

Ambient vibration based damage diagnosis using statistical modal filtering and genetic algorithm: A bridge case study

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Abstract. The authors recently developed a damage identification method which combines ambient vibration measurements and a Statistical Modal Filtering approach to predict the location and degree of damage. The method was then validated experimentally via ambient vibration tests conducted on full-scale reinforced concrete laboratory specimens. The main purpose of this paper is to demonstrate the feasibility of the identification method for a real bridge. An important challenge in this case is to overcome the absence of vibration measurements for the structure in its undamaged state which corresponds ideally to the reference state of the structure. The damage identification method is, therefore, modified to adapt it to the present situation where the intact state was not subjected to measurements. An additional refinement of the method consists of using a genetic algorithm to improve the computational efficiency of the damage localization method. This is particularly suited for a real case study where the number of damage parameters becomes significant. The damage diagnosis predictions suggest that the diagnosed bridge is damaged in four elements among a total of 168 elements with degrees of damage varying from 6% to 18%.

Keywords: Ambient vibration, damage detection, localization, quantification, modal filtering, residual, test statistics, experimental validation

1. Introduction

Current non-destructive testing methods for the monitoring and the diagnosis of structures, such as acoustic, ultrasonic, electromagnetic and radiographic methods, are very useful for the evaluation of the state of health of structures. but sometimes unsuited for continuous monitoring. They are considered as being local methods since they require detailed inspection of small parts of the structure and assume that the damaged zones are a priori known. The need for more global methods of damage diagnosis led to the development of dynamic evaluation methods based on vibration measurements. These techniques rely on the fact that damage modifies the rigidity, the mass or the property of energy dissipation which influence the dynamic response of the structure [1]. Such techniques enable to detect possible degradations which can be internal, invisible and localized anywhere in the structure.

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For civil engineering structures, ambient vibration tests are preferred over forced vibration ones because the artificial excitation of large structures having low natural frequencies is quite difficult and expensive. In the ambient vibration tests, operation disturbances can be avoided and the measured response is representative of the actual operating conditions of the structures which vibrate due to natural excitation. A number of methods rely on the identification of the modal parameters of the structure before and after damage, based on output only measurements. In these so-called modal based methods, damage is estimated according to several criteria which may include the change in frequencies [2,3], mode shapes [4,5], mode shape curvatures and Ritz vectors [6]. The main disadvantage of modal methods is the loss of information due to data compression resulting from the computation of modal parameters [1,2].

To overcome this disadvantage, time domain methods may constitute an alternative to modal methods. Time domain methods make use of vibration measurements before and after damage and evaluate appropriate residuals in terms of either direct measured data alone or combined with identified modal parameters. Residuals can be based on: (i) Auto-regressive moving-average vectors, auto-regressive vectors and auto-regressive with exogenous inputs models [7]; (ii) an appropriate distance between measurements before and after damage such as the Mahalanobis distance [8] and the mean strain energy measure which uses response data for a specified time period [9]; (iii) model-based state space observers where the design of the damage detection filter consists of finding feedback gains and a filter output gain [10]; (iv) a stochastic subspace-based identification method and a statistical local approach [11]; (v) a statistical modal filtering method [12].

In previous studies [12,13], the authors proposed a statistical modal filtering (SMF) method to estimate a new residual for damage identification (i.e., detection, localization and quantification) based on generalized likelihood ratio tests. Constraints were added to the tests in order to allow only physically feasible damage. The damage methodology was tested on simulation examples of simple structures and was then validated experimentally using ambient vibration tests performed on laboratory-scale reinforced concrete beams and slabs with artificially created damage [13].

In this paper, further enhancements to the damage identification method are proposed to enable its application on a real case study, namely the Béni-Khiar bridge, a four-span bridge located in the North-East of Tunisia. Such enhancements include dropping the requirement that the reference state of the structure be the undamaged state, in addition to the incorporation of a genetic algorithm to improve the damage localization capability for complex structures.

This paper is organized as follows. The enhanced damage identification method is presented in Section 2. Section 3 describes the case study and its finite element model. The ambient vibration tests and the modal identification results are summarized in Section 4. Damage identification results are presented and discussed in Section 5. Finally, concluding remarks are given in Section 6.

2. Enhanced damage identification method

2.1. Accounting for the absence of undamaged state measurements

Structural health monitoring consists of confronting the signal y_d measured at a current state with a reference signal y_0 . Both signals are measured at r points and contain vibration measurements made at N different instants of time. Vibration data can be measurements of accelerations, velocities or displacements according to the type sensors used. The state of the structure can be described in its finite element model by a set of physical and/or geometrical parameters which form a vector θ of dimension p . When $\theta = \theta_0$, the structure is at its reference state which, ideally and in the original version of the diagnosis algorithm, corresponds to an undamaged state. The state defined by r represents the current state of the structure. The monitoring of the system consists of detecting changes in the vector θ .

The absence of vibration measurements at the intact state considered as the reference state is one of the difficulties encountered in real case studies. The proposed damage identification method is enhanced by extending it to the present and frequently encountered situation where the intact state has not been subjected to measurements.

In the modal filtering approach, the residual generation is based on the estimation of the error between the measurements taken on the structure at the current state and their projections onto the incomplete modal basis of the

structure as computed from the finite element model. The expression of the residual $\xi_N(\theta_0)$, proposed in [12,13], is given by

$$\xi_N(\theta_0) = \sqrt{N} \dot{P}(\theta_0)^T (\hat{R}_d \otimes I_r) P(\theta_0) \tag{1}$$

where

$$P(\theta) = \text{vec}(\Gamma(\theta)) \tag{2}$$

$$\dot{P}(\theta) = \text{vec}(\nabla \Gamma(\theta)|_\theta) \tag{3}$$

$$\hat{R}_d = \frac{1}{N} \sum_{t=1}^N y_{t,d} y_{t,d}^T \tag{4}$$

in which $\Gamma(\theta) = I_r - \Phi(\theta)\Phi^+(\theta)$ is an $r \times r$ matrix, $\Phi(\theta)$ is the incomplete $r \times m$ modal matrix of the parameterized structure whose columns represent the m lowest dominant vibration modes and, with $r \geq m$, the $m \times r$ matrix $\Phi^+(\theta)$ is the pseudo-inverse of $\Phi(\theta)$. The matrices P and \dot{P} are, respectively, of size $r^2 \times 1$ and $r^2 \times p$. The $r \times r$ matrix \hat{R}_d corresponds to the current signal y_d which contains measurements $y_{t,d}$ at N different instants of time, the $r \times 1$ vector $y_{t,d}$ being the measurement vector at an instant t . I_r is the identity matrix of dimension $r \times r$. the symbol \otimes denotes the Kronecker tensor product; the operator $\text{vec}()$ is a column stacking operator, it reshapes the $(r \times r)$ matrix $\Gamma(\theta)$ to a vector $P(\theta)$, of dimension $r^2 \times 1$, and the $(r \times r) \times p$ array $\nabla \Gamma(\theta)|_\theta$ to a matrix $\dot{P}(\theta)$, of dimension $r^2 \times p$.

When the projection of the measurement vector in the current state is made on the mode shapes of the same state of the structure, the average of the corresponding residual $\xi_N(\theta_d)$ is approximately zero

$$E_{\theta_d}(\xi_N(\theta_d)) \approx 0, \tag{5}$$

where E_{θ_d} is the expectation operator when the measurement vector is y_d .

Assuming $\theta_d = \theta_0 + \frac{\eta}{\sqrt{N}}$, the vector $E_{\theta_d}(\xi_N(\theta_d))$ can be written as

$$E_{\theta_d}(\xi_N(\theta_d)) = E_{\theta_d} \left(\xi_N \left(\theta_0 + \frac{\eta}{\sqrt{N}} \right) \right) = E_{\theta_d}(\xi_N(\theta_0)) + \nabla E_{\theta_d}(\xi_N(\theta))|_{\theta=\theta_0} \frac{\eta}{\sqrt{N}}, \tag{6}$$

where η is the damage parameter vector of size p .

Thus, it results from Eqs (5) and (6) that

$$E_{\theta_d}(\xi_N(\theta_0)) = J \eta, \tag{7}$$

where J is the $p \times p$ sensitivity matrix which is given by the expression

$$J = -\frac{1}{\sqrt{N}} \nabla E_{\theta_d}(\xi_N(\theta))|_{\theta=\theta_0} = -[[P(\theta_0)^T [\hat{R}_d \otimes I_r]] \otimes I_p] \ddot{P}(\theta_0) - \dot{P}(\theta_0)^T [\hat{R}_d \otimes I_r] \dot{P}(\theta_0), \tag{8}$$

in which I_p is the $p \times p$ identity matrix and $\ddot{P}(\theta) = \nabla(\text{vec}(\dot{P}(\theta)^T))|_\theta$, \ddot{P} is of size $pr^2 \times p$.

Based on the central limit theorem and a local asymptotic approach [3], the residual $\xi_N(\theta_0)$, denoted ξ , is normally distributed with zero mean at the reference state and a non zero mean at the current, possibly damaged, state and with constant covariance. For low damage levels, the statistical test can, therefore, be formulated as follows:

$$\xi \mapsto \begin{cases} \mathcal{N}(0, \Sigma) & \text{under } H_0 \\ \mathcal{N}(J \eta, \Sigma) & \text{under } H_1 \end{cases} \tag{9}$$

where $H_0 : \eta = 0$ is the null hypothesis and $H_1 : \eta \neq 0$ is the alternative hypothesis which describe the states of absence and presence of damage, respectively, and $\Sigma(\theta_0)$ is the asymptotically determined covariance matrix of size $p \times p$.

The evaluation of the residual is formulated within a stochastic framework in order to detect, locate and quantify damage in structures, as indicated in [12,13] and combined with a genetic algorithm to solve the damage localization problem, as described in the following section.



Fig. 1. Photographic views of the Béni Khiar bridge.

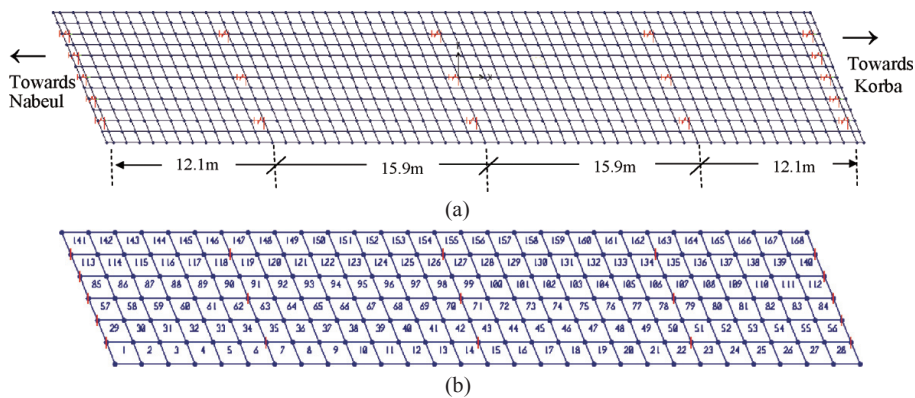


Fig. 2. The bridge deck model. (a) Two-dimensional finite element model of the tested deck; (b) Quadruplet numbering.

2.2. Damage localization using a genetic algorithm

The idea of damage localization is to perform as many sensitivity tests as the number of combinations of parameters, leading to a family of hypotheses pairs. Each pair represents a parameter combination to be studied and the results are interpreted based on the test values. Therefore, damage localization requires numerous analyses of structures modeled by a large number of parameters. To reduce computational time, we propose to use a genetic algorithm to solve the localization problem within the SMF method. This type of optimization methods is inspired from the mechanisms of evolution of species according to the processes of natural selection, namely: selection, crossover and mutation [14]. These methods are particularly well adapted to problems where the search space is irregular and contains many local maxima.

The problem consists of searching for the maximum value of sensitivity tests for all possible combinations of damage. This makes it possible to extract directly, among the p parameters, k parameters which are most relevant. The relevance is translated here by the value of sensitivity test. The k parameters to be determined represent the damaged elements.

Based on numerical simulation results, it can be noted that a crossover probability ranging between 0.7 and 0.9 allows a fast convergence of the algorithm. In addition to pointing towards local maxima, The mutation allows creation of desirable genes that are absent in the initial generation. The probability of mutation must however remain low to ensure the convergence of the genetic algorithm. The value adopted in our case study is fixed around 0.03, which belongs to the interval [0.01, 0.05] as recommended by [14].

3. Description and modeling of the case study

The bridge of Béni Khiar, constructed in the early 90's, is located between the Tunisian cities of Korba and Nabeul near the town of Béni Khiar. A photograph of the bridge is shown in Fig. 1. The deck of this case study is made of

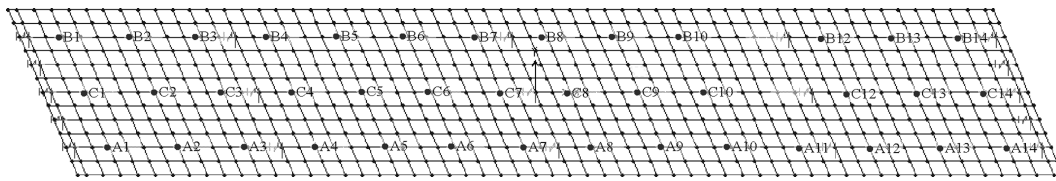


Fig. 3. Location of measurement points.



Fig. 4. Vibration sensors and data acquisition system.

a continuous reinforced concrete slab skewed at an angle of 22° . It consists of four spans of length 12.1, 15.9, 15.9 and 12.1 m, respectively. The slab thickness and width are 0.6 m and 10 m, respectively.

The finite element model of the bridge deck includes 696 shell elements and 767 nodes, which corresponds to 2301 degrees of freedom. As depicted in Fig. 2, the finite elements are grouped into 168 quadruplets of adjacent elements and the damage is assumed to be uniformly distributed within each quadruplet. It should be noted that, among the 696 shell elements, the 24 elements which are located outside of the support lines are assumed to be undamaged.

The elastic modulus at the reference state is estimated at $E = 32.2$ GPa based on the empirical expression $E = 11000 \sqrt[3]{f_{c28}}$ taken from the French code BAEL [15] where f_{c28} is the concrete compressive strength expressed in MPa. The Poisson's ratio and the density are estimated from the literature as $\nu = 0.2$ and $\rho = 2500$ kg/m³. The locations of the supports are marked in Fig. 2 by star symbols. The stiffness of the supports is evaluated as $K_{bearing} = 80$ GN/m based on vibration measurements performed below and above support bearings.

Since no vibration measurements have been taken on the bridge prior to the testing campaign, the reference state is defined by the ideal state represented by the finite element model of the undamaged structure. The damage parameters are defined as the bending stiffness EI of the quadruplets, where the inertia I is considered per unit width. The degree of damage will be expressed in terms of reduction in stiffness.

4. Ambient vibration tests and modal identification

Ambient vibration tests were conducted on the bridge using a sixteen-channel data acquisition system with nine force-balance uniaxial accelerometers. The sensors are capable of measuring accelerations within the range of ± 0.25 g with a resolution of $0.1 \mu\text{g}$. Signal conditioners were used to improve the quality of the signals by filtering undesired frequency contents. As shown in Fig. 3, vibration measurements were conducted at 40 selected points during a period of 10 minutes at a sampling frequency of 100 Hz with a low-pass filter of 40 Hz. In each point, two components of the acceleration vector are measured: the vertical component and the horizontal component which is perpendicular to the axis of the bridge. The vibration measurement equipment is depicted in Fig. 4 and a typical sample of acceleration record is shown in Fig. 5.

The program ARTeMIS Extractor [16] was used to identify the modal properties of the bridge, based on the Frequency Domain Decomposition technique as depicted in Fig. 6. The first four identified natural frequencies are 4.98, 6.54, 8.59 and 9.13 Hz. These frequencies represent the most dominant vibration modes; therefore they are retained for the identification task. The first four computed frequencies are: 5.04, 6.71, 8.85 and 9.51 Hz. Comparison of experimental and numerical frequencies indicates that the model input uncertainties, the modeling errors and the

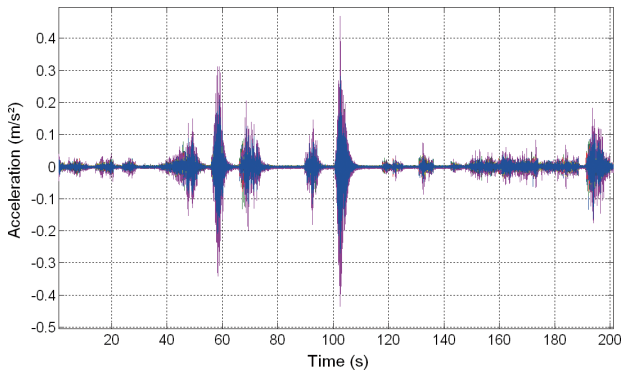


Fig. 5. Sample of an acceleration measurement under ambient excitation.

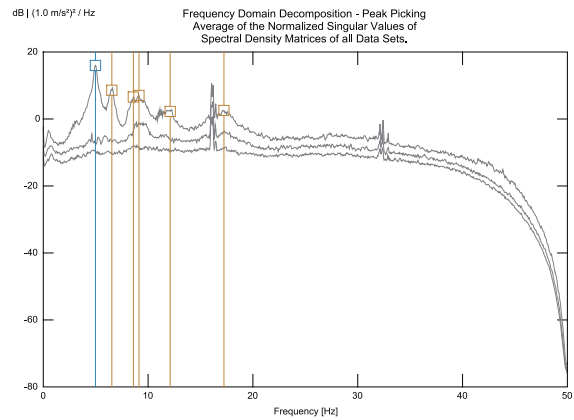


Fig. 6. Average of normalized singular values of spectral density matrices of all data sets using the FDD technique.

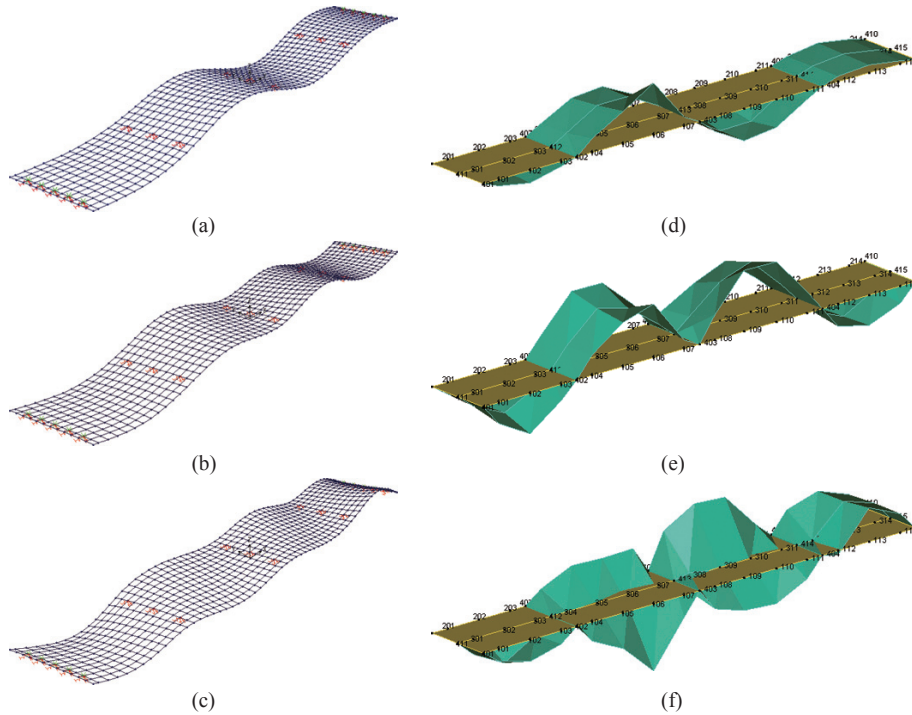


Fig. 7. Measurement-based mode shapes identified using the FDD technique and computed mode shapes. (a) First computed mode shape; (b) Second computed mode shape; (c) Third computed mode shape; (d) First measured mode shape; (e) Second measured mode shape; (f) Third measured mode shape.

possible damage amount to less than 4.2% error in the identification of frequencies. The measurement-based mode shapes and the computed mode shapes for the first, second and third modes are shown in Fig. 7. The values of the Modal Assurance Criterion MAC were estimated to predict the degree of matching between the experimental and computed mode shapes. The diagonal terms of the MAC matrix are 96.1, 94.4, 60.5 and 46.0% for the 1st, 2nd, 3rd and 4th mode, respectively. Thus, comparison of the first two mode shapes indicates that the finite element model reproduces reasonably well the dynamic properties of the structure. However, for the third and fourth modes, the correspondence between the experimental and computed vibration modes deteriorates.

Table 1
Damage quantification results using six datasets

Degree of damage (%)	Element	Number of dataset						Mean value	Standard deviation
		1	2	3	4	5	6		
	E 110	19.59	17.81	20.34	17.33	18.42	16.98	18.41	1.31
	E 118	5.46	4.31	7.94	5.91	7.59	5.12	6.06	1.43
	E 126	10.73	10.86	8.24	9.41	11.45	9.45	10.02	1.19
	E 163	14.12	16.02	9.68	8.83	10.21	9.99	11.47	2.89

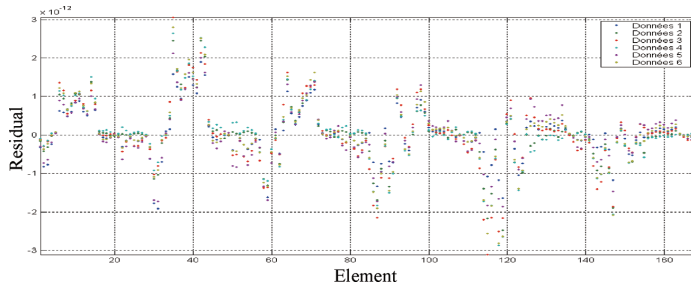


Fig. 8. Residual evaluated using the six datasets.

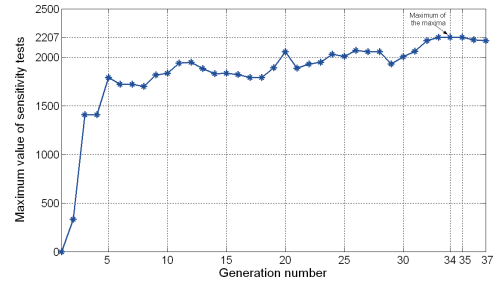


Fig. 9. Maximum value of sensitivity tests according to generations.

5. Results and discussion

Since the basis of the proposed SMF method is the estimation of residual vectors, Fig. 8 shows examples of residuals estimated from six datasets. It can be seen that the mean of the residual vectors is different from zero, which indicates that the structure is potentially damaged. This information remains insufficient, though, and subsequent statistical tests need to be applied to confirm the damage identification. The value of the global test estimated from the first dataset is equal to 2347. This value, confirmed by the other datasets, indicates that the bridge is damaged since it is much larger than the threshold value of 213.6 taken from χ^2 table for 168 degrees of freedom and a probability of false alarm $\alpha = 1\%$.

Before the application of the sensitivity tests for damage localization, it is interesting to group together elements which are attributed close test values. A vector quantization method is employed to determine the clusters of elements given that each hypothesis of simple damage can be represented by a change vector $V_i = \Sigma^{-1/2} J_i / (J_i^T \Sigma^{-1} J_i)^{1/2}$ [17,18]. In this case study, the 168 elements are grouped into 133 clusters of elements.

The search for damage locations by the genetic algorithm consists of determining the clusters which give the maximum value of sensitivity tests. The adopted algorithm employs a fixed size for the generations that is chosen equal to ten. The maximum number of generations is taken equal to fifty. The damage is assumed to be multiple and of maximum number k equal to six. Figure 9 illustrates the maximum value of sensitivity tests according to the generations using the first dataset. After 37 generations, the adopted algorithm reached the maximum of sensitivity tests. Therefore, to find the maximum value, the number of tests carried out was equal to 370. This number is about half the number of tests that would be required if a branch-and-bound strategy [12,19] were used. The element clusters corresponding to the maximum, and thus representing the damaged ones, are the clusters numbered 91, 79, 22 and 57. These clusters correspond to the elements E110, E126, E118, E106 and E163. This localization result is confirmed by using the other datasets.

The quantification of damage is carried out using the six datasets and The results are summarized in Table 1. The average values obtained in elements E106, E110, E118, E126, and E163 are, respectively, 0.00, 18.41, 6.06, 10.02 and 11.47%. The zero value determined in element E106 indicates that this element is intact. The degree of damage obtained in element E163 is the least accurate, with a standard deviation equal to 2.89. This is expected since the corresponding diagonal term in the Fisher information matrix is small indicating a large uncertainty. As depicted in Table 1, the coefficients of variation for the degrees of damage obtained in the other damaged elements are small, which indicates that the evaluation of damage for these elements was relatively accurate.

6. Conclusion

In this paper, our interest is concerned with damage diagnosis in a four-span reinforced concrete bridge using ambient vibration tests. These tests, are simpler to realize, cause no disturbance to the operation of the structures and provide rich information on their behavior. The identification method used is the statistical modal filtering approach, enhanced by a genetic algorithm in order to localize damage in a reasonable computational time and extended to allow the diagnosis when no measurements are made on the intact structure. The results of the damage diagnosis suggest that the bridge is damaged in elements E110, E118, E126 and E163 with the degrees of damage 18.41, 6.06, 10.02 and 11.47%, respectively. Having completed the global identification of damage including location and degree of damage, it is interesting to carry out an identification test by ultrasound in order to confirm these results and to refine the location of damage. This operation becomes relatively simple since it is sufficient to test well defined zones rather than the entire bridge. The obtained results can be taken into account in evaluating the residual capacity of the structure and studying possible repair scenarios.

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