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Testing Quantum Mechanics in the Neutral Kaon System

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Abstract

The neutral kaon system is a sensitive probe of quantum mechanics. We revive a parametrization of non-quantum-mechanical effects that is motivated by considerations of the nature of space-time foam, and show how it can be constrained by new measurements of $K_L \to 2\pi$ and $K_{L,S}$ semileptonic decays at LEAR or a ϕ factory.

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1 Introduction and Summary

The neutral kaon system is a textbook example of a microscopic quantum-mechanical system that exhibits a rich variety of physical phenomena. Its resolution of the tau-theta puzzle was a manifestation of parity violation [1]. It is still the only place where CP violation has been observed in the laboratory [2], and the suppression of $K_L \to \mu^+\mu^-$ decays was one of the primary motivations for charm [3]. It offers one of the most sensitive available tests of the CPT invariance that is inherent to local quantum field theory [4]. It has provided very elegant quantum-mechanical interference effects [5]. Indeed, several years ago, two of us (J.E. and D.V.N.) argued in a paper with Hagelin and Srednicki [6] that the neutral kaon system was, together with long-range neutron interferometry, one of the two most sensitive probes of a possible breakdown of conventional quantum mechanics suggested by investigations of local field theory in the presence of microscopic event horizons.

It was observed some time ago [7] that black holes apparently required a mixed statistical description, understood intuitively as being due to the loss of information across the event horizon. Hawking later suggested [8] that pure initial states could evolve into mixed final states in the presence of a microscopic event horizon. He proposed a density matrix formalism in which ρ_{in} and ρ_{out} were linearly related by a $\not{S}$-matrix that could not be factorized as the product of S- and $S^\dagger$-matrix elements as expected in conventional local quantum field theory:

$$\rho_{out} = \not{S}\rho_{in} \qquad : \qquad \not{S} \neq SS^\dagger \tag{1}$$

Ref. [6] then pointed out that in this case the normal Liouville equation that describes the time-evolution of the quantum-mechanical density matrix would also require modification by the addition of an extra linear term:

$$\partial_t \rho = i[\rho, H] + \not{\delta}H\rho \tag{2}$$

It was shown explicitly that addition of the $\not{\delta}H$ term would allow an initially pure state to evolve into a mixed state with positive entropy. The extra term in equation (2) is characteristic of open quantum mechanical systems [9]. However, we regarded it as necessary because of the intrinsic impossibility of measurements within the event horizon. Bounds on this type of non-quantum-mechanical behaviour were derived from the agreement of observations of neutral kaons and of neutrons with conventional quantum mechanics. Both systems were used [6] to obtain similar upper limits on the hadronic matrix elements of $\not{\delta}H$ of order $10^{-20}\ GeV$.

Although very small in a microscopic system, such effects would be magnified in a macroscopic system with Avogadro's number of elementary particles [10]. They could even engender the transition from quantum-mechanical to classical behaviour in large systems [10]. Indeed, the modification (2) has a form similar to that postulated for this purpose in ref. [11] without any microscopic justification. Thus the modification (2) constitutes a possible realization of the idea, advocated more recently by Penrose [12], that quantum gravity might explain the classical behaviour of large systems. It could be interesting to test this possibility in macroscopic quantum-mechanical laboratories such as SQUIDs [10].

Lately, the possibility of a microscopic violation of the laws of quantum mechanics has been re-examined in the context of string theory [13]. Specifically, studies of scattering and decay processs in a spherically-symmetric string black hole background have not revealed any loss of quantum coherence [14]. We have attributed this to the presence in string theory of an infinite set of local symmetries that include a $W_{1+\infty}$-algebra [13]. This in turn contains an infinite-dimensional Cartan subalgebra of charges that are in involution with the Hamiltonian, and hence conserved. Thus they provide an infinite set of W-hair that characterizes the black hole state, preserves information, and hence maintains quantum coherence. Thus we find no evidence for the modifications (1,2) of conventional quantum mechanics in scattering off one particular topologically non-trivial space-time background.

However, this does not mean that the S-matrix of quantum field theory and conventional quantum mechanics are sacrosanct. The symmetries that preserve quantum coherence relate states with different masses [13] : in particular, they relate the light particles that appear in laboratory experiments to Planck mass string states. Since realistic measurements are conducted with a truncation of the full physical string spectrum, they do not include all observables. The connections between light and massive states mean that the former should be considered as an open system as in equation (2), with the possibility of apparent information loss [15]. Thus it is relevant to test the general formalism (2) also in the context of string theory.

Two new experimental tools to do this in the neutral kaon system have become available since ref. [6] was written. One is the CP-LEAR experiment [16], in which copious tagged K^0 decays are available, and the other is the DAϕNE ϕ-factory now under construction [17], which will provide copious coherent K-$\overline{K}$ pairs. In both experiments, it will be possible to observe CP-violating asymmetries in K_S decays, and hence new tests of CPT invariance can be made [18]. The purpose of this paper is to point out that the types of measurements proposed as tests of CPT invariance also serve as probes for violations of quantum mechanics.

In section 2 we remind the reader of basic features of the modification (2) of quantum mechanics, with reference to the neutral kaon system in which $\not\delta H$ has three possible matrix elements to be bounded by experiment [6]. Then, in section 3 we show explicitly how two of them can be disentangled by measurements of CP-violating $K_{L,S}$ semileptonic decay asymmetries and $K_L \to 2\pi$ decays. One of these parameters bears a phenomenological resemblance to the CPT-violating parameter introduced in ref. [18], but the other appears in a different way. Finally, in section 4 we comment on the outlook for such probes of quantum mechanics.

2 Formalism for the Violation of Quantum Mechanics in the Neutral kaon System.

This is described in the usual quantum-mechanical framework [5] by a phenomenological Hamiltonian with hermitian (mass) and antihermitian (decay) components:

$$H = \left(\begin{array}{cc} M - \frac{1}{2}i\Gamma & M_{12}^{*} - \frac{1}{2}i\Gamma_{12} \\ M_{12} - \frac{1}{2}i\Gamma_{12} & M - \frac{1}{2}i\Gamma \end{array} \right) \tag{3}$$

in the $(K^0, \overline{K}^0)$ basis. When H is not hermitian, the time-evolution of the density matrix ρ is ordinarily given by

$$\partial_t \rho = -i(H\rho - \rho H^{\dagger}) \tag{4}$$

and the state is pure if $Tr\rho^2 = (Tr\rho)^2$, which it remains forever it started pure. We define components of ρ and H by

$$\begin{aligned} \rho &\equiv \frac{1}{2}\rho_\alpha \sigma_\alpha \\ H &\equiv \frac{1}{2}h_\beta \sigma_\beta \end{aligned} \tag{5}$$

where we use the Pauli σ-matrix basis, and the ρ_α are real but the h_β are complex.

It is convenient for our subsequent discussion to use the CP eigenstate basis $K_{1,2} = \frac{1}{\sqrt{2}}(K^0 \pm \overline{K}^0)$, in which we can represent the ordinary evolution (4) by $\partial_t \rho_\alpha = h_{\alpha\beta}\rho_\beta$ where

$$h_{\alpha\beta} = \left(\begin{array}{cccc} -\Gamma & -Re\Gamma_{12} & Im\Gamma_{12} & 0 \\ -Re\Gamma_{12} & -\Gamma & 0 & -2ImM_{12} \\ Im\Gamma_{12} & 0 & -\Gamma & -2ReM_{12} \\ 0 & 2ImM_{12} & 2ReM_{12} & -\Gamma \end{array} \right) \tag{6}$$

At large t, ρ decays exponentially to

$$\rho \propto \left(\begin{array}{cc} 1 & \epsilon^* \\ \epsilon & |\epsilon|^2 \end{array} \right) \tag{7}$$

which corresponds to the usual pure long-lived mass eigenstate K_L, with the CP impurity parameter ϵ given by

$$\epsilon = \frac{\frac{1}{2}iIm\Gamma_{12} - ImM_{12}}{\frac{1}{2}\Delta\Gamma - i\Delta M} \tag{8}$$

where $\Delta M \equiv M_L - M_S$ is positive and $\Delta\Gamma \equiv \Gamma_L - \Gamma_S$ is negative.

We now consider the possible addition to $h_{\alpha\beta}$ (6) of a modification of the form (2), which we parametrize as $\not{h}_{\alpha\beta}$. As discussed in ref. [6], we assume that the dominant violations of quantum mechanics conserve strangeness, in which case $\not{h}_{1\alpha} = 0$, and therefore that $\not{h}_{0\alpha} = 0$ to conserve probability. One can show that $\not{h}_{\alpha\beta}$ must be a negative matrix, and hence in turn that $\not{h}_{\alpha 1} = \not{h}_{\alpha 0} = 0$. Therefore we arrive at the general parametrization [6]

$$h_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix} \tag{9}$$

where the negativity of $\not{h}_{\alpha\beta}$ further imposes $\alpha,\ \gamma > 0$ and $\alpha\gamma > \beta^2$. The equations of motion for the components of ρ are

$$\begin{aligned}
\partial_t \rho_{11} &= -(\Gamma + Re\Gamma_{12})\rho_{11} - \gamma(\rho_{11} - \rho_{22}) - 2ImM_{12}Re\rho_{12} - (Im\Gamma_{12} + 2\beta)Im\rho_{12} \\
\partial_t \rho_{12} &= -(\Gamma - 2iReM_{12})\rho_{12} - 2i\alpha Im\rho_{12} + (ImM_{12} - \frac{1}{2}iIm\Gamma_{12} - i\beta)\rho_{11} \\
&- (ImM_{12} + \frac{1}{2}iIm\Gamma_{12} - i\beta)\rho_{22} \\
\partial_t \rho_{22} &= -(\Gamma - Re\Gamma_{12})\rho_{22} + \gamma(\rho_{11} - \rho_{22}) + 2ImM_{12}Re\rho_{12} \\
&- (Im\Gamma_{12} - 2\beta)Im\rho_{12}
\end{aligned} \tag{10}$$

and it is clearly possible in principle to determine independently the three parameters α, β, γ by measurements of the evolution of the density matrix over all times.

3 Constraining the Parameters that violate Quantum Mechanics

Since the time-evolution (10) is described by a 4×4 linear matrix equation, the general solution can be written in the form

$$\rho_\alpha(t) = \sum_{j=1}^{4} c_{\alpha j} exp(\lambda_j t) \tag{11}$$

where the coefficients $c_{\alpha j}$ depend on the initial conditions, e.g., tagged K_0 or $\overline{K_0}$ beam. However, it is clear that the large-time behaviour is dominated by the eigenvector whose eigenvalue λ_j has the least negative real part, corresponding to the conventional K_L component. On the other hand, the eigenvector whose eigenvalue has the most negative real part, corresponding to the conventional K_S component, can only be probed at short times. Interference effects at intermediate times can in principle probe the other two eigenvectors. The feasibility of this possibility depends crucially on the nature of the experiment, and we will not discuss such interference measurements here. Nor we will discuss measurements where the correlations between K_0 and $\overline{K}_0$ particles emanating from ϕ decay play a crucial way. We will concentrate on the information that can be obtained from measurements of individual kaons at large and small times.

It is easy to check that, for large t, ρ decays exponentially to [6]

$$\rho \propto \begin{pmatrix} 1 & \frac{-\frac{1}{2}i(Im\Gamma_{12}+2\beta)-ImM_{12}}{\frac{1}{2}\Delta\Gamma+i\Delta M} \\ \frac{\frac{1}{2}i(Im\Gamma_{12}+2\beta)-ImM_{12}}{\frac{1}{2}\Delta\Gamma-i\Delta M} & |\epsilon|^2+\frac{\gamma}{\Delta\Gamma}-\frac{4\beta ImM_{12}(\Delta M/\Delta\Gamma)+\beta^2}{\frac{1}{4}\Delta\Gamma^2+\Delta M^2} \end{pmatrix} \tag{12}$$

where the CP impurity parameter ϵ is given as usual by equation (8). The density matrix (12) describes a mixed state with $Tr\rho^2 < 1$ when ρ is normalized so that $Tr\rho = 1$. It corresponds to a *mixture* of a conventional K_L beam with a low-intensity K_S beam. Conversely, if we look for a solution of the time-evolution equation (10) with $\rho_{11} << \rho_{12} << \rho_{22}$, corresponding to what would conventionally be a K_S beam, we again find a mixed state:

$$\rho \propto \begin{pmatrix} |\epsilon|^2+\frac{\gamma}{|\Delta\Gamma|}-\frac{4\beta ImM_{12}(\Delta M/\Delta\Gamma)+\beta^2}{\frac{1}{4}\Delta\Gamma^2+\Delta M^2} & \epsilon-\frac{i\beta}{\frac{\Delta\Gamma}{2}-i\Delta M} \\ \epsilon^*+\frac{i\beta}{\frac{\Delta\Gamma}{2}+i\Delta M} & 1 \end{pmatrix} \tag{13}$$

We note that the signs of the terms in (13) that are linear in β, relative to those of ImM_{12} and $Im\Gamma_{12}$, are reversed with respect to the corresponding terms in the "K_L" density matrix (12).

The experimental value of an observable O is given in this formalism by

$$\langle O\rangle = Tr(O\rho) \tag{14}$$

as for a conventional mixed quantum-mechanical state. The $K\to 2\pi$ observable is represented in our $K_{1,2}$ basis by

$$O_{2\pi} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{15}$$

Therefore the rate of $K\to 2\pi$ decays is given in the long lifetime "K_L" limit by

$$|\epsilon|^2+\frac{\gamma}{\Delta\Gamma}-\frac{4\beta ImM_{12}(\Delta M/\Delta\Gamma)+\beta^2}{\frac{1}{4}\Delta\Gamma^2+\Delta M^2} \tag{16}$$

and is hence not a direct measurement of the CP-violating parameter ϵ. Previously [6], we discarded the β-dependent terms in (16), assuming that β was similar in magnitude to γ. Here we will be more general, keeping the term in (16) that is linear in β.

Other observables that are useful in constraining theories of CP violation and, in our case, looking for a deviation from quantum mechanics, are semileptonic $K^0/\overline{K}^0$ decays. The $K \to \pi^- l^+ \nu$ observable is

$$O_{\pi^- l^+ \nu} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{17}$$

whilst the $K \to \pi^+ l^- \overline{\nu}$ observable is

$$O_{\pi^+ l^- \overline{\nu}} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \tag{18}$$

Hence the CP-violating observable δ is given by

$$\delta \equiv \frac{\Gamma(\pi^- l^+ \nu) - \Gamma(\pi^+ l^- \overline{\nu})}{\Gamma(\pi^- l^+ \nu) + \Gamma(\pi^+ l^- \overline{\nu})} \tag{19}$$

in both the "K_L" and "K_S" limits (12) and (13). In the usual quantum-mechanical formalism, we would simply find

$$\delta \simeq 2Re\epsilon \tag{20}$$

where the phase ϕ_ϵ of ϵ is determined with high precision, from measurements of $\Delta\Gamma$ and ΔM. However, using equations (12, 13, 19) we find that

$$\delta_{L,S} \simeq 2Re[\epsilon(1 - \frac{i\beta}{ImM_{12}})], 2Re[\epsilon(1 + \frac{i\beta}{ImM_{12}})] \tag{21}$$

So far, there has not been any high-statistics measurement of δ_S. Checks of the standard phenomenology of CP violation have been made by combining measurements of δ_L and $K \to 2\pi$ decays in the long lifetime limit. In our case, comparing (16) and (21), we see that

$$(\frac{\delta^2}{4} - R_{2\pi} cos^2\phi_\epsilon) = -\frac{\gamma}{|\Delta\Gamma|} cos^2\phi_\epsilon - \frac{\beta}{|\Delta\Gamma|} 8|\epsilon| cos^2\phi_\epsilon sin\phi_\epsilon \tag{22}$$

where $R_{2\pi} \equiv$BR($K \to 2\pi$). Thus measurements of these two quantities cannot determine both β and γ, and the basic geometry of the problem is shown in the figure. Any discrepancy between the measurements of "ϵ" from $K \to 2\pi$ decays and of "$Re\ \epsilon$" from semileptonic K decays could be taken as evidence against conventional quantum mechanics, but its origin would be ambiguous. In fact, putting in the latest experimental values [19] $\sqrt{R_{2\pi}} \simeq |\epsilon| = (2.265 \pm 0.023) \times 10^{-3}$, $\phi_\epsilon = 43.73 \pm 0.15^o$, and $\delta = (3.27 \pm 0.12) \times 10^{-3}$, we find,

$$(-0.006 \pm 0.204) \times 10^{-6} = -0.522 \frac{\gamma}{\Delta\Gamma} - (6.54 \times 10^{-3}) \frac{\beta}{\Delta\Gamma} \tag{23}$$

and hence there is currently no evidence for a violation of quantum mechanics. If we ignore the contribution of β in (23), and use the experimental value $\Delta\Gamma = 737 \times 10^{-17} GeV$ [19], we find that

$$\gamma = (0.01 \pm 0.39) \times 10^{-6} |\Delta\Gamma| \simeq (0.1 \pm 3) \times 10^{-22} GeV \quad (24)$$

whereas we find

$$\beta = (0.01 \pm 0.31) \times 10^{-4} |\Delta\Gamma| \simeq (1 \pm 23) \times 10^{-20} GeV \quad (25)$$

if we ignore the contribution of γ in (23).

It is possible to disentangle the parameters β and γ by also measuring the semileptonic asymmetry in "K_S" decays. The geometrical rôle of this measurement is also shown in the figure. Taking the difference between the two asymmetries in (21), we find that

$$\delta_L - \delta_S = \frac{8\beta}{|\Delta\Gamma|} \frac{sin\phi_\epsilon}{\sqrt{1 + \sin^2 \phi_\epsilon}} = \frac{8\beta}{|\Delta\Gamma|} sin\phi_\epsilon cos\phi_\epsilon \quad (26)$$

Therefore a measurement of the difference between the semileptonic decay asymmetries in the long- and short-lifetime limits is directly sensitive to the parameter β that violates quantum mechanics. This measurement has also been mentioned previously as a way to look for a violation of CPT invariance [18]. This is hardly a coincidence, since it is known that a breakdown of quantum mechanics leads in general to a weakened form of the CPT theorem of conventional local field theory [20]. On the other hand, we have seen that another quantum-mechanics-violating parameter γ does not have the same CPT-violating signature.

As mentioned above, it is in principle possible to determine also the parameter α by measurements in the intermediate time region where other eigenvectors of the time-evolution matrix equation (11) play a rôle. Correlation measurements may also be interesting. However, we will not discuss these possibilities here.

4 Outlook

We have shown in this paper that the neutral kaon system is a uniquely precise and sensitive microscopic probe for possible violations of quantum mechanics. It is possible to set up a theoretically-motivated and well-defined parametrization of non-quantum-mechanical terms in the time-evolution equation for the density matrix of the neutral kaon system [6]. High-precision experiments already constrain these parameters (25), and have the exciting prospect of further constraining them in the future (26). The interpretation of the bound (25) would benefit from a theoretical estimate of the likely magnitude of non-Hamiltonian matrix elements that violate quantum mechanics. *A priori*, one might expect any such matrix elements in individual hadronic states to be suppressed by some power of m_{proton}/M_{Planck}, although we are not yet in a position to calculate them. As we have mentioned earlier, quantum coherence is maintained in string theory by virtue of symmetries linking light particles to massive states [13], and such apparently non-quantum-mechanical terms can arise when unmeasured observables associated with massive string states are summed over [15]. Therefore we consider it very important to take a phenomenological attitude, and analyze this possibility from a strictly experimental point of view. The present CP-LEAR and future DAϕNE experiments are well-placed to contribute to this programme, since they have the possibility to measure an asymmetry in semileptonic decays in the short life-time limit, as well as examine interference effects that can in principle unravel all the non-quantum-mechanical parameters.

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Figure Caption

The geometry of the tests of quantum mechanics proposed in this paper. The rate $R_{2\pi}$ of $K_L \to 2\pi$ decays is not just given by the magnitude $|\epsilon|$ of the CP-violating mass mixing parameter [see equation (16)] and the CP-violating $K_{S,L}$ leptonic decay asymmetries $\delta_{S,L}$ are not just 2Re ϵ [see equation (21)].