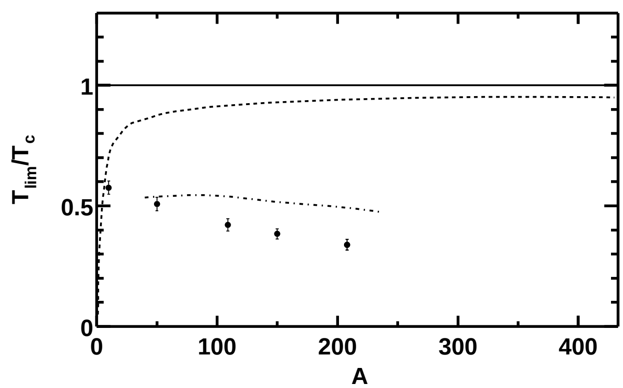

FIG. 2: Theoretical variation of the ratio T_{lim}/T_c with mass along the line of beta stability. The solid line indicates the reference value of T_c. The short dashed line shows the effect of finite size scaling derived from an Ising model[38]. The line with alternating short and long dashes depicts the ratio of the nuclear binding energy per nucleon to the bulk binding energy per nucleon, 16 MeV. Points with uncertainties are derived from the model calculations in references [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35].

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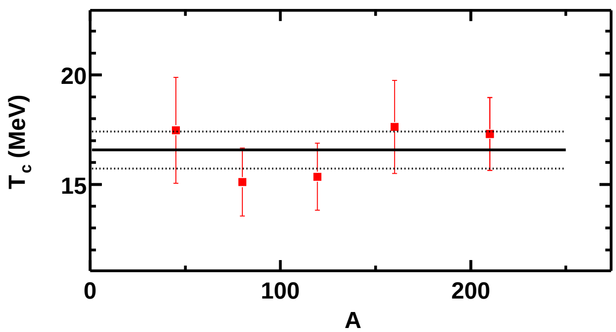

FIG. 3: Derived values of the critical temperature of symmetric nuclear matter. Values derived from data in five different mass regions are presented. The mean value of 16.6 MeV is indicated by the horizontal solid line. The range corresponding to $\pm$ one standard deviation from this mean value is shown by the thin dotted lines.

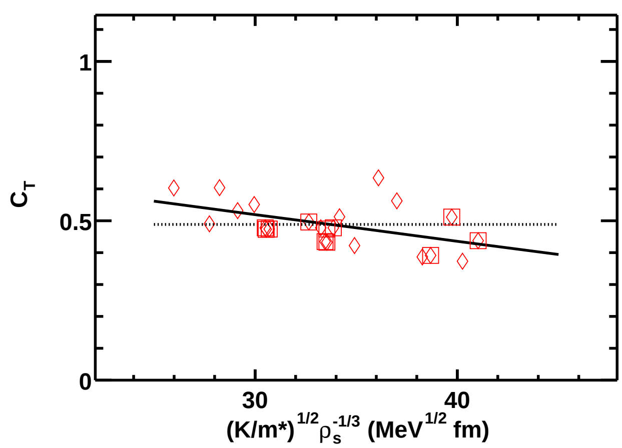

FIG. 4: The constant C_T of equation (1), evaluated from various microscopic calculations. C_T is plotted against $(K/m^*)^{\frac{1}{2}}\rho_s^{-\frac{1}{3}}$. Derived values of C_T are indicated by open diamonds. The dotted horizontal line indicates the mean value of C_T. The solid line represents the linear least squares fit to the derived values. Values of C_T obtained from Skyrme and Gogny interactions are further identified by open squares placed around the diamonds.

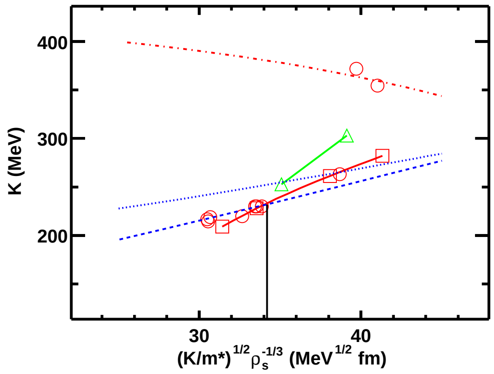

FIG. 5: Compressibility modulus, K, as a function of $(K/m^*)^{\frac{1}{2}}\rho_s^{-\frac{1}{3}}$. The values obtained for Gogny interactions of reference 9 are represented by open squares ($\alpha = \frac{1}{3}$) and open triangles ($\alpha = \frac{2}{3}$). Symbols for each set are connected by thin solid lines. Results using different Skyrme interactions are represented by open circles. The other lines represent generalized calculations using Skyrme interactions [13] with $\sigma = \frac{1}{6}$(short dashed line), $\sigma = \frac{1}{3}$ (dotted line)and $\sigma = 1$(alternating dashes and dots).