SU-ITP-93-14

Quasi-Fermi Distribution and Resonant Tunneling of Quasiparticles with Fractional Charges

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(May 18, 1993)

Abstract

We study the resonant tunneling of quasiparticles through an impurity between the edges of a Fractional Quantum Hall sample. We show that the one-particle momentum distribution of fractionally charged edge quasiparticles has a quasi-Fermi character. The density of states near the quasi-Fermi energy at zero temperature is singular due to the statistical interaction of quasiparticles. Another effect of this interaction is a new selection rule for the resonant tunneling of fractionally charged quasiparticles: the resonance is suppressed unless an integer number of *electrons* occupies the impurity. It allows a new explanation of the scaling behavior observed in the mesoscopic fluctuations of the conductivity in the FQHE.

The question regarding the extent to which the fractionally charged quasiparticles proposed by Laughlin [1] are real and whether they can be observed individually, was recently resolved by experiments [2]. In these experiments FQHE samples with constriction were studied, in order to observe essentially one-particle tunneling processes of the quasiparticles. In particular, the frequencies of the mesoscopic fluctuations of the longitudinal resistance in the FQHE with $\nu = 1/3$ were compared to those in the Integer QHE.

Previously, Jain and Kivelson [3] suggested that the the resonant tunneling of electrons from one edge to another through an impurity could cause an enhancement of the dissipative resistance in the IQHE samples with a narrow constriction. Kivelson and Pokrovsky [4] proposed an analogous mechanism for the fractionally charged quasiparticles in the FQHE. Their model implied simple scaling laws for the periods of the mesoscopic oscillations in the vicinity of the state with filling factor $\nu \equiv p/q$: $\Delta B \propto q$ at fixed gate voltage V_G, and $\Delta V_G \propto p$ at fixed magnetic field B. Both predictions have been confirmed experimentally [2].

Nevertheless, the theoretical understanding of this scaling can not be considered as satisfactory. In particular, Kivelson [5] derived quasiclassical quantization rules for a multi-anyon bound state at the impurity allowing for the statistical interaction; his scaling relations are different from the observed ones. P. Lee [6] supported Kivelson's result from the position of the theory of edge quasiparticles. He accounted for the obvious discrepancy with the experiment with the Coulomb blockade.

Another problem in understanding these experiments is that the mechanism of resonant tunneling usually implicates the existence of a Fermi level for excitations. It clearly exists for the case of the IQHE but is much less obvious for the FQHE. Recently Haldane [7] defined the generalized Pauli principle for anyons. This principle, however, does not imply the existence of the distinct Fermi level required to explain the resonances in tunneling.

The purpose of this work is to elucidate these general questions and give a new explanation for the experimental result.

Let us start with the quasiclassical quantization rule derived by Kivelson. It reads

$$\Phi = m\phi_0^* + N\phi_0, \tag{1}$$

where Φ is the total magnetic flux through an area A surrounded by the trajectory of the quasiparticle, $\phi_0 = hc/e$ is the flux quantum for an electron, $\phi_0^* = q\phi_0$ is the flux quantum for a quasiparticle (anyon) with charge $e^* = e/q$, N is the number of quasiparticles captured by the impurity, and m is the angular momentum of the tunneling quasiparticle. The first term in the r.h.s. of equation (1) is required by the gauge invariance, while the second one simply shows that each quasiparticle is bound with one flux quantum. The same quantum spectrum arises in the exact solution for a system of N anyons in a quadratic potential [8].

At a given gate voltage V_G, the area A enclosed by the trajectory, corresponding to the Fermi level, is the same for any quantized value of Hall conductivity. This is true because the Laughlin liquid is incompressible. Therefore, the intervals of the magnetic field between consequent bound states of a quasiparticle are $\Delta B_q = \phi_0^*/A$ if the number of quasiparticles N is fixed. The scaling, consistent with these intervals, was observed experimentally.

However, during the tunneling the number of quasiparticles N coupled with the impurity changes by one. It corresponds to the change of the flux Φ by a single flux quantum ϕ_0 instead of $\phi_0^* = q\phi_0$. Corresponding periods $\Delta B_1 = \phi_0/A$ have *not* been observed experimentally.

The solution to this puzzle lies in the fractional statistics of quasiparticles. Consider the situation where N quasiparticles are initially bound to the impurity, and the tunneling quasiparticle arrives at an orbit enclosing all of them. In the quasiclassical approximation, the wave function of this quasiparticle will gain a phase factor $z = exp(i2\pi N/q)$ after each complete revolution around the quantized orbit. More accurately, it is multiplied by $z(1-\gamma/2)$, where γ is the total probability of tunneling from the impurity to either left or right edge. The total tunneling amplitude contains a series

$$t_{LR} = \sum_k z^k (1-\gamma/2)^k = \frac{1}{1 - z(1-\gamma/2)}. \tag{2}$$

Usually resonant enhancement of the tunneling happens when all the amplitudes, corresponding to different numbers of revolutions in (2) are coherent, *i.e.* $z = 1$. This is obviously the case for the usual Fermi quasiparticles ($\nu = 1$). For a fractional value of ν, the contributions of q consequent revolutions almost cancel each other. Thus the *resonant* tunneling is suppressed unless N/q is an integer. In other words, the tunneling of an anyon is resonantly enhanced only if an integer number of electrons are already bound to the impurity. This simple selection rule restores the scaling suggested in ref. [4] and agrees with experiment.

The scaling of the oscillation intervals on the gate voltage ΔV_G [4] is also easily reproduced. Indeed, at a fixed magnetic field B the change ΔV_G corresponding to a new resonance is determined by the change of the area

$$\Delta A = \Delta\Phi/B, \tag{3}$$

where $\Delta\Phi$ is the change of the flux through the trajectory. As we have already established, $\Delta\Phi = q\phi_0$ for the resonant tunneling at $\nu = p/q$. On the other hand, the value of the magnetic field B_ν, corresponding to the filling factor ν, is approximately $1/\nu$ times B_1. As a result we obtain $\Delta A_\nu = p\Delta A_1$ and $\Delta V_G^\nu = p\Delta V_G^1$

These intuitive and semi-classical arguments are supported by direct calculations in the framework of Wen's theory of edge excitations [9]. Simultaneously, we find the distribution of edge quasiparticles over momenta to confirm the conjecture of its Fermi-like character [4]. All calculations have been performed for special values of $\nu = 1/q$, where q is an odd integer.

In Wen's theory the operator creating a quasiparticle

$$\psi^\dagger(x,t) = :e^{i\phi(x,t)}: \tag{4}$$

at the point x of the edge is associated with the chiral Bose field $\phi(x,t)$ of an edge magnetoplasmon. This field obeys the commutation relationship

$$[\phi_{x,t}, \phi_{x',t}] = -i\pi\nu\,\mathrm{sign}(x - x') \tag{5}$$

and is related to the charge density $\rho = e/2\pi\,\partial\phi/\partial x$ at the edge. The permutation relations of the anyon operators (4) are

$$\psi^\dagger(x,t)\psi^\dagger(x',t) = e^{i\pi\nu\,\mathrm{sign}(x-x')}\psi^\dagger(x',t)\psi^\dagger(x,t). \tag{6}$$

To find the distribution of edge quasiparticles over momenta, it is necessary to calculate the Fourier-transformation $\tilde{G}_p$ of the simultaneous correlation function $G(x-x') = \langle\psi^\dagger(x,t)\psi(x',t)\rangle$. We have performed this calculation explicitly with the following result: $\tilde{G}_p$ can be represented as the product

$$\tilde{G}(p') = g_T(p')\frac{1}{\exp(\beta p' v)+1}, \tag{7}$$

where the momentum $p' = p - p_F$, p_F is a Fermi momentum, and v is the drift velocity along the edge for both the chiral field and the anyons. The second factor is the usual Fermi-distribution, while the first one can be treated as the temperature-dependent density of states; it is an even function of p'. At $T = 0$ the density of states $g_T(p')$ has a singularity $\propto |p'|^{\nu-1}$ and diverges at the Fermi-level. The singularity is smeared out at a finite temperature. Details of the calculations will be published elsewhere.

To investigate the tunneling, we modelled the impurity as a void in the incompressible quantum Hall liquid, its edge being an additional environment for the edge quasiparticles. The perimeter L_i of this edge was assumed to be small enough to neglect the probability of thermal excitation of states with non-zero angular momentum m. The outer edge, on the contrary, was assumed to be in the thermodynamical limit; it serves as a thermostat. The many-body quantum mechanical states at the impurity are well-defined in the limit of a small tunneling coupling

$$H_t = \int dxdyt(x,y)\psi^\dagger(x)\psi_i(y) + h.c., \tag{8}$$

where $\psi_i(y)$ is the annihilation operator for the edge quasiparticles at the impurity. This limit allows us to reduce the evolution equation of the density matrix at the impurity to a set of kinetic equations, describing the evolution of probabilities $W_N = \langle\hat{\mathcal{P}}_N\rangle$ to have exactly N quasiparticles at the impurity, where $\hat{\mathcal{P}}_N$ is the appropriate projection operator.

The transition amplitudes are connected with different equilibrium averages similar to $\langle\psi_i^\dagger(y)\psi_i(y')\hat{\mathcal{P}}_N\rangle$. In the generic case they are periodic only at the q-fold boundary [10], gaining the phase $2\pi N/q$ in each cycle. This phase, similar to the Berry phase, is the exact consequence of the fractional statistics and does not depend on the distribution of edge phonons. As usual, the broken symmetry leads to the selection rule for the allowed transitions; namely, transitions $N \longrightarrow N+1$ are suppressed unless N/q is integer. This statement coincides with our conclusion extracted from the semi-classical model. In contrast to the previous derivation, we made no assumptions about the geometrical properties of the orbit of the tunneling quasiparticle.

The factorization (7) yields the Boltzmann distribution for the probabilities of different many-particle states of the impurity in equilibrium. In the presence of the inter-edge potential difference, however, the impurity population depends on the tunneling probabilities. The current-voltage dependence (see Figure 1) is highly non-linear and asymmetric, especially in the vicinity of the resonance.

In conclusion, we have found new selection rules for the resonant tunneling of quasiparticles in the FQHE, arising from the broken symmetry specific to anyons. The equilibrium momentum distribution of the edge quasiparticles has quasi-Fermi properties with the temperature-dependent density of states. This explains the appearance of resonant tunneling effects in the anyonic systems and, in particular, the scaling properties of the mesoscopic pattern measured in the experiment [2].

Upon completion of this work, we received the preprint [11], where the RG equations for resonant inter-edge tunneling are solved numerically in a different geometry. The authors considered neither scaling properties of the resonant effects, nor the momentum distribution of quasiparticles. Their main emphasis was the line shape.

V. L. P. is indebted to Steve Kivelson for numerous discussions and to J. S. Langer and Institute of Theoretical Physics in Santa-Barbara for the hospitality extended to him at the initial stage of this work. L. P. P. wishes to thank the Soros Foundation for partial financial support under grant # S92.56, and Jared Levy for valuable comments on the manuscript.

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FIGURES

FIG. 1. Non-linear resonant tunneling current I versus the inter-edge voltage V expressed in the units of the temperature T at different values of the one-particle energy E of the bound state at the impurity.