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$N = 2$ SU Quiver with USP Ends or SU Ends with Antisymmetric Matter

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Abstract: We consider the four dimensional scale invariant $N = 2$ SU quiver gauge theories with $USp(2N)$ ends or $SU(2N)$ ends with antisymmetric matter representations. We argue that these theories are realized as six dimensional A_{2N-1} $(0, 2)$ theories compactified on spheres with punctures. With this realization, we can study various strongly coupled cusps in moduli space and find the S-dual theories. We find a class of isolated superconformal field theories with only odd dimensional operators $D(\phi) \geq 3$ and superconformal field theories with only even dimensional operators $D(\phi) \geq 4$.

Keywords: S-duality, M5 brane, Riemann surface.

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Contents

1. Introduction

Recently, Gaiotto [1] proposed a remarkable method to describe four dimensional scale invariant $N=2$ quiver gauge theories with bi-fundamental and fundamental fields. Four dimensional $N=2$ quiver gauge theories are defined as the compactification of six dimensional A_N $(0,2)$ theories on Riemann surfaces with punctures. The marginal couplings of the quiver gauge theory are determined as the moduli of punctured Riemann surfaces. The punctures are labeled by Young Tableaux and there is a correspondence between puncture type with the flavor symmetry of the four dimensional quiver gauge theory. The various S-dual frames were shown to correspond to different degeneration limits of this punctured Riemann surface. This generalized the previous observation of Argyres-Seiberg dualtiy [2] on the infinite strongly-coupled region of SU(3) gauge theory with six fundamental hypermultiplet matters: they found a S-dual theory in which a weakly coupled $SU(2)$ theory coupled with a isolated interacting E_6 superconformal field theory in which a $SU(2)$ subgroup is gauged. Gaiotto's construction generalized their result and is used to find a new family of isolated superconformal field theories with $SU(N)^3$ flavor symmetry [1].

$D=6$ is the maximal dimension in which we can formulate a superconformal field theory. The six dimensional $(0,2)$ superconformal field theory has the famous ADE classification. The compactification of six dimensional theory on a Riemann surface provides a lot of insights on four dimensional conformal field theory [3]. For instance, if we compactify A_{N-1} theory on a smooth torus, the $SL(2,Z)$ invariance of the four dimensional $N=4$ SU(N) gauge theory is directly related to the $SL(2,Z)$ modular group of the torus. Gaiotto's construction provides another six dimensional framework to understand S-dualities of four dimensional $N=2$ scale invariant theory. Here we allow codimension two defects [4] of this A_{N-1} theory. These defects have singularities at the punctures from which we can also read the flavor symmetries of four dimensional theory.

It is definitely interesting to extend this analysis to other four dimensional $N = 2$ scale invariant theories. It is the main aim of this note to extend this analysis to the $N = 2$ SU linear quiver gauge theories with $USp(2n)$ group on the end or with $SU(2n)$ group on the end with antisymmetric matter representation. These theories have a Type IIA brane construction involving two $O6$ orientifold planes. In type IIA theory, we have a $NS5 - D4$ system in the background of $O6$ planes and D6 branes. The $NS5 - D4$ system lifts to a single M5 brane wrapped on a smooth Riemann surface in an M theory background describing the M theory lift of O6 planes and D6 branes. The Riemann surface is identified with the Seiberg-Witten curve. We can rewrite the Seiberg-Witten curve in a way along with Gaiotto's construction on an ordinary SU quiver. It can be shown that these theories can be realized as the compactification of A_{k-1} theory on spheres with punctures. In particular, we confirm that SU quiver gauge theory with USp ends falls in the same duality web as the quiver gauge theory with pure SU nodes [1]. We also identify the dual quiver to the theory with SU ends with antisymmetric representations. We will study the infinite strongly coupled region of some theories and we can see the emergent weakly coupled node coupled to an isolated E_6 or E_7 superconformal theory [5, 6]. We also find an family of isolated superconformal field theories with only odd dimensional operators $D(\phi) \geq 3$ and superconformal field theories with even dimensional operators $D(\phi) \geq 4$.

This note is organized as follows: in section 2, we review Gaiotto's construction on A_k theory; In section 3, we describe the brane construction of our model and rewrite the Seiberg-Witten curve in a form which makes the description of compactification of the six dimensional $(0, 2)$ theory manifest; In section 4, we describe explicitly the six dimensional description of some specific examples. In section 5, we study the various degeneration limits of our theories. Finally, we give our conclusion.

2. Review of $(0, 2)$ A_{k-1} Theory on Punctured Riemann Surfaces

We consider a four dimensional $N = 2$ linear quiver gauge theory with a chain of SU groups

$$SU(n_1) \times SU(n_2) \times ... \times SU(n_{n-1}) \times SU(n_n), \tag{2.1}$$

and bifundamental hypermultiplets between the adjacent gauge groups and k_a extra fundamental hypermultiplets for $SU(n_a)$ to make the gauge couplings marginal. The marginality of gauge couplings imposes the constraints on the number of fundamentals:

$$k_a = (n_a - n_{a-1}) - (n_{a+1} - n_a), \tag{2.2}$$

we define $n_0 = 0, n_{n+1} = 0$. Since k_a is nonnegative, we have

$$n_1 < n_2 < ...n_r = ..n_l > n_{l+1}...n_n. \tag{2.3}$$

Let's first consider the left tail. Let us denote $N = n_r = ... = n_l$, so that the non-increasing number $(n_a - n_{a-1}), a \leq r$ satisfies the relation $\sum_{a=1}^{a=r}(n_a - n_{a-1}) = N$. For the right tail, the non-decreasing number $n_a - n_{a+1}$ starting from $a = n$ also satisfies the relation $\sum_{a=l}^{a=n}(n_a - n_{a+1}) = N$. So we associate a Young Tableaux with total boxes N for each tail (see Figure

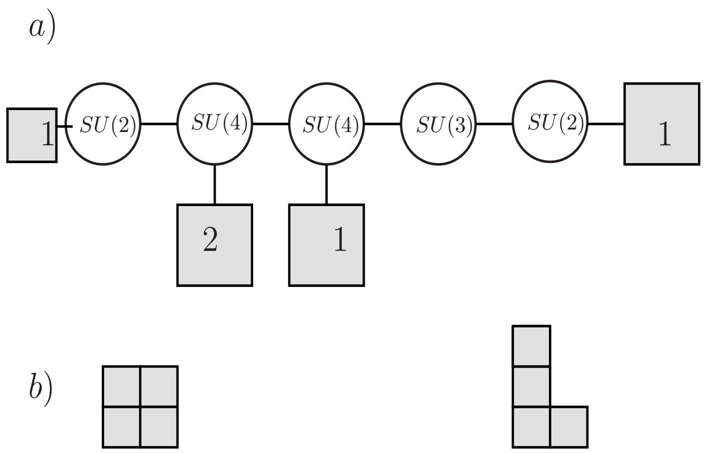

Figure 1: a) A $N = 2$ linear quiver with $N = 4$; b) The Young Tableaux associated with left and right tail.

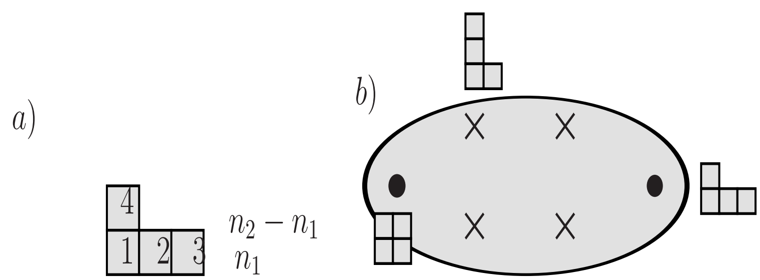

Figure 2: a) Young Tableaux associated with the tail in a linear quiver gauge theory with $N = 4$, $p_1 = 1 - 1 = 0$, $p_2 = 2 - 1 = 1$, $p_3 = 3 - 1 = 2$, $p_4 = 4 - 2 = 2$, the flavor symmetry is $SU(2)$; b) The punctured sphere for $(0, 2)$ A_3 theory compactification, each puncture is labeled by a Young-tableaux

1a). The flavor symmetry of this linear quiver is $U(1)^{n+1} \times \sum_{a=1}^{r} SU(k_a) \times \sum_{a=l}^{n} SU(k_a)$, which can be read explicitly from the quiver diagram.

The Seiberg-Witten curve for this theory has been solved in [7]. The curve is rewritten in the following form [1]:

$$t^N + \sum_{i=2}^{N} \phi_i(x) t^{N-i} = 0, \tag{2.4}$$

where x is the coordinate on a sphere; and the Seiberg-Witten differential is simply $\lambda = tdx$.

$\phi_i(x)dx^i$ are degree i differentials on the sphere with poles at $n + 3$ punctures, say with $x = 0, \infty$ and $x_1 ... x_{n+1}$. The poles at $x_1 ... x_{n+1}$ are of order $p_i = 1$, and are called basic punctures; the pole at $t = 0$ has order $p_i = i - s$, where s is the height of the Young Tableaux we have just constructed. The pole at $t = \infty$ can be similarly determined from the Young Tableaux associated to the other tail.

This motivates the following six dimensional description of these four dimensional $N = 2$ superconformal field theories: the linear theory is realized as a six dimensional A_{N-1} theory compactified on a sphere with $n+3$ punctures, and the punctures are labeled by the Young Tableaux (see Figure 1b). There are also defects at the punctures. Recall that six dimensional A_{N-1} theory has operators of dimension $2, 3, ..., N$. Compactification on Riemann surface involves the ordinary twisting to preserve supersymmetry, this twisting turns the dimension i operators into a degree i meromorphic differential $\phi_i dx^i$ on Riemann surface.

The orders of the poles are determined from the Young Tableaux by using the formula $p_i = i - s$, where i is the label of the ith box and s is the height of ith box in the Young tableaux (see Figure 2a). The dimension of the space of these meromorphic differentials is given by

$$dimension\ of\ \phi_i = \sum_{punctures\ d=1}^{n+3} p_d^{(i)} + 1 - 2i \tag{2.5}$$

The parameters of these differential are identified with dimension i operators of the four dimensional theory, i.e. the parameters for the Coulomb branch. The Seiberg-Witten curves describing the low energy effective theory of these models are also expressed in terms of these operators:

$$t^N + \sum_{i=2}^{N} \phi_i(x) t^{N-i} = 0 \tag{2.6}$$

The gauge coupling constants are identified with the moduli of the punctured sphere M. The moduli space for a sphere with $n+3$ punctures is n dimensional which is identified with the n coupling constants of our linear quiver. The duality group is $\pi_1(M)$.

We can also determine the flavor symmetry from the punctures. This can be determined by studying the mass-deformed theory. The Seiberg-Witten curve for mass-deformed theory has the additional term $\phi_1 t^{N-1}$; we can do a linear transformation on t to eliminate this linear term. Additionally, the Seiberg Witten curve is changed to $\lambda = t' dx$, where t' is the new variable. The change of Seiberg Witten curve only redefines the mass parameters. The mass parameters are now identified as the residue of this new Seiberg-Witten differential at the puncture. The pattern of the residue is in one-to-one correspondence with the Young Tableaux of this puncture. For each column of Young Tableaux with height l_h, there are l_h same residue for Seiberg Witten differential. The flavor symmetry of this puncture is

$$S(\prod_{l_h>0} U(l_h)). \tag{2.7}$$

(See Figure 2a for an example). One can check that this characterization of flavor symmetry matches that read from the linear quiver gauge theory. If the Young Tableaux of a puncture has only one row and the flavor symmetry is then $SU(N)$, we call the puncture a full puncture; if the Young Tableaux has two columns, one of them has $N - 1$ boxes, the other has one box, and the flavor symmetry is $U(1)$, then we call this a basic puncture. The punctures associated with the tails are called generic punctures.

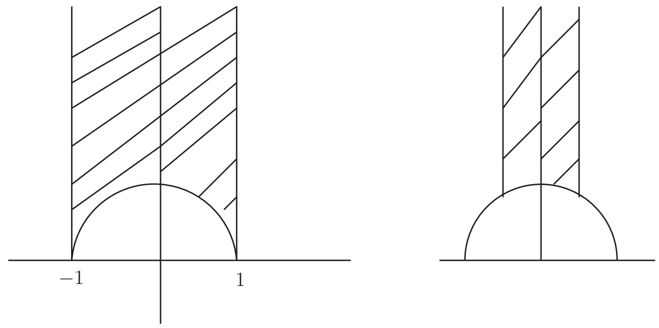

Figure 3: a) The fundamental domain of $\frac{H}{\Gamma(2)}$. Here H is the upper half plane, $\Gamma(2)$ is the duality group of a sphere with four punctures; b)The fundamental domain of $H/SL(2,z)$.

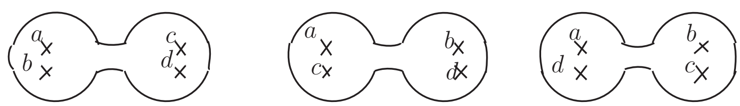

Figure 4: The various weakly coupled limit of SU(2) theory with four fundamental matter. The narrow strip denotes the weakly coupled SU(2) gauge group. The punctures are associated with flavor symmetry $SU(2)$.

This then gives a complete description of four dimensional $N = 2$ quivers from six dimensional point of view. One of the great virtues of this description is that we can derive various weakly coupled descriptions of these superconformal field theories by studying various degeneration limits of the punctured sphere.

We first study the simplest case of four dimensional scale invariant SU(2) theory with four fundamental matter hypermultiplets. The full flavor symmetry is $SO(8)$, we can write it in a form with only manifest $SU(2)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d$ flavor symmetry. This description will make the six dimensional interpretation manifest.

It is shown in [8] that the duality group of this theory is $SL(2,z)$. The duality group is the combination of $\Gamma(2)$ (the symmetry group of a sphere with four punctures) and the triality of $SO(8)$ flavor symmetry. The triality of $SO(8)$ flavor symmetry permutates four manifest $SU(2)$ flavor groups.

Before we use the triality symmetry, let us consider the moduli space which is shown in Figure 3a). The six dimensional description is shown in Figure 4. We have three different weakly coupled descriptions as the different degeneration limit of the punctured sphere. These three weakly coupled descriptions correspond to the three cusps $(1,0), (-1,0), \infty$ in the moduli space in Figure 3a). After using the permutation symmetry, the four punctures are identical and the duality group is enhanced to $SL(2,z)$. The moduli space becomes $H/SL(2,z)$, which is shown in Figure 3b). The three weakly coupled descriptions are identical and we have only one weakly coupled description, which corresponds to the only cusp ∞ in the new moduli space.

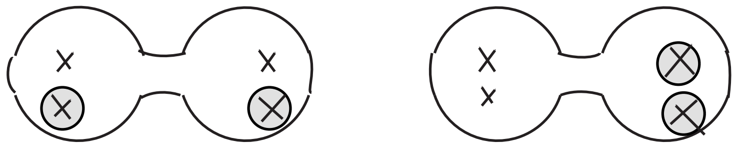

Figure 5: a) The weakly coupled SU(3) description, here the cross denotes $U(1)$ puncture and circle cross denotes $SU(3)$ puncture; b) Description with weakly coupled SU(2) gauge group

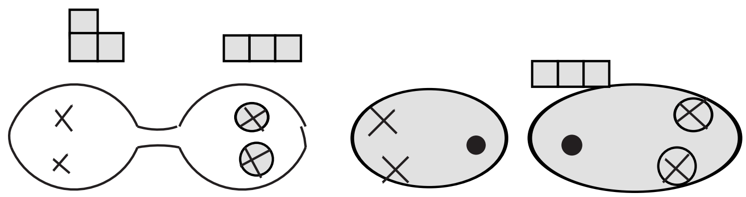

Figure 6: The collision of two basic puncture create a new puncture when we turn off the weakly coupled gauge group, we also draw the Young Tableaux associated with this new created puncture, in our particular case, the flavor symmetry is $SU(3)$.

Next we study the $SU(3)$ theory with six fundamentals. This is the case considered by Argyres and Seiberg [2]. The moduli space of this theory is depicted in Figure 3a). The manifest flavor symmetry is $U(1)^2 \times SU(3)^2$ in our description. The two different degeneration limits of six dimensional description are depicted in Figure 5. Figure 5a) is ordinary description in which the $SU(3)$ gauge coupling can be made arbitrarily weak. The description shown in Figure 5b) is rather surprising, since this description corresponds to the infinitely strongly coupled region of the description Figure 5a). However, it is quite natural from the six dimensional point of view, it is just one degeneration limit of the punctured Riemann surface. This gives a complete picture of various weakly coupled corner in the moduli space, since these are the only cusps in the moduli space shown in Figure 3a)(two of the cusps are identical).

Actually, we can determine which gauge group is becoming weakly coupled and what kind of puncture is left when we completely turn off this gauge group. When a basic puncture collides with a generic puncture, then the gauge group becoming arbitrarily weak is $SU(n_1)$, where n_1 is the first row of the Young Tableaux associated with the generic puncture. When we completely turn off this weakly coupled gauge group, we leave a puncture with a Young Tableaux whose first row has n_2 boxes, while the other rows are not changed. In our particular case, the generic puncture is a simple puncture, with $n_1 = 2$, $n_2 = 3$, after decoupling, we create a new puncture with only one row $n_2 = 3$. The flavor symmetry is $SU(3)$ according our rules. This is shown in Figure 6. The theory associated with three punctures with $SU(3)$ gauge group is an interacting superconformal field theory. For a sphere with three punctures, there is no moduli so this theory is an isolated fixed

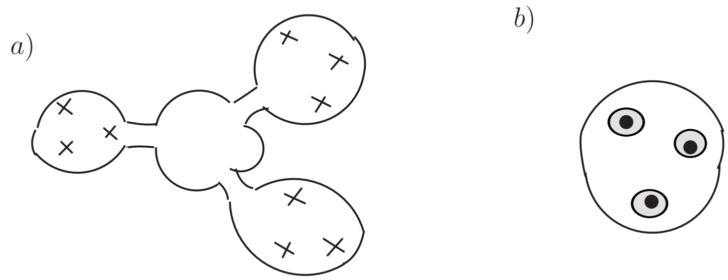

Figure 7: a) The collision of three groups of $N-1$ basic punctures; b) T_N theory with three full punctures with $SU(N)$ flavor symmetry

point. As we described earlier, the Seiberg Witten curve is

$$t^3+\frac{U}{(x-1)^2x^2}=0 \tag{2.8}$$

U is the dimension 3 operator of this theory. The manifest flavor symmetry is $SU(3)^3$, which is enhanced to E_6. This is the famous $N=2$ E_6 superconformal field theory. The S-dual theory of $SU(3)$ theory is recovered easily in this way. It is a $SU(2)$ theory with one fundamental and coupled with E_6 superconformal field theory, and a $SU(2)$ subgroup of E_6 is gauged and identified with the weakly coupled $SU(2)$ gauge group. The number of fundamentals can be either determined from the three punctured sphere, or from the counting of the conformal anomaly of the $SU(2)$ theory: the $SU(3)$ puncture provides the conformal anomaly equal to three fundamental of $SU(2)$, so we need one extra fundamental to compensate the conformal anomaly.

Similarly, we can also construct isolated superconformal theories with flavor symmetry $SU(N)^3$ by considering the degeneration limit of superconformal quiver $SU(2)\times SU(3)...SU(N)^{N-2}\times SU(N-1)...SU(2)$[1]. We have a total of $3N-3$ basic punctures on the sphere. It is easy to deduce that when $N-1$ basic punctures collide (we can collide basic punctures step by step), a $SU(N-1)$ gauge group becomes weakly coupled and if we turn off this gauge coupling we are left a $SU(N)$ puncture. In the core of the degenerated sphere, we have a theory with three $SU(N)$ punctures. See Figure 7. The supergravity dual of this T_N theory is found in [9].

We have known how a basic puncture collide with a generic puncture. It is interesting to consider the collision of two generic punctures. It is shown in [1] that when two generic $SU(2)$ punctures associated with a Young Tableaux with two columns of equal heights collide, an USp gauge group becomes weakly coupled. We will check in this paper that this conjecture is correct. We will also consider the collision of a special $U(1)$ puncture and a $SU(2)$ puncture.

The above construction can be generalized to four dimensional SU quiver with loops [1]. It can also be generalized to D_N theory [10] and there is also brane web construction

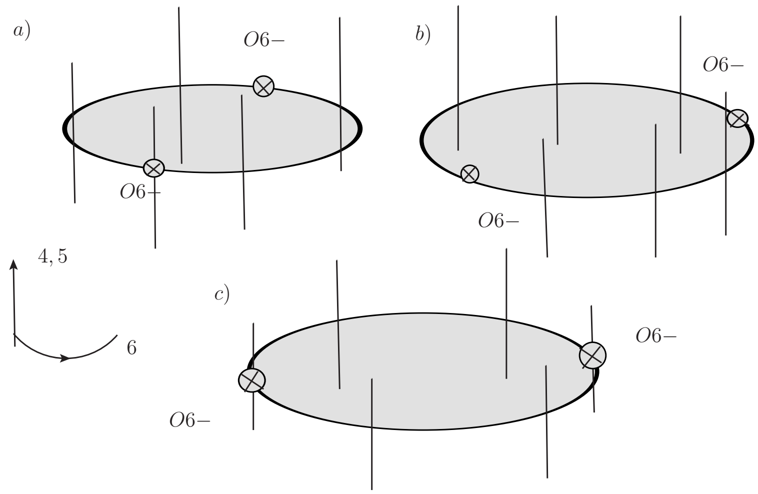

Figure 8: The tree families of brane configurations in the background of two negatively charged O6-planes. The short vertical lines represent the NS branes, the crossed circles are the orientifold planes. The D6 branes is put in between the NS branes, we omit them in the picture.

in [11]. There is an interesting relation between four dimensional $N = 2$ gauge theories and liouville correlation function [12].

3. $N = 2$ SU Quiver with USp Ends or SU Ends with Antisymmetric matter

Four dimensional $N = 2$ superconformal SU field theory with USp ends or SU ends with antisymmetric representations can be derived by adding orientifold six planes to Type IIA D4-NS5 brane system [13]. The solution of the model [14] can be found after lifting the above brane configuration to M theory along the similar line as in [7]

We first consider D4 and NS5 branes system in type IIA theory; We also include two orientifold six planes and 8 D6 branes so that the net RR charges cancels. The k four branes lie along the directions x_0, x_1, x_2, x_3, x_6; we take x^6 coordinate compact. The NS5 branes lie along $x^0, x^1, x^2, x^3, x^4, x^5$ directions. The orientifold six planes extend along $x_0, x_1, x_2, x_3, x_7, x_8 x_9$ directions. It corresponds to the space time transformation

$$h : (x_4, x_5, x_6) \to (-x_4, -x_5, -x_6), \tag{3.1}$$

together with the world sheet parity Ω and $(-1)^{F_L}$. The D6 branes are parallel to $O6$ planes.

There are three main families of $N = 2$ quiver gauge theory with these brane configurations, depending on the positions of the NS branes:

i) The number of NS branes is odd, $N = 2r + 1$. Only one NS5 brane intersect with the orientifold plane. One typical brane configuration is depicted in Figure 8a. The quiver gauge theory is

$$USp(k) \times SU(k)^{r-1} \times SU(k), \tag{3.2}$$

k must be even since for USp group the rank must be even. We have the bifundamental matter fields between the adjacent group. Two fundamentals are attached at the USp node and we have two fundamentals and one antisymmetric hypermultiplet at the last $SU(k)$ node. The flavor symmetry is $SO(4) \times U(1)^r \times SU(2) \times U(1)$. The $SO(4)$ flavor symmetry is from the two fundamentals of USp node, and the last $SU(2) \times U(1)$ is from the two fundamentals of the SU ends.

Note that the antisymmetric representation of $SU(k)$ is real, so the flavor symmetry of this representation is $Usp(2) = SU(2)$. In this paper, however, we do not consider the mass deformation of antisymmetric matter, so we do not include the flavor symmetry associated with it. Use the isomorphism $SO(4) = SU(2) \times SU(2)$, the total flavor symmetry is $SU(2) \times SU(2) \times U(1)^r \times SU(2) \times U(1)$.

ii)The number of NS branes is even. $N = 2r$, and there are no NS branes intersecting the O6-planes. One example is shown in Figure 8b. The quiver gauge theory is

$$USp(k) \times SU(k)^{r-1} \times USp(k). \tag{3.3}$$

We have the bifundamentals between the adjacent group and two fundamentals at the first and last USp gauge factor. The flavor symmetry is $SU(2) \times SU(2) \times U(1)^r \times SU(2) \times SU(2)$.

iii)The number of NS branes is even $N = 2r$. There are two NS branes intersecting with the O6-planes. One configuration is shown in Figure 1c. The quiver gauge theory is

$$SU(k) \times SU(k)^{r-1} \times SU(k) \tag{3.4}$$

Besides the bifundamental matters, we have two fundamentals and one antisymmetric at the first and the last SU factor. The flavor symmetry is $U(1) \times SU(2) \times U(1)^r \times SU(2) \times U(1)$.

When $r = 0$, the above theories are degenerate as

i) $USp(k)$ with a traceless-antisymmetric and 4 fundamentals.

ii) Also a $USp(k)$ with traceless-antisymmetric and 4 fundamentals, this is only for the massless antisymmetric matter, the mass deformation for this matter is not allowed.

iii) $SU(k)$ with 2 antisymmetric hypermultiplets and 4 fundamentals.

The Seiberg-Witten curves for those theories are derived by lifting the Type IIA configuration to M theory [14]. Here we briefly review the derivation. The NS5-D4 brane configuration is lifted to a single M5 brane wrapped on a Riemann surface in $O6 - D6$ background. In lifting to M theory, we grow a circular dimension x_{10} with radius R. Define the variables

$$v = x^4 + ix^5, \quad s = (x^{10} + ix^6)/(2\pi R) \tag{3.5}$$

Before orbifolding, the background space is $\tilde{Q} = C \times T^2$. The Z_2 identification of the orientifold is $(v, s) \simeq (-v, -s)$. The M theory background is therefore the orbifold space $Q = \tilde{Q}/Z_2$.

We only need the complex structure of this orbifold background. To do this, we first write an algebraic equation of torus. The torus can be written as an complex curve in the weighted projective space $CP^2_{(1,1,2)}$. $CP^2_{(1,1,2)}$ is defined as the space $(w, x, y)/(0, 0, 0)$ modulo the identification

$$(\lambda\omega, \lambda x, \lambda^2\eta) \simeq (\omega, x, y), \quad \lambda \in C^*. \tag{3.6}$$

The torus is represented as

$$\eta^2 = \prod_{i=1}^{4}(x - e_i\omega), \tag{3.7}$$

where the numbers e_i encode the complex structure τ of the torus in usual way.

The Z_2 automorphism of the torus is $\eta \to -\eta$ with ω and x fixed. The Z_2 identification of the orientifold background becomes $(v, \omega, x, \eta) \simeq (-v, \omega, x - \eta)$. The fixed points are

$$(0, 1, e_i, 0) \qquad i = 1, 2, 3, 4, \tag{3.8}$$

we write it in $\omega = 1$ patch.

Let us define Z_2 invariant variables

$$y \equiv \eta v, \qquad z = v^2, \tag{3.9}$$

the orbifolded background Q (without mass deformation for the fundamental matter) is

$$y^2 = z\prod_{i=1}^{4}(x - e_i\omega). \tag{3.10}$$

In the following, we write all the formulas in the patch $\omega = 1$, so the orbifold equation is

$$y^2 = z\prod_{i=1}^{4}(x - e_i). \tag{3.11}$$

The mass deformed (which corresponds to mass deformation to four fundamental matters induced by D6 branes) background is

$$y^2 = z\prod_{i=1}^{4}(x - e_i) + Q(x) \tag{3.12}$$

and

$$Q(x) = \sum_{j=1}^{4}\mu_j^2\prod_{k\neq j}[(x - e_k)(e_j - e_k)] \tag{3.13}$$

The Seiberg Witten curve for those field theories is a Riemann surface embedded into above background. We can first write the Seiberg Witten curve for the brane configuration before orbifolding, which is just the elliptic model in [7], and then require the curve invariant under the Z_2 transformation. For the elliptic model, the bifundamental masses satisfy the relation $\sum_\alpha m_\alpha = 0$, so to get the most generic mass-deformed theory, the background is not simply $C \times T_2$ but an affine model. There is no such problem for our model; before orbifolding, the relation $\sum_\alpha m_\alpha$ still applies, however, after orbifolding, the bi-fundamental masses are all independent(the orbifold images of D4 branes have opposite v coordinates, so the bi-fundamental mass for two images are opposite). We do not need to change the background to an affine bundle to allow most generic mass deformation for the bifundamental matters. The situation is different if we want to turn on mass deformation for anti-symmetric matter, the background is an affine bundle. We will not discuss this complication in this paper.

The Seiberg-Witten curve of the above quiver gauge theories without mass deformation is

$$z^n + A(z) + \sum_{s=1}^{r} \frac{B_s(z) + yC_s(z)}{x - x_s} + \sum_{p=1}^{q} \frac{yD_p(z)}{x - e_p} = 0, \quad k = 2n, \tag{3.14}$$

here x_s are positions of NS5 branes which don't intersect with the orientifold; q is the number of NS branes which intersect with the orientifold planes and e_p are positions of $NS5$ branes stuck at orientifold. This is natural since e_p are fixed points under the orbifold action. $A(z)$ and $B_s(z)$ are polynomials in z

$$A(z) = \sum_{l=1}^{n} A_l z^{n-l}, \quad B_s(z) = \sum_{l=1}^{n} B_{sl} z^{n-l}, \tag{3.15}$$

and C_s and D_p are polynomials in z

$$C_s(z) = \sum_{l=2}^{n} C_{sl} z^{n-l} \quad D_p(z) = \sum_{l=2}^{n} D_{pl} z^{n-l}. \tag{3.16}$$

We also have the constraint:

$$\sum_{s=1}^{r} C_s(z) + \sum_{p=1}^{q} D_p(z) = 0. \tag{3.17}$$

This curve can be derived by first write the Seiberg-Witten curve of elliptic model, and then impose the orbifold invariance and finally express it in terms of orbifold invariant variable. The mass-deformed Seiberg-Witten curve is

$$z^n + A(z) + \sum_{s=1}^{r} \frac{B_s(z) + yC_s(z)}{x - x_s} + \sum_{p=1}^{q} \frac{(y - y_p)D_p(z)}{x - e_p} = 0 \tag{3.18}$$

where $y_p = \sqrt{Q(e_p)}$. $A(z), B(z), C(z), D(z)$ are polynomials in z of order $n-1$.

The Seiberg-Witten differential is given by

$$\lambda = \frac{ydx}{\prod_{i=1}^{4}(x - e_i)}. \tag{3.19}$$

We will rewrite the above curve in a form along the way in [1]. Let's first consider case ii) with two USp ends, which corresponds to $q = 0$. We rewrite the Seiberg-Witten curve in a form which makes the interpretation with the A_{2n-1} theory compactification on a punctured sphere manifest. Expanding the Seiberg-Witten curve in terms of polynomial of z, we have

$$z^n + \sum_{l=1}^{n} \frac{p_r^l(x)}{\Delta'} z^{n-l} + \sum_{l=2}^{n} \frac{yp_{(r-2)}^l(x)}{\Delta'} z^{n-l} = 0, \tag{3.20}$$

here $\Delta' = (x-x_1)....(x-x_r)$ and $p_r^l(x)$ are polynomials with order r; $p^l_{(r-2)}$ are $r-2$ order polynomials. Define $z = \prod_{i=1}^4 (x-e_i)t^2$, then

$$y = t\prod_{i=1}^{4}(x-e_i) \tag{3.21}$$

The Seiberg-Witten differential becomes

$$\lambda = tdx \tag{3.22}$$

and the Seiberg-Witten curve is

$$t^{2n} + \sum_{l=1}^{n} \frac{p_r^l(x)}{\Delta' \prod_{i=1}^4 (x-e_i)^l} t^{2n-2l} + \sum_{l=2}^{n} \frac{p_{r-2}^l(x)}{\Delta' \prod_{i=1}^4 (x-e_i)^{l-1}} t^{2n-2l+1} = 0 \tag{3.23}$$

With this form, we conclude that this theory can be realized as the six dimensional A_{2n-1} theory compactified on a sphere with r basic punctures $x_1, ...x_r$ (see Figure 9a) for the Young Tableaux and 4 generic punctures $e_i, i = 1, ..4$ with Young Tableaux in Figure 9b). The defects at the punctures are:

$$\phi_{2l} = \frac{p_r^l(x)}{\Delta' \prod_{i=1}^4 (x-e_i)^l} dx^{2l}, \quad \phi_{2l-1} = \frac{p_{r-2}^l(x)}{\Delta' \prod_{i=1}^4 (x-e_i)^{l-1}} dx^{(2l-1)}. \tag{3.24}$$

To clarify one point, x is a coordinate on C, and since we do not put any singularity at ∞, we can add a point at ∞ to C and compactify it to a sphere. This does not change the Seiberg-Witten differential and other properties of our model.

Several checks can be made about this conclusion:

a)The moduli space of the sphere with $r+4$ punctures has dimension $r+1$ which can be identified with the coupling constant of gauge groups in the quiver.

b)The various differentials have pole $p_i = 1$ at punctures at x_s , which can be associated with the flavor symmetry $U(1)$, where the Young Tableaux is shown in Figure 9a). The punctures e_i has order $p_l = \frac{l}{2}$ when l is even, $p_l = \frac{l-1}{2}$ when l is odd. This puncture can be represented as a Young Tableaux with two columns of height n in Figure 9b). These poles correspond to $SU(2)$ flavor symmetry. The total flavor symmetry is then $SU(2)^4 \times U(1)^r$, which matches the flavor symmetries read from the quiver diagram.

c)For the differential ϕ_{2l}, the dimension is $4l + r - 2(2l) + 1 = r + 1$, which matches the dimension of the polynomials p_r^l. For differential ϕ_{2l-1}, the dimension is $r-1$, which also matches the parameters needed for the polynomial p_{r-2}^l.

d)When the mass deformation is turned on, we have the t^{2n-1} term. Do a linear transformation on $t = t' + \alpha$ to eliminate this term. And keep the Seiberg-Witten differential as $\lambda = t'dx$. One can check the residue of the punctures x_s and e_p have the same patter as determined by the Young Tableaux.

Next, we consider case i) for which only one NS5 brane intersects with the O6 plane. The Seiberg-Witten curve is

$$z^n + A(z) + \sum_{s=1}^{r} \frac{B_s(z) + yC_s(z)}{x - x_s} + \frac{yD_1(z)}{x - e_1} = 0, \quad k = 2n \tag{3.25}$$

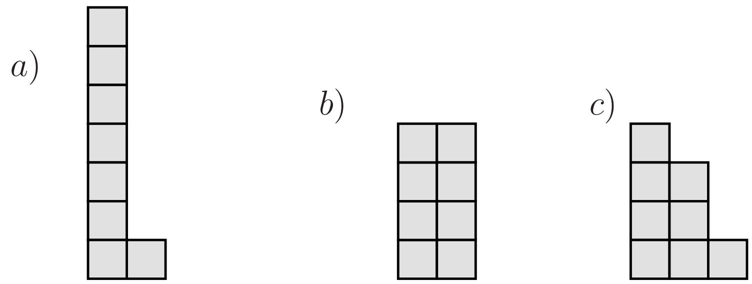

Figure 9: Young-Tableaux for a): Puncture with $p_i = 1$ b): Puncture with $p_l = \frac{l}{2}$ for even l, $p_l = \frac{(l-1)}{2}$ for odd l; c): Puncture with $p_l = \frac{l}{2}$ for even l, $p_l = \frac{(l+1)}{2}$ for odd l.

Expand the curve in the polynomial of z and define $z = \prod_{i=1}^{4}(x-e_i)t^2$, the curve becomes

$$t^{2n} + \sum_{l=1}^{n} \frac{p_r^l(x)}{\Delta' \prod_{i=1}^{4}(x-e_i)^l} t^{2n-2l} + \sum_{l=2}^{n} \frac{p_{r-1}^l(x)}{\Delta' \prod_{i=2}^{4}(x-e_i)^{l-1}(x-e_1)^l} t^{2n-2l+1} = 0 \quad (3.26)$$

Similarly, we conclude that this theory can be realized as the six dimensional A_{2n-1} compactified on a sphere with r punctures at x_s and 3 punctures at $e_i, i = 2,3,4$, we also have a different puncture at e_1 with Young Tableaux in Figure 9c). The defects at the punctures are:

$$\phi_{2l} = \frac{p_r^l(x)}{\Delta' \prod_{i=1}^{4}(x-e_i)^l} dx^{2l}, \quad \phi_{2l-1} = \frac{p_{r-1}^l(x)}{\Delta' \prod_{i=2}^{4}(x-e_i)^{l-1}(x-e_1)^l} dx^{(2l-1)} \quad (3.27)$$

Similar checks can be made:

a)The dimension of moduli space of the punctured sphere is $r+1$ which is identified with the $r+1$ coupling constants of gauge groups.

b)The flavor symmetries correspond to x_s are U(1), while $e_i, i = 2,3,4$ represent flavor symmetry $SU(2)$. The e_1 puncture has pole $p_l = \frac{l}{2}$ for l even, and $p_l = \frac{(l+1)}{2}$ for odd l. This can be represented by the Young Tableaux in Figure 9c). The flavor symmetry of this puncture is $U(1)$. Therefore, the total flavor symmetry is $U(1)^r \times SU(2)^3 \times U(1)$, Which matches our counting from the quiver diagram. Note that the Young Tableaux for the $U(1)$ from the two fundamentals on the SU ends is different from the $U(1)$ punctures for the bi-fundamental matter.

c)The dimension of ϕ_{2l} is $r+1$, and ϕ_{2l-1} has dimension r, which matches the parameters needed for the polynomial $p_r^l(x)$ and $p_{r-1}^l(x)$.

d)The flavor symmetry can be checked from the mass deformed theory.

Finally, let's consider the quiver in case iii); the Seiberg-Witten curve can be written as

$$t^{2n} + \sum_{l=1}^{n} \frac{p_r^l(x)}{\Delta' \prod_{i=1}^{4}(x-e_i)^l} t^{2n-2l} + \sum_{l=2}^{n} \frac{p_{r-1}^l(x)}{\Delta' \prod_{i=3}^{4}(x-e_i)^{l-1}(x-e_1)^l(x-e_2)^l} t^{2n-2l+1} = 0 \quad (3.28)$$

Similarly, this theory can be written as the six dimensional A_{2n-1} theory compactified on Riemann surface with punctures e_i and x_s. The defects at the punctures are:

$$\phi_{2l} = \frac{p_r^l(x)}{\Delta' \prod_{i=1}^4 (x-e_i)^l} dx^{2l}, \quad \phi_{2l-1} = \frac{p_{r-1}^l(x)}{\Delta' \prod_{i=3}^4 (x-e_i)^{l-1}(x-e_1)^l(x-e_2)^l} dx^{(2l-1)} \quad (3.29)$$

One can check along the similar line that this is the correct interpretation.

4. Some Special Examples

We want to mention some special examples which are of later interest for us. We first analyze $SU(2n)$ with two-antisymmetric matter and four fundamentals, this corresponds to $r=0, q=2$. The Seiberg-Witten curve is

$$0 = t^{2n} + \sum_{l=1}^{n} \frac{A_l}{\sum_{i=1}^4 (x-e_i)^l} t^{2n-2l} + \sum_{l=2}^{n} \frac{D_l}{\sum_{i=3}^4 (x-e_i)^{l-1}(x-e_1)^l(x-e_2)^l} t^{2n-2l+1} \quad (4.1)$$

So this theory can be represented as the A_{2n-1} theory compactified on a sphere with four punctures, two of which have the form as Figure 9a, and two of which have the form as Figure 9b.

We then study the quiver gauge theory corresponding to $r=1, q=0$, the quiver gauge theory is $USp(2n) \times USp(2n)$. The flavor symmetry in this case is $SU(2)^4 \times SU(2)$. The last $SU(2)$ comes from the bifundamental matter which now furnish a real representation of quiver theory. Naively, we identify this theory as A_{2n-1} compactified on a sphere with four punctures e_i and one basic puncture x_1. The manifest flavor symmetry from this representation is $SU(2)^4 \times U(1)$.

Finally, we consider the quiver corresponding to $r=0, q=1$, this is a $USp(2n)$ theory with four fundamental and one-antisymmetric hypermultiplet. The Seiberg-Witten curve is

$$t^{2n} + \sum_{l} \frac{p_l}{\prod_{i=1}^4 (x-e_i)^l} t^{2n-2l} = 0 \quad (4.2)$$

This theory is represented as A_{2n-1} theory compactified on sphere with four identical puncture with $SU(2)$ flavor symmetry. Combined with the permutation symmetry of this four identical punctures, we expect that this theory has the $SL(2,Z)$ duality. Notice that the above curve can be written as

$$(t^2 + \frac{q}{\prod_{i=1}^4 (x-e_i)})^n = 0. \quad (4.3)$$

It is amusing to note that for $SU(2)$ theory with four foundamentals, the Seiberg Witten curve is

$$t^2 + \frac{q}{\prod_{i=1}^4 (x-x_i)} = 0. \quad (4.4)$$

So the Seiberg-Witten curve for $USp(2n)$ theory with four fundamentals and one traceless anti-symmetric representation is tensor product of that of $SU(2)$ theory with four fundamentals [15].

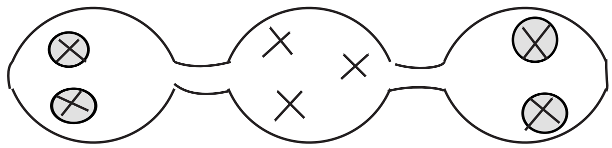

Figure 10: The degeneration limit corresponds to two weakly coupled USp group, two $SU(2)$ punctures are colliding.

5. Degeneration Limit

In reference [1], it is conjectured that the SU quiver gauge theory $SU(2)\times SU(4)\times SU(6)...\times SU(2n)^{m-2n+4} \times ...SU(4) \times SU(2)$ is S dual to quiver gauge theory $SU(2) \times SU(3) \times SU(4) \times ...SU(2n)^{m-2n+3} \times USp(2M)$. The SU quiver gauge theory is realized as the compactification of A_{2n-1} theory compactified on a sphere with $m+1$ basic punctures and two special puncture with $SU(2)$ flavor symmetry. When two special punctures collide, a USp gauge group is decoupled. More generally, when both ends are associated with USp group, it is related to a sphere with four $SU(2)$ punctures and several basic punctures.

This conjecture is proved in this paper by rewriting the Seiberg-Witten curve of $USp\times SU^{r-1}\times USp$ quiver in a form which makes the above interpretation manifest. We show that the quiver with two USp ends are associated with the sphere with several basic punctures and four $SU(2)$ punctures. When two $SU(2)$ punctures collide with each other, the gauge coupling of the USp group becomes weakly coupled. The linear quiver with two USp ends associated with the degeneration limit is shown in Figure 10. When we turn off one of weakly coupled gauge coupling, we are left with a puncture associated with the flavor symmetry $SU(2n)$, which can be seen from the linear quiver.

The theory with only one USp node can be derived by colliding several basic punctures. It is associated with a sphere with two $SU(2)$ punctures and several basic punctures, see Figure 11a). The Young tableaux associated with $SU(2)$ flavor symmetry implies the tail $n_1 = 2, n_2 = 4, ..n_k = 2k, n_n = 2n$, if we collide a basic puncture with a $SU(2)$ puncture, a $SU(2)$ gauge group becomes weakly coupled, we are in another corner of moduli space around which we have the weakly coupled description. In this description, the quiver becomes $SU(2)\times SU(4)\times SU(6)...\times SU(2n)^{m-2n+4}\times ...SU(4)\times SU(2)$. The degeneration limit of the punctured sphere corresponding to two quiver are shown in figure 11b).

The T_N theory can be derived by using a sphere with four $SU(2)$ punctures and $N-1$ basic punctures. Figure 12 shows how we can get the T_N (N must be even) theory by colliding the punctures.

We can also study the degeneration limit of quiver theory which has SU ends with antisymmetric matter. The linear quiver with two such SU ends is depicted as the six

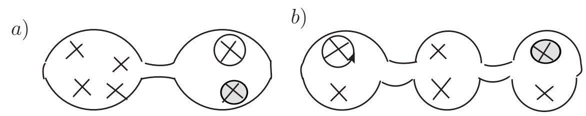

Figure 11: a) Weakly coupled description with USp ends; b) Weakly coupled description with all SU chain groups with bifundamental and fundamental matters.

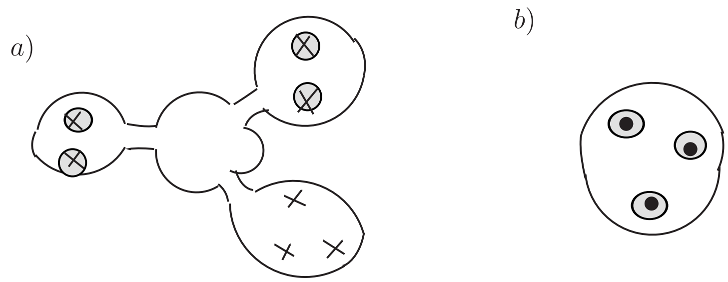

Figure 12: a) One degeneration limit of quiver gauge theory with USp ends and $N-2$ SU gauge groups, here $N=4$; b)T_N theory when we turn off the weak gauge couplings completely.

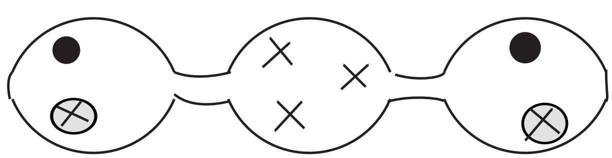

Figure 13: The degeneration limit corresponds to two weakly coupled SU group with antisymmetric representation, one SU(2) puncture and a special $U(1)$ puncture are colliding.

dimensional theory compactified on a sphere with several punctures in Figure 13. We can similarly conclude that the linear quiver with one SU ends with antisymmetric matter is associated with a sphere with several basic puncture, one special puncture and a SU(2) puncture. This can be shown by colliding several basic punctures of above theory with two special SU ends. When special $U(1)$ puncture collides with $SU(2)$ puncture, a $SU(2n)$ gauge theory becomes weakly coupled. When we turn off this coupling completely, we are left with a $SU(2n)$ puncture, which can be seen from our linear quiver.

Motivated by the discussion of USp theory, We can conclude from the form of two

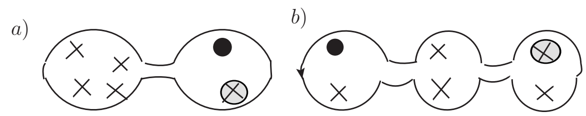

Figure 14: a) Linear quiver with weakly coupled SU ends with antisymmetric representation, b) S dual quiver to theory a with SU chain of gauge groups with bi-fundamental and fundamental matters.

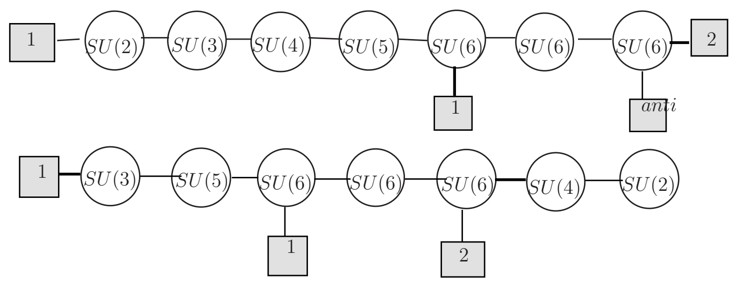

Figure 15: a)Linear quiver with weakly coupled SU ends with antisymmetric representation, b)S dual to a with SU chain of gauge groups

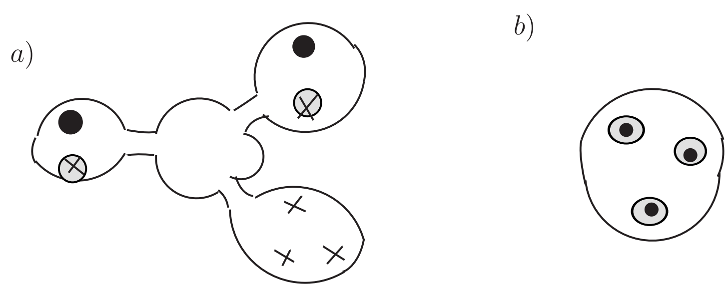

Figure 16: T_{2n} theory from $SU(2n) \times SU^{n-2} \times SU(2n)$ quiver.

special punctures that this quiver is S dual to linear quiver with two tails, one tail associated with the Young Tableaux of special U(1), the other associated with $SU(2)$. For the special $U(1)$, the Young-Tableaux implies that the tail has the form $n_1 = 3, n_2 = 5, ..n_k = 2k+1, ..n_{n-1} = 2n-1, n_n = 2n$, while the other tail has the form $n_1 = 2, n_2 = 4, ..n_k = 2k, n_n = 2n$. Two different degeneration limits are depicted in figure 14. We also write the two different linear quivers associated with these punctured spheres in Figure 15 with $2n = 6$. Similarly, we can find T_{2n} theory from $SU(2n) \times SU^{n-2} \times SU(2n)$, see figure 16. We can also find T_{2n} from theory with form $SU(2n) \times SU^{(}n-2) \times USp(2n)$ by colliding appropriate punctures.

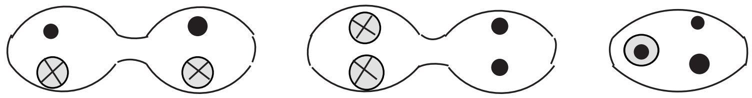

Figure 17: a) Theory with weakly coupled SU group; b) Theory with weakly coupled USp group; c)E_6 theory.

Next, let's have some fun with other isolated superconformal field theory, as in the examples outlined in [16]. First, let's consider $SU(4)$ theory with four fundamentals and two antisymmetric hypermultiplets. The weakly coupled $SU(4)$ description is depicted in Figure 17a). It is associated with A_3 theory compactified on a sphere with two $SU(2)$ punctures and two special $U(1)$ punctures. There is another degeneration limit, which is depicted in Figure 13b). As we discussed earlier, when two $SU(2)$ punctures collide, a $USp(4)$ group becomes weakly coupled, if we turn off this gauge coupling, we are left with a $SU(4)$ puncture. The resulting three punctured sphere is shown in Figure 13c).

For the three punctured spheres, the order of poles at each puncture are

$$SU(4)\ puncture\ : p_2 = 1, p_3 = 2, p_4 = 3; \tag{5.1}$$

$$Special\ U(1)\ puncture\ : p_2 = 1, p_3 = 2, p_4 = 2. \tag{5.2}$$

The Seiberg-Witten curve for the tree punctured sphere can be read from our rules (we put three punctures at $x = 1, 0, \infty$:

$$t^4 + \frac{U}{(x-1)^2x^2}t = 0 \tag{5.3}$$

One can check that the other differentials are vanishing by using formula (2.5). This theory has a dimension 3 operator. The curve reduces to

$$t^3 + \frac{U}{(x-1)^2t^2} = 0 \tag{5.4}$$

which is exactly same as the E_6 superconformal field theory. The manifest flavor symmetry is $SU(4) \times U(1) \times U(1)$, which is a subgroup of E_6. We expect the flavor symmetry is enhanced to E_6 and we identify this theory as E_6 superconformal field theory.

More generally, if we consider $SU(2n)$ theory, then the theory associated with three punctured sphere has the manifest flavor symmetry $SU(2n)\times U(1)\times U(1)$. The original theory has operators $D(\phi) = 2, 3...2n$. In the dual description, a $USp(2n)$ group is becoming weakly coupled, and it has even dimensional operators. Therefore, this isolated superconformal field theory only has odd dimensional operators with $D(\phi) \geq 3$, its Seiberg-Witten curve can be written from the puncture types according to our general rule.

Then let's consider the quiver $USp(4)\times USp(4)$ which has been described in section 4. This theory is associated with A_3 theory compactified on a sphere with four $SU(2)$ puncture and a $U(1)$ puncture. These two weakly coupled USp group description correspond to the degeneration limit shown in Figure 18a). There is another degeneration limit shown in

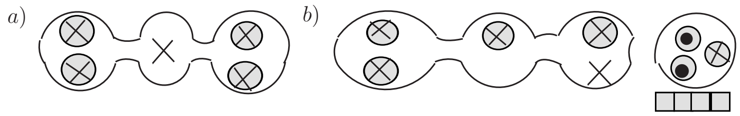

Figure 18: a)Theory with weakly coupled SU group b) Theory with weakly coupled USp group, c)E_6 theory.

Figure 18b). Here two $SU(2)$ punctures collide and one $SU(2)$ puncture collide with other $U(1)$ punctures. When two $SU(2)$ puncture collide, a $USp(4)$ gauge group becomes weakly coupled. When it is completely turned off, a $SU(4)$ puncture is left. On the other hand, when a $SU(2)$ puncture collides with a basic puncture, a $SU(2)$ gauge group becomes weakly coupled. When we turn off this weakly coupled gauge group, a puncture with Young Tableaux of only one row $n_1 = 4$ is created, this is associated with a $SU(4)$ puncture.

The resulting three punctured sphere has two $SU(4)$ punctures and one $SU(2)$ puncture. This theory has only one dimension 4 operator. The original quiver has two dimension 2 and two dimension 4 operators; in the dual description, we have a dimension 2 operator for $SU(2)$ group and a two dimensional and four dimensional operators for $USp(4)$ group, so we are left a dimension 4 operator for this isolated superconformal field theory.

The A_3 theory compactified on such punctured sphere is identified as E_7 superconformal field theory [1][2]. Note that the manifest flavor symmetry is $SU(4) \times SU(4) \times SU(2)$, which is a maximal subgroup of E_7.

More generally, when we consider $USp(2n) \times USp(2n)$ theory, the three punctured spheres have flavor symmetry $SU(2) \times SU(2) \times SU(2n) \times SU(2)$. For generic n, the collision of basic puncture and $SU(2)$ puncture creates a puncture with flavor symmetry $SU(2) \times SU(2)$. This theory has only even dimension operators $D(\phi) \geq 4$.

6. Conclusion

In this paper, we have studied $N = 2$ linear SU quiver gauge theory with USp ends or SU ends with antisymmetric representations. We rewrite the Seiberg-Witten curve in a form which makes manifest the interpretation of six dimensional A_N theory compactification on punctured sphere. We identified the flavor symmetry of the theory with the punctures. We then study the degeneration limit of those theories and identify the weakly coupled description in various cusps of moduli space. For the USp ends, we check the previous conjecture; For SU ends, we conjecture a dual quiver with ordinary SU chain with bi-fundamental and fundamental matters. Finally, we have seen how E_6 and E_7 superconformal field theories come from the degeneration limit of certain special field theories. We also found a class of isolated superconformal field theories with odd dimension operators starting from dimension 3 and superconformal field theory with even dimension operators starting from dimension 4.

We only considered massless antisymmetric matter in this paper. It would be interesting to study the mass deformed theory and identify the six dimensional description. The mass deformed antisymmetric matter theory changes the background from a product manifold $C \times T^2$ to an affine bundle , which is similar to the most generic mass deformed elliptic model. The addition of mass deformation of antisymmetric matter may change the picture dramatically. We can see this from the elliptic model with only one gauge group. Without mass deformation, the four dimensional theory is defined as the six dimensional $(0, 2)$ theory compactified on a smooth torus, and we have $N = 4$ supersymmetry. Now if we turn on the mass deformation for the adjoint hypermultiplet, this theory is described by a torus with one puncture, and we only have $N = 2$ supersymmetry.

It is also interesting to study the quiver with SO node and SU node with symmetric representation. The Type IIA brane construction [13] involves a orientifold six plane with positive charge and a negative charged orientifold six plane. It would be interesting to find a six dimensional description.

Acknowledgments

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