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New supersymmetric contributions to $t \to cV$

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Abstract

We calculate the electroweak-like one-loop supersymmetric contributions to the rare and flavor-violating decay of the top quark into a charm quark and a gauge boson: $t \to cV$, with $V = \gamma, Z, g$. We consider loops of both charginos and down-like squarks (where we identify and correct an error in the literature) and neutralinos and up-like squarks (which have not been calculated before). We also account for left-right and generational squark mixing. Our numerical results indicate that supersymmetric contributions to $t \to cV$ can be upto 5 orders of magnitude larger than their Standard Model counterparts. However, they still fall short of the sensitivity expected at the next-generation top-quark factories.

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1 Introduction

The discovery of the top quark by the CDF and D0 Collaborations [1] at Fermilab, and its subsequent mass determination ($m_t = 175 \pm 6\,\mathrm{GeV}$) have initiated a new era in particle spectroscopy. However, unlike the lighter quarks, the top quark is not expected to form any bound states, and therefore its mass and decay branching ratios may be determined more precisely, both theoretically and experimentally. Upcoming (Run II – Main Injector) and proposed (Run III – TeV33) runs at the Tevatron will yield large numbers of top quarks, as will be the case at the LHC, turning these machines effectively into 'top factories.' Even though higher precision in the determination of m_t is expected (it is already known to 3%), more valuable information should come from the precise determination of its branching fractions into tree-level and rare (and perhaps even 'forbidden') decay channels.

The purpose of this paper is to study one class of such rare decay modes: $t \to cV$, with $V = \gamma, Z, g$. The particular case of $t \to cg$ has received some phenomenological attention recently as a means to probe the scale at which such new and unspecified interactions might turn on [2]. Our purpose here is to consider an explicit realization of this coupling within the framework of low-energy supersymmetry. This is different in spirit from the line of work in Ref. [2], as the effective mass scale at which such vertices 'turn on' is determined here by the interactions of presumably rather light sparticles. Within supersymmetry, the $t \to cV$ vertex was first contemplated in Ref. [3], where the one-loop QCD-like (loops of gluinos and squarks) and electroweak-like (loops of charginos and down-like squarks) contributions were calculated. The QCD-like supersymmetric corrections were subsequently re-evaluated and generalized in Ref. [4], which pointed out an inconsistency in the corresponding results of Ref. [3]. Here we study the electroweak-like supersymmetric contributions to $t \to cV$. We reconsider the chargino–down-like-squark loops and point out and correct an inconsistency, essentially a lack of gauge invariance because of the apparent omission of a term, in the corresponding results of Ref. [3]. We also consider for the first time the neutralino–up-like-squark loops, and include the effects of left-right and generational squark mixing.

Our numerical results indicate that for typical values of the parameters one gets a large enhancement over Standard Model predictions of top-quark decays to gauge bosons [5]. For the most optimistic values of the parameters the enhancement can be as large as five orders of magnitude. However, even for the most optimistic values of the parameters, such rare decay channels fall short of the expected sensitivity of the next-generation top factories.

2 Analytical Results

In this section we obtain the one-loop electroweak-like supersymmetric effective top-quark-charm-quark-gauge-boson vertex by considering loops involving charginos and neutralinos, including the effects from left-right and generational squark mixing. We then present the decay rates of the top-quark to the charm-quark and a gauge boson.

The invariant amplitude for top-quark decay to a charm-quark and a gauge boson can be written as

$$M = M_0 + \delta M \tag{1}$$

where M_0 is the tree-level amplitude and δM is the first-order supersymmetric correction. As there are no explicit flavor-violating tcV couplings in the Lagrangian $M_0 = 0$, whereas δM is given by

$$i\delta M = \bar{u}(p_2)\, V^\mu u(p_1)\, \epsilon_\mu(k,\lambda)\,, \tag{2}$$

where p_1, p_2, and k are the momenta of the incoming top-quark, outgoing charm-quark, and outgoing gauge boson respectively, and $\epsilon_\mu(k,\lambda)$ is the polarization vector for the outgoing gauge boson. The vertices V^μ may be written as

$$V^\mu(tcZ) = -i\gamma^\mu(P_L F_{Z1} + P_R F'_{Z1}) + k_\nu \sigma^{\mu\nu}(P_R F_{Z2} + P_L F'_{Z2})\,, \tag{3}$$
$$V^\mu(tc\gamma) = -i\gamma^\mu(P_L F_{\gamma 1} + P_R F'_{\gamma 1}) + k_\nu \sigma^{\mu\nu}(P_R F_{\gamma 2} + P_L F'_{\gamma 2})\,, \tag{4}$$
$$V^\mu(tcg) = -iT^a\gamma^\mu(P_L F_{g1} + P_R F'_{g1}) + T^a k_\nu \sigma^{\mu\nu}(P_R F_{g2} + P_L F'_{g2})\,. \tag{5}$$

where as usual we have defined $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$ and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. T^a are the generators of $SU(3)_C$. The form factors $F_{1,2}$ and $F'_{1,2}$ encode the loop functions and depend on the various masses in the theory. The Feynman rules used to obtain them are given in Refs. [6, 7] and the corresponding Feynman diagrams are shown in Fig. 1. The vertices are derived assuming $p_1 - p_2 - k = 0$.

The form factors for the electroweak-like corrections due to loops involving charginos and strange- and bottom-squarks are given by

$$\begin{aligned}
F^c_{i1} = {} & \frac{1}{4\pi^2}\sum_{j=1}^{2}\sum_{\rho=1}^{2}\sum_{l=1}^{2}\sum_{\epsilon=1}^{2}\sum_{m=1}^{2}\Big\{ A_c^{j,\rho,l} D_c^{j,\epsilon,m} E_{ic}^{\rho,\epsilon}\,[m_t^2(c_{12}+c_{23}-c_{11}-c_{21}) - 2c_{24} \\
& + m_c^2(c_{23}-c_{12})] + B_c^{j,\rho,l} C_c^{j,\epsilon,m} E_{ic}^{\rho,\epsilon} m_c m_t\,(2c_{23}-c_{11}-c_{21}) \\
& + A_c^{j,\rho,l} C_c^{j,\epsilon,m} E_{ic}^{\rho,\epsilon} m_t m_{\chi_j^\pm}(c_0+c_{11}) \\
& + B_c^{j,\rho,l} D_c^{j,\epsilon,m} E_{ic}^{\rho,\epsilon} m_c m_{\chi_j^\pm}(c_0+c_{11})\Big\}^{(-p_1,k,\chi_j^\pm,\tilde{\epsilon}_m,\tilde{\rho}_l)} \\
& + \frac{1}{2\pi^2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{\rho=1}^{2}\sum_{l=1}^{2}\Big\{ B_c^{j,\rho,l} C_c^{k,\rho,l} F_{ic}^{j,k} m_c m_t\,(c_{21}+c_{22}-2c_{23}) \\
& + A_c^{j,\rho,l} D_c^{k,\rho,l} E_{ic}'^{j,k}\,[m_c^2 c_{22} - m_i^2(c_{12}+c_{23}) - 2c_{24} + \tfrac{1}{2}] \\
& + B_c^{j,\rho,l} D_c^{k,\rho,l} F_{ic}^{j,k} m_c m_{\chi_k^\pm}(c_{12}-c_{11}) \\
& + A_c^{j,\rho,l} C_c^{k,\rho,l} F_{ic}^{j,k} m_t m_{\chi_j^\pm}(c_{12}-c_{11}) + A_c^{j,\rho,l} D_c^{k,\rho,l} F_{ic}^{j,k} m_{\chi_j^\pm} m_{\chi_k^\pm} c_0\Big\}^{(-p_1,p_2,\chi_k^\pm,\tilde{\rho}_l,\chi_j^\pm)} \\
& + \frac{1}{2\pi^2}\frac{1}{m_t^2-m_c^2}\sum_{j=1}^{2}\sum_{\rho=1}^{2}\sum_{l=1}^{2}\Big\{ A_c^{j,\rho,l} D_c^{j,\rho,l} H_{ic} m_t^2(-B1) + A_c^{j,\rho,l} C_c^{j,\rho,l} H_{ic} m_t m_{\chi_j^\pm} B0
\end{aligned}$$

$$
\begin{aligned}
&+B_c^{j,\rho,l}C_c^{j,\rho,l}H_{ic}m_c m_t(-B1)+B_c^{j,\rho,l}D_c^{j,\rho,l}H_{ic}m_c m_{\chi_j^\pm}B0\Big\}^{(-p_1,\chi_j^\pm,\tilde{\rho}_l)}\\
&+\frac{1}{2\pi^2}\frac{1}{m_t^2-m_c^2}\sum_{j=1}^2\sum_{\rho=1}^2\sum_{l=1}^2\Big\{A_c^{j,\rho,l}D_c^{j,\rho,l}H_{ic}m_c^2B1+A_c^{j,\rho,l}C_c^{j,\rho,l}H_{ic}m_t m_{\chi_j^\pm}(-B0)\\
&+B_c^{j,\rho,l}C_c^{j,\rho,l}H_{ic}m_c m_t B1+B_c^{j,\rho,l}D_c^{j,\rho,l}H_{ic}m_c m_{\chi_j^\pm}(-B0)\Big\}^{(-p_2,\chi_j^\pm,\tilde{\rho}_l)}, \qquad (6)
\end{aligned}
$$

$$
F'^c_{i1} = F^c_{i1}(A,B,C,D,E',F,H\to B,A,D,C,F,E',G)\,, \qquad (7)
$$

$$
\begin{aligned}
F^c_{i2} =\;& \frac{1}{4\pi^2}\sum_{j=1}^2\sum_{\rho=1}^2\sum_{l=1}^2\sum_{\epsilon=1}^2\sum_{m=1}^2\Big\{A_c^{j,\rho,l}D_c^{j,\epsilon,m}E_{ic}^{\rho,\epsilon}m_t(c_{12}+c_{23}-c_{11}-c_{21})\\
&+A_c^{j,\rho,l}C_c^{j,\epsilon,m}E_{ic}^{\rho,\epsilon}m_{\chi_j^\pm}(c_0+c_{11})+B_c^{j,\rho,l}C_c^{j,\epsilon,m}E_{ic}^{\rho,\epsilon}m_c(c_{23}-c_{12})\Big\}^{(-p_1,k,\chi_j^\pm,\tilde{\epsilon}_m,\tilde{\rho}_l)}\\
&+\frac{1}{2\pi^2}\sum_{j=1}^2\sum_{k=1}^2\sum_{\rho=1}^2\sum_{l=1}^2\Big\{A_c^{j,\rho,l}D_c^{k,\rho,l}E'^{j,k}_{ic}m_t(c_{11}+c_{21}-c_{12}-c_{23})\\
&+B_c^{j,\rho,l}C_c^{k,\rho,l}F_{ic}^{j,k}m_c(c_{22}-c_{23})+A_c^{j,\rho,l}C_c^{k,\rho,l}E'^{j,k}_{ic}m_{\chi_k^\pm}(-c_{11}-c_0)\\
&+A_c^{j,\rho,l}C_c^{k,\rho,l}F_{ic}^{j,k}m_{\chi_j^\pm}c_{12}\Big\}^{(-p_1,p_2,\chi_k^\pm,\tilde{\rho}_l,\chi_j^\pm)}, \qquad (8)
\end{aligned}
$$

$$
F'^c_{i2} = F^c_{i2}(A,B,C,D,E',F\to B,A,D,C,F,E')\,, \qquad (9)
$$

In these expressions $i=Z,\gamma,g$, the sums over $j,k=1,2$ run over the two chargino mass eigenstates, $\rho,\epsilon=2,3$ represent strange- and bottom-squarks (we ignore the mixing with the down-squark), and $l,m=1,2$ represent a sum over squark mass eigenstates which are obtained from the $\tilde{q}_{L,R}$ gauge eigenstates via: $\tilde{q}_{\rho1}=\cos\theta_\rho\tilde{q}_{\rho L}+\sin\theta_\rho\tilde{q}_{\rho R}$ and $\tilde{q}_{\rho2}=-\sin\theta_\rho\tilde{q}_{\rho L}+\cos\theta_\rho\tilde{q}_{\rho R}$. The various B and c functions in the above expressions are the well documented Passarino-Veltman functions [8] (adapted to our metric where $p_i^2=m_i^2$); the arguments of the B and c functions are indicated by the superscripts on the braces in Eqs. (6)–(9). For example, the arguments of the c functions that appear within the first brace of Eq. (6) are $(-p_1,k,m_{\chi_k^\pm},m_{\tilde{q}_{\epsilon m}},m_{\tilde{q}_{\rho l}})$ while the arguments of the B functions that appear in the third brace of Eq. (6) are $(-p_1,m_{\chi_k^\pm},m_{\tilde{q}_{\rho l}})$. (Note that the Passarino-Veltman functions depend only on the square of their arguments.) The Passarino-Veltman functions contain infinities which cancel each other out, as they should since there is no t-c-V vertex in the Lagrangian. The coefficient functions are given by

$$
\begin{aligned}
A_c^{j,\rho,l} =\;& \frac{g}{2}\Bigg[-U_{j1}(K\Gamma_2^\dagger)_{2\rho}\left\{\begin{matrix}\cos\theta_\rho\ (l=1)\\ -\sin\theta_\rho\ (l=2)\end{matrix}\right\}\\
&+\frac{1}{\sqrt{2}m_W\cos\beta}U_{j2}(KM_dB_2^\dagger)_{2\rho}\left\{\begin{matrix}\sin\theta_\rho\ (l=1)\\ \cos\theta_\rho\ (l=2)\end{matrix}\right\}\Bigg], \qquad (10)
\end{aligned}
$$

$$
B_c^{j,\rho,l} = \frac{g}{2}\left[\frac{m_c}{\sqrt{2}m_W\sin\beta}V_{j2}^*(K\Gamma_2^\dagger)_{2\rho}\left\{\begin{matrix}\cos\theta_\rho\ (l=1)\\ -\sin\theta_\rho\ (l=2)\end{matrix}\right\}\right], \qquad (11)
$$

$$
\begin{aligned}
C_c^{j,\rho,l} &= \frac{g}{2}\left[\frac{m_t}{\sqrt{2}m_W \sin\beta}V_{j2}(\Gamma_2 K^\dagger)_{\rho 3}\begin{Bmatrix}\cos\theta_\rho\ (l=1)\\ -\sin\theta_\rho\ (l=2)\end{Bmatrix}\right], && (12)\\
D_c^{j,\rho,l} &= \frac{g}{2}\Bigg[-U_{j1}^*(\Gamma_2 K^\dagger)_{\rho 3}\begin{Bmatrix}\cos\theta_\rho\ (l=1)\\ -\sin\theta_\rho\ (l=2)\end{Bmatrix} \\
&\qquad +\frac{1}{\sqrt{2}m_W\cos\beta}U_{j2}^*(B_2 M_d K^\dagger)_{\rho 3}\begin{Bmatrix}\sin\theta_\rho\ (l=1)\\ \cos\theta_\rho\ (l=2)\end{Bmatrix}\Bigg], && (13)\\
E_{Zc}^{\rho,\epsilon} &= -\frac{g}{\cos\theta_W}\left[-\frac{1}{2}\sum_{p=2}^{3}\Gamma_{QL}^{\rho p}\Gamma_{QL}^{*\epsilon p}+\frac{1}{3}\sin^2\theta_W\delta^{\rho\epsilon}\right], && (14)\\
E_{\gamma c}^{\rho,\epsilon} &= \frac{e}{3}\delta^{\rho\epsilon}, && (15)\\
E_{gc}^{\rho,\epsilon} &= -g_s\delta^{\rho\epsilon}, && (16)\\
E_{Zc}^{\prime j,k} &= \frac{g}{2\cos\theta_W}\left[-U_{j1}^*U_{k1}-\frac{1}{2}U_{j2}^*U_{k2}+\sin^2\theta_W\delta^{jk}\right], && (17)\\
E_{\gamma c}^{\prime j,k} &= -\frac{e}{2}\delta^{jk}, && (18)\\
E_{gc}^{\prime j,k} &= 0, && (19)\\
F_{Zc}^{j,k} &= \frac{g}{2\cos\theta_W}\left[-V_{j1}V_{k1}^*-\frac{1}{2}V_{j2}V_{k2}^*+\sin^2\theta_W\delta^{jk}\right], && (20)\\
F_{\gamma c}^{j,k} &= -\frac{e}{2}\delta^{jk}, && (21)\\
F_{gc}^{j,k} &= 0, && (22)\\
G_{Zc} &= -\frac{g}{2\cos\theta_W}\left(-\frac{2}{3}\sin^2\theta_W\right), && (23)\\
G_{\gamma c} &= -\frac{e}{3}, && (24)\\
G_{gc} &= -\frac{g_s}{2}, && (25)\\
H_{Zc} &= -\frac{g}{2\cos\theta_W}\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right), && (26)\\
H_{\gamma c} &= -\frac{e}{3}, && (27)\\
H_{gc} &= -\frac{g_s}{2},\,. && (28)
\end{aligned}
$$

The chargino mixing matrices U_{ij} and V_{ij} and the generational mixing matrices K, Γ_2 and B_2 which appear in these expressions are defined in Ref. [6], M_d is the diagonal matrix (m_d, m_s, m_b) and Γ_{QL} is the squark mixing matrix defined in Ref. [7].

In deriving the above form factors, we have used the following relations:

$$
\begin{aligned}
\sum_\lambda k^\mu k^\nu \epsilon_\mu(k,\lambda)\epsilon_\nu(k,\lambda) &= 0, && (29)\\
\bar{u}(p_2)p_1^\mu P_{R,L}u(p_1) &= \bar{u}(p_2)[m_t\gamma^\mu P_{L,R}+ip_{1\nu}\sigma^{\mu\nu}P_{R,L}]u(p_1), && (30)\\
\bar{u}(p_2)p_2^\mu P_{R,L}u(p_1) &= \bar{u}(p_2)[m_c\gamma^\mu P_{R,L}-ip_{2\nu}\sigma^{\mu\nu}P_{R,L}]u(p_1), && (31)
\end{aligned}
$$

$$
\begin{aligned}
\bar{u}(p_2)(p_1+p_2)^\mu P_{R,L} u(p_1)\epsilon_\mu(k,\lambda) &= \bar{u}(p_2) 2p_1^\mu P_{R,L} u(p_1)\epsilon_\mu(k,\lambda) \\
&= \bar{u}(p_2) 2p_2^\mu P_{R,L} u(p_1)\epsilon_\mu(k,\lambda) \\
&= \bar{u}(p_2)[m_t\gamma^\mu P_{L,R} + m_c\gamma^\mu P_{R,L} \\
&\qquad + ik_\nu\sigma^{\mu\nu} P_{R,L}] u(p_1)\epsilon_\mu(k,\lambda)\,. \qquad (32)
\end{aligned}
$$

(The first two equalities in Eq. (32) are only true when one takes the absolute square of both sides of the equation and uses Eq. (29).)

Our results above for the chargino-squark loops disagree with those of Ref. [3] in the limit of $m_c = 0$. We have an additional term

$$
\frac{1}{2\pi^2}\sum_{j=1}^{2}\sum_{\rho=1}^{2}\sum_{l=1}^{2}\sum_{\epsilon=1}^{2}\sum_{m=1}^{2}\left\{A_c^{j,\rho,l} D_c^{k,\rho,l} F_{ic}^{j,k} m_{\chi_j^\pm} m_{\chi_k^\pm} c_0\right\}^{(-p_1,p_2,\chi_k^\pm,\tilde{\rho}_l,\chi_j^\pm)} \qquad (33)
$$

in F_{i1}^c. This term is required by gauge invariance, *i.e.*, it is needed to ensure that the coefficient of γ^μ in Eqs. (4) and and (5) vanishes [9] for the massless gauge bosons.

The form factors for the electroweak-like corrections due to loops involving neutralinos and top- and charm-squarks are given by Eqs. (6)–(9) with $m_{\chi^\pm}$ replaced by m_{χ^0} and with the coefficients A_c–F_c replaced by

$$
\begin{aligned}
A_n^{j,\rho,l} &= -\frac{1}{\sqrt{2}}\left[\frac{2}{3}eN'_{j1} + \frac{g}{\cos\theta_W}\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)N'_{j2}\right](\Gamma_1^\dagger)_{2\rho}\left\{\begin{matrix}\cos\theta_\rho\ (l=1)\\ -\sin\theta_\rho\ (l=2)\end{matrix}\right\} \\
&\quad -\frac{1}{\sqrt{2}}\left[\frac{g}{2m_W\sin\beta}N_{j4}\right](M_u B_1^\dagger)_{2\rho}\left\{\begin{matrix}\sin\theta_\rho\ (l=1)\\ \cos\theta_\rho\ (l=2)\end{matrix}\right\}, \qquad (34) \\
B_n^{j,\rho,l} &= -\frac{1}{\sqrt{2}}\left[\frac{g}{2m_W\sin\beta}N_{j4}^*\right](M_u\Gamma_1^\dagger)_{2\rho}\left\{\begin{matrix}\cos\theta_\rho\ (l=1)\\ -\sin\theta_\rho\ (l=2)\end{matrix}\right\} \\
&\quad +\frac{1}{\sqrt{2}}\left[\frac{2}{3}eN'^*_{j1} - \frac{g}{\cos\theta_W}\left(\frac{2}{3}\sin^2\theta_W\right)N'^*_{j2}\right](B_1^\dagger)_{2\rho}\left\{\begin{matrix}\sin\theta_\rho\ (l=1)\\ \cos\theta_\rho\ (l=2)\end{matrix}\right\}, \qquad (35) \\
C_n^{j,\rho,l} &= -\frac{1}{\sqrt{2}}\left[\frac{g}{2m_W\sin\beta}N_{j4}\right](\Gamma_1 M_u)_{\rho 3}\left\{\begin{matrix}\cos\theta_\rho\ (l=1)\\ -\sin\theta_\rho\ (l=2)\end{matrix}\right\} \\
&\quad +\frac{1}{\sqrt{2}}\left[\frac{2}{3}eN'_{j1} - \frac{g}{\cos\theta_W}\left(\frac{2}{3}\sin^2\theta_W\right)N'_{j2}\right](B_1)_{\rho 3}\left\{\begin{matrix}\sin\theta_\rho\ (l=1)\\ \cos\theta_\rho\ (l=2)\end{matrix}\right\}, \qquad (36) \\
D_n^{j,\rho,l} &= -\frac{1}{\sqrt{2}}\left[\frac{2}{3}eN'^*_{j1} + \frac{g}{\cos\theta_W}\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)N'^*_{j2}\right](\Gamma_1)_{\rho 3}\left\{\begin{matrix}\cos\theta_\rho\ (l=1)\\ -\sin\theta_\rho\ (l=2)\end{matrix}\right\} \\
&\quad -\frac{1}{\sqrt{2}}\left[\frac{g}{2m_W\sin\beta}N_{j4}^*\right](B_1 M_u)_{\rho 3}\left\{\begin{matrix}\sin\theta_\rho\ (l=1)\\ \cos\theta_\rho\ (l=2)\end{matrix}\right\}, \qquad (37) \\
E_{Zn}^{\rho,\epsilon} &= -\frac{g}{\cos\theta_W}\left[\frac{1}{2}\sum_{p=2}^{3}\Gamma_{QL}^{\rho p}\Gamma_{QL}^{*\epsilon p} - \frac{2}{3}\sin^2\theta_W\delta^{\rho\epsilon}\right], \qquad (38) \\
E_{\gamma n}^{\rho,\epsilon} &= -\frac{2}{3}e\delta^{\rho\epsilon}\,, \qquad (39) \\
E_{gn}^{\rho,\epsilon} &= -g_s\delta^{\rho\epsilon}\,, \qquad (40)
\end{aligned}
$$

$$
\begin{aligned}
E'^{j,k}_{Zn} &= \frac{g}{2\cos\theta_W}\left[\frac{1}{2}N'^*_{j3}N'_{k3} - \frac{1}{2}N'^*_{j4}N'_{k4}\right], && (41)\\
E'^{j,k}_{\gamma n} &= 0\,, && (42)\\
E'^{j,k}_{gn} &= 0\,, && (43)\\
F^{j,k}_{Zn} &= \frac{g}{2\cos\theta_W}\left[-\frac{1}{2}N'_{j3}N'^*_{k3} + \frac{1}{2}N'_{j4}N'^*_{k4}\right], && (44)\\
F^{j,k}_{\gamma n} &= 0\,, && (45)\\
F^{j,k}_{gn} &= 0\,. && (46)
\end{aligned}
$$

The neutralino mixing matrices N_{ij} and N'_{ij} and the generational mixing matrices Γ_1 and B_1, are defined in Ref. [6], and M_u is the diagonal matrix (m_u, m_c, m_t). In Eqs. (6)–(9) the sums over j, k now run from 1–4 over the four neutralino mass eigenstates, and $\rho, \epsilon, p = 2, 3$ represent charm- and top-squarks (we ignore the mixing with the up-squark); $l, m = 1, 2$ represent a sum over squark mass eigenstates, as earlier. The coefficients G and H are unaltered.

While we have included the charm-quark mass in the form factors above for completeness, we set $m_c = 0$ hereafter. The supersymmetric electroweak-like contribution to the decay rates is then given in terms of the form factors obtained above, as follows

$$
\begin{aligned}
\Gamma(t \to cZ) &= \frac{1}{32\pi m_t^3}(m_t^2 - m_Z^2)^2\Big[(2 + \frac{m_t^2}{m_Z^2})(F_{Z1}^2 + F'^2_{Z1})\\
&\quad -6m_t\,(F_{Z1}F_{Z2} + F'_{Z1}F'_{Z2}) + (2m_t^2 + m_Z^2)(F_{Z2}^2 + F'^2_{Z2})\Big]\,, && (47)\\
\Gamma(t \to c\gamma) &= \frac{m_t}{32\pi}\Big[(2(F_{\gamma 1}^2 + F'^2_{\gamma 1})\\
&\quad -6m_t\,(F_{\gamma 1}F_{\gamma 2} + F'_{\gamma 1}F'_{\gamma 2}) + 2m_t^2(F_{\gamma 2}^2 + F'^2_{\gamma 2})\Big]\,, && (48)\\
\Gamma(t \to cg) &= \frac{m_t}{24\pi}\Big[(2(F_{g1}^2 + F'^2_{g1})\\
&\quad -6m_t\,(F_{g1}F_{g2} + F'_{g1}F'_{g2}) + 2m_t^2(F_{g2}^2 + F'^2_{g2})\Big]\,. && (49)
\end{aligned}
$$

In these expressions each form factor receives contributions from both chargino-squark and neutralino-squark form factors. It is not hard to verify that for $m_c = 0$, the chargino contributions to $F'_{1,2}$ vanish. (In Ref. [3] there is a factor of π missing in the expression for $\Gamma(t \to cZ)$ and a factor of $\frac{1}{3}$ missing in $\Gamma(t \to cg)$.)

3 Numerical Results

Before we attempt to evaluate the rather lengthy expressions given above, we would like to consider qualitatively the possibility of dynamical enhancements of the loop amplitudes. Experience with similar diagrams contributing to the self-energy of the top quark in supersymmetric theories [10] indicates possible large corrections when the mass of the top quark equals the sum of the masses of the other particles leaving the vertex involving the top quark.[1] In the present case we have vertices with top quarks and: (i) gluinos and top-squarks in the case of QCD-like contributions (calculated in Refs. [3, 4]); (ii) charginos and down-like squarks in the case of 'charged' electroweak-like corrections; and (iii) neutralinos and up-like squarks in the case of 'neutral' electroweak-like corrections. Given the presently-known lower bounds on the squark (excluding $\tilde{t}$) and gluino masses (*i.e.*, $m_{\tilde{q}}, m_{\tilde{g}} > 175\,\mathrm{GeV}$; $m_{\tilde{q}} \approx m_{\tilde{g}} > 230\,\mathrm{GeV}$ [11]), this type of enhancement might only be present in the third type of contribution when $m_t \approx m_\chi + m_{\tilde{t}_1}$, which requires a light top-squark whose mass is constrained experimentally to $m_{\tilde{t}_1} \gtrsim 60\,\mathrm{GeV}$ [12]. The 'neutral' electroweak-like contributions might also be enhanced by large GIM-violating top-squark–charm-squark mass splittings. This latter enhancement is also present in the 'charged' electroweak-like contributions. However, the 'charged' contributions fall short of the 'neutral' electroweak-like corrections for the gluon and photon cases. Interestingly, the 'charged' contribution for the Z is higher than in the 'neutral' case. We first address the neutral electroweak-like contributions and comment on the charged contributions afterwards.

The neutral electroweak-like contributions might be enhanced as discussed above, but this is subject to other mixing factors in the $A_n - F_n$ coupling functions in Eqs. (34)–(46) being unsuppressed. At the root of this question is whether the quark-squark-neutralino couplings might be flavor non-diagonal as a result of their evolution from the unification scale down to the electroweak scale. This question might be explored by considering the squared squark mass matrices at the electroweak scale that are obtained by renormalization group evolution of a universal scalar mass at the unification scale [6, 13]:

$$\widetilde{X}^2_{iR} = M_W^2\, \mu^{(0)}_{iR} I + \mu^{(1)}_{iR} X_i X_i^\dagger \qquad (i = 1, 2) \tag{50}$$

$$\widetilde{X}^2_{1L} = M_W^2\, \mu^{(0)}_{1L} I + \mu^{(1)}_{1L} X_1 X_1^\dagger + \mu^{(2)}_{1L} X_2 X_2^\dagger \tag{51}$$

$$\widetilde{X}^2_{2L} = M_W^2\, \mu^{(0)}_{2L} I + \mu^{(1)}_{2L} X_1 X_1^\dagger + \mu^{(2)}_{2L} X_2 X_2^\dagger \tag{52}$$

where $i = 1$ ($i = 2$) corresponds to up-type (down-type) flavors, the $\mu^{(0,1,2)}$ are RGE-dependent coefficients, and X_1 (X_2) are the up-type (down-type) Yukawa matrices. The matrices $B_i \equiv \widetilde{U}_i^* U_i^T$, appearing in the equations in Sec. 2 above, are obtained from the $\widetilde{U}_i$ matrices that diagonalize $\widetilde{X}^2_{iR}$, and the U_i matrices that diagonalize the right-handed quark mass matrices. Because of the simple form for $\widetilde{X}^2_{iR}$ in Eq. (50), it can be shown that $\widetilde{U}_i = U_i$ and therefore $B_i = I$ [6]. [Note that the quark mixing matrices U_i and V_i mentioned in this section are different from the chargino mixing matrices U_{ij} and V_{ij} mentioned in Section 2.]

The other relevant set of matrices are $\Gamma_i \equiv \widetilde{V}_i V_i^\dagger$, obtained from the $\widetilde{V}_i$ matrices that diagonalize $\widetilde{X}^2_{iL}$ and the V_i matrices that diagonalize the left-handed quark mass matrices. In the case of $\lambda_t \gg \lambda_b$, which requires $\tan\beta \sim 1$, the $X_2 X_2^\dagger \propto \lambda_b^2$ term

[1] The sign of these corrections depends on the observable being calculated: in the case of $B(t \to cV)$ they are positive, whereas in one-loop supersymmetric corrections to $\sigma(p\bar{p} \to t\bar{t})$ [10] they were negative.

in Eqs. (51,52) is small compared to the $X_1 X_1^\dagger \propto \lambda_t^2$ term, and therefore the former may be neglected. This implies that both $\widetilde{X}_{1L}^2$ and $\widetilde{X}_{2L}^2$ are diagonalized by the same matrix: $\widetilde{V}_1 = \widetilde{V}_2 = V_1$, and therefore $\Gamma_1 = \widetilde{V}_1 V_1^\dagger = I$, whereas $\Gamma_2 = \widetilde{V}_2 V_2^\dagger = V_1 V_2^\dagger = K$ reduces to the regular CKM matrix [6]. As the quark-squark-neutralino couplings in flavor space are proportional to Γ_i, we see that for $\lambda_t \gg \lambda_b$ there are no flavor off-diagonal couplings in the up-quark sector, as required for a unsuppressed contribution to the 'neutral' electroweak-like contributions to $t \to cV$.

One might consider instead a scenario where $\lambda_t \sim \lambda_b$, as would be consistent with $\tan\beta \gg 1$. In this case the $X_2 X_2^\dagger \propto \lambda_b^2$ term in Eqs. (51,52) is no longer negligible and $\Gamma_1 \neq I$ is expected. The precise form of Γ_1 requires a complicated calculation, essentially solving the matrix renormalization group equations. For our purposes here it suffices to consider the following effective form

$$\Gamma_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & -\epsilon & 1 \end{pmatrix} , \tag{53}$$

where ϵ parametrizes the size of the ratio λ_b/λ_t. For moderate values of $\tan\beta$ this form should be adequate (*i.e.* ϵ not too close to 1). We still expect $\Gamma_2 \approx K$. We assume that the lower (2×2) right corner of V_1 is approximately the identity to relate Γ_{QL} to Γ_1.

The above forms for $\Gamma_{1,2}$ plus the result $B_{1,2} = I$ above, allow us to evaluate numerically the branching ratios of Sec. 2. Perhaps the most optimistic top factory being contemplated at the moment is a high-luminosity upgrade of the Tevatron, where studies show that one might be sensitive to $B(t \to c\gamma) \approx 4\times 10^{-4}\,(8\times 10^{-5})$ [14], $B(t \to cZ) \approx 4 \times 10^{-3}\,(6 \times 10^{-4})$ [14], and $B(t \to cg) \approx 5 \times 10^{-3}\,(1 \times 10^{-3})$ [2] with an integrated luminosity of $10\,(100)\,\mathrm{fb}^{-1}$, where the branching ratios are with respect to $\Gamma(t \to bW)$. These expected sensitivities will not allow direct tests of the Standard Model predictions for these processes: $B(t \to c\gamma)^{\mathrm{SM}} \sim 10^{-12}$, $B(t \to cZ)^{\mathrm{SM}} \sim 10^{-12}$, and $B(t \to cg)^{\mathrm{SM}} \sim 10^{-10}$ [5], but might uncover virtual new physics effects that enhance these rates over Standard Model expectations.

Indeed, we generally find that $B(t \to cV)$ greatly exceeds the corresponding Standard Model contribution, but unfortunately falls below the expected experimental sensitivities, as was observed also in previous studies of the QCD-like corrections [4]. Specifically, concentrating on the 'neutral' electroweak-like corrections, we have as the dominant inputs the masses of the charm-squark and top-squark, the top mixing angle, the mass of the neutralino(s), and the neutralino composition. The results for $V = g, \gamma$ scale with ϵ^2 as defined in Eq. (53); we take $\epsilon = 0.5$ for concreteness. (For $V = Z$, the cross section increases montonically with ϵ for $\epsilon \leq 0.5$.) Numerically[2] we find that when $m_t \approx m_\chi + m_{\tilde{t}_1}$, the branching ratios are enhanced compared to off-resonance values by a factor of 2-10. This factor depends on the specific combination of neutralino and top-squark masses that satisfy this relation (all other parameters

[2]We used the software package `ff` [15] to evaluate the Passarino-Veltman functions.

being kept fixed); the enhancement decreases with increasing stop mass and so is maximised when $m_{\tilde{t}_1}$ is 60 GeV. The off-resonance values themselves are larger than the Standard Model predictions for not-too-heavy sparticles. We also verify that large $m_{\tilde{c}} - m_{\tilde{t}_1}$ mass splitting enhances the results, because of its GIM-violating effect. (A useful test of our code is that the branching ratios go to zero due to the GIM mechanism, if we set squark masses equal.) With regards to neutralino composition, the largest branching ratios are obtained for neutralinos with comparable bino and higgsino admixtures. Increasing the scale of the sparticle masses typically leads to a rapid decrease in the branching ratios for g and γ. For the Z the decrease is very gradual, as has also been noted in Ref. [4]. For g and γ the cross-section seems to be maximised for stop mixing angles close to 0 or π. The effects of mixing for Z are very dependent on the other parameters chosen, such as the neutralino composition, etc. The effect of varying $\tan\beta$ is of $O(1)$.

In varying the different parameters above we have worked in the most general framework of the MSSM, in which the various parameters can be varied independently. In a more specific model, such as one with universal scalar masses and radiative electroweak symmetry breaking, these parameters are not all independent. Although our choice of mixing matrices was motivated by certain specific scenarios we vary our parameters freely so as to look for the maximal supersymmetric contributions. Furthermore, we choose $m_{\tilde{c}_{1,2}}$ and $m_{\tilde{t}_2}$ to be $\sim$ 1TeV.

For the most optimistic values of the parameters, *i.e.*, when the above enhancing circumstances all simultaneously occur, we find $B(t \to c\gamma) \lesssim 2 \times 10^{-7}, B(t \to cZ) \lesssim 4 \times 10^{-7}$ and $B(t \to cg) \lesssim 3 \times 10^{-5}$. We see that $B(t \to cg)$ is the one closest to the level of experimental sensitivity expected at the Tevatron, so perhaps it would be the mode to be first observed at a future sufficiently sensitive machine. This hope is further enlarged by recalling that $B(t \to cg)$ receives comparable contributions from the QCD-like supersymmetric corrections [4], which we have not evaluated here. Eventually, such a process can be a possible test for supersymmetry.

We have also evaluated the charged electroweak-like corrections and have found them to be smaller (typically by a factor of 10 or more) than the neutral electroweak-like corrections discussed above for g and γ, but a factor of 10 higher for the Z. (Again we assume that the lower (2×2) right corner of V_2 is approximately the identity to relate Γ_{QL} to Γ_2.) In the case of universal squark masses at the unification scale, GIM-violating bottom-squark strange-squark mass differences are generated by RGE evolution, resulting in shifts to the left-handed down-like squark mass matrices. The dominant term is from the second term of Eq. (52) which may be rewritten as $-|c|K^\dagger(\widehat{m}_u)^2K$, where $\widehat{m}_u = \{m_u, m_c, m_t\}$ and $|c| \leq 1$ is an RGE-dependent constant. Inserting the values of the CKM matrix elements we find (approximately): $m^2_{\tilde{s}_L} \to m^2_{\tilde{s}_L} - |c|(m_t/5)^2$ and $m^2_{\tilde{b}_L} \to m^2_{\tilde{b}_L} - |c|m_t^2$. Choosing the maximal $(|c| = 1)$ mass splittings, we find $B(t \to c\gamma) \lesssim 10^{-8}$, $B(t \to cZ) \lesssim 2 \times 10^{-6}$, and $B(t \to cg) \lesssim 10^{-7}$ for squark masses as low as experimentally allowed. (These results are not much altered even if one drops the assumption of universal squark masses at the unification scale.) The numerical results for the charged electroweak-like corrections cannot be compared with the corresponding ones in Ref. [3] because, as we explained above, the formulas presented in Ref. [3] are inconsistent with gauge invariance constraints.

We finally try to connect up with the recent literature in Ref. [2], where in addition to the $t \to cg$ decay mode, people have considered hadronic processes like $p\bar{p} \to t\bar{c}$, which might be more easily detectable. These works ignore any substructure that the t-c-g vertex might have, and replace it all by an effective scale Λ, defined for instance by the relation $\Gamma(t \to cg) = 8\alpha_s m_t^3/3\Lambda^2$. A given branching ratio obtained in the supersymmetric theory then corresponds to a scale Λ in the effective theory. Dividing this expression by $\Gamma(t \to bW)$ we find that $\Lambda \approx 1\,\mathrm{TeV}/\sqrt{B(t \to cg)}$. Therefore a supersymmetric prediction of $B(t \to cg) \sim 10^{-5}$ corresponds to $\Lambda \sim 300\,\mathrm{TeV}$. The point of this exercise is to note how misleading such estimates of new-physics scales might be, as this actually corresponds in our case to sparticle masses of a few hundred GeV!

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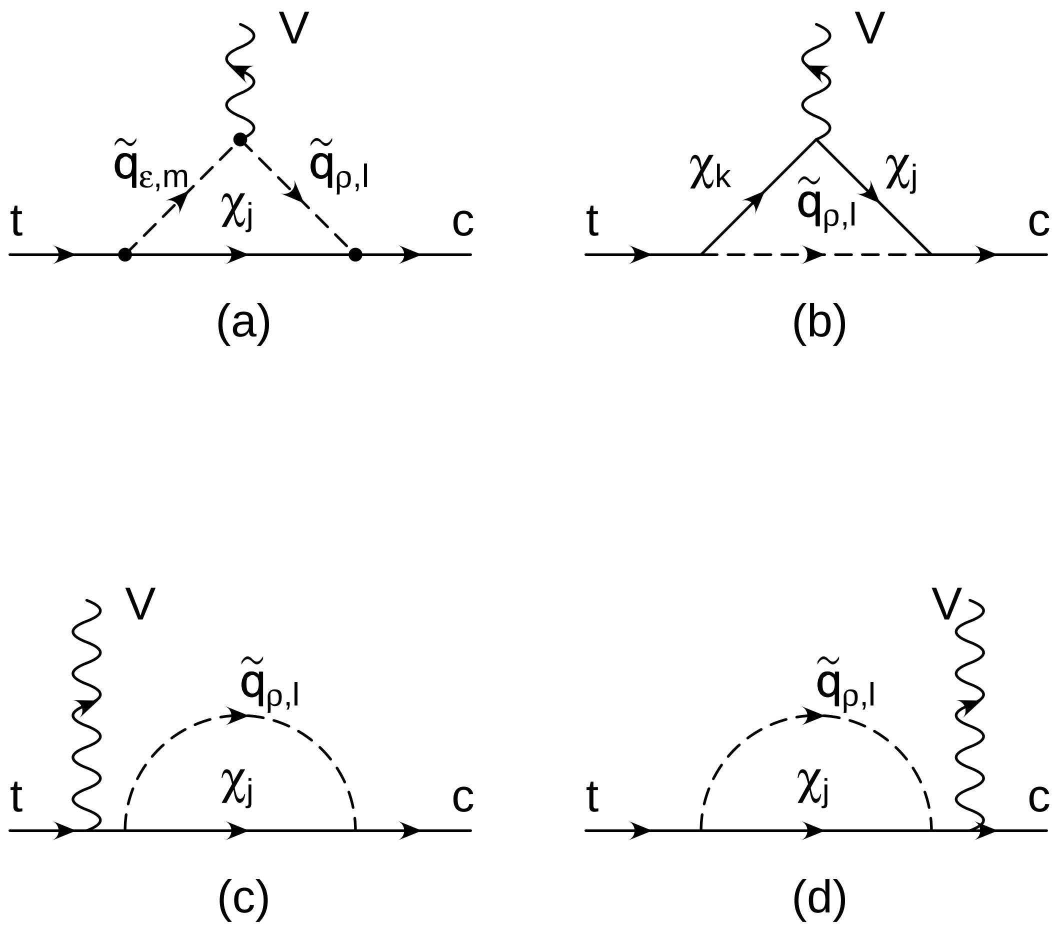

Figure 1: Feynman diagrams for one-loop electroweak-like supersymmetric contributions to the t-c-V ($V = \gamma, Z, g$) vertex. In the figure χ represents the chargino or the neutralino and $\tilde{q}$ represents the down-type or up-type squarks respectively. The subscripts are explained in the text. The arrows on the squark lines indicate the direction of flow of flavor; the arrows on the gauge bosons indicate the direction of momentum flow. Diagram (b) is absent for $V = g$, and for $V = \gamma$ when $\chi = \chi^0$.